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ABSTRACT

Qualifications for teaching the courses described in "A Transfer Curriculum in Mathematics for Two Year Colleges" is discussed. Important components of the teacher's education are seen to include apprenticeship in teaching and specific mathematics courses, identified by reference to other Committee on the Undergraduate Program in Mathematics (CUPM) reports. (MM)

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MATHEMATICAL ASSOCIATION OF AMERICA

COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS

QUALIFICATIONS FOR TEACHING UNIVERSITY PARALLEL MATHEMATICS COURSES IN TWO-YEAR COLLEGES

August 1969

QUALIFICATIONS FOR TEACHING UNIVERSITY PARALLEL MATHEMATICS COURSES IN TWO YEAR COLLEGES

Report of the
Ad Hoc Committee on Qualifications for a
Two Year College Faculty in Mathematics

COMMITTEE ON THE UNDERGRADUATE PROGRAM
IN MATHEMATICS
Mathematical Association of America
August 1969

Committee on the Undergraduate Program in Mathematics

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AD HOC COMMITTEE ON QUALIFICATIONS FOR A TWO YEAR COLLEGE FACULTY IN MATHEMATICS

I. INTRODUCTION

The Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America has directed its attention for a number of years to the course material in mathematics that should be taught in colleges and universities. Following the publication of several reports devoted to the needs of students concentrating in special areas, CUPM addressed its attention to the problem of constructing a general mathematics program compact enough to be within the means of small four-year colleges, and yet flexible enough to meet the needs of students taking mathematics for a wide variety of reasons. The resulting report, *A General Curriculum in Mathematics for Colleges*, commonly referred to as the GCMC report, was published in 1965. (It is currently being revised.)

CUPM subsequently published reports on the qualifications needed by teachers of the GCMC curriculum[1] and on the adaptation of that curriculum to the circumstances of university parallel programs in two-year colleges.[2] The present report is an effort to describe the qualifications desirable for faculty members teaching courses in the university parallel or transfer programs in two-year colleges.

Our comments and recommendations are addressed to administrators of two-year colleges, to university mathematics departments, to mathematics teachers in two-year colleges, and to those contemplating careers as mathematics teachers in two-year colleges. The concluding section of our report offers specific advice to each of these four groups.

We discuss the qualifications of teachers of the following set of courses, whose subject matter can be thought of as a working definition of university parallel mathematics.

Mathematics 0: Elementary Functions — A one-semester course in coordinate geometry and the properties of the elementary functions.

1. *Qualifications for a College Faculty in Mathematics, 1967.*
2. *A Transfer Curriculum in Mathematics for Two Year Colleges,* 1969. Copies of these and other CUPM reports may be obtained without charge from CUPM, P. O. Box 1024, Berkeley, California 94701.

Mathematics A: Elementary Functions with Algebra and Trigonometry — A slower paced version of Mathematics 0 in which are embedded some topics from high school algebra and trigonometry. This course is to be thought of as extending over more than one semester.

Mathematics B: Introductory Calculus — An intuitive one-semester course covering the basic concepts of single variable calculus.

Mathematics C: Mathematical Analysis — A two-semester course completing the study of elementary calculus.

Mathematics L: Linear Algebra — A sophomore level one-semester introduction.

Mathematics PS: Probability and Statistics — An Elementary one-semester course (not having calculus as a prerequisite) suitable for students in business and the social sciences.

Mathematics NS: The Structure of the Number System — A two-semester course as recommended by the CUPM Panel on Teacher Training for beginning the preparation of elementary school teachers.

The detailed discussion of these courses will be found in the CUPM report, *A Transfer Curriculum in Mathematics for Two Year Colleges*, mentioned above. In addition, this report suggests that, under certain circumstances, it may be advisable for a two-year college to offer additional courses and suggests a selection from among the following: further courses for elementary school teachers; finite mathematics; a calculus-based course in probability; numerical analysis and intermediate differential equations (or differential equations with topics from advanced calculus).[1]

1. Note that our working definition does not include courses in computer science. We refer the reader to *Curriculum 68*, Communications of the ACM, 11(1968). Copies of this report are available for one dollar from the Association for Computing Machinery, 211 East 43 Street, New York, N. Y. 10017.

Our recommendations are intended to apply to all instructors who teach *any* such university parallel courses. We are aware of the great importance in two-year colleges of courses in mathematics for students in occupational and technical curricula and of courses designed for students lacking even basic mathematical skills. We are also aware of the existence of difficult and challenging pedagogical and curricular questions related to such courses. We have chosen to wait until there is a better resolution of these questions before seeking to formulate recommendations about the proper qualifications for teaching courses that are not parallel to those commonly offered by four-year colleges and universities.

The university parallel role of the two-year college is of increasing importance in the educational system. For example, in California 86% of all freshmen in publicly supported institutions in 1966-67 were in two-year colleges. The percentage of college students enrolled in two-year colleges has been increasing rapidly both in California and elsewhere. There are already a number of universities in which the junior class is larger than the freshman class. Moreover, a majority of students entering two-year colleges intend to continue their education at least to the bachelor's degree. Thus, university mathematics departments must recognize that the university parallel courses taught in two-year colleges are becoming an integral part of the university program in mathematics.

Conversely, recent trends in four-year institutions are placing new demands on teaching of mathematics in two-year colleges. As students come to colleges with better preparation in mathematics, many courses are moving downward toward the freshman year. It should be recognized that those now being trained or hired as teachers in two-year colleges must be prepared to deal at some time in the near future with subjects that are now thought of as belonging to the junior or senior years.

The degree most commonly held by mathematics teachers in two-year colleges is the master's degree, but this degree is of such varying quality that it is scarcely useful as a measure of qualification for appointment, promotion or tenure. We feel it necessary to make recommendations which are independent of degrees held or of total credit hours earned in mathematics courses but which deal, rather, with the substance of the mathematical training of prospective faculty members.

It should be understood that no academic program or degree in itself qualifies an individual to teach effectively at any level unless this preparation is accompanied by a genuine interest in teaching and by professional activities reflecting continuing mathematical growth. These activities may assume many forms:

(a) taking additional course work,

(b) reading and studying to keep aware of new developments and to explore new fields,

(c) engaging in research for new mathematical results (even when unpublished),

(d) developing new courses, new ways of teaching and new classroom material,

(e) publishing expository or research articles,

(f) participating in the activities of professional mathematical organizations.

This list reflects our conviction that an effective teacher must maintain an active interest in the communication of ideas and have a dedication to studying, learning, and understanding mathematics at levels significantly beyond those at which he is teaching.

A two-year college mathematics department, whose staff members are engaged in activities such as those described above and have the academic qualifications to be described below, should have confidence in its ability to provide the quality of teaching required of it.

II. THE FORMAL EDUCATION OF MATHEMATICS TEACHERS IN TWO YEAR COLLEGES

The university parallel courses in mathematics that a teacher in a two-year college should be able to teach effectively have been described in the previous section. We shall now set forth our recommendations for the mathematical qualifications for the teachers of these courses.

This mathematical background falls into two distinct components: a basic component which consists of a strong mathematics major program and a graduate component which embodies the requirement that a teacher at a two-year college must have a knowledge of mathematics well beyond that which he will be asked to teach.

BASIC COMPONENT

The basic component of mathematics courses for the two-year college teacher is most succinctly described as a solid grounding in analysis and algebra, with additional courses in geometry, computer science and probability providing greater breadth of knowledge.

We assume that the prospective teacher has mastered the following lower division undergraduate material, as described in the CUPM publication, *A General Curriculum in Mathematics for Colleges.*

* Calculus courses in one and several variables including an introduction to differential equations (GCMC 1, 2, 4).

* The fundamentals of computer science, including experience in programming as well as the use of a computer.

* A semester course in linear algebra employing both matrices and a basis-free, linear transformation approach (GCMC 3).

* A course in probability and statistics that presupposes a course in calculus (GCMC 2P).

In addition, the prospective teacher should attempt to obtain as many of the following upper division courses as he can at the undergraduate level.

* A semester course in advanced multivariable calculus, covering differential and integral vector calculus, including the theorems of Green and Stokes, and an introduction to Fourier series and boundary value problems (GCMC 5).
* A year's work in abstract algebra, treating the important algebraic systems (groups, rings, modules, vector spaces, and fields) and thoroughly developing the basic concepts of homomorphism, kernel and quotient construction with applications and consequences of these ideas. (This course is described in the CUPM report, *Preparation for Graduate Study in Mathematics*. We include a detailed outline in the Appendix.)
* A thorough year's course dealing with the important theorems in real analysis, with emphasis on rigor and detailed proofs. The treatment should use metric space notions and should lead to a detailed examination of the Riemann-Stieltjes integral (GCMC 11-12; see the Appendix).
* A semester course in complex analysis, covering Cauchy's Theorem, Taylor and Laurent expansions, the calculus of residues, and analytic continuation, with application of these ideas to transforms and boundary value problems (GCMC 13).
* A semester course in applied mathematics. The student should be introduced to applications of mathematics in order that his teaching might better reflect the relevance of mathematical ideas (GCMC 10).
* A semester course in which the student studies some geometric subject such as topology, convexity, affine and projective geometries, differential geometry or a comparative investigation of Euclidean and non-Euclidean geometries (GCMC 9).

* A semester course in probability and statistics that builds on the student's lower division course in probability and statistics and reflects the growing importance of this subject to the biological and social sciences, the management sciences, and engineering (GCMC 7).

If the student has not completed all of the upper division courses of this strong mathematics major as an undergraduate, then he should cover material comparable to that of the omitted courses during his graduate training.

GRADUATE COMPONENT

Graduate (one semester) courses which are especially appropriate for the graduate component are:

P Measure and Integration
Q Functional Analysis
R Complex Analysis
S General Topology
T Homology and Multivariable Integration
U Topology and Geometry of Manifolds
V Galois and Field Theory
W Ring Theory and Multilinear Algebra
X Advanced Ordinary Differential Equations with Applications
Y Problem-Oriented Numerical Analysis
Z Seminar in Applications

Of these, P, S, and X should be in the program of every prospective two-year college teacher.

Detailed descriptions of these courses are given in the CUPM report, *A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates* (for courses P, S, and X, these are reproduced in the Appendix). The program presented there is designed to prepare teachers to function in the first two years of a four-year college with occasional teaching of upper division courses. It does not differ greatly from our program: In the four-year college report, instead of P, S, and X, the courses P, Q, S, and T are regarded as essential, and the material on probability and statistics, applied mathematics, and differential equations serves as a pool of courses on the applications of mathematics from which a year sequence is to be elected by the student. These differences are due to the fact that a two-year college teacher has less access to the services of experts in specific fields and, consequently, needs a somewhat broader training.

It should be emphasized that course X in differential equations is not a second undergraduate course in the subject, but is to be a genuine graduate course at least on the same level as the course in measure and integration. The graduate course in applied mathematics which the Committee most strongly favors is one (not yet commonly offered) which stresses the formulation and analysis of mathematical models in diverse fields, using the calculus, probability, and linear algebra of the first two undergraduate years. Course Z of the report is of this type.

Students who plan to continue into advanced graduate work and to specialize in some area of pure mathematics are advised to take as many as possible of the other courses in the list. Other students may substitute electives to obtain a deeper knowledge of some other area of mathematics or computer science.

Undergraduate mathematics, especially in the lower division, is heavily slanted toward real analysis; courses in general topology and measure theory provide essential background for teaching courses in calculus and probability. If further work in analysis is elected, we recommend a study of functional analysis (Course Q) in preference to complex analysis (Course R) as a sequel to measure theory, as the former will further develop the ideas of linear algebra and the concept of uniform convergence. In spite of its importance for more advanced work in pure mathematics, a second year of abstract algebra to follow the strong undergraduate algebra course described in the basic component is not recommended as essential for teachers of mathematics in a two-year college.

The graduate component of courses should be augmented by two particular features to prepare the prospective teacher for the two-year college mathematics faculty.

First, a year's work focused on the problems of lower division undergraduate teaching, such as an apprenticeship in teaching as described below, preferably carried out at a nearby two-year college.

Second, a comprehensive examination designed specifically to test the breadth and depth of a candidate's understanding of mathematics relevant to the undergraduate curriculum.

A student who has a strong undergraduate major in mathematics will be able to complete the program in one year, even if he has not completed quite all of the courses listed in the basic component. For a student whose prior training is not substantially that of the basic component, the completion of the graduate component may require two years of study beyond his bachelor's degree. For example, if his undergraduate major does not include strong preparation in algebra and analysis, his program might be as follows:

First Year (both semesters)
- Abstract Algebra
- Real Analysis (GCMC 11-12)
- Probability and Statistics (GCMC 2P, 7)
- Apprenticeship in Teaching

Second Year

First Semester
- Measure and Integration (P)
- Complex Analysis (GCMC 13).
- Topology (S)
- Apprenticeship in Teaching

Second Semester
- Applied Mathematics (GCMC 10 or Z)
- Advanced Differential Equations (X)
- Apprenticeship in Teaching
- Comprehensive Examination

The mathematical background in the graduate component, if satisfactorily completed, will permit the student to teach with confidence the university parallel courses of the two-year college. Moreover, new courses, as they arise, should be well within his competence to prepare.

APPRENTICESHIP IN TEACHING

An important component in the training of teachers of mathematics for two-year colleges is an understanding of the teaching and learning processes as they apply to these institutions. One of the best ways for the potential instructor to gain this kind of knowledge and experience is through a supervised teaching activity. This activity preferably should take place in a two-year college, but it can, if necessary, be carried out in a four-year institution in appropriate courses.

Most value will be obtained if apprentice teachers receive experience in a variety of courses involving a heterogeneous group of students with differing career aspirations, comparable to the situation that they will encounter in most two-year colleges.

The success of an apprenticeship program will depend significantly upon the attitude of the graduate faculty. If effective teaching is regarded as an important function of the department, and if senior mathematicians encourage excellence in teaching by precept and by example, the apprentice teachers will respond accordingly.

The work assignment of the apprentice should be carefully graduated and should always involve close contact with and supervision by a senior colleague. The apprentice should have frequent opportunities to go over purposes, methods, and content with his supervisor. Arrangements should be made for frequent post-teaching conferences in which the teaching and learning problems encountered are reviewed and solutions suggested. This can be done individually or in a group for all apprentices in the program. Valuable contributions can be made to such seminar sessions by mathematics instructors from two-year colleges and by experts in curricular construction and evaluation.

In total, the apprenticeship in teaching should constitute approximately one quarter of the work load of the student during his graduate experience.

Adequate budgetary provisions should be made for the extra burden of the apprenticeship program on the senior mathematicians, as well as for the financial support for the apprentices.

An apprenticeship system has a great potential for preparing two-year college mathematics teachers having a real attachment to the discipline and an understanding of the values and the rewards of the teaching profession. Done poorly, it will discourage candidates from the field. Done well, it will attract and retain competent and interested persons.

III. COMPOSITION OF A TWO YEAR COLLEGE MATHEMATICS FACULTY

Although mathematics teachers at two-year colleges are called upon to teach specialized courses for a variety of students (remedial, general education, technical-occupational), our attention in the present report continues to be focused upon qualifications of persons who teach courses in the university parallel curriculum.

It is our recommendation that all teachers of university parallel courses at a two-year college have the mathematical preparation equivalent to our graduate component. Although the university parallel courses that a two-year college teacher may be called upon to offer today are principally like those described in the introduction under A, O, B, and C, it seems reasonable to expect that courses in finite mathematics, linear algebra, probability and statistics, and mathematics for prospective elementary school teachers will be standard offerings in two-year colleges in the near future. Our recommendation reflects a belief that the teacher of university parallel courses should have mathematical training well beyond the course he is teaching. Moreover, the mathematical background we recommend will permit all faculty members to participate in knowledgeable discussions of curricular changes, both internally and with faculty members of four-year colleges and universities. The mathematical preparation we recommend will permit a faculty member to prepare new courses with confidence. Moreover, it will provide the individual faculty member with a basis for effective participation in mathematical organizations, which in turn will help him to maintain the intellectual curiosity and interest in mathematics that is essential to a successful mathematics teacher.

The committee believes that a universally well qualified faculty for the university parallel courses is most important, with each member able to make a contribution in all of the ways already indicated. We do not, however, envision that all two-year college staff members will have exactly the same mathematical background. The choice available for individual preferences in the graduate component allows a staff which includes people with varying interests, and hence people especially well prepared to teach linear algebra or probability and statistics or computer science.

A two-year college may not be able at this time to recruit all such staff members from candidates with preparation equivalent to our recommended graduate component. In this case, they might seek on a temporary basis new candidates who have the mathematical preparation equivalent to our basic component, and these candidates could be assigned to teach courses O, A and B. New faculty members whose qualifications are not equivalent to our graduate component should be required to augment their mathematical background so that in time they would be better prepared to have responsibility for any of the university parallel courses.

IV. RECOMMENDATIONS TO FOUR GROUPS

(a) To Administrators of Two Year Colleges

These recommendations are addressed to those who appoint and promote two-year college faculty members and to those who, through accreditation and certification, influence the setting of qualifications for such teachers.

Although it has been traditional for college policies on the appointment, promotion and tenure of faculty to include certain degree requirements, it is a fact that the course requirements for a particular degree in mathematics vary considerably from one institution to another, and even minimum standards are not well defined. This is especially true of the master's degree. The Committee strongly encourages those concerned to note that this report recommends a set of courses which prospective members of a mathematics faculty should have taken. Successful completion of these courses should ensure that the faculty member is adequately prepared, in terms of subject matter, to teach university parallel courses.

The Committee urges all administrators to recognize proficiency in the content of the courses recommended in this report rather than academic degrees as a basis for faculty appointments and advancement. For example, graduate mathematics training of secondary school teachers, customarily and properly, differs from the training we have described. The Committee suggests that faculty members in mathematics be relied upon to determine the degree of proficiency possessed by those under consideration. Furthermore, it is recommended that orientation programs be developed for new faculty members who have had no previous experience teaching in two-year colleges.

(b) To University Mathematics Departments

University mathematics departments should realize from the preceding sections that the major role in the training of mathematics instructors for two-year colleges is theirs. They must accept responsibility for establishing formal programs for the training of new instructors for two-year colleges and for retraining instructors who are now teaching in these institutions.

We believe that this can be done within existing frameworks of mathematics departments whose course offerings approximate in depth the detailed outlines to which we have referred. For such departments, this will not require extensive changes in curricula, except possibly for the introduction of a program for apprenticeship in teaching. However, it is necessary that the mathematics faculty be fully aware of the particular complexion, problems, and status of two-year colleges throughout the country. The mobility of instructors suggests the need for a national point of view. Moreover, in order to fulfill their responsibility, the university faculty must recognize and respect the basic role of two-year colleges and be mindful of the problems that will be faced by mathematics instructors in two-year colleges.

(c) To Those Currently Teaching Mathematics in Two Year Colleges

All college teachers of mathematics, at one time or another, find it necessary to supplement their own mathematical training. Rapid changes are taking place in college mathematics. Hence, increasing numbers of college teachers are continuing their mathematical development by individual study and additional formal course work in mathematics.

Teachers of mathematics in two-year colleges should find that the course outlines referred to in this report provide useful guidelines for individual study, faculty seminars and additional course work.[1]

To serve the mathematical needs of two-year college students, a faculty member must maintain an awareness of contemporary curricula in both secondary schools and four-year colleges. He will find that the recommended courses provide a basis for effective communication with staff members of mathematics departments of four-year colleges. Personal knowledge of mathematics courses at four-year colleges is needed in order to be aware of the demands that will be made upon students after they transfer. This knowledge and the

1. For detailed course descriptions, see the Appendix and the CUPM report, *A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates, 1969.*

recommended strong preparation in mathematics make possible the necessary continuous evaluation and development of mathematics courses in two-year colleges.

(d) To Prospective Teachers of Mathematics in Two Year Colleges

The two-year college teacher of university parallel mathematics courses has the responsibility for training students with a wide variety of goals. Some could be mathematicians, some scientists or engineers; there are others who will use mathematics in economics, psychology or other social sciences. One who intends to teach mathematics in a two-year college could well use our description of a program of mathematics courses and apprenticeship in teaching as a guide in planning his own graduate work. He should also be aware that the program we have outlined is substantially different both in nature and extent from what we would regard as an optimal graduate program for teachers in secondary schools.

APPENDIX

The following course outlines are referred to frequently in the report. They are reproduced from the CUPM documents, *A General Curriculum in Mathematics for Colleges* (Mathematics 11-12), *Preparation for Graduate Study in Mathematics* (Mathematics D-E), *A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates* (Mathematics P, S, X).

Mathematics 11-12. Real Variable Theory (6 semester hours).
First Semester — 39 lessons.

a. **Real numbers** (6 lessons). The integers; induction. The rational numbers; order structure, Dedekind cuts. The reals defined as a Dedekind complete field. Outline of the Dedekind construction. Least upper bound property. Nested interval property. Denseness of the rationals. Archimedean property. Inequalities ([7] is a good source of problems). The extended real number system.

b. **Complex numbers** (3 lessons). The complex numbers introduced as ordered pairs of reals; their arithmetic and geometry. Statement of algebraic completeness. Schwarz inequality.

c. **Set theory** (4 lessons). Basic notation and terminology; membership, inclusion, union and intersection, cartesian product, relation, function, sequence, equivalence relation, etc.; arbitrary unions and intersections. Countability of the rationals; uncountability of the reals.

d. **Metric spaces** (6 lessons). Basic definitions: metric, ball, boundedness, neighborhood, open set, closed set, interior, boundary, accumulation point, etc. Unions and intersections of open or closed sets. Subspaces. Compactness. Connectedness. Convergent sequence, subsequence, uniqueness of limit. A point of accumulation of a set is a limit of a sequence of points of the set. Cauchy sequence. Completeness.

e. **Euclidean spaces** (6 lessons). R^n as a normed vector space over R. Completeness. Countable base for the topology. Bolzano-Weierstrass and Heine-Borel-Lebesgue theorems. Topology of the line. The open sets; the connected sets. The Cantor set. Outline of the Cauchy construction of R. Infinite decimals.

f. **Continuity** (8 lessons). (Functions into a metric space). Limit at a point, continuity at a point. Continuity; inverses of open sets, inverses of closed sets. Continuous images of compact sets are compact. Continuous images of connected sets are connected. Uniform continuity; a continuous function on a compact set is uniformly continuous. (Functions into R) Algebra of continuous functions. A continuous function on a compact set attains its maximum. Intermediate value theorem. Kinds of discontinuities.

g. **Differentiation** (6 lessons). (Functions into R) The derivative. Algebra of differentiable functions. Chain rule. Sign of the derivative. Mean value theorems. The intermediate value theorem for derivatives. Lhospital's rule. Taylor's theorem with remainder. One-sided derivatives; infinite derivatives. (This material will be relatively familiar to the student from his calculus course, so it can be covered rather quickly.)

Second Semester - 39 lessons

h. **The Riemann-Stieltjes integral** (11 lessons). [Alternative: the Riemann integral.] Upper and lower Riemann integrals. [Existence of the Riemann integral: for f continuous; for f monotonic.] Monotonic functions and functions of bounded variation. Riemann-Stieltjes integrals. Existence of $\int_a^b f\,d\alpha$ for f continuous and α of bounded variation. Reduction to the Riemann integral in case α has a continuous derivative. Linearity of the integral. The integral as a limit of sums. Integration by parts. Change of variable. Mean value theorems. The integral as a function of its upper limit. The fundamental theorem of calculus. Improper integrals. The gamma function. ([11], 367-378; [10], 285-297).

i. **Series of numbers** (11 lessons). (Complex) Convergent series. Tests for convergence (root, ratio, integral, Dirichlet, Abel). Absolute and conditional convergence. Multiplication of series. (Real) Monotone sequences; lim sup and lim inf of a sequence. Series of positive terms; the number e. Stirling's formula, Euler's constant ([11], 383-388. Again, see [7] for problems).

j. Series of functions (7 lessons). (Complex) Uniform convergence; continuity of uniform limit of continuous functions. Equicontinuity; equicontinuity on compact sets. (Real) Integration term by term. Differentiation term by term. Weierstrass approximation theorem. Nowhere-differentiable continuous functions.

k. Series expansions (10 lessons). Power series, interval of convergence, real analytic functions, Taylor's theorem. Taylor expansions for exponential, logarithmic, and trigonometric functions. Fourier series: orthonormal systems, mean square approximation, Bessel's inequality, Dirichlet kernel, Fejér kernel, localization theorem, Fejér's theorem. Parseval's theorem.

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Mathematics D-E. Abstract Algebra

The purpose of this year course is to introduce the student to the basic structures of abstract algebra and also to deepen and strengthen his knowledge of linear algebra. It provides an introduction to the applications of these concepts to various branches of mathematics.

1. **Groups** (10 lessons). Definition. Examples: Vector spaces, linear groups, additive group of reals, symmetric groups, cyclic groups, etc. Subgroups. Order of an element. Theorem: Every subgroup of a cyclic group is cyclic. Coset decomposition. Lagrange theorem on the order of a subgroup. Normal subgroups. Homomorphism and isomorphism. Linear transformations as examples. Determinant as homomorphism of GL(n) to the non-zero reals. Quotient groups. The first two isomorphism theorems. Linear algebra provides examples throughout this unit.

2. **Further group theory** (10 lessons). The third isomorphism theorem. Definition of simple groups and composition series for finite groups. The Jordan-Hölder theorem. Definition of solvable groups. Simplicity of the alternating group for $n > 4$. Elements of theory of p-groups. Theorems: A p-group has nontrivial center; a p-group is solvable. Sylow theory. Sylow theorem on the existence of p-Sylow subgroups. Theorems: Every p-subgroup is contained in a p-Sylow subgroup; all p-Sylow subgroups are conjugate and their number is congruent to 1 modulo p.

3. **Rings** (10 lessons). Definition. Examples: Integers, polynomials over the reals, the rationals, the Gaussian integers, all linear transformations of a vector space, continuous functions on spaces. Zero divisors and inverses. Division rings and fields. Domains and their quotient fields Examples: Construction of field of four elements. embedding of complex numbers in 2 x 2 real matrices, quaternions. Homomorphism and isomorphism of rings. Ideals. Congruences in the ring of integers. Tests for divisibility by 3, 11, etc., leading up to Fermat's little theorem, $a^{p-1} \equiv 1 \pmod p$, and such problems as showing that $2^{32} + 1 \equiv 0 \pmod{641}$. Residue class rings. The homomorphism theorems for rings.

4. **Further linear algebra** (continuing Mathematics 3) (12 lessons). Definition of vector space over an arbitrary field. (Point out that the first part of Mathematics 3 carries over verbatim and use the opportunity for some review of Mathematics 3.) Review of spectral theorem from Mathematics 3 stated in a more sophisticated form (e.g., as in reference [4]). Dual-space adjoint of a linear transformation, dual bases, transpose of a matrix. Theorem: Finite-dimensional vector spaces are reflexive. Equivalence of bilinear forms and homomorphism of a space into its dual. General theory of quadratic and skew-symmetric forms over fields of characteristic not two. The canonical forms. (Emphasize the connections with corresponding material in Mathematics 3.) The exterior algebra defined in terms of a basis—two-and three-dimensional cases first. The transformation of the p-vectors induced by a linear transformation of the vector space. Determinants redone this way.

5. **Unique factorization** domains (12 lessons). Primes in a commutative ring. Examples where unique factorization fails, say in $Z[\sqrt{-5}]$. Definition of Euclidean ring, regarded as a device to unify the discussion for Z and F[x], F a field. Division algorithm and Euclidean algorithm in a Euclidean ring; greatest common divisor; Theorem: If a prime divides a product it divides at least one factor; unique factorization in a Euclidean ring. Theorem: A Euclidean ring is a principal ideal domain. Theorem: A principal ideal domain is a unique factorization domain. Gauss's lemma on the product of two primitive polynomials over a unique factorization domain. Theorem: If R is a unique factorization domain so is R[x].

6. **Modules over Euclidean rings** (14 lessons). Definition of module over an arbitrary ring viewed as a generalization of vector space. Example: Vector space as a module over F[x] with x acting like a linear transformation. Module homomorphism. Cyclic and free modules. Theorem: Any module is a homomorphic image of a free module. Theorem: If R is Euclidean, A an n x n matrix over R, then by elementary row and column transformations A can be diagonalized so that diagonal elements divide properly. Theorem: Every finitely generated module over a Euclidean ring is the direct sum of cyclic modules. Uniqueness of this decomposition, decomposition into pri-

mary components, invariant factors and elementary divisors. Application to the module of a linear transformation, leading to the rational and Jordan canonical forms of the matrix. Several examples worked in detail. Similarity invariants of matrices. Characteristic and minimal polynominals. Hamilton-Cayley Theorem: A square matrix satisfies its characteristic equation. Application of module theorem to the integers to obtain the fundamental theorem of finitely generated abelian groups.

7. **Fields** (10 lessons). Prime fields and characteristic. Extension fields. Algebraic extensions. Structure of F(a), F a field, a an algebraic element of some extension field. Direct proof that if a has degree n, the set of polynomials of degree n-1 in a is a field, demonstration that $F(a) \cong F[x]/(f(x))$, where f is the minimum polynomial of a. Definition of (K:F), where K is an extension field of F. If $F \subset K \subset L$ and (L:F) is finite, then (L:F) =(L:K) (K:F). Ruler-and-compass constructions. Impossibility of trisecting the angle, duplicating the cube, squaring the circle (assuming π transcendental). Existence and uniqueness of splitting fields for equations. Theory of finite fields.

References:

1. Barnes, W. *Introduction to Abstract Algebra.* Boston, Massachusetts: D. C. Heath and Company, 1963.
2. Birkhoff, G. and MacLane, S. *A Survey of Modern Algebra.* New York: The Macmillan Company, 1965.
3. Herstein, I. N. *Topics in Algebra.* New York: Blaisdell Publishing Company, 1964.
4. Hoffman, K. and Kunze, R. *Linear Algebra.* Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1961.
5. Lewis, D. *Introduction to Algebra.* New York: Harper and Row, 1965.
6. Mostow, G., Sampson, J. H. and Meyer, J. P. *Fundamental Structures of Algebra.* New York: McGraw-Hill Book Company, Inc., 1963.
7. van der Waerden, B. L. *Modern Algebra,* Vol. I, rev. ed. New York: Ungar Publishing Company, 1953.

Mathematics P. Measure and Integration

This course provides an introduction to and essential background for Course Q, can be used in Course R, and is naturally useful in more advanced courses in real analysis. We present two outlines, which represent different approaches and a somewhat different selection of material. [1] . If presented in the right spirit, a course in measure and integration provides insights into the material of lower division courses that the student will have to teach.

First Outline

1. **The limitations of the Riemann integral.** Examples of a series that fails to be integrable term by term only because its sum is not integrable; of a differentiable function with a non-integrable derivative. Limitations of integration in general: there is no countably additive, translation-invariant integral for all characteristic functions of sets (the usual construction of a non-measurable set will serve).

2. **Lebesgue integration on the line.** Outer measure; definition of measurable sets by means of outer measure. Measurability of sets of measure 0, of intersections and unions, of Borel sets. Countable additivity. Application: the Steinhaus theorem on the sets of distances of a set of positive measure. Measurable functions, Borel measurability, measurability of continuous functions. Egoroff's theorem. Definition of the integral of a bounded measurable function as the common value of inf $\int \psi(x)dx$ for simple majorants ψ of f and sup $\int \varphi(x)dx$ for simple minorants φ . Riemann integrable functions are Lebesgue integrable. Bounded convergence and applications (necessary and sufficient condition for integrability; $\log 2 = 1 - 1/2 + 1/3 - \ldots$). Integrability of non- negative functions, Fatou's lemma, monotone convergence, integrability of general functions. A non-negative function with zero integral is zero almost everywhere.

3. **L^p spaces, with emphasis on L^2; motivation from orthogonal series.** Schwarz inequality; with little extra effort one gets (via convex functions) the Hölder, Minkowski and Jensen inequalities. L^∞ as a formal limit of L^p via $(\int f^p)^{1/p} \to$ ess sup f as $p \to \infty$. Parseval's theorem, Riesz-Fischer theorem. Rademacher

1. Only the first outline is reproduced here. The reader is referred to the original report for the second outline.

functions; proof that almost all numbers are normal. Convergence of $\Sigma \pm \frac{1}{n}$ and other series with random signs. Proof (by Bernstein polynomials or otherwise) that continuous functions on an interval are uniformly approximable by polynomials. Hence continuous functions are dense in L^p.

4. **Differentiation and integration.** Proof that an indefinite integral is differentiable almost everywhere and its derivative is the integrand; the Lebesgue set. Equivalence of the properties of absolute continuity and of being an integral.

5. **Lebesgue-Stieltjes integral with respect to a function of bounded variation.** A rapid survey pointing out what changes have to be made in the previous development. Applications in probability, at least enough to show how to treat discrete and continuous cases simultaneously. Riesz representation for continuous linear functionals on C[a, b].

6. **General measure spaces.** Definition of the integral and convergence theorems in the general setting; specialization to n-dimensional Euclidean space. Fubini's theorem. Application to convolutions and to such matters as gamma-function integrals and $\int e^{-x^2}dx$. The one-dimensional integral as the integral of the characteristic function of the ordinate set.

7. **Complex measures.** (If time permits) Decompositions. Radon-Nikodym theorem.

References

1. Asplund, E. and Bungart, L. *A First Course in Integration.* New York: Holt, Rinehart and Winston, 1966.

2. Hartman, S. and Mikusinski, J. *The Theory of Lebesgue Measure and Integration.* Oxford: Pergamon Press, 1961.

3. Hewitt, E. and Stromberg, K. *Real and Abstract Analysis.* New York: Springer-Verlag, 1965.

4. Kingman, J. F. and Taylor, S. J. *Introduction to Measure and Probability.* Cambridge: University Press, 1966.

5. Natanson, I. P. *Theory of Functions of a Real Variable.* New York: Frederick Ungar, 1955.

6. Riesz, F. and Sz.-Nagy, B. *Functional Analysis.* New York: Frederick Ungar, 1955.

7. Royden, H.L. *Real Analysis,* 2nd ed. New York: Macmillan Company, 1968.

8. Rudin, W. *Real and Complex Analysis.* New York: McGraw-Hill, 1966.

Mathematics S. Topology

We assume that the students have made a brief study of metric spaces, Euclidean spaces and the notion of continuity of functions in metric spaces. (This material is covered in sections d, e, and f of GCMC 11-12.)

1. **Basic topology.** Topological spaces, subspace topology, quotient topology. Connectedness and compactness. Product spaces and the Tychonoff Theorem. Separation axioms, separation by continuous functions. Local connectedness and local compactness. Metric spaces, completion of metric spaces, uniform continuity. Paracompactness, continuous partitions of unity.

2. **Applications to calculus.** Use the results above to re-prove the basic topological results needed for calculus and the Heine-Borel and Bolzano-Weierstrass Theorems.

3. **Fundamental group.** Homotopies of maps, homotopy equivalence. The fundamental group π_1, functorial properties, dependence on base point. Show that $\pi_1(S^1) = Z$.

4. **Applications of the fundamental group.** Brouwer fixed point theorem for the disk D^2. R^2 is not homeomorphic with R^3. Relevance of the fundamental group to Cauchy's residue theorem. Fundamental theorem of algebra.

5. Covering spaces. Covering spaces, homotopy lifting and homotopy covering properties. Regular coverings, existence of coverings, universal covering. Factoring of maps through coverings. Relation with Riemann surfaces.

Mathematics X. Advanced Ordinary Differential Equations with Applications.

This course is designed to provide background for teaching the topics in differential equations that occur in the lower division GCMC courses; to give further exposure to applications via one of the most intensively used classical routes; and to provide a first course for students who may be interested in specializing in this area. Because of the nature of the subject, many different good course outlines are possible, but in any case, emphasis should be put on efficient ways of obtaining from differential equations useful information about their solutions, as distinguished say from methods for finding baroque solution formulas of little practical value.

1. **Fundamentals.** The vector differential equation $\dot{x} = f(t,x)$; prototypes in physics, biology, control theory, etc. Local existence (without uniqueness), by the Cauchy construction, when f is continuous. Prolongation of solutions and finite escape times. Properties of integral funnels (e.g., Kneser's theorem); extreme solutions when $n = 2$. Jacobian matrix of f locally bounded $\rightarrow$ Lipschitz condition $\rightarrow$ uniqueness $\rightarrow$ continuous dependence on initial values and parameters. Effects of stationarity.

2. **Numerical integration.** Euler, Runge-Kutta, and other methods; elements of error analysis for these methods. Practical machine computation.

3. **Linear equations.** Discussion of physical and other real-world models leading to linear equations. Linearization. Structure of the solution set of the vector equation (1)$\dot{x} = A(t)x + b(t)$; variation of parameters formula; the fundamental matrix. Matrix exponentials; thorough treatment of (1), using Jordan canonical form,

when A is constant. Applications in engineering systems theory. Floquet's theorem.

4. **Sturm-Liouville theory.** The two-point boundary value problem for second order self-adjoint equations and how it arises. Existence of eigenvalues. Comparison, oscillation, and completeness theorems. Orthogonal expansions. Green's function. Applications to diffusion and wave equations. Some special functions.

5. **Stability.** Liapunov, asymptotic, and orbital stability; uniform properties. Basic theorems of Liapunov's direct method. Extensive treatment of the linear case. Applications in control theory.

6. **Phase-plane analysis.** Geometric treatment of second-order stationary systems. Classification of simple equilibrium points. Closed orbits and Poincaré-Bendixson theory.

Optional Topics

7. **Power series solutions.** Classification of isolated singularities of linear equations; formal solutions; Frobenius method. Asymptotic series.

8. **Carathéodory theory.** (Prerequisite: Lebesgue Integration).

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2. Carrier, G. F. and Pearson, C. E. *Ordinary Differential Equations.* New York: Blaisdell, 1968.
3. Coddington, E. and Levinson, N. *Theory of Ordinary Differential Equations.* New York: McGraw-Hill, 1955.
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Interscience, 1963.
8. Zadeh, L. A. and Desoer, C. A. *Linear System Theory,* New York: McGraw-Hill, 1963.