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ABSTRACT

The goal of this study was to investigate improvement in the usage of equivalence and order relations under specified instructional conditions. Forty-eight upper middle class kindergarten children were given a 14-session pretreatment designed to define two equivalence and four order relations of length and matching, followed by tests of conservation and transitivity. On the basis of the transitivity test scores, the children were then assigned by matched pairs to two treatments: a Classification Treatment in which an object was considered as representing a collection of like objects, and a Standard Treatment in which no such assumption was made. Posttests of conservation, transitivity, and use of the asymmetric property were then given. Analysis by sign test and contingency tables showed that both treatments were effective but not significantly different. No evidence was found that acquisition of matching relations preceded acquisition of length relations. Also, possession of transitivity appeared unstable and unrelated to conservation. (MM)

An Investigation in The Learning of Relational Properties by Kindergarten Children

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The basic goal of this study was to investigate improvement in the usage of properties of equivalence and order relations by five-year-old children under specified instructional conditions. Two relational categories, matching relations and length relations, were chosen for study because such relations are, on a logical basis, foundational to number and length. Specifically, the purposes of the study were fourfold. First, the effects of two treatments were contrasted on the ability of five-year-old children to use the transitive and asymmetric properties and conserve relations. Second, the effects of the two treatments were assessed in changing children from a state of evidence absent to a state of evidence present in the case of the ability to use transitivity and conserve relations. Third, interrelationships of the abilities to conserve relations and use transitivity were assessed before treatments and fourth, interrelationships of the abilities to conserve relations, use transitivity, and use asymmetry were assessed after treatments.

Relations

Sinclair (1971, p. 173) has pointed out that length is achieved later than number with a time lag in achievement of between six months and a year. She also pointed out that an even greater time lag exists (two to three years) between acquisition of conservation of length and numerical conservation. Without going into great detail, it can be noted that ordinal and cardinal number (and thereby counting) are inextricably bound up with rudimentary aspects of length. For if a collection of objects A exist which are to be counted, a succession of ordered subsets of A can be determined by successive selection of elements. "One", after the selection of the first element, has both cardinal and ordinal characteristics in that it tells how many elements have been selected and also that the first one has been selected. A subset of the collection A of one element has also been determined. "Two," after the selection of the next element also has both cardinal and ordinal characteristics. It tells how many elements have been selected and also that the second one has been selected. The subset corresponding to "two" is an ordered set, is a subset of the collection A, and contains the set consisting of the first element. It is ordered by the relation "precedes," which is transitive, asymmetrical, and nonreflexive (and is thereby a strict partial ordering). If this counting process is continued until the set A is exhausted (say A contains n elements), then $A = \{a_1, a_2, \ldots, a_n\}$ has been well-ordered by the relation "precedes." A chain of sets has been established in that if A_1 is the set which contains a_1, A_2 the set which contains a_1 and a_2, etc., then $A_1 \subset A_2 \subset \ldots \subset A_n$. In this sense, "one" is included in "two," "two" is included in "three," etc., which is how the Genevans (Sinclair, 1971, p. 150) view number.

If the set A is considered to be a polygonal path, the line segments of which are the same length, then the same conceptual analysis as was applied immediately above can be applied in finding how many line segments are included in A. If the unit of measurement is taken to be congruent to each line segment of the polygonal path, then (with the assumption that all line segments of A are congruent) the length of the polygonal path can be determined through counting. A well-developed concept of a unit of measurement and a means to determine the congruency of the line segments of the polygonal path are assumed as foundational to a successful completion of the task of finding the length of the polygonal path through counting.

Considering a line segment as one entity is distinct from considering it as a unit of measurement. In each conceptualization, however, structural similarities exist. Those structural similarities include ordered sets, set inclusion, a chain of sets, and relations based on one-to-one correspondence and congruence for line segments. While one-to-one correspondence is fundamental to congruence for line segments, operationally, set equivalence and congruence for line segments may be considered as distinct for children. One-to-one correspondence is logically necessary to "one," and congruence to a unit of measurement. "One," if viewed as an equivalence class, is based directly on the equivalence relation of one-to-one correspondence. A unit of measurement can be logically considered as an equivalence class of segments based on the congruence relation for segments. Conceptually, a given cardinal number or a given unit of measurement have no unique characteristics in that they may be considered as part of a system of classes determined by the relations and the set of objects on which the relations are defined. Relations, then, are inextricably connected with counting and measurement processes and with number (cardinal and ordinal) and length.

When Sinclair (1971) noted that a time lag existed between achievement of length and number, she was considering length as being a product of measurement. In short, length was being rightfully considered as number. The question of an existence of an analogous time lag in the usage of properties of equivalence and order relations across the relational categories of matching and length relations has not been satisfactorially answered. There are a priori reasons to suspect that such a time lag would exist. Length relations are based on congruence for line segments, a concept that assumes distance. For two point sets F and F' are <u>congruent</u> if and only if there is a one-to-one correspondence between their points that preserves distance (Gans, 1969, p. 69). Matching relations are based on one-to-one correspondence and set inclusion. Piaget (1970, p. 37) contends that Russell and Whitehead, while they have based number on one-to-one correspondence and classes, have introduced number into classes by considering one-to-one correspondence as not depending on the qualities of elements. Regardless of whether one accepts Piaget's point of view of number or not, length relations (by definition) must involve distance. It would seem, thereby, that length relations would be a later acquisition than would matching relations if the same pattern of acquisition is present as was present in studies which based acquisition of length on measurement processes. Three hypotheses of this study then are: (1) the ability of five-year-old children to conserve matching relations precedes an ability to conserve length relations; (2) the ability of five-year-old children to use the asymmetric property appears for matching relations before it does for length relations; and (3) the ability of five-year-old children to use the transitive property appears for matching relations before it does for length relations.

Training Studies

During the decade of 1960-70, a substantial number of training studies were conducted surrounding acquisition of logical thinking. As Beilin (1971, p. 85) has pointed out, most of these studies center around the conservation phenomenon because of the ease it offers in making a crucial test of Piagetian theory. In the same research review, Beilin (1971, p. 105) pointed out that few data exist on the training of classification and relational skills except for studies surrounding class inclusion. Data which do exist, however, "leads to the conclusion that these operations are as subject to the effects of instruction as are those of conservation. . . . Whether the attainments are truly operational is again open to interpretation and may be a function of the criteria used to judge the responses. . . (Beilin, 1971, pp. 107-108)." Two studies particularly noteworthy were conducted by Almy and her associates (1966; 1970). These studies have been summarized elsewhere (Almy, 1971). The intent of the later study was to determine whether prescribed instruction in mathematics and science programs in kindergarten would lead to more advanced logical thinking capabilities in second grade than would be the case where no prescribed instruction was present. Almy (1971) reported that ". . . when the performances of the children who had prescribed lessons beginning in kindergarten are compared with performances of the group about whose kindergarten experience information is available but who did not have the prescribed lessons, we see that the latter do as well as their counterparts who had the lessons (p. 235)." The variables Almy reported information on are conservation of number and weight, class inclusion, seriation, ordination, reordering, and transitivity. Rather than dwell on the lack of overall significance, an explanation of which would be based on a lack of experimental control, over-all performances and the interrelationships of the data of Almy's study relevant to the present study are discussed.

Standardized interview procedures were used where it was possible to employ a 0-1 dichotomy (not operational vs. operational) to the data. Utilizing this scoring procedure, Almy (1971, p. 233) reported data on 629 second grade children. Of these 629, 366 displayed operational performance on conservation of number, 221 operational conservation of weight, 50 operational class inclusion, 189 operational seriation, 253 operational ordination, 62 operational reordering, and 161 operational transitivity. The greatest task intercorrelation was .32 between conservation of number and conservation of weight. Even though Almy is quite cautious in her interpretation of these low inter-task correlations in that they may be a result of the standardized interview procedures, the low correlations do not seem to support Piaget's (1950) structural approach to intelligence. That is, if a child displays operational performance on a task which is in the scope of concrete operations, he should be able to display operational performance on any other such task. At the beginning of the concrete operational period, however, Piaget (1952, p. 204) contended as early as 1941 that logical forms such as transitivity have to be reacquired for each new content the child encounters. Other authors have addressed themselves to intertask consistency problems. Flavell and Wohwill (1969) have commented that ". . . during this period in which the newly emerging structures are in process of formation. . . the child's responses may be expected to oscillate from one occasion to the next, to be maximally susceptible to the effects of task related variables. . .(p. 95)." Yet, because Almy (1) did not detect improvement in logical reasoning at the beginning of second grade for children who engaged in prescribed mathematics and science programs over children for whom no prescribed instruction was present; (2) obtained low intertask correlations; and (3) obtained low frequencies of children who were operational with regard to most tasks used, a pervasive well-controlled experimental study is needed at the kindergarten level in which improvement in the usage of relational properties is of basic concern. It is not at all clear that such a study is doomed to failure at the outset. If children below age five were used as subject, the chances for success would be drastically reduced. However, at age five good chances for success with a long-term intervention study are present for it is at this age that children generally are in a transitional stage.

Piaget (1971) has stated that "From this general observation (acceptance of the existence of a maturing process)*,..., to an acceptance of inflexible stages characterized by invariable chronological age limits and by a permanent thought content is a long way indeed (p. 171)."

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*Added by the authors.

Treatments

Even though alternate treatments were not of major interest, two treatments were generated, one of which was called the Classificational Treatment and the other the Standard Treatment. The treatments are subject to the same criticism made by Beilin (1971) against Kohnstamm's(1968) now infamous class-inclusion study. That is, "Some studies, such as Kohnstamm's . . . contain so many experimenter manipulations as to make them of limited value as scientific instruments, although they may be noteworthy as educational polemics (Beilin, 1971, pp. 96-97)." The treatments, however, were not constructed to test Piagetian theory in a cricial experiment nor to uncover mechanisms of transition between stages in cognitive development, which seem to be the basic purpose of the Genevan's present training studies. Rather, the treatments were constructed more in the philosophy of Cronbach (1966, p. 77) when he suggested that researchers search for limited generalizations where particular content and inductive experiences produce some pattern of responses in pupils at a given level of development. The treatments were each homogeneous enough, however, to warrant serious speculation as to what may cause differences in performance, if differences occur.

A pretreatment, consisting of fourteen instructional sessions, was administered to each child in the experiment. It was designed to define, operationally, the two equivalence relations and the four order relations of interest. No attempt was made to go beyond what Piaget (1964) has termed physical experience, although particular children may have. The highest level of activity expected of the children was discrimination among the three matching relations and among the three length relations in situations not involving conservation of the relations. An example task at the highest level of activity is where a child had to select from various bags a bag of objects containing more objects than a given collection, one containing fewer, and one containing as many.

Classificational Treatment. The Classificational Treatment consisted of twelve instructional sessions and was based on the partitioning induced on a collection of objects by an equivalence relation defined on that collection. The children in this treatment overtly sorted linear objects into classes on the basis of "as long as" or sets of objects into classes on the basis of "as many as." Direct instruction then included instruction on (a) conserving subclass membership, (b) the substitutional property of an object (or set) for any object (or set) in its equivalence class, and (c) the asymmetric property of the order relations and transitive properties of the equivalence relations, all in the context of equivalence classes children formed through sorting activities. The direct instruction involved "programmed" questioning and discussion techniques as well as reversibility techniques. The following learning task is an example. It contains explicit guidelines to the experimenter on how to proceed.

> Give the children six shoeboxes on which are fastened "standard sets" of five, six, seven, eight, nine, or ten objects with one and only one standard set on each box. Also give each a collection of bags of objects each of which contains one of five, six, seven, eight, nine or ten objects. They are to select a bag from the collection and determine which shoebox it goes into by comparing the objects in it with the standard sets on the shoebox. A bag of objects is placed into a shoebox if and only if the bag contains as many objects as does the standard set on the shoebox.
>
> Start the activity by selecting a bag of objects and then finding which box it goes into. Be sure to pair the objects in the bag with the objects of standard sets with which there are not as many. Tell the children why the bag does not go into such shoeboxes. Say, e.g.,
>
> > "This bag does not go into this box because there are more (objects-bag) than (objects-shoebox)."
>
> Always point to the respective collection that you name. When you pair the objects of the selected bag with the objects on the shoebox to which it belongs, say,
>
> > "These (objects-bag) go into this box because there are as many (objects-bag) as (objects-shoebox).'

When they are done, select a box and take a bag from it. Ask,

> "Are there as many (objects-bag) as (objects-shoebox)?"

Always point to the respective collections that you name. The children should respond affirmatively. If not, have a child compare the objects to reestablish the relation. Select another bag from the box. Ask,

> "Are there as many (objects-bag) as (objects-shoebox)?"

Always point to the respective collection that you name. The children should respond affirmatively. If not, have a child compare the objects to reestablish the relation. Select another bag from the box. Ask,

> "Are there more (objects-bag) than (objects-shoebox)?"

The children should respond NO. If not, then have a child reestablish the relation between the two collections. Select another bag from the box. Ask,

> "Are there fewer (objects-bag) than (objects-shoebox)?"

The children should respond NO. If not, then have the child reestablish the relation between the two collections.

Select one collection of bags. Pointing to the standard set for this collection, say,

> "Find a bag of objects with as many things in it as (objects-shoebox)."

Attempt to have the children perform the task without actually making an overt comparison between the two collections. Ask several children to do the task. If the children cannot perform this activity, re-establish the relation between the standard set and several bags of objects in the box. Then ask the children to perform the task.

The objectives for the Classificational Treatment were as follows, stated in terms of the matching relations category.

1. A child should be able to sort sets of objects into collections on the basis of "as many as," given "standard sets" on which to base the sorting.

2. After sorting, a child should be able to determine that there are just as many objects in any set in a given collection as there are objects in the respective "standard set," and not more nor fewer.

3. After sorting, a child should be able to determine that for any two sets in a given collection, the relation "as many as" holds between the two collections and that the two relations "more than" and "fewer than" do not hold.

4. After sorting, a child should be able to determine that any bag of objects taken from a particular collection has more or fewer objects in it than a bag taken from a different collection by establishing the respective relations between the two standard sets.

5. After sorting, a child should be able to determine that there are more (fewer) objects in any set taken from a particular collection than there are in any other set taken from a different collection and the relation "as many as" does not hold between the two given sets of objects, by having established the respective relations between the two representative standards.

Standard Treatment. The Standard Treatment consisted of twelve instructional sessions and was based on a technique involving possible combinations of two variables: (a) perceptual screening vs. no perceptual screening and (b) a physical transformation which produced a conflictive situation and a physical transformation which produced a nonconflictive situation. The conflictive and nonconflictive situations can be described as situations in which a child had to conserve a relation versus a situation in which he did not, respectively. In these contexts, direct instruction was given on usage of the transitive property of the two equivalence relations and the asymmetric property of the four order relations. One major difference of this treatment and the Classificational Treatment is in the functional role of an object. In the latter treatment, an object (an object can be considered as a collection) was to be considered as representing a collection of like objects whereas in the Standard Treatment no attempt was made to consider an object as representational. Programmed questioning and disucssion techniques were utilized which involved reversibility procedures. The first learning task in the following sequence of two is an example of direct instruction on the asymmetric property involving perceptual screening and physical transformations which produced conflictive situations. The second learning task is an example of the same property with no perceptual screening and physical transformations which produced conflictive situations.

1. Give each child twelve model Indians and ten model cowboys. Say,

"Compare the Indians and cowboys. Make pairs."

Be sure each child thinks he/she has more Indians than cowboys. Then have each child place a cover over the Indians and cowboys and mix them up under the cover. After each child is done, ask individually,

"Do you have more cowboys than Indians?"

If a child responds correctly, say,

"That's right, you do not have more cowboys than Indians because you have more Indians than cowboys."

If a child responds incorrectly, say,

"Uncover the cowboys and Indians and make pairs."

Then ask,

"Do you have more Indians than cowboys?"
"Do you have more cowboys than Indians?"

Have the child cover the Indians and cowboys and mix them up under the cover. Then ask,

"Can you tell me if you have more cowboys than Indians?"

2. Give each child a piece of cardbaord on which are fastened twelve tile and on which are painted ten dots as follows:

Tile

• • • • • • • • • •   Dots

Also, give each child a cup containing ten discs. Say,

"Compare the discs and tile."

Be sure each child establishes that he has fewer discs than tile. Then have them move the discs to cover the dots. Ask each child individually,

"Do you have fewer tile than discs?"

If the child responds correctly, say,

"That's right. You have fewer discs than tile. You do not have fewer tile than discs."

If the child responds incorrectly, have him put the discs back on the tile. Ask,

"Do you have fewer discs than tile?"

Then have the child put the discs back on the dots. Ask,

"Do you have fewer tile than discs?"

For each relational category, the first three instructional sessions dealt with the transitive property and the last three dealt with the asymmetric property. In the case of each property, the first session involved a nonconflictive situation and the last two conflictive situations. The objectives of the Standard Treatment were as follows, stated in terms of matching relations:

1. A child should be able to use the transitive property of "as many as."
2. A child should be able to use the asymmetric property of "fewer than" and "more than."

The conjecture was made that the Classificational Treatment would produce more capabilities in the children than would the Standard Treatment. The basis for such a conjecture was the functional role of the objects in each treatment.

Testing Instruments

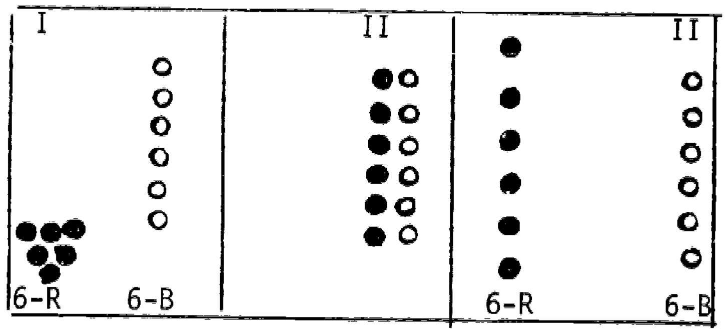
Six item sets were designed, three for each relational category. A conservation of matching relations test,a transitivity of matching relations test, and an asymmetric property of matching relations test were deisgned. The conservation of matching relations test included a test of the ability of a child to establish matching relations. Analogous instruments were designed for the length relation category. A description of items used in each test follows.

Conservation of Matching Relations. Six items were designed each in which the child constructed the matching. To construct a matching, the child had to pair the objects of two collections and then respond correctly to two questions. One collection was then physically transformed by the experimenter and the child was asked the same two questions.

Item 1 (As Many As). The experimenter (E) stated, referring to the objects in frame I of Figure I, "Pair the red discs and blue discs." After a child (S) made the pairing, E asked in a random order (frame II): (a) "Are there as many red discs as blue discs?" and (b) "Are there more red discs than blue discs?"

E then transformed the red discs as in frame III and asked questions (a) and (b) again, also in random order.

Figure I



Item 2 (As Many As). Seven black checkers were attached in a circular pattern to a piece of white cardboard. Seven red checkers were matched with the black checkers and then transformed into a circular pattern with diameter less than the diameter of the circular pattern of the black checkers. The two questions; "Are there as many red checkers as black checkers?" and "Are there fewer red checkers than black checkers?" were asked.

Items 3 and 4 (More Than). Each of the two relations "fewer than" and "as many as" were paired with the relation "more than" to generate the questions asked. In Item 3, "as many as" was used and in Item 4, "fewer than" was used. The perceptual cues after transformation always favored the relation which did not hold. Five blue discs and six red discs were used as objects in Item 3, and seven black checkers and eight red checkers were used as objects in Item 4.

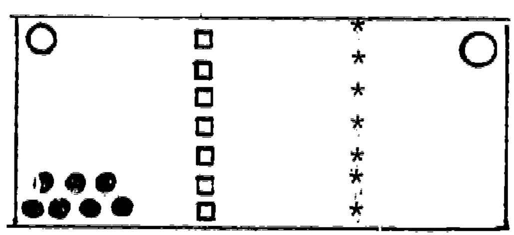
Items 5 and 6 (Fewer Than). Each of the two relations "more than" and "as many as" were paired with the relation "fewer than" to generate the questions asked. In Item 5 "as many as" was used and in Item 6 "more than" was used. The perceptual cues after transformation always favored the relation which did not hold. Five red discs and six blue discs were used as objects in Item 5 and seven red checkers and eight black checkers were used as objects in Item 6.

Conservation of Length Relations. The conservation of length relations test was structu y isomorphic to the conservation of matching relations test. The perceptual cues after transformation were biased in favor of the incorrect relation. Two basic tr nsformations were used. The first was a lateral translation of one of the two linear objects and the second was a transformation which left the objects in a ⊢ formation.

Transitivity Test. Eight transitivity items were used in the study; four items in the matching relations category and four in the length relations category.

Item 1 (As Many As). Seven tile and seven jacks were arranged on a piece of cardboard in such a way no apparent perceptual bias was present. A collection of seven checkers were placed in a pile in one corner of the cardboard. The arrangement looked as in Figure II.

Figure II



A styrofoam cup was placed at each corner of the cardboard. E stated, "Pair the checkers and the tile." After S was done, E asked, "Are there as many checkers as tile?" E then placed the checkers in a cup and stated, "Pair the tile with the jacks." After S was done, E asked, "Are there as many tile as jacks?" E then placed the jacks into the remaining cup and asked (in a random order) "Are there as many checkers as jacks?" and "Are there more checkers than jacks?"

Item 2 (As Many As). Item 2 was analogous to Item 1 except eight red discs, eight blue discs, and eight green discs were used and the "more than" questions was changed to "fewer than."

Item 3 (More Than). Item 3 was similar to Item 1 except eight checkers, seven tile, and six jacks were used. The two key questions after the objects had been placed into the cups were: "Are there more checkers than jacks?" and "Are there fewer checkers than jacks?"

Item 4 (Fewer Than). Item 4 was similar to Item 1 except six checkers, seven tile, and eight jacks were used. The two key questions after the objects had been placed into the cup were: "Are there fewer checkers than jacks?" and "Are there more checkers than jacks?"

Item 5 (As Long As). A red, a blue, and a green stick, all of the same length, were placed in such a way that no apparent perceptual bias was present. E stated, "Compare the red stick and the blue stick." After S had done so, E asked, "Is the red stick as long as the blue stick?" E then placed the red stick in a small box after which he stated, "Compare the blue stick and the green stick." After S had done so, E asked, "Is the blue stick as long as the green stick?" E then placed the green stick in another small box. E then asked in random sequence, "Is the red stick as long as the green stick?" and "Is the red stick shorter than the green stick?"

Item 6 (As Long As). Item 6 was analogous to Item 5 except soda straws were used instead of sticks.

Item 7 (Longer Than). Item 7 was similar to Item 5 except the relation involved was "longer than" and the two key questions were "Is the red stick as long as the green stick?" and "Is the red stick longer than the green stick?"

Item 8 (Shorter Than). Item 8 was similar to Item 5 except the relation involved was "shorter than" and the two key questions were "Is the red stick as long as the green stick?" and "Is the red stick shorter than the green stick?"

Asymmetric Property Test. Four items were constructed, one for each order relation involved. Item descriptions follow.

Item 1 (More Than). Nine tile, eight checkers, and two styrofoam cups were used. E stated, "Pair the tile and the checkers." After S was done, E asked, "Are there more tile than checkers?" E then placed the tile in one cup and the checkers in the other. E then asked, "Are there more checkers than tile?" and "Are there more tile than checkers?"

Item 2 (Fewer Than). Item 2 was analogous to Item 1 except eight tile and nine checkers were used and "fewer than" was substituted for "more than."

Item 3 (Longer Than). A green straw, a blue straw longer than the green straw, and two boxes into which the straws could be placed were used. E stated, "Compare the blue straw with the green straw." After S was done, E asked, "Is the blue straw longer than the green straw?" E then placed the straws into the two boxes and asked, "Is the green straw longer than the blue straw?" and "Is the blue straw longer than the green straw?"

Item 4 (Shorter Than). Item 4 was analogous to Item 3 except the relation "shorter than" was replaced for "longer than."

Procedures

The subjects for the study were 48 upper middle class kindergarten children enrolled in two private kindergartens in Athens, Georgia; Beech Haven Baptist Kindergarten and St. Joseph's Catholic Kindergarten. The experiment began on April 5, 1971 and ran through May 14, 1971. At the initiation of the experiment the pretreatment was administered to the 48 children. Upon completion of the pretreatment the tests of conservation of matching relations, conservation of length relations, and items 1, 3, 4, 5, 7, and 8 of the transitivity test were administered. The Classificational and Standard Treatments were then administered after which the tests of conservation of matching relations, conservation of length relations, transitivity, and the asymmetric property were administered.

Testing Procedures. Each item in each test was individually administered by specially trained testers.* A particular test was administered at one sitting where the sequence in which the items were administered was randomized. In the case of CMR and CLR two scores were obtained from each item. If a child correctly answered the two questions asked after he had made the initial comparison, he was given credit for having established a matching or length relation, as the case may be. If a child also correctly answered the two questions asked after E performed the physical transformation, he was given credit for having conserved a matching or length relation, as the case may be. In essence, in order to conserve a relation, (a) the child had to establish the relation which actually held between two objects (or collections) and (b) display evidence that he perceived that the relation established was the only relation which held. If a child just guessed at correct answers, the probability was .25 he would successfully complete an item. For six items, the probability of successfully completing at least four out of six items was less than .05. For the nonconserving child, such a probability would be less than .05. If a child, then, successfully completed at least four out of six conservation items for a given relational category, evidence was strong that he was able to conserve relations of that category. In the case of not successfully completing at least four out of six items in a relational category, the child was classified as an indeterminant; that is, no evidence existed that he was able to conserve relations of that category.

*The experimenters are indebted to Ms. Judy Caddell, Ms. Marie Vitale, Mr. Francis Kidder, and Mr. Charles Lamb, all graduate students at the University of Georgia and to Mr. Douglas T. Owens, now at the University of British Columbia, for their assistance in the conduction of the experiment.

For a child to successfully complete a transitivity item, he had to (1) establish the relations and (2) correctly answer each of the last two questions after the objects were screened from view. If a child guessed on the last two questions, the probability that he would successfully complete an item was .25. In such a case, the probability of successfully completing at least four out of six items was less than .05 and the probability of successfully completing at least five out of eight items was less than .05. In the case of the transitivity pretest, then, evidence was strong that a child used transitivity if he successfully completed at least four out of six items. On the transitivity posttest, the evidence was strong that a child used transitivity if he successfully completed at least five out of eight items. If a child successfully completed no more than three items on the pretest, he was classified as an indeterminant and if he successfully completed no more than four items on the posttest, he was classified as an indeterminant.

For a child to successfully complete an asymmetric item, he was required to correctly answer each of the last two questions after the objects were screened from view. The evidence was strong (probability less than .05) that a child used the asymmetric property if he successfully completed at least three out of four items. If a child successfully completed no more than two out of four items, he was classified as an indeterminant.

Design and Data Analysis

The children were instructed in groups of six. After the first test administration, the children were assigned by match pairs to each treatment within each school. The pairs were formed on the basis of the transitivity test scores. In Beechhaven School, there were six children who met criterion on the transitivity pretest and in St. Joseph School, there were ten children who met criterion on transitivity.

Contingency tables were formed and used in testing the hypotheses concerning the predicted lag in acquisition of length relations relative to matching relations. In order to test the relative effectiveness of the treatments, the sign test (Siegel, 1956, pp. 68-75) was utilized. The following procedures were followed in its application. Two scores existed for each child for transitivity, conservation of length relations, and conservation of matching relations; a pretest scores and a posttest score. A child was assigned a score of 1 if he met criterion and a 0 otherwise. On any one test, four possibilities existed for each child for a pretest-posttest score; 0-0, 0-1, 1-0, and 1-1. If a child changed from 0 to 1, a "+" was assigned; if he changed from 1 to 0, a "-" was assigned; and in the two other cases, a "0" was assigned. A sign was then assigned to each matched pair of children. If each child of a pair had the same sign or a zero assigned, a tie was awarded that pair. If the children of a pair had different signs or a zero and a sign assigned, a "+" or "-" was assigned. A "+" was assigned if the ordered pairs (+,0),(+,-) or (0,-) occurred and a "-" if the ordered pairs (-,0), (-,+), or (0,+) occurred, where the first member of a pair designated the score of a child from the Classificational Treatment.

In case of the asymmetric property, posttest scores only were available. Signs were attached to the pairs of children in the following way. A "+" was assigned to (1,0), a "-" to (0,1), and a tie awarded to the pairs (0,0) and (1,1).

In the sign test, ties are disregarded. The remaining N pairs then have different signs. The probability of getting at most r "+" signs out of N pairs, assuming "+" and "-" are equally likely, is just $1/2^N \sum_{i=0}^{r} \binom{N}{i}$, for a one-tailed test of the null hypothesis, which is the number of "+" signs is equal to the number of "-" signs.

It must be emphasized that the sign test was employed to test the <u>relative</u> effectiveness of the treatments. It did not test gains from pre- to posttests the way it was used. To make such tests, the McNemar test for significance of changes (Siegel, 1956, pp. 63-67) was used. The null hypothesis tested is the probability that a child will change from noncriterion to criterion is equal to the probability he will change from criterion to noncriterion. The alternative hypothesis tested is the former probability is greater than the latter.

Results

Interrelationships of Relational Properties

There was no evidence to support the hypothesis that conservation of matching relations precedes conservation of length relations on either the pretest or posttest. If conservation of matching relations precedes conservation of length relations, the frequency of children meeting criterion for conservation of matching relations but not for conservation of length relations should have

Table 1

Contingency Tables: Matching by Length Relations*

	Conservation				Transitivity				Asymmetric	
	Pretest		Posttest		Pretest		Posttest		Posttest	
M \ L	C	~C	C	~C	C	~C	C	~C	C	~C
C	19	8	30	8	10	11	24	8	25	4
~C	7	14	7	3	11	15	8	9	12	8

*C denotes criterion and ~C noncriterion exceeded the frequency of children meeting criterion for conservation for length relations but not for conservation of matching relations. There were eight children in the former category and seven in the latter category on each of the pretest and posttest (Table 1), which just does not support the hypothesis.

There also was no evidence to support the hypothesis that transitivity of matching relations precedes transitivity of length relations on either the pre- or posttest. There was some evidence to support the reversal of the hypothesis that the asymmetric property of matching relations precedes the asymmetric property of length relations. Twelve children met criterion on the asymmetric property of the length relations (two out of two items correct) but not the asymmetric property of matching relations whereas only four children met criterion for matching but not length.

Treatment Contrasts

There was no evidence to support the conjecture that the Classificational Treatment would produce more capabilities in the children than would the Standard Treatment. For conservation of matching relations, there were 15 out of 24 pairs with a "+" or "-" assigned. Seven of these pairs had a "+" assigned, an event which could occur with probability .50 under the hypothesis of equally likely occurrence of "+" and "-" for each pair. For conservation of length relations, only seven pairs out of 24 had a "+" or "-" assigned of which at most three were "+". This event could occur with probability .5 under the hypothesis of equally likely occurrence of "+" or "-" for each pair. In case of the transitive property, only eight pairs out of 24 had a "+" or "-" assigned of which at most three were "-". This event could occur with probability .36 under the hypothesis of equally likely occurrence of "+" and "-". In case of the asymmetric property ten pairs out of 24 had a "+" or "-" assigned of which at most five were "+". This event could occur with probability .63 under the hypothesis of equally likely occurrence of "+" or "-".

Hence the null hypothesis that the number of pairs with "+" assigned is equal to the number of pairs with a "-" assigned is not rejected for any of the four tests run.

Treatment Effectiveness

Because the treatment contrasts were not significant does not mean that the treatments were not effective. In order to assess treatment effectiveness, the subjects in the two treatments were pooled and the McNemar test for significance of changes was run. In case of the transitivity test, 22 children changed from noncriterion to criterion from the pretest to the posttest, eight remained at the noncriterion level on the posttest, and 33 children met criterion on the posttest. Two children had missing data but both met criterion on the posttests.

Table 2

Contingency Tables for Changes: Pretest by Posttest

	Conservation				Transitivity*	
	Length*		Matching*			
Pre \ Post	C	~C	C	~C	C	~C
C	24	2	25	3	11	5
~C	13	9	13	7	22	8

*p<.01 McNemar test for significance of changes.

The null hypothesis that the probability of changing from noncriterion to criterion is equal to the probability of changing from criterion to noncriterion is rejects (p<.01) and the alternative hypothesis is accepted for transitivity.

In case of conservation of length and matching relations the alternative hypothesis was also accepted. Thirteen children changed from noncriterion to criterion for each of length and matching and 37 and 38 children met criterion on the posttest, respectively.

Post-Hoc Analyses

Approximately 73 percent of the children met criterion on the transitivity posttest. Because instruction was given only on transitivity of the equivalence relations, a separate analysis of the equivalence and order relations was conducted to ascertain whether performance was uniform across equivalence and order relations. Criterion performance on the pretest was set at two correct out of three items and on the posttest three correct out of four items. On the

Table 3

Contingency Tables for Transitivity: Equivalence by Order Relations

Pretest

α	C	~C
C	5	6
~C	6	29

Posttest

α	C	~C
C	20	3
~C	18	7

pretest there was no indication that transitivity of equivalence relations was any more or less difficult than transitivity of order relations. However, on the posttest, while 20 children met criterion on transitivity for both equivalence and order relations, 18 met criterion for equivalence but not for order. This latter result can be attributed directly to the experimental sessions as transitivity of order relations was not considered in either treatment. Of these 18 children, 12 did not meet criterion on the transitivity pretest and six did. Of the 20 children who met criterion on transitivity of equivalence and order relations on the posttest, 10 did not meet criterion on the transitivity pretest. It does not appear, then, to be possible to make a strong prediction of a transitivity pretest score based on knowledge of whether or not a child was able to use transitivity of order on the posttest (given that he met criterion for transitivity of equivalence on the posttest). A trend was present in the data, however, in that the odds were two to one that a child did not meet criterion on the transitivity pretest given that he met criterion for transitivity of equivalence but not order on the posttest.

If a child could conserve relations on the pretest it would seem that such a child could learn transitivity more easily than those children for whom evidence was not present that they could conserve relations. In Table 4, a child was categorized as being able to conserve relations if he could conserve both order and length relations. For these 20 children who did not meet criterion for either conservation or transitivity on the pretest, 70 percent met criterion

Table 4

Contingency Table: Conservation by Transitivity Pre- and Posttests

Pretest		Posttest: Trans.	
Cons.	Trans.	C	~C
C	C	8	1
	~C	7	3
~C	C	4	4
	~C	14	6

on the transitivity posttest. Given that a child did not meet criterion on the transitivity pretest then, whether or not he met criterion on the conservation pretest seemed to not be related to whether or not he met criterion on the transitivity posttest.

Eight children existed who did not meet criterion on the conservation pretest but did meet criterion on the transitivity pretest. Of these eight, six met criterion on either of conservation of length or matching. Of the two children who met criterion on neither conservation of length nor matching, one met criterion on the transitivity posttest and one did not..

Discussion

The results of this study, when contrasted with the results of Almy's (1971) study, give a different perspective of possible effects of prescribed lessons in mathematics or science on the child's logical reasoning. As noted in Almy's study, only 161 out of 629 second grade children displayed an ability to use the transitive property, where no differences existed in a group of children who had prescribed instructional experiences in the kindergarten vs. children who did not have prescribed instructional experiences in kindergarten. The task Almy (1971, p. 229) used to assess transitivity involved "longer than" in the context of the Müller-Lyer illusion, where she did not demand a child give verbal reasons for his answer. Even though the tasks used in this study to assess transitive reasoning were given in a different contextual setting, eight tasks were used across two relational categories and six relations with a performance criterion set. That is, the criterion set for a child to be classified as being able to use transitivity was much more stringent in the present study than in Almy's. Yet approximately 73 percent of the kindergarten children in the present study met criterion on the transitivity posttest as opposed to approximately 25 percent of the second grade children in Almy's study. Moreover, approximately 77 percent of the children met criterion on the conservation of matching relations posttest whereas in Almy's study approximately 58 percent of the second grade children were classified as operational conservers of number. It appears then, that under controlled instructional conditions, the logical thinking of children as it pertains to properties of matching and length relation can be improved long before second grade. However, such an assertion must be duly tempered by the following observations. First, the transitivity items used to test transitivity of the two equivalence relations must be considered as direct achievement items because direct instruction was administered on transitivity of those two equivalence relations. When the data on transitivity of the order relations is considered, only approximately 50 percent of the children met criterion. Yet, prospects were quite good that the remaining 50 percent could be trained to criterion without a great deal of effort. Second, the experimental design has not yet allowed for a retention test to be given at the beginning of second grade. It may well be the case that the children in the sample will have regressed dramatically by then. Third, whether a trained knowledge of transitivity makes other mathematical concepts more easily acquired which are logically based on transitivity is yet to be explored. Fourth, a crucial aspect is whether or not the children in the study were operational in their use of transitivity using Genevan standards.

A rather important aspect of the study was that no evidence was present that acquisition of matching relations preceded acquisition of length relations. At first sight, it appears these results are at variance with past research. However, as already noted, such a time lag was reported in studies which involved measurement. Structural similarities between counting and measurement has been noted earlier, similarities which include the relations studied here. Operational usage of the length relations are included but do not exhaust operational measurement procedures. One retarding factor in the measurement process may be in the unit iteration procedure. It seems it would be easier to just count singular elements than to iterate a unit of measurement. Certainly, the results of this study indicate that the observed time lag is not due to a time lag in the development of logical forms such as transitivity and asymmetry across differing relational content.

The issue of what constitutes operationality is far from settled. In this study operational definitions were given in order to make evidence as strong as possible for classifying a child as being able to conserve relations, use the transitive property, and use the asymmetric property. In case of the transitive property, children were forced to use a transitive inference in order to correctly respond to an item. A difference between the criteria for a child to successfully complete a transitivity item here and the criteria used by Smedslund (1963) is in the child's justification of the response. The issue of operativity has been studied by Divers (1970), Owens and Steffe (1972) and Owens (1972). While the data cannot be simply described, transitivity items not requiring verbal justification which involve a screened stimulus and a conflictive stimulus are about of equal difficulty (Divers, 1970; Owens and Steffe, 1972). Verbal justifications given by children cannot always be sensibly interpreted. Items which do require verbal justification do not necessarily turn out to be more difficult than items which do not require verbal justification. Moreover, a high consistency exists as to whether or not a child can use transitivity in the structural interview as used in this study and in a transitivity problem involving verbal justification (Owens, 1972).

A surprising result of the study was contained in the post-hoc analyses. It was observed that given that a child did not meet criterion on the transitivity pretest, whether or not he met criterion on the conservation pretest, seemed to bear little relationship to whether or not he met criterion on the transitivity posttest. These results are at variance with those of Garcez (1969) as reported by Beilin (1971) in the case of conservation and transitivity of weight. An explanation of the results may reside in the procedures used in the treatments in that children were presented with situations which contained opportunities for them to conserve the relations involved.

If it could be unequivocally stated that 73 percent of the sample could indeed use transitivity across the six relations included in the study, the educational implications would be indeed strong. Because some equivocation creeps into the results as to whether the subjects were operational on Genevan standards, more work needs to be done in experimental contexts utilizing multiple tests of transitivity and other properties in order to obtain a clearer picture of the scope of the learning. Moreover, the sample characteristics of the children need varied. Owens (1972) has obtained quite different results using an all black sample whereas Johnson (1971), utilizing a black and white sample, did not observe differential effects on classification, seriation, and transitivity tests. Treatment conditions also need varied to obtain critical variables in learning logical forms. In the face of nonsignificant treatment contrasts, such variables may not be easily produced.

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