DOCUMENT RESUME

61 261 TM 001 165

OR	Primoff, Ernest S.
E	Preliminary Report on Use of Self-Ratings to Provide J-Coefficient Data.
ITUTION	Civil Service Commission, Washington, D.C. Bureau of Policies and Standards.
DATE	Jan 71
	21p.
PRICE	MF-$0.65 HC-$3.29
RIPTORS	*Achievement Rating; Correlation; Evaluation Criteria; *Occupational Tests; Rating Scales; Reliability; *Self Evaluation; Standardized Tests; Statistical Analysis; Test Reliability; *Test Results; *Validity
TIFIERS	*J Coefficient

RACT

This report shows how Beta weights for the efficient may be easily developed without a formal validity y, and indicates how indications of ability other than tests can sed to measure the same abilities that are measured by tests. See TM 001 163-64,166 for further information on job elements cale) procedures. (Author/DLG)

Enc. #3

ED 061261

PRELIMINARY REPORT ON USE OF SELF-RATINGS TO PROVIDE J-COEFFICIENT DATA

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January 1971

PRELIMINARY REPORT ON USE OF SELF-RATINGS
TO PROVIDE J-COEFFICIENT DATA

INTRODUCTION

Herb Ozur and his staff have computed the correlations and Beta weights for data collected by Don Wagner on five tests and six self-ratings. The tests (or test parts) are Spelling, Reading, Grammar, Word Meaning, and Filing. Self-ratings were for spelling ability, reading ability, grammar ability, word meaning ability, filing ability, and amount of education.

Table 1 shows all the inter-correlations between test scores and ratings.

TABLE 1

Intercorrelations

N = 204, except for Education when N = 197

	Read. Test	Gram. Test	Word Test	Fil. Test	Rating Spel.	Rating Read.	Rating Gram.	Rating Word	Rating Fil.	Rating Educa.
Spelling Test	.55	.56	.69	.27	.63	.45	.47	.46	.21	.40
Reading Test	-	.66	.74	.34	.49	.51	.55	.57	.33	.49
Grammar Test	.66	-	.70	.17	.47	.43	.53	.52	.31	.51
Word Meaning Test	.74	.70	-	.27	.56	.55	.62	.65	.41	.50
Filing Test	.34	.17	.27	-	.23	.14	.19	.17	.31	.18
Self-Rating in Spelling Ability	.49	.47	.56	.23	-	.61	.67	.61	.41	.41
Self-Rating in Reading Ability	.51	.43	.55	.14	.61	-	.65	.73	.37	.43
Self-Rating in Grammar Ability	.55	.53	.62	.19	.67	.65	-	.76	.53	.51
Self-Rating Word Abil.	.57	.52	.65	.17	.61	.73	.76	-	.42	.49
Self-Rating Filing Abil.	.33	.31	.41	.31	.41	.37	.53	.42	-	.28
Self-Rating in amount of education	.49	.51	.50	.18	.41	.43	.51	.49	.28	-

Note that the rating which correlated highest with the spelling test was the rating in spelling ability. The rating which correlated highest with the grammar test was the rating in grammar ability. The rating which correlated highest with the word meaning test was the rating in word meaning ability. The rating which correlated highest with the filing test was the rating in filing ability.

Table 2 shows the Beta weights of the tests on the elements.

TABLE 2

Beta Weights

	β_1 Spel.	β_2 Read.	β_3 Gram.	β_4 Word Mean.	β_5 Filing	β_6 Education	R
Spelling Test	.54	.04	.01	.06	-.10	.15	.66
Reading Test	.11	.11	.12	.20	.04	.23	.64
Grammar Test	.14	-.03	.16	.18	.02	.29	.62
Word Mean. Test	.16	.03	.12	.32	.08	.19	.72
Filing Test	.16	-.05	-.10	.01	.29	.11	.35

Part I. Validity Coefficients

How J-Coefficients Might Be Derived

Suppose that it is decided that for a particular job, either using job element blank or other analytic method, that the job criterion of success would be as follows:

Element	Weight
Education	4
Reading	10

In traditional validation studies, it would be necessary to rate people in education and reading on the job and derive a criterion, administer tests, and secure validity coefficients. This is not necessary with the J-Coefficient technique. There are only three simple steps, once tables like the above are available.

STEP 1

Compute the standards deviation of the composite for standard scores for education and reading. In standard score form, the standard deviation of education would be 4 and of reading would be 10. The standard deviation of the composite is the square root of the sum of the squares of the standard deviations plus twice the product of the intercorrelation times standard deviations, in this case: 1/

$$\sqrt{4^2 + 10^2 + 2\,(.43)\,(4 \times 10)} = 12.264$$

The .43 came from Table 1, the correlation between education and reading.

STEP 2

We need the correlations of each element with the composite criterion. The numerator of each correlation is the sum of the products of the weight for the selected elements times the correlation of the particular element with the selected element.

The denominator of each correlation is the standard deviation of the composite, found in Step 1. Note that the standard deviation of each element is one, since elements are regarded to be in standard score form. Therefore, the standard deviation for each individual element can be ignored in the element. For example, the correlation between th[illegible] for spelling ability and the composite is figured as follows:

$$\frac{(.61 \times 10) + (.41 \times 4)}{12.264} = .6311$$

Following are the correlations between the rating on each element and the composite criterion:

Spelling ability	.6311
Reading ability	.9600
Grammar ability	.6963
Word meaning ability	.7550
Filing ability	.3930
Education	.6768

Note that for J-Coefficient purposes, diagonal entries in Table 1 are unity; lack of reliability has already influenced the Beta weights, insofar as relation of test to job is concerned. (See page 5)

1/If there are more than two elements, there are $\frac{n(n-1)}{2}$ products, one for each correlation.

STEP 3 -- Finding the J-Coefficients

The J-Coefficients are sums of products of the test Beta weight on each element times the correlation of each element with the composite criterion. For example, the J-Coefficient for the spelling test is the sum of the following products:

Element	Beta Weight From Table 2	Correlation of Element With Composite Computed Above
Spelling	.54	.6311
Reading ability	.04	.9600
Grammar ability	.01	.6963
Word meaning ability	.06	.7550
Filing ability	-.10	.3930
Education	.15	.6768

The sum of the products--.54 x 6311, .04 x 9600, etc., is .4940. This is the J-Coefficient for the spelling test.

The J-Coefficients for all the tests are as follows:

Test	J-Coefficient
Spelling	.494
Reading	.581
Grammar	.511
Word Meaning	.615
Filing	.179

Test Coverage By Elements

The formula for the J-Coefficient, as shown in Basic Formulae for the J-Coefficient (1955), has $\Sigma \beta r$ in the numerator, and R_t R_j in the denominator.

The denominator is the product of the multiple R of jobs on the elements, times the multiple R of tests on the elements.

The following paragraph on pages 7-8 of Basic Formulae for the J-Coefficient deals with this denominator in the final formula (formula 17):

> "With respect to the denominator of (17), it is not necessary that the checklist should provide enough elements to cover all test and job variances. It is enough to know that the multiple R of a complete set of elements with test or with criterion is 1. However, if some job-test covariance is not covered in the elements, that is, if the checklist is incomplete in some abilities present in both job and test, then the numerator of (17) will not be completely represented by data. This incompleteness may be adjusted in at least two ways, as it becomes apparent in validity studies. An obvious way to increase the coverage of the checklist is by adding new elements. An example will be given later. If, however, it appears for a certain test or kind of job that the job-test covariance for elements present in the list is, although incomplete, proportional to total covariance (that is, that calculated J-coefficients are a multiple of validity coefficients), then adjustment may be made for a test or kind of job by a compensating factor."

Note that any correction in this regard would raise the J-Coefficient. Thus, the J-Coefficient is a minimum bound of the validity coefficient.

If, for a particular test, and set of elements, J-Coefficients are obviously too small because of inadequate number of elements, the best answer would be to add elements so that the R's will be closer to 1. Another alternative would be to place the R of the test in the denominator, as indicated by the formula. This would raise the J-Coefficient, implying that relationship between job and test on unknown elements is the same as the relationship for known elements. Before placing R in the denominator, unreliability of test would be considered as an element not correlated with other elements. According to standard test theory, the Beta weight and correlation of unreliability with the test would be the square root of unity minus the square of the reliability coefficient. Since the multiple R is the square root of Beta r products, the multiple R of test on all elements plus unreliability is the square root of R^2 plus $(1 - r^2)$, where R is the multiple R of test on elements and r is reliability of test.

With respect of R for job, there is no statistical problem, since the composite criterion is made up of the given elements by the same kind of decision as in any validity study. If additional elements are needed to define the job, they should be added to the list, rather than compensated by increasing the J-Coefficient to provide for their absence.

Part II. Use of J-Coefficients to Analyze Tests

Before the J-Coefficient was available, it was thought necessary that analytical factors be uncorrelated, so that their products could be summed to equal test intercorrelations. This required that factors be uncorrelated.

As Thurstone pointed out, elements in real life are not uncorrelated. For example, he pointed out that the dimensions of boxes are correlated. Boxes with large widths tend to have large lengths. The need for orthogonal factors resulted in artifacts.

The J-Coefficient formula makes it possible to deal with elements that are correlated. Thus, the procedure begins with elements that can be understood by raters, and the weights are based on the meaning given these elements by raters. (Of course, if elements are independent, Beta weights and r's are identical, and such elements would be orthogonal, with loadings equal to Betas and to r's.)

The discussion of the table on page 2 illustrates some of the insights that can be gained by J-Coefficient analysis of the relation between tests and elements. Further analysis can be done by comparing actual test intercorrelations with J-Coefficients between tests.

When validity coefficients were computed, reliability of tests was not considered because unreliability of a test was lumped with other elements in the test that were not in the job.

With tests, however, there are elements that are not in the job elements, and to analyze these elements, it is necessary to subtract out unreliability.

For the spelling, reading, grammar and word meaning tests, the Kuder-Richardson formula based on number of items, standard deviation and mean was used, since these tests are power tests. The filing test is a speed test. For this test, a reliability formula developed by Primoff which compares each response in the test with all other responses, and derives a reliability coefficient for a hypothetical set of measures, was used. Following are the reliability coefficients:

Test	Reliability Coefficient
Spelling	.79
Reading	.74
Grammar	.70
Word Meaning	.90
Filing	.605

In test theory, unreliability of the test is considered an element uncorrelated with any other element, and correlated only with the particular test. Thus, the Beta and the r for unreliability are identical. If unreliability is considered to be an element, Table 2 would have an additional entry, unreliability, with a Beta weight w, and Table 1 would have an entry for unreliability with an r equal to w. Now the multiple R shown in Table 2 would be greater. The product for the Beta times r for unreliability would have to be added to the other products of Beta times r to give the augmented R, considering unreliability as an element.

Similarly, the reliability of the test can be considered to be the correlation and Beta weight of the test on a single "reliable" element. Thus, the square of reliability plus the square of unreliability equals one. Thus, both the Beta and the r for unreliability are:

$$\beta = r = \sqrt{1 - \text{reliability}^2}$$

The augmented multiple R for a test including unreliability as an element will be the sum of all Beta-r-products. R^2 represents the Beta-r products for the elements in Table 2. The additional product for unreliability will be one minus the square of the reliability coefficient. (Multiplying Beta for unreliability by r for unreliability.) Thus,

$$(\text{Augmented R})^2 = 1 - \text{reliability}^2 + (\text{original R})^2$$

The J-Coefficient shows the test interrelationships: $J = \dfrac{\Sigma \beta r}{\text{Augmented R Product}}$

$\Sigma \beta r$ is the sum of the products of test Beta weight on element times test correlation on element. For example, for the spelling and reading tests, the Beta for the spelling test on Element 1, .54 is multiplied by the r of the reading test on Element 1, .49; the Beta .04 is multiplied by the r .51, the Beta .01 by the r .55, the Beta .06 by the r .57, the Beta minus .10 by the r .33, and the Beta .15 by the r .49. The products are summed to give $\Sigma \beta r$

(Except for rounding errors, an identical sum would be obtained by using Betas for the reading test and r for the spelling test.)

To obtain J-Coefficients between tests, each sum of Beta r products is divided by the product of the augmented R's.

Table 3 compares the J-Coefficients between tests with actual test intercorrelations. Test intercorrelations are shown in Table 1, but in no way affected the Beta weights or r's between tests and elements which were used in computing J-Coefficients.

TABLE 3

J-Coefficients Between Tests Compared to Actual Correlations

Test	$\Sigma \beta r$	Augmented R	J-Coefficient	Actual Intercorrelation
Spelling Test vs.		.90		
Reading Test	.37	.93	.44	.55
Grammar Test	.35	.95	.41	.56
Word Mean. Test	.41	.84	.54	.69
Filing Test	.14	.87	.18	.27
Reading Test vs.		.93		
Spelling Test	.37	.90	.44	.55
Grammar Test	.39	.95	.45	.66
Word Mean. Test	.46	.84	.59	.74
Filing Test	.15	.87	.19	.34
Grammar Test vs.		.95		
Spelling Test	.35	.90	.41	.56
Reading Test	.39	.93	.45	.66
Word Mean. Test	.44	.84	.55	.70
Filing Test	.15	.87	.18	.17
Word Mean. Test vs.		.84		
Spelling Test	.41	.90	.54	.69
Reading Test	.46	.93	.59	.74
Grammar Test	.44	.95	.55	.70
Filing Test	.18	.87	.25	.27
Filing Test vs.		.87		
Spelling Test	.14	.90	.18	.27
Reading Test	.15	.93	.19	.34
Grammar Test	.15	.95	.18	.17
Word Mean. Test	.18	.84	.25	.27

The first column in Table 3 lists the tests being compared. The second column gives $\Sigma \beta r$ for each interrelation. The third column shows the augmented R, computed as explained above on the basis of the R in Table 2 and the reliability coefficient. The next column shows the J-Coefficient, computed by dividing each $\Sigma \beta r$ by the product of the augmented R's for the tests being compared. For example, for the Spelling Test vs. the Reading Test, $J = \frac{.37}{.90 \times .93} = .44$

The last column shows the actual intercorrelation between the tests, as given in Table 1. As noted before, these intercorrelations had not been used in calculating β's or J's. Aside from the fact that correspondence between J-Coefficients for tests and actual intercorrelations of the tests is a check of the operation of the J-Coefficient formula, the comparison between J's and intercorrelation is an analytical device.

There is a close correspondence in relative order between both the unmodified $\Sigma \beta r$ or the J-Coefficients and the original intercorrelations. The modification by augmented R affected the ranks very little, but did affect means and standard deviations of the measures:

r between J-Coefficient and actual intercorrelations	.9696
r between $\Sigma \beta r$ and actual intercorrelations	.9743

Mean, actual intercorrelations	.495
Mean, J-Coefficients	.378
Mean, $\Sigma \beta r$	.304

Standard deviation, actual intercorrelations	.2014
Standard deviation, J-Coefficients	.1552
Standard deviation, $\Sigma \beta r$	.1256

Table 3 should be considered along with Tables 1 and 2. It can be seen in Table 3 that the elements listed in Table 2 practically determine the intercorrelation between the Filing Test and the Grammar Test (J-Coefficient .18, actual intercorrelation .17), and the Word Meaning Test (J-Coefficient .25, actual intercorrelation .27). On the other hand, there are other elements common to the Filing Test and the Spelling Test (J-Coefficient .18, actual intercorrelation .27), and the Filing Test and the Reading Test (J-Coefficient .19, actual intercorrelation .34). Similarly, the extent of common coverage by the given set of elements can be seen for other combinations of tests.

The advantage of this type of analysis is that the elements are elements that can be understood by people in general, since the Beta weights were determined by correlations with the elements as understood by people in general.

This kind of information is obviously useful in developing new tests, improving old tests, and looking for the actual meaningful elements that make up tests.

Part III - Use of J-Coefficient Analysis in the Job Element Procedure

The original development of the J-Coefficient procedure, released in 1955, required replications of validity studies, which made it impractical to develop test Betas for other than a specified set of elements. The use of self-ratings to obtain test Betas makes it possible to analyze tests in terms of the most appropriate elements for a particular set of jobs, and thus makes J-Coefficient analysis appropriate for all job-element examining.

With the job-element procedure, specific elements are chosen for particular jobs. Ability in the elements may be measured by all available evidences, one evidence being a written test. Other evidences include past accomplishments and training.

Exact Criterion in Terms of Job-Elements

The first question is the validity of a test for a given requirement in terms of a particular job-element pattern.

In Part I of the present report, it was assumed that the criterion is simply the sum of abilities in elements, weighted for a particular job. This is inexact, since the elements correlate.

For example, if a criterion should include ability in elements A, B, and C, with unit weights applied to the standardized rating in each element, and if elements A and B correlate .80 together, but 0 with element C, the correlation between element A and the criterion will be (letting a, b, and c be standardized ratings in elements A, B and C):

$$\frac{\Sigma a(a+b+c)}{N\sigma_A\sqrt{\sigma_A^2+\sigma_B^2+\sigma_C^2+2r_{AB}(\sigma_A\sigma_B)+2r_{AC}(\sigma_A\sigma_C)+2r_{BC}(\sigma_B\sigma_C)}}$$

Since the ratings are standardized, $\sigma_A = \sigma_B = \sigma_C = 1.$

Also, the problem states that r_{AB} is .80, while $r_{AC} = r_{BC} = 0$.

Making these substitutions and bringing N to the numerator, the above expression becomes:

$$\frac{\frac{\Sigma a^2}{N}+\frac{\Sigma ab}{N}+\frac{\Sigma ac}{N}}{\sqrt{3+1.60}}$$

Since ratings are standardized, $\frac{\Sigma a^2}{N} = \sigma_A^2 = 1.$

$$\frac{\Sigma ab}{N} = r_{AB} = .80.$$

$$\frac{\Sigma ac}{N} = r_{AC} = 0.$$

Making these substitutions, the above expression for the correlation between Element A and the criterion is $\frac{1.80}{\sqrt{4.60}} = .84$

On the other hand, the correlation between ability C and the criterion will be dependent on $\Sigma c\,(a + b + c) = \Sigma ac + \Sigma bc + \Sigma c^2$, where $\Sigma ac = \Sigma bc = 0.$ $\Sigma c^2 = N\sigma_c^2$ in which σ_c^2 is 1, so $\frac{\Sigma c^2}{N}$ will be 1.
The numerator of the correlation is 1; the denominator is still the standard deviation of the composite criterion.

Thus, ability C will correlate with the criterion only $\frac{1}{\sqrt{4.60}} = .47$

The relative importance of an element in a criterion may be measured by the square of its correlation with the criterion. This is because the square of the correlation is the proportion of variance in the criterion that is calculable by regression from the particular element.. If elements were independent, the squares of the correlations would add to unity, and be obvious proportional contributions to variance. When elements are not independent, the squares of the correlations still represent the relative extent of independent contribution of each element to the criterion.

Thus, although Elements A, B, and C were given equal unit weights in determining the criterion, Element A (and similarly Element B) has a relative importance of $.84^2 = 71$ in the criterion, while Element C has a relative importance of only $.47^2 = .22$ in the criterion, only 31% as much.

This example demonstrates that when elements are combined into a criterion, those elements which have the greatest general communalities with other elements obtain greater importance than the weight being applied.

For greatest accuracy, which is important in fair testing, each element should contribute to the total criterion in proportion to its importance as determined by careful analysis.

Example of an Exact Criterion in Terms of Job Elements

For example, suppose that a simple mail and file clerk job has been analyzed, and the consensus of informed opinion is that overall ability in the job is made up of the following pattern:

Element	Estimated Importance
Spelling Ability	3
Reading Ability	2
Ability to Use Grammar	0
Understanding Word Meaning	1
Filing Ability	10
Education	2

The importance values in the second column may be obtained as an average of judgments, using one of the job analysis procedures of the job-element method. By evaluating elements in terms of importance, judges usually mean that the elements should be represented in the criterion in the same relative proportions. They mean that spelling ability is three times as important as understanding of word meaning, etc. They do not usually mean that the criterion is determined by adding three parts of spelling ability to two parts of reading ability, etc. Statistically, this implies that the estimated importances express the desirable influence that each ability should have in a criterion; the proportion that each ability would contribute if all the abilities were independent

The ideal criterion would then be one in which the importance of each ability in criterion variance would be in proportion to its estimated importance. That is, if each ability is correlated with the criterion, the squares of the correlations would be in the proportions of the above estimated importances.

The question then becomes: What are the weights to be applied to standardized ratings in the elements which will result in each element having an influence on the criterion in proportion to the estimated importances?

Let such weights for an exact criterion be S for spelling ability, R for reading ability, G for ability to use grammar, W for understanding of word meaning, F for filing ability, and E for education.

If $\bar{s}$, $\bar{r}$, $\bar{g}$, etc. are standardized ratings in each element, the correlation between the element spelling ability and the criterion equals:

$$\frac{\Sigma \bar{s}(S\bar{s} + R\bar{r} + G\bar{g} + W\bar{w} + F\bar{f} + E\bar{e})}{N\sigma_s \sqrt{S^2+R^2+G^2+W^2+F^2+E^2+2(r_{SR}SR + r_{SG}SG + r_{SW}SW + r_{SF}SF + r_{SE}SE + r_{RG}RG + r_{RW}RW + r_{RF}RF + r_{RE}RE + r_{GW}GW + r_{GF}GF + r_{GE}GE + r_{WF}WF + r_{WE}WE + r_{FE}FE)}}$$

This becomes:

$$\frac{\frac{S\Sigma\bar{s}^2}{N} + \frac{R\Sigma\bar{r}\bar{s}}{N} + \frac{G\Sigma\bar{g}\bar{s}}{N} + \frac{W\Sigma\bar{w}\bar{s}}{N} + \frac{F\Sigma\bar{f}\bar{s}}{N} + \frac{E\Sigma\bar{e}\bar{s}}{N}}{\sigma_s\sqrt{S^2 + ----}}$$

Where $\frac{\Sigma\bar{s}^2}{N} = \sigma_s^2 = 1.$

$\frac{R\Sigma\bar{r}\bar{s}}{N} = Rr_{RS}.$

$\sigma_s = 1.$

So correlation between element and criterion becomes:

$$\frac{S + Rr_{RS} + Gr_{GS} + Wr_{WS} + Fr_{FS} + Er_{ES}}{\sqrt{S^2 + R^2 + ------}}$$

This is the correlation between spelling ability and the criterion. Similar expressions can be found for the correlation of each of the other elements with the criterion.

The identical denominator would appear in each correlation, since the denominator is N times the product of the standard deviation of the element, which is 1 since ratings are standardized, times the standard deviation of the criterion which is the portion under the square root sign.

Sets of simultaneous equations may then be developed, proportions of squared correlations being set equal to proportions of estimated importances.

For example, if r_{SC} is the correlation between spelling ability and the criterion, and r_{RC} is the correlation between reading ability and the criterion, then

$$\frac{r^2_{SC}}{r^2_{RC}} \text{ should be } \frac{3}{2}$$

in proportion to the importances of spelling ability and of reading ability on page 12, so that

$$\frac{r_{SC}}{R_{RC}} = \sqrt{\frac{3}{2}}$$

Similarly, if r_{WC} is the correlation between understanding word meaning and the criterion, then -

$$\frac{r^2_{RC}}{r^2_{WC}} = \frac{2}{1} \text{ and } \frac{r_{RC}}{r_{WC}} = \sqrt{2}$$

There are six unknowns - S, R, G, W, F, and E. Therefore, we need six linear independent equations. These may be set up in any convenient order, proportioning two correlations at a time, and equating to the square root of the proportions of estimated importance. For the various intercorrelations, the intercorrelations among ratings shown for the elements in Table 1 are used. (For example, r_{SR} is .61; r_{SG} is .67, etc.) In each proportion, the denominators of the correlations are ignored since denominators cancel when one correlation is divided into another.

Following are six equations which might be set up, following the generalized formula for a correlation between a single element and the criterion on page 13.

eq. 1, based on $\left[\frac{r_{SC}}{R_{RC}} = \sqrt{\frac{3}{2}} = 1.225\right]$

$$\frac{S + .61R + .67G + .61W + .41F + .41E}{R + .61S + .65G + .73W + .37F + .43E} = 1.225$$

eq. 2, based on $\left[\frac{r_{RC}}{R_{WC}} = \sqrt{2} = 1.415\right]$

$$\frac{R + .61S + .65G + .73W + .37F + .43E}{W + .61S + .73R + .76G + .42F + .49E} = 1.415$$

eq. 3, based on estimated importance of grammar to be 0 so that

$r_{GC} = 0$:

$$G + .67S + .65R + .76W + .53F + .51E = 0$$

eq. 4, based on $\left[\frac{r_{FC}}{r_{WC}} = \sqrt{10} = 3.163\right]$

$$\frac{F + .41S + .37R + .53G + .42W + .28E}{W + .61S + .73R + .76G + .42F + .49E} = 3.163$$

eq. 5, based on $\left[\frac{r_{EC}}{r_{FC}} = \frac{1}{\sqrt{5}} = .447:\right]$

$$\frac{E + .41S + .43R + .51G + .49W + .28F}{F + .41S + .37R + .53G + .42W + .28E} = .447$$

The last equation, equation 6, is defined by the expression which **represents** the variance of the criterion, in the denominator of the correlation between element and criterion (for example, between spelling and criterion on page 13). In order that **each numerator** actually be a correlation coefficient, it would be necessary that this variance be 1.

Thus, equation 6 is:

$$S^2 + R^2 + G^2 + W^2 + F^2 + E^2 + 2\,(.61SR + .67SG + .61\,SW + .41SF + .41SE + .65RG + .73RW + .37RF + .43RE + .76GW + .53GF + .51GE + .42\,WF + .49WE + .28FE) = 1$$

For hand operation, it is probably easiest to solve first for S using equation no. 1; substituting this in equation 2 and solving for R, etc. It is desirable at each step to check the partial answers in the original equations, substituting 1 for the letters still unknown.

Because the expression in equation 6 was set equal to 1, each numerator and each denominator in equations 1 to 5 represents a correlation of an element with the criterion. After the simultaneous equations are solved, the correlations are found to be:

Element	Correlation with Criterion
Spelling ability	.377
Reading ability	.308
Ability to use grammar	.000
Understanding word meaning	.217
Filing ability	.687
Education	.307

Finding the J-Coefficients for the Exact Criterion

Now, using the Beta weights for tests on the elements which have been given in Table 2, multiplying by the above correlations to get cross products, and summing the cross products provides the following J-Coefficients:

Test	J-Coefficient (Validity)
Spelling Test	.206
Reading Test	.217
Grammar Test	.185
Word Meaning Test	.252
Filing Test	.280

The highest J-Coefficient, as one might expect, is for the Filing Test.

Remember that the job analysis indicated that the element Filing ability was to count most. If we wanted only the test that has the highest validity for filing ability, regardless of other abilities, Table 1 shows that the test with the highest correlation for Filing ability would be the Word Meaning Test, with correlation of .41. The Filing Test had a correlation of only .31 with the Element Filing ability, less even than the Reading Test which was .33. However, because the Word Meaning Test also taps irrelevant abilities to an unnecessary extent, the above J-Coefficients show that it would correlate with a fair criterion of exactly proportioned content only .252, while the Filing Test has the highest J-Coefficient, .280 with this criterion. In order to get a high score on the Word Meaning Test, an applicant must be highly qualified on elements unrelated to the job. When a criterion is set up in which the elements are emphasized in terms of their actual importance on the job, the Word Meaning Test correlates with this criterion lower than it would with Filing ability alone.

Other Indications of Ability

The Job-Element Procedure permits all evidences to be used to evaluate applicants. In the present experiment, there was only a limited amount of non-test evidence, but for explanatory purposes this evidence will be explored.

In the present examination, applicants were asked to tell whether they had a typing or stenography job where filing was important. One might expect persons who claim high proficiency in filing, especially if they have had relevant experience, to be worthy of special consideration in evaluating the elements, even if they do not achieve a high score on the filing test. Such special consideration might be checking with previous supervisors where they had related employment, or use of performance tests where they did not. Applicants who demonstrate ability on the elements through non-test evidences may be considered qualified along with those who achieve high test scores.

The following table illustrates some of the features of the problem. It shows cumulative frequencies for test scores for five groups: Total competitors (204); competitors who claimed to be highly qualified in filing ability and who claimed filing experience (30); competitors who claimed to be highly qualified in filing ability but without experience (42); competitors who claimed to be experienced but not highly qualified in filing ability (34); and the remainder (98).

TABLE 4

Cumulative Frequencies on Filing Test
For People Claiming To Be:

Test Score	Total N = 204	Highly qual. in filing & experienced N = 30	Highly qual. alone--no experience N = 42	Experienced alone - no claim for high qual. N = 34	Remainder N = 98
30	2 = 1%	-	2 = 5%	-	-
28	4 = 2%	1 = 3%	3 = 7%	-	-
26	6 = 3%	2 = 7%	4 = 10%	-	-
25	8 = 4%	-	-	-	2 = 2%
24	9 = 4%	-	-	-	3 = 3%
23	10 = 5%	-	-	-	4 = 4%
22	15 = 7%	4 = 13%	5 = 12%	-	6 = 6%
20	16 = 8%	-	-	-	7 = 7%
19	26 = 13%	6 = 20%	8 = 19%	-	12 = 12%
18	41 = 20%	9 = 30%	11 = 26%	2 = 6%	19 = 19%
17	50 = 25%	12 = 40%	15 = 36%	3 = 9%	20 = 20%
16	60 = 29%	14 = 47%	18 = 43%	-	25 = 26%
15	77 = 38%	16 = 53%	22 = 52%	8 = 24%	31 = 32%
14	99 = 49%	19 = 63%	24 = 57%	13 = 38%	43 = 44%
13	118 = 58%	23 = 77%	26 = 62%	19 = 56%	50 = 51%
12	133 = 65%	25 = 83%	28 = 67%	21 = 62%	59 = 60%
11	143 = 70%	-	30 = 71%	22 = 65%	66 = 67%
10	148 = 73%	26 = 87%	32 = 76%	-	68 = 69%
0-9	204=100%	30 = 100%	42 = 100%	34 = 100%	98 = 100%

Note that applicants in columns 2 and 3 do have better test results than others, as may be expected.

The mean test score was 13.01; the standard deviation 5.72. If we expect to interview for employment all who make at least 2 σ above the mean (upper quantile of normal distribution), we would 3 include all with a test score of 17 or over.

In column 2 are 18 applicants with test scores below 17 but who claim filing experience and a high degree of filing ability. Those of this group whose claims are verified by checks with employers could be added to the 50 who received a test score of 17 or over.

If practicable, applicants in columns 2 and 3 with less than a score of 17 on the test, for whom an employer check is not feasible, might be given a performance test in ability to file. If a performance test is used, the resulting score could be scaled similarly to the Filing Test.

Part IV - Derivation of J-Coefficient in Terms of Elementary Algebra

The derivation of the J-Coefficient published in 1955 involved calculus and determinants. It is characteristic of scientific formulas originally derived by calculus that as their use becomes better understood, it becomes possible to develop derivations in terms of algebra. The equations in Part III suggest a development which requires no more than algebraic manipulation of summed products.

Let:

A_i, B_i, C_i -- be standardized ratings of applicant i on **Elements A', B', C'--**

a, b, c--- be tentative weights applied to the standardized scores A, B, C-- on Elements A', B', C'-- to determine a score in Test T.

X_i be a standardized score on a criterion for applicant i.

The correlation between test and criterion will be (where $\sigma_{d'}$ is the **S.D. of the** derived composite test score) - -

$$\text{J-Coefficient} = \frac{\Sigma(aA+bB+cC)X}{N\sigma_X\sigma_{d'}} = \frac{\frac{a\Sigma AX}{N}+\frac{b\Sigma BX}{N}+\frac{c\Sigma CX}{N}+\cdots}{\sigma_X\sigma_{d'}}$$

$$\doteq \frac{\frac{a\Sigma AX}{N}+\frac{b\Sigma BX}{N}+\frac{c\Sigma CX}{N}+\cdots}{\sigma_X\sigma_{d'}}$$

where $\frac{a\Sigma X}{N}$ **is the correlation between Element A' and the criterion, the other two terms in the numerator are similarly correlations between Elements B' and C' and the criterion, and** σ_X **is 1. Thus, the J-coefficient between Test T and criterion X is:**

$ar_{A'X}+br_{B'X}+cr_{C'X}$

A special element is assumed for the test -- an error element, having no correlation with any other element or with the criterion. As in general test theory, with a set of standarized test scores, the **correlation between test score and error element** is $\sqrt{1\text{-reliability}^2}$. This would not affect the numerator, because the product of the weight for error, times the correlation of error with criterion, is 0, since the correlation of error with criterion is 0. On the other hand, the error element does affect $\sigma_{d'}$, since the square of its weight adds into $\sigma_{d'}$. **(Since error is independent of other elements, Beta weight and r for error element on test are equal).**

Let each of the tentative test weights a, b, c -- be divided by $\sigma_{d'}$, so that the adjusted weights are $\frac{a}{\sigma_{d'}}$, $\frac{b}{\sigma_{d'}}$, $\frac{c}{\sigma_{d'}}$, etc.

Now, bringing $\sigma_{d'}$ into the numerator:

$$J_{TX} = \frac{a}{\sigma_{d'}} r_{A'X} + \frac{b}{\sigma_{d'}} r_{B'X} + \frac{c}{\sigma_{d'}} r_{C'X}$$

Thus, the J-Coefficient is the sum of the products of adjusted test weight in each element times the correlation of the element with the criterion.

Where the test weights completely determine the test scores, except for unreliability,

$\frac{a}{\sigma_{d'}}$, $\frac{b}{\sigma_{d'}}$, etc. will be test Beta weights on the elements,

with the situation that the augmented R as described on page 7 is unity.