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ABSTRACT

A group of 47 kindergarten and first grade children from a school chiefly composed of Negro children from low-income families was involved in this study which followed a Solomon four group design. All children received a pretreatment of 17 lessons on matching relations and length relations. The pretest group was then given six tests on relations, conservation, and transitivity. The treatment group received nine lessons on conservation of matching relations and the transitive properties; the control group followed normal classroom activities except for two additional lessons on matching relations. Twelve posttests were then administered, including the six used as pretests in a slightly different form. The scores were analyzed by ANOVA and MANOVA using the factors of Treatment, Grade and Pretesting. On MANOVA, only the Grade factor approached significance. On ANOVA, the Grade factor was significant on two tests of matching and conservation, and the Treatment factor was significant on one of the transitivity tests. No pretest effect was found. Analysis of contingency tables failed to show any consistent pattern relating children's performance on the conservation and transitivity tests. (MM)

THE EFFECTS OF SELECTED EXPERIENCES ON THE ABILITY OF DISADVANTAGED KINDERGARTEN AND FIRST GRADE CHILDREN TO USE PROPERTIES OF EQUIVALENCE AND ORDER RELATIONS*

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The purposes of this study were fourfold: (a) to determine the effectiveness of a set of activities designed to teach conservation and the transitive property of the matching relations "as many as," "more than," and "fewer than," to a group of economically disadvantaged five- and six-year-old children; (b) to determine the effect of the learning activities on the ability of the children to use properties of matching relations other than the specific properties upon which instruction was given; (c) to determine the effect of the learning activities on the ability of the children to conserve and use relational properties of the length relations "as long as," "longer than," and "shorter than;" (d) to determine relationships among conservation, and relational properties of the matching relations and the length relations.

Operations and Structures

An operation, a concept central to Piaget's (1970, pp. 21-23) developmental theory, has four properties. First, an operation is an action which can be carried out in thought as well as executed physically. The second characteristic of an operation is that it is reversible; the action can be carried out in one direction and in the opposite direction. Thirdly, an operation always [illegible] some conservation (invariant). The fourth property is that every operation is related to a system of operations called a structure.

Piaget (Beth & Piaget, 1966, pp. 172-178) believes that all the mental structures of the stage of concrete operations (from age 7 to age 12, approximately) may be reduced to a single model which he has termed a "grouping" structure. Piaget has suggested eight groupings, four dealing with operations of classes and four dealing with operations of relations. In Grouping I: Primary Addition of Classes, grouping elements are operations of classes ordered in a chain of inclusions. Combination of grouping elements is interpreted as the union of classes, and reversibility is by inversion which may be interpreted as

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taking complements relative to a class supraordinate in the chain. Elements of Grouping V: Addition of Asymmetrical Relations are operations of order relations, which are sometimes considered in a series, $0 < A < B < C$, etc. Combinations of elements, "$(A < B) + (B < C) = (A < C)$ [p. 177.]," exhibit the transitive property.

Reversibility in Grouping V is by reciprocity, of which there are three forms (Beth & Piaget, 1966, pp. 176-178). Consider $A < B$, where "<" is an order relation. Reciprocity consists of permuting the terms $(B < A)$, reversing the relation $(A > B)$, or both $(B > A)$. In this study, reversibility was considered in the third sense which is the logical equivalent of $A < B$.

Development of Number and Measurement Concepts

From Piaget's (1970, pp. 37-38) analysis of children's mental processes, it was concluded that the development of the concept of number is a synthesis of operations of class inclusion and operations of order. So long as the elements of a class have their qualities, Grouping I and Grouping V cannot be applied to the same elements simultaneously, but the basis of the notion of number is that the elements are stripped of their qualities, such that each element becomes a unit. As soon as the qualities of the elements are abstracted, Grouping I and Grouping V can no longer function separately but must necessarily merge into a single new structure (Beth & Piaget, 1966, pp. 259-67). "Class inclusion is involved in the sense that two is included in three, three is included in four, etc. [Piaget, 1970, p. 38]." Since the elements are considered to be equivalent the only way to tell the elements apart is to introduce some order. The elements are arranged one after another spatially, temporally or in the counting sequence.

Van Engen (1971, pp. 37-40) disagrees with Piaget's notion of number. "The difficulty with this conception of number is that it does not distinguish between the elements of a set and the relation that exists between two or more elements of the set. [p. 40]." Van Engen suggests that, from a mathematical point of view, the cardinal numbers can be defined by sets of a particular kind. For example, $5 = \{0, 1, 2, 3, 4\}$. To determine the cardinality of a set S, it is only necessary to find one of the standard sets to which S is equivalent.

Van Engen (1971, pp. 35-37) suggests that the matching relations, "as many as," "more than," and "fewer than" are the basis of the development of number in children. These relations may be operationally defined between two finite sets,

A and B, of objects as follows: place an a beside a b until all the a's or b's are exhausted. If both sets are exhausted simultaneously, then there are as many a's as b's. If set B is exhausted and set A is not exhausted, there are more a's than b's and fewer b's than a's.

The relation "as many as" is thus another way of expressing set equivalence and is an equivalence relation. Suppose "there are as many a's as b's" is indicated by $A \sim B$ for equivalent sets A and B. The following properties of "as many as" are easily verified: (a) reflexive, $A \approx A$; (b) symmetric, $A \approx B$ implies $B \approx A$; and (c) transitive, $A \approx B$ and $B \approx C$ imply $A \approx C$. The relations "more than" and "fewer than" are order relations. Suppose $A > B$ indicates "there are more (or fewer) a's than b's." Then the following properties obtain: (a) non-reflexive, $A \not> A$; (b) asymmetric, if $A > B$ then $B \not> A$; (c) transitive, $A > B$ and $B > C$ imply $A > C$. The relations "more than" and "fewer than" are examples of asymmetrical transitive relations of which Piaget wrote. They also exhibit the reversibility property: if there are more a's than b's, then there are fewer b's than a's, and conversely. Thus, it appears that from the mathematical point of view of Van Engen and from the psychological perspective of Piaget, the matching relations are involved in the devolopment of the number concept.

Measurement has been defined as "a process whereby a number is assigned to some object [Steffe, 1971, p. 335]." From this definition it follows logically, that number is a prerequisite of length, a measurement concept. Sinclair (1971) has stated that the ". . . first measurement concept (length) is achieved rather later than that of number; [p. 153]." Sinclair presented empirical evidence to confirm the hypothesis that number conservation precedes length in development. Sinclair maintains that "although the psychological construction is parallel, dealing with continuous elements is very much more difficult than dealing with discontinuous units [p. 153]."

Relations also provide a basis for the development of measurement in elementary school children. For a definition of the length relations, "as long as," "longer than," and "shorter than," consider two segments A and B. A is as long as B, if whenever (transformations of) A and B lie on a line such that two end points (right or left) coincide, the remaining two end points coincide. A is longer than B and B is shorter than A, if and only if the remaining end point of B coincides with a point between the end points of A. The relation "as long as" is an equivalence relation, and it has the reflexive, symmetric, and transitive properties. The relations "longer than" and "shorter

than" are order relations and the non-reflexive, asymmetric and transitive properties obtain. While these relations are defined on segments, children use physical representations such as sticks.

Smedslund (1963b) has argued that from a logical point of view, conservation precedes transitivity in the child's development. Consider three quantities which are related by a transitive relation @. Assume that a child established A @ B. B (or A) must undergo some transformation, T, before B is compared with C; otherwise, A and C can be compared perceptually. Hence B = T(B) must hold from one comparison to the other. While the emphasis of the study was on relational properties, conservation was considered a requisite concept.

In Piaget's (1952) classical conservation of number tasks, a child is asked to establish that there are as many objects in a set A as in set B. Then one of the collections, say A, is taken through a physical transformation. Then the child is asked, "Are there as many a's as b's, or does one have more?" Van Engen (1971, p. 43) has argued that this task may be measuring whether or not the child conserves the one-to-one correspondence rather than conservation of number. In this study a task similar to the above example is considered to be a measure of conservation of the relation "as many as." It is not necessary that conservation be limited to cases of equivalence. Order relation conservation was included in the present study. Also included is the analogous conservation of the three length relations.

Method

The subjects of the study were 23 kindergarten and 24 first grade children from an elementary school in Atlanta, Georgia. Kindergarten children were randomly selected from 35 children of two classes whose ages were in the range (5;1)* to (5;10) at the outset of the study. Grade one children were randomly chosen from 48 children of three classes with ages between (6;1) and (6;10), inclusively. The school was chiefly composed of Negro children from low income families. With one exception, the children in the sample were Negro.

Tests

Thirteen tests, described below, were constructed to measure the abilities of the children to establish relations, conserve relations, and use relational properties.

*5 years, 1 month

The Matching Relations (MR) Test was designed to measure the ability of a child to establish matching relations. The Conservation of Matching Relations (CMR) Test was designed to measure the ability of a child to conserve a matching relation, provided that he can establish the relation. These two tests were administered simultaneously. In the example presented in Figure 1, a child was given five blue discs glued on a piece of cardboard and six red discs (i). He was instructed to pair the red discs and the blue discs. After the pairing (Figure 1--ii), the examiner asked two questions, "Are there as many red discs as blue discs?" and "Are there more red discs than blue dsics?" After the second response the examiner rearranged the red discs (Figure 1--iii) and repeated the same two questions. In each case the correct answer to one question was "yes" and to the other was "no," and in each item, the rearrangement was perceptually biased in favor of the incorrect conclusion. The first two questions comprised an item of the MR Test. All four questions were considered in the CMR Test.

Insert Figure 1 about here

The purpose of the Transitivity of Matching Relations (TMR) Test was to measure a child's ability to use the transitive property of matching relations. On a TMR item a child was presented three collections A, B, C, of physical materials, arranged in clusters. Suppose, for example, that there were fewer a's than b's and fewer b's than c's. The child was instructed to pair the a's and b's and was then asked, "Are there fewer a's than b's?" The examiner then put the a's into a cup which sat nearby and said, "Pair the b's and c's." After the pairing the examiner asked, "Are there fewer b's than c's?" The examiner then placed the c's in another cup and asked, "Are there fewer a's than c's?" and "Are there more a's than c's?" (or "Are there as many a's as c's?") Note that the sets A and C were not "paired" and that the objects were screened at the time of the transitive inference.

The purpose of the Symmetric Property of the Matching Relations (SMR) Test was to determine the child's ability to use the symmetric property of the relation "as many as." For an item of SMR test the child was presented two collections A and B of objects and instructed to pair the objects. After the pairing the examiner asked two questions: "Are there as many a's as b's?" (Response.) "Are there more (or fewer) a's than b's?" (Response.) Then the examiner put the two collections into two cups and asked, "Are there as many

b's as a's?" (response), and "Are there more (or fewer, as before) a's than b's?"

The Test of the Asymmetric Property of the Matching Relations (AMR) was designed to measure the ability of a child to use the asymmetric property of the relations "more than," and "fewer than." The child was presented two collections, for example, with more a's than b's and instructed to pair them. After the pairing the examiner asked, "Are there more a's than b's?" After the response the examiner placed the two collections into two cups and asked, "Are there more b's than a's?" and "Are there more a's than b's?"

The Reversibility of Matching Relations (RMR) Test was designed to measure the child's ability to use the following property: if there are more (fewer) a's than b's, then there are fewer (more) b's than a's. On a given item, the child was presented with two collections A and B of objects such that an order relation held. After the child had paired the objects, the examiner asked, "Are there more (fewer) a's than b's?" The examiner then put the objects into two cups and asked, "Are there fewer (more) b's than a's?" and "Are there as many b's as a's?"

Tests, corresponding to the above, were constructed to measure the abilities of the children to use length relations, conservation and properties. These were Length Relations (LR), Conservation of Length Relations (CLR), and Transitivity of Length Relations (TLR), Symmetric Property of the Length Relation (SLR), Asymmetric Property of Length Relations (ALR), Reversibility of Length Relations (RLR) tests, respectively. In the respective cases the tests were analogous to the corresponding tests of matching relations. A description may be obtained by replacing [illegible] A and B by "sticks" A and B, and the relations "as many as," "more than," and "fewer than," by "as long as," "longer than," and "shorter than," respectively. In each case the relations tests (MR, LR) involved situations under which the stimuli were arranged to aid the child in establishing the relation. The questions of the conservation tests (CMR, CLR) were administered under conditions of perceptual conflict. All other items were administered under screened stimuli conditions. The children were not asked to give reasons for the answers on any of these structured tests.

There were a total of 58 items. On each of the MR, LR, CMR, CLR, TMR, and TLR tests there were two items which exhibited each relation. Thus, there were six items on each of these tests. The SMR and SLR tests contained three items each. The AMR, ALR, RMR, and RLR tests had two items for each of two order relations, or four items per test.

The Transitivity Problem was designed to measure the ability of a child to solve a problem which involved transitivity of a matching relation with minimum guidance from the examiner. The situation involved a cardboard box from which the front and top were removed. The box was divided into halves by a partition as shown in Figure 2. Ten checkers were attached to the bottom inside one half of the box and ten tiles were attached in the other side. Twelve buttons lay on the table in front of the box. After the objects were identified, the examiner said, "Find out if there are as many checkers as tile. You may use the buttons to help you find out." In general the examiner gave as little guidance as was possible, but if the child failed to respond at some point, the examiner directed the next step toward solution. When a response was given, the examiner asked for an explanation.

Insert Figure 2 about here

Scoring Tests

An item was scored "pass" provided that a child answered correctly all the questions contained in the item and "fail" otherwise. The number of items scored "pass" by a child on each test was considered to be his score on the test. For the purpose of comparing the results of the structured tests and the Transitivity Problem, it was desirable to distinguish children who can use a property from those who cannot use the property. This was accomplished by setting a criterion score on the CMR and TMR tests at four of the six items. The probability of reaching this criterion by guessing was less than .038.

For the Transitivity Problem, the following four levels of ability to apply the transitive property were identified: 1--the child neither consistently established relations nor used the transitive property; 2--the child established relations but did not use the transitive property; 3--the child both established relations and used the transitive property; 4--the child established relations, used transitivity, and gave adequate justification for his conclusion. The consensus of two of three judges' ratings, based on transcripts of audio tapes, was taken as the child's rating on the Transitivity Problem.

Instructional Activities

All of the instructional activities were designed for use in small instructional groups and involved manipulative materials. In some activities

each child had his own set of materials, and other activities involved one set of materials for the entire group. In the latter case, the instructor or one child performed the manipulations, and all of the children entered into discussion. Materials for instruction varied from desirable materials, such as small toys, to neutral materials such as checkers, tiles, and colored wooden discs. Colored sticks, straws, etc., represented segments for length comparison.

The purpose of Unit I, Matching Relations, was to develop the ability of the children to establish matching relations. The relations were introduced by having the children pair the objects from two finite sets. It was noted that the sets might or might not have been in one-to-one correspondence. When the sets A and B were equivalent, the situation was labeled, "there are as many a's as b's." "More than" was introduced second, and "fewer than" was introduced as the reverse of "more than." It was emphasized that if a relation held between two sets (in a fixed order), then no other relation held.

Unit II, Length Relations, was designed to develop the ability of children to establish length relations. The relations were introduced by placing the ends of two sticks together, observing the remaining ends, and associating the name of the appropriate relation. After "longer than" was discussed, "shorter than" was introduced as the reverse. The equivalence relation "as long as" was the third length relation considered.

The purpose of Unit III, Conservation of Matching Relations, was to develop the ability of children to maintain matching relations between sets when the physical matching of the objects was destroyed. The principle of reversibility of a transformation was emphasized by having the children return the objects, following a transformation, to the position in which the relation was established. Combinations of perceptual screening, perceptual conflict, child transformations of his own materials, and instructor transformations in a group situation were used in Unit III and Unit IV, Transitivity of Matching Relations. Unit IV was designed to develop the ability of children to use the transitive property of the matching relations. The chief method of the transitivity training was what has been termed "fixed practice" with "empirical control" (Smedslund, 1963c). The instructor gave explicit instructions for comparing sets A and B, then B and C. Sets A and C were compared after the child had made a prediction of the relation between them.

Design

The main purpose of the study, to determine the effect of the instruction,

implied that Treatment was one of the factors under consideration. Since age is obviously related to cognitive development, Grade was a second major factor. Furthermore, in most of the learning research based on cognitive development theory, a pretest is given to all subjects. Then important questions of the facilitating effect of the pretest and of an interaction of pretest and treatment, can be raised. The Solomon Four Group Design is the appropriate design to determine those effects (Campbell & Stanley, 1963), and was used in the study. One-half of the treatment group and one-half of the control group at each grade level were chosen at random to take the pretest.

Procedure

Children in experimental and control groups had experience in establishing relations. Ten lessons on matching relations from Unit I and seven lessons from Unit II on length relations were given. Then the tests on relations (MR, LR), conservation (CMR, CLR), and transitivity (TMR, TLR) of each relational category were administered as pretests to the pretest group while the no-pretest group had only the relations tests. Following the pretests the differential treatment began. The treatment group had four lessons on conservation of matching relations (Unit III) and five lessons on the transitive property (Unit IV). Near the end of this instructional period the control group had two additional lessons on matching relations, but the remainder of the treatment period was spent in normal classroom activities.

Each lesson was of 20-30 minutes duration. There were four to six children in an instructional group. The investigator and two teachers' aides served as instructors and testers. Instructional groups were rotated among instructors each day. During testing experimental and control groups were balanced among testers in five untimed interviews per child. However, the (sixth) Transitivity Problem session was held entirely by the investigator. The test items, given during a test session, were randomly ordered for each child, independently of other children, and each pair of test questions of an item were randomly ordered for each item and each child.

Near the end of the study it was apparent that the treatment had not extensively changed the language patterns of the children with regard to relational terminology. The investigator felt that strict adherence to predetermined terminology could make the tests invalid in terms of the concepts measured. Approximations to desired terminology, for example, "the same" for "as many as," were accepted in the posttests. Further, if a child were giving a "no--no" or a "yes--yes" response set to an item, the question

was repeated using an alternate terminology. This was the only way in which the posttests differed from those tests which were given as pretests, but this was considered to be sufficient to make the pretest data invalid. Thus, Pretest was retained as a factor but the data were disregarded.

Statistical Analyses

Data from each individual consisted of a vector of 12 posttest scores and a rating on the Transitivity Problem. The score on a test was the number of items passed by a child on that test. A multivariate analysis of variance (MANOVA) and 12 univariate analyses of variance (ANOVA's) with the factors Treatment and Grade and the two-way interaction were performed on the 12 test scores. Also a Treatment by Pretest MANOVA and 12 associated ANOVA's were conducted. Each factor contained two levels: Treatment--treatment and control groups; Grade--kindergarten and first grade; and Pretest--pretest group and no-pretest group. The 12 variables were Matching Relations (MR), Conservation of Matching Relations (CMR), Transitivity of Matching Relations (TMR), Symmetric Property of the Matching Relation (SMR), Asymmetric Property of Matching Relations (AMR), Reversibility of Matching Relations (RMR), Length Relations (LR), Conservation of Length Relations (CLR), Transitivity of Length Relations (TLR), Symmetric Property of the Length Relation (SLR), Asymmetric Property of the Length Relations (ALR), and Reversibility of Length Relations (RLR). Table 1 contains a diagram of the statistical design for the 2 X 2 multivariate and univariate analyses.

Insert Table 1 about here

Calculations for all of the MANOVA's and ANOVA's were performed by computer with the use of the computer program MUDAID (Applebaum and Bargmann, 1967). MUDAID provides multivariate and univariate analyses of variance for pairs of factors and pairwise interactions. Also, each multivariate pass provides matrices of intercorrelations among the response variables.

Each covariance matrix in a multivariate analysis contains estimates of the variances of the variables on the main diagonal and estimates of the covariances for pairs of variables in the off-diagonal positions. Each covariance matrix has an associated matrix of sums of squares and cross products. The sums of squares of error and sums of products of error are the residuals after the effects of the factors and interactions have been removed by subtraction of their sums of squares and sums of products from the respective

totals. The correlations reported in this study were calculated from the covariance matrix derived from the matrix of sums of squares and products of error in the Treatment X Grade analysis.

Chi-square tests for independence (Ferguson, 1966, pp. 192-208) were used to determine whether a relationship existed between levels of performance on the Transitivity Problem and grade levels or treatment groups. Chi-square tests for relationship were made between levels on the Transitivity Problem and criterion on CMR or TMR and between conservation and transitivity within a relational category. Chi-squares were calculated on the 2 X 2 and 2 X 3 tables of frequencies of criterion levels by levels of performance.

Results

Multivariate Analyses

None of the F ratios for any factor or two-way interaction were significant at the .05 level of significance in the multivariate tests. However, the F statistic for the main effect of Grade was 1.95 in the Treatment versus Grade multivariate analysis with 12 and 32 df. The critical value ($p < .05$) of F with 12 and 32 df is 2.07. Thus, the factor Grade approached significance, but no interpretation was made.

Univariate Analyses

Analyses of Variance for which F ratios were significant in the Treatment versus Grade analyses are reported in Table 2. Table 3 contains analyses of variance for the cases of significance in the Treatment versus Pretest analysis. Group means, as percentages, for significant main effects in the absence of interaction are presented in Table 4. Grade was the only significant ($p < .01$) effect for the variables matching relations, MR, and conservation, CMR. In the first case, the first grade group mean was 87% and the kindergarten group mean was 59%. On conservation the first grade group performed at a mean of 62% and the kindergarten group mean was 36%. It was not anticipated that treatment would be significant for MR since all children had received instruction in matching relations.

Insert Tables 2, 3 and 4 about here

Treatment was a significant ($p < .01$) main effect for the transitivity variable, TMR. The Treatment group had a mean of 58% and the mean performance of the control group was 36%. Treatment was also a significant ($p < .05$)

factor for the variable AMR, in the Treatment versus Pretest analysis, and was close to significance at the .05 level in the Treatment versus Grade analysis. In this case the means were 73% and 52% for the treatment and control groups, respectively. Grade was also a significant ($p < .05$) main effect for AMR as it was for SMR ($p < .01$) and RMR ($p < .05$). In each of these cases the first grade group performed at a higher level than the kindergarten group.

The F statistic for the factor Grade and the variable LR, length relations, was significant. However, in Bartlett's test (Ostle, 1963, pp. 136-37) the hypothesis of homogeneity of variances was rejected. Thus, no interpretation of the ANOVA can be made.

There were no significant interactions in the Treatment versus Grade Analyses. There were, however, two significant Pretest X Treatment interactions ($p < .05$) for the variables SLR and RLR. Pretest was not a significant main effect in the absence of interaction in any analysis. The cell means for the significant interactions are presented in Table 5. In each case the greatest mean was that of the treatment group which had no pretest, and the least mean was that of the treatment group which had pretests. One possible interpretation of this interaction is that the pretests interfered with the effect of the treatment. However, this may be a misinterpretation since instruction was not given on the symmetric and reversibility properties of either category of relations, nor was there any indication of transfer to the properties of length relations from the instruction which was given. The interpretation which is accepted here is that the pretests had essentially no effect on the subjects' performances on the posttests.

Insert Table 5 about here

Grade was the most general effect in the study, but Grade was not significant for any length relational variable. Means, as percentages, are presented by grade levels and totals for the length relational variables in Table 6 for the purpose of comparison with means for matching relational variables. A grand mean of 87% for the variable Length Relations was equal to the mean for the first grade group on MR which was significantly greater than the mean for the kindergarten group on MR. The grand mean of 57% for CLR was between the means of 62% for the first grade and 36% for the kindergarten group on CMR. For TLR the mean of 49% was between the different

means on TMR for the treatment and control groups. The means of 60%, 61%, and 67% for SLR, ALR, and RLR, respectively, were between the respective matching relational means which were different for the two grades because of a grade effect in each case. Also, in each case the mean for the kindergarten group was higher for length than for the corresponding matching relations variable. Thus, while no factors were significant for the length relational variables, overall performance in each case was not decidedly different from performance on the corresponding matching relations variable. No formal statistical tests were made between variables across relational category.

Insert Table 6 about here

Transitivity Problem Results

In order to test the relationship between performance on the Transitivity Problem and the factors, Treatment and Grade, chi-square tests for independence, were performed on contingency tables. The frequencies of Transitivity Problem ratings for the treatment groups are presented in Table 7, and ratings by Grade frequencies are found in Table 8. Two children used counting and no ratings were possible. While it is of interest to see the number of children at each of the four levels on the Transitivity Problem, categories 3 and 4 were combined into a single category, 3 or 4--the child used transitivity, for the chi-square tests. This was necessary to increase the expected frequency for some cells. Frequencies are presented both ways but the chi-square tests were performed on the 2 X 3 tables.

Insert Tables 7 and 8 about here

The chi-square calculated for Table 7 was 3.62 with 2 df which is not significant at the .05 level. Thus, while there appeared to be a tendency for more treatment group children to get a rating of 3 or 4 and more control group children to get a rating of 1 or a rating of 2, the hypothesis of independence was not rejected. The chi-square calculated for Table 8 was 8.97 with 2 df which is significant at the .02 level. Thus, the null hypothesis of independence was rejected, and the existence of a relationship between grade and the level of performance on the Transitivity Problem was accepted. There was a tendency for first grade children to have the higher rating of 3 or 4, and for the kindergarten children to have the lower ratings of 1 and 2.

While the treatment was effective in improving the abilities of the children to perform the transitivity tasks of TMR, the treatment was not related to level of performance on the Transitivity Problem. On the other hand, there was no significant difference between grades in performance on TMR, but grade level was related to level of performance on the Transitivity Problem. These results raise a question about the relationship between performance on the Transitivity Problem and the more structured tests.

Relationships Among the Variables

Chi-square tests were used to test for a relationship between level of performance on the Transitivity Problem and criterion performance on TMR and CMR. The frequencies of the ratings on the Transitivity Problem versus meeting the criterion on TMR and CMR are presented in Table 9 and Table 10, respectively. Four levels of ratings are shown in the Tables, but the chi-square tests were made on the 2 X 3 tables.

Insert Tables 9 and 10 about here

The chi-square calculated for Table 9 was 5.45. The critical value of chi-square with 2 df is 5.99 ($p < .05$). Thus, the chi-square for level of performance on the Transitivity Problem versus transitivity as measured by TMR test was near significance at the .05 level, but independence was accepted. The chi-square calculated for Table 10 was 22.43 ($p < .001$). There was a strong relationship between ratings on the Transitivity Problem and achieving the criterion on the CMR test.

The product-moment correlations in the present study were calculated by using the error covariance matrix from the Treatment versus Grade analysis. The reason for using this error matrix to calculate the correlations is that the effects of Treatment and Grade were statistically removed by subtraction, and only the (nonsignificant) effects of Pretest remain. The correlations are presented in Table 11. Since df for error in the analyses of variance are 43, there are 42 df associated with each correlation of Table 11. The critical values for correlations significantly different from zero are .30 ($p < .05$) and .39 ($p < .01$).

Insert Table 11 about here

Inspection of Table 11 revealed that 47 of the 66 correlations were significantly different from zero and all were positive. Only two correlations were greater than .60 and 16 others were greater than .50. Of the 19 nonsignificant correlations, 12 were with or between LR and TLR. It was interesting that the only length variable with which LR was correlated was CLR. Indeed, each item of the CLR Test was dependent upon an item of the LR Test. It appears that there is little relationship between each of LR and TLR and the remaining variables. In addition to TLR, three variables are not correlated with CLR. The nonsignificant correlations of SMR with RMR and RLR indicates a lack of relationships between the symmetric property of "as many as" and the reversibility property of either relational category. The additional nonsignificant correlation was between TMR and AMR. The remaining correlations with each matching relational variable were significant. It is interesting to note that CMR was correlated with each variable across both relational categories.

Whether or not a child in the present study attained the criterion on a particular test is a measure of the child's ability to use the relational property of the test. In order to examine the hypothesis that conservation ability precedes the ability to use the transitive property within a category of relations, 2 X 2 frequency tables, of those who met and who did not meet the criterion on conservation and transitivity, were prepared. The frequencies of children meeting criterion on CMR versus those meeting criterion on TMR, are presented in Table 12. Table 13 contains the frequencies of children in the sample who met the criterion on conservation versus those who met the criterion on transitivity of length relations. The nonsignificant chi-squares indicate independence between the ability to use conservation and the ability to use transitivity within the respective relational categories. These results are not consistent with the significant product-moment correlation of .51 between CMR and TMR for the matching relations. However, in the case of length relations, the result is consistent with the nonsignificant correlation between conservation and transitivity.

Insert Tables 12 and 13 about here

Examination of Tables 12 and 13 revealed that about one-half of the children who could use the transitive property within a relational category failed to conserve the relations of the same category. Thus, no evidence is

provided by these data that, for the children in this study, the ability to conserve relations preceded the ability to use the transitive property.

Presumably, a solution of the Transitivity Problem required use of the transitive property of the relation "as many as." However, other abilities were necessary for a solution. Thus, the fact that some children achieved the criterion on TMR but did not reach a solution in the Transitivity Problem is consistent with the logical conclusion. What appears inconsistent with the logical conclusion is that seven children solved the Transitivity Problem but failed to reach the criterion on the transitivity (TMR) test. Of these seven, however, four made a score of three on the TMR test and thus gave evidence of some facility in transitivity. The failure of the other three children may be attributed to inaccuracy of measurement in the TMR test.

Another discrepancy between the data and the logical conclusion is the fact that 8 children used the transitive property (as defined by the criterion on TMR), but did not conserve matching relations. It is interesting to note that 5 of these 8 children were in the treatment group. It is also of interest to observe that in the entire study, 13 children who had the treatment achieved the criterion on TMR while only 4 control group children did so.

Discussion and Conclusion

The Effectiveness of the Treatment

The mean performance of the treatment group was significantly greater than the mean performance of the control group on the Transitivity of Matching Relations Test. This was an indication that the treatment was effective in improving the ability of the children in the treatment group in using the transitive property of these relations. However, the results from the Transitivity Problem indicated no relationship between a student's membership in a treatment group and his level of performance on the Transitivity Problem. This apparent discrepancy may be interpreted by an examination of the tasks and the instructional activities. In the instructional setting the children were instructed to compare two sets, say A and B, and B and a third set, C. The sets were constructed in such a way that the same relation existed between B and C as between A and B. The children were then asked to predict the relation between A and C and were given an opportunity to verify their prediction. Each item of the structured transitivity test followed this same procedure except that on the test the child did not have the opportunity to verify his conclusion. Also, in the testing situation the objects were

screened at the time of the transitive inference, whereas this was not always the case in instruction. In the Transitivity Problem, the child was required to compare sets A and B and sets A and C where A contained two more objects than B or C. He then was required to remove (either physically or mentally) two objects from the set A, to form a new set which was equivalent to B and C, before applying the transitive property of "as many as," to conclude that B was equivalent to C. The reasonable conclusion then, is that the treatment improved the ability of the children to perform tasks very much like the treatment activities, but this improvement did not generalize to the Transitivity Problem, a higher order task.

These results were consistent with previous transitivity training studies. In a study with five- to seven-year-old children, Smedslund (1963a) found that none of the children acquired transitivity of weight due to practice. In another study he (Smedslund, 1963c) found that about 30% of a group of eight-year-old children acquired transitivity of weight by practice, while only 12.5% of a control group acquired transitivity. Thus, behavior indicative of transitivity has been obtained in some training studies, but it appears to be difficult to induce transitivity by practice.

It appears from Piaget's theory that if a child's cognitive structure contains the grouping of addition of asymmetrical transitive relations, he can use the transitive property of any such relations, regardless of the concrete embodiment. Piaget (1952, p. 204) has indicated, on the contrary, that a formal structure of transitivity is not acquired all at once, but it must be reacquired every time a new embodiment is encountered. Sinclair (1971) has further suggested that such properties of the concrete embodiments as discrete or continuous will effect the attainment of psychologically parallel concepts.

In the present study, experiences in length relations were given to introduce an embodiment of transitive relations in addition to the matching relations, but no instruction was given in transitivity of the length relations. The results indicate that while the treatment improved the ability to use transitivity of the matching relations, there was no corresponding improvement in the ability of the children to use transitivity of length relations. Thus, the conclusion was reached that the treatment was rather task specific and no generalized scheme of transitivity was induced.

This conclusion is consistent with Piaget's conjecture, and with the results of training studies in conservation. For example, Beilin's (1965) subjects improved in conservation of number and length when experiences were

given. However, the training was not sufficient to foster generalization to conservation of area.

The results of the Asymmetric Property of the Matching Relations Test indicate that the treatment was effective in improving the ability of the children in the treatment group in using the asymmetric property of the matching (order) relations. This may be interpreted, not as a transfer of training, but as a direct consequence of the instructional activities. In each activity, the instructors stressed the relations which did not hold as well as the relation which did hold. Consider, for example, an activity in the differential treatment in which there were more a's than b's. After the transitive inference or conservation question, "Are there more a's than b's?" the instructor also asked "Are there as many a's as b's?" and "are there fewer a's than b's?" If a child failed to answer "no" to each of these latter two questions, the instructor corrected the child by using the materials. The statement that there are not fewer a's than b's is equivalent to the statement that there are not more b's than a's. This logical equivalent that there are not more b's than a's is precisely the asymmetrical inference from the relation which does hold: there are more a's than b's. This situation may have been interpreted in this way by the children, so that the treatment effect was obtained for the asymmetric property.

The differential treatment contained four lessons on conservation of matching relations and five lessons on transitivity of matching relations. The conservation portion of the treatment was not successful in improving the conservation ability of the children in the treatment group. Many of the conservation training studies previously reported have indicated that conservation ability was improved (see Beilin, 1971). The conservation treatment in the present study was apparently either too short, or the activities were inappropriate for the subjects of the study. Another possible factor was that the transitivity instruction intervened between the conservation instruction and the testing period. This delayed the testing on conservation for one more week after instruction than the testing on transitivity. There remains the possibility that the conservation lessons were instrumental in fostering the improvement of performance of the treatment group in the transitive and asymmetric properties.

Matching and Length Relational Properties

The mean performance of the first grade group was higher than the mean performance of the kindergarten group on all matching relations tests except

transitivity. It is not surprising that these cognitive abilities improved between the ages of five and six. The amazing result is that age had no significant effect on the abilities of the children in using any of the length relational properties. Consideration of the means indicated that performance on length relational properties was at about the same level as performance on matching relational properties. Thus, from the point of view of relations rather than number and length, Sinclair's (1971) hypothesis is not confirmed for the children in this study.

Conservation and Transitivity Attainment

The result that about one-half of the children who used the transitive property in each relational category failed to use conservation of that respective category is at variance with results of previous studies. Smedslund (1963b) found only 4 of 160 subjects who passed the test on transitivity and failed on conservation of discontinuous quantities, and only 1 subject was in the corresponding cell for length. Owens and Steffe (in press) observed only 4 of 126 instances (among 42 subjects) in which transitivity of a matching relation preceded conservation of that relation. Divers (1970) found that in 87% of the cases where transitivity of a length relation was attained, the relation was also conserved. In the studies cited, the results consistently indicated that attainment of conservation preceded attainment of the transitive property. None of the studies involved instruction or practice, and the present results may be interpreted in terms of the treatment effect. The treatment was effective in improving performance on the test of the transitive property while the treatment had no effect on conservation performance for matching relations. Thus, some children in the treatment group met the criterion on the transitivity test who otherwise might not have attained transitivity. Only two children who used transitivity on the Transitivity Problem failed to exhibit conservation. This explanation applies, however, only to the matching relational category, because the treatment was not effective in improving the performance on transitivity of length relations.

Perhaps an interpretation can be made in terms of the characteristics of the children in the sample. Skypek (1966) conducted a study which involved both middle and lower socio-economic status children. It was found that among the low status children, the developmental pattern of cardinal number conservation was erratic. While the present study included no middle class group for comparison, it appears that the pattern of attainment of conservation and relational properties was irregular for these low economic status subjects.

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Table 1

Diagram of the Design

Factors and Levels		Twelve Response Variables											
Treatment	Grade	MR	CMR	TMR	SMR	AMR	RMR	LR	CLR	TLR	SLR	ALR	RLR
Treatment	Kinder-garten												
	First												
Control	Kinder-garten												
	First												
Treatment	Pretest												
Treatment	Pretest												
	No Pretest												
Control	Pretest												
	No Pretest												

Table 2

Treatment Versus Grade Analyses of Variance with Significant Effects

Source of Variation	Response Variable							
	Matching Relations (MR)		Conservation of MR (CMR)		Transitivity of MR (TMR)		Symmetric Property of MR (SMR)	
	M.S.	F	M.S.	F	M.S.	F	M.S.	F
Treatment (T)	.44	.16	2.19	.62	19.90	8.34**	2.32	2.69
Grade (G)	31.53	11.79**	28.88	8.18**	2.36	.99	11.10	12.89**
T X G	.15	.06	1.49	.42	.16	.07	.15	.18
Error	2.67		3.53		2.38		.86	

	Asymmetric Property of MR (AMR)		Reversibility of MR (RMR)		Length Relations (LR)	
	M.S.	F	M.S.	F	M.S.	F
Treatment	7.69	3.98	2.60	1.97	1.00	1.01
Grade	10.39	5.37*	8.74	4.79*	5.87	5.89*
T X G	.14	.07	.47	.26	.53	.53
Error	1.93		1.83		1.00	

*p < .05 **p < .01

Note: Each factor and interaction had 1 df: error 43 df.

Table 3

Treatment versus Pretest Analyses of

Variance with Significant Effects

Source of Variation	Response Variable							
	Transitivity of MR (TMR)		Asymmetric Property of MR (AMR)		Symmetric Property of LR (SLR)		Reversibility of LR (RLR)	
	M.S.	F	M.S.	F	M.S.	F	M.S.	F
Treatment (T)	20.41	8.45**	8.37	4.10*	.21	.18	1.71	1.21
Pretest (P)	1.19	.49	5.39	2.64	4.71	4.09*	3.53	2.31
T X P	.00	.00	.39	.19	4.92	4.29*	9.54	6.26*
Error	2.42		2.04		1.15		1.52	

*p < .05 **p < .01

Note: Each factor and interaction had 1 df: error 43 df.

Table 4

Group Means, as Percents, for Significant Main Effects in the Absence of Interaction

Effect	Group	MR	CMR	TMR	SMR	AMR	RMR
Treatment	Treatment	--	--	58	--	73	--
	Control	--	--	36	--	52	--
Grade	First	87	62	--	81	74	72
	Kindergarten	59	36	--	48	50	50
Totals		73	49	47	65	62	61

Table 5

Cell Means as Percents: Treatment X Test Interactions

	SLR			RLR		
	Pretest	No Pretest	Total	Pretest	No Pretest	Total
Treatment	42	85	62	54	91	72
Control	58	58	58	67	58	63
Total	50	71	60	60	74	67

Table 6

Means, As Percents, for Length Relational Variables by Grades

Grade	LR	CLR	TLR	SLR	ALR	RLR
First	93	63	50	67	68	65
Kindergarten	81	51	48	54	53	70
Total	87	57	49	60	61	67

Table 7

Contingency Table: Transitivity Problem

Ratings Versus Treatment Group

Treatment Group	Rating				
	1	2	3 or 4	3	4
Treatment	4	7	10	6	4
Control	7	12	5	4	1

Table 8

Contingency Table: Transitivity Problem

Ratings Versus Grade Level

Grade Level	Rating				
	1	2	3 or 4	3	4
First	3	7	12	8	4
Kindergarten	8	12	3	2	1

Table 9

Contingency Table: Ratings on Transitivity Problem Versus Criterion on Transitivity of Matching Relations

TMR Criterion Level	Rating 1	2	3 or 4	3	4
Criterion	1	7	8	5	3
Not Criterion	10	12	7	5	2

Table 10

Contingency Table: Ratings on Transitivity Problem Versus Criterion on Conservation of Matching Relations

CMR Criterion Level	Rating 1	2	3 or 4	3	4
Criterion	0	5	13	8	5
Not Criterion	11	14	2	2	0

Table 11

Intercorrelations Among the 12 Variables

	CMR	TMR	SMR	AMR	RMR	LR	CLR	TLR	SLR	ALR	RLR
MR	73**	57**	32*	59**	59**	22	35*	16	49**	55**	40**
CMR		51**	37*	59**	48**	30*	41**	40**	56**	48**	39**
TMR			44**	28	47**	07	24	28	51**	59**	32*
SMR				46**	27	43**	41**	36*	55**	51**	17
AMR					45**	43**	40**	26	54**	52**	41**
RMR						07	27	25	54**	46**	50**
LR							46**	15	27	24	20
CLR								11	53**	25	39**
TLR									47**	33*	23
SLR										64**	57**
ALR											40**

*p < .05

**p < .01

Note: decimals are omitted.

Table 12

Contingency Table: Criterion on CMR Versus Criterion on TMR

Conservation of Matching Relations (CMR)	Transitivity of Matching Relations (TMR)	
	Criterion	Not Criterion
Criterion	9	10
Not Criterion	8	20

Table 13

Contingency Table: Criterion on CLR Versus Criterion on TLR

Conservation of Length Relations (CLR)	Transitivity of Length Relations (TLR)	
	Criterion	Not Criterion
Criterion	9	15
Not Criterion	9	14

Figure 1

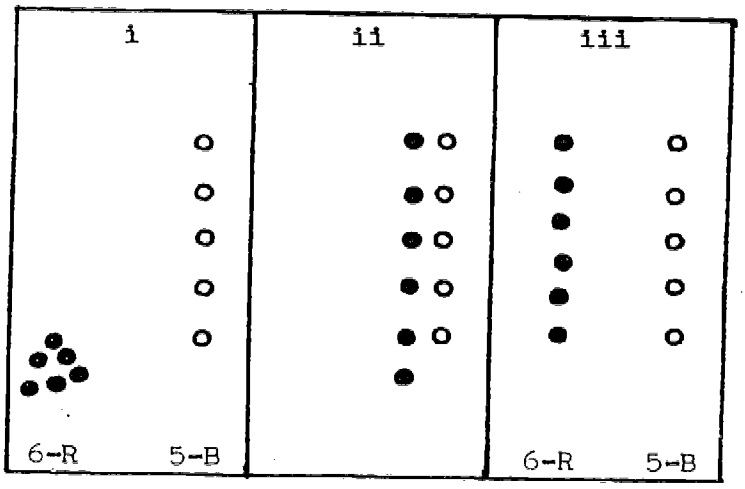

Figure 2