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ABSTRACT

This is the first issue of a new quarterly series containing expanded abstracts of recent research in mathematics education. (Previous issues were occasional publications of the School Mathematics Study Group.) The twenty abstracts review research articles published in the past two years, and are classified in four sections: (1) Aspects of mathematics learning, (2) Mathematics instruction and instructional materials, (3) Mathematics achievement and its correlates, and (4) Teacher education and evaluation. Each abstract includes an objective indication of the purpose, rationale, research design and procedure, findings, and interpretation of the investigation as originally reported, together with a critical analysis by the abstractor. All abstractors are mathematics educators in professional positions. For ease of reference, each abstract includes the ERIC accession numbers, descriptors, and identifiers of the article. Future issues of this publication will include abstracts of non-journal material available through the ERIC service. (MM)

Vol. 5, No. 1, Winter, 1972

INVESTIGATIONS IN MATHEMATICS EDUCATION

Expanded Abstracts and Critical Analyses of Recent Research

Center for Science and Mathematics Education
The Ohio State University
in cooperation with
the ERIC Science, Mathematics and Environmental Education Clearinghouse

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Winter, 1972

SECTION 4: TEACHER EDUCATION AND EVALUATION

NOTES . . .

from the Editor

INVESTIGATIONS IN MATHEMATICS EDUCATION was originated as an occasional, in-house publication:

> The Advisory Board of the School Mathematics Study Group believes that knowledge of the results of research in mathematics education can be helpful and should be used in the development of programs for the improvement of mathematics education. The purpose of this journal is to make such knowledge more readily available to all those involved in SMSG activities.

This purpose was achieved in several ways, but principally through invited abstracts of published research reports which satisfied each of the following as minimal criteria:

1. The report must be readily available to anyone who wishes to read it.

2. The report must make clear the purpose of the investigation which should be concerned directly with, or have definite implications for one or more of the following (independently, or interactingly):

   a. the mathematics or mathematics education program: its content, organization, etc.;

   b. the learner;*

   c. the teacher;*

   d. instructional methods, materials, activities, and environment;*

   e. organization for implementing instruction;*

   f. cultural, demographic, etc. variables.*

   [* within the context of mathematics or mathematics education]

3. The report must include some information pertaining to the research design and procedure for the investigation, and to its scope and delimitation (which may range anywhere from the preschool level through grade 14, or may be at the pre- or in-service level if concerned with teacher education).

4. The report must include some degree of objective evidence from observed findings in support of conclusions or inferences or implications drawn from the investigation.

Each abstract included (in one way or another) an objective indication of the (1) purpose, (2) rationale, (3) research design and procedure, (4) findings, and (5) interpretations of a particular investigation as reported by the investigator(s)--insofar as such information was made explicit in the research report. Finally, an abstracter was given an opportunity to comment upon or to raise questions pertaining to the reported research in the concluding section of his abstract: Abstracter's Notes.

It soon became evident that INVESTIGATIONS IN MATHEMATICS EDUCATION was of interest to many members of the mathematics education community apart from those persons who were directly involved in SMSG activities. Now, after four editions[1] as an occasional journal under SMSG sponsorship, INVESTIGATIONS IN MATHEMATICS EDUCATION is being reorganized as a four-times-a-year publication of the Center for Science, Mathematics and Environmental Education, The Ohio State University, with the cooperation of ERIC/SMEAC.

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[1]Volume 1, January 1969; Volume 2, July 1969; Volume 3, January 1970; and a brief transitional Volume 4, November 1971. Each of these has been available for purchase from A. C. Vroman, Inc., 2085 East Foothill Boulevard, Pasadena, CA 91109.

Journal policy will be guided by the same Editorial Committee as before: Professors E.G. Begle, J. N. Payne, and L. Pikaart. The new Editor of the journal will be Professor J. L. Higgins. The past editor, Prof. J. F. Weaver, will serve--at least for the present--as an editorial consultant for the reorganized journal.

This issue reflects the transition of the journal from an occasional publication to a quarterly publication. It reviews research articles that have appeared during the past two years in mathematics education journals. This emphasis provides a continuity of coverage for previous readers of the journal. The next issue (vol. 5, no. 2) will broaden the scope of coverage to include non-journal material such as final research reports, research papers presented at meetings, etc. The coverage of non-journal material will be restricted to documents available through the ERIC

Document Reproduction Service (EDRS)--thus maintaining our policy of abstracting only reports which are readily available to our readers.

In order to emphasize links with the ERIC system, each abstract will be prefaced with the ERIC system accession information assigned to the paper when it was announced in RESEARCH IN EDUCATION or CURRENT INDEX TO JOURNALS IN EDUCATION. For journal-published articles the accession information includes the complete journal citation as well as ERIC Descriptors and identifiers. The latter provide the reader with words which describe major ideas found in the paper, and are useful for purposes of rapid scanning. Reprints of journal articles are not available from EDRS. For complete journal articles readers should contact their library, or the journal involved.

All critical abstracts published in INVESTIGATIONS IN MATHEMATICS EDUCATION are prepared by mathematics educators in professional positions. The abstractors donate their time and talents so that this journal may be published at minimum cost as a service to the mathematics education community. It is our hope that this publication will continue to provide a research dialogue that is beneficial to all aspects of mathematics education.

J. F. Weaver
Guest Editor

SECTION 1:

ASPECTS OF MATHEMATICS
LEARNING

EJ 039 647 310 SE 503 265

INTERACTIONS BETWEEN "STRUCTURE-OF-INTELLECT" FACTORS AND TWO METHODS OF PRESENTING CONCEPTS OF MODULAR ARITHMETIC--A SUMMARY PAPER. Behr, Merlyn J., Journal for Research in Mathematics Education v1 n1, pp29-43, Jan '70

Descriptors--*Aptitude, *Instruction, *Learning, *Mathematical Concepts, *Number Concepts, Arithmetic, Programed Instruction, Research

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Kenneth B. Henderson, University of Illinois

1. Purpose

The basic purpose of this study was to seek information that might suggest whether some of the mental factors Guilford identifies in his matrix-model of the intellect correlate with success in usual school learning situations. More specifically, Behr sought to "suggest answers to the questions, namely, (1) Are the unique mental factors significantly correlated with, or significant predictors of, success in usual school learning situations? and (2) Would it be possible to design instructional materials in a way that would suit the learner's mental ability profile?"

2. Rationale

One test of the worth of an empirical theory is its ability to yield confirmed predictions. Particularly, a test of Guilford's conceptualization of the intellect is its ability to yield confirmed predictions of the behavior of individuals in certain situations. From Guilford's matrix-model it is reasonable to predict that a person who manifests aptitude in figural, semantic, or symbolic domains will be able to learn better if the content is presented respectively in these modes. Behr sought to test this hypothesis for figural- vs. verbal-symbolic aptitudes. His behavioral definition of 'learn better' was in terms of the time used by the students to complete a programmed unit, their scores on tests of their knowledge of structural properties of the modulus-seven system, and their scores on tests of computational skill in this system.

3. Research Design and Procedure

Using the subject matter of modulus-seven arithmetic, Behr wrote two programs, both having the same content. One, utilizing the figural mode, i.e., using diagrams, was regarded as figural-symbolic; the other utilizing the verbal mode was regarded as verbal-symbolic. Behr regarded the amount of symbolic content as the same in both programs. These two treatments were independent variables. Other independent variables were the profiles the subjects manifested on a subset of the mental factors in Guilford's matrix, viz., the triples formed by combining figural and semantic in the content classification with cognition and memory in the operations classification and with units, classes, relations, and systems in the products classification. Tests used to determine the profiles were those suggested by Guilford and Hoepfner.

The criterion (dependent) variables were the behaviors mentioned in the previous section. The two tests were administered two days after completion of the program. Two (similar?) tests were administered two weeks later. Thus initial learning and retention both of knowledge and skill were measured.

The subjects were 228 college students in preservice mathematics courses designed for elementary school teachers at Florida State University. After their profiles in the mental factors had been determined, they were randomly assigned to one of the two treatment groups, viz., figural-symbolic presentation or verbal-symbolic presentation.

Regression analysis, using Cronbach and Gleser's discussion, was used to test the hypotheses. Rejection of the null hypothesis implied that a significant interaction occurred between the particular mental aptitude and the two modes of presentation.

4. Findings

There was a significant interaction between the modes of presentation and the following mental factors: cognition of figural transformations, cognition of semantic units, memory of semantic relations, and cognition of semantic classes. Behr concluded that the figural factor was a significant predictor for learning only in the figural-symbolic mode, and that the semantic factor was a significant predictor for learning only in the verbal-symbolic mode.

Concerning the specific questions Behr sought to answer (See the purpose above), he felt that, although a conclusive answer to the questions cannot be given, the weight of the evidence suggests affirmative answers.

5. Interpretations

Behr feels that the findings support Guilford's model of the intellect. He also feels that different mental factors may predict different "orders" of learning, e.g., skill vs. understanding. He agrees that further research on this conjecture is needed.

Abstracter's Notes

The lack of precision in the definitions of some of the categories in Guilford's model haunts this study. Behr adds nothing to clarifying the categories. One wonders how the symbolic and semantic categories can be disjoint as Guilford's model presumes. (A symbol is a symbol of something and thus has semantic content.) And were the figural and semantic contents in Behr's programs disjoint as implied by Guilford's model? And was the figural content Behr used like that Guilford describes? Perhaps in the more extensive report of the study of which the present article is a summary, these and other questions are answered. Finally, to conceive of figural-symbolic and verbal-symbolic is to change Guilford's model.

Behr did not say what kind of program was used. If it was linear, the principal findings are what one would expect. And no matter what kind of program was used, we need evidence that this is typical of "usual school learning situations." And is the sample of students used such that one need not restrict the generalizations?

In light of the inconsistency of the findings, the abstracter cannot agree that the experiment supports Guilford's theory. Nor does it weaken it.

Kenneth B. Henderson
University of Illinois

EJ 039 649 310 SE 503 279
LINEAR MEASUREMENT IN THE PRIMARY GRADES: A COMPARISON OF PIAGET'S DESCRIPTION OF THE CHILD'S SPONTANEOUS CONCEPTUAL DEVELOPMENT AND THE SMSG SEQUENCE OF INSTRUCTION Huntington, Jefferson R., Journal for Research in Mathematics Education, vl n4, pp219-232, Nov '70

Descriptors--*Elementary School Mathematics, *Learning, *Mathematics Concepts, *Mathematics Materials, Grade 1, Grade 2, Grade 3

Expanded Abstract and Analysis Prepared Especially for I.M.E. by L. D. Nelson, University of Alberta

As the title implies, the purpose of this study was to compare the spontaneous development of ideas of linear measurement of children in grades one to three as described by Piaget with linear measurement instruction provided for these grades in the S.M.S.G. books.

Despite the fact that the author has sorted out material concerning linear measurement from Piaget's rather extensive works on space and geometry and that the linear measurement material from the S.M.S.G. books is also only part of a much larger set of material, both Piaget and the S.M.S.G. committee seem to agree that most children have gained a fairly clear insight into the idea of linear measurement by about the end of grade three (age nine). The main difference, of course, is that Piaget claims that true understanding of linear measurement arises spontaneously out of the child's experience, maturation, social interaction, and a phenomenon he calls equilibration, while the S.M.S.G. writers believe that these ideas are gained through a program of specific instruction in a specific logico-mathematical sequence.

Huntington has made a careful analysis of the S.M.S.G. instructional sequence in linear measurement from the beginnings of the concept of line segment and the use of the straightedge in grade one, through to the use of standard units in measuring perimeters in grade three. He has also carefully traced Piaget's claims of how the two operations: subdivision and change of position develop in the child, how they are finally co-ordinated into a single operation by unit iteration, and how this results in a true understanding of linear measurement. He has taken pains to discuss the semantic and other problems which plague attempts to link child development research with curriculum construction. Finally, the author has constructed a useful chart showing at which grade levels the Piagetian concepts of change of position, spontaneous measurement, conservation of distance, and conservation of length, would most likely occur.

A few implications of how Piaget's research would affect the sequencing of learning material such as that provided by the S.M.S.G. committee are discussed. Huntington, however, leaves us with the question of precisely how a sequence of instruction in linear measurement can be devised which would attempt to make full use of Piaget's findings.

Abstracter's Notes

The question that Huntington asks is a most fundamental one to mathematics curriculum workers. If Piaget is correct, does it mean that we must only wait until the concepts develop sponteneously, and all our efforts at providing a content-specific sequence of instruction would be in vain? Or is there a content-general sequence which would aid in the development of those mathematical concepts?

It is the quality of information and the careful analysis of the differing points of view provided by articles such as this which will be most helpful in finding some satisfactory answers to these fundamental questions.

L. D. Nelson
University of Alberta

EJ 012 476 310 SE 500 889

NEGATION IN THE MAJOR PREMISE AS A FACTOR IN CHILDREN'S DEDUCTIVE REASONING Roberge, James J., Sch Sci Math v69 n8, pp715-722, Nov '69

Descriptors--*Deductive Methods, *Elementary School Students, *Learning, *Logical Thinking, Critical Thinking, Elementary School Mathematics, Mathematical Logic

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Thomas C. O'Brien, Southern Illinois University, Edwardsville

1. Purpose

To investigate the effects of the inclusion of the word not in the major premise of an argument on subjects' performance on written class logic and conditional logic tests; to assess differences in these effects over grade level; to assess whether these effects differ according to the test used.

2. Rationale

Two previous researches--[Hill (1961) and Paulus (1967)]--reported conflicting conclusions regarding the effect of the use of not on subjects' performance on deductive reasoning tests. Since the researchers used different types of items and tested at different grade levels, "the need for a more extensive and refined measure of the effects of negation in the major premise both at specific grade levels and for different types of deductive reasoning was apparent."

3. Research, Design, and Procedure

Two corresponding paper-and-pencil logic tests--the Paulus-Roberge Class Reasoning Test and the Paulus Conditional Reasoning Test--each consisting of four items in each of six principles (four valid, two not) were administered in random order to 57 subjects in each even numbered grade from 4 to 10 in three public school systems in southeastern Connecticut. The test items were "concrete familiar," i.e. the items had conclusions which stated facts "not within the students' experience, e.g., 'This picture belongs to Mary'." Corresponding examples of items from the two tests are given below.

Principle	Valid?	Class	Conditional
1	Yes	All of Joan's friends are going to the	If Joan goes to the museum, then she

museum today.	will meet her friend Sue.
Pat is a friend of Joan.	Today, Joan is going to the museum.
∴ Pat is going to the museum today.	∴ Today, Joan will meet her friend Sue.

In order to measure the effect of negation in the major premise, the researcher "systematically introduced" the word <u>not</u> to produce items such as the following:

Principle	Valid?	Class	Conditional
1	Yes	All Chevy trucks do not belong to Mr. Jones.	If the car is a Ford, then it does not belong to Mr. Smith.
		This truck is a Chevy.	The car is a Ford.
		∴ This truck belongs to Mr. Jones.	∴ The car belongs to Mr. Smith.
2	No	All the lamps that are not broken are plugged in.	If the radio is not plugged in, then you may listen to the phonograph.
		The lamp is plugged in.	You may listen to the phonograph.
		∴ The lamp is not broken.	∴ The radio is not plugged in.

Not given in the present article but available from another source [Roberge (1970)] is a description of the item format. Each item was followed by multiple choice options, A. Yes, B. No, C. Maybe, the latter response defined for subjects as meaning "It may be true or it may not be true. You weren't told enough to be certain whether it is 'yes' or 'no'."

A three-way analysis of variance--<u>G</u>rade X <u>T</u>ype of Reasoning X Presence of <u>N</u>egation--with "repeated measures on two of the factors [T and N]," was conducted with cell entries consisting of subjects' scores in the negation and no negation items on the class logic and conditional logic tests according to grade. In the analysis, then, the possible range of a student's score for each cell was 0-12.

4. Findings

Grade level, types of reasoning, and negation effects were all statistically significant, as were T X N and G X T effects ($p<.001$). Mean scores are given below.

Cross Tabulation of Means for Analysis of Variance

Grade Level	Type of Reasoning		Combined
	Class	Conditional	
4	4.89	4.62	4.76
6	4.97	4.58	4.78
8	5.64	6.65	6.14
10	7.30	7.10	7.20
Combined	5.70	5.74	

Grade Level	Negation		Combined
	Yes	No	
4	4.13	5.39	4.76
6	4.28	5.27	4.78
8	5.34	6.95	6.14
10	6.43	7.96	7.20
Combined	5.05	6.39	

Type of Reasoning	Negation		Combined
	Yes	No	
Class	4.76	6.64	5.70
Conditional	5.33	6.14	5.74
Combined	5.05	6.39	

5. Interpretations

The finding of statistically significant N effects supports Hill's conclusion that negation in the major premise does influence subjects' ability to recognize valid deductions. Further, the lack of a statistically significant G X N term suggests that the relative difficulty of items with and without negation in the major premise tends to remain constant over grade level. The statistically significant T X N term suggests that the effect of negation in the major premise is not the same for the class logic and conditional logic items. A possible explanation lies in the inherent ambiguity of statements of the form "all...are not...," such as those used in this study.

Further research, it is suggested, should be given to the effects of negation (1) as used in various inference patterns, (2) in terms of the number of nots in an argument, and (3) in terms of the location of not in the major premise.

Abstracter's Notes

1. As the researcher suggests, the ambiguity of statements of the form "All...are not..." raises questions regarding the central issue of this research. Given that many items in the class logic test may have been ambiguous, it is not clear just what the finding of significant main and interaction effects might mean. The trend of data within the non-ambiguous conditional logic items suggests that N effects might be fruitfully assessed here, but this was an issue that was not investigated.

2. Several confounding issues may exist even if the ambiguity question were to have been resolved.

2.1 T or T X N differences could be attributed to the fact that item content may have differed from one test to another; i.e. "All of Joan's friends are going to the museum today," versus "If Joan goes to the museum, then she will meet her friend Sue." From the listing of sample items in the article, one is not able to assess the extent of content differences.

2.2 Similarly, item content differences could confound N effects, i.e., "If Joan goes to the museum, then she will meet her friend Sue," versus "If the car is a Ford, then it does not belong to Mr. Smith."

2.3 Not may not have been systematically placed in the class logic items. As suggested by the instances given in the report, it may be that items such as "All X's that are not Y's are Z's" were used in the same class as "All X's are not Y's." While both premises contain the word not, the logical structures of the two items are not the same. Unfortunately, the extent of this potential confounding factor is not discernable from the sample item descriptions given in the article.

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EJ 039 514 270 SE 503 272

THE FEASIBILITY OF INDUCING NUMBER CONSERVATION THROUGH TRAINING ON REVERSIBILITY Sparks, Billie Earl; And Others, Journal for Research in Mathematics Education, v1 n3, pp134-143, May 70

Descriptors--*Conservation (Concept), *Instruction, *Number Concepts, *Research, Elementary School Mathematics, Mathematical Concepts

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Marilyn N. Suydam, The Pennsylvania State University

1. Purpose

The purpose was to investigate the feasibility of inducing number conservation through training on reversibility tasks that are not related to the criterion measure.

2. Rationale

Piaget stresses the acquisition of reversible operations as the foundation of conservation development, which may be of considerable importance in the child's development of a stable concept of number as well as achievement in school. Researchers have been successful in inducing conservation by training on reversibility, but the training was so closely related to the test that the results might be due to a practice effect.

3. Research Design and Procedures

From 46 children, 19 five-year-olds and 18 four-year-olds were classified as non-conservers on the basis of a ten-item test, and were randomly assigned to a control or an experimental group for each age level. After a two-day interval, the experimental groups were trained on a reversibility task using clay. A second training, with clay and then with pennies, followed after another two-day interval. The training involved a progression from continuous to discontinuous quantity; the tests contained only items involving discontinuous quantities.

The posttest, a parallel form of the pretest using blocks, knobs, and marbles, was administered immediately after the second training session to all groups. After intervals of one and three weeks, parallel retention tests were given. Due to absence on training or testing days, data were available for only eight five-year-olds and six four-year-olds in each group. The data were analyzed with Fisher's exact probability test.

4. Findings

Seven of the five-year-olds who were trained evidenced conservation on the posttest, while all eight of the children who were not trained remained non-conservers ($p = .007$). On the two retention tests, six of the trained group continued to evidence conservation; one child who was not trained also attained conservation ($p = .02$). None of the four-year-olds evidenced conservation on any test.

5. Interpretations

Experiences in reversibility can induce conservation in children who are already on the threshold of achieving conservation. There is doubt regarding the feasibility of inducing conservation in four-year-olds; however, they may have been trained for an insufficient length of time. More research is needed to determine whether reversibility is the sufficient criterion for conservation.

Implications for the classroom teacher include: (1) attempts to force number concepts on a non-conserver will probably end in failure (this is implied by the instability of the child's number concept) and (2) the described program could be used by teachers, naturally and over a longer period of time.

Abstracter's Notes

The number of children involved is limited (16 five-year-olds and 12 four-year-olds). The amount of time devoted to the study was limited (two 20-minute training periods plus four testing sessions). It would seem that the interpretation of the findings should take into account these two limitations. Would the results have been similar with a larger sample? Would the results have been similar with more training? What is the effect of the attrition of nine children?

As is usual in most of the Piagetian-oriented research, little attempt is made to select children so that the findings are more generalizable; rather, the nearest available group of children is used. True, evidence of conservation level was ascertained; it is obviously a necessary consideration to obtain groups who can be dichotomized. But would the results be the same if random selection techniques to obtain the sample were employed?

The dichotomization may leave something to be desired. Is it impossible to make finer distinctions--i.e., not merely making a conserver/non-conserver dichotomy, but ascertaining levels within each? How many items separated conservers from non-conservers? Why did one child who was not trained apparently attain conservation? As the authors point out, it is possible that all of the children attained conservation "spontaneously", (as this one child apparently did), and not as a result of training.

Is the first interpretation statement valid--will attempts to force number concepts on a non-conserver end in failure? Has the relationship between "reversibility" and "number concepts" been established clearly enough to warrant this interpretation?

Is the second interpretation statement plausible--should the training procedures be used in the classroom? Should the teacher attempt to induce reversibility? The authors should be commended for attempting to make an interpretation for classroom teachers--but, more definitive research is needed before direct application of training procedures used in research is recommended for classroom practice.

Marilyn N. Suydam
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EJ 039 648 310 SE 503 273
DIFFERENTIAL PERFORMANCE OF FIRST-GRADE CHILDREN WHEN SOLVING ADDITION PROBLEMS Steffe, Leslie P., Journal for Research in Mathematics Education v1 n3, pp144-161, May 70
Descriptors--*Addition, *Elementary School Mathematics, *Problem Solving, *Research, Arithmetic, Conservation (Concept), Grade 1, Learning

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Shirley Hill, University of Missouri, Kansas City

1. Purpose

The purpose of the study was to isolate the effects of some variables that influence children's success in solving addition word problems in a curriculum using the intuitive set approach. The "intuitive set approach" is characterized by the use of set theory as a basis for natural numbers and their operations, beginning with physical objects and proceeding through pictorial representations to numerals only.

2. Rationale

The variables studied are: IQ, quantitative comparisons, visual aids and transformation. Quantitative comparisons are classified as gross, intensive, or extensive. Elkind (1961) defines gross quantitative comparisons as "single perceived relations between objects (larger than, fewer than) which are not coordinated with each other." When coordination begins the child is making intensive quantitative comparisons (Piaget, 1952) and when there is the potential of grasping proportionality of differences, the child can make extensive comparisons (Piaget, 1952). At this stage the concept of unit becomes possible.

Tests of quantitative comparisons and IQ measure different aspects of ability as measured by low, but significant, correlations.

In an addition problem representing the union of sets, the described sets may be static and the union implied only by the question. Or the union may be suggested by describing a movement of bringing the sets together. The second possibility describes a "transformation."

Previous studies (Smedslund, 1964; Zweng, 1963) have investigated effects of visual aids on problem solving. Differing conclusions are not necessarily contradictory because different ages of children were involved.

3. Research Design and Procedure

From a total population of 2166 first-grade children of one school district, an ordered sample of 341 was randomly selected. The sample was partitioned into three groups on the basis of Kuhlmann-Anderson Intelligence Test (Form A) IQ: 1) range of 114-140, 2) range of 101-113, 3) range of 78-100.

Each child was tested individually by the same experimenter on three successive tests. The first test was of quantitative comparisons. Each part involved a comparison between two sets of blocks and the questions: Are there more here_______? or _______here? or are there the same number? The blocks were arranged so that some items could be answered correctly by making a gross comparison while others require an extensive comparison. The latter includes one-to-one correspondence and comparison by counting. The former includes comparison by relative size or by relative density.

On the basis of this test children were assigned to four levels: Level 1) each item of each of the three parts answered correctly, Level 2) each item of exactly two parts answered correctly, Level 3) each item of exactly one part answered correctly, Level 4) no items answered correctly. Then the 341 children tested in the IQ range of 78-140 were assigned randomly to the 12 groups (IQ by Level).

The second test consisted of 18 addition problems, six with accompanying physical aids, six with pictorial aids and six with no aids. Nine problems involved a transformation.

The third test was of addition combinations that appeared in the eighteen problems.

A mixed design with repeated measures on the two problem-solving variables was used to detect differences in the means of, or interactions among, the four levels of quantitative comparisons, the three IQ groups, the transformation vs. no transformation problems, the three levels of visual aids. A two-way analysis of variance was used to detect differences in performance on the test of addition facts in the four levels of quantitative comparison and three IQ groups. Correlation coefficients were calculated between total scores on the problem-solving test and total scores on the addition-facts test.

4. Findings

Low frequencies for the 38% of the children who were in Level 4, under Item 4 of each part of the test of quantitative comparisons indicate that these children were, in general, using gross quantitative comparisons rather than the extensive comparisons required on these items.

Internal-consistency reliability coefficients ranged from .64 to .83 on the various tests or subtests.

The mean score (69%) of the children in Level 4 was significantly different ($p<.01$) from the mean scores of the other levels, which did not differ significantly from each other. The mean score (75%) of the children in IQ group 78-100 was significantly different ($p<.05$) from the mean scores of the children in the two higher IQ groups, which did not differ significantly from each other.

Means of the subdivisions of the problem-solving test based on aids were: physical aids, 86%; pictorial aids, 85%; and no aids, 72%. The mean of 72% differed significantly from both of the other means, which did not differ from each other.

Means of levels based on transformations were: transformation, 85%; no transformation, 77%. The difference was significant ($p<.01$).

The ANOVA showed significant differences on the addition-facts test in both the levels and IQ categories.

Correlation coefficients between scores on problem-solving and scores on addition-facts were: .49 ($p<.01$) for the 132 children; .68 ($p<.01$) for Level 4; .05 (not significant) for Level 3; .39 ($p<.05$) for Level 2; .39 ($p<.05$) for Level 1; .60 ($p<.01$) for IQ group 78-100; .36 (not significant) for IQ group 101-113; .23 (not significant) for IQ group 114-140.

5. Interpretations

There were three categories of children for which the evidence suggests that experiences in the mathematics curriculum produced differential results:

(1) Children in Levels 1, 2 and 3 in IQ group 101-113 and 114-40 and Level 1 in IQ group 78-100, who had mean scores greater than 80% on the problem-solving test.

(2) Children in Level 4 in IQ groups 101-113 and 114-140 and Levels 2 and 3 in IQ group 78-100, who had mean scores between 74 and 79%.

(3) Children in Level 4 in IQ group 78-100 who had a mean score of 58%.

General intelligence apparently plays a vital role in problem-solving ability. Problems with no visual aids were more difficult than those with aids. Problems with a described transformation were easier than those without. However, the lack of an interaction between the transformation variable and levels was surprising.

Abstractor's Notes

This is a well-designed, fairly complex study which succeeded in its purpose of isolating the effects on problem-solving of several important variables. While many of its results probably could have been expected, some were surprising and the study does call attention to some important considerations in teaching addition to first graders. It is not altogether clear, however, how the results relate to the particular curriculum described, i.e., the intuitive set approach. One wonders, in fact, whether the findings might not be expected in other curricula as well.

One of the results unexpected by the investigator was the fact that the correlation of scores on problems with aids and addition-facts test was not much different from the correlation of scores on problems without aids and the addition-facts test. The investigator's interpretation that this is additional support for the conclusion that visual aids facilitate problem-solving may not be entirely warranted. The particular result requires further study.

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SECTION 2:

MATHEMATICS INSTRUCTION AND INSTRUCTIONAL MATERIALS

EJ 039 516 270 SE 503 278

BEHAVIORAL OBJECTIVES AND FLEXIBLE GROUPING IN SEVENTH GRADE MATHEMATICS Bierden, James E., Journal for Research in Mathematics Education, vl n4, pp207-217, Nov '70

Descriptors--*Behavioral Objectives, *Class Organization, *Instruction, *Secondary School Mathematics, Attitudes, Grade 7, Grouping, Research

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Alan R. Osborne, The Ohio State University

1. Purpose

To explore and develop a management plan for instruction based upon using behavioral objectives to guide in establishing intra-class grouping. Attention was directed to four major questions:

a) What changes occurred in achievement, attitudes, and test anxiety in mathematics?

b) Was intra-class grouping an important part of the "experimental" procedure?

c) What were the reactions of students to intra-class grouping, the use of behavioral objectives, and the classroom management procedures?

d) What modifications in the classroom management procedures are needed to increase its effectiveness?

2. Rationale

Intra-class grouping was cited as the most frequently reported means of providing for individual differences. Recognizing the promise of this means of individualizing instruction, three guidelines were established as particularly important in assuring success of intra-class grouping:

a) Flexibility--the capability of moving a student from group to group

b) Accuracy--correctness in assessing student needs, abilities, and interests in meeting the demand of flexibility

c) Appropriate materials--availability of materials for groups with a correct level and content

The use of behavioral objectives á la Mager was deemed particularly appropriate in meeting the guidelines.

3. Research Design and Procedure

Two intact seventh grade classes (n = 44) were taught by the investigator using the guidelines for intra-class grouping. The year's course was broken into twenty-four topics. Each topic had two instructional phases. Phase I was initiated by giving each student a set of objectives and assignments classified into three levels: Basic, Intermediate, and Advanced. An instructional period of four days was followed by testing directed toward the objectives. The testing determined whether the student was placed in the Basic, Intermediate, or Advanced group for Phase II instruction. Phase II instruction was economically directed toward unattained objectives. A test was given at the end of the topic unit.

A pretest-posttest design was used and analyzed by means of a t-test for differences between correlated means. Data were compared to those determined in two previous experiments reported in 1964 and 1966.

Achievement was tested by means of the California Achievement Test and the SMSG Mathematics Inventory. The Aiken-Drager Attitude Scale and Sarason's Test Anxiety Scale provided effective measures. Children were surveyed concerning the "experimental" procedure.

4. Findings

a) Differences in pre-post test measures of achievement, attitude, and test anxiety were significant in the preferred direction.
b) Students reacted more favorably in May than in November to a questionnaire concerning intra-class grouping, use of behavioral objectives, and classroom management techniques.
c) Attitude scores compared favorably to the attitude scores of the four comparison groups. Achievement differences were not significant.
d) The typical student changed groups 7.9 times.

5. Interpretations

The investigator refrained from over-interpreting the results concluding "...they [the results] do indicate that the development of methods built around flexible intra-class grouping and behavioral objectives has potential for improving... aspects of mathematics instruction."

Abstractor's Notes

This study is an exploratory, developmental study. It does not fit the traditional experimental group versus control group model desired by many experimentalists which is our inheritance from psychology. Unfortunately, some at best specious comparisons to other seventh grade groups of a different school generation were forced on the reader. Given the other groups' participation in experiments and attitude being a major variable, the comparisons are simply not interpretable. Why must developmental studies be forced into a comparison mode of thinking?

The procedures for classroom management, for determining and using objectives, and designing instruction are only outlined. Insufficient information is provided for faithful replication. If a developmental study establishes promise for a treatment, then it is incumbent on the researcher to report comprehensively in order that subsequently a comparative study may be conducted. The researcher who conducts an exploratory study assumes a responsibility for establishing a procedural base for future comparative studies.

It is disconcerting to find numerical errors in tables and trivial misprints in the text of a journal of the potential stature of the JRME.

Alan R. Osborne
Ohio State University

EJ 039 515 270 SE 503 274

THE EFFECTS OF STUDYING DECIMAL AND NONDECIMAL NUMERATION SYSTEMS ON MATHEMATICAL UNDERSTANDING, RETENTION, AND TRANSFER

Diedrich, Richard C.; Glennon, Vincent J., Journal for Research in Mathematics Education, v1 n3, pp162-172, May '70
Descriptors--*Elementary School Mathematics, *Instruction, *Number Systems, *Research, Grade 4, Number Concepts

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Marilyn J. Zweng, University of Iowa

1. Purpose

This investigation examines the effect of studying nondecimal numeration systems and of studying decimal numeration only, on fourth graders' understandings of (a) the decimal system, (b) processing decimal numerals (i.e. computation), and (c) place-value systems in general.

2. Rationale

The authors quote three prior studies related to teaching nondecimal numeration systems: Brownell's (1964), Jackson's (1965), and Schlinsog's (1965). With one exception, thre three investigators found that a study of nondecimal numeration was no more effective than the study of decimal numeration alone in furthering an understanding of decimal numeration concepts and/or their application to computation. Jackson's study was the exception. He found that with fifth graders, studying numeration systems with bases different from ten increased the subjects' understanding of base ten. His findings for seventh graders, however, concur with the other two investigators. Brownell and Schlinsog studied, respectively, second graders and sixth graders.

With respect to other literature, the authors note, particularly, the position of Dienes who claims that in order to form true abstractions rather than simply associations, students should be exposed to the greatest possible number of instances of involved variables, which in this study would imply the greatest possible number of bases.

3. Research Design and Procedure

Four fourth grade classes participated in the experiment. One class (GO) served as the control group and received no instruction in numeration. The class designated G1 received instruction in decimal numeration only. G3 received instruction in bases three, five and ten. The subjects in group G5 were taught bases three, five, six, ten and twelve. One of the two experimenters taught all four groups.

In each of the three experimental groups, the same concepts were taught and the same teaching aids were used, with the exception that they were adapted to the treatment.

Instruction took place over a period of nine consecutive school days for 30 minutes per day.

A pre-test, post-test and retention test were administered to each subject. The pre-test and retention test were the same form, and the post test an equivalent form, of a 48-item multiple choice test. The retention test was given six weeks after the end of instruction. Each test consisted of three 16-item subtests designed to measure the three understandings indicated under "Purpose" above.

In addition, IQ scores, obtained from the Otis Mental Ability Test, and mathematics achievement scores, obtained from the Stanford Achievement Test, were available.

Pre-test scores were used as co-variate measures.

A separate 4 x 3 x 3 analysis was performed for each of the three "understandings": 4 treatments x 3 levels of IQ x 3 levels of mathematics achievement. In addition to the multivariate analysis of co-variance, a univariate analysis of co-variance was utilized in instances where the multivariate analysis indicated that differences existed.

4. Findings

The findings below have been selectively chosen by the abstracter on the basis of their importance.

Understanding the Decimal Numeration System

On post-test measures of "Understanding the Decimal Numeration System" the totality of the three groups studying numeration performed better than the control group. However, there was no difference between the groups studying nondecimal bases (G3 and G5) and the group studying decimal numeration only (G1).

There was no treatment effect on retention test scores.

Understanding Processing of Numerals (Computation)

No differences among treatments were observed on either the post-test or the retention test with respect to understanding computation.

Understanding a Place-Value System in General

On the post-test of "Understanding Place-value Systems," it was found that (a) the three groups studying numeration performed better than the control group, and (b) the two groups studying only decimal numeration.

On the retention test there was no observed treatment effect.

5. Interpretations

The authors state, "If one wishes to foster, at the fourth-grade level, understanding of the decimal system, the available evidence suggests that only the decimal system need be taught. Also, if one wishes to foster understanding of both decimal and nondecimal systems, the implication is that both decimal and nondecimal systems should be taught." They also observe the following:

(1) There is no adverse effect from teaching (nondecimal) numeration to fourth graders.

(2) Understanding computation with decimal numerals appears to be more difficult than understanding the decimal system itself.

(3) In order for children to retain their understandings of numeration, a maintenance program must be implemented.

(4) Children of one IQ or mathematical achievement level do not benefit more than others from a study of numeration, either decimal or nondecimal.

Abstracter's Notes

In view of this and other studies it appears that mathematics educators should reappraise their position that nondecimal numeration systems should be taught in order to help children better understand the decimal numeration system. We now have considerable evidence that this "better understanding" simply does not occur.

If the time spent on nondecimal bases and the good techniques utilized in their instruction, such as actual grouping of objects or pictures of objects into sets corresponding to the powers of the base, were devoted to base ten concepts we might expect much better competence and understanding of decimal numeration than is the present outcome of our "modern mathematics" programs.

This last comment is not supported by the observations of the investigators since their group studying only decimal numeration achieved no better than the other two experimental groups; however, when one considers the cumulative effects of a maintenance program over a period of three or four years, the observation seems warranted.

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EJ 013 673 310 SE 500 959

THE EFFECTS OF SMSG TEXTS ON STUDENTS' FIRST SEMESTER GRADE IN COLLEGE MATHEMATICS Flanagan, S. Stuart, School Science and Mathematics, v69 n9, pp817-820, Dec '69

Descriptors--*Academic Achievement, *College Mathematics, *Learning, *Mathematics Education, Educational Background, Student Evaluation, Undergraduate Study, [School Mathematics Study Group (SMSG)]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by F. Joe Crosswhite, The Ohio State University

1. Purpose

The purpose of this study is adequately described by the title of the report. The basic question addressed was, "Does studying from SMSG texts in high school, as opposed to traditional texts, significantly affect the performance of students (as indicated by course grades) in the first semester of college calculus."

2. Rationale

The author refers to the implicit assumption that students who study from SMSG textbooks would achieve significantly higher in college and to the limited research basis testing this assumption as providing a rationale for the study.

3. Research Design and Procedure

Subjects (730) for the study were drawn from entering freshmen in the College of Arts and Sciences at the University of Virginia in the 1966-67 academic year. Subgroups for analysis were defined according to their study of SMSG texts in high school: (A) at least one but less than four semester, (B) four semesters or more, or (C) no study from SMSG texts--and by their SAT mathematics score--(G) greater than or equal to 680, (H) less than 680 but greater than 600, or (I) less than or equal to 600.

The author indicates that a predictor equation using final grade in first semester calculus as the criterion variable and high school mathematics background, class rank, and SAT and CEEB scores was generated. Neither the prediction equation or correlations are reported, and the relation of this investigation to the analyses which are reported is not clear.

The author indicates that covariance was used for statistical control of differences in "intelligence" when comparisons were made. The definition of "intelligence" is not given, no covariates are identified, and no covariance tables are reported.

4. Findings

No significant differences between groups A, B, and C with respect to achievement in first semester calculus were found. The hypothesis of no significant difference between groups A, B, and C within groups G, H, and I was rejected at the .05 level. Subsequent analyses revealed a significant difference only for groups B and C within group I. The only data reported are group means on the criterion variable.

Abstractor's Notes

This article reveals the serious limitations of such an abbreviated research report. The article is essentially the same as the author's dissertation abstract (see D.A. XXIX, 1433-A). It elaborates on the abstract only through the insertion of introductory paragraphs and a table of group means on the criterion variable. Both editors and authors should consider whether an abstract provides sufficient information to constitute a research article in a journal. In this case the almost complete absence of data, incomplete description of sample, inadequate presentation of analyses, etc., simply leave too many gaps for the reader to fill. Obvious questions are suggested which may have been adequately answered in the dissertation. Among the more fundamental of such questions are:

1. Was the categorization of high school texts really well defined? Could all non-SMSG texts be accurately described as "traditional"?
2. Since the only significant difference reported involved groups B and C within group I, shouldn't one consider the possible bias introduced by a researcher imposed minimal high school mathematics background (at least four semesters) for group B? If adjustments were made for differences in high school background they are not revealed in the report.
3. How is the reader expected to judge the validity of the design, analyses, or conclusions when so little data is reported and the descriptions of sampling procedures, sample characteristics, treatment, etc., are so limited?

F. Joe Crosswhite
Ohio State University

EJ 039 510 270 SE 503 266
ATTITUDE CHANGES IN A MATHEMATICS LABORATORY UTILIZING A MATHEMATICS-THROUGH-SCIENCE APPROACH Higgins, Jon L., Journal for Research in Mathematics Education, v1 n1, pp43-56, Jan '70
Descriptors--*Instruction, *Research, *Secondary School Mathematics, *Student Attitudes, Learning Laboratories, Mathematics Education

Expended Abstract and Analysis Prepared Especially for I.M.E. by Thomas E. Kieren, University of Alberta

1. Purpose

The study was designed to explore the nature of naturally occurring interest groups with respect to an activity approach to mathematics instruction. Specifically it was designed to seek the existence of such groups and to relate group membership to mathematics achievement and attitude.

2. Rationale

The study had a practical and a theoretical rationale. The study was part of an evaluation of SMSG Mathematics Through Science (MTS). It arose partly as a follow-up to teacher comments as to the mixed reaction to MTS on the part of students. In theory, it has been suggested that manipulative materials and activity contribute to mathematics learning at the junior high level both as a basis of ideas and a motivating force. Within the context of specific scientifically oriented material, this study attempted to more specifically define the motivational and attitudinal contribution of such activity.

3. Research Design and Procedures

The study was conducted in 29 volunteer grade 8 classrooms. The teachers were experienced and many had experience teaching science. The students had not used MTS previously and most of the classrooms had programs using the same text (20 of 29). Because of previous work and teacher comments, the chapter "Graphing, Equations and Linear Functions" was used. The teachers in this study received four in-service sessions.

The study was conducted over a five-week period in the spring of the year, with four weeks of instruction and one of testing. Three achievement tests and 17 attitude scales chosen from the National Longitudinal Study of Mathematical Abilities (NLSMA) test batteries were used as pre- and post-tests.

The analysis proceeded in three stages. In the first stage, tests for correlated samples were used to try to identify tests and scales on which significant pre-post mean differences existed. This stage was used to look for achievement gains and particularly to identify attitude scales on which changes were indicated.

Given that attitude scales existed on which significant change occurred, the second stage of analysis sought to identify naturally occurring interest groups. This was done by a proceudre called heirarchical group analysis, a procedure developed and modified by Ward, Ward and Hook, and Johnson. This analysis starts by considering single students as groups and then stepwise, successively pairing at least two groups in order to minimize the increase of within group variance based on profile vectors of attitude scores. The process stops when the increase in variance becomes too large in the eyes of the researcher. Because of prohibitive execution time this procedure was carried out only on two random samples each consisting of 3 to 4 students per class. Given the existence of such groups an interpretive analysis was carried out to characterize the groups.

The third stage of analysis, using one way ANOVA sought to find differences among groups on achievement and attitude scales.

4. Findings

The initial analysis showed the following results. Significant gains were reported on all three achievement tests. Of the 18 attitude scales, attitudes were significantly lower as measured by five scales such as "Fun vs. Dull", "Ideal Math Self Concept" and "Take more Math". None of the actual score differences were large with respect to the standard deviations. On the scale "I think Father uses Mathematics", the attitude score increased significantly.

The hierarchical group analyses used these six significant scores as profile vectors. It was stopped after reducing the number of groups to eight in both samples after which the within group variance increase was considered too large. At this point univariate ANOVA on the six attitude scales revealed between group differences only the "I think Father uses Math" scale. This difference occurred in both samples and seemed to be due to low scores in one small group in each sample. In characterizing groups it was found that in each sample four of the groups accounted for most of the students. A small group in each sample reflected strong negative attitude changes and a small group reflected strong positive changes.

On the remaining 12 non-profile attitude scales univariate analysis revealed only one significant difference, that on

"Debilitating anxiety" in sample 1. The unfavorable attitude group appeared to have a markedly high debilitating anxiety score. On the three pre and post treatment achievement tests, between attitude group differences occurred only on one scale for one sample. Again the unfavorable attitude group plus one other small group appeared to have the markedly low scores.

5. Interpretations

The author makes the following interpretations

Although attitude clusters existed they did not seem to be related to achievement

There appeared to be groups who really liked and really disliked the MTS unit, but these were small groups.

Most attitude changes were small.

The test did not differentiate between mathematics and the mathematics classroom mentioning only the former.

Students appeared to learn mathematics from the unit.

Abstracter's Notes

The hierarchical group analysis is an interesting procedure. In this case it did not seem to produce groups interpretable by their attitudes. Perhaps other procedures would.

The author's notion of the importance of relating classroom characteristics to attitude is highly important in studying laboratories.

Perhaps the most significant feature of the study was that it sought to define explicit student characteristics and relate these to outcome performances. Other studies with more clearly defined class activity characteristics, with different student characteristics, or with a greater variety in types of outcome variables may produce relationships which were not clear here.

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EJ 013 672 310 SE 500 956

WHEN TO CORRECT ARITHMETIC PROBLEMS--A CRITICAL VARIABLE

Johnston, James O.; and others, School Science and Mathematics, v69 n9, pp799-805, Dec '69

Descriptors--*Academic Achievement, *Arithmetic, *Elementary School Mathematics, *Elementary School Students, *Learning, Performance Factors, Problem Solving

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Loye Y. Hollis, University of Houston

1. Purpose

The purposes of this study were:

(a) To determine whether knowledge of results after each response on an arithmetic exercise would result in performance different from that obtained when knowledge of results is provided following completion of the exercise.
(b) To determine whether knowledge of results after each response on an arithmetic exercise would result in performance different from that obtained when knowledge of results is provided after a 24-hour delay.
(c) To determine whether knowledge of results after completion of an arithmetic exercise would result in performance different from that obtained when knowledge of results is provided after a 24-hour delay.

2. Rationale

Research concerning knowledge of results has typically focused on the question, "What effect does delaying of results after a response have on learning?" Such studies have usually been conducted in a laboratory and have dealt with very short delay periods. Some studies have found that delaying knowledge of results impedes learning. However, others have failed to find this effect. One study suggests the effect of delayed knowledge of results upon learning may be due to the activity taking place between the response and the knowledge of results rather than the amount of elapsed time.

3. Research Design and Procedure

Three fourth-grade arithmetic classes were used as subjects. Group 1 consisted of 25 pupils, Group 2 of 21 pupils, and Group 3 of 22 pupils. The content used was an introduction to fractions. All groups were taught by the same experimenter and were assigned identical worksheets consisting of 18 problems per day for the seven days of the experiment. The content of the worksheets increased in difficulty each day.

The procedures used with each group in the 45-minute lesson period was for instruction concerning the concepts and procedures for solving the problems was given orally. To control for extraneous sources of knowledge of results, pupils were not permitted to ask questions. Worksheets containing the lesson problems were then distributed. Group 1 was given the answer to each problem as it was solved. Group 2 was given the answers after all the problems were solved. Group 3 was given the answers to the problems the following day. All groups corrected their own papers.

Scores on the seven daily assignments were summed and considered as a single criterion measure. Because of the differences found to exist between groups on a pre-test, criterion measure scores were analyzed as a single classification analysis of covariance. Differences among treatments were found to be significant ($p < .005$).

In order to determine the appropriateness of the analysis of covariance technique, a test was made to determine whether the assumption of homogeneity of within-class regressions had been satisfied. The obtained F ($F = 2.43$) was not significant ($p > .05$), indicating that this assumption had been satisfied.

4. Findings

The results of the study indicated the following:

(a) Performance for the per problem knowledge of response group was superior to performance by the end of the period knowledge of response ($p < .05$).
(b) Performance by the per problem knowledge of response group was superior to performance by the 24-hour delayed knowledge of response group ($p < .01$).
(c) The 24-hour delayed knowledge of response group did not differ significantly from the end of period knowledge of response group.

5. Interpretations

(a) This study supports the position that keeping the delay period free of formal activity improves performance.
(b) Though no attitude measures were employed, differences in attitude and motivation were observed between the per-problem group and the other two groups.
(c) Since some research presents evidence that with verbal materials and highly verbal older children and college students, delayed knowledge of response may be as effective as immediate knowledge of response, research

should be carried out to determine how the age variable interacts with knowledge of results conditions.

(d) If other research should substantiate the results reported here, educational practice should be modified to permit this learning principle.

Abstracter's Notes

This experiment, although a rather sophisticated statistical analysis was employed, could be classified as action research. And, this type of activity should be encouraged.

Several questions could be raised concerning the design and procedure.

1. How were the subjects (classes) selected for the treatments?
2. How valid are scores received from pupils scoring their own work?
3. Was the time of day when the pupils were given the treatments considered?
4. Would a post-test over all the concepts taught have yielded any additional data?

It would seem that the authors could have developed additional interpretations and/or implications from the study. For example, the study would seem to support the use of programmed materials.

Loye Y. Hollis
University of Houston

EJ 011 987 140 SE 500 887
EVALUATING MATHEMATICS COURSES FOR PROSPECTIVE ELEMENTARY SCHOOL TEACHERS Moody, William B.; Wheatley, Grayson H., School Science and Mathematics, v69 n8, pp703-707, Nov '69
Descriptors--*College Mathematics, *Course Evaluation, *Evaluation, *Elementary School Teachers, *Teacher Education, Undergraduate Study, [University of Delaware]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Roland F. Gray, University of British Columbia

1. Purpose

The purpose of this study, as stated, was "to examine the efficacy of activity-oriented instruction in the learning of multiplication."

2. Rationale

Piaget was cited as suggesting the probable superiority of activity learning: "To know an object is to act upon it...to modify, to transform the object, and to understand the process of transformation, and as a consequence to understand the way the object is constructed." The authors noted a lack of research in the area of activity learning and postulated that if activity-oriented instruction is superior to conventional instruction then it should result in either higher original learning, higher transfer, or higher retention. Otherwise prediction of such increments by Piaget, Dienes, Bruner and others is open to question.

3. Research Design and Procedure

Hypotheses to be tested in this study were stated in the null form as follows:

There is no difference between activity-oriented and conventionally instructed students in:

a. original learning
b. transfer of learned multiplication facts
c. retention of learned multiplication facts

The basic design was that of a four-treatment group study conducted over a period of four weeks. All groups were taught by the same instructor. A sample consisting of 90 grade three pupils with no previous instruction in multiplication, of lower-middle socio-economic background, and with a mean I.Q. of 95 were randomly assigned to four treatments as follows:

a. Treatment group A received daily multiplication instruction beginning with activity in which subjects manipulated instructional materials. Subjects were taught to respond to computational examples by appropriate arrangement of manipulative objects. No emphasis was placed on memorization.
b. Treatment group B pupils were taught basic multiplication facts by rote memorization procedures. No models or manipulative materials were used.
c. Treatment group RW pupils were taught by the same approach as group R, but with the added practice of solving word problems.
d. Treatment group C were taught addition.

Pre-tests, post-tests, retention-tests and transfer tests were administered to all subjects. Original learning was measured by a computation type test and transfer by a word problem test. Retention of original learning and of transfer learning were measured after 6 and 8 weeks respectively in which intervals no multiplication was taught.

Three orthagonal comparisons between treatment means were performed on four sets of scores to test the three hypotheses. The A, R, and RW scores were contrasted with the Treatment C scores to ascertain whether learning, transfer and retention occurred as a function of instruction. Treatment R scores were contrasted with RW scores for effects of instruction on solving word problems. Treatment A scores were compared with R and RW scores for differential effects of activity-oriented instruction.

4. Findings

The findings are summarized serially as follows:

a. Original learning occurred as a function of instruction ($p < .02$)
b. Transfer of learned facts did not occur for the instructed subjects ($p > .05$)
c. RW subjects were not superior in word problem performance ($p > .05$)
d. All instructed subjects lost original advantages over the non-instructed group over the retention interval ($p > .05$)
e. Activity-oriented instruction did not result in superior original learning, transfer or retention ($p > .05$)

Consequently, the three null hypotheses were not rejected.

5. Interpretations

The authors note the limitations of small size, atypical characteristics of the sample and low level of original learning of all treatment groups. They conclude "If further research on the effect of activity-oriented instruction replicate the findings presented, then some theoretical 'appearance saving' may be called for by those systems which predict the superiority of activity learning."

Abstractor's Notes

To the limitations noted above, one must add the short period of the experiment. It may turn out with further study that any superior effects of activity-oriented instruction will be more nearly related to more thoroughly developed concept understandings which may render different computational situations specific cases of generalized and internalized concept systems. This, in fact, may more closely approach the meaning and intent of such activity advocates as Piaget and the others cited than what the authors have suggested in the introduction to this study. Four weeks may indeed be too short a period of time for any such outcome to become manifest.

On the other hand, this study certainly points to the need for further experimentation to determine what outcomes, if any, may be related to activity-type procedures. This study does not support any immediate conclusion of their innate superiority. In view of a growing trend toward activity-type instruction we could find ourselves, without care, once again pursuing activity for its own sake.

Roland F. Gray
University of British Columbia

EJ 039 002 110 SE 503 277

PARTS OF A SYSTEMS APPROACH TO THE DEVELOPMENT OF A UNIT IN PROBABILITY AND STATISTICS FOR THE ELEMENTARY SCHOOL Shepler, Jack Lee, Journal for Research in Mathematics Education, v1 n4, pp197-205, Nov '70

Descriptors--*Curriculum Development, *Elementary School Mathematics, *Instructional Materials, *Probability Theory, Grade 6, Instruction, Research

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jack E. Forbes, Purdue University, Calumet Campus

1. Purpose

To test the feasibility of teaching topics in probability and statistics to a class of sixth-grade students and to construct a set of materials and procedures in probability and statistics for sixth-grade students using the content outline, task analysis, and grade level recommendations developed by Shepler, Harvey, and Romberg and the recommendations of Bloom for producing mastery learning.

2. Rationale

The absence of instruction in concepts in probability and statistics in elementary schools although research has shown such instruction possible and recommendations for such instruction have been made is attributed partly to lack of adequate materials. The construction of adequate materials and the demonstration of their adequacy could lead to removing this instructional deficiency.

3. Research Design and Procedure

A pilot study with five sixth-grade students was used as part of the process of establishing and sequencing behavioral objectives and of selecting or developing materials and procedures within the overall framework of Shepler et al and Bloom. The resulting four-week unit was taught to 25 sixth-grade students (mean I.Q. = 117.7, S.D. 6.8) who "possessed no reading difficulty" by a project assistant who was a certified elementary school teacher with an undergraduate concentration in mathematics and two years of teaching experience. The investigator worked with the teacher and with the class.

A criterion test of 72 items to inventory student behavior relative to 14 behavioral objectives was constructed by the investigator and used as a pre-and posttest. Both an "arbitrary" 90/90 criterion (90% of the students should score 90% or better relative to each objective) and a "practical" criterion requiring that at least 22 of the 25 students score at or above the following levels on an objective measured by n items:

$$\frac{n-1}{n} \quad \text{for } n \geq 5$$

$$\frac{n}{n} \quad \text{for } n < 5$$

were used to determine the achievement of the goals of the instruction.

4. Findings

Pretest and posttest means and variances on the 72 item test were 27.28, 66.8 and 74.13 and 11.17 respectively. The class satisfied the "practical" criterion relative to ten of the fourteen objectives and was "very close" on an eleventh. It satisfied the 90/90 criterion relative to five of the objectives.

5. Interpretations

The author concludes that the data establish the feasibility of teaching most of the included topics to sixth grade students. He attributes much of the success of the materials to the systematic approach to their development and to the mastery learning techniques employed in their use. He observes that the special features of the study (above-average students, special training for an already well-trained teacher, direct involvement of the investigator in the classroom) preclude extrapolation from this study to "regular" sixth-grade classroom situations. He recommends further developmental work with materials in this area and application of the procedures used in the construction of these materials to other curriculum areas.

Abstracter's Notes

The author's claim that the absence of instruction in probability and statistics in elementary school is "partly due to a lack of adequate materials" is no doubt true. However, it seems equally likely that it is due to the inability or unwillingness of teachers to teach concepts from these areas. This conjecture is reinforced by some of the author's comments about the study.

Concerning the evaluational instrument, he states "The test had content validity since the items were criterion items based on the instructional analysis and materials. They were written to test specific behavioral objectives of the instructional treatment." Later, he observes, in regard to the objectives relative to which the instruction failed, "A close analysis of the items measuring the three objectives...indicated that poor wording of some items...were plausible for not achieving the objectives." Could not "poor wording" (e.g. contextual, semantic, or sequence cuing) have produced correct responses for other items? In short, is a question a criterion question or a test a criterion test because one intends it to be? While these may not be questions of "content validity" one would feel more secure if the test had been subjected to something like a "consensus of experts" evaluation of what level of what behaviors were really being measured, regardless of intent.

The investigator's observations about the 90/90 criterion when few test items are used and his rationale for opting for his "practical criterion" are valid and important.

No doubt this study was a valuable learning experience for the investigator. It is one thing to construct behavioral analyses of some piece of the curriculum on a theoretical level. It is quite another to subject these analyses to the crucial test of real classrooms with real teachers and real students. Unfortunately, like many such studies, this one was carried out under a design in which neither teachers nor students were "really real."

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EJ 010 396 110 SE 500 852
WHAT MATHEMATICS SHALL WE TEACH THE LOW ACHIEVER? Weiss, Sol,
The Mathematics Teacher v62 n7, pp571-575, Nov '69
Descriptors--*Curriculum, *Low Achievers, *Program Content, *Secondary School Mathematics, Junior High Schools, Low Ability Students, Mathematics Education

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Donovan A. Johnson, The University of Minnesota

The purpose of this survey was to determine what mathematical topics are recommended by mathematics educators for inclusion in mathematics courses for low achievers in the junior high school.

A low achiever is defined as a student who (1) is achieving below his assigned grade level, (2) is not mentally retarded, and (3) has no serious emotional problems.

The recommendations were obtained by sending a questionnaire to 200 leading mathematics educators in the United States. Of these 200 questionnaires, 172 were returned, and of these, 155 were usable for recording responses. The questionnaires contained a list of 47 mathematical topics. The respondents were asked to rate these on a scale from 1 (n0) to 5 (yes) as to whether or not, in their opinion, each of these topics should be included in the mathematics program for low achievers.

The average response was computed for each topic. A topic was classified as recommended if (1) the average response was above 3.5, (2) more than half rated the topic 5, and (3) the number of 5's (yes) was twice the number of 1's (no). The results are recorded for each of the 47 topics.

On the basis of these criteria, 29 of the 47 topics were recommended. Only 3 topics were rejected namely, vectors, linear programming and truth tables. The remaining topics were classified as undecided.

The study tends to confirm the existence of widely differing views of what mathematics is most suitable for low achievers. This is particularly evident with respect to the so called "social arithmetic" topics. Comments by respondents indicate that many consider that what content is taught is less important than how the content is taught to low achievers. Others felt that it was the depth, organization, or applications of the topic which was the determining factor.

Abstracter's Notes

The study indicates the opinions of 155 experts on 47 topics for the low achiever in junior high school. The topics recommended do not necessarily provide the best or the complete program for low achievers. The study does not identify the source of difficulty of low achievers nor does it suggest that it is possible for low achievers to learn these recommended topics. Topics not included such as game theory, codes, space curves, topology, magic squares, nomographs, projections might be of equal relevance.

Donovan A. Johnson
University of Minnesota

SECTION 3:

MATHEMATICS ACHIEVEMENT AND ITS CORRELATES

EJ 038 737 010 SE 503 275

THE RELATIONSHIP BETWEEN A SEVENTH-GRADE PUPIL'S ACADEMIC SELF-CONCEPT AND ACHIEVEMENT IN MATHEMATICS Bachman, Alfred Morry, Journal for Research in Mathematics Education, v1 n3, pp173-179, May '70

Descriptors--*Achievement, *Mathematics, *Research, *Self Concept, *Social Studies, Ability, Grade 7, Secondary School Mathematics

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Kenneth J. Travers, University of Illinois

1. Purpose

To investigate relationships between mathematics achievement and self-concept in several subject-matter areas, particularly mathematics, at the seventh-grade level.

2. Rationale

The study continues the work of Brookover, Patterson and Thomas (W. B. Brookover et al., Self-Concept of Ability and School Achievement, Cooperative Research Project 845, Office of Research and Publications, East Lansing: Michigan State University, 1962) but restricts achievement measures to mathematics. That work is reported to hypothesize that self-concept of ability is a functionally limiting factor in school achievement.

3. Research Design and Procedure

This is a correlational study, using Pearson rs and partial correlations. That is, linearity of relationships is assumed.

Three instruments were administered to a sample of 408 seventh-grade pupils (210 males and 194 females--according to Table 3 reproduced below) in the Portland, Oregon, public schools during 1966-1967, resulting in the following scores:

a. Academic self-concept. The General Self-Concept of Ability Scale as developed by Brookover et al. (1962). Sections of self-concept in mathematics and social studies were also used.
b. Mathematics achievement. Portland Elementary School Mathematics--Upper Test, developed by the Portland, Oregon, School District.

c. Intelligence. School College Ability Test (SCAT 4A), taken by all seventh-grade pupils in the fall of 1966.

4. Findings

a. Mathematics achievement vs. self-concept

Substantial correlations (.45 or greater) were found between mathematics achievement and both general self-concept scores. The social studies correlations were weaker, but still exceeded critical values (f0r df = 200 and an alpha of .05, a critical r of .14 is reported).

The effects of school ability (intelligence) were then removed from the correlations. Smaller but reportedly significant coefficients were obtained between mathematics self-concept were found for the males on the full correlations and for both sexes on the partial correlations.

b. Self-concept interrelationships

Substantial pairwise intercorrelations (.50 or greater), all significant at the .05 level, were reported between general, mathematics and social studies self-concept measures.

c. Intelligence, mathematics achievement and mathematics self-concept

The following table (Table 3, p. 177 in the report) summarizes these data.

CORRELATIONS AMONG MATHEMATICS ACHIEVEMENT, MEASURED INTELLIGENCE, AND MATHEMATICS SELF-CONCEPT

	Correlation coefficients				
	Zero order: No variable controlled		Partial: Third variable controlled		
Variables correlated	Males N=210	Females N=194	Variables controlled	Males N=210	Females N=194
Math Ach--Intelligence	.77	.78	Math SC	.72	.76
Math Ach--Math SC	.48	.55	Intelligence	.29	.36
Math SC--Intelligence	.40	.34	Math Ach	.05	.16

df = 200.

$r_{.05} = .14$.

5. Interpretations

The study revealed statistically significant relationships between seventh-grade pupils' mathematics achievement and both mathematics and general self-concept measures. It is suggested that if pupil achievement can be improved by improving self-concept, then this might best be done by emphasizing self-concept in the subject area.

Abstracter's Notes

This study makes extensive use of two instruments which are not well known--the Portland Elementary Mathematics--Upper Test and the Brookover *et al*. General Self-Concept of Ability Scale. Psychometric characteristics of these tests (reliability, validity, etc.) would have been extremely useful information. For example, to what extent have the reported correlations been attenuated due to errors of measurement (i.e., low reliability in the instruments)?

The investigator utilized partial correlations in an attempt to represent more clearly the underlying relationships between the measures. It should be noted that the degrees of freedom and the standard error term for a partial correlation differ from those for full correlations (see, for example, Guilford, 1965, p. 341). A more enlightening procedure may have been to employ multivariate generalizability theory. (For an introductory treatment of this topic, see, for example, Rajaratnam, Cronbach, and Gleser, *Psychometrika*, 30: 395-418, 1965). This technique enables one to estimate interrelationships between variables when effects of errors of measurement have been removed.

Finally, further information on method of selection of pupils (random within schools? stratified to provide socio-economic representativity?) would be helpful. And since stability of the measures is of interest, were the tests administered at the same time of year? Under comparable conditions?

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University of Illinois

EJ 010 075 010 SE 500 841
REFLECTIVENESS/IMPULSIVENESS AND MATHEMATICS ACHIEVEMENT
Cathcart,W. George; Liedtke, Werner, Arithmetic Teacher v16 n7, pp563-567, Nov '69
Descriptors--*Achievement, *Mathematics Education, *Reaction Time, *Response Mode, *Testing, Comparative Analysis, Grade 2, Grade 3

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Paul C. Burns, The University of Tennessee

1. Purpose

Cathcart and Liedtke investigated the hypothesis that reflective (slower-answering) pupils would be higher achievers in mathematics than impulsive (faster-answering) pupils, since they tend to reflect upon the quality of their answer in contrast to the more unconsidered response of the impulsive pupils.

2. Rationale

The context of the study was centered about Jerome Kagan's ideas about differing cognitive styles in pupils. Kagan had found that differences in cognitive style might exist even if children "are of equal intelligence...". Furthermore, research by Kagan reported in 1965 reported some interesting relationships between speed of response and various aspects of reading skills at the primary level, reflective children achieving better word recognition scores.

3. Research Design and Procedures

The sample consisted of 46 grade 2 pupils and 12 grade 3 pupils from a county school in the suburban area of Edmonton, Alberta.

The steps involved in the actual conduct of the investigation involved:

a) development of a mathematics achievement test which included subject items to measure understanding and application of mathematical concepts, word problems, and basic facts. Reliability oi the overall test was 0.69. Reliabilities for subtests were: concepts, 0.78; problem solving, 0.91; basic facts, 0.98.
b) development of test items adopted from Kagan's "Matching Familiar Figures" (MFF) to enable the examiners to identify different cognitive styles (reflective-impulsive) of pupils.

c) development of tests of length conservation and area conservation similar to those developed by Piaget.
d) administration of MFF and conservation tests individually to subjects by four examiners; of the achievement test to subjects as a group test.

To analyze the data, the investigators identified the impulsi.e subject as one who responded to MFF test in less than 17.4 seconds on the average and who made 7 or more errors on the six item test. The reflective subject was defined as one who took 17.4 seconds or more to respond but made 6 or fewer errors. This provided 26 "impulsive" and 18 "reflective" pupils.

The chi-square statistic was used as an indication of the relationship between cognitive style and the variables of sex, age, intelligence, grade level, and conservation. The same type of design was used in a two-way analysis of variance, using as criteria the four aspects of achievement: concepts, problem solving, basic facts, and total achievement.

4. <u>Findings</u>

The findings included:

a) grade 2 subjects in the sample tended to be impulsive
b) grade 3 subjects were predominantly reflective (Kagan also found that cognitive style is a function of age.)
c) an "apparent trend" existed for high intelligence groups (115.8 or more) to be more reflective than the lower intelligence group (108 or less)
d) no significant amount of interaction existed between cognitive style and sex, grade, age, intelligence, or conservation
e) a statistical significance existed between the means of the two groups on the achievement subtests (problems, basic facts, and total score) when sex, age, intelligence, and conservation scores were controlled in the two-way analysis. (There was no significant differences between the two groups on application of concepts.)

5. <u>Interpretations</u>

The investigators concluded that reflective pupils obtain higher mathematics achievement scores than impulsive pupils, suggesting that speed of response is not a valid criterion of ability to achieve in mathematics at the primary school level, and that sometimes standardized mathematics tests may put the reflective child at a disadvantage.

Abstractor's Notes

This study should be of interest to the classroom teacher as well as the researcher in the sense that mathematics instruction may be increased by awareness of the existence of differences in cognitive styles within the classroom.

Points or questions may be raised regarding aspects of this investigation, though most of them are recognized by the researchers:

1. How was the original sample selected?
2. To what extent were the results influenced by the fact that:
   a. the grade 3 sample was small (8 subjects) with only one classified as "impulsive"?
   b. there were few items on each subtest of the mathematics achievement test? (A better description of the achievement test, including arguments for its validity as an achievement measure, would have been helpful.)
   c. the mean intelligence quotient was 111.9?
   d. a "median split" on both errors and average time on the MFF test was not used to identify impulsive and reflective subjects?
   e. the MFF and conservation tests were administered by different examiners?

Paul C. Burns
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EJ 038 738 010 SE 503 281
A TWO-STATE SEQUENTIAL STRATEGY IN THE PLACEMENT OF STUDENTS IN AN UNDERGRADUATE MATHEMATICS CURRICULUM Fujita, George Y.; O'Reilly, Joseph P., Journal for Research in Mathematics Education vl n4, pp241-250, Nov '70
Descriptors--*Ability, *College Freshmen, *College Mathematics, *Predictor Variables, *Research, Mathematics

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Donald J. Dessart, The University of Tennessee

1. Purpose

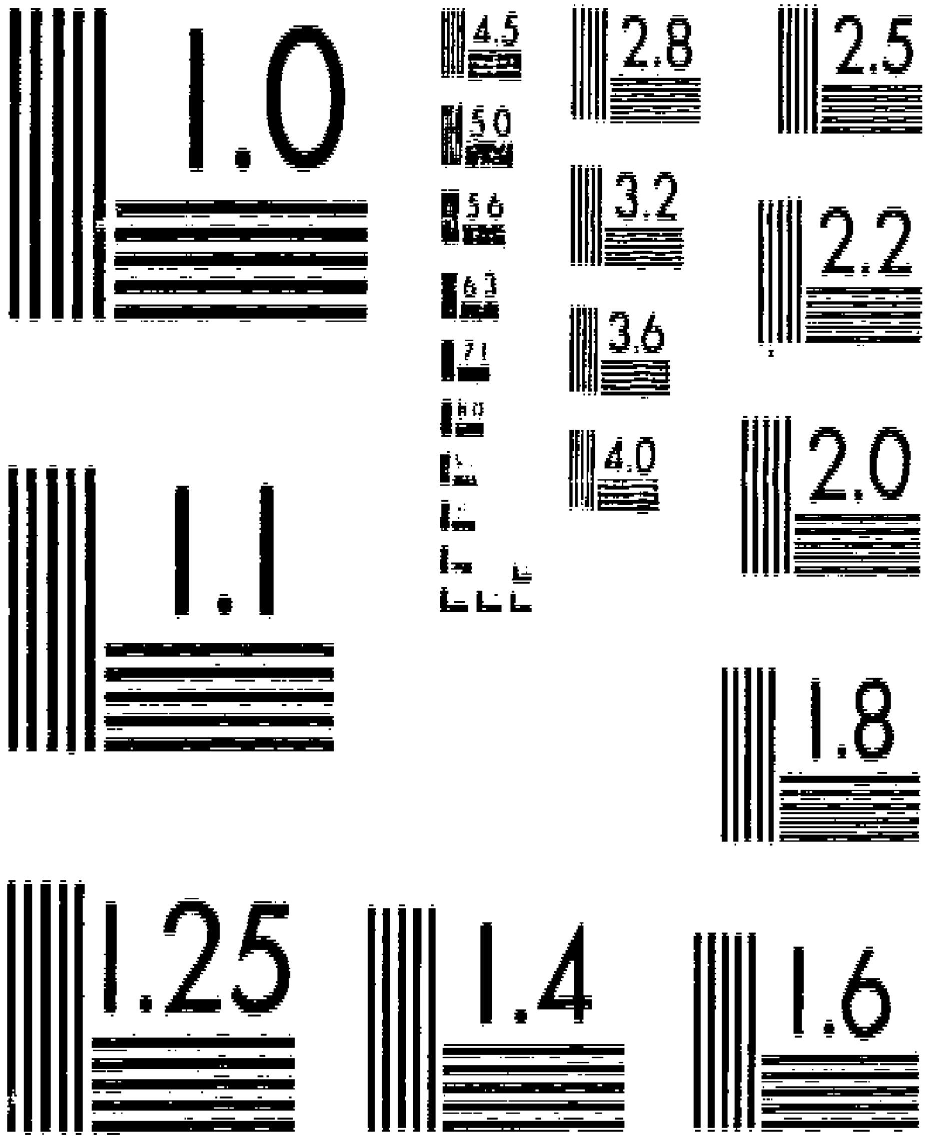
The purpose of this study was: (a) to evaluate various measures of mathematical ability for predicting the academic performance of students in a precalculus course at a large state university; and (b) to develop and test two sequential decision models which could be used in the placement of students.

2. Rationale

A two-stage "standard sequential decision model" proposed by Cronbach and Gleser in which students, after taking an initial test, are classified into three categories: (a) those who are accepted, (b) those who are not accepted, and (c) those who are to be given additional testing, was modified by the investigators. In the modification, called a Pre-Accept Selection Strategy, some students are accepted after the initial testing as in the previous model, but all others are provided an opportunity for additional testing. The acceptability of students is then determined by use of a composite formula developed by analyzing the scores of several tests and a criterion variable through a multiple regression analysis.

3. Research Design and Procedure

The following examinations were available as predictors: (a) the mathematics section of the Sequential Test of Educational Progress (STEP-M); (b) the Scholastic Aptitude Test of Mathematical Ability (SAT-M); (c) the Scholastic Aptitude Test of Verbal Ability (SAT-V); and (d) the Mathematics Test of the Cooperative Test Service (COOP-M). Of 1,139 students enrolled in the precalculus course during the semester under investigation, only 842 received a final grade (A to F), which was used as a measure of performance in the course; and of these 842



MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

students, 460 had previously taken the COOP-M and SAT tests and 325 students had taken all of the instruments listed in (a) through (d) above.

A correlation analysis was performed on the data available for the latter two groups. A chi-square analysis was utilized to help establish upper and lower cut-off scores for the SAT-M and COOP-M tests, and a multiple regression analysis was used to determine a composite prediction formula based on these two tests. These criteria were then employed in the standard sequential decision strategy and the Pre-Accept Selection Strategy to determine the placement of students in the course.

4. Findings

The two predictors having the greatest correlation with the final course grades were the COOP-M ($r = 0.42$) and the SAT-M ($r = 0.40$). The next three predictors in order of magnitude of their correlation coefficients were STEP-M ($r = 0.30$), SAT-V ($r = 0.24$), and sex ($r = 0.22$), with all coefficients significantly different from zero ($p < .01$). Furthermore, the multiple regression analysis, employing all variables and using final grades as a criterion, revealed that the best predictors of final grades were COOP-M, SAT-M, and sex.

With the number of desired predictors reduced to three, the chi-square analysis was used to help determine cut-off scores. In an application of the standard sequential decision model, the SAT-M scores were used in the initial screening with a lower cut-off score of 510 and an upper score of 600. (Note: the graphs for Figures 3 and 4, pp. 247-248, appear to have been interchanged in the journal report.) In the first screen, a total of 49% of the students would have been accepted (21%) or rejected (28%) without further testing, and 51% would have been subjected to further testing. However, 38% of the students, who would have been rejected in the first screen, actually achieved at a level of C or better in the course.

In an application of the Pre-Accept Selection Strategy, an additional 28% of the students (those who were rejected in the standard model) would have received an opportunity for further testing. It was estimated that one-half of the 28% would probably have elected to take the second test; so that the Pre-Accept Strategy would have resulted in a final decision for 35% of the students in the initial screen as compared to 49% in the standard strategy. The final decision for acceptance or rejection of the remaining 65% would have been based on a composite of the SAT-M and COOP-M scores.

5. Interpretations

The investigators concluded that "The advantage of the two-stage sequential selection model is the gain in predictability of course performance, coupled with the most efficient administration of placement tests." Furthermore, since the SAT is generally available for most students, it is a logical test for the first-stage selection process. However, in this study it was found that the use of the SAT-M as a first stage tool would have resulted in a 38% error among those not recommended for placement in the course indicating an apparent inability of the SAT-M to identify low performing groups effectively. On the other hand, the authors noted that "Significant improvement in predicting the math grades was achieved with the addition of a second test, here the COOP-M." (Unfortunately, the reader is provided little information concerning the errors in prediction based upon the composite use of the SAT-M and the COOP-M in the second stage of either selection strategy.)

Abstracter's Notes

Questions may be raised concerning the appropriateness of the sample of students utilized for this study. At the outset of the study, 1,139 students were enrolled in the precalculus course of which 297 had withdrawn before the conclusion of the term. The investigators noted that the students who had withdrawn were similar in every way to the deficiency (D or F) students, which is somewhat surprising as one might have expected that some students would have withdrawn for reasons other than academic. Of the 842 remaining students who completed the course, only 460 had taken both the COOP-M and SAT tests. The authors used this group of students in the study because of their availability, but the representativeness of this sample might be seriously questioned, particularly if the group became the basis for developing selection criteria which presumably could be applied in the placement of a population of students similar to those of the study. If the 460 students cannot be regarded as a random selection from the initial population, it would seem that a bias could easily be introduced, which could lead to inappropriate or inefficient selection criteria.

In the sequential decision models, the SAT-M was used as the first screening tool and a composite of SAT-M and COOP-M was used as the second screen. The SAT-M appeared to lack the ability to identify adequately the low performing group, whereas the COOP-M did not seem to suffer as greatly from this deficiency. Since the first screen is applied to a much larger group of students, one might wonder if the COOP-M would be preferable for the initial test to the SAT-M. Furthermore, since high school grade point averages or high school mathematics course grade averages are usually available, it would seem that their use in the model might have some merit in addition to or in place of the SAT-M.

This study seemed to demonstrate that sequential decision models for placement of undergraduates in mathematics classes offer considerable promise for the future. But as the authors observed, most contemporary mathematics tests are not completely adequate for measuring achievement of students in the newer instructional programs. Since such instruments constitute the very heart of a placement procedure, more research is needed to develop better instruments.

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EJ 002 320 140 SE 500 213

STUDENT MATHEMATICS ACHIEVEMENT AS RELATED TO TEACHING IN-SERVICE WORK Norris, Fletcher R., Mathematics Teacher v62 n4, pp321-327, Apr '69

Descriptors--*Academic Achievement, *Effective Teaching, *Inservice Programs, *Teacher Education, Achievement, Comparative Analysis, Elementary School Teachers, Mathematics Teachers

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Robert E. Pingry, The University of Illinois

1. Purpose

The purpose of this study was to compare the competence and knowledge of two groups of sixth grade teachers; one group involved in an in-service course on mathematical content and the other group of teachers who did not take part in the course.

2. Rationale

The researcher was interested in the effectiveness of an in-service teacher education lecture series. He decided to measure this effectiveness by two tests:

a) one test was administered to the teachers to measure teacher knowledge, and
b) one test was administered to the pupils to measure student achievement.

One of the underlying assumptions of the study was that a teacher's effectiveness can be measured to a certain extent by the achievement of the teacher's students. So to determine whether or not in-service training is effective, the researcher decided to measure pupil achievement. Most previous reports written on in-service experiences state that appreciable gains resulted for the teachers in the criterion used, but no attempt was made to determine whether or not the pupils benefited by the experience.

3. Research Design and Procedure

There were two groups of sixth grade teachers from Nashville, Tennessee in this experiment. One group, the experimental group, attended a series of in-service lectures in which the teachers made an in-depth study of the concepts contained in the pupil textbook. Elementary Mathematics, Patterns and Structures, Book 6. This book was newly adopted in the schools in Nashville and was being used by the teachers for the first time. The course consisted of 14 weekly meetings lasting about two and one-half hours each. No instructional techniques were consciously presented in the course. The emphasis was on mathematical concepts. The classes met from September 12, 1967 to December 12, 1967.

The other group, the comparison group, did not attend the lectures. In an attempt to equalize interest in mathematics teaching in the two groups, the comparison group was asked to fill out a questionnaire or progress report relating to the mathematics they had been teaching. This served to alert the comparison group and remind them of the importance of mathematics.

A 60-item multiple-choice test was constructed for use with the teachers. It was administered to both groups of teachers three times: a pretest at the beginning of the in-service course, a posttest at the end of the course in December, and again at the end of the school year.

A 50-item multiple-choice test was constructed for use with the pupils. It was administered twice: a pretest at the beginning of the school year and a posttest at the end of the school year.

The data analysis was performed by a 2 x 3 analysis of covariance design using the pretest scores as the independent variable. The two treatments used were the experimental treatment (in-service course) and the comparison treatment (no in-service course). The 3 levels used in the design were 3 I.Q. levels: high, middle, and low.

The researcher reported that the analysis of the teacher data on the three test administrations was performed by the method of orthogonal comparisons. The number of students involved was 844; there were 493 in the experimental group and 351 in the comparison group. By the end of the study there were 18 teachers in the experimental group and 15 in the comparison group.

POSTTEST PUPIL MEANS AND VARIANCES
ADJUSTED ON PRETEST SCORES

Ability Levels		Treatments: Experimental	Treatments: Comparison	Ability Level Means
High	Mean	30.23	30.02	30.13
	s^2	24.93	31.41	
Middle	Mean	27.16	24.56	25.86
	s^2	37.42	27.60	
Low	Mean	23.71	22.08	22.90
	s^2	32.69	33.80	
Treatment Means		27.03	25.55	

a. There were significant differences ($p < .05$) between the adjusted means in the three ability levels. The adjusted mean of the high ability level was greater than that of the middle level, and the mean of the middle level was greater than that of the low level. There was a positive relationship demonstrated between achievement and ability.

b. There was a significant difference ($p < .05$) between the adjusted means of the two treatment groups. The adjusted mean of the experimental group was greater than that of the comparison group.

c. There was no significant interaction between the two factors of the design.

The orthogonal comparisons on teacher test scores resulted in three significant comparisons.

a. For both groups of teachers combined there was significant mean gain from the initial administration of the test to the other two administrations.

b. For both groups combined, there was significant mean gain from the post-test administration to the administration of the end-of-year test.

c. The comparison showed that the difference between mean initial scores and subsequent weighted mean scores was significantly greater in the experimental group than the same difference in the comparison group.

5. Interpretations

The researcher concluded that for this study that the in-service study geared to concepts the teachers were teaching was of benefit to the pupils. This was true for all pupils involved, regardless of ability level. The researcher made the conjecture that it would seem from this study that pupils learned more from teachers who have thorough, early, intensive exposure to the entire spectrum of the subject area to be taught than those who haven't had such an experience.

Abstracter's Notes

From the description written by the author of this article it was difficult to determine just what was done in certain aspects of this research.

It was not clear what sampling procedures were used. Did the researcher random sample students and assign them at random to treatments, or did he random sample teachers and assign some to go to the in-service course and some assigned not to go to the course? Is it possible that the teachers who took the course had elected to take it? It was not clear from the description what the number of measures was in each of the cells of the analysis of variance and covariance tables.

Was the same pupil and teacher test used repeatedly or was another equivalent form of the test used? It appears from the description that the same test was used.

Was there much communication during the school day between teachers taking the course and teachers not taking the course? Since blocks of students were assigned to a single teacher was it not possible to separate out the variance attributed to the teacher differences?

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EJ 040 101 520 SE 503 276
THE COMPREHENSIVE MATHEMATICS INVENTORY: AN EXPERIMENTAL INSTRUMENT FOR ASSESSING THE MATHEMATICAL COMPETENCIES OF CHILDREN ENTERING SCHOOL Reys, Robert E.; Rea, Robert E., Journal for Research in Mathematics Education, v1 n3, pp180-186, May '70
Descriptors--*Diagnostic Tests, *Evaluation, *Elementary School Mathematics, *Mathematical Concepts

[See also Rea, Robert E. & Reys, Robert E. Mathematical competencies of entering kindergartners. Arithmetic Teacher 17: 65-74, 1970.]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Vincent J. Glennon, University of Connecticut

1. Purpose

The purpose of the study was to develop a set of test items for assessing the mathematical abilities of children on entering school--The Comprehensive Mathematics Inventory (CMI).

2. Rationale

The mathematical capabilities of young children has been an reas of speculative inquiry for hundreds of years. As an area of scientific study it had its beginnings in the early work of the eminent psychologist G. Stanley Hall, 1883 and subsequent years. In recent years the work has been carried on by the distinguished scholars William A. Brownell and Jean Piaget as well as many others.

Proceeding on the reasonable premise that recent innovations in mass communications have enriched the lives of children beyond that of children of prior generations, Reys and Rea sought to build a more valid instrument than those presently available which teachers "might use to diagnose areas of strength and weakness of entering kindergartners".

3. Research Design and Procedure

Test items were constructed in seven curriculum strands (topics). The strands selected and the number of items in each were number (50), money (22), measurement (34), pattern identification (7), recall (20), vocabulary (27), and geometry (34). Six open-ended questions were also included but not treated in the data analysis.

A precise protocol was established for each item (as evidenced in the seven sample items in Table 1) to ensure uniformity in the administration of the CMI. All materials used in the items were judged to be well within the experiential background of the children.

Following a three-day training session, forty-three research assistants from two campuses of the University of Missouri administered the CMI to 727 entering kindergartners in the metropolitan St. Louis area in the fall of 1968. The six schools containing 30 kindergarten classes were "carefully selected to be broadly representative of subpopulations found in urban areas".

Because of the length of the instrument (200 items), the CMI was administered individually in two test sessions per child requiring a total time of 35-40 minutes.

4. Findings

Reliabilities ranged from .91 to .94 for Part I (money, number and vocabulary), and from .83 to .87 for Part II (geometry, measurement, pattern identification, and recall). All correlations among the seven subscales were significantly greater than zero ($p < .01$). Number had the highest correlation with the other variables (.93); pattern identification had the lowest (.45).

The authors hypothesize that the high intercorrelations among the seven subtexts is due to a general intelligence factor (g) which they suggest might be labelled 'numerosity'.

5. Interpretations

Reys and Rea believe that the construction of the CMI and other similar instruments by other investigators will contribute to the task of developing sound mathematical programs for young children. Too, they believe that such instruments have potential both as diagnostic tools on which to base remedial programs and for identifying the high achiever who needs an enriched program. The authors are continuing their efforts in the further development of the CMI.

Abstracter's Notes

(1) The authors state that there was "no attempt to initially establish a broad curriculum matrix for the selection of test items". But would the CMI be a more valid instrument if it included those additional strands (topics) that are a legitimate part of an early school mathematics program--such as classifying, seriation, set equivalence, set inclusion, counting, and Piagetian conservation tasks?

(2) The seven illustrative test items shown in the report

utilize the enactive and iconic modalities. Yet some kindergarten children are on the symbolic mode. Should not some (few?) test items reflect this modality? Else, how can the teacher determine the upper limits of these children?

(3) Shouldn't a test which is "comprehensive" include an equitable distribution of items over the dimension of types of cognitive learning--*á la* Brownell's four types or Gagne's seven types? No information is provided the reader on the question of how or if such a distribution was built into the CMI.

(4) In addition to the carefully prepared protocol for each item (verbal communication), the reliability of the test is affected in large part by the non-verbal communication between tester and testee. It is a fair question to ask how the research assistants were trained to ensure control over this variable. The report, while it mentions use of videotapes in the training program, does not inform the reader on this important question.

Vincent J. Glennon
University of Connecticut

EJ 038 735 010 SE 503 267
THE EFFECTS OF TWO SEMESTERS OF SECONDARY SCHOOL CALCULUS ON STUDENTS' FIRST AND SECOND QUARTER CALCULUS GRADES AT THE UNIVERSITY OF UTAH Robinson, William Baker, Journal for Research in Mathematics Education, v1 n1, pp57-60, Jan '70

Descriptors--*Academic Achievement, *College Mathematics, *Grade Prediction, *Research, *Secondary School Mathematics, Analytic Geometry, Calculus, Mathematics, Mathematics Education

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Arthur F. Coxford, The University of Michigan

1. Purpose

The purpose of the study was to determine whether the completion of two semesters of calculus in high school had a significant effect on the grades students received in first quarter and in second quarter calculus at the University of Utah.

2. Rationale

Many students complete the Advanced Placement Program course in calculus in high school and do not receive either advanced placement or collegiate credit for their work. These students must, if they do collegiate level mathematics, repeat in college the mathematics they studied in high school. Although opinions abound, there is little evidence upon which one can base judgments concerning the effects of the previous work on the collegiate work.

3. Research Design and Procedure

Graduates for the years 1965, 1966, and 1967 who had completed one semester of analytic geometry but no calculus in high school were selected from five Utah high schools. These students had completed two quarters of calculus at the University of Utah. For these students the independent variables (1) high school grade in analytic geometry, (2) average high school grade in mathematics, (3) rank in high school graduating class, (4) ACT English score and (5) ACT Mathematics were used to predict first and second quarter grades in college calculus. The method used was the Wherry-Doolittle Test Selection Method.

Students who graduated in the years 1965-1967 from the five high schools and who had completed two semesters of calculus in high school and had repeated part of this work at the University of Utah were identified. They were partitioned into three subgroups:

- A11: Completed analytic geometry and one quarter of calculus
- A12: Completed analytic geometry and two quarters of calculus
- A22: Completed two quarters of calculus but not analytic geometry

The previously constructed regression equations were used to predict the grades for first quarter and second quarter collegiate calculus for each group (when possible). These grades were subtracted from the grades actually received. These sets of signed differences were tested by the Wilcoxon Matched-Pairs Signed-Ranks Test and by the t test for significance at the 5 per cent level.

4. Findings

The investigator reported the following findings:

(a) For each subgroup (A11, A12, A22), the mean difference (grade received minus predicted grade) in first quarter calculus grades was significant ($p < .01$) on each statistical test. The grades received were higher than those predicted.
(b) For the subgroup A12, the mean difference in second quarter calculus grades was significant ($p < .01$) on each statistical test. The grades received averaged nearly two-thirds of a point higher than those predicted.
(c) For the subgroup A22, the mean difference in second quarter calculus grades was significant ($p < .05$) on the Wilcoxon test, but not significant on the t test ($p > .05$). The predicted grades were again lower than the grades received.

5. Interpretations

The investigator concluded that the "...results of this investigation indicate that students at the University of Utah will benefit from the high school courses even if they must repeat the courses in college (assuming that "higher grades" are a measure of "benefit"). The author recognized the limitation that the results may be local in nature.

Abstracter's Notes

The problem of gathering evidence concerning the effects of completing two semesters of calculus in high school on similar collegiate work is a difficult one to investigate experimentally. Ideally one would want to randomly assign students wishing to take high school calculus to two treatments: no calculus and calculus. But even here difficulties arise because not all the calculus students repeat the work in college. The present study used regression techniques to predict grades of calculus repeaters. The over-riding question this procedure raises is whether or not the students used to develop the regression equation were similar to those whose grades were predicted. If they were, then one still is faced with the question of how well the regression equations predicted grades. These questions need to be considered by future investigators.

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SECTION 4:

TEACHER EDUCATION AND EVALUATION

EJ 013 951 450 SE 500 954
A STUDY OF SELECTED CHARACTERISTICS OF SECONDARY MATHEMATICS TEACHERS Kerr, R. D.; and others. <u>School Science and Mathematics</u>, v69 n9, pp781-790, Dec '69
Descriptors--*Mathematics Teachers, *State Surveys, *Secondary School Mathematics, *Teacher Characteristics

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Gerald R. Rising, State University of New York at Buffalo

1. <u>Purpose</u>

The purpose of the study was to gather information about "selected background and professional characteristics of mathematics teachers and [to] compare and contrast the characteristics of mathematics teachers in schools located in urban, suburban and rural areas."

2. <u>Rationale</u>

The authors relate the study to statements in the literature on the need for pre-service and in-service training and adequate school size to support change in the mathematics program.

3. <u>Research Design and Procedure</u>

Questionnaires were distributed to a random sample of 250 Missouri secondary school mathematics teachers, no more than one to a school. [The sample of schools thus includes between a third and a half of those in the state.] Follow-up letters and phone calls brought 233 returns.

Returns are related to school type: urban (in a metropolitan area of over 35,000 population), suburban (non-urban schools in districts within six miles of an urban district), and rural.

Most reports are in percentages. One way analysis of variance, Chi Square, and a non-parametric median test were used in data analysis.

4. <u>Findings</u>

a. A mean of 29 undergraduate semester hours earned in mathematics with almost a third having less than 25 and a tenth less than 17. Missouri certification requirement: 24.
b. Professional activities: 35% had attended one or more NSF institutes; half had no graduate work at all [while a third appear to have nine or more

graduate hours in mathematics alone]; less than 40% belonged to any professional organization related to mathematics; less than half read a related journal; and only 4% had been "involved in innovative program development in mathematics sponsored by outside agencies," although 80% expressed interest in such participation.

c. Significant differences (.05) were noted in favor of urban and suburban teachers over rural teachers on: number of graduate hours; master's degree held; NSF institute attendance; memberships; journal reading; and participation in innovative programs.

d. "Some evidence [suggests] turnover in rural school districts was greater than in urban and suburban districts."

5. Interpretations

"Encouraging:...the extent to which teachers in urban and suburban districts availed themselves of opportunities to improve in service...." Disheartening: relatively small percentage of rural teachers taking advantage of continuing education opportunities. "The solution to the problem may lie both in school district reorganization to allow...sufficiently large high schools...and in increased efforts to upgrade the in-service opportunities available to teachers in rural areas."

Abstracter's Notes

This is a straightforward study that provides good base-line data about mathematics teacher characteristics as well as an example of publication in the right journal. Perhaps as a sign of my own age, I interpret some of the data as more encouraging than do the authors: I believe that it suggests improvement over a more disheartening past and Missouri teacher professionalism that probably compares favorably with the rest of the country. In this regard interesting comparisons could be drawn with more self-satisfied states like New York and with those that have had intensive activity like California and Minnesota.

In any brief report some choices must be made. An unfortunate omission here is the distribution of the 233 teachers among school types. I agree with the authors' choice of median (as opposed to mean) time in district in analysing teacher mobility.

Since rural does not necessarily imply small for school size, I cannot extend the argument for school reorganization; this may, of course be an artifact of an unstated fact about Missouri schools.

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