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ABSTRACT

This handbook presents statistical procedures that will assist State and local school officials in making longrange projections for a decade or more. The author suggests several seemingly appropriate procedures but leaves it to the State and local officials to select the procedures that appear most suitable for their specific local conditions. This document is organized around eight chapters that (1) make general observations on statistical projections, (2) examine local school district histories to appraise the problem of applying statistical projection techniques to them, (3) present some summary materials on procedures for making shortrun projections, (4) discuss methods for making unified projections for the State and all its political units, and (5) present materials to aid local districts in making longrange projections. (Author/JF)

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HANDBOOK OF STATISTICAL PROCEDURES FOR LONG-RANGE PROJECTIONS OF PUBLIC SCHOOL ENROLLMENT

by A. J. Jaffe
Bureau of Applied Social Research, Columbia University

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE / Office of Education
Office of Program Planning and Evaluation

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FOREWORD

It is expected that this methodology handbook will be of considerable aid to State and local school officials in preparing long-range enrollment estimates. A lead time of several years is often required to build or enlarge facilities, obtain staff, and plan educational programs.

Dr. A. J. Jaffe, Director of the Manpower and Population Program of Columbia University's Bureau of Applied Social Research, is a well-known demographer and statistician. Among his many writings is the *Handbook of Statistical Methods for Demographers*, published by the U.S. Census Bureau (and issued by the Government Printing Office in 1951). Any reader who is interested in pursuing the various methodological problems of making projections could profitably refer to this earlier volume.

Dr. Jaffe introduces in the present volume a variety of statistical methods which are applicable to different situations. The author suggests the use of several procedures which may be most appropriate under different conditions in a school district, but the reader must select the procedures which appear to him to be most useful for his situation. Local conditions are so variable that no hard and fast and immutable "rules" can be laid down. Each official using this volume must take into consideration his knowledge about his State or local conditions, select the method—or methods—which he wants to use to make forecasts, and then interpret the resulting statistical projections in light of his intimate knowledge of his particular conditions. Fortunately, enough States and local school districts have enough in common so that a few general procedures will fit most projection needs.

JOSEPH FROOMKIN,
Assistant Commissioner for Program Planning and Evaluation,
Office of Education.

June 1969

PREFACE

State and local school officials need estimates of long-range future enrollment in public schools for a variety of planning purposes. Accordingly, we are here presenting an array of statistical procedures which can be used for making projections a decade or longer into the future.

There is no statistical formula which will foretell the future precisely. The best that we can hope to attain is some reasonable estimate which may serve as a basis for drawing plans for construction, recruitment of teachers, and so forth. The statistical procedures for obtaining this "reasonable estimate" vary greatly from one State or local school district to another insofar as the history, conditions, and information available for each area are different from others. Therefore, we present a variety of methods with suggestions as to the type of condition under which each may be most appropriate. The State and local school officials must then choose that method, or methods, which seems most suitable for their specific local conditions.

After applying that "best" procedure and obtaining a long-range projection, State and local school officials must then evaluate the statistical results in light of all their knowledge of the local community. No statistical formula can take into consideration every item of knowledge available to the local residents. Therefore the judgment of the State and local officials, based on their intimate knowledge of local conditions, must be applied to an appraisal of any statistical results.

This Handbook is organized as follows: In chapter 1, we make some general observations on statistical projections. In chapter 2, we examine the history of local school districts in the United States in an effort to appraise the general problem of applying statistical projection techniques to them.

Chapters 3 and 4 present some summary materials on the procedures for making shortrun projections. All of these methods have been used by local school districts and States, and work well in the short run. However, they are of questionable value for longrun projections.

In chapters 5, 6, and 7, we discuss methods for making unified projections for the State and all its political units. We reason as follows: It is relatively easy to make reasonably accurate projections for a State since it is such a large unit. True, some States, such as California, pose methodological problems because of the unusually large number of in-migrants; nevertheless, it is easier to estimate future migration into the entire State than into any particular part of it. Therefore, we first project school enrollment for the entire State.

We also know that the total number of pupils enrolled in each of the public schools of the State must equal the total number enrolled in the State. Therefore, we can use the projected number in the State as a standard in calculating the projected number in each local political unit. By equating the sum of the local units with the State total, we reduce the average error in each political subunit. Chapter 5 contains a discussion of this.

Not all States will prepare unified projections for their subdivisions. In some cases, local school districts will find it necessary to prepare their own projections. Chapter 8 presents some materials which will aid the local districts to make long-range projections. We prefer the unified projections, but if they are not available, then the procedures outlined in chapter 8 can be substituted.

Finally, several appendixes are included, containing additional methodological materials.

DR. A. J. JAFFE,
Director of the Manpower and Population Program,
Columbia University, Bureau of Applied Social Research.

June 1969

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Local School offices

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In addition, we wish to thank the following persons from Columbia University: Jerome B. Gordon, who assisted on all phases of this handbook; Dr. John U. Farley and Joseph Lopatin for assistance with the exponential smoothing procedures shown in chapter 7 and appendix C; Melvin Loos for his editorial efforts; and Fred Morgan and Orlando Rodriguez for their clerical and statistical assistance in the preparation of various parts of this report.

Finally, we wish to thank the following for permission to quote from their works:

Connecticut State Department of Education, Maurice J. Ross, Chief, Bureau of Research, Statistics and Finance, Hartford, Conn. *Instructions for Using the Estimate of Future Enrollments* and *Connecticut's Need for New Teachers, 1968–1982* (Research Bulletin No. 3, April 1967).

John K. Folger and the Southern Regional Education Board, "Cohort Survival Method" Chapter IV of a mimeographed, undated Southern Regional Education Board report entitled *Some Methods for Projecting School and College Enrollments* by John K. Folger.

Dr. Francis Duehay and the Harvard University Graduate School of Education, *Watertown: Its Schools and Needs*, Cambridge, 1966.

Dr. E. Brewin, Dr. A. R. Post, and the Fels Institute of Local and State Government, University of Pennsylvania, *Estimate of Future Population Growth by School District, Bucks County, Pa.*, June 1967.

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CHAPTER 3

SHORT-RANGE SCHOOL ENROLLMENT PROJECTION TECHNIQUES: COHORT-SURVIVAL

Perhaps one of the most frequently used techniques in the projection of school enrollments is the cohort-survival or grade persistence method. The technique derives its name from the use of grade-to-grade survival or persistence ratios, easily computed from historical series of enrollment by individual grades—data which most local school districts and State departments of education should have on hand.

Basically, only two inputs are required to make enrollment forecasts using the technique. The first is the number of residential births for the State or local school district, which is obtained from vital statistics data compiled by local or State boards or departments of health. The second is the array of projections of grade-to-grade survival ratios; for example, the probabilities or chances of a given cohort of new enrollees "surviving" from birth to kindergarten, or from fifth to sixth grade. The grade-survival ratios may be less or more than one, or unity. Grade-survival ratios of less than one indicate the net effects of deaths, out-transfers to private schools, net out-migration from the community, or dropouts. Grade-survival ratios of more than one indicate the net effects of in-transfers from private schools and net in-migration into the State or community. Projections of enrollments are made by applying, consecutively, the individual grade-to-grade-survival ratio to each entering cohort—for example, new enrollees in kindergarten to first grade.

The simplest version of the cohort-survival method can be illustrated as follows: Suppose that in 1960, 1,000 infants are born in community X. In 1965, 800 enter kindergarten. The survival ratio from birth to kindergarten is 800 divided by 1,000, or .80.

Next, suppose that in 1965 there were 600 children in kindergarten, and in 1966, 650 in first grade. The survival ratio from kindergarten to first grade is 1.083.

These ratios can be calculated between each two grades all the way to graduation from high school—completion of the 12th grade.

Furthermore, in an effort to obtain more stable ratios, the numbers can be averaged for several years. Thus, in the appended article, "Connecticut's Need for New Teachers, 1968–1982," 5-year enrollment averages were used (table 4 of article).

Long-Range Projection Difficulties

Long-range projections can be made by simply continuing the process of applying the survival ratios until, at least, those alive at the initial date have completed the 12th year of school; the arithmetic is simple. One major problem which arises in making long-range projections is that of estimating future numbers of births in order to begin the successive entry cohorts, for example, the numbers in kindergarten or first grade at each successive year. This is a complicated job, and to do it properly requires more personnel and machine resources than most school districts have available. On the other hand, the methods to be proposed in chapters 5 to 8 make full use of the long-range population projections which the Census Bureau has prepared, thus greatly minimizing the work required at the local level.

A second major problem is that of estimating the future population of school-going-age which will live within that school district. Simple extrapolation of the survival ratios assumes that there will be no drastic changes in the volume or direction of migration. Yet extensive in- or out-migration can affect the survival ratio. (Drastic change in the balance of public and private school enrollment can also alter the survival

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ratio.) One way of trying to deal with this problem is to apply linear regression techniques, as shown in the appended paper by the Southern Regional Education Board. If the community has had extensive in-migration during the past decade or longer, for example, then the regression line will project increasing survival ratios in future years; if there has been out-migration, it will project decreasing survival ratios. In short, linear regression assumes that future migration patterns will be similar to those in the past.

Unquestionably, migration is, at least, as difficult to extrapolate as are the numbers of future births. It becomes even more difficult to do so when a small geographic entity, as a local school district, is treated by itself, and without taking into account its relationship to the larger geographic unit—county, standard metropolitan area, or State—of which it is a part. By utilizing the population projections for States which the U.S. Census Bureau has prepared, in the manner suggested in chapters 5 to 8, the local school district is projected within the framework of the county and the State. Thus, the migration element in the future population is more likely to be taken into account. Mechanical projection of past trends without taking into account the relation of the school to the county and State of which it is a part, can lead to the anomaly illustrated below with the following hypothetical, but not unlikely case:

1. School district x contains 25,000 population in 1970.
2. County X of which it is a part contains 100,000 persons in 1970.
3. During the 10 years preceding, school district x grew at a rate of 10 percent a year, and the county as a whole at a rate of 1 percent a year.
4. Projecting these rates of growth we have:

	10 years ahead	15 years ahead	20 years ahead
School district x	65,000	105,000	168,000
County X	110,000	116,000	122,000

It is highly unlikely that the school district will have as large a population as does the entire county, some 16 or 17 years hence. What seems more likely is that during the particular decade under study, 1960 to 1970, that portion of the county containing school district x happened to receive a large number of migrants, perhaps as overflow from a neighboring large central city. During the next decade, 1970 to 1980, there is no reason to believe that the school district will continue that rapid rate of growth, unless there are factors which will lead to rapid population growth in the entire county. This is the sense in which projections of a smaller geographic area within the framework of a larger area are likely to be more accurate.

Appended Articles[1]

"Connecticut's Need for New Teachers, 1968–1982" by Maurice J. Ross (Hartford: Connecticut State Department of Education, Research Bull. No. 3, April 1967), illustrates the application of the cohort-survival technique to an entire State to project about 15 years. The numbers of classrooms and teachers to be needed are derived from these projected enrollments. These projection techniques can be applied to a single school district, as is shown in the sheets provided by the Connecticut State Department of Education, Bureau of Research, Statistics and Finance, and entitled "Instructions for Using the Estimate of Future Enrollments," Exhibit 1.

The paper "The Cohort-Survival Method" prepared by the Southern Regional Education Board, illustrates how projected survival ratios can be modified by means of linear regression methods. These modifications are called the "ratio method" in that paper. Note that judgmental decisions are needed in projecting survival ratios; mechanical projection alone, by means of a regression formula, can lead to trouble.

The last paper, the Harvard Graduate School of Education study, depicts the detailed steps in the development of enrollment forecasts for Watertown, Mass., utilizing U.S. Census and Commonwealth of Massachusetts materials on birth and fertility characteristics of the community. In this paper the cohort-survival method is called "percentage of survival technique."

[1] Tables appearing in the appended materials have been numbered in sequence with others contained in the text of this handbook.

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CONNECTICUT'S NEED FOR NEW TEACHERS 1968–82

By Maurice J. Ross, *Chief*
Bureau of Research, Statistics and Finance

(Table numbers ours)

Introduction

Enrollments in the public schools, including the endowed and incorporated academies of Connecticut, have been increasing and they will continue to increase during the next decade. Indications are, however, that the increases will be quite modest compared to increases in the past decade. If Connecticut can solve the problems involved in matching the numbers of new teachers needed to the areas of subjects in which they are needed, the perennial teacher shortage may at long last be alleviated, at least temporarily. However, the introduction of more "head start" pupils and more kindergarten pupils may well increase the number of teachers needed. Other educational changes are operating to reduce the ratio of children to teachers and so increase the demand for teachers. We need more experience before we can make estimates which take these changes into account. Meanwhile, the projected teacher needs may be considered as minimal.

This report is the 1967 revision of the study of teacher needs. More recent information on births and enrollments in Connecticut schools have been used in this revision. Estimated enrollments are different from those in the previous reports. Actual births have varied from what was expected and estimated births have been revised. The percentage of children attending kindergarten is increasing. Public school enrollments, including special classes, passed 500,000 in 1961–62; they will pass 620,000 in 1969 and 700,000 by 1980.

These predictions are more accurate for the earlier years than they are for the later years. This is due to the fact that projections for the later years rely more heavily on predicted births. In the decade 1950–1960, population experts expected the birth rate and the number of annual births to decrease. This did not occur; in fact, a new record high in births to residents of Connecticut was established each year from 1952 through 1957. (Table 3) There were small decreases in the number of births in 1958, 1959 and 1960 compared with 1957. An all time record high in the number of births occurred in 1961. The birth rate in 1962 was substantially less than the previous year. Birth rates continued to decline through 1966. The Connecticut State Department of Health anticipates a gradual increase in the birth rate, but not to the level of the 1950's. It is anticipated that the increasing population of Connecticut, even with lower birth rates than we had in the 1950's, will lead to an increasing number of births from 1967 on. These births will lead to increased school enrollments a few years later.

Enrollments in non-public schools are also increasing. The pattern of enrollments in these schools may become clearer in the years ahead and this pattern will be reflected in the ratio of the number of children in public school to the number of births some years earlier and in the ratio of the number of children in public school grade in a particular year to the number of children in the previous grade one year earlier.

It should be noted that children not officially enrolled in a particular grade, kindergarten through twelve, are being separately accounted for. There are now about 6,000 such pupils in special classes or groups outside the regularly designated grade groups. This constitutes a number equal to one percent of the "regular" enrollment. This percentage has been used in making forecasts. As more children needing special programs are identified, more teachers may be needed for the small classes which are customary in these programs. At a ratio of 15 pupils per teacher, about 400 teachers are needed for these pupils. It should also be noted that the needs for non-teaching personnel, e.g., guidance counselors, school social workers, school psychologists, psychometrists, librarians, super-

TABLE 3.—*Connecticut population, birth rates and births, 1940–76*

Year	Connecticut population [1]	Connecticut birth rate	Births to Connecticut residents [1]
1940	1,712,000	14.6	25,074
1941	1,761,000	16.5	28,996
1942	1,824,000	20.3	37,059
1943	1,869,000	20.8	38,880
1944	1,894,000	17.9	33,986
1945	1,909,000	17.5	33,409
1946	1,916,000	21.5	41,131
1947	1,932,000	23.4	45,181
1948	1,948,000	21.5	41,965
1949	1,980,000	20.6	40,819
1950	2,016,000	20.1	40,485
1951	2,052,000	21.2	43,506
1952	2,106,000	22.1	46,537
1953	2,173,000	22.1	47,996
1954	2,246,000	22.5	50,428
1955	2,298,000	22.8	52,330
1956	2,342,000	22.9	53,584
1957	2,393,000	23.8	56,909
1958	2,456,000	22.9	56,244
1959	2,498,000	22.6	56,423
1960	2,544,000	22.3	56,659
1961	2,604,000	21.9	57,046
1962	2,663,000	20.8	55,480
1963	2,711,000	20.8	56,476
1964	2,775,000	20.4	56,611
1965	2,825,000	19.2	54,208
1966	2,873,000	18.2	52,289
1967	2,929,000	18.4	53,894
1968	2,986,000	18.6	55,540
1969	3,043,000	18.8	57,208
1970	3,100,000	19.1	59,210
1971	3,158,000	19.3	60,949
1972	3,216,000	19.5	62,712
1973	3,274,000	19.7	64,498
1974	3,332,000	19.9	66,307
1975	3,390,000	20.2	68,478
1976	3,448,000	20.2	69,650

[1] Population figures for years ending in 0 are from the U.S. Census. Population and birth data for interim years and for 1966 on are based on revised calculations and/or predictions supplied by the Connecticut State Department of Health. Population to nearest thousand. Births to nearest hundred.

visors and administrators, are not taken into account in this bulletin.

It is suggested that the data in this bulletin be carefully studied by citizens and school personnel of Connecticut. It is believed that a knowledge of the facts herein presented will alert the State of Connecticut to the need for finding satisfactory solutions to the problems of teacher education and placement.

Projections should not be considered exact predictions; predictions cannot be accurate to the last digit. A reasonable allowance for error is five percent either way.

Procedures

1. The kindergarten enrollments from 1963 through 1967 and the births from 1957 through 1961 were used to determine the percent that kindergarten enrollment was of the number of births five years earlier. The mean percent was 88.3 (see table 4).

2. The persistence from grade to grade was determined by calculating what percent the enrollment in any given grade during the five-year period 1963–1967 was of the enrollment of the previous grade for the five-year period 1962–1966. The mean persistence from grade to grade is as follows (see table 4):

	Percent
1st grade of kindergarten	111. 3
2nd grade of 1st	94. 5
3rd grade of 2nd	98. 8
4th grade of 3rd	99. 9
5th grade of 4th	99. 6
6th grade of 5th	99. 1
7th grade of 6th	99. 1
8th grade of 7th	99. 6
9th grade of 8th	105. 5
10th grade of 9th	95. 1
11th grade of 10th	94. 0
12th grade of 11th	92. 9

3. The number of pupils to be expected in kindergarten through grade 12 for the years 1967–68 through 1981–82 was calculated by applying the average percent found in 1 above to births and estimated births from 1962 through 1976 and the percentage found in 2 above to enrollments and estimated enrollments from 1967–68 through 1981–82 (see tables 5 and 6 and Exhibit 2).

4. The total number of teachers needed in kindergarten, grades 1–6, grades 7–8, grades 9–12 and distinctive classes for each year has been determined by:

a. Dividing the estimated number of kindergarten pupils found for each year by 40 (see table 7 and Exhibit 3).

b. Dividing the estimated number of pupils in grades one through eight for each year by 25 and for grades nine to twelve for each year by 20 (see table 7 and Exhibit 3).

c. Dividing the estimated number of distinctive (special and ungraded) pupils by 15 (see table 7 and Exhibit 3).

5. The total number of new teachers needed in Connecticut elementary and secondary schools from 1967–68 through 1981–82 has been summarized in table 8 and Exhibit 4. This number is the algebraic sum of the number of new teachers needed because of changing enrollments and a turnover of 5 percent of the number of teachers in-service one year previously.

TABLE 4.—*Enrollments [1] and persistence in Connecticut public schools, 1962–67*

Year	Number births 5 years earlier	Number in kindergarten	Number in grade 1	Number in grade 2	Number in grade 3	Number in grade 4	Number in grade 5	Number in grade 6	Number in grade 7	Number in grade 8	Number in grade 9	Number in grade 10	Number in grade 11	Number in grade 12	Distinctive classes [2]	Total grades K–6	Total grades 7–8	Total grades 9–12	Total enrollment	High school graduate
1961-62	53, 584	45, 075	49, 710	45, 161	42, 471	42, 142	39, 253	37, 067	36, 846	36, 908	39, 296	33, 102	25, 296	22, 613	....	300, 879	73, 754	120, 307	494, 940	22, 081
1962-63	56, 909	47, 425	50, 918	46, 950	44, 528	42, 513	42, 019	38, 960	36, 837	36, 415	38, 302	37, 273	30, 885	23, 167	....	313, 313	73, 252	129, 627	516, 192	22, 478
1963-64	56, 244	48, 672	53, 782	48, 070	46, 524	44, 221	42, 322	41, 582	38, 661	36, 541	39, 201	37, 437	35, 924	29, 650	....	325, 173	75, 202	142, 212	542, 587	27, 929
1964-65	56, 423	49, 810	54, 025	50, 846	47, 412	46, 615	44, 187	41, 939	41, 004	38, 491	38, 375	36, 137	34, 131	32, 642	....	334, 834	79, 495	141, 285	555, 614	31, 729
1965-66	56, 659	51, 204	54, 916	51, 118	49, 902	47, 350	46, 203	43, 690	41, 594	41, 046	40, 531	36, 667	34, 300	31, 948	5, 956	344, 383	82, 640	143, 446	576, 425	30, 815
1966-67	57, 046	53, 066	55, 808	51, 924	50, 789	50, 026	47, 208	45, 847	43, 240	41, 643	43, 501	38, 578	34, 498	31, 690	6, 971	354, 668	84, 883	148, 267	[3] 596, 762	[4] 30, 549
1st 5-year total		242, 186	263, 351	242, 145	230, 837	222, 841	213, 984	203, 238	194, 942	189. 401	195, 705	180, 616	160, 536	140, 020	5, 956	1, 618, 582	384, 343	676, 877	2, 685, 758	135, 032
Last 5-year total		250, 177	269, 449	248, 908	239, 155	230, 725	221, 939	212, 018	201, 336	194, 136	199, 910	186, 092	169, 738	149, 097	12, 927	1, 672, 371	395, 472	704, 837	[3] 2, 787, 580	143, 500
Mean percent persistence		88. 3	111. 3	94. 5	98. 8	99. 9	99. 6	99. 1	99. 1	99. 6	105. 5	95. 1	94. 0	92. 9						[5] 96. 4

[1] Enrollments as of Oct. 1 of each year. Statewide figures are higher since postgraduate students are not included in these figures—a total of some 110 additional students annually. In addition prekindergarten classes are being established, but there is not yet sufficient data on which to base estimates.

State support figures are based on a resident average daily membership figure which will be still higher since aggregate membership is divided by 180 and many school districts operate their schools for more than 180 days, more and more school districts are offering free summer schools, and prekindergarten classes are being established.

[2] Distinctive classes—e.g., sum of ungraded pupils and special-class pupils.

[3] Includes nursery enrollment of 1,973 for 1966–67.

[4] Estimated.

[5] First 5-year total of high school graduates divided by the 5-year total of seniors for the same 5 years.

CHAPTER 1

INTRODUCTION

Very large amounts of energy, human resources, and money are being invested in elementary and secondary schools. Forty-five million or more children and youth, and some 2 million teachers are involved in the public schools alone. Almost $30 billion a year are being spent on these pupils and schools. Another 6 to 7 million students are enrolled in private elementary and secondary schools. In future years the numbers of pupils and teachers and the amounts of expenditures will increase considerably.

Future increases in public school enrollment will vary considerably from one part of the United States to another. In some areas there will be large increases in the numbers of students, and accordingly in the number of teachers and amounts of expenditures, whereas in other parts there may be declines. In order to plan realistically for coming events in any given part of the United States, it then becomes necessary to estimate the number of pupils which there is likely to be. The future pupils, so to speak, are the beginning of the process; their number determines the teaching and other facilities which will be needed. Therefore, a crucial question becomes: How many pupils will there be in any given area at some specified future date?

The facilities which will be needed cannot be created overnight. There is a lag of several years between the time at which more teachers, buildings and other facilities will be needed, and the time that they can be available. The essence of planning then, is to anticipate these future needs sufficiently in advance so that the teachers and facilities will be there when the pupils arrive.

Projections for Areas and Grades

The future, for projection purposes, means at least a decade ahead, say, 1980 (at this writing). The problems of such longrun estimation are quite different from the problems encountered in estimating school enrollment next year, or even 2 years ahead; these shortrun problems will be discussed only summarily in this report. Accordingly, our major emphasis is on these longrun projections, both for the total State and for its components—counties, groups of counties, or school districts, insofar as the latter may be meaningful to study. Ordinarily one thinks of the school district as the basic educational administrative unit for which planning should be undertaken. Actually, as we shall see in chapter 2, many school districts are so ephemeral that planning can be done only at a higher level, the county or State. Nevertheless, we shall show how individual school district projections can be made if the local community deems such calculations worthwhile.

Since projections for the totality of public school enrollment, grades kindergarten through 12, are of only partial value, we are setting forth projection techniques for three groups of grades: 1 to 4, 5 to 8, and 9 to 12. Projection of the numbers of kindergarten children are of dubious use since any numbers will depend on the extent of such kindergarten facilities which the school will provide, and the fact that kindergarten attendance is voluntary in so many States. Beginning with first grade, however, attendance is compulsory; hence, projections are feasible. High school, grades 9 to 12, obviously should be separated out since its building and staffing needs are so different from the elementary grades. To a lesser extent the building and staffing needs of grades 5 to 8 are different from those of the lower grades.

Data Needs and Sources

A fundamental need for making a projection for any specific area of the United States is knowledge of the past history of that area. This seems obvious, yet we have observed that some school districts have very poor historical records, and some State

Exhibit 1

CONNECTICUT STATE DEPARTMENT OF EDUCATION
Bureau of Research, Statistics and Finance

Instructions for Using the Estimate of Future Enrollments

1. In the column headed "allocated births" place the annual number of such births. These official figures are obtainable from the Bureau of Vital Statistics of the Connecticut State Department of Health or from its official reports. In most towns the data may be secured from the town clerk also. Figures beyond the last completed calendar year, if used, will be estimated. Allocated births are births to residents of the town in which the birth itself occurs.

2. In the grade enrollment columns, write your grade enrollments as of October 1 of the 6 school years indicated. The figures for the current year represent your town totals from the age-grade tables as they appear in your Connecticut School Register. Prior to October 1966, the data were reported to us on the REPORT OF CONDITION OF PUBLIC SCHOOLS (now known as ED 001, END OF YEAR SCHOOL REPORT). Beginning in October 1966, the data were reported on ED 006, "FALL SCHOOL REPORT."

3. You are now ready to perform a number of additions. An adding machine or calculator will be helpful.

a. Total the allocated births for the bottom 5 years in the 6-year period for which you have exact data.[1]

b. Write the total in the margin at the left of the line titled, "Bottom 5-year total."[2]

c. Total the enrollment in grade 1 (or kindergarten) as of October 1 for the bottom 5 years.

d. Write this total in the correct column and in the line titled, "Bottom 5-year total."

e. In each grade column find the total enrollment for the first five of the 6-year period for which you have data and write this total in the line titled, "Top 5-year total."[2]

f. In each grade column find the total enrollment for the last five of the 6-year period for which you have data and write this total in the line titled, "Top 5-year total."

g. Perform "c" and "f" for each grade except for P.G. (postgraduate) and Spec. (special students).

4. You are now ready to calculate the percentage of persistence (percent persistence).

a. Find the percentage to the nearest tenth of 1 percent that the figure in 3d above is of 3b above.

b. Write this figure in the column headed "1" or "k" on the line titled "percent persistence."

c. To find the percent persistence for each of the grades 2 through 12 (1 through 12 if you have kindergartens) divide the bottom 5-year total for the selected grade by the top 5-year total for the preceding grade. E.g., to find the percent persistence for grade 5, divide the "Bottom 5-year total" for grade 5 by the "Top 5-year total" for grade 4.

5. Multiply the percent persistence figure in the grade 1 (or kindergarten) column by the birth figure for the years corresponding to the school years for which you are making predictions. Write each product in the grade 1 (or kindergarten) column opposite the year on which the calculation is based. You now have your estimates of grade 1 (or kindergarten) enrollments for the years to come. Round figures off to the nearest whole.

6. Proceed to make the estimates for the other grades 2 (or 1) to 12 in order as follows:

a. To find the predicted enrollment of a given grade for a future year, multiply the percent persistence figure for the given grade by the enrollment in the preceding grade in the preceding year.

7. P.G. (postgraduate) and Spec. (special students) figures may be projected as annual averages based on experience for the years for which you have data or modified as local practice seems to indicate.

8. The columns headed "Total Enrol." may be used to indicate enrollments for grades in which you are particularly interested, e.g., K to 6, K to 8, 1 to 8, 7 to 12, or 9 to 12, etc.

9. The lines headed "Total Known Period," "Average Known Period," and "Total Estimated Period," may be disregarded.

[1] "Bottom 5-years," count up five from the bottom in the 6-year period for which you have data.

[2] "Top 5-years," count down five from the top in the 6-year period for which you have exact data.

education offices have no information whatsoever—such as statistics in properly tabulated and usable form—about the counties and school districts in their States. It is our impression that usable historical information on enrollment by grade fortunately does exist in most parts of the Nation. We can advise any local area which wishes to prepare a projection to make certain that it has the necessary data on past enrollment, and if not, to resurrect the statistics from the archives.

One of the most time-consuming and laborious aspects of making longrun school enrollment projections is the preparation of future population estimates. Since the U.S. Bureau of the Census makes such projections, we have utilized them and built our school enrollment projection techniques so as to include them.

Summary description of the methods which the Census Bureau uses for making its population projections appears in appendix A. Unfortunately the Census Bureau has not supplied the projected population information in sufficient age detail. Accordingly we include a set of procedures, the Sprague Multipliers in appendix B, which can be used to subdivide the census data into the age groups desired. An example of the large amount of work involved in preparing population projections is given in chapter 3.

Making Alternative Projections

There are three uncertainties which make public school enrollment projections so problematic for any one small part of the Nation, such as a county or local school district. For the entire Nation, on the other hand, it is much easier to anticipate and calculate these uncertainties: extent and direction of internal migration; levels of and changes in the birth rate; extent of attendance at public schools, including possible shifts between public and private schools, and particularly the retention of high school students until graduation.

The only way of trying to anticipate these uncertainties is by making alternative projections. The projection giving the greatest increase in enrollment, for example, might be based on the assumptions of large-scale immigration, a high birth rate, shift from private to public schools, and the retention of all students until graduation from high school. On the other hand, the projection which gives the least increase in enrollment might be based on the assumptions of little immigration (or even out-migration), a low birth rate, perhaps shifts from public to private schools, and continued dropouts before high school graduation.

The correct future enrollment is likely to lie between these high and low projections. Accordingly, further refinements can be made by utilizing other assumptions somewhere between the high and low. The final result might be a series of four or five projections. The "best" projection is selected through the making of successive projections and the intimate knowledge of local conditions, as described in following sections.

Internal migration

Every year about one child in every 16 of school age—5 to 17 years—moves across a county line and almost invariably moves from one school jurisdiction to another. Another one child in 10 changes residence within the same county each year; an unknown number also change local school districts. As a result of such extensive migration, it is possible that the majority of the children attend school in at least two separate school districts sometime between kindergarten and high school graduation.

We know something about past migrations, but it is impossible to predict precisely what migration will occur in the future.[1] How many people will move into or out of one particular part of the country—State, county, or local school district—during a specified time period in the future, can be guessed at but never predicted exactly. Historical migration is an approximate guide.

In making projections of public school enrollment for any local areas, we are, to a large extent, projecting past migration patterns together with the host of socioeconomic and other factors which underlie this past migration. Since it is very unlikely that any given historical migration pattern will continue unchanged for at least a full decade into the future, we should make alternative projections. Each set of procedures used has implicitly in it, different assumptions as to future migration. By calculating future public school enrollment using two or more sets of procedures, we then have a set of possibilities within which planning can be carried on. Whatever the pattern of future migration may reasonably be, we have taken it into account when making a range of estimates.

1 The reader who is interested in pursuing this topic further is referred to Henry S. Shryock, *Population Mobility within the United States*, 1964; see also "Migration" in *International Encyclopedia of the Social Sciences*, 1968.

For use with towns having no kindergartens

Town......................

CONNECTICUT STATE DEPARTMENT OF EDUCATION
Bureau of Research, Statistics, and Finance

Estimate of Future Enrollments

Allocated births		School year	Enrollments by grades as of Oct. 1														Total enrollment to—	Total enrollment to—	Total enrollment to—	Total enrollment to—	Grand total	School year
Years	No.		1	2	3	4	5	6	7	8	9	10	11	12	P.G.	Spec.						
1955		1961-62																				1961-62
1956		1962-63																				1962-63
1957		1963-64																				1963-64
1958		1964-65																				1964-65
1959		1965-66																				1965-66
1960		1966-67																				1966-67
Top 5-year total																						
Bottom 5-year total																						
Percent persistence																						
1961		1967-68																				1967-68
1962		1968-69																				1968-69
1963		1969-70																				1969-70
1964		1970-71																				1970-71
1965		1971-72																				1971-72
1966		1972-73																				1972-73
1967		1973-74																				1973-74
1968		1974-75																				1974-75
1969		1975-76																				1975-76
1970		1976-77																				1976-77
1971		1977-78																				1977-78
1972		1978-79																				1978-79
1973		1979-80																				1979-80
Total known period																						
Average known period																						
Total estimated period																						

Sources: Births: Bureau of Vital Statistics, Connecticut State Department of Health. School enrollments: ED 001 end of year school report—age grade report.
Percent persistence or projecting percent determined by: Bottom 5 total for any given grade divided by the top 5 total for the preceding grade. Births—bottom 5-year total of grade I enrollment divided by the 5-year total of births 6 years earlier.

The birth rate

The changing birth rate also contributes to the problems of projection. This is one of the main reasons why the Census Bureau makes alternative population projections, ones assuming both high and low levels of future fertility. Fortunately for the local school administrator—and we say fortunately only in a relative sense—changes in the birth rate tend to be more or less similar throughout the Nation. Unlike migration patterns, changes in the birth rate generally are not unique to particular areas. The decrease in the U.S. birth rate since the late 1950's has been evident in all parts of the country, and if it should turn up at any time, it will turn up in all areas. Of course the changes are not of similar magnitude in all areas, but at least they are in the same direction. This is more than can be said for migration.

Attendance at public schools

Changes in the proportion of the school age population going to public school are probably the least of the analyst's problems. Relative to the problems posed by migration, the problems of anticipating future attendance rates are minimum. One place where trouble can arise is in estimating the possible shift between public and private schools. In many communities the proportion attending public schools has been fairly constant for a long period of time and there is no anticipation of a change. Only where it is believed that there is likely to be a significant change in the distribution of pupils between public and private schools, must the statistician take this factor into account in the projection.

Practically 100 percent of the children of elementary school age attend such schools, both public and private. Hence, the projection problem for grades 1 to 8 is minimized. At the high school level, however, dropouts are too prevalent. In the mid-1960's only some two-thirds of the population of high school age was actually attending school. Clearly, there is room for increased attendance rates in future years, and such increases are already underway. Another projection problem then, becomes that of trying to estimate how close to 100 percent of the youth will graduate from high school by the target date.

Making Successive Projections

A final projection is never made. Instead, successive projections are made until the target date is reached. Every 2 or 3 years new projections to the original target date should be made, and each successive one is likely to be more accurate since it takes into account more historical data, and projects a shorter period ahead. For example, if we make enrollment projections in 1970 for 1980, we should repeat them not later than 1973, and again in 1975 or 1976. At both of these later dates, in addition to making projections for 1980, we should also make first projections for 1985 and 1990.

When making population projections for the total United States, the Census Bureau aims at making new ones about every 3 years. Since internal migration is not a problem in making population projections for the Nation, successive projections 3 years apart may be sufficient. When making projections for a local area, however, where migration can be so important, successive projections 2 years apart are more likely to provide a better picture of the target date.

These successive projections serve as guides to the school administrators in planning their building, personnel, and other needs. For example, suppose that a projection made in 1970 indicates that by 1980 there will be a 30 percent increase in school enrollment. It will not require 10 years, however, to build the facilities needed for such an increase. Instead, the school authorities can undertake their facility expansion in steps. In 1970 plans for a 10 percent increase by 1975 could be made. If the projection made in 1973 still indicates the same size expansion (30 percent by 1980), then in 1973 plans for a further increase in facilities can be made. If the projection made in 1975 for the year 1980 indicates that the increase over 1970 will be only 20 percent, instead of the original 30 percent, then no additional facilities beyond those planned for in the 1973 projection need be planned. If, however, the third projection still indicates a 30 percent increase, then there still is time enough in the latter half of the 1970's to build all the facilities needed for 1980.

Need To Know Local Conditions

The projection techniques shown in this handbook must be considered only as first approximations. No statistical formula can take into account all the information available about a local school district, a county, or a State. The school administrator, after seeing the results of a statistical projection, must then review the statistical findings in light of all other information

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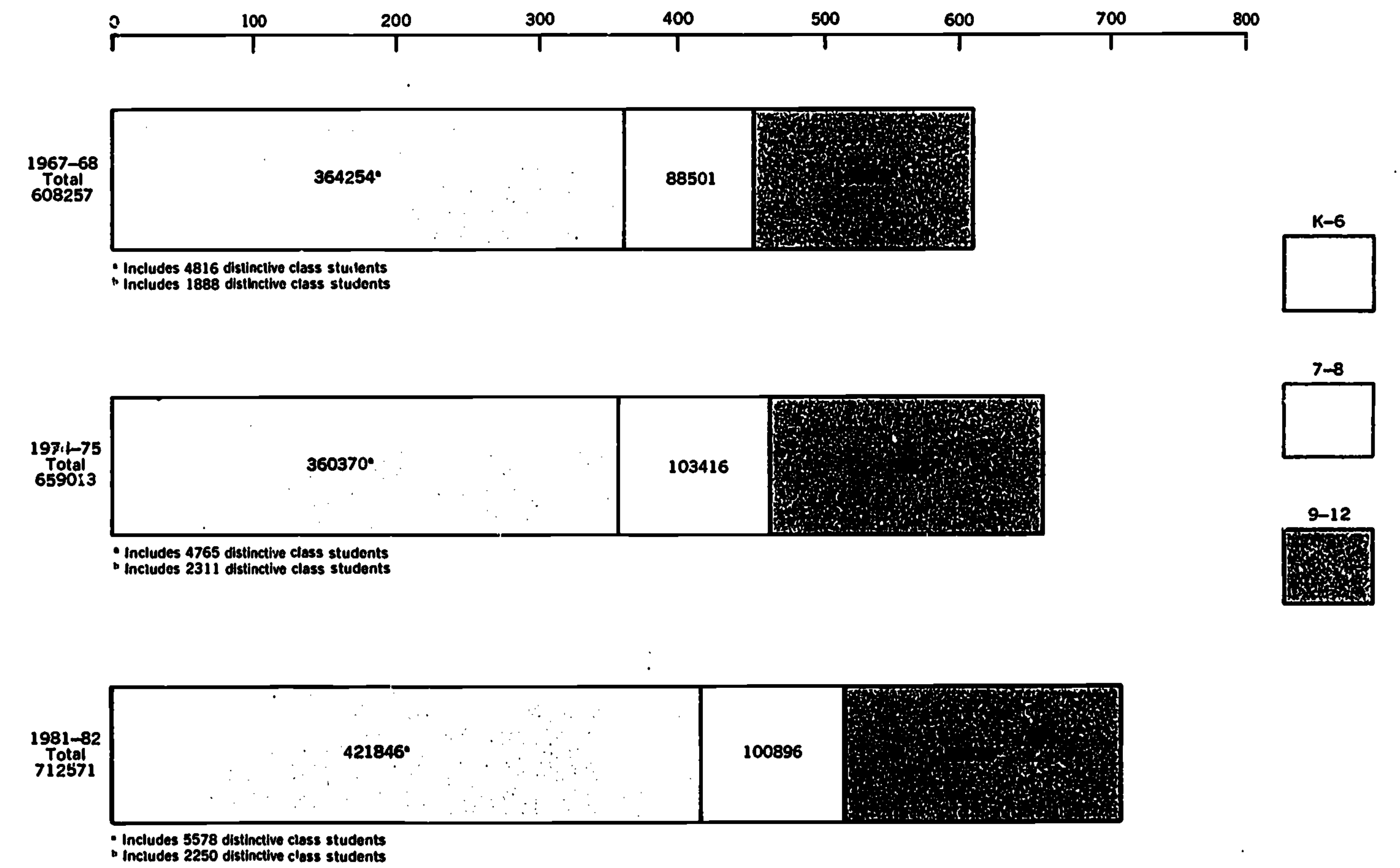
TABLE 5.—*Anticipated enrollments in Connecticut public schools, 1968–82*

Year	Number in kindergarten	Number in grade 1	Number in grade 2	Number in grade 3	Number in grade 4	Number in grade 5	Number in grade 6	Number in grade 7	Number in grade 8	Number in grade 9	Number in grade 10	Number in grade 11	Number in grade 12	Number in distinctive classes [1]	Total enrollment	High school graduate
1967–68	48,989	59,062	52,739	51,301	50,738	49,826	46,783	45,434	43,067	43,933	41,369	36,263	32,049	6,704	608,257	30,895
1968–69	49,868	54,525	55,814	52,106	51,250	50,535	49,378	46,362	45,252	45,436	41,780	38,887	33,688	6,832	621,713	32,475
1969–70	49,988	55,503	51,526	55,144	52,054	51,045	50,080	48,934	46,177	47,741	43,210	39,273	36,126	6,936	633,737	34,825
1970–71	47,866	55,637	52,450	50,908	55,089	51,846	50,586	49,629	48,738	48,717	45,402	40,617	36,485	6,986	640,956	35,172
1971–72	46,171	53,275	52,577	51,821	50,857	54,809	51,379	50,131	49,430	51,419	46,330	42,678	37,733	7,003	645,673	36,375
1972–73	47,588	51,388	50,345	51,946	51,769	50,654	54,375	50,917	49,930	52,149	48,899	43,550	39,648	7,022	650,180	38,221
1973–74	49,042	52,965	48,562	49,741	51,894	51,562	50,198	53,886	50,713	52,676	49,594	45,965	40,458	7,031	654,287	39,002
1974–75	50,515	54,584	50,052	47,979	49,691	51,686	51,098	49,746	53,670	53,502	50,095	46,618	42,701	7,076	659,013	41,164
1975–76	52,282	56,223	51,582	49,451	47,931	49,492	51,221	50,638	49,547	56,622	50,880	47,089	43,308	7,125	663,391	41,749
1976–77	53,818	58,190	53,131	50,963	49,402	47,739	49,047	50,760	50,435	52,272	53,848	47,827	43,746	7,186	668,364	42,171
1977–78	55,375	59,899	54,990	52,493	50,912	49,204	47,309	48,606	50,557	53,209	49,711	50,617	44,431	7,278	674,591	42,831
1978–79	56,952	61,632	56,605	54,330	52,441	50,708	48,761	46,883	48,412	53,338	50,602	46,728	47,023	7,397	681,812	45,330
1979–80	58,549	63,388	58,242	55,926	54,276	52,231	50,252	48,322	46,695	51,075	50,724	47,566	43,410	7,509	688,165	41,847
1980–81	60,466	65,165	59,902	57,543	55,870	54,059	51,761	49,800	48,129	49,263	48,572	47,681	44,189	7,668	700,068	42,598
1981–82	61,501	67,299	61,581	59,183	57,485	55,647	55,572	51,[illegible]95	49,601	50,776	46,849	45,658	44,296	7,828	712,571	42,701

[1] For details, see table 6.

Exhibit 2

Anticipated Enrollments in Connecticut Public Schools
(in thousands)



which he has about his community. The illustrative projections for 1980 which we show for Maryland (chapter 7) are the first approximations to 1980. We do not have all the knowledge which the State and county educators have. They must review the projections and decide for themselves whether they are probable or not.

At this point the coordination of local school district projections at the State level becomes important. Insofar as the Census Bureau is able to make usable projections of State population by age, the enrollment in public schools in the entire State can be projected with a minimum of error. This means, then, that the sum of the local school districts must equal the State total. If, for example, in the illustrative case of Maryland, every county board of education should decide that its enrollment will be 10 percent above that indicated by the statistical projections, some counties will be in deep trouble. We know fairly well what the 1980 school enrollment is likely to be in Maryland. All the counties cannot have 10 percent more than the State. The State department of education must reconcile such diverse adjustments of the original projected 1980 figures.

Table 6.—*Further details on anticipated enrollments in Connecticut public schools, 1968–82*

Year	Total grades K–6	Total grades 7–8	Total grades 9–12	Distinctive classes: Total ungraded: Elementary	Distinctive classes: Total ungraded: Secondary	Distinctive classes: Total special: Elementary	Distinctive classes: Total special: Secondary	Grand total
1967–68	359,438	88,501	153,614	395	121	4,421	1,767	608,257
1968–69	363,476	91,614	159,791	400	126	4,471	1,835	621,713
1969–70	365,340	95,111	166,350	402	131	4,494	1,909	633,737
1970–71	364,382	98,367	171,221	401	135	4,482	1,968	640,956
1971–72	360,949	99,561	178,160	397	139	4,440	2,027	645,673
1972–73	358,065	100,847	184,246	394	143	4,404	2,081	650,180
1973–74	353,964	104,599	188,693	389	147	4,354	2,141	654,287
1974–75	355,605	103,416	192,916	391	148	4,374	2,163	659,013
1975–76	358,182	100,185	197,899	394	149	4,406	2,176	663,391
1976–77	362,290	101,195	197,693	399	149	4,456	2,182	668,364
1977–78	370,182	99,163	197,968	407	149	4,553	2,169	674,591
1978–79	381,429	95,295	197,691	420	146	4,692	2,139	681,812
1979–80	392,864	95,017	192,775	432	144	4,832	2,101	688,165
1980–81	404,766	97,929	189,705	445	144	4,979	2,100	700,068
1981–82	416,268	100,896	187,579	458	144	5,120	2,106	712,571

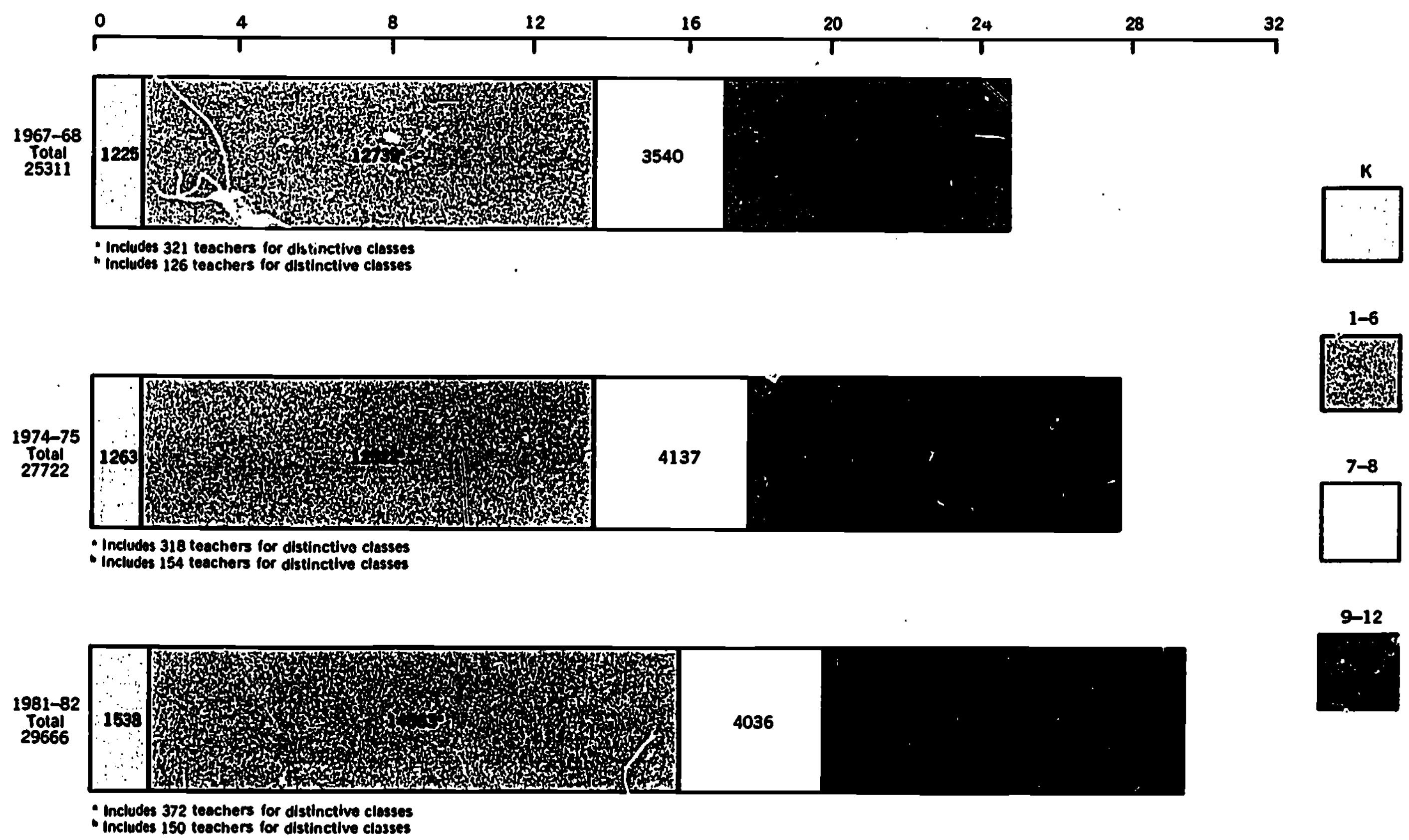
Table 7.—*Total number of classroom teachers needed for Connecticut public schools, 1967–82*

Year	Kindergarten ratio 40:1	Grades 1–6 ratio 25:1	Grades 7–8 ratio 25:1	Grades 9–12 ratio 20:1	Distinctive classes ratio 15:1	Total[1]
1966–67	1,327	12,064	3,395	7,413	465	24,664
1967–68	1,225	12,418	3,540	7,681	447	25,311
1968–69	1,247	12,544	3,665	7,990	455	25,901
1969–70	1,250	12,614	3,804	8,318	463	26,449
1970–71	1,197	12,661	3,935	8,561	466	26,820
1971–72	1,154	12,591	3,982	8,908	466	27,101
1972–73	1,190	12,419	4,034	9,212	469	27,324
1973–74	1,226	12,197	4,184	9,435	469	27,511
1974–75	1,263	12,204	4,137	9,646	472	27,722
1975–76	1,307	12,236	4,007	9,895	475	27,920
1976–77	1,345	12,339	4,048	9,885	479	28,096
1977–78	1,384	12,592	3,967	9,898	486	28,327
1978–79	1,424	12,979	3,812	9,885	494	28,594
1979–80	1,464	13,373	3,801	9,639	501	28,778
1980–81	1,512	13,772	3,917	9,485	512	29,198
1981–82	1,538	14,191	4,036	9,379	522	29,666

[1] Exclusive of nursory school.

Exhibit 3

Total Number of Classroom Teachers Needed for Connecticut Public Schools (in thousands)



1967–68: [a] Includes 321 teachers for distinctive classes; [b] Includes 126 teachers for distinctive classes

1974–75: [a] Includes 318 teachers for distinctive classes; [b] Includes 154 teachers for distinctive classes

1981–82: [a] Includes 372 teachers for distinctive classes; [b] Includes 150 teachers for distinctive classes

CHAPTER 2

HOW USEFUL ARE SCHOOL DISTRICTS AS BASIC UNITS FOR PROJECTION PURPOSES?

The major part of our efforts is devoted to the presentation of methods for projecting enrollment for single counties, or groups of contiguous counties. Secondary effort will be given to the methodology of projections for individual school districts. The reasons for giving first priority to counties is that it is much easier to make reasonable projections for them, whereas there are a number of factors which make it difficult to analyze individual school districts.

TABLE 1.—*Number of local basic administrative units (school districts), and number of public and nonpublic elementary and secondary schools: United States, 1929–30 to 1965–66*

School year	Public school systems				Nonpublic schools[1]	
	School districts[2]	Elementary schools		Secondary schools	Elementary	Secondary
		Total	1-teacher			
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1929-30	(3)	238, 306	149, 282	23, 930	9, 275	3, 258
1931-32	127, 531	232, 750	143, 391	26, 409	9, 734	3, 289
1933-34	(3)	236, 236	139, 166	24, 714	9, 992	3, 327
1935-36	(3)	232, 174	131, 101	25, 652	9, 992	3, 327
1937-38	119, 001	221, 660	121, 178	25, 467	9, 992	3, 327
1939-40	117, 108	(3)	113, 600	(3)	11, 306	3, 568
1941-42	115, 493	183, 112	107, 692	25, 123	10, 285	3, 011
1943-44	111, 383	169, 905	96, 302	28, 973	10, 285	3, 011
1945-46	101, 382	160, 227	86, 563	24, 314	9, 863	3, 294
1947-48	94, 926	146, 760	75, 096	25, 484	10, 071	3, 292
1949-50	83, 718	128, 225	59, 652	24, 542	10, 375	3, 331
1951-52	71, 094	123, 763	50, 742	23, 746	10, 666	3, 322
1953-54	63, 057	110, 875	42, 865	25, 637	11, 739	3, 913
1955-56	54, 859	104, 427	34, 964	26, 046	12, 372	3, 887
1957-58	47, 594	95, 466	25, 341	25, 507	13, 065	3, 994
1959-60	40, 520	91, 853	20, 213	25, 784	13, 574	4, 061
1961-62	35, 676	81, 910	13, 333	25, 350	14, 762	4, 129
1963-64	31, 705	77, 584	9, 895	26, 431	(3)	4, 451
1965-66	26, 983	73, 216	6, 491	26, 597	15, 340	4, 606

[1] Data for most years are partly estimated.
[2] Includes operating and nonoperating districts.
[3] Data not available.

NOTE.—Beginning in 1959-60, includes Alaska and Hawaii.

Source: U.S. Department of Health, Education, and Welfare, Office of Education, "Biennial Survey of Education in the United States," chapters on Statistical Summary of Education; "Statistics of State School Systems"; "Statistics of Nonpublic Elementary Schools"; and "Statistics of Nonpublic Secondary Schools."

Some Distruptive Factors

Changing Boundaries

To begin, any projection must be based on past events which occurred in the specific geographic area—school district or county or State. Without this historical information any statistical projection will be only an untutored guess. Furthermore, statistical projections are impossible to make on the basis of historical data referring to a changing geographic area. Yet very many of the school districts have had changing boundaries over the last few years, and apparently such changes will continue into the future. This can be seen in the fact that over the last generation the number of school districts in the Nation has declined from about 127,000 in 1931 to about 23,500 in 1966. There was a consistent decline throughout this period, and the end does not yet appear in sight. (See table 1.)

The number of operating school districts and the percent decline for the years 1962 to 1966 are as follows:

School year	Number of operating school districts	Percent decrease
1962	28,859	---
1963	27,763	4
1964	25,991	6
1965	24,446	6
1966	21,685	11

Obviously the decreasing numbers of school districts have come about as a result of consolidations of previously existing districts. For purposes of statistical projection-making we would need to know precisely which school districts were combined in the past, and which will be combined in the future. Such information, if available at all, is

Exhibit 4

Total Number of New Teachers Needed in Connecticut Elementary and Secondary Public Schools

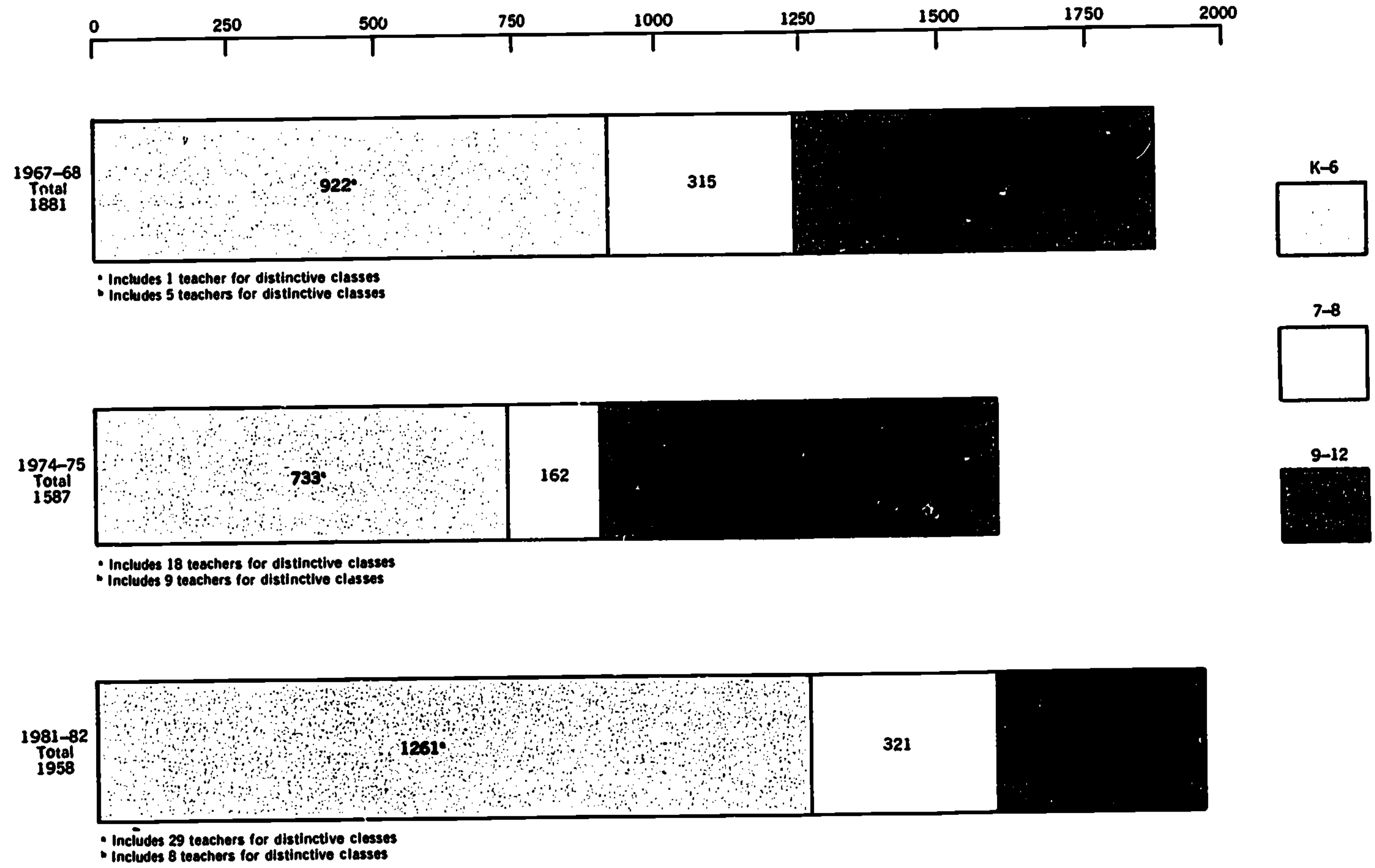


TABLE 8.—*Total number of new teachers needed in Connecticut public elementary and secondary schools,[1] 1968–82*

Year	Kindergarten to grade 6	Grades 7-8	Grades 9-12	Distinctive classes	Total [2]
1967-68	921	315	639	6	1,881
1968-69	830	302	693	30	1,855
1969-70	762	322	728	30	1,842
1970-71	688	321	650	25	1,603
1971-72	580	244	775	23	1,622
1972-73	552	251	749	26	1,578
1973-74	495	352	684	24	1,555
1974-75	715	162	683	27	1,587
1975-76	749	77	731	27	1,584
1976-77	818	241	485	28	1,572
1977-78	976	121	507	31	1,635
1978-79	1,126	43	482	32	1,683
1979-80	1,154	180	248	32	1,614
1980-81	1,189	306	328	36	1,850
1981-82	1,232	321	368	37	1,958

[1] Grades 9-12, 5 percent of 7,681 (1967-68) plus the difference between 7,990 (1968-69) and 7,681 (1967-68), (384+309) or 693 new teachers needed in 1968-69. (See table 7.)

[2] Exclusive of nursery school.

available only locally. Presumably some historical records are available in State or local offices and could be used, if all past consolidations were made by combining whole school districts. But if any previous school districts were split and apportioned to other districts, it becomes a very difficult task to reconstruct constant geographic entities.

Of major importance for making statistical projections is the fact that future consolidations are largely unknown at present. In some States consolidations and the redrawing of school boundaries are in the hands of county or State authorities and can be imposed on the school district. In other cases the decision to consolidate or not is made by the school districts concerned. In either event it is difficult to predict exactly which school district boundaries will change, and which will not. We hazard a guess that ultimately the number of school districts in the United States will approach the number of counties—about 3,000—but we have no idea when.

The number of counties in the United States and their boundaries have remained largely unchanged for a number of decades, and all our present information strongly suggests that few, if any, boundary changes are contemplated in the foreseeable future. Accordingly, then, the county rather than the school district is preferable for making statistical projections.

Size of School Districts

Another statistical problem is that many school districts (as of fall 1966) were too small to permit making reliable projections. The enrollment in a school district with but a few hundred students, or even a very few thousand, could vary enormously and unpredictably. In general, the larger the enrollment, the more likely are the possible future fluctuations to be minimized. Part of the problem with school districts which contain small populations lies in the possible effects of migration. If one business establishment or factory or government installation moves out, it is possible that half of the school children may leave with their parents. Conversely, the opening of a large facility can double enrollment overnight via in-migration. A large school district, on the other hand, cannot be so affected.

Only about 1,400 school districts in total United States in 1966–67 had an enrollment of 6,000 or more students. This is a rule of thumb minimum number needed to minimize chance and unpredictable fluctuations. Another 1,700 school districts had between 3,000 and 6,000 students. The remaining 20,000 school districts (including the nonoperating ones) are really too small to permit making reliable longrun projections. (See table 2.)

On the other hand, about 75 percent of all pupils were enrolled in school districts containing 3,000 or more students, and almost 60 percent in districts with 6,000 or more. Hence, projections covering very large numbers of students are possible even if only a small proportion of the school districts are included.

Most counties have large numbers of elementary and secondary pupils. Nevertheless, a significant number (as in Texas, for example) had well under 3,000 pupils in 1960, and many had under 1,000. For such small counties statistical projections are likely to be no more valid than for similarly small school districts. Combining the counties into clusters is required.

Overlapping School Districts

A number of geographic areas have several overlapping school districts. There may be two or more districts for grades kindergarten to 4 or 6 or 8, and then one consolidated district for high school (or junior plus senior high). Situations of this type do not lend themselves easily to making projections. Many of such overlapping school districts are probably quite small in size and were considered previously in the section "Size of School Districts."

TABLE 2.—*Number of public school systems and number of pupils enrolled, by size of system: United States, 1966–67*

Enrollment size [1]	School systems		Pupils enrolled	
	Number	Percent	Number (in thousands)	Percent
(1)	(2)	(3)	(4)	(5)
Total	23,390	100.0	[2] 43,842	100.0
25,000 or more	170	.7	12,590	28.7
12,000 to 24,999	350	1.5	5,730	13.1
6,000 to 11,999	880	3.8	7,293	16.6
3,000 to 5,999	1,726	7.4	7,178	16.4
1,800 to 2,999	1,819	7.8	4,251	9.7
1,200 to 1,799	1,636	7.0	2,416	5.5
600 to 1,199	2,838	12.1	2,437	5.6
300 to 599	2,723	11.6	1,185	2.7
150 to 299	2,091	8.9	459	1.0
50 to 149	2,230	9.5	209	.5
15 to 49	2,673	11.4	71	.2
1 to 14	2,386	10.2	22	.1
None [3]	1,868	8.0		

[1] Based on the number of pupils enrolled in October 1966.

[2] Includes 992,000 students enrolled at the college level.

[3] Systems not operating schools.

NOTE.—Because of rounding, detail may not add to totals.

Source: U.S. Department of Commerce, Bureau of the Census, "1967 Census of Governments," CG-P-3, "Public School Systems in 1966-67."

THE COHORT-SURVIVAL METHOD[4]

By John K. Folger

Southern Regional Educational Board

We used the cohort survival method to project public school enrollment. We also plan to use a modification of the cohort method to revise our college enrollment projections, which were made by the ratio method. The cohort method is probably more accurate, at least for short-range projections, than the ratio method. The tests which we made in planning this study showed that a two-year forecast by the cohort method had a percent of error only about half as large as the ratio method when compared with actual enrollments for the two years. (See Tables 9 and 10).

The basic data employed in our estimates made by the cohort method were enrollments in each single grade for the past 20 years. The definitions used for enrollment and adjustments of the enrollment figures were discussed in Chapter II. In addition to the enrollment data, we used information on the births occurring during the past seven years to make estimates of the number of pupils who would enter school in each of the next six years. We corrected all of the data on births as indicated in Chapter II. We also adjusted births from calendar-year basis to a school year (October 1 to September 30) basis.

We computed survival rates from each grade to the next higher grade the following year with the adjusted date. The survival rate is obtained by dividing the enrollment in a given grade into the enrollment in the next higher grade the following year. This rate indicates the proportion of the cohort that survives to the following year. Usually the survival rate is less than one; but in states like Florida that have a heavy in-migration, the grade group can actually get larger as it progresses through the early elementary grades and the survival rates will be above one.

[4] Source: John K. Folger, *Higher Education in the South*, Southern Regional Education Board, Atlanta, 1954, ch. IV.

The survival rate measures the combined effects of migration, deaths, dropouts, and retardation. The data are not available to permit an exact analysis of the importance of these various factors in the survival rate; however, we can indicate some of the major variations in the survival rates and their probable causes.

Survival from one grade to the next is generally very high for the elementary grades. For most Southern states the survival of whites from grades two to three, three to four, and four to five is between 97 and 100 percent, and is as high as 102 percent in Florida. The grade-to-grade survival rates were computed for all years from 1932 to 1952. In most states the survival rates for these early elementary grades showed only slight or no increase during the 20-year period. The survival rates for Negroes for grades two to three, three to four, and four to five were generally lower than those for the whites and showed more improvement during the two decades. By 1950–52 survival rates for Negroes were generally about 90 to 97 percent for the early elementary grades. The difference between the white and Negro survival ratios probably represents both the greater amount of retardation among Negro pupils and the higher nonwhite out-migration from the South in recent years.

Survival rates from grades one to two are always lower than for the other elementary grades because of the great amount of retardation in the first year of school. For all Southern states the greatest improvement in survival rates during the last 20 years has been in the survival from grade one to grade two. In 1932 some states had four times as many Negroes enrolled in grade one as in grade two the following year, representing a survival rate of 25 percent. By 1950–52 the survival from grade one to

TABLE 9.—*Percentage[1] difference between cohort-survival and ratio estimates of enrollment and actual enrollment in North Carolina, 1951 and 1952, by race*

Grade	WHITES				NEGROES			
	1951		1952		1951		1952	
	Ratio	Cohort	Ratio	Cohort	Ratio	Cohort	Ratio	Cohort
1	−2.3	−1.6	−1.7	−4.8	−10.7	5.9	−7.7	7.2
2	−6.3	1.3	−0.1	−0.2	−9.9	−7.0	−6.1	−1.1
3	0.2	−0.9	−4.2	0.8	−4.8	−0.1	−7.6	−5.0
4	−1.6	−1.5	−1.8	−1.6	−7.6	−2.5	−6.6	0.3
5	−1.3	−1.3	−0.2	−2.5	−4.7	−1.2	−4.4	−3.6
6	−3.3	−2.1	−0.9	−2.6	−6.4	−0.8	−8.4	−2.1
7	−5.2	−2.9	−4.2	−3.8	−1.0	−3.5	−7.6	−4.8
8	4.8	−2.3	1.7	−5.1	−16.3	−3.8	−19.5	2.8
9	−0.6	−1.2	0.1	−1.7	−35.6	−6.9	−33.9	−1.0
10	12.0	0.0	10.8	−0.8	−38.9	−3.5	−30.6	−9.2
11	−13.8	−0.2	−16.9	1.2	−34.5	−6.6	−38.6	−7.7
12	3.3	−1.0	12.9	3.2	1.7	−0.8	1.0	−4.4
Total	1.5	−1.2	−0.4	−1.8	−11.4	−1.3	−12.2	−1.4

[1] Minus sign indicates that estimate was lower than actual enrollment. actual enrollment is the base for all percentages. Estimates for 1951 and 1952 were computed using data up to 1950, and were then compared with actual enrollment in 1951 and 1952.

TABLE 10.—*Percentage[1] difference between cohort-survival and ratio estimates of enrollment and actual enrollment in South Carolina, 1951 and 1952 by race*

Grade	WHITES				NEGROES			
	1951		1952		1951		1952	
	Ratio	Cohort	Ratio	Cohort	Ratio	Cohort	Ratio	Cohort
1	−1.4	9.1	−3.7	3.7	−5.4	4.4	2.7	2.8
2	−4.0	−4.7	−4.0	2.2	−13.6	−2.9	−15.6	−5.6
3	−2.7	−1.0	−5.9	−6.2	−13.1	1.6	−17.6	1.3
4	−1.7	0.1	−4.8	−2.5	−12.5	0.5	−14.9	2.0
5	−1.5	−2.1	−3.3	−3.8	−11.2	−0.7	−13.0	−0.4
6	−0.7	−1.4	−2.8	−4.2	−5.9	2.5	−8.5	3.4
7	−4.2	0.2	−2.3	−3.1	−5.4	3.5	−5.3	6.0
8	−5.2	−0.5	−9.6	−1.2	6.0	−8.1	−2.4	−8.6
9	−6.3	−3.4	−6.5	−3.4	−2.8	0.5	−0.5	−1.9
10	−7.1	−2.7	−8.7	−4.4	−10.1	−1.4	−7.9	−0.3
11	−8.5	1.8	−10.6	−0.9	−8.6	3.2	−12.4	6.0
12	−9.9	−7.5	−13.2	−5.2	−20.3	−7.3	−26.2	−2.8
Total	−3.6	−0.4	−5.4	−2.1	−8.4	0.6	−8.9	0.4

[1] Minus sign indicates that estimate was lower than actual enrollment, actual enrollment is the base for all percentages. Estimates for 1951 and 1952 were computed using data up to 1950, and were then computed with actual enrollment in 1951 and 1952.

Impacted School Districts

The enrollment in a number of school districts—probably among the smaller ones—ebbs and flows as the Federal government opens, enlarges, contracts, and closes facilities. Enough personnel and their families, including school-age children, are involved so that total school enrollment can be significantly affected. Often the local school district is not notified very much ahead of time, certainly never a decade in advance. Thus long-run planning is impossible for such a school district. An impacted district is particularly dependent on the county or State for assistance with financing, personnel, and even buildings. For example, it may be necessary to recruit large numbers of teachers on short order, or conversely, to find jobs for a large number elsewhere if the Federal Government suddenly decides to contract or close a facility.

Case Studies of Small School Districts

One such district in the New Jersey portion of the New York metropolitan area has about 1,500 students enrolled in public elementary and high school. The community is an old one, dating back to before the American Revolution, and consists in large part of single-family houses. As of 1968 it appears to be almost all built up; land use surveys do not indicate any large growth in population. It is thought that if present zoning continues, the number of families and of children enrolled in public schools will remain largely unchanged. In fact, it can be argued that the largest future variations in the numbers of public school students will come about from changes in the life cycles of the present families. As older families whose children are grown up are replaced by younger families with school-age children, the number of students will increase, and the reverse.

However, even this reasonably "certain and predictable" school district contains some unknowns that can upset a longrun projection. First, there is no guarantee that a decade hence the zoning laws will not be changed to permit multi-family buildings, and hence more children. Two sets of projections can be made, one assuming a continuation of the single family house as the predominant type, and a second assuming the construction of some specific number of multi-family buildings.

A second unknown is the ratio of public to private school enrollment. Heretofore, this has tended to be fairly constant. But it is not inconceivable that the ratio can change sufficiently to make long-range projections uncertain. In a school district of this size, the shift of even 100 or 200 pupils between public and private schools can have a measurable impact on the public school facilities.

Finally, there is the unknown factor of the high school and its future. The present building (there is only one high school) needs replacing. Education experts have suggested that the community has too small an enrollment to permit having an efficient high school. It is thought that not enough diversity of courses can be provided; it is very difficult to attract and keep high quality teachers and administrators, and so forth. Accordingly, it was recommended that this community's high school students be sent to a larger neighboring community. As of this time, 1968, the community has not decided what to do with its high school students. Enrollment projections for 1980 can be quite different for this and the larger neighboring communities, depending on where the high school students will go.

A contrasting situation is that of another district in the Westchester County portion of the New York metropolitan area which has a total enrollment of 5,500 students (1968–69). The district lies across two communities—one in the early stages of economic decay typical of river communities along the upper reaches of the Hudson River Valley—and another which is one of the more affluent ex-urban communities in the central area of Westchester County. The district is primarily a post-World War II development.

The housing stock in the district ranges from high-density apartment units to low-density single family dwellings with a minimum zoning of 2 acres each. Because of recent down-zoning in the more affluent community, the present population level of 18,000 in that community is thought to be at or near two-thirds of the ultimate saturation level as projected in some recent local land use surveys. Thus the contribution to future growth in enrollments from the more affluent community in the district is not anticipated to be sizable over the next decade. The older, and less well-off community, comprising the other half of the district, is going through the throes of urban renewal and the potential impact of an expressway to be built on land adjacent to the Hudson River. These prospective changes are thought to affect the future growth of moderate and high-density two for Negroes had risen to 90 percent in at least one state and was above 50 percent in all states but one.

Survival rates for the upper elementary grades have also improved considerably in the last 20 years, and for whites are in the 90 to 98 percent range for the last few years. For Negroes the survival rates in the upper elementary grades are still generally between 85 and 90 percent although they have been improving during the last 20 years. Survival rates in high school grades have not increased much for whites in the last 20 years. The proportion of students who reach high school has greatly increased due to increased survival in the earlier grades, but the proportion of high school freshmen who complete high school has not increased much. For Negroes there has been more increase in survival through high school, but they still lag behind the whites in the proportion that remains in school. High school survival rates in the Southern states between 1950–52 ranged between .80 and .90 for whites and between .70 and .85 for Negroes.

For each state, we plotted the survival rates for the last 20 years on graph paper and examined them for trends and irregularities. For most grades in most states the survival rates were consistent from year to year. We computed the linear regression [5] of the survival ratios for the last 20 years and plotted the regression line on the same graph with the actual survival ratios. For most of the survival ratios, a straight line fitted the actual plotted points very well. In these cases, we projected the linear regression into the future to obtain future values of the survival ratios. Certain restrictions were placed on all these linear projections. None of the survival rates were allowed to go above a limit determined separately for each state from inspection of the survival rates for the early elementary grades. For example, in Alabama survival rates for the early elementary grades appeared to be fairly stable between 97 and 98 percent. Therefore, we chose 97.5 percent as an upper limit for the survival rates for Alabama. For Florida, this upper limit was 102.5 percent, the in-migration to Florida making the grade cohorts actually grow in size as they progressed through the elementary grades. As a final restriction, projected survival rates for Negroes were not allowed to exceed the white survival rate for the same grade and year.

A simpler procedure, and one which will probably be as accurate, is to decide on the restrictions to be observed in projecting the trend, and then draw in a freehand curve, using the past data and the restrictions on the future trends. For the survival rates which were not appropriate for linear regression, we adopted several different procedures. Some of the rates were projected by assuming that their future pattern would be like some adjacent grade where the data were regular. Some were projected by using linear regression over a few of the more recent years. Survival rates which did not seem to exhibit any upward trend during the past 20 years were projected by averaging survival rates in the last few years and projecting the average into the future. A few of the survival rates were so irregular that no method of projection gave much confidence that we would be able to estimate the future ones accurately, but for the most part the survival rates exhibited regular trends.

The most difficult problem in the use of the cohort method is the determination of the size of the entering first grade in each future year. Births can be "survived" to first grade enrollment six years later, but these survival rates fluctuate much more than those from one grade to the next. Investigation showed that it was more accurate to project births seven years and relate them to second grade enrollment than to relate births to the first grade enrollment six years later. Even though second grade enrollment provides a better base for computing the entering cohort, it would also be a good idea to check each entering cohort against the estimated population six,

[5] For a description of linear regression and computing procedures see any standard statistics text. For example: Hagood, Margaret J. *Statistics for Sociologists*. (New York: Henry Holt and Co. 1941 Chapter XXI.)

TABLE 11.—*Alabama white elementary enrollment*

Grade	Year											
	1940–41	1941–42	1942–43	1943–44	1944–45	1945–46	1946–47	1947–48	1948–49	1949–50	1950–51	1951–52
1	56,258	55,654	54,609	54,024	54,763	53,721	52,513	52,042	52,711	64,053	45,495	47,957
2	45,835	46,602	46,718	45,460	44,998	45,821	45,015	45,717	46,142	47,162	56,881	42,411
3	48,143	45,400	45,577	44,614	43,857	44,225	44,373	44,164	44,345	45,185	46,280	54,828
4	47,033	46,350	44,221	43,562	43,172	42,637	42,568	43,250	42,726	43,054	43,982	44,818
5	46,851	44,889	44,810	41,945	41,335	41,256	40,850	41,446	41,861	41,710	41,938	42,726
6	44,431	43,563	42,024	40,942	38,572	38,762	38,368	38,747	39,228	39,908	40,205	40,352
7	38,707	40,787	40,538	38,603	37,726	36,405	36,460	36,753	37,274	38,043	38,724	38,979
8	30,729	32,133	34,287	32,687	31,774	32,333	31,140	31,726	32,228	33,282	33,690	34,422
	SURVIVAL RATES[1]											
Survival from:												
1–2		0.8284	0.8394	0.8325	0.8329	0.8367	0.8379	0.8706	0.8866	0.8047	0.8880	0.9322
2–3		.9905	.9780	.9550	.9647	.9828	.9684	.9811	.9700	.9793	.9813	.9639
3–4		.9628	.9740	.9558	.9677	.9722	.9625	.9747	.9674	.9709	.9734	.9684
4–5		.9544	.9668	.9485	.9489	.9556	.9581	.9736	.9679	.9762	.9741	.9714
5–6		.9298	.9362	.9137	.9196	.9378	.9300	.9485	.9465	.9555	.9639	.9622
6–7		.9180	.9306	.9186	.9214	.9438	.9406	.9579	.9620	.9698	.9681	.9695
7–8		.8302	.8406	.8063	.8231	.8570	.8554	.8702	.8769	.8920	.8858	.8889

[1] Survival rates are computed by dividing the enrollment in a given grade by the enrollment in the next lower grade the year before. For example, $\frac{46,602}{56,258}=$ 0.8284.

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apartment units which often yield fewer numbers of pupils per unit.

There is a great deal of uncertainty about the annual size of enrollments, particularly in the elementary grades, as well as the future existence of the district itself, because:

First, New York State allows tuition transfers between adjacent, and in some instances, noncontiguous school districts depending upon vacancy levels. These tuition transfers have contributed to the annual variations of elementary enrollments.

Second, several parochial school systems in the area serviced by the school district have, due to financial uncertainties, recommended to parents that they enroll their children in public school for the first grade and then transfer them into the parochial school system for the continuation of their education. This has the effect of overloading existing capcity levels in the kindergarten and first grades and drastically reducing capacity levels in the second and later grades.

Third, the district has several wards in the poorer of the two communities that are predominantly nonwhite. As a result several of the neighborhood schools have reached unsafe white/nonwhite enrollment proportions as designated by the New York State Department of Education. As a result of this, redistribution through busing of pupils to other schools in the district and adjacent districts is being contemplated.

Fourth, New York State Department of Education facility planners have informed the local school board that several of the existing facilities have become superannuated. The State Education Department consultants have recommended to local school board officials a possible merger with an adjacent school district with newer, but relatively underutilized capacity levels.

All of these factors: the noncontiguity of local political units and school districts; the economic, demographic and housing characteristics in the communities; the age and capacity of existing and contemplated additions or changes in school plants; the very existence of the school district as a distinct entity; contribute to the difficulty of generating reasonable, reliable, and comparable enrollment statistics series, and projections.

To some extent every school district is unique and has its own history. We have no reason to believe that the two districts described are particularly unusual. They simply illustrate some of the problems involved in trying to make enrollment projections for these semipermanent local public school districts.

seven and eight years old. If an error is made in determining the size of the beginning group, it will be survived through all the succeeding grades to affect all the projections which contain this particular cohort. Therefore, it is important to double check the entering cohort if possible.

The survival rates from births to second grade enrollment were projected by linear regression like the other survival rates. These rates, unlike the grade to grade survival rates, were allowed to decrease if previous trends in the state indicated that further decrease was likely.

When all the survival rates were projected, we computed future enrollment estimates. We computed the estimate of enrollment for the first future year by applying the proper survival rates to the actual enrollments of the last year for which they were available. The enrollment estimates for the second future year were computed from the estimated enrollments of the first future year, the third future year from the second future year, and so on until all the cohorts were survived through school. The computation of enrollment for a single state is illustrated below.[6]

Step 1. Copy down the enrollment data and compute the survival rates grade by grade. Table 11 shows the elementary grades for the period from 1941–1952. From inspection of the graph of the survival rates, select the years which will be used to compute the trend.

Step 2. Copy down the births. Adjust them for under-registration and on school year basis. Since the minimum age at which children are admitted to the first grade varies, adjustment to a school year basis should reflect the regulations of the state defining the eligible group. For example, if a child must be six before October 1 to be admitted to school in September, the adjustment consists of taking ¼ the previous year's births and ¾ of the current year's births. These adjustments are shown in table 12.

[6] A more complete description of computing procedures can be found in *Classrooms For How Many?* by the State of New York Commission on School Buildings.

TABLE 12.—*Adjustment of births for under-registration and to a school-year basis*

Year	Reported births	Percent of under-registration	Corrected birth	One-quarter of previous year	Three-quarters of present year	Adjusted to school year
1935	38,530	81.2	47,480	....	....	47,480
1936	36,928	82.3	44,868	11,870	33,651	45,521
1937	38,286	83.5	45,867	11,217	34,396	45,613
1938	39,045	84.6	46,151	11,467	34,613	46,080
1939	37,840	85.8	44,121	11,538	33,091	44,629
1940	39,001	86.8	44,929	11,030	33,697	44,727
1941	40,536	88.2	45,968	11,232	34,476	45,708
1942	46,430	89.5	51,862	11,492	38,896	50,388
1943	50,317	90.9	55,340	12,965	41,512	54,477
1944	47,190	92.2	51,201	13,837	38,401	52,238
1945	43,884	93.2	47,088	12,800	35,316	48,116
1946	50,978	94.4	53,986	11,772	40,490	52,262
1947	57,132	95.4	59,874	13,496	44,906	58,402
1948	53,387	96.2	55,522	14,968	41,642	56,610
1949	51,810	96.7	53,572	13,880	40,179	54,059
1950	49,512	97.1	50,997	13,393	38,248	51,641
1951	50,548	97.4	51,898	12,749	38,924	51,673
1952	50,720	97.6	51,977	12,974	38,983	51,957

TABLE 13.—*Computation of survival ratios, births to second grade enrollment, Alabama whites*

Year of birth	Year of 2d grade enrollment	Number of adjusted births	2d grade enrollment	Enrollment divided by births	Years coded
				Y	X
1935	1942–43	47,480	46,718	0.9840	−3
1936	1943–44	45,521	45,460	.9987	−2
1937	1944–45	45,613	44,998	.9865	−1
1938	1945–46	46,080	45,821	.9944	0
1939	1946–47	44,629	45,015	1.0086	1
1940	1947–48	44,727	45,717	1.0221	2
1941	1948–49	45,708	46,142	1.0095	3
				$\Sigma Y = 7.0038$	$\Sigma X = 0$
1942	1949–50	50,388	47,162	.9360	....
1943	1950–51	54,477	56,881	1.0041	....
1944	1951–52	52,238	42,411	.8119	....
1945	1952–53	48,116	48,116	1.000	....
1946	1953–54	52,262	52,262	1.000	....
1947	1954–55	58,402	58,402	1.000	....
1948	1955–56	56,610	56,610	1.000	....
1949	1956–57	54,059	54,059	1.000	....
1950	1957–58	51,641	51,641	1.000	....
1951	1958–59	51,673	51,673	1.000	....
1952	1959–60	51,957	51,957	1.000	....

Step 3. Compute percent of births that enter the second grade seven years later. This is shown in table 13.

Step 4. Project survival rates for each grade into the future, using linear regression. Computations for births to second grade enrollment are shown below; the data are taken from table 13. The data for the school years 1949–50, 1950–51, and 1951–52 have been omitted from the calculations, because there were changes in the age at entrance which affected the size of the entering cohorts from 1950 to 1952. The necessary figures for a regression equation, taken from table 13, are shown below.

$$\Sigma X = 0 \qquad \Sigma Y = 7.0038 \qquad A = Y/n = 1.0005$$

$$\Sigma X^2 = 28 \qquad \Sigma XY = .1454 \qquad b = \frac{\Sigma XY}{\Sigma X^2} = .0052$$

$$N = 7$$

After A and b are determined, the regression equation $Y = A + bX$ is used to estimate values of Y for future years. For example, we obtain a ratio of 1.0369 for 1952–53, 1.0421 for 1953–54, etc.

Step 5. Consider the reasonableness of your projection. Is it in line with projections in other states? Does it seem logical? In a state like Alabama, for instance, which has had a history of out-migration, it seems unlikely that the number of students in the second grade will exceed the number of births seven years previously by any appreciable amount. Instead of using the regression line as a projection, the average ratio for the seven years from 1943–1949 seemed a better type projection. Therefore the average 1.0005 is rounded off to 1.000 for ease in computation and projection.

Step 6. Using the projected survival rates, estimate the future enrollment by applying the survival rates to the present enrollment and births. (See bottom half of table 13.)

DEMOGRAPHY[7]

Harvard Graduate School of Education

The projection method used by the Harvard Study Staff is a percentage of survival technique. This method, described in this appendix, involves the computation of the number of public-school students in a given geographical area who in the past have reenrolled in the public schools the following year. This computed figure is then used to predict future enrollments.

An alternative method of projection which the Harvard Study Staff seriously considered was a multi-variable technique which isolates all of the factors involved and treats them separately. Limitations in the data available from the public and nonpublic schools enrolling Watertown students prevented the use of this method.

The assumptions underlying any demographic projection must be understood by the users if undue reliance on them is to be avoided and if the figures are to be reasonably adjusted should unforeseen events occur. In calculating a percentage of survival from, for example, the Phillips School first grade to the second grade, the basic assumption is that factors which have in the past prevented first-grade Phillips students from enrolling in the second grade the next year will continue to have the same overall effect. Thus a child might not enroll in the second grade because of retention in the first grade, dropping-out or exclusion from the public schools, transfer to nonpublic schools or a different Watertown school or the schools in a different town, death, or physical incapacitation. In addition, a student who had not been in the first grade of the Phillips School the previous year might show up for the second grade if his family moved into the Phillips School district, if he transferred from a nonpublic school, or if he was left back from the previous year's second grade. The percentage of survival technique assumes that the net effect of all these factors will remain in the future as it has in the past. If any of these factors changes radically in the future, then the projections based upon this assumption will have to be altered accordingly.

The largest potential error in the projections lies in the assumption that the relationship between the public and private schools of Watertown will remain unchanged. Historically, about 80 percent of Watertown's students have attended public schools. (See table 14.) After discussions with officials of the Archdiocese of Boston and the principals of local parochial and private schools, it was determined that there are at present neither plans for new construction nor plans to phase-out any grade levels for these schools. To the extent that plans change, the projections will have to be altered.

It has been the experience of the Watertown public schools that the construction of high-rise apartments has had little or no effect on school enrollments. These apartments have tended to be single- or double-bedroom units; families which occupy them are generally without children. In addition, conversion of single-family homes to two-family units is minimal in Watertown.

The Watertown School Department should be alert to possible future changes in these trends. Special attention should be paid to the disposition of the Arsenal site and the MBTA car barn.

Birth-to-kindergarten projections rest largely on the assumption that the fertility ratio of Watertown women in each age group will remain at the 1965 level. A simple but important yearly check on the projections would be to compare the actual births each year in the 1966–71 period with the predicted births. To the extent that these predictions are inaccurate, the enrollment projections, beginning five years afterward, should be adjusted accordingly.

It is suggested that all projections should be updated on a yearly basis and particularly at such time as final planning is made on any given facility or program. An annual census of school-age and preschool-age children living in Watertown would provide this information most accurately. Close cooperation between the schools and the office of the Town Clerk would provide the schools with the necessary information on Watertown births. Much work is needed to update the recordkeeping system for providing information on enrollments of all Watertown children and all children attending schools in Watertown. A thorough study of the whole recordkeeping system should be made with a view to using data-processing techniques.

Methodology.—A basic consideration in the development of a methodology for the projection of school enrollments is the need to provide information on public school enrollments in a form which will allow the development of plans for the districting of the town. This objective required the division of projected school enrollments according to some geographical sectioning of the town. The method for the distribution of present enrollments in Watertown follows.

TABLE 14.—*Distribution of Watertown children in public and nonpublic schools*

Year	Minors 5–16	Public schools	Private schools	Vocational and special schools	Percent in public schools
1961-62	6,851	5,263	1,567	21	77
1962-63	7,081	5,558	1,507	16	78
1963–64	7,222	5,487	1,726	9	76
1964–65	7,150	5,513	1,614	23	77
1965–66	7,255	5,445	1,793	14	75

Source: 1961-62 through 1964–65 from annual reports of the Department of Education, Public Document No. 2, the Commonwealth of Massachusetts. 1965-66 obtained directly from Department of Education.

TABLE 15.—*Construction of new dwelling units*

Year:	*Number of units*
1961	104
1962	295
1963	366
1964	52
1965	137
1966	32

Source: Watertown Town Reports.

[7] Source: Harvard Graduate School of Education; *Watertown: Its Schools and Needs*, Cambridge, 1966. App. A.

Initially an attempt was made to have the districts used in the projections conform exactly to the eight present elementary school districts. It was found, however, that often exact school district lines are unclear. In some cases, children living on opposite sides of a border street attend the same school; in others, such children attend different schools. Therefore, for projecting enrollments by districts, district lines were regarded as dividing border streets; children living on opposite sides of border streets were considered to be attending schools in different districts. The eight school districts were then divided into a total of sixty-nine subdivisions.

With the aid of computers, school enrollments for 1966–67 and Watertown births for 1958 through 1965 were distributed into the eight districts and sixty-nine subdivisions by address of parent. Addresses for children in kindergarten through grade 9 were obtained through a special school census administered by the Watertown teachers at the request of the Study Staff. Addresses for children in grade 10 through grade 12 were obtained from data available at the New England Educational Data Systems. Parent addresses for birth data were provided by the office of the Town Clerk. Projected births for 1966 through 1971 were also distributed geographically.[8] The births for 1962 through 1965, the projected births for 1966 through 1971, and the present school enrollments form the basic data used to project enrollments for the next ten years by the percentage of survival method.

The method described results in a grade-by-grade distribution on enrollments over the sixty-nine subdivisions of the town for each year of the projections. Such data allowed the Study Staff to investigate the implications of various districting patterns and grade organizations.

Although these data represent the best estimate of the school population of the individual subdivisions of the town, they must be viewed with some caution. Trying to predict the number of children who will reside in a small geographic area as much as ten years from now is very difficult. The greater the number of subdivisions that are combined, the greater is the expected validity of the projection. Such estimates are made to provide data for overall planning purposes. The validity of any individual subdivision projection is not to be relied upon heavily in the overall planning. Therefore, too great a reliance should not be placed upon the *exact* boundaries described in the long-term districting recommendations.

Predicting the Number of Births.—In order to predict how many students there will be in Watertown Kindergartens in 1976, it must first be predicted how many Watertown children will be born in 1971. Rather than trying merely to establish a trend from a table of the number of births in Watertown in the past, the Study Staff has used a more complex method.

Census reports include the age of Watertown women for those years. Using a percentage of survival technique similar to the one just described, 1965 data can be used to predict the age distribution of women in Watertown until 1971. This set of figures and a calculated fertility rate for these women can be used to predict the number of births. A more detailed explanation follows.

[8] Since these births represent future births, no addresses exist for them. The distribution of these births into the sixty-nine subdivisions was determined by an analysis of the pattern of distribution of births over the same subdivisions during the period 1958–1965.

Table 16 shows the age distribution of women in Watertown according to census reports.

TABLE 16.—*Age distribution of women*

Age group	1955[1]	1960[2]	1965[3]
10 to 14	1,346	1,565	1,481
15 to 19	1,073	1,283	1,424
20 to 24	1,409	1,251	1,651
25 to 29	1,633	1,471	1,666
30 to 34	1,607	1,383	1,201
35 to 39	1,409	1,372	1,204
40 to 44	1,429	1,316	1,311

[1] *The Decennial Census*, 1955, Mass., Sect. of the Commonwealth.
[2] *U.S. Census of Population, Mass. General Population Characteristics.*
[3] From the 1965 Mass. census, not yet published.

Table 17 shows the calculation of a percentage of survival of the number of women in a five-year age group to the number of women in the next age group five years later. In table 18, these empirical percentages are applied to the 1965 age distribution to obtain a 1970 distribution.

It can be assumed that ⅕ of the change in the number of Watertown women between 1965 and 1970 took place each year and that the same ⅕ change can be applied to the 1970 figure to get a figure for 1971. In this manner, table 19, showing the number of women in Watertown by age group, was compiled.

TABLE 17.—*Percentage of survival*

Age group	1955–60 percent survival	1960–65 percent survival	Sum	Average percent survival
10–14 to 15–19	1,283/1,346=0.9532	1,424/1,565=0.9099	1.8631	0.9316
15–19 to 20–24	1,251/1,073=1.1659	1,651/1,283=1.2868	2.4527	1.2264
20–24 to 25–29	1,471/1,409=1.0440	1,666/1,251=1.3317	2.3757	1.1879
25–29 to 30–34	1,383/1,633=0.8469	1,201/1,471=0.8165	1.6634	.8317
30–34 to 35–39	1,372/1,607=0.8538	1,204/1,383=0.8706	1.7244	.8622
35–39 to 40–44	1,316/1,409=0.9340	1,311/1,372=0.9555	1.8895	.9448

TABLE 18.—*1970 predicted age distribution of women*

1965 age group	Number in 1965	Percent survival	Prediction for 1970	1970 age group
10 to 14	1,481	0.9316	1,481 (0.9316)=1,380	15 to 19.
15 to 19	1,424	1.2264	1,424 (1.2264)=1,746	20 to 24.
20 to 24	1,651	1.1879	1,651 (1.1879)=1,961	25 to 29.
25 to 29	1,617	.8317	1,617 (0.8317)=1,345	30 to 34.
30 to 34	1,201	.8622	1,201 (0.8622)=1,036	35 to 39.
35 to 39	1,204	.9448	1,204 (0.9448)=1,138	40 to 44.

TABLE 19.—*Women in Watertown by age group, 1966–71*

Age group	1966	1967	1968	1969	1970	1971
15 to 19	1,415	1,406	1,398	1,389	1,380	1,371
20 to 24	1,670	1,689	1,708	1,727	1,746	1,765
25 to 29	1,725	1,784	1,843	1,902	1,961	2,020
30 to 34	1,230	1,259	1,287	1,316	1,345	1,374
35 to 39	1,170	1,137	1,103	1,070	1,036	1,002
40 to 44	1,276	1,242	1,207	1,173	1,138	1,103

The second major task in predicting the number of births is to compute fertility ratios (number of births per thousand women) for these age groups. As can be seen from table 20, the number of births in Watertown was fairly stable until 1965.

TABLE 20.—*Number of births in Watertown, 1950–65*

Year	Births	Year	Births
1950	807	1958	975
1951	864	1959	991
1952	840	1960	970
1953	858	1961	1012
1954	916	1962	925
1955	931	1963	955
1956	952	1964	929
1957	986	1965	831

Source: Town Clerk's Office. A check of the actual birth-record certificates occasionally revealed a higher figure than that supplied to us by the clerk. In such a case, the higher figure was used.

Fertility ratios have been estimated for 1955, 1960, and 1965. The rates for "1955" and "1960" were computed by reference to the following three-year averages of births to help offset any unusual deviation for those years.

Year	Births
"1955" births	
1954	916
1955	931
1956	952
Total	2, 799
Average	933
"1960" births	
1959	991
1960	970
1961	1, 012
Total	2, 973
Average	991

This procedure could not be used in calculating the 1965 fertility ratios because the 1966 birth data are not yet complete. For reasons explained below, however, the actual figure of 831 births was used in 1965 fertility ratio calculations.

The estimation of fertility ratios is based on the assumption that fertility ratios between age groups remain in a constant proportion to each other. Fertility weights previously computed[9] were multiplied by a common multiplicative factor in each of the years 1955, 1960, and 1965 to calculate the fertility rate for that year. The general formula to compute the factor is:

$$\frac{1000N}{F_1 \cdot W_1 + F_2 \cdot W_2 \ldots F_n \cdot W_n},$$

where N = the number of births, the $F_1, F_2, \ldots F_n$ are the number of females in each age group, and the $W_1, W_2, \ldots W_n$ are the fertility weights for the corresponding age group. For 1955, the factor is $\frac{1{,}000 \times 933}{1{,}490} = 626$.

For 1960 it is 729; and for 1965, 556. The resulting set of fertility ratios is shown in table 21. The fertility ratios multiplied by the number of females in each age group give a total number of births equal to that observed in the year.

Although on the surface, the 831 births in Watertown in 1965 represent a remarkable 11 per cent decrease from the preceding year, it in fact reflects a national trend. The decrease in births for the United States as a whole was 9 per cent, a figure that was exceeded in many nearby towns. For example, Dedham experienced a 12 per cent decrease in births the same year. The trend has continued nationally into 1966, although there are indications that an eventual increase in the number of women between the ages of 15 and 44 may again set off rises in the number of births. The decrease in fertility ratios observed in table 21 also reflects a national trend and is expected to continue. The 1965 drop in Watertown exceeds the national average and may not be repeated for some years. Therefore it was assumed that Watertown's fertility ratios through 1971 would be equal to those estimated for 1965.

The predicted number of births for 1966–1971 can be calculated by multiplying the 1965 fertility ratios by the number of women by age groups.

[9] Fertility weights computed by Pascal K. Whelpton in *Forecasts of the Population of the United States 1945–1975* (Washington, 1947), p. 21, were verified for the New England area. Harvard University Study Staff, *A Report on the Schools of Boston*, May, 1962, p. A–3.

TABLE 21.—*Fertility ratios*

Age group	Female population			Fertility weights [1]	Fertility ratios			Births		
	1955	1960	1965		1955	1960	1965	1955	1960	1965
15 to 19	1, 073	1, 283	1, 424	0. 08	50	58	44	54	74	63
20 to 24	1, 409	1, 251	1, 651	. 30	188	219	167	265	274	276
25 to 29	1, 633	1, 471	1, 666	. 30	188	219	167	307	322	278
30 to 34	1, 607	1, 383	1, 201	. 20	123	146	111	201	202	133
35 to 39	1, 409	1, 372	1, 204	. 10	63	73	56	89	100	67
40 to 44	1, 429	1, 316	1, 311	. 02	13	15	11	19	20	14
Total births								[2] 935	[2] 992	831

[1] The weights are proportional to fertility ratios in each age group and add up to 100.
[2] Sum differs from actual "1955" and "1960" figures because of rounding.

TABLE 22.—*Projected births by age group, 1966–71*

Age group	1966	1967	1968	1969	1970	1971
15 to 19	62	62	62	61	61	60
20 to 24	279	282	285	288	292	295
25 to 29	288	298	308	318	327	337
30 to 34	137	140	143	146	149	153
35 to 39	66	64	62	60	58	56
40 to 44	14	14	13	13	13	12
Total births	846	860	873	886	900	913

Summary: Projection of births.

Year:	*Births*
1966	846
1967	860
1968	873
1969	886
1970	900
1971	913

The Percentage of Survival Method.—The October 1 attendance figures as reported by the principals have been used with the following two corrections:

(*a*) The sixth grade of the Phillips school has attended Parker school in 1965 and 1966. To provide figures for the Phillips and Parker school districts, these children have been counted according to the district in which they reside.

(*b*) Hosmer school was not open in 1966. To provide projections for Hosmer school district and to avoid letting the Hosmer children affect the calculations in other districts, the number of children living in the Hosmer school district has been subtracted from the totals of all other schools and credited to the Hosmer district.

Many other demographic studies have used a birth-to-first-grade percentage of survival calculation because of great fluctuations in the rate of survival from kindergarten to first grade. This latter rate in Watertown, however, is relatively stable. After comparing both techniques, it was felt that considering kindergarten enrollments the previous year would be a better predictor of first-grade enrollments than six-year-old birth data. Therefore a birth-to-kindergarten percentage of survival was used as the basis for the projections.

The percentage of survival was based on a four-year period because the data on the distribution of births by school district extended back only to 1958. This data would relate to the 1963 kindergarten class. Since there is kindergarten enrollment data for 1963, 1964, 1965, and 1966, at most a four-year average could be utilized. An example follows:

Births in Phillips school district		Kindergarten enrollment		B-K percent survival
1958	129	1963	111	111/129=0.86047
1959	118	1964	91	91/118= .77119
1960	115	1965	97	97/115= .84348
1961	121	1966	99	99/121= .81818
Sum				3.29332
Average				.8233

Therefore the birth-to-kindergarten survival rate for the Phillips school district is .8233.

To predict the kindergarten enrollment in the Phillips school in 1973, it is necessary to utilize 1968 projected births. It is anticipated that there will be 873 births in Watertown in 1968. That year, it is expected that .1425 of Watertown's births will occur in the Phillips school district, or $873(.1425)=124$. Thus .8233 of this amount, or $124(.8233)=102$, will enroll for kindergarten in 1973.

For all other calculations besides the birth-to-kindergarten percentage of survival, a percentage of survival technique based on a five-year average has been used. The Staff could go no farther back than 1961 because, effective with the opening of schools in 1961, the boundary lines for Cunniff, Browne, and Lowell Schools were changed.[10] (A change effective in 1962, involving Browne and Cunniff schools, which allowed students living on certain streets who would be in the sixth grade in 1961 to remain in their old schools rather than transfer, will not affect these calculations as sixth grade figures for 1961 are not used in the calculations.) An example follows:

	Grade 5	Grade 6	Percent survival
1961	57	(1)	....
1962	60	57	57/57=1.00000
1963	46	64	64/60=1.06666
1964	64	42	.91304
1965	61	63	.98437
1966	(1)	65	1.06557
Sum			5.02964
Average			1.0059

[1] Not relevant to this percentage of survival calculation.

Thus the percentage of survival for Cunniff, from fifth- to sixth-grade, is 1.0059. For example, since it is predicted that there will be fifty-four students in the Cunniff fifth grade in 1968, we would expect $54(1.0059)=54$ students in the sixth grade there in 1969. A similar calculation was performed for all districts, K–6.

The students from Cunniff, Browne, Lowell, Marshall Spring, and Parker schools will attend West Junior High school. The students from Hosmer and Coolidge will attend East Junior High school. Students from Phillips may attend either junior high school. Although there is a junior-high-school district line which runs down Common Street, into Mount Auburn, and then into Irving Street, cutting Phillips school district in two, in reality all the children in the district have the choice of attending either junior high school.

From data supplied by the Phillips principal, the percentage of children who entered West and the percentage who entered East were computed. In this manner, a composite percentage of survival for Watertown sixth graders entering the seventh grade was computed for each junior high school. Percentage of survivals calculated for each junior high school were applied to all districts feeding that junior high school. Town-wide average survival rates were applied to the Phillips school district. Survival rates calculated for the high school were applied to all sections of the town. Table 23 shows the survival rates calculated.

Projections.—In the manner described above, the Study

[10] See School Committee Minutes, 4/12/61.

Staff has projected enrollments for the Watertown public schools for each of the years 1967 through 1976 for each of the eight elementary school districts and sixty-ni· · subdivisions. Tables 24–25 show system-wide grade-by-grade summaries for those years and projected total enrollments under the proposed 4-4-4 organizational pattern.

TABLE 23.—*Average percentage of survivals*

SCHOOL	B–K	K–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12
BROWNE	0.7288	0.9097	0.8565	0.8983	1.0014	0.9450	1.0067	1.0033	1.0105	1.0137	0.9974	1.0320	0.9674
COOLIDGE	.6954	.9132	.8205	.9100	.8834	.9515	.9183	.9907	1.0161	1.0057	.9974	1.0320	.9674
CUNNIFF	.9240	.9095	.9121	.9902	.9844	.9509	1.0059	1.0033	1.0105	1.0137	.9974	1.0320	.9674
HOSMER	.6397	.8755	.8969	1.0035	.9533	.9902	1.0017	.9907	1.0161	1.0057	.9974	1.0320	.9674
LOWELL	.5567	.9051	.8085	.9636	1.0807	.9601	.9736	1.0033	1.0105	1.0137	.9974	1.0320	.9674
MARSHALL SPRING	.6031	.7008	.9315	.9337	.9946	.9868	.9809	1.0033	1.0105	1.0137	.9974	1.0320	.9674
PARKER	.7726	.6097	.8844	.9826	.9088	1.0555	1.0946	1.0033	1.0105	1.0137	.9974	1.0320	.9674
PHILLIPS	.8233	.8604	1.0341	.9088	1.0654	1.3970	1.0001	.9970	1.0129	1.0099	.9974	1.0320	.9674
E. JR. HIGH								.9907	1.0161	1.0057	.9974	1.0320	.9674
W. JR. HIGH								1.0033	1.0105	1.0137	.9974	1.0320	.9674
SR. HIGH											.9974	1.0320	.9674
Town Total	.6886	.8630	.9023	.9508	.9705	.9784	.9909	.9970	1.0129	1.0099	.9974	1.0329	.9674

TABLE 24.—*Projections of Watertown public school enrollments by grade, 1967–76*

Year	K	1	2	3	4	5	6	7	8	9	10	11	12	Total
1967	632	574	508	511	514	505	498	487	460	433	461	459	470	6,512
1968	673	547	510	487	514	506	498	407	487	460	433	462	456	6,530
1969	655	588	494	490	489	501	501	501	500	488	460	440	461	6,568
1970	582	571	531	475	496	477	502	502	507	503	488	476	433	6,543
1971	590	502	522	502	473	482	469	501	509	509	501	504	460	6,524
1972	597	510	460	496	503	461	478	470	506	515	509	517	488	6,510
1973	607	519	464	438	495	492	457	478	475	509	514	518	501	6,467
1974	617	522	466	442	436	486	486	457	483	483	508	521	509	6,416
1975	631	537	480	450	442	426	480	485	462	489	482	526	513	6,403
1976	637	544	488	460	451	434	424	479	494	467	488	498	508	6,372

TABLE 25.—*Projections of enrollments in 4–4–4 organizational patterns, 1967–76*

Year	K-4	5-8	9-12	Total
1967	2,730	1,950	1,823	6,512
1968	2,740	1,988	1,811	6,530
1960	2,716	2,003	1,840	6,568
1970	2,655	1,988	1,900	6,543
1971	2,580	1,961	1,974	6,524
1972	2,566	1,915	2,020	6,510
1973	2,523	1,902	2,042	6,467
1974	2,483	1,912	2,021	6,416
1975	2,540	1,853	2,010	6,403
1976	2,580	1,831	1,961	6,372

CHAPTER 4

SHORT-RANGE SCHOOL ENROLLMENT PROJECTION TECHNIQUES CONTINUED: DWELLING-UNIT ENROLLMENT-YIELD MULTIPLIERS

Another approach for making shortrun projections is through the use of information on the inventory of housing in the community, and the average number of pupils yielded by each type of housing, for example, one-family dwellings vs. apartment houses. This approach has particular merit for those local school areas undergoing substantial growth in the development of the land-space of the community or for those communities whose annual fluctuations in enrollment are geared to significant fluctuations in the employment size of the local work force; for example, "federally-impacted" communities, areas with a substantial segment of the employment base consisting of civilian or military personnel assigned to federal facilities. For these communities, the dwelling-unit, enrollment-yield method of forecasting school enrollment often can provide useful information for planning the timing for school construction, as well as for school personnel recruitment and development.

Ideally, a community should use both approaches—the cohort-survival and the dwelling-unit enrollment. Each method is fairly independent of the other, thereby providing a check. Both techniques should provide reasonably similar projections for the target date in order to insure confidence in the estimates. This is especially so if the target date is no more than 5 years in the future. If, for example, both indicate that enrollment will increase about 15 percent in the following 5 years, then school officials can be reasonably confident of the projection. On the other hand, if one technique suggests an increase of about 5 percent, and the other one of 15 percent, then the school officials are in a dilemma. Neither set of techniques can be demonstrated to be inherently more accurate than the other. Accordingly, only a careful subjective appraisal using all information available to the community can suggest what may be the best estimate.

In this chapter we shall cover some basic topics along with two examples of the application of the approach. They are: first, general informational and organizational requirements of the approach; second, the derivation of pupil-yield multiples for estimating school facility capacities in Montgomery County, Md., and for local areas in California.

Housing Inventories and Land-Use Surveys

The technique, "dwelling-unit enrollment-yield multipliers," taking as it does existing and anticipated construction into account, is really a land-use survey applied to school enrollment projections. Many communities conduct such surveys for purposes of planning roads, police and fire service, and other public services. Private organizations use information on how the land is being utilized at present, and how it might be utilized in the future, for planning the opening of retail stores to the installation of telephone cables, and so forth. By the simple device of using the type of dwelling unit to estimate the probable number of children enrolled in public schools, the land-use survey becomes a technique for projecting school enrollment.

Note also that this technique is most useful for the shortrun period, perhaps 2 to 5 years ahead. It is useful to the extent that construction work has begun, building permits issued, or housing development seriously contemplated. For longer-range periods, the land-use survey becomes largely judgmental. When—5, 10, 20 years hence—will a given parcel of land be developed? And will it be for single-family detached structures or some other construction? Zoning laws stipulate the type of permissible structure, but what is to prevent the present zoning laws from being changed 10 years from now?

An interesting methodology is being tested in Bucks County, Pa., for making longrun projections based on present and future housing supply. The report—"Estimate of Future Population Growth by School District, Bucks County, Pennsylvania"—describing the methodology is given in appendix D. The authors wrote:

> This estimate of population growth by school district depends on a *methodology which is still under development* (italics ours) but is consistent with results produced by special censuses taken in Bucks County since 1960.

How useful these procedures may be for other counties or communities is difficult to say since, apparently, Bucks County and the rest of the Philadelphia Standard Metropolitan Area is the only part of the country where this has been tried. Perhaps one of the reasons why other counties had not tested it since 1960 is that special censuses—past 1960—are required, as well as numbers of building permits. Furthermore, intimate and detailed knowledge of the local area is required. Perhaps after the 1970 Decennial Census results become available, other communities may be able to test the method.

It should also be noted that it is not certain that this method can be applied to a small area without taking into consideration the county or region of which it is a part. The authors wrote:

> It is an open question whether there is any such thing as a purely local trend in the development of a small area's population. The population growth of a small area appears to depend not only on the growth of its own housing supply but also on the provision of housing in many, many other small areas.

General Information Requirements

To begin, there must be considerable cooperation and communication with the local construction and real estate industry. Foreknowledge of future real estate development expansion, in conjunction with the pupil-yield characteristics of the intended housing, will aid school planners in the location and acquisition of site facilities and determination of local school transportation needs. A good example of this is the enabling legislation of Dade County, Fla. which requires submission of real estate development plans to the county school planners. Thus, they can evaluate the effects of the intended development upon future school enrollment, facilities, and other needs.

Second, information also should be obtained on the destruction of housing. If it is known that a large area of houses is to be cleared in order to provide new highways or bridge approaches, that information may be as important, or more so, than the numbers of new dwelling units scheduled for construction. Furthermore, if existing buildings are to be torn down and replaced with a different type of dwelling unit, that too should be known in advance. With such information, estimates of the probable loss in public school enrollment can be set alongside information on the possible gain.

Third, the user of the dwelling-unit enrollment-yield multiplier approach must be intimately aware of the housing cycle of the local community. This is perhaps best illustrated by the case of Montgomery County, Md. The 1960 median family income of $9,340 and educational attainment levels of 13.3 years for men and 12.6 years for women (population aged 25 and over) places the county among the highest in the Nation in these two characteristics. Until the early 1960's the county served largely as a bedroom for Federal Government employees. However, with the rapid growth of a federally-supported science-based industrial complex, the community grew more rapidly and took on a different character. Let us examine some aspects.

In the lower third of the county, immediately adjacent to the District of Columbia, older, previously expensive housing began to depreciate in value, falling to a price range which relatively young large families could afford. The result was that schools in this area experienced a relatively rapid increase in enrollments, particularly in the elementary grades.

In another portion of the lower third of the county, a somewhat different phenomenon was going on. Surveys of an intensive built-up area comprised of high-rise apartment units and populated largely by young families of moderate income levels consistently overestimated the number of transfers of pupils from kindergarten to first grade. In one survey an expected number of 400 first grade entrants in a particular school from the preceding spring term of kindergarten dwindled to less than 40 by the start of the fall term. The apparent cause of this massive out-migration was that many families with children, ages 4 to 6, purchased single-family dwellings and moved to other areas of the county served by other schools.

Since the several school buildings were part of the same school district, however, the total enrollment for the entire district was not affected.

The county development board approved the creation of a new form of dwelling unit known as an apartment hotel. School planners in Montgomery County, after consultation with several developers interested in the construction of such units, anticipated a substantial drop in the pupil-yields from such units. The major reason is that the units are geared to the demands of middle-to-older-age persons and to couples who are well past the child-rearing stage of the family life cycle.

Another example of the effects of the family life cycle is given in the Bucks County report (appendix D). Despite the increase in population between 1960 and 1965, school enrollment increased only by the amount expected on the basis of the number of preschool age children, under age 5, living in the county in 1960. The authors explain this apparent contradiction by stating that, "The major trend 1960–1965 in school enrollments has had to do with the aging of the resident population."

It would thus appear that intimate knowledge of the local housing cycle in conjunction with some general knowledge of family life-cycle behavior—particularly on the question of timing of housing purchases—would aid school planners in the design and conduct of dwelling-unit surveys, as well as the exploitation of local building-development information.

Use of Dwelling-Unit Pupil-Yield Multipliers

Montgomery County, Md.

The Montgomery County Educational Services Administration through its Division of Planning has had considerable experience using the dwelling-unit pupil-yield multiplier approach for projecting enrollments for its 160 or more schools.

To accomplish this the Division of Planning divided the county into 12 educational planning regions. Each planning region consists of several elementary and secondary schools, as well as a number of kindergarten units. In some cases planning regions were established in portions of the county which have only recently (latter 1960's) been developed, but which are expected to grow during the next 5 to 7 years. As a result several of these planning regions have only kindergarten and elementary facilities. However, sites for the acquisition of secondary facilities are already, or are currently (1968) in various stages of being programed.

The planning division has placed the planning regions onto several dozen grid squares comparable with the 1:200 scale maps used by the County Planning Commission for land zoning and development. Information on the state of construction activity, the inventory of occupied and vacant dwellings, location of present and future educational facilities, and their current capacity levels is entered on the grid squares comprising the individual planning region. A small number of items of information are entered on one acetate sheet. These acetate overlays are superimposed on the County Planning Commission land zoning maps. Inspection of these acetate sheets then gives clues as to the next steps to take—site inspection and acquisition, and other plans.

Information on the housing construction activity levels of each grid square in the planning regions are obtained from building permit data supplied by the County Clerk's Office, site visits by planning division staff members to individual real estate developers, and dwelling-unit surveys made by the Planning Division and conducted through the schools. Capacity information on the individual schools are obtained from normal operating reports submitted to the County Educational Services Administration by school principals.

In the above way data on the existing and expected numbers of dwelling units by type—single family detached, single-family attached, etc.—are obtained. If we now know on the average approximately how many pupils will live in each type of dwelling unit, we can calculate total school enrollment. Such information sometimes can be obtained from the decennial census; more likely a special local survey will be needed. Knowing the number of dwelling units of a specific type, and the number of children enrolled in public school and living in that type of dwelling unit, average yield per residential dwelling can be calculated. An example for Montgomery County is given in table 26. Note that a single-family detached unit provides on the average eight times as many public kindergarten and elementary school pupils as does a high-rise apartment—.80 pupils as compared with 0.10.

Unfortunately, these average yields are not fixed values over time, but are subject to change. Therefore, periodic surveys are required. There are two reasons why they vary. One is the family

TABLE 26.—*Public student yields per residential dwelling unit, Montgomery County, Md.*

	Population per dwelling unit [1]	Total with kindergarten	Total without kindergarten	Kindergarten and elementary	Kindergarten	Elementary school	Junior high school	Senior high school
1 and 2 family units (averages all zones)...	3.7	1.44	1.31	0.80	0.13	0.67	0.34	0.30
Single-family detached units....	3.7	1.44	1.31	.80	.13	.67	.34	.30
Single-family attached units....	3.6	1.18	1.07	.65	.11	.54	.28	.25
Apartment units (averages all zones)...	2.4	.28	.25	.15	.03	.12	.07	.06
Low-rise apartments.......	2.8	.56	.51	.31	.05	.26	.13	.12
High-rise apartments.......	2.2	.19	.17	.10	.02	.08	.05	.04

[1] The dwelling unit factors (population per dwelling) and the school children factors (students per dwelling) apply to total dwellings, both occupied and vacant (assuming normal vacancy ratios). The school children factors are for public school only: they do not take into account parochial and private school enrollments.

Source: Research and Special Studies Branch, Advance Planning Section, Maryland-National Capital Park and Planning Commission.

cycle; a family may continue living in its single-family detached house long after its children have completed secondary school. A second reason is changes in the birth rate. Even among families of an age likely to have children in elementary or secondary school, the number of children will reflect the general level of, and changes in, the national birth rate. Beginning in the latter 1950's in the United States, the birth rate began to decline. Beginning in the early 1960's then, it is likely that the average yield per residential dwelling unit began to decline. This decline, of course, must have occurred at different times in different parts of the country, and must have varied from one county or local school district to another.

State of California

The Bureau of School Planning in the State of California Department of Education reviews the facilities plans of individual school districts in the State school system. As part of the normal review procedures of the Bureau, several series of enrollment projections are made using the dwelling-unit pupil-yield approach. The period of projection varies with the school level: 3 years for an elementary, and 4 years for a secondary school facility; for land acquisition purposes, the projection is 7 years.

The dwelling-unit pupil-yield projections are used in conjunction with other enrollment projections—most notably cohort-survival enrollment projections—to provide bureau staff members working with local district officials with a means of portraying local conditions.

As in the case of Montgomery County, basic enrollment or attendance data are derived from normal operating information compiled by the local school districts and supplied to the State Department of Education. Data on housing units under construction are obtained from field inspection made by the local district official or bureau staff members, building permits, and discussion with developers. Dwelling-unit pupil-yield multipliers are obtained from statistical analysis of previously conducted field surveys. These multipliers have been developed for several classes of housing, as well as for individual grades within each class of housing.

Exhibit 5 is a worksheet covering computation made for the elementary level projections, while exhibit 6 covers computations on procedures for the secondary school projections. Exhibit 7 shows the procedures, factors, and computations for the land or site acquisition surveys.

Bureau of School Planning
California State Department of Education

Form SP-1E (Rev. 6/66)
Education Code
Chapter 10, Division 14

EXHIBIT 5—Projected Average Daily Attendance

____________________ School district ____________________ County

Grades maintained: ________ to ________ inclusive

ENROLLMENT FROM FORM R-30 ________ Month ________ Year:

Grade	K	1	2	3	4	5	6	7	8	Special	Ungraded	Total
Enrollment												

1. Number of adults and nonresident pupils ________
2. TOTAL REMAINING ENROLLMENT, excluding item 1 above . . . . ________
3. Three times first grade enrollment . . . . ________
   Total enrollment in three highest grades maintained . . . . ________
   First grade enrollment minus enrollment of three grades . . . . ________
4. Number of children on kindergarten waiting list . . . . ________
5. Number of resident pupils attending out of district . . . . ________
6. Number of houses under construction ________ on ____ Month ____ Day ____ Year
7. Number of pupils to be housed. (Number of houses × house factor for grades to be housed)
   Kindergarten (No. of houses × .16) ________ pupils
   Grades 1–6 (No. of houses × .84) ________ pupils
   Grades 7–8 (No. of houses × .21) ________ pupils
   Total . . . . ________

HOUSE FACTORS

Kindergarten	.16	Grade Three	.15	Grade Six	.12
Grade One	.15	Grade Four	.14	Grade Seven	.11
Grade Two	.15	Grade Five	.13	Grade Eight	.10

8. Special Education (Number determined by Division of Special Schools and Services)
   a. Newly identified pupils included in item 2 above . . . . ________
   b. Authorized pupils not included in item 2 above . . . . ________
9. *Total projected enrollment* . . . . ________
10. SPECIAL EDUCATION ENROLLMENT BREAKDOWN
    EMR 1-3 ____ SMR ____ Deaf ____ HH ____ Blind ____
    EMR 4-8 ____ Part. See. ____ CP ____ OH ____ EH ____
    Total Special Education . . . . . . ________
11. Total enrollment, exclusive of Special Education . . . . ________
12. Total units of a.d.a. exclusive of Special Education—by grade levels

Grade	Enrollment		Units of a.d.a.[1]
Kindergarten	________	× .97	________
Grades 1–3	________	× .97	________
Grades 4–6	________	× .97	________
Grades 7–8	________	× .97	________
Total	________	× .97	________ units of a.d.a

[1] Transfer average daily attendance figures to bottom of Form SP-LAD 100).

Certified as correct

Approved by State Department of Education

________________ Authorized Agent of School District ________ Date

________________ Field Representative ________ Date

Bureau of School Planning
California State Department of Education

Form SP-1S (Rev. 6/67)
Education Code
Chapter 10, Division 14

EXHIBIT 6—Projected Average Daily Attendance

______________________ School district ______________________ County

Grades maintained: __________ to __________ inclusive

ENROLLMENT FROM FORM R-30 __________ Month ______ Year:

Grade	3	4	5	6	7	8	9	10	11	12	Special Ed.	Total
Enrollment												

1. Enrollment earned by adult classes, evening classes, and non-resident pupils. . . . . . . __________

2. Total enrollment, excluding units earned by adult classes, evening classes, and non-resident pupils - Grades ______ to ______ in applicant district. . . . . __________

3. (a) Total enrollment of four highest grades in elementary district or districts included in applicant district . . . . __________
   (b) Total enrollment of four highest grades in applicant district. __________
   (c) Difference of item (a) minus (b). . . . . __________

4. Resident pupils attending out-of-district . . . . __________

5. HOUSE FACTORS

   Number of houses under construction __________ on __________ (date)

   Grade 7 - _____ houses x .11 = _____ pupils   Grade 10 - _____ houses x .09 = _____
   Grade 8 - _____ houses x .10 = _____ pupils   Grade 11 - _____ houses x .08 = _____
   Grade 9 - _____ houses x .10 = _____ pupils   Grade 12 - _____ houses x .06 = _____

   Total __________

6. Sum of items 2, 3, 4, and 5 . . . . __________

7. Adjustment for dropouts (______ to ______ grades) _____ x _____ (factor) . . . . __________

8. Special education (number determined by Bureau of Special Education)
   (a) Newly identified pupils included in Item 2 above . . . . __________
   (b) Authorized pupils not included in Item 2 above . . . . __________

9. Estimated enrollment (Item 6 minus Item 7 - plus Item 8b if applicable) . . . . __________

10. Special education enrollment breakdown

    EMR 7-8 _____   EMR 9 _____   EMR 10-12 _____
    SMR _____   Deaf _____   Hard of Hearing _____
    Blind _____   Partial Seeing _____   CP _____ OH _____

    Total Special Education __________

11. Total enrollment exclusive of Special Education . . . . __________

12. Total units of ADA exclusive of Special Education - by grade levels

    Enrollment 7-8 __________ x .97 = __________ units of ADA
    Enrollment 9 __________ x .97 = __________ units of ADA
    Enrollment 10-12 __________ x .97 = __________ units of ADA
    Enrollment 9-12 __________ x .97 = __________ units of ADA

13. Total ADA Item 12 plus Special education Item 10 . . . . __________
    Transfer Item 13 to Form SP-LAD 1003, Column 4.

Approved by State Department of Education

______________________ Field Representative ______ Date

Certified as correct

______________________ Authorized Agent of School District ______ Date

State Department of Education
Bureau of School Planning
Rev. 8/61

Form SP-1 Site (Secondary)
State Aid Chapter 10
Education Code, Division 14
Section 19577

EXHIBIT 7—7-Year Projection of A.D.A. (for purchase of sites)

______________ School District ______________ County Grades Served ______

Existing district enrollment (excluding adult and evening classes)...... ______
Attach R-30 report.

Enrollment in component elementary districts according to latest October or March R-30 data, or latest monthly report. Attach R-30 report.

K	1	2	3	4	5

Projected enrollment 7 years hence: grades 2 through 5 become grades 9 through 12. Grade 1 becomes grades 7 and 8. If kindergarten is greater than grade 1, it becomes grade 7. Special education remains the same as existing enrollment in special education.

7	8	9	10	11	12	Sp.Ed.	Total

1. Total enrollment estimated 7 years hence.......................... ______
2. House count ______ x ____ * (for factor see below).................. ______
   (House count embodies foundations or excavations for a house through construction stage to recently completed, but not having yet been occupied.)
3. Subdividers' statements of intent to build:
   Number of houses ______ x ____ * (for factor see below)............ ______
4. Plans filed for subdivision with zoning authorities:
   Number of houses ______ x ____ * (for factor see below)............ ______
5. Number of zoned residential lots ______. Number estimated houses to be built within 7 years ______ x ______ * (for factor see below) ______
   Supporting document required from Planning Commission.

TOTAL ESTIMATED ENROLLMENT IN DISTRICT by ______ (Year) (Enrollment) [ ]

*House count factors: Grades 7-8 = .21; 9 = .10; 10-12 = .23

	Enrollment		A.D.A.**
7-8	______	x .97	______
9	______	x .97	______
10-12	______	x .97	______
Total	______		______

(A.D.A. Units) [ ]

**Transfer these figures to top of Balance Sheet.

Submitted by:

______________ Authorized Agent ______ Date

Approved by Department of Education:

______________ Field Representative ______ Date

CHAPTER 5

LONGRUN PROJECTION TECHNIQUES: INTEGRATED STATE AND LOCAL AREA SCHOOL ENROLLMENT

The previous chapters described some of the problems confronting school planners in the area of enrollments projection, analyzed trends in the form of administrative organization of the school systems in the United States, and described currently used short-range enrollment projection techniques. In this chapter the reader is introduced to the concept of long-range school enrollment projections used throughout the balance of this report. The general structure of the integrated State-local area school enrollment projection technique will be discussed, and the detailed mechanics of the approach will be shown in the following chapters.

State and Local Area Projection Technique

The Concept

The general approach consists of first making projections of public school enrollment for the State, and then working from the State downward to the county and local school district. In this way the local unit is fitted into the framework of the larger geographic area, and full advantage taken of all available knowledge regarding possible future population movements. In this respect our approach is somewhat analogous to that used by the U.S. Census Bureau in making population estimates for metropolitan areas. The Census Bureau commented as follows:

In the present report, however, since estimates have been developed for all metropolitan areas and for the nonmetropolitan remainders of the States, it was possible to take advantage of the availability of independent State totals developed as part of the Census Bureau's regular estimates program. The independent State figures are believed to have a much higher degree of accuracy than estimates of subareas of States. Adjustments to State totals should provide, therefore, an improvement in estimates for individual areas, on the average. Consequently, the estimates for metropolitan areas and nonmetropolitan parts of each State were summed and adjusted to State totals. (Quoted from: *Population Estimates*, Series P-25, No. 371, August 14, 1967, p. 11.)

Underlying this concept is the knowledge that almost all children between the ages of perhaps 5 and 17 are enrolled in elementary or secondary school. Hence, the first major job in making a school enrollment projection is that of making a population projection for children and teenagers; at this point, school enrollment projections are barely different from population projections. However, the large majority, but not all, of the children are enrolled in public schools. This fact calls for additional techniques in order to project enrollment in public schools; on the other hand, the fact that so large a proportion is in public school makes the projection job relatively easy, once the population projection has been made.

Because of the foregoing, the integrated projection is meant to be used for long-range projections, beyond 5 to 10 years. In effect, it is to be used for a time period well beyond the limits of locally generated vital statistics data, grade-to-grade survival patterns, and dwelling-unit multipliers. Indeed there is no need to use this method for shortrun projections since previous empirical evidence has shown that both the dwelling-unit pupil-yield approach and the grade-survival methods are fairly accurate for the shortrun time periods.

Advantages of this Technique

The advantages of the integrated State and local area school enrollment projection technique may be summarized as follows: First, a range of enrollment projections rather than single estimates for future enrollments can be generated. This is accomplished (1) through the introduction of alternate series of population projections for the State, based on different assumptions of population growth; and (2) the introduction of alternate age-specific enrollment estimates and projections. The projective power can be refined by experimentation with alternate statistical trend functions for the projection of the proportion of total State enrollment accounted for by the localities. Further, some flexibility is afforded by the ability to project for several possible levels of aggregation for both school enrollment and geographic detail.

Second, the technique can be updated frequently for revision of enrollments projections. What is required is the addition of data to the historical base of enrollment proportions, and/or the availability of revised population forecasts. Mechanically, the process involves the refitting of trend functions to the local area-State enrollment proportions, the possible adjustment of age-specific enrollment ratio growth patterns, and the adjustment of the Census age-grade matrix.

For the local school district, a range of projected school enrollment estimates will satisfy most, if not all, needs for planning purposes. If the community can have reasonable assurance that enrollment will increase not less than some given amount, and not more than some other amount, it can draw its plans accordingly. For example, if the minimum increase is thought to be 15 percent, and such an increase would require the construction of an additional 10 schoolrooms, then this is the minimum school building program to undertake. When undertaking this minimum construction, however, the community must understand that enrollment could increase by 35 percent, which would mean building a total of 20 to 25 classrooms instead of 10. Any decision to build more than 10 classrooms, however, can be held up until an updated projection has been made.

State Population Data and Projections

A number of State and local agencies have developed or are developing their own series of population projections. In addition, the Census Bureau makes population projections periodically for States as part of its continuing program of demographic projections for the Nation.[1] The former set of projections varies in relative quality and accuracy. (See appendix A.) More frequently than not, the production of alternate series of population projections based on different assumptions of population change and growth is well beyond the capacities of State and local agencies. This is not to say that State and local agencies have not in some instances done a commendable job in the production of population forecasts, but rather that the State population forecasts prepared by the Census Bureau provide a uniform and sufficiently differentiated series upon which to base enrollment forecasting. Thus the technique displayed here has the advantage of being able to use both the Census- and State-produced series of population projections. In effect, a variety of population projections is presented to State school planners from which they can choose that which appears to be appropriate on the basis of conformity with State budgetary and fiscal practices and local conditions.

Furthermore, this technique uses the State data on numbers of children enrolled in public schools, by age of child and grade in which enrolled, available at each decennial period, together with intercensal data derived from local school records and estimates of enrollment by age for total United States also provided by the Census Bureau. State school planners can adjust these census data to meet their own changing situation and local conditions. This will be shown when we discuss the use of local quotients—for example, the State age-specific enrollment ratio divided by the national age-specific enrollment ratio—and other approaches for adjusting data to local conditions.

Local Area Projections

The basic historical information is the proportion of the State's total which attends public school in a specific local area. This proportion is then projected and converted into estimated school enrollment by applying it to the projected State enrollment. This can be done for any combination of grades.

The projections can be made with various trend lines. If electronic processing is not available, trend lines of the first and second degree can be fitted through least squares by persons with a minimum of training in statistical techniques and using desk calculators. Work sheets detailing the specific steps in each phase of the projection can be developed fairly simply for this purpose. If volume production of projections is desired, or if the production of projections is considered as part of a more complex model of school planning, or if more complex trend lines are to be fitted, the work can be programed for a computer.

[1] See for example, *Current Population Reports, Population Estimates*, "Revised Projections of the Population of States. 1970 to 1985," Series P-25, No. 375, Oct. 3, 1967.

Like most forecasts of this type, the extrapolation of trends is dependent on the historical base of information available. In this case, it is assumed that the share of total State enrollment attributed to the locality will follow the growth pattern embedded in the trend function. The stability of these underlying relationships is a function of the size of the historical data base. The larger the number of years for which historical information is at hand, the greater will be the degree of reliability or confidence that one can place in the projections. The estimates or projections get substantially less reliable the farther into the future they extend.

The Model

Each part of the overall model has a great deal of built-in flexibility and affords the school planner some choice in the range of population projection series used, age-specific enrollment rates assumed, and the statistical tools used to project local area shares of State school enrollment. Even the form of enrollment organization, whether total enrollment, elementary-secondary, 4-4-4 or individual grade, can be used. The choice will depend upon the needs of the planner and the common sense guidelines of statistical significance and reliability of the estimates. Table 27 portrays the range of projection alternatives for each part of the State-local area school enrollment projection technique. Let us turn to each of the parts of the overall model and discuss some of its characteristics.

The model consists of several parts. The first portion is a State enrollment projection submodel. It integrates alternative population forecasts of the State, made by the U.S. Bureau of the Census or a State agency, with projection of the Decennial Census age-grade matrices. To obtain the projected matrices, the decennial census matrices are modified in the light of Current Population Survey (C.P.S.) trends on school enrollment by age, and such other information as may be available to the State.

The second part of the model is concerned with the projection of the local area proportion of total State graded enrollment. As can be seen from examination of table 27, there is a wide variety of "tools" for accomplishing this task. The simplest involves the application of least-squares trend equations to the projection of the local area proportion of total State graded enrollment. Each of the means for projecting the local area share of State enrollment will be discussed in detail in the ensuing chapters and their predictive accuracy evaluated.

The third and final part of the State-local area enrollment projection model by grade consists of nothing more than joining together the two halves in producing future enrollments estimates. Specifically, this involves apportioning the projected State grade enrollment estimates among the local areas in accordance with their projected share (or percentage) of the State total. This procedure is repeated for each available alternative population projection. Statistical adjustments are made to each local area graded enrollment projection series to be consistent with the independently derived State level graded enrollment projection.

In summary, the integrated State-local area school enrollment projections model consists of the following several steps:

1. The development of an adjusted age-grade matrix for school enrollment for the State, based on U.S. Decennial Census data.

2. The projection of the Decennial age-grade

TABLE 27.—*Structure, component methodology and projection possibilities of integrated State-local area school enrollment long-range projection technique*

Population projection series

U.S. Bureau of the Census State population projection series or suitable State agency population projection series.

Age-interval smoothing technique

Linear interpolation of 5- or 10-year age detail into single years of age intervals.

"Sprague" third-degree polynomial interpolation coefficients for smoothing 5- or 10-year age detail into single years of age intervals. (See appendix B.)

Age-grade enrollment matrix—U.S. decennial census

Total State school enrollment, public and private combined, by sex, by single grades, by single years of age, and by color.

Total State school enrollment, public school *only*, by sex, by single grades, by single years of age, and by color.

Total State school enrollment, private schools only, by sex, by single grades, single years of age, and by color.

Post decennial age-grade enrollment patterns

U.S. Bureau of the Census—*Current Population Survey* fall school enrollment trends by age and grade groups.

State public school enrollment statistics by age and grade.

Adjustment of State-national age-grade enrollment patterns

Guestimates.

Use of age-grade specific State-national location quotients using Decennial Census materials.

Form of enrollment organization

All grades (K-12).

Elementary-secondary (K-6, 7-12).

"4-4-4" (1-4, 5-8, 9-12).

Individual grade (1, 2, 3 to 12).

Projection of Local Area Shares of Total State Enrollment

Least squares trend lines fitted to historical data

1. Simple linear.
2. Curvilinear.
3. Log linear.
4. Log log.
5. Hyperbolic.
6. Exponential.

Quadratic exponential smoothing functions fitted to historical data

1. Double exponential smoothing.
2. Triple exponential smoothing.

Local area aggregation

State.

State planning or statistical areas.

State enrollment areas.

Counties.

Districts.

353-581 O - 70 - 4

matrix for the State using suitable adjusted growth rates in specific age-grade grouping enrollment cells derived from the U.S. Census C.P.S. statistical series on fall school enrollments, as well as other locally available information.

3. Application of projected age-grade matrix for the State to detailed age-projections of its population to derive estimated enrollment by grade group. These serve as controls for the local political unit projections, derived as follows:

4. The fitting of statistical trend functions to local political unit-State enrollment ratios.

5. The projection of local political unit-State enrollment ratios through interpolation of statistical trend functions derived in step 4.

6. Application of projected local political unit-State enrollment ratios, derived in step 4, to obtain estimated detailed enrollments.

7. Adjustment of detailed enrollment estimates derived in step 6 to overall State estimates obtained in step 3.

The next two chapters deal with the detailed description of the development of the integrated State-local projection model.

Chapter 6 considers the development of State level graded enrollment projections. Material covered includes development and modification of Decennial Census age-grade enrollment matrices and application to State projections of the school-age population. Examples and work sheets are shown for each step in the computations. The States of California and Maryland serve as test cases for this phase of the approach and remaining technical chapters.

Chapter 7 shows the development and projection of local area proportions of total State enrollment. A variety of statistical trend techniques are discussed and applied to data on enrollment by State Statistical Area for California and county for Maryland. These projected shares are then applied to the projected State enrollment to arrive at projected local area enrollment.

CHAPTER 6

DEVELOPMENT OF STATEWIDE ENROLLMENT PROJECTIONS

This chapter covers the first portion of the integrated State-local area enrollment projection outlined in chapter 5. In particular, the following procedures in making the statewide enrollment forecasts are discussed.

1. Development of State-level age-grade specific enrollment ratios for use in making statewide projections.

2. Modification and projection of statewide age-grade specific enrollment ratios for use in conjunction with population projections for the State.

3. Application of statewide projected age-grade enrollment ratios to statewide population projections by age to derive final estimates of future school enrollment.

The relative accuracy of this approach will be evaluated, using some comparison with actual enrollment data. Further, possible modifications of the approach in the light of trends in public and private school enrollment will be discussed. For the interest of the user, work tables depicting step-by-step computations will be displayed.

Projections by sex[1] and color are possible insofar as the basic decennial census data contains this information. However, as will be pointed out in chapter 7, historical data on public school enrollment must also be available by sex and color in order to make such enrollment projections. In addition, projected population data by age and color are needed. The Census Bureau, however, provides projected age by color estimates only for States which had 250,000 or more nonwhite population in 1960. Accordingly, before attempting any enrollment projections by color, the State technician should ascertain from the Census Bureau whether he can obtain the data needed for his target date.

[1] As a practical measure there is probably little value in making projections for boys and girls separately, so that the sex dichotomy can be ignored.

Population and school enrollment data for this and the succeeding chapters covering the procedures in the integrated State-local level school enrollment projection approach are for the States of Maryland and California. School enrollment forecasts cover the period from 1965 to 1985. The results are shown at 5-year intervals. The form of school enrollment used in the illustration is the 4–4–4 arrangement—grades 1–4, 5–8, and 9–12.

Statewide Age-Grade Specific Enrollment Ratios

Age-grade specific enrollment matrices will be developed for the 1960 public school population of Maryland. This table, when adjusted for trends in certain age-grade cells in the matrices, will be used with Census Bureau population projections, subdivided into single years of age, to make school enrollment projections. The modification of the final age-grade enrollment tables and their application to the Census population projections by single years of age will be the subject of the next section.

Basic data for the development of the age-grade enrollment matrix are obtained from the most recent U.S. Census volume on detailed characteristics of the population in each State; for example, Series PC(1)-D in 1960. The specific tables used in constructing the 1960 matrix are: (1) "Year of School in which Enrolled for Persons 5- to 34-years-old by Single Years of Age, Color, Sex, and Type of School (Public and Private), for the State: Urban and Rural: 1960;" (2) "Single Years of Age, by Color, Nativity, and Sex, for the State: 1960."

Age-grade data on school enrollments for 1960 are shown for each sex separately and for all

TABLE 28.—*Age by sex by grade, public school enrollment, State of Maryland, 1960*

Sex and grade	Age								
	5, 6	7 to 9	10 to 13	14, 15	16, 17	18, 19	20, 21	22 to 24	Total
Males, total	22,307	76,334	102,561	38,736	32,511	7,313	1,133	1,077	281,972
1 to 4	22,307	75,185	14,259	382	434	111	45	67	112,790
5 to 8	0	1,149	85,551	12,317	1,429	430	204	205	101,285
9 to 12	0	0	2,751	26,037	30,648	6,772	884	805	67,897
Females, total	22,008	74,680	97,778	37,473	30,875	5,366	771	750	269,701
1 to 4	22,008	73,521	10,236	261	311	177	38	22	106,574
5 to 8	0	1,159	84,900	8,405	863	240	81	149	95,797
9 to 12	0	0	2,642	28,807	29,701	4,949	652	579	67,330
Both Sexes, total	44,315	151,014	200,339	76,209	63,386	12,679	1,904	1,827	551,673
1 to 4	44,315	148,706	24,495	643	745	288	83	89	219,364
5 to 8	0	2,308	170,451	20,722	2,292	670	285	354	197,082
9 to 12	0	0	5,393	54,844	60,349	11,721	1,536	1,384	135,227
Total population	138,316	191,344	249,252	95,309	96,756	82,348	75,226	113,057	1,041,608
Males	69,751	96,498	127,525	48,429	48,934	40,720	37,760	57,237	526,854
Females	68,565	94,846	121,727	46,880	47,822	41,628	37,466	55,820	514,754

schools, public, and private schools. The age detail for 1960 ranges from age 5 to 34. We are concerned only with the population ages 5–24 for the purposes of making the final matrix.

The first step is to draw up work tables for making the basic calculations. Table 28 illustrates the procedures with data for Maryland, 1960; the distribution of males and females, ages 5–24 by grade, the total number enrolled by grade in each group, and the total population in each age group are copied from the appropriate Census table.

Step two is the calculation of age-grade coefficients. This is accomplished by dividing the total number in each age-grade cell in table 28 by the total number at each age level. Observe the Maryland example (in table 28); the total population aged 5–6 years is 138,316; the total number of public school pupils age 5–6 years is 44,315. Dividing 44,315 by 138,316 gives a coefficient of 0.3204 which means that 32.04 percent of the pupils aged 5–6 years in 1960 were enrolled in grades 1–4

TABLE 29.—*Age-grade matrix, proportions enrolled in public school, both sexes, State of Maryland, 1960*

Grades	Age							
	5, 6	7 to 9	10 to 13	14, 15	16, 17	18, 19	20, 21	22 to 24
Total enrolled	0.3204	0.7893	0.8037	0.8000	0.6561	0.1539	0.0253	0.0161
1 to 4	.3204	.7772	.0983	.0067	.0077	.0035	.0011	.0008
5 to 8	0	.0121	.6838	.2179	.0247	.0081	.0038	.0031
9 to 12	0	0	.0216	.5754	.6237	.1423	.0204	.0122

in public school in Maryland. Thus you divide each entry in the column by the total at the bottom of each column. Note that the coefficients are calculated to four places, 0.3204. Enter the age-grade coefficients in the appropriate cells in table 29.

In step three, add the coefficients in each cell in the columns, exclusive of the line marked "total enrolled," to check on your calculations.

Modification and Projection of Statewide Age-Grade Matrix

Because not all persons of elementary and secondary school age are at present (1969) attending public school, and because we always anticipate change, the age-grade matrix as of the last decennial census, may be deemed inappropriate for projection purposes. Actually, in some States the 1960 matrix may continue well into the future substantially unchanged. On the other hand, in some States it may be known that substantial changes had occurred within a few years after the last decennial census, and further changes are expected. Therefore, the first task for State education officials is to decide whether or not the age-grade matrix as given in the last census can be used as is, or must be modified for projection purposes.

If it is thought that modifications are needed, then several procedures are available for doing so. The first is to make changes within the State in accordance with nationwide changes. A second is to adopt the matrix (from the last Decennial Census) for some other State which can be used as a model. A third alternative is for the State to tabulate its own records, age by grade, and use this information for changing the last Decennial Census matrix. Finally, State officials may simply assume some model age-grade matrix which they expect to achieve by the time of the target date.

Note that at this point, judgment of State and local conditions becomes important in estimating a future age-grade matrix for public schools. For example, in 1960 in Maryland, among 5- and 6-year-olds, 0.3204 were enrolled in public schools; in California the figure was 0.3461 and for total United States, 0.3076. There is no guaranteed statistical procedure for estimating what this ratio, 0.3204, will be in Maryland in 1980. Will it continue to be above the national average? Will it equal California's ratio of 0.3461? Or will it be some other value? Judgment about local conditions is the best guide.

In the following materials we are focusing our attention on the use of nationwide changes to indicate changes within the State. The last three alternatives require no additional technical explanations; the mechanics for carrying them out are similar to those for the first alternative.

Two sets of procedures for projecting the age-grade matrix using nationwide changes are shown. The first set is recommended. The second set is included to show a possible alternative way of projecting the matrix; due to lack of sufficient information, however, it is probable that few States will be able to apply this second set of procedures. Nevertheless, some States may be able to use this second set, or some adaptation of it, depending on the data which may be locally available.

First Set of Procedures

In the following illustration for Maryland, we have made the basic assumption that the 1980 goal for the State will be the national average as of 1960; the only exception is the 5- and 6-year age group which, in 1960, was already above the national average. For California, which was already above the national average in 1960, we assumed no further changes. In reality, these assumptions need not hold for either of these two States or any other State; State officials must decide for themselves what the appropriate goals may be. Whatever goals are used, however, the mechanics to be followed in extrapolating the last Decennial Census matrix are exactly the same as illustrated in the following pages. In case of doubt as to which standard is the most appropriate, two or more alternative ones may be used and a range of estimated changes calculated.

Two steps are involved in this procedure. First, State-national location quotients, as of the time of the most recent census, must be calculated for each total-age enrollment group. This is necessary to ascertain the degree of adjustment to be made in the projection of specific age-grade enrollment ratios. Second, the basic matrix may be projected using the State-national total-age enrollment group location quotient computed in the first step.

The first step in the adjustment of the age-grade matrix, the development of the State-national age enrollment location quotient, is fairly simple. Table 30 displays the necessary computations for Maryland and California. Column a shows for total United States, the proportion enrolled in each age group. Column b contains the same proportions computed for 1960 public school enrollment for Maryland as given in table 29. By dividing each of the entries in column a by the respective entries in column b, we obtain the ratios shown in column c. These ratios are the State-national enrollment-location quotients mentioned previously. They reflect the degree to which the State public school age-enrollment patterns are either greater or smaller than the 1960 national public school age-enrollment levels.

If the proportion in a given age group in a State has a ratio below 1.0 as shown in column c, of table 30, then the State has a larger proportion enrolled than does the Nation. Just the opposite is true for those ages with age-enrollment ratios greater than one, or unity. In these ages, the proportions enrolled are below prevailing national age-enrollment levels.

The reader is warned that these age-enrollment adjustment factors should not be applied mechanically. State education planners should be knowledgable about local conditions and graded enrollment trends. Thus, the adjustment factors should be further manipulated by State planners to reflect sudden changes in, for example, the distribution of the total school enrollment population between public and private systems; such a factor cannot be treated mechanically. Thus, good common sense and judgment must be employed in the use and application of the State-national age-enrollment adjustment factors.

For the projections of the Maryland and Cali-

TABLE 30.—*Computations for projecting 1960 age-grade matrix of public school enrollment, States of Maryland and California*

State and age	Age enrollment ratios		col. a ÷ col. b	Age enrollment ratios projected (See text)			
	U.S.	Maryland		1965	1970	1975	1980
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Maryland							
5 and 6 years...	0.3076	0.3204	0.9600	1.0200	1.0401	1.0601	1.0802
7 to 9 years.....	.8317	.7893	1.0537	1.0134	1.0269	1.0403	1.0537
10 to 13 years...	.8411	.8037	1.0465	1.0116	1.0232	1.0349	1.0465
14 and 15 years.	.8445	.8000	1.0556	1.0139	1.0278	1.0417	1.0556
16 and 17 years.	.7208	.6561	1.0986	1.0246	1.0493	1.0739	1.0986
18 and 19 years.	.1937	.1539	1.2586	1.0646	1.1293	1.1939	1.2586
20 and 21 years.	.0343	.0253	1 3557	1.0880	1.1778	1.2668	1.3557
22 to 24 years..	.0210	.0161	1.3043	1.0761	1.1521	1.2281	1.3043
	U.S.	California					
California							
5 and 6 years...	.3076	.3461	.8888	....	....	....	....
7 to 9 years.....	.8317	.8732	.9525	....	....	....	....
10 to 13 years...	.8411	.8785	.9574	....	....	....	....
14 and 15 years.	.8445	.8794	.9603	....	....	....	....
16 and 17 years.	.7208	.7505	.9604	....	....	....	....
18 and 19 years.	.1937	.1576	1.2291	1.0573	1.1146	1.1718	1.2291
20 and 21 years.	.0343	.0375	.9147	....	....	....	....
22 to 24 years...	.0210	.0256	.8203	....	....	....	....

fornia age-grade matrices, the following procedure was adopted. In the instance where the State-national location quotients (column c of table 30) exceeded unity or one, the extent to which unity was exceeded was linearly interpolated over the projection period—from 1960 to 1980. For example, the State-national enrollment ratio for ages 20 to 21 for Maryland in table 30 (column c) is 1.3557. Using the interpolation assumption, 25 percent of the 0.3557 differential would be allocated to 1965, 50 percent to 1970, 75 percent to 1975, and so forth. The adjustment or projection factors for the affected age groups derived in this manner for Maryland and California are shown in columns d through g of table 30.

The projection factors (displayed in columns d to g, inclusive) were applied to the original 1960 age-grade matrices shown in table 29. The final projected enrollment ratios are then obtained, and are shown in tables 31 and 32. This procedure adjusts the original Census State age-grade matrix for Maryland up to the 1960 national public school enrollment norm over the prospective period of projection.

In those instances where the State-national enrollment ratios fell below unity, the original 1960 age-grade matrix was left unadjusted. This was the case with most of the California age groups—the one exception being ages 18 to 19. In the case of Maryland, the youngest age-group, ages 5 to 6, was projected forward using the 1960 California age-enrollment ratio in place of the national norm.

It was assumed then that the 1960 California age-enrollment patterns, with the exception of the 18 to 19 age group were near the upper limits of possible school attachment patterns, given the existence of an alternative—private and parochial schools.

Second Set of Procedures

A possible alternative procedure for the enrollment ratios of the age-grade matrix is the derivation of growth rates for the total United States to be applied to the age-grade enrollment ratios of a particular State. The example shows how we develop national age-grade enrollment-growth rates for grades 9 to 12, for age groups 14 to 15, and 16 to 17. Data are drawn from the fall school enrollment series of the Census Bureau's *Current Population Survey*, P–20 Series.

TABLE 31.—*State of Maryland projected age-grade matrices, 1965–80*

Years and grades	5, 6 years	7 to 9 years	10 to 13 years	14, 15 years	16, 17 years	18, 19 years	20, 21 years	22 to 24 years
1965								
1 to 4.......	0.3272	0.7880	0.0994	0.0068	0.0079	0.0037	0.0012	0.0009
5 to 8........		.0123	.6917	.2209	.0253	.0086	.0041	.0033
9 to 12........			.0219	.5934	.6390	.1515	.0222	.0131
Total.	.3272	.8003	.8130	.8111	.6722	.1638	.0275	.0173
1970								
1 to 4.......	.3340	.7983	.1006	.0069	.0081	.0040	.0013	.0009
5 to 8........		.0124	.6996	.2240	.0259	.0091	.0045	.0035
9 to 12........			.0221	.5913	.6544	.1607	.0240	.0141
Total.	.3340	.8107	.8223	.8222	.6884	.1738	.0298	.0185
1975								
1 to 4.......	.3408	.8096	.1017	.0070	.0083	.0042	.0014	.0010
5 to 8........		.0126	.7076	.2270	.0265	.0097	.0048	.0038
9 to 12........			.0224	.5994	.6698	.1698	.0258	.0150
Total.	.3408	.8212	.8317	.8334	.7046	.1837	.0320	.0198
1980								
1 to 4.......	.3476	.8190	.1029	.0071	.0085	.0044	.0015	.0010
5 to 8........		.0127	.7156	.2300	.0271	.0102	.0052	.0043
9 to 12........			.0226	.6074	.6852	.1791	.0276	.0159
Total.	.3476	.8317	.8411	.8445	.7208	.1937	.0343	.0209

TABLE 32.—*State of California projected proportion enrolled in public schools, ages 18 and 19*

Grades	1960	1965	1970	1975	1980
1 to 4....................	0.0035	0.0040	0.0042	0.0044	0.0046
5 to 8....................	.0046	.0079	.0083	.0087	.0092
9 to 12....................	.1495	.1547	.1632	.1716	.1799
Total..............	.1576	.1666	.1757	.1847	.1937

Columns a and b in table 33 present the proportion of the United States population aged 14-15 and 16-17, enrolled in grades 9-12, over the period from 1960 to 1966. Inspection of the data for the 14- and 15-year-olds shows them to be linear; accordingly, we fit a least square trend line of the form: $y=a+bx$ (see appendix C). An annual growth rate is computed from the trend line by dividing the "b" value by the "a" value. For this age group the average annual gr vth rate is: $0.0050 \div 0.8338 = 0.006$ or 0.6 percent.

For the 16- and 17-year-olds the enrollment ratios do not appear to be linear. Between 1960 and 1962 there is very little change; between 1962 and 1963 there is a large increase in the population enrolled in grades 9-12. Then, between 1963 and 1966, there is very little increase again. A straight line fitted to the years 1963-66 gives an average annual increase of 0.3 percent.

These growth rates can now be used as follows: Let us begin with our age-grade ratios for 1960. At that time in Maryland in grades 9-12, 0.5754 of all 14- and 15-year-olds were enrolled, and 0.6237 of all 16- and 17-year-olds were enrolled. We shall project to 1980, or 20 years ahead. Accordingly, we can compute the total growth by means of tables showing annual interest compounded; such tables are generally available in banks.

For the 14- and 15-year-olds, among whom the growth rate was 0.6 percent, the entry for 20 years is 1.127, and for 16- and 17-year-olds, who had a growth rate of 0.3 percent, the entry for 20 years is 1.06173.

Multiplying 0.5754 by 1.127 gives us an estimated age-grade ratio of 0.6485 in 1980. The first set of procedures described gave a 1980 estimate of 0.6074 (table 31).

TABLE 33.—*Projection of age-grade public school enrollment ratios, ages 14-15 and 16-17, for grades 9-12, United States*

Year	Proportion of age group enrolled in grades 9 to 12	
	14 to 15 years (a)	16 to 17 years (b)
1960	0.8176	0.7725
1961	.8217	.7835
1962	.8338	.7874
1963	.8342	.8316
1964	.8389	.8264
1965	.8388	.8267
1966	.8517	.8403

Age 14-15: $Y=0.8338+0.0050X$ (origin 1963, for years 1960-66).
Age 16-17: $Y=0.8312+0.0022X$ (origin 1964½, for years 1963-66).

For ages 16 and 17, we multiply 0.6237 by 1.06173 and obtain an estimate of 0.6622 for 1980. The first set of procedures give an estimate of 0.6852.

At present (1969), there is insufficient experience and national data to permit recommending the second set of procedures. Perhaps in another 5 years or so, when we have at least 10 years of data on age-grade enrollment ratios, this second set may prove to be useful. The main reason for presenting it is to show how these procedures could be applied. Some States may wish to use United States experiences as guides to adjust their State data; others may simply wish to compare changes in their own States with national changes.

Furthermore, some States may have their own data on age-grade enrollment ratios which could be extrapolated in the same way as we described for the 14- and 15-year-olds. Indeed, if a State tabulates its public school enrollment by age and grade for years after the Decennial Census, it can construct its own matrix by using the population data provided by the Census Bureau, showing the age composition of States subsequent to the last Decennial Census. An example of such population data is given in *Population Estimates*, "Estimates of the Population of States, by Age, 1960 to 1966," Series P-25, No. 384, February 13, 1968. Before undertaking the construction of such a matrix, it would be advisable to discuss it with Census Bureau personnel.

Private School Information

For projecting the State's age-grade matrix, it will be helpful to have information about the private school enrollment, particularly that in Catholic schools, since this is the single largest component of the private school population. A State which historically has had a large private school enrollment may very well have an age-grade matrix for public schools which is well below the national average, and may never reach the national level. State education personnel will be familiar with the size and type of the private school enrollment within their States, and should be able to take this factor into account when projecting the age-grade matrix.

At the time of the Decennial Census, age-grade matrices for the private school population are available in the same Census table as that for the public schools. This decennial information can be combined with locally available information to estimate changes in the age-grade matrix for the private school population for other than Census years.

Use of Projected Age-Grade Ratios

In order to make our final projections of school enrollment, we apply the projected age-grade ratios to the projected population. The latter figures are obtained from the Census Bureau in the form of 5-year age groups. These are then split into single years of age using the Sprague multiplyers technique as described in appendix B.

Testing 1965 Projections

This section deals with an evaluation of results of the application of previously obtained projected age-grade ratios to the 1965 population projections to derive 1965 statewide enrollment estimates. The results presented in this section cover projections made using an adjusted age-grade matrix for public school for California and Maryland over the period 1960 to 1965. For California, the population estimate for July 1, 1965, prepared by the State Population Research Unit, Department of Finance, was used for the projections of school enrollment. In the case of Maryland, Census Bureau population estimates were used to derive the 1965 estimates of school enrollment. Since the B–1 Series, a high estimate, and D–1, a low estimate, were virtually identical, only one population series is shown in table 34. Actual public school enrollments for both States were obtained from annual reports of fall school enrollment reported to the State education departments by the local public school systems. The steps in making the 1965 projections are shown in table 34.

TABLE 34.—*Steps in projecting 1965 public school enrollment, State of Maryland*

	Total[1] population	Age grade matrix			Projected 1965 enrollment		
		1 to 4	5 to 8	9 to 12	1 to 4 axb	5 to 8 axc	9 to 12 axd
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Total	1,314,878				262,134	233,588	189,471
5, 6 years	163,020	0.3272			53,340		
7–9 years	225,876	.7880	0.0123		177,990	2,778	
10–13 years	283,317	.0994	.6917	0.0219	28,162	195,970	6,205
14, 15 years	133,968	.0068	.2200	.5834	911	29,504	78,157
16, 17 years	128,996	.0079	.0253	.6390	1,019	3,264	82,428
18, 19 years	120,641	.0037	.0086	.1515	446	1,038	18,277
20, 21 years	110,975	.0012	.0041	.0222	133	455	2,464
22–24 years	148,085	.0009	.0033	.0131	133	489	1,940

[1] The B–1 and D–1 estimates were virtually identical, hence only one is given here.

How do the 1965 projected enrollments compare with the actual reported enrollment? The projected number was 12,000 short, almost all in secondary schools (table 35). The average error was less than 2 percent; only in grades 9 to 12 was the error a little over 5 percent.

The error for the secondary school level may come from one or two sources. The population may be slightly off, but we think this is minor. More likely, a larger proportion of the teenagers continued in high school. We assumed only that Maryland retention rates would approach that of the Nation (table 30); perhaps Maryland State officials who are thoroughly familiar with the State situation would have chosen another standard that would have taken into account the increased holding power of grades 9 to 12.

We made similar 1965 projections for California using the age distribution calculated by the State. When applied to the projected age-grade matrix, the projected enrollment turned out to have an error of less than one-half of 1 percent. Grades 1 to 4 were overestimated by well under 2 percent (table 35).

TABLE 35.—*Comparison of actual vs. projected public school enrollment, States of Maryland and California, 1965, grades 4–4–4*

Grades	Enrollment		Percent error
	Actual	Projected	
Maryland:			
1–4	261,017	262,134	0.43
5–8	235,981	233,588	−1.01
9–12	200,390	189,471	−5.45
Total	697,388	685,193	−1.75

Grades	Actual	Projected		Percent error	
		I & II	D-1	I & II	D-1
California:					
1–4	1,387,699	1,412,570	1,406,690	1.79	1.37
5–8	1,258,414	1,235,583	1,232,897	−1.81	−2.03
9–12	1,110,513	1,099,695	1,100,059	−0.97	−0.94
Total	3,756,626	3,747,848	3,739,646	−0.23	−0.45

NOTES.—Population projections used to compute future estimates of enrollment for Maryland were obtained from the U.S. Bureau of the Census, and divided into single years-of-age estimates using the procedures explained in appendix B.

Population projections used to compute future estimates of enrollment for California were obtained from the Department of Finance, Financial and Population Research Section, State of California publication, *California Population to the Year 2040*. Population projection data for the year 1965 was derived from unpublished estimates supplied by the Financial and Population Research Section. The two series are: (a) State of California Series I and II combined and (b) State of California Series equivalent to the U.S. Bureau of the Census Population Projection Series D-1.

Longrun Projections

The steps shown in table 34 can be repeated for each future 5-year period, 1970, 1975 and 1980. For each time period the appropriate populations and age-grade matrix is inserted, and the calculations made as in table 34. The results of such calculations are given in table 36 for 1980, for Maryland and California.

It is possible to construct several projections, giving a range of estimates resulting from use of two different population projections, and two or more age-grade matrices to encompass various possibilities. Thus, one could have projected enrollments: (*a*) high population and high proportions enrolled; (*b*) high population and low proportions enrolled; (*c*) low population and high proportions enrolled, and (*d*) low population and low proportions enrolled.

Projection "*a*" would provide the maximum enrollment and projection "*d*", the minimum.

TABLE 36.—*1980 projections of statewide public school enrollments, by grade groups, States of Maryland and California*

[Numbers in thousands]

Grades	Maryland		California	
	High population estimate	Low population estimate	High population estimate	Low population estimate
	B-1	D-1	I	II
1-4	345.9	266.6	1853.3	1812.6
5-8	287.3	252.9	1649.0	1618.0
9-12	262.9	258.2	1603.4	1578.2
Total	896.1	777.7	5105.7	5008.8

CHAPTER 7

MAKING LOCAL AREA SCHOOL ENROLLMENT PROJECTIONS

In the preceding chapter we showed how to make long-range projections of public school enrollment for the State. Now we shall show how these statewide estimates can be subdivided into local units. For purposes of demonstrating these procedures we are using counties and State planning regions. Any State which has the necessary data for local school districts can use them in exactly the same way as we show for counties.

The general idea consists of calculating the historical proportions, or shares, of the State's public school enrollment which is found in each of the local units. These shares are then projected to the target date, and converted into absolute numbers on the basis of the State's total enrollment. In this chapter we shall show some of the statistical approaches which can be used for projecting the local area shares. We shall continue using the three grade groups, 1 to 4, 5 to 8, and 9 to 12.

Introduction

Statistical Area Concepts Used

The local area concept used for illustrative projections in the case of Maryland is the county. In California, it was the State statistical area, comprising combinations of California's 58 counties. The justification for the selection of these statistical areas was several-fold: first, suitable historial series of fall school enrollment were available; second, proportions of total State enrollment were relatively stable and did not fluctuate widely from year to year at these levels of aggregation; third, the future estimates of absolute enrollment derived from projections of these statistical areas could be used as "benchmarks" or "control totals" for further prorating to contiguous school districts or educational planning regions; fourth, these statistical and political area concepts were considered by State education system planners as relevant for planning purposes. Exhibits 8 and 9 show these local political units and statistical areas for both Maryland and California.

California State Statistical Areas

Area	Name	Counties included
1	North Coast	Del Norte
		Humboldt
		Lake
		Mendocino
2	Sacramento Valley	Butte
		Colusa
		Glenn
		Sacramento
		Sutter
		Tehama
		Yolo
		Yuba
3	Mountain	Alpine
		Amador
		Calaveras
		El Dorado
		Inyo
		Lassen
		Mariposa
		Modoc
		Mono
		Nevada
		Placer
		Plumas
		Shasta
		Sierra
		Siskiyou
		Trinity
		Tuolumne
4	San Francisco Bay	Alameda
		Contra Costa
		Marin
		Napa
		San Francisco
		San Mateo
		Santa Clara
		Solano
		Sonoma

California State Statistical Areas—Continued

Area	Name	Counties included
5	Central Coast	Monterey San Benito San Luis Obispo Santa Cruz
6	San Joaquin Valley	Fresno Kern Kings Madera Merced San Joaquin Stanislaus Tulare
7	Santa Barbara-Ventura	Santa Barbara Ventura
8	Los Angeles Metropolitan Area.	Los Angeles Orange
9	San Diego Metropolitan Area.	San Diego
10	Southeast	Imperial Riverside San Bernardino

Historical Enrollment Series

Basic data for the development of projections of local area shares of total State-graded enrollment are shown in appendix E, tables E–1 to E–6. These consist of historical series of total State enrollment—using the "4–4–4" grade organization and the proportional distribution by counties for Maryland, and State Statistical Areas for California.

The date on enrollments from which the proportional distributions were computed is based on annual reports of fall public school enrollment compiled by the State Departments of Education. These enrollments are generally those recorded on or about October 31 of each year. From the methodological viewpoint, the date of the enrollment statistics is immaterial; the only requirement is that the same date be used each year. The middle or end of October, the beginning of the school year, is generally used.

Historical data, if they are to be analyzed, must be comparable over time. The two adjustments made on the Maryland data illustrate how we attempt to achieve comparability and make the resulting data more suitable for our purposes. 1. Public school enrollments by county, 1956 to 1958, excluded enrollments at several campus facilities attached to the Maryland State College System. Since the location of the facility is known, its numbers of pupils were added into the appropriate county totals. 2. Inmates of institutions (reform schools, orphan homes, and so forth) who were pupils in schools were included in the published county totals; since separate information was given for each institution, we subtracted the numbers from the county totals in order to have the noninstitutional school enrollment. For projection purposes, the noninstitutional population appears more relevant; the institutional population can change by edict, such as the building of a new or enlarged institution, or the removal of an institution from one county to another. Hence, there may be little point in trying to project the enrollment in institutions.

These particular adjustments may not be required for any other State; we made no adjustments to the California data. Only detailed study of a State's historical series will indicate whether adjustments are needed, and if so, what kind.

Projection Techniques

General Outline

There are four basic steps in preparing the final projected enrollments for 1980, or any other year. The first step consists of projecting the share or proportion of the total State enrollment which each county has in order to select that statistical device which seems to give the best fit. This is done by analyzing a series of years which end some 3 to 5 years before the last year for which enrollment data are available by county. For example, for Maryland we fitted lines to the period 1956 to 1962 and projected them to 1965; the projected shares could then be compared with the actual 1965 shares.

The second step consists of fitting lines to the entire series of available years, and projecting to the target date. For Maryland, lines were fitted to the period 1956 to 1966 and projected to 1980.

The third step consists of adjusting these projected shares so that the three grade groups will be of approximately the same size. For example, if a county has 10 percent of all pupils enrolled in grades 1 to 4, it is likely to have close to 10 percent of those enrolled in grades 5 to 8, and 9 to 12.

The fourth and last step then consists of applying these adjusted shares to the projected total State enrollment (see chapter 6) in order to estimate enrollment in each county.

Line Fitting

In the material that immediately follows this discussion, two basic approaches, including several different formulas, will be applied to the projec-

Exhibit 8

Maryland—Counties, Places of 25,000 or More, and Standard Metropolitan Statistical Areas

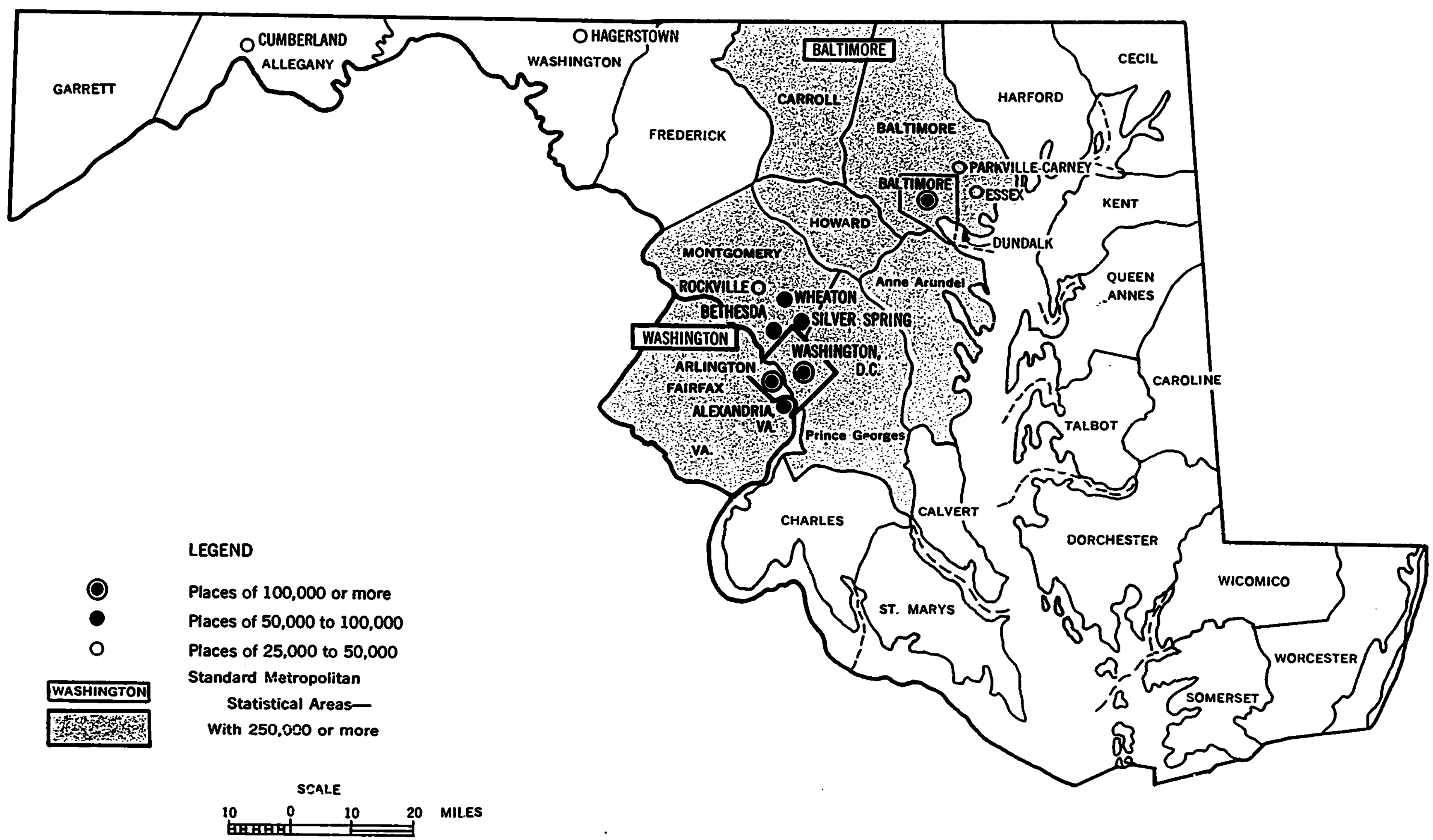

Exhibit 9

California State Statistical Areas

tion of local area enrollment shares for Maryland and California. They are: first, various forms of the least-squares trend lines, and second, the use of triple-exponential-smoothing prediction equations. In appendix F there is a description of a possible third approach, the use of multiple regression analysis techniques.[1]

One way to decide which type of trend line to fit is to apply several to the historical data and then pick out that one that fits best to use for projecting. The line which fits best is that one which has the minimum difference from the observed data; we take the difference between the observed historical datum and that produced by the formula, square it, and then add all these differences squared. After the projections to the target date have been made, another test can be applied to help evaluate the quality of the projections. This test is explained when we describe the 1980 projections for the two States.

There is no single formula which can be guaranteed to give the best fit in all cases. Therefore trial and error is the only way of selecting a formula for projecting a particular county. The reasoning is simple; if one formula describes the historical data better, we assume that that formula will be best for prediction purposes. The examples for Maryland and California will illustrate how we choose the particular formula to be used for projecting.

Least-squares trend lines.—One of the more commonly used methods of statistical forecasting is time series analysis or the fitting of trend lines. In this approach, we study movement and changes from one year to the next.

There are several alternative types of trend lines that can be fitted to time series data using the least squares criterion. Those which we used, and which are identified by number in the several tables of this chapter and appendix E, are:

1. Straight line (see appendix C):
   $y=a+bx$, where: y=enrollment share
   a=intercept
   b=slope of line, or amount of change per year
   x=time in years (1,2,3, etc.)
2. Exponential: $y=a^{(bx)}$
3. Logarithmic straight line: $y=a(x+b)$
4. Rectangular hyperbola: $y=a+(b/x)$
5. Hyperbolic: $y=l/a+bx$
6. Saturation: $y=x/[a(x+b)]$

Detailed explanation of these formulas and their use can be found in a variety of statistical texts including: Mordecai J. B. Ezekiel and Karl Fox, *Methods of Correlation and Regression Analysis*, John Wiley and Sons, New York, p. 77 ff; Samuel B. Richmond, *Statistical Analysis*, 2nd Ed. The Ronald Press, New York, pp. 358-366, 1964; F. E. Croxton and D. J. Cowden, *Practical Business Statistics* (various editions).

Triple exponential smoothing.—In its most basic form exponential smoothing is nothing more than a sophisticated averaging technique. It involves a weighted moving average scheme whereby the more recent data in a time series receive proportionately more weight than do the older observations in calculating the future elements of the series. To illustrate this, suppose that 1 year ago the county "Z" enrollment share for grades 1 to 4 for State "X" was 0.0300 and we used this figure as the best estimate of what this year's enrollment ratio could be. However, it turned out that this year's enrollment ratio for grades 1 to 4 in county "Z" was 0.0350 instead of the anticipated 0.0300. In estimating for the next year, we would want to take into account this new enrollment figure, as well as the previously calculated one. Assume for the purposes of this illustration that 10 percent (the smoothing constant) of the forecast should be based on the most recent demand figure while 90 percent would be predicted on the older average. Our average of next year's enrollment ratio would then be: future enrollment ratio estimate= (0.10) (0.0350) + (0.90) (0.0300) = 0.0305.

In the calculations made for California, we used three smoothing constants, 0.40, 0.50, and 0.75. As will be seen in subsequent discussion of this State, the smallest smoothing constant appears to give the best results.

Additional information on the fitting of triple exponentials is given in appendix C.

Using the Techniques

Applying Techniques to Maryland

The first step.—Utilizing the historical enrollment ratio data for Maryland. (See appendix E, tables E–1 to E–3), two sets of projections were made using the least squares trend line fitting

[1] Multiple regression analysis will not be considered further since this approach was not particularly useful for Maryland or California. How useful it may be in other States can be determined only empirically. We are including it in appendix F so that it can be tested elsewhere if a State wishes to do so.

approach. The first set consisted of trend lines fitted to the historical enrollment ratio data for each of the "4-4-4" grade organization for each of the 24 counties from 1956 to 1962. Optional or "best-fit" trend lines from this first data series were then projected to 1965 to derive 1965 estimates to compare with actual 1965 county enrollment shares. The equations thus derived are displayed in appendix E, table E-7.

Trend line fits were made for both the 1965 estimates and the subsequent 1980 projections using the G. E. BASIC time-sharing computer programs.

The projected 1965 shares may need a slight adjustment before they can be compared with the actual proportions. For example grades 1 to 4, the projected 1965 shares added to 0.9988; grades 5 to 8, the projected 1965 shares added to 1.0071, and grades 9 to 12, the projected 1965 shares added to 0.9768.

The calculated shares then must be increased or decreased so as to add to 1.0000, after which they can be compared with the observed shares in 1965. (Appendix E, table E-8 gives the unadjusted sums for each grade group for 1965 and 1980, for both Maryland and California.)

The comparison between the 1965 actual and 1965 projected enrollment ratios (based on projections of the 1956-1962 trend lines) for the 24 counties in Maryland are given in table 37.

The projected county enrollment ratios were next applied to the estimates of the total State-graded enrollment in Maryland, as shown in chapter 6, table 34, to derive 1965 estimates of absolute enrollment for each of the 24 counties. The comparison between the 1965 "actual" graded enrollment and the "estimated" absolute-graded enrollment is shown in table 38. Table 39 summarizes the results of the enrollment ratio and absolute-enrollment comparisons made in tables 37 and 38.

The second step.—The above procedures were duplicated for the historical data series of county enrollment ratios for Maryland over the period 1956 to 1966. The best trend line equations derived for these 11 years were extrapolated to produce forecasts of county enrollment shares in 1980. (See appendix E, table E-9.) These enrollment ratios were then adjusted in accordance with the data in appendix E, table E-8, in order to have them add to unity, or 100 percent. These shares are shown in table 40, columns a, b, and c.

The third step.—Inspection of the proportions in columns a, b, and c of table 40 suggest that within each county the grade-group shares diverge too much. For example, Allegany County had 0.0144 in grades 1 to 4, 0.0065 in grades 5 to 8, and 0.0172 in grades 9 to 12. We can measure this divergency as follows:

Obtain the average of the three shares,

TABLE 37.—*Actual and projected enrollment shares by county, "4-4-4" organization of enrollment, State of Maryland, 1965*

County	Actual shares			Projected shares			Percent error		
	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12
Allegany	.0210	.0223	.0276	.0199	.0237	.0283	−5.2	6.3	2.5
Anne Arundel	.0834	.0704	.0685	.0832	.0760	.0741	−0.2	−4.3	8.2
Baltimore City	.2117	.2241	.2391	.2448	.2437	.2350	15.6	8.7	−1.7
Baltimore	.1510	.1517	.1650	.1522	.1480	.1648	0.8	−2.4	−0.1
Calvert	.0078	.0074	.0056	.0084	.0071	.0057	7.7	−4.1	1.8
Caroline	.0068	.0072	.0061	.0070	.0070	.0063	2.9	−2.8	3.3
Carroll	.0188	.0178	.0161	.0171	.0164	.0168	−9.0	−7.9	4.3
Cecil	.0165	.0127	.0132	.0170	.0129	.0137	3.0	1.6	3.8
Charles	.0150	.0151	.0118	.0133	.0148	.0100	−11.3	−2.0	−7.6
Dorchester	.0089	.0097	.0082	.0091	.0092	.0088	2.2	−5.2	7.3
Frederick	.0238	.0234	.0233	.0218	.0224	.0238	−8.4	−4.3	2.1
Garrett	.0074	.0073	.0062	.0069	.0071	.0063	−6.8	−2.7	1.6
Harford	.0323	.0305	.0264	.0284	.0270	.0305	−12.1	−11.5	15.5
Howard	.0157	.0152	.0143	.0153	.0151	.0139	−2.5	−0.7	−2.8
Kent	.0052	.0049	.0039	.0053	.0048	.0043	1.9	−2.0	10.3
Montgomery	.1221	.1360	.1454	.1228	.1443	.1477	0.6	6.1	1.6
Prince Georges	.1650	.1492	.1399	.1419	.1369	.1312	−14.0	−8.2	6.2
Queen Annes	.0061	.0064	.0052	.0065	.0062	.0053	6.5	−3.1	1.9
St. Marys	.0127	.0106	.0123	.0105	.0091	.0098	−17.3	−14.1	−20.3
Somerset	.0062	.0069	.0053	.0064	.0070	.0058	3.2	1.4	9.4
Talbot	.0066	.0062	.0059	.0069	.0067	.0054	4.5	8.1	−8.5
Washington	.0286	.0288.	.0295	.0280	.0277	.0299	−2.1	−3.8	1.3
Wicomico	.0186	.0170	.0130	.0188	.0180	.0143	1.1	0.6	2.9
Worcester	.0088	.0091	.0073	.0085	.0089	.0074	−3.4	−2.2	−1.4
Total	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000			

TABLE 38.—*Actual and projected absolute enrollments by county, "4-4-4" organization of enrollment, State of Maryland, 1965*

County	Actual enrollment, grade groups			Projected enrollment, grade groups			Percent error		
	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12
Allegany	5,471	5,263	5,657	5,216	5,536	5,362	−4.7	5.2	−5.2
Anne Arundel	21,781	18,737	14,781	21,810	17,753	14,040	.1	−5.3	−5.0
Baltimore City	55,274	52,918	44,439	64,170	56,925	44,525	16.1	7.6	.2
Baltimore	39,410	35,807	32,255	39,897	34,572	31,225	1.2	−3.4	−3.2
Calvert	2,030	1,743	1,264	2,202	1,658	1,080	8.5	−4.9	−14.6
Caroline	1,776	1,695	1,381	1,835	1,635	1,194	3.3	−3.5	−13.5
Carroll	4,896	4,204	3,553	4,482	3,831	3,183	−8.5	−8.9	−10.4
Cecil	4,307	3,569	2,726	4,456	3,013	2,596	3.5	−15.6	−4.8
Charles	3,917	2,998	2,424	3,486	3,457	2,065	−11.0	15.3	−14.8
Dorchester	2,334	2,279	1,854	2,385	2,149	1,667	2.2	−5.7	−10.1
Frederick	6,208	5,515	4,887	5,715	5,232	4,509	−7.9	−4.9	−7.7
Garrett	1,923	1,726	1,418	1,809	1,658	1,194	−5.9	−3.9	−15.8
Harford	8,420	7,189	5,607	7,445	6,307	5,779	−11.6	−12.3	3.1
Howard	4,085	3,590	3,100	4,011	3,527	2,634	−18.1	−1.7	−15.0
Kent	1,362	1,151	882	1,389	1,121	815	2.0	−2.6	−7.6
Montgomery	31,884	32,107	29,215	32,190	33,707	27,985	1.0	5.0	−4.2
Prince Georges	43,074	35,213	28,415	37,197	31,979	24,859	−13.6	−9.2	−12.5
Queen Annes	1,586	1,512	1,128	1,704	1,448	1,004	7.4	−4.2	−9.0
St. Marys	3,316	2,497	1,937	2,752	2,126	1,857	−17.0	−14.9	−4.1
Somerset	1,637	1,632	1,214	1,678	1,635	1,099	2.5	.2	−9.5
Talbot	1,722	1,462	1,208	1,809	1,565	1,023	5.0	7.0	−15.3
Washington	7,452	6,803	6,230	7,340	6,470	5,665	−1.5	−4.9	−9.1
Wicomico	4,857	4,234	3,166	4,928	4,205	2,709	1.5	−.7	−14.4
Worcester	2,295	2,137	1,649	2,228	2,070	1,402	−2.9	−2.7	−15.0
Total	261,017	235,981	200,390	262,134	233,588	189,471	0.0	−1.0	−5.4

column d; express the share in each grade group as a proportion of the average, columns e, f, and g. (See table 40.)

Historically, divergencies as great as those shown in columns e, f, and g do not occur. Analysis of the divergencies for the years 1956 to 1966 gives a standard deviation of about 0.06 around a mean of 1.00. This empirical finding appears reasonable since the age compositions of individual counties ordinarily do not differ so greatly as to result in disproportionate enrollments in one grade group or another. If, for example, in a particular county all children were aged 5 to 13, that county might have a sizable proportion of the State's enrollment in grades 1 to 4 and 5 to 8, but zero share in grades 9 to 12. In actuality, such a peculiar age distribution rarely occurs. As a result, a county tends to have about the same proportion of the State's enrollment in each grade group.

TABLE 39.—*Summary comparisons* of projected and observed shares and enrollment, by grade groups, by counties, Maryland, 1965*

	Shares			Enrollment		
	1-4	5-8	9-12	1-4	5-8	9-12
Mean percent error X	5.94	4.75	5.28	6.54	0.23	9.38
Standard deviation S	4.00	3.42	4.83	5.41	4.35	4.70
Coefficient of variation V	.82	.72	.91	.83	.70	50

*Summaries of the columns showing percent error, in tables 37 and 38.

Hence, one test of the goodness of fit of a projection is the amount of divergency resulting among the three grade groups. The divergency shown in table 40 is too great; various other lines could have been fitted in an effort to reduce this divergency. However, any final decision as to which projections might be best must always be made in light of all knowledge available to State and local educators. Since we are simply illustrating a methodology, we shall not fit additional lines, but show how the final results can be adjusted.

Since the standard deviation of the historical divergencies was 0.06, we can set the upper limit as 1.06, and the lower as 0.94, for the ratios shown in columns e, f, and g. Any share in columns a, b, or c which falls within these limits is accepted. If it is larger than 1.06, it is reduced to 1.06; and if it is smaller than 0.94, raised to 0.94. For example, in Allegany County, grade 1 to 4, the projected proportion was 0.0144 (column a); this was 1.134 (column e) of the average 0.0127 (column d). We multiply 0.0127 by 1.06 to get an adjusted share of 0.0135 (column h). Similarly, the original projected share for grades 5 to 8 is raised from 0.0065 to 0.0119. The final results of this first adjustment are shown in columns h, j, and k of table 40.

TABLE 40.—*Projected public school fall enrollment shares by grade group and county, and method of adjustment, State of Maryland, 1980*

County	Projected shares 1-4 (a)	Projected shares 5-8 (b)	Projected shares 9-12 (c)	Projected shares Av. (d)	Percent of mean 1-4 (e)	Percent of mean 5-8 (f)	Percent of mean 9-12 (g)	Projected shares first adjustment * 1-4 (h)	Projected shares first adjustment * 5-8 (j)	Projected shares first adjustment * 9-12 (k)	Projected shares second adjustment 1-4 (l)	Projected shares second adjustment 5-8 (m)	Projected shares second adjustment 9-12 (n)
State total	1.0000	1.0000	1.0000	1.0000				1.0096	.9976	1.0057	1.0000	1.0000	1.0000
Allegany	.0144	.0065	.0172	.0127	1.134	.512	1.354	.0135	.0119	.0135	.0134	.0119	.0134
Anne Arundel	.0181	.1043	.0721	.0948	1.140	1.100	.761	.1000	.1000	.0901	.0990	.1002	.0896
Baltimore City	.0978	.1462	.1607	.1349	.725	1.084	1.191	.1268	.1430	.1430	.1256	.1434	.1422
Baltimore	.1652	.1497	.1612	.1587	1.041	.943	1.016	.1652	.1497	.1612	.1636	.1501	.1603
Calvert	.0081	.0071	.0090	.0081	1.000	.877	1.111	.0081	.0076	.0086	.0080	.0076	.0086
Caroline	.0045	.0064	.0043	.0051	.882	1.255	.843	.0048	.0054	.0048	.0048	.0054	.0048
Carroll	.0204	.0161	.0118	.0161	1.267	1.000	.733	.0171	.0161	.0151	.0169	.0161	.0150
Cecil	.0166	.0134	.0094	.0131	1.267	1.023	.718	.0139	.0134	.0123	.0138	.0134	.0122
Charles	.0189	.0126	.0106	.0140	1.350	.900	.757	.0148	.0132	.0132	.0147	.0132	.0131
Dorchester	.0058	.0072	.0042	.0057	1.018	1.263	.737	.0058	.0060	.0054	.0057	.0060	.0054
Frederick	.0222	.0180	.0171	.0191	1.162	.942	.895	.0202	.0180	.0180	.0200	.0180	.0179
Garrett	.0053	.0032	.0038	.0041	1.293	.780	.927	.0043	.0039	.0038	.0043	.0039	.0038
Harford	.0421	.0279	.0344	.0348	1.210	.802	.989	.0369	.0327	.0348	.0365	.0328	.0346
Howard	.0215	.0235	.0184	.0211	1.019	1.114	.872	.0215	.0224	.0198	.0213	.0225	.0197
Kent	.0036	.0031	.0019	.0029	1.241	1.069	.655	.0031	.0031	.0027	.0031	.0031	.0027
Montgomery	.1206	.1458	.1658	.1441	.837	1.012	1.150	.1355	.1458	.1527	.1342	.1462	.1517
Prince Georges	.2450	.2353	.2404	.2402	1.020	.980	1.000	.2450	.2353	.2404	.2426	.2360	.2390
Queen Annes	.0050	.0055	.0037	.0047	1.064	1.170	.787	.0050	.0050	.0044	.0050	.0050	.0044
St. Marys	.0165	.0101	.0100	.0122	1.353	.828	.820	.0129	.0115	.0115	.0128	.0115	.0114
Somerset	.0024	.0047	.0026	.0032	.750	1.467	.813	.0030	.0034	.0030	.0030	.0034	.0030
Talbot	.0044	.0036	.0037	.0039	1.128	.923	.949	.0041	.0037	.0037	.0041	.0037	.0037
Washington	.0263	.0205	.0201	.0223	1.179	.919	.901	.0236	.0210	.0210	.0234	.0211	.0209
Wicomico	.0185	.0171	.0107	.0154	1.201	1.110	.695	.0163	.0163	.0145	.0161	.0163	.0144
Worcester	.0068	.0121	.0069	.0087	.781	1.390	.793	.0082	.0092	.0082	.0081	.0092	.0082

*Adjusted to maximum of 1.06 and minimum of 0.94.

It is seen that these last three columns do not quite add to 1.0000. Accordingly, we raise or lower them so that they add to unity; the final results (second adjustment) are shown in columns l, m, and n.

The fourth step.—This consists of applying the final shares (columns l, m, and n of table 40) to the projected State enrollment by grade, as given in table 36. The results for 1980 are shown in table 41.

TABLE 41.—*1980 projected enrollment by grade groups, State of Maryland, by counties*

[Numbers in thousands]

County	Based on *high* population projections, Series B-1 1-4	5-8	9-12	Total	Based on *low* population projections, Series D-1 1-4	5-8	9-12	Total
State total	345.9	287.3	262.9	896.1	266.6	252.9	258.2	777.7
Allegany	4.6	3.4	3.5	11.5	3.6	3.0	3.5	10.1
Anne Arundel	34.2	28.8	23.6	86.6	26.4	25.3	23.1	74.8
Baltimore City	43.4	41.2	37.4	122.0	33.5	36.3	36.7	106.5
Baltimore	56.6	43.1	42.1	141.8	43.6	38.0	41.4	123.0
Calvert	2.8	2.2	2.3	7.3	2.1	1.9	2.2	6.2
Caroline	1.7	1.6	1.3	4.6	1.3	1.4	1.2	3.9
Carroll	5.8	4.6	3.9	14.3	4.5	4.1	3.9	12.5
Cecil	4.8	3.8	3.2	11.8	3.7	3.4	3.2	10.3
Charles	5.1	3.8	3.4	12.3	3.9	3.3	3.4	10.6
Dorchester	2.0	1.7	1.4	5.1	1.5	1.5	1.4	4.4
Frederick	6.9	5.2	4.7	16.8	5.3	4.6	4.6	14.5
Garrett	1.[illegible]	1.1	1.0	3.6	1.1	1.0	1.0	3.1
Harford	12.6	9.4	9.1	31.1	9.7	8.3	8.9	26.9
Howard	7.4	6.5	5.2	19.1	5.7	5.7	5.1	16.5
Kent	1.1	0.9	0.7	2.7	0.8	0.8	0.7	2.3
Montgomery	46.4	42.0	39.9	128.3	35.8	37.0	39.2	112.0
Prince Georges	84.0	67.8	62.7	214.5	64.8	59.6	61.7	186.1
Queen Annes	1.7	1.4	1.2	4.3	1.3	1.3	1.1	3.7
St. Marys	4.4	3.3	3.0	10.7	3.4	2.[illegible]	2.9	9.2
Somerset	1.0	1.0	0.8	2.8	0.8	0.9	0.8	2.5
Talbot	1.4	1.1	1.0	3.5	1.1	0.9	1.0	3.0
Washington	8.1	6.1	5.5	19.7	6.2	5.3	5.4	16.9
Wicomico	5.6	4.7	3.8	14.1	4.3	4.1	3.7	12.1
Worcester	2.8	2.6	2.2	7.6	2.2	2.3	2.1	6.6

Applying Techniques to California

The first step.—Initially trend lines using least squares formulas, both straight line and linear, were applied to the counties and State statistical areas. The results were inferior to those obtained in Maryland. For example, in the North Coast Statistical Area, grades 1 to 4, the linear projection was in error by 47.7 percent, and the curvilinear by 41.3. For all State Statistical Areas the average percent error was 22.8 (appendix E, table E-10). The errors for individual counties in some cases were considerably greater than for State Statistical Areas; obviously, the least squares formulas are not very useful.

Following the principal of trying various formulas until we found one which gave a good fit, we tried the triple-exponential-smoothing equations. We could have tried the several other formulas, described earlier, but decided not to do so since they were tested in Maryland. Two sets of equations were calculated for the period 1947 to 1960, from which 1965 was projected; one used a 0.50 smoothing constant and the other 0.75.

The 1965 projected shares were then adjusted in accordance with the data in appendix E, table E–8 so as to add to unity.

The 1965 prediction equations derived from the 1947–1960 data series are displayed in appendix E, table E–11. The comparison of the 1965 enrollment ratio estimates derived using the two smoothing constant assumptions are shown in table 42. The corresponding absolute enrollment estimates for 1965 (using two different population projections) and their comparison with actual 1965 estimates are shown in appendix E, tables E–12 and E–13.

It is clear that use of the triple exponential formula gives a much better fit to the historical data. The average error for grades 1 to 4, for example, is about 7 percent, varying slightly, depending on the smoothing constant and population projection used. The least squares formula, as we saw previously, gave an error of 22.8 percent (appendix E, table E–10). There is little to choose between the 0.50 and 0.75 smoothing constants. The former gives closer fits for grades 1 to 4 and 5 to 8, whereas the latter gives a better fit for grades 9 to 12.

The second step.—On the basis of these results we proceeded to use this formula for making projections to 1980. Projection equations and the projected enrollment ratios for 1980 for the 10 State Statistical Areas in California are displayed in appendix E, table E–14.

The third step.—This consists of adjusting the projected shares so that within each county, the shares in each grade group will be approximately the same. We repeat for California, for the shares projected with the 0.500 smoothing constant, the calculations for Maryland shown in table 40, columns e, f and g. Inspection of columns e, f, g in table for California, appendix E, table E–15, suggests that the variance among grades is large, perhaps too large for comfort. A simple way of testing is to calculate the standard deviation of the 30 entries in these three columns; this turns out to be 0.24.

Inspection of the projected shares calculated with the 0.75 smoothing constant in appendix E, table E–14 suggests that the standard deviation is even greater.

Accordingly, we decide to return to the second step and apply an equation using a smoothing constant of 0.400. This produces the shares shown in table 43, columns a, b, and c. We next calculate the columns e, f, and g, proportion of the mean; the standard deviation of these 30 numbers is 0.13. The extent of variation from one grade group to the next is obviously much smaller with the 0.400 smoothing constant. Accordingly, we use this for our 1980 projections, instead of either the 0.50 or the 0.75.

For the 10 Statistical Areas of California, the historical divergencies among grades had a standard deviation of about 0.03. Accordingly, we assumed a range of 0.96 to 1.04 as marking the limits within which the projected proportions would be allowed to vary. Any share in columns a, b, or c of table 43 which deviated from the mean by a

TABLE 42.—*Actual and projected enrollment shares by area, "4–4–4" organization of enrollment, State of California, 1965*

Statistical Areas	Actual enrollment shares			Projected enrollment shares						Percent error of projections					
				0.50 Constant			0.75 Constant			0.50 Constant			0.75 Constant		
	1–4	5–8	9–12	1–4	5–8	9–12	1–4	5–8	9–12	1–4	5–8	9–12	1–4	5–8	9–12
North Coast	.0109	.0116	.0120	.0100	.0136	.0106	.0094	.0144	.0103	−8.3	17.2	−11.7	−13.8	24.1	−14.2
Sacramento Valley	.0553	.0556	.0573	.0603	.0604	.0567	.0624	.0622	.0593	10.9	8.6	−1.1	12.8	11.9	3.5
Mountain	.0209	.0226	.0238	.0209	.0233	.0209	.0186	.0236	.0225	0.0	3.1	−12.2	−11.0	4.4	−5.5
San Francisco Bay	.2228	.2232	.2274	.2200	.2249	.2396	.2204	.2145	.2483	−1.3	0.8	5.4	−1.1	−3.9	9.2
Central Coast	.0244	.0242	.0230	.0253	.0247	.0232	.0242	.0233	.0230	3.7	3.7	0.9	−0.8	−3.7	0.0
San Joaquin Valley	.0992	.1009	.0967	.1099	.1055	.0931	.1134	.1106	.0936	10.8	10.5	−3.7	14.3	11.0	−3.2
Santa Barbara-Ventura	.0352	.0330	.0310	.0323	.0314	.0293	.0317	.0316	.0298	−8.2	−4.9	−5.5	−9.9	−4.2	−3.9
Los Angeles-Long Beach Metropolitan Area	.4035	.4037	.4060	.3936	.3916	.3959	.4036	.4023	.3893	−2.5	−3.0	−2.5	0.0	−0.4	−4.1
San Diego Metropolitan Area	.0615	.0603	.0601	.0691	.0663	.0730	.0621	.0595	.0679	12.4	10.0	21.5	1.0	−1.3	13.0
Southeast	.0662	.0647	.0629	.0586	.0583	.0577	.0542	.0580	.0560	−11.5	−9.9	−8.3	−18.1	−10.4	−11.0

proportion greater than 1.04 was reduced, and any which was below 0.96 was increased. Thus, for example, in the San Francisco Bay Area, the projected share in grades 1 to 4 is 0.2324 (column a). The mean share for this area is 0.2023 (column d); grades 1 to 4 are thus 1.148 of the mean (column e). Since this latter number is greater than 1.04, we multiply 0.2023 by 1.04 to obtain the value of .2104 (column h).

The sum of columns h, j and k do not quite add to 1.0000. Accordingly, we adjust them so that they add to unity; the final results (second adjustment) are shown in columns l, m, and n (table 43).

The fourth step.—This consists of applying the final shares (columns l, m, and n of table 43) to the projected State enrollment by grade as given in table 36. The final results for 1980 are shown in table 44.

TABLE 43.—*Projected public school fall enrollment shares by grade group and area, and method of adjustment, State of California, 1980*

Statistical Areas	Projected shares				Percent of mean			Projected shares, First adjustment *			Projected shares, Second adjustment		
	1-4	5-8	9-12	Average	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(j)	(k)	(l)	(m)	(n)
State total	.9609	.9503	.9569	.9560				.9574	.9568	.9638	1.0000	1.0000	1.0000
North Coast	.0108	.0091	.0117	.0105	1.028	.866	1.114	.0108	.0101	.0109	.0113	.0106	.0113
Sacramento Valley	.0432	.0383	.0424	.0413	1.046	.927	1.026	.0430	.0396	.0424	.0449	.0414	.0440
Mountain	.0238	.0247	.0348	.0278	.856	.888	1.251	.0267	.0267	.0289	.0279	.0279	.0300
San Francisco Bay	.2324	.1938	.1806	.2023	1.148	.957	.892	.2104	.1942	.1942	.2198	.2030	.2015
Central Coast	.0315	.0304	.0353	.0324	.972	.938	1.089	.0315	.0311	.0337	.0329	.0325	.0350
San Joaquin Valley	.1247	.1083	.1342	.1224	1.018	.884	1.096	.1247	.1175	.1273	.1302	.1228	.1321
Santa Barbara-Ventura	.0509	.0597	.0491	.0562	1.065	1.062	.873	.0584	.0584	.0540	.0610	.0610	.0560
Los Angeles-Long Beach Metropolitan Area	.3086	.3599	.3752	.3479	.887	1.034	1.078	.3340	.3599	.3618	.3489	.3761	.3753
San Diego Metropolitan Area	.0472	.0450	.0198	.0376	1.255	1.220	.526	.0391	.0391	.0361	.0408	.0409	.0375
Southeast	.0788	.0802	.0738	.0776	1.015	1.033	.951	.0788	.0802	.0745	.0823	.0838	.0773

* Adjusted to maximum of 1.04 and minimum of .96.

TABLE 44.—*Projected enrollment by grade groups, State of California, by areas, 1980*

[Numbers in thousands]

Statistical Areas	Based on *high* population projections, Series B-1				Based on *low* population projections, Series D-1			
	1-4	5-8	9-12	Total	1-4	5-8	9-12	Total
State total	1853.3	1640.0	1603.4	5105.7	1812.6	1618.0	1578.2	5008.8
North Coast	20.9	17.5	18.1	56.5	20.5	17.2	17.8	55.5
Sacramento Valley	83.2	68.3	70.5	222.0	81.4	67.0	69.4	217.8
Mountain	51.7	46.0	48.1	145.8	50.6	45.1	47.3	143.0
San Francisco Bay	407.4	334.7	323.1	1065.2	398.4	328.5	318.0	1044.9
Central Coast	61.0	53.6	56.1	170.7	59.6	52.6	55.2	167.4
San Joaquin Valley	241.3	202.5	211.8	655.6	236.0	198.7	208.5	643.2
Santa Barbara-Ventura	113.1	100.6	89.8	303.5	110.6	98.7	88.4	297.7
Los Angeles-Long Beach Metropolitan Area	646.6	620.2	601.9	1868.7	632.3	608.4	592.4	1833.1
San Diego Metropolitan Area	75.6	67.4	60.1	203.1	74.0	66.2	59.2	199.4
Southeast	152.5	138.2	123.9	414.6	149.2	135.6	122.0	406.8

CHAPTER 8

PROJECTING A SINGLE SCHOOL DISTRICT

The previous chapters were concerned with procedures which could be utilized by a central State agency for preparing projections for each of its counties and school districts. Our reasons for preferring a statewide approach were set forth in chapter 2, where we discussed the difficulties of studying individual school districts, especially those which are very small or whose geographic boundaries have been changed in recent years, or expect boundary changes within the foreseeable future. It is recognized, nevertheless, that in many instances no estimates prepared by a central State agency will be available to the local school district, and the local unit will wish to have some estimate of its possible elementary and high school enrollment for a decade or longer. Accordingly, in this chapter we are presenting some procedures which are modified versions of those previously described, and which can be used by any school district. Short-term projections, 1 to 5 years in the future, were described in chapters 3 and 4 and need no further elaboration here.

Types of School Districts

Before describing any projection techniques, it is best to review the several types of school districts, because each type requires its own kind of projection methods. The first major distinction is between those districts which have maintained constant geographic boundaries over a number of years, and expect to keep those boundaries intact, at least, until the projection date; and those districts which have had, or expect to have, significant boundary changes.

Within each of these two types are those school districts which provide education from kindergarten through high school; and those which provide a certain number of years schooling after which the students are sent to another district. Perhaps the most common form is that in which the local district provides the elementary schooling and a consolidated district provides the high school. Thus, for the purposes of projecting enrollment, two or more school districts are involved.

Within each of these latter two types, the districts can be subdivided further into at least three size groups: very small, enrollment under 1,000 students per year; intermediate, from 1,000 to 100,000; very large districts such as New York City, Chicago, or any similar metropolis.

Further complications to plague the statistician are sometimes introduced when the geographic boundaries are changed and simultaneously, the district moves from a kindergarten through high school program to one covering only part of the entire elementary and high school education. Such districts can only be handled on an ad hoc basis and will not be described here.

Some school districts have boundary lines which do not coincide with any political or census boundaries. Such places have additional difficulties in obtaining the basic population data needed for making projections; we shall offer some suggestions on how to cope with this problem.

Finally, there are some districts which are nonoperating. The making of enrollment projections for such places may be simple for the nonoperating unit, but poses some problems for the district which does the actual teaching. We shall suggest ways of handling this type of situation. In summary, because of the large variety of school district types, we shall discuss a variety of approaches. Officials of a school district can then select the procedure, or combination of procedures, most appropriate to their particular needs.

Basic Projection Techniques

Two variants of what we call the Basic Projection Technique, and the circumstances under which each is applicable, will be described. Three conditions are necessary for the application of both of these variants, and these are:

1. They apply to school districts having between 1,000 and 100,000 pupils. School officials from smaller or larger districts can use them also if they so desire; however, these very small and very large districts often have unique situations which do not lend themselves readily to the application of a standard technique. Indeed, even a district which falls between the above range, and to which the Basic Projection Technique presumably can be applied, should consider whether or not it has unique situations or problems which would preclude the use of a standard technique. For example, if the officials of a school district containing 1,200 pupils know that several large enterprises are slated to move into the area and can be expected to double its population within, say, the next 5 years, then they must take this into account in projecting school enrollment. The application of a standard technique cannot take such an unusual event into account.

2. The school district must have had constant and unchanging boundaries for a significantly long period, and the expectation is that the boundaries will continue unchanged until, at least, the projection date. The first variant of our Basic Projection Technique uses Decennial Census data; this requires that the boundaries have remained unchanged for at least three Census periods and will continue unchanged. In the illustrations used here, we shall employ data from the 1940, 1950 and 1960 population censuses and shall project to 1980; hence we must have constant boundaries over the 40 year period, 1940 to 1980. When the 1970 Census results become available, data for 1950, 1960, and 1970 can be used for projecting to 1980. In the event that the district's boundaries have changed, a modification in the procedures is required and will be discussed in a section entitled "Changing Boundaries."

The second variant of the Basic Projection Technique uses locally available school enrollment data. For this, data should be available for at least 10 years, and preferably more, for a district having unchanging boundaries. In the illustration given, historical data are presented for the period 1955 to 1968, 14 years, and then projected to 1980.

3. For the first variant, we also must have districts whose boundary lines conform to recognized political lines such as a county or city, or conform to the boundaries of one or more minor civil divisions as published in the Decennial Population Censuses of the United States. For such school districts it is possible to obtain the necessary data from the published Census volumes. The treatment of those districts which do not have recognized political boundary lines are discussed in the section on nonconforming boundary lines.

For the second variant it is possible to use school districts whose boundaries do not conform to recognized political lines. Nevertheless, it is desirable that such districts lie within one county, although projections can be made—albeit with more work—if they cross county lines.

Finally, we should note that it is desirable for a school district to make projections using both variants, if possible. They should be compared, scrutinized carefully, and evaluated in terms of land use information and other facts known to the local community and which cannot be taken into account via general statistical procedures. On the basis of such an examination, a decision can be made as to which projection to accept, or how to modify them in light of additional locally available information.

Variant One

Procedures covering grades 1 through 12

This can be used in school districts whose boundaries conform to recognized political boundaries. The general idea consists of relating the population of school age in the local school district, to the same age population in the county for three Census periods, using published decennial data. The population of school age in the county is then related to the same population in the State. Projections to 1980 follow immediately since we have population projections for all States, prepared by the U.S. Census Bureau. We can then work from these State figures to the local school district. The sequence of steps (shown in table 45) is as follows:

1. We wish to use data from the most recent three Decennial Population Censuses; for present purposes we shall use data for 1940, 1950, and 1960.

2. Copy out the numbers of persons aged 5 to 17 years living in the State at each date. For 1940 the data will be found in Vol. II, in the appropriate State section; 1950 data appear in the P–B series, and 1960 statistics in the PC (1) B series.

3. Next copy out the numbers of persons aged

5 to 17 years living in the county at each date. Use the same sources as for step 2.

4. We next wish to have the numbers in the 5- to 17-year age group living in the school district at each date. As long as the school district boundaries coincide with political boundaries, Census data showing age distributions will be available, in one table or another of the mentioned State volumes. (If the school district crosses political boundaries, use the second variant and see the section entitled "*Boundary line Problems.*") Most likely the Census tables will not show the age group 5 to 17 for smaller communities, for example, those having under 10,000 total population in 1960. The ages likely to be shown are 5-year groups, and sometimes 10-year groups. In such cases it is necessary to use the Sprague multipliers (explained in appendix B) to estimate the numbers. For example, let us assume that a given school district coincides with a census minor civil division as shown in the 1960 census (Series PC (1) B) for each State. For such a school district we have the number by sex, aged 5 to 14, and 15 to 24 years. We use the Sprague multipliers to estimate the number aged 15 to 17 inclusive, and add that number to the number aged 5 to 14.

If a school district coincides with an urban place between 2,500 and 10,000 population as reported in the 1960 Census, the age groups shown are 5 to 9, 10 to 14, and 15 to 19. In this case we divide the 15 to 19 age group to obtain the estimated number aged 15 to 17, and add this to the number aged 5 to 14.

Note that statistics on age distributions are not uniformly presented in all Decennial Censuses. Nevertheless, using the Sprague multipliers any age distribution containing 5- or 10-year groups can be subdivided into the required 5- to 17-year inclusive total.

5. The above items of information can now be assembled as in table 45.

Part I of Table 45.—In column a enter the numbers aged 5 to 17 in the State at each Census period. In column b enter the numbers aged 5 to 17 in the county at each Census period. In column c (and subsequent columns) enter the numbers aged 5 to 17 in the school district(s) within the the county in column b being studied, at each Census date. In this example, both Leonia and Tenafly are in Bergen County. The next step consists of dividing the entry in column b by that in column a in order to obtain the percent of the State's total in the county. The percent is shown in column e. We now need the school district population aged 5 to 17 as a percent of that in the county. Thus for Leonia, we divide column c by column b and enter the result in column f. For Tenafly, we divide column d by column b and enter the percentage in column g.

Part II of table 45.—Since we wish to project to 1980, we obtain the State projections for the age group 5 to 17 as prepared by the Census Bureau. The ones used in this example were taken from the Bureau's *Current Population Reports, Population Estimates*, "Revised Projections of the Population of States, 1970 to 1985," Series

TABLE 45.—*Projecting school districts 1960 to 1980, an example using New Jersey*

Part I, past (Census data)	Number of persons aged 5 to 17				County as percent of State (b+a)	School district as percent of county	
	State	Bergen County	School districts			Leonia (c+b)	Tenafly (d+b)
			Leonia	Tenafly			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
1940 [1]	838, 186	[4] 82,485	[4] 1, 133	[4] 1, 531	0. 841	1. 374	1. 850
1950 [2]	834, 286	93,733	[4] 1, 245	[4] 1, 954	11. 235	1. 328	2. 085
1960 [3]	1, 367, 953	135,785	[4] 1, 807	3, 600	13. 581	0. 973	1. 938
Part II, projections, 1980 [5]	From census	Column a× column e	Column b× column f	Column b× column g		From table 46	
Series IB	2, 045, 000	351,000	2, 100	7, 310	17. 162	0. 625	2. 08 3
Series IIB	2, 012, 000	345,000	2, 100	7, 190	17. 162	. 625	2. 083
Series ID	1, 746, 000	300,000	1, 875	6, 250	17. 162	. 625	2. 083
Series IID	1, 719, 000	295,000	1, 840	6, 140	17. 162	. 625	2. 083

[1] Source: 1940 Population Census, Vol. II, Part 4, State of New Jersey. State data from table 7, county data from table 22, and local community data from table 30.

[2] Source: 1950 Population Census, P-B30, State of New Jersey. State data from table 15, county data from table 41, and local community data from table 38.

[3] Source: 1960 Population Census, PC(1) 32B, State of New Jersey. State data from table 16, county data from table 27, and local community data from table 22.

[4] Sprague multipliers used to obtain 5 to 17 year groups.

[5] U.S. Census Bureau, Series P-25, No. 375, table 5.

P-25, No. 375, October 3, 1967, table 5. (From time to time the Census Bureau issues new projections and it is best to use the latest available.) Four series are shown, varying from maximum to minimum numbers; these are shown in column a as IB, IIB, ID, and IID.

We next obtain columns e, f, and g on the basis of the calculations shown in table 46. The entries for the first three lines in table 46 are taken from columns e, f, and g of table 45. Lines 4 to 9 inclusive are calculated as directed in table 46. The final entries in line 9 are then entered in table 45, part II, columns e, f, and g. Note that in table 46 we have simply fitted a straight line using a simplified version of the procedures shown in appendix C.

TABLE 46.—*Work sheet for calculating projection equations for table 45*

	County as percent of State	Leonia as percent of county	Tenafly as percent of county
1) First census, 1940	9.841	1.374	1.856
2) Second census, 1950	11.235	1.328	2.085
3) Third census, 1960	13.581	0.973	1.938
4) Line 3 minus line 1	3.740	−.401	.082
5) Line 4 divided by 2	1.870	−.200	.041
6) Sum of lines 1, 2 and 3	34.657	3.675	5.879
7) Line 6 divided by 3	11.552	1.225	1.960
8) Line 5 times 3	5.610	−.600	.123
9) Line 7 plus line 8	17.162	0.625	2.083

The entries in column a are multiplied by the percentages in column e and the numbers entered in column b.

The numbers in column b are next multiplied by the percentages in columns f and g for the school district(s), and entered in columns c and d.

Column c of part II then shows the final projections for the school district of Leonia in 1980. If New Jersey should have the maximum population growth, then Leonia will have the maximum number of persons aged 5 to 17, or 2,190. Minimum population growth will result in a projected number of 1,840.

In order to obtain the estimated percent increase in the numbers enrolled in public school, we divide the estimated 1980 numbers for the school district by the 1960 numbers as shown in table 45. For Leonia we have: 2,190÷1,807=21% increase (Series IB); 2,160÷1,807=13% increase (Series IIB); 1,875÷1,807=4% increase (Series ID) and 1,840÷1,807=2% increase (Series IID).

For Tenafly the respective increases are: 103 percent, 100 percent, 74 percent, and 70 percent. These percentage increases are then multiplied by the number of residents actually enrolled in the public school system(s) in 1960 in order to obtain the projected numbers in public school in 1980. For these purposes it is preferable to use the numbers enrolled in the spring 1960 (if available) rather than the fall 1960 enrollment, since the census was taken in April 1960.

The final numbers are then the first approximations to 1980 enrollment and must be evaluated in terms of locally known information. For example, in 1960 the Leonia school district included 111 students from other school districts. If Leonia should continue to receive nonresident students in 1980, then some estimate of this number will be needed in order to obtain a total enrollment figure. Whether a statistical projection of the numbers of nonresident pupils in 1980 can be made, depends on who these nonresidents are and what arrangements have been made between the two (or more) school districts.

How the final statistical projection can be evaluated in terms of locally known information can be illustrated with Leonia. The 1980 projections suggested that the increase in public school enrollment may vary between 2 and 21 percent, depending on the growth of New Jersey. A survey of the Leonia public school system was made in early 1968 (Educational Developments Associate, April 8, 1968). After examining all locally available data the investigators wrote " . . . Assuming no change in these conditions, the public school population of Leonia, through 1975, will increase by only 78 students" (over the number enrolled in 1967, p. 16). The total expected increase between 1960 and 1975 is about 12 percent, and falls well within the limits of our statistical projections. The survey took into consideration land use information, together with the history of the Leonia school system over the last generation, and what is referred to as the "character" of the community.

Other Grade Groupings

Kindergarten.—No concrete suggestions are offered about how to make long-range projections for kindergarten. This is because there is considerable variation in the prevalency of kindergartens. One school district may have all its 5-year-olds enrolled in kindergarten while a neighboring district is still debating as to whether or not to open a kindergarten, and a third school district attracts only a fraction of its resident 5-year-olds to kindergarten.

The problems of projecting future birth rates accurately enough for purposes of estimating kindergarten enrollment are too uncertain to warrant doing so. Only shortrun projections, for a year or two ahead, are feasible. Generally the local school district personnel can use their most recent experience with kindergarten enrollment to make estimates for the immediate future. If information is available to the school district on numbers of births, or if a local census has been taken which provides the numbers of children under 6 years of age by single years of age, such information can be combined with previous experience to make the projections.

Special grade groupings.—Some school districts may enroll only pupils in grades 1 through 4 or 6, or sometimes 8. Others may enroll pupils only in grades 6 or 8 or 9 through 12. There are a large variety of possible grade combinations, and we cannot specify precisely how each should be handled. The general idea consists of selecting out those ages which most nearly fit the specific grade groupings, and then treating the data in precisely the same way as was illustrated previously in tables 45 and 46. For example, if a school district had only grades 1 to 8 inclusive, we should take the ages 5 to 14 inclusive and substitute that for the age group 5 to 17 as given in tables 45 and 46. If the school district has only grades 1 to 6 inclusive we should take ages 5 to 12 inclusive; in this event it would probably be necessary to use the Sprague multipliers to estimate the ages 10 to 12 inclusive.

Variant Two

Procedures covering grades 1 through 12

In the event that a school district's boundaries do not correspond to recognized political lines, it can only use variant two, providing that it has school enrollment data for a significant period and has unchanging boundaries. School districts which can use variant one can also use variant two if they have locally available school enrollment statistics. The general idea consists of relating the numbers enrolled in public school in the local school district to the numbers enrolled in the county. The numbers enrolled in the county are then related to the number aged 5 to 17. Projections of the county population aged 5 to 17 are then made as variant one. The projected—for example, 1980—county population in this age group is then converted to the numbers enrolled in public school in the county. From this latter figure we obtain the estimate of projected public school enrollment in the local school district. The sequence of steps is as follows:

1. Part I of table 47. In column a enter the numbers aged 5 to 17 in the county, as obtained from the Decennial Census volumes for 1950 and 1960 (or 1960 and 1970). The source and procedures are the same as in variant one.

2. In column b enter the numbers enrolled in public school in the county, grades 1 to 12 inclusive, as determined from local school records for the same 2 years.

3. Divide column b by column a to obtain the enrollment as a percentage of the population.

4. Part II of table 47, projections to 1980. The projections of the county population aged 5 to 17 are made exactly the same as in variant one, and are shown in column a; these are based on data from three census periods.

5. We need an estimate of public school enrollment in the county in 1980 as a percent of the population aged 5 to 17. Since most of the children are in public school in most districts, we arbitrarily assumed 90.0 percent to be the value for 1980. Actually, since in many school districts large numbers of children attend private schools, judgment must be used in projecting the 1980 percent in public schools. Such judgment takes into consideration the observed percentages for the years for which both population and public school enrollment are available (1950 and 1960 in table 47), and the fact that the percentage will not be greater than 100. If the school officials feel doubtful about the 1980 percentage, two different percentages can be inserted into the formula, one giving the largest percentage which is thought to be possible, and the other, the smallest. The calculations described for column b would then be carried out twice, once for each of the two 1980 projected percentages. This will provide a range of projected school enrollment in the county.

TABLE 47.—*Procedures for converting population, aged 5 to 17, to numbers enrolled in public schools, grades 1 to 12, for counties, hypothetical data*

Part I, past	Population aged 5-17	Number in public schools, grades 1-12	Percent enrolled b÷a	School district enrollment as percent of county enrollment	Enrollment in school district
	(a)	(b)	(c)	(d)	(e)
1950	70,000	61,000	87.1		
1960	90,000	79,000	87.8		
Part II, projections to 1980		Column a × column c			Column b × column d
Series IB	160,000	144,000	90.0	4.822	6,940
Series IIB	155,000	140,000	90.0	4.822	6,750
Series ID	115,000	104,000	90.0	4.822	5,010
Series IIID	111,000	100,000	90.0	4.822	4,820

6. Multiplying the data in column a by that in column c provides the projected numbers enrolled in public school in the county in 1980, shown in column b. The alternative population projections (shown in column a) provide alternative enrollment projections.

7. From table 48 we obtain the projected percentage of the county public school enrollment which is in the local school district. This is entered in column d.

8. The projected 1980 enrollment in the local school district is then obtained by multiplying the data in column b by that in column d, and is shown in column e, table 47, part II.

9. Table 48 shows the calculations for projecting the percentage of the county public school enrollment, grades 1 to 12, which the local school district is likely to have. The data in columns a and b come from local records. In the example illustrated, 1980 is the 26th year; at that date the school district is likely to contain 4.822 percent of the county's enrollment. A straight line was fitted to the data in column c by means of procedures shown in appendix C.

TABLE 48.—*Procedures for projecting local school district enrollment from estimated county enrollment data, grades 1 to 12, hypothetical data*

Year	Numbers enrolled in public schools, in: County	Numbers enrolled in public schools, in: Local school district	Y=percent enrolled in school district, b÷a
	(a)	(b)	(c)
1955	70,000	2,800	4.00
1956	71,500	2,890	4.04
1957	72,000	2,940	4.08
1958	74,000	3,050	4.12
1959	77,800	3,230	4.15
1960	79,000	3,300	4.18
1961	82,000	3,450	4.21
1962	82,500	3,510	4.25
1963	83,000	3,550	4.23
1964	84,000	3,640	4.33
1965	85,000	3,690	4.34
1966	86,500	3,770	4.36
1967	88,000	3,870	4.40
1968	88,000	3,900	4.43

NOTE.—Percent enrolled in school district: $Y=3.00+.032X$ (years 1955 to 1968, origin 1955).

Projecting to 1980: $X=26$ in 1980; $Y=3.00+26\ (.032)=4.822$.

Other Grade Groupings

These are treated for variant two in exactly the same manner as for variant one.

Very Small School Districts

These are defined as districts having under 1,000 pupils in all grades as of the most recent date. Actually many such districts have fewer than 100 students. Included in this very small category are the nonoperating school districts; generally, they have too few children enrolled to warrant operating a separate school system and instead, pay a neighboring district to provide the schooling.

Projections for these districts can be made using the procedures previously shown in tables 45, 46, and 47. Actually, it is probably more realistic to make ad hoc projections in terms of the local conditions as they are known to the community. We can arbitrarily divide these districts into the following three classes:

1. Those which have had a reasonably constant enrollment over the last decade, and the community sees no prospects of drastic change between the present and the projection date, for example, between 1969 and 1980. In such a case where, for example, enrollment fluctuated between 100 and 125 students, and the school officials see no prospects for change, the 1980 projection can be taken simply as between 100 and 125.

2. Those districts which have had rather steady increases in enrollment over the last decade or longer, and the school officials have reason to believe that such increases will continue to the projection date. In such case a line can be fitted to the historical public school enrollment data and extrapolated. As an example let us assume the following data for local school district XYZ:

1955	98	1960	107	1965	118
1956	100	1961	104	1966	121
1957	99	1962	107	1967	120
1958	102	1963	109	1967	120
1959	105	1964	114	1968	122

Fitting a straight line by means of the procedures shown in appendix C, we have: $Y=94.23+1.97\ X$ (years 1955 to 1968 with origin 1955). Projection to 1980 ($X=26$) gives an estimate of about 145 pupils to be enrolled at that date. The community and school officials can now examine this statistic and evaluate it in terms of all other information available about the possible future growth of the community.

3. Those districts which have had rather consistent decreases in enrollment, and are likely to have further decreases. In this case exactly the same procedures suggested in step 2, are applied to arrive at an estimate for 1980. In this case the value for b will be negative.

Some districts may have mixed experiences and must be treated on an individual basis. For example, a district may have had a constant enrollment of between 175 and 200 students every year for the last 20 years; it is known, however, that a new factory will be built and the population can be expected to increase by about 50 percent over the next decade. In such an event, a projection to 1980 can be made by arbitrarily increasing enrollment by 50 percent, or to between 250 and 300 pupils.

In the final analysis, projecting the enrollment in very small school districts (including nonoperating districts) involves judgment. There is no mechanical or statistical procedure which will automatically provide the "correct" projected number. All statistical projections must be evaluated judgmentally in light of all other information available.

Very Large School Districts

These are defined as districts having 100,000 or more pupils; there are only between 20 and 25 such districts in the United States. It is possible to apply the basic projection technique as previously described to such districts. Whether or not this will be satisfactory in any particular school district can be determined only by making the projection and then evaluating it in terms of all other known factors. Since only a few moments are required to make the calculations, it is advisable to do so, whether or not the results finally are accepted or rejected.

As a general rule, the larger the school district, the more reason there is for treating it in the same way as a State. The procedures described in chapters 6 and 7 for making State projections can be applied to large cities. The basic problems consist of: obtaining a distribution of single years of age by grade enrolled and by color (if this characteristic is desired); obtaining the 1980 (or other future date) age distribution. These two problems can be handled as follows:

Data showing the distribution by age and grade can be obtained from the Census Bureau's Decennial Census data. If the desired tabulation is not published, perhaps a special tabulation can be purchased. If this should turn out to be not feasible, the school district can tabulate its own data to show the numbers by single years of age by single grades, and by color, if desired. This should be made for a Census year. From the published Census statistics the numbers by color in each single year of age in the total population can be obtained. If the data are shown only in 5-year age groups, Sprague multipliers can be used to obtain single years of age. Using the numbers in the total population and the numbers enrolled in each grade, the age-grade matrix can be calculated, and then projected in exactly the same manner as for a State.

The second problem, that of obtaining a future population projection, is more complicated. When making State enrollment projections, we relied on the State population projections made by the Census Bureau and regularly published. As of 1968, the Census Bureau has not published population projections for large cities, but is familiar with the procedures for doing so. Accordingly, we recommend that a school district official interested in obtaining projections for his district, discuss this question with Census Bureau personnel. A variety of ways are available for making the calculations, and the Census Bureau statisticans can advise him as to the best procedures to apply to his particular city.

In some States, a central State office prepares population projections for cities and sometimes counties. If this is not done, then the local school district personnel would have to prepare the projections. Whether it has the trained personnel and equipment needed to make such projections using the same general procedures as those used by the Census Bureau, can be determined only by the individual local school board. The interested reader will find descriptions of the Census Bureau's methods in its publication P–25, No. 375 included in appendix A, and in *Population Estimates*, "Projections of the Population of the United States by Age, Sex, and Color to 1990, with Extensions of Population by Age and Sex to 2015," Series P–25, No. 381, December 19, 1967; pp.

1-50; and in *Population Estimates*, "Summary of Demographic Projections," Series P-25, No. 388, March 14, 1968, pp 22-32, also included in appendix A.

Boundary Line Problems

Changing Boundary Lines

The first requirement in making projections is that the boundary lines for the projection date, for example, 1980, are known. If boundary lines are subject to unpredictable changes, and the local school district officials have no assurance regarding future boundaries, then there is no point in making a projection. If it is believed that present (i.e., at the time of making the projection) boundaries will be unchanged at the projection date, and it is known that during the last several years there have been one or more significant boundary changes, then the school district has a problem which is generally solvable.

Most school districts have boundaries which correspond to political boundaries. For such areas it is possible to obtain data on population by age from the Decennial Census volumes. Therefore, knowing the present boundaries of the district, it is often possible to reconstruct the population at previous Census periods for the identical present boundaries. This only involves locating the various geographic areas which comprise the present district, and adding the Census data. Once having obtained the numbers aged 5 to 17 at three successive Census dates, it is then possible to apply variant one of the Basic Projection Technique; if the district is very small, the modified version, as described under "Very small school districts," can be used. If local school enrollment data are also available, then variant two also can be used.

Nonconforming Boundary Lines

If the public school enrollment data are available, then variant two of the Basic Projection Technique can be applied. If this information is not available, then statistical projections are difficult to make. Under these latter circumstances, the local school district officials must have knowledge regarding the number of children living in their area. They will know the current number enrolled in the public schools by age and grade; indeed, this information may be available for a short number of years prior, as well as the present. The officials may not know how many youngsters who live within their school district area attend private schools; nor are they likely to have any statistical basis for making projections using the formula shown in tables 47 and 48 (or any other statistical projection formula).

Two devices that can help the school officials in making judgments about possible future enrollment are the taking of a local population census, and the making of a land use survey (see also chapter 4). The former describes the demographic situation as of the time of the census, and by containing information on the numbers of preschool children by age, permits some estimates of public school enrollment to be made for the 5 years following the date of the local census.

A land use survey fundamentally consists of a series of judgments as to what may happen in the community within the next several years, 10 or 20 or more years, or whatever period the land use survey is designed to cover. Employing information on the amount of land available for commercial or industrial vs. residential use, together with guesses as to how rapidly people may move into (or out of) the community, judgments are made as to the possible number of school age children at some future date. Combining this information with the data obtained from the local census which shows the division between public and private school enrollments, some guesses can be made as to future public school enrollment.

Projections by Color

Whether such projections can be made depend in large measure on the availability of population and public school enrollment by color. Census projections of State population by age are available by color for the States having large numbers of nonwhites; *Current Population Reports, Population Estimates*, "Revised Projections of the Population of States, 1970 to 1985," (Series P-25, No. 375, October 3, 1967) contains projections of the nonwhites by age only for States having 250,000 or more nonwhites in 1960. Data are available by color for the larger cities from the census publications.

Variant one of the Basic Projection Technique can be applied to the whites and nonwhites separately for any local school district for which census data by color can be obtained (and to which variant one is applicable). The techniques are identical to those described in tables 45 and 46.

If variant one cannot be used, then variant two can be applied, perhaps, if county and school district enrollments are available by color. In this case the procedures described in tables 47 and 48 can be used.

If data by color cannot be obtained, then we are forced to try to adapt the suggestions given in "Nonconforming boundary lines." These include the taking of local censuses and the use of judgment in guessing at future enrollments.

APPENDIXES

Appendix A

CURRENT POPULATION ESTIMATES AND PROJECTIONS PROVIDED BY STATE AGENCIES AND THE U.S. BUREAU OF THE CENSUS

We mentioned that the job of preparing population projections was difficult and required more professional personnel and resources than most local school districts have available. Accordingly, we recommended that maximum use be made of the projections provided by the U.S. Bureau of the Census. In this appendix A, then, we are presenting extracts on the work of State agencies and the Census Bureau, and guides to the use of census data.

We recommend that any State agency interested in making its own population projections should consult with Census Bureau personnel before undertaking the job.

First let us distinguish between "current population estimates" and "projections." Complete counts of the population nationwide are taken every 10 years and the results are published in the Decennial Census volumes. The last census—or count—was taken in 1960. Hence, any estimate for a year after 1960, but not later than the current year, as 1966, for example, is a "current population estimate." A "projection" is an estimate for any year after the current year. In terms of this monograph, any year after 1968 is a projection.

Current Population Estimates

Detailed methodology on how a local community can make "current population estimates" is given in *Current Population Reports, Population Estimates*, "Methods of Population Estimation: Part I," Series P-25, No. 339, June 6, 1966, issued by the U.S. Bureau of Census, Washington, D.C. Subsequent reports are being prepared and will be available from the Census Bureau. If a local school district board of education is interested in preparing a current population estimate for its own use, the above mentioned report will be found highly useful.

Information on the work of State and local agencies as of 1965 is presented in the census report, *Population Estimates*, "Inventory of State and Local Agencies Preparing Population Estimates, Survey of 1965," Series P-25, No. 328, March 8, 1966. Excerpts from this report, including the names and addresses of State agencies making current population estimates, are given in the following reference materials.

Projections

Excerpts from the basic methodology used by the Census Bureau in making State projections are given in the following reference materials. These are taken from *Population Estimates*, "Revised Projections of the Population of States, 1970 to 1985," prepared by Meyer Zitter, Series P-25, No. 375, October 3, 1967. This report presents State projections by age for each of the several series. Additional data and information for States are available from the Census Bureau.

The Census Bureau has also prepared projections on school enrollment, included in *Popu-*

lation Estimates, "Summary of Demographic Projections," Series P-25, No. 388, March 14, 1968. These are available only for total United States and for public and private schools combined. Accordingly, they cannot be used for making State and local district public school enrollment projections. However, description of the methodology, as given in the following excerpts, will be of interest to the reader. (*Table numbers ours.*)

Work of State Agencies [1]

Sources of Estimates

In all but one State, North Dakota, some State agency reported making population estimates for counties or other local areas. In a number of instances, census counts rather than estimates are available. Thus, the State of Kansas takes a State census every year as of March 1. Massachusetts takes one in years ending in 5; the results of the last one, taken as of January 1, 1965, have recently become available. The Washington State Census Board counts the population in selected places and supplements these counts with estimates of the population of other cities and towns. The State of Rhode Island in 1965 contracted with the U.S. Bureau of the Census to conduct a special census covering the whole State. (See, *Current Population Reports*, Special Censuses, Series P-28, No. 1393.) In all other instances, the data reported here represent population estimates derived by various methods.

As in earlier surveys, the State departments of health lead other types of statewide agencies in the preparation of local population estimates. Out of a total of 66 different State agencies making such estimates, 27 were departments of health. This is approximately the same number of State departments of health reported as preparing estimates in our earlier surveys. State universities are the second most important source of such estimates; 21 such agencies reported making population estimates. Ten of these were Bureaus of Business Research at State universities and the remainder were represented by Departments of Sociology and newly established Population Study Centers. Other types of agencies preparing estimates were: economic development commission (6), employment security commissions (4), State planning commissions (3), and other agencies. These agencies include the State Census Boards in Oregon and Washington. (The Oregon State Census Board has recently been replaced by the Center for Population Research and Census at Portland State College.) In the State of California, population estimation is the responsibility of the Population Research Unit in the Department of Finance. In Utah, an interagency committee has the responsibility for such estimates.

Table A-1 below summarizes the sources of population estimates by type of agency preparing such estimates. The information for each state is given in Table A-4. The results from the earlier surveys are also shown for comparative purposes. In general, the changes reported over time are true representations of shifts in responsibility of preparing such estimates. It is quite possible, however, that the increase in the total number of agencies reporting work in this area since 1960 reflects the more extensive coverage of the 1965 survey.

In 15 States, more than one agency is involved in population estimation work. This apparent dual responsibility may involve some duplication; at times it represents supplemental estimates by a second agency to meet its own specific needs, but overlapping responsibility does appear to be the situation in about 10 of these States.

Methods Used

The methods used by State agencies to make population estimates for local areas are summarized in table A-2. Explanations of the various methods listed are given in a later section of this report.

TABLE A-1.—*State agencies making population estimates for local areas: Periodic surveys, 1955 to 1965*

Agency	1965	1960	1957-58	1955
Total	66	57	62	46
Department of health	27	27	30	31
State university	21	16	19	9
Bureau of business research	10	10	15	7
Other department	11	6	4	2
Planning commission or economic development agency	9	5	3	1
Employment security office	4	2	4	2
Other	[1] 5	7	6	3

[1] Includes California State Department of Finance, Kansas State Board of Agriculture, Utah Population Committee, Washington State Census Board, and the Office of the Secretary of the Commonwealth of Massachusetts (census every 10 years).

[1] Source: U.S. Bureau of the Census, *Population Estimates*, "Inventory of State and Local Agencies Preparing Population Estimate, Survey of 1965," Series P-25, No. 328, March 8, 1966, pp. 2-11. (*Table numbers ours.*)

Table A-2.—*Methods used by State agencies to make population estimates for local areas: Survey of 1965*

Method:	Number of agencies
Agencies reporting, total	66
Agencies preparing estimates of total population	[1] 60
State census	2
Component method	34
Bureau of the Census: Method II	15
Age or grade progression	3
Average of Method II and other methods	4
Other [2]	12
Composite method	5
Bogue-Duncan	3
Other variation	2
Censal ratio method	10
Simple	2
Complex	8
Vital rates	3
Ratio-correlation	3
Other	2
Natural increase	3
Arithmetic or geometric extrapolation	4
Other	2

[1] Excludes three agencies involved in supplementing the estimates of total population prepared by other agencies, and three agencies that have not prepared estimates in this decade.

[2] Includes net migration based on past trends (4); net migration based on other symptomatic data (3); Census Bureau Method I (1); and other variations.

More than half of the agencies preparing estimates of the total population of counties use some form of component method in which the primary components of population change, that is, births, deaths, and net migration are estimated separately. Included are methods in which net migration is based on current series of symptomatic data or simply on the extrapolation of past levels or trends. Component Method II, or an adaptation of the method, was the principal specific component method used, accounting for close to half of all the component method variants.

The censal ratio method was the second most frequently cited method. Here, too, the symptomatic data series used varies from State to State, with vital statistics being very frequently used. Three agencies use estimates based solely on natural increase without any allowance for net migration, and four agencies were reported as using arithmetic or geometric extrapolation. These last methods are considered substantially less reliable than the other methods shown, in that they do not make use of any current indicators which may reflect population change due to net migration. In only four instances was an average of the results of two or more methods used, although many of the single-technique estimates use more than one series of data to estimate either net migration or the distribution of total population among geographic areas. Recent studies conducted by the Bureau of the Census suggest that the averaging of the results of two or more independent methods of relatively the same level of accuracy tend to produce estimates of lower average error than estimates produced by a single method.

The assignment of the method to a particular classification is based on a description of the method provided by the local agencies. In this respect, then, there had to be a fair degree of guesswork in the assignment process. However, within broad categories, it may be possible to see to what extent techniques used by State agencies have changed in the 10 years since the first survey was initiated in 1955. Table A-3 below summarizes the results of the four surveys available to date.

Table A-3.—*Summary of methods used by State agencies to make population estimates for local areas: Periodic surveys, 1955 to 1965*

Method	1956	1960	1957-58	1955
Total	60	57	62	44
State census	2	2	2	2
Component method	34	24	28	17
Composite method	5	6	5	...
Censal ratio	10	14	7	3
Natural increase	3	4	5	7
Proration	...	3	6	5
Extrapolation	4	4	4	8
Other	2	...	5	2

Classification of Methods

The questionnaire asked each respondent to describe the methods used to prepare estimates, including the kinds of symptomatic data and the particular way in which they were used. It also asked whether net migration was measured separately. This information was used to classify each of the replies into the methods listed below. A brief explanation of these methods is as follows.

Component or migration-and-national-increase methods are those in which the components of population change since the last census—that is, births, deaths, and net migration—are estimated separately.

In *Component Method II** of the Bureau of the Census, net migration is estimated on the basis of school enrollment or school census data from the difference between the actual population of elementary school age and the population of school age expected on the basis of the most recent Census and births since the census. (In *Component Method I* of the Bureau of the Census, the net migration rate for a given area is estimated, on the basis of school enrollment or school census data, as the difference between the percent change in the population of school age for the area and the corresponding change for the United States.)

In other component methods, net migration is estimated in various other ways, such as the use of data on school enrollment for successive school years by grade (the *grade-progression* method), the use of reported migration of previous decades, and other kinds of data, such as information on residential utility installations and dwelling units.

In the *composite method,** estimates of various age groups are derived separately and are then summed to secure a total for all ages. One method is to use age-specific deaths and death rates; another, developed by Bogue and Duncan,[2] uses death statistics to estimate ages 45 and over, birth statistics to estimate ages 18 to 44 and under 5, and Component Method II (see above) to estimate ages 5 to 17 years. Other variations of the composite method employ other combinations of "indicators" for various age groups. These methods permit the choice of those indicators best suited to a given age range and provide some age detail as a by-product.

In the *censal ratio methods*, current symptomatic data are multiplied by the ratio of population to the same symptomatic datum at the last census for the area for which the estimate is being made. A variation of the method allows for a trend in this ratio between the census and the estimate date. Sometimes the initial estimates of population by the censal ratio method for the constituent parts of an area are adjusted so as to add to an independent estimate for the entire area. A censal ratio method may be simple, using one indicator, or complex using two or more indicators. The *vital rates method** is a ratio method in which two sets of estimates, based, respectively, on birth and death statistics, are averaged. The *ratio-correlation method** is a censal ratio method in which the estimate is obtained by use of a formula for multiple regression between change in the proportion of the State's population in a county and the change in a number of censal ratios for the last decade. The *dwelling unit method** is yet another censal ratio method, in which the estimates of population are based on estimates of dwelling units. The latter, in turn, are based on data on building permits issued, or on data on electric, gas, or water meter connections. In this method, allowance may or may not be made for the trend in the size of household in the area. Because of the lack of the necessary data to measure changes in the number of dwelling units for rural areas, the method is generally limited to urban areas.

The *proration method* involves the distribution of an estimated total for some large area among the constituent parts, as an independently estimated State total among counties, on the basis of the population at the last census or on the basis of current symptomatic data, such as school data, births, or deaths. This procedure implicitly assumes that the State's population is currently distributed in the same proportion over the counties as at the last census, or that the ratio of population to the symptomatic item is the same for all of the counties. (In this respect, the proration method differs from the censal ratio method. In the latter, the ratio of population to the symptomatio item is derived separately for each county.)

Arithmetic extrapolation assumes that the yearly amount of population change in an area in the postcensal period equals the average yearly amount of change in the area in a recent past period—usually the most recent intercensal period. In *geometric extrapolation*, the average yearly rate of change is assumed to remain the same as in the past period.

The *natural increase method* involves merely adding natural increase (births minus deaths) to the census figure. It assumes, therefore, that net migration since the last census equals zero.

*Detailed descriptions of methods noted by an asterisk (*), as used by the Bureau of the Census, are given in: U.S. Bureau of the Census, *Current Population Reports*, Series P-25, No. 324, January 20, 1966, and No. 298, February 12, 1965.

[2] See, Donald J. Bogue and Beverly Duncan, "A Composite Method for Estimating Postcensal Population of Small Areas by Age, Sex, and Color," National Office of Vital Statistics, *Vital Statistics—Special Reports*, Vol. XLVII, No. 6 (August 24, 1959).

TABLE A-4.—*Population estimates prepared by State agencies: Survey of 1965*

(Agencies reporting that they prepare population "projections" are noted by an asterisk (*).

State	Name and address of agency preparing estimates	Areas and detail available	Method or types of data used	Source and frequency
Alabama	Bureau of Business Research* University of Alabama P.O. Box KK University, Alabama 35486	Counties	Component—Method II	Published annually in *Alabama Business*.
Alaska	Research and Analysis Section* Employment Security Division Department of Labor P.O. Box 2661 Juneau, Alaska 99801	Election districts	Component—Method II	Published annually in *Current Population Estimates—Alaska, by Election District*.
Arizona	Arizona State Employment Service* 1717 West Jefferson Street Phoenix, Arizona 85007	Counties (Cities irregularly upon request).	Component—Estimate of net migration based on dwelling units and State income tax returns.	Published annually in *Arizona Basic Economic Data*.
Arkansas	Bureau of Business and Economic Research College of Business Research University of Arkansas Fayetteville, Arkansas 72703	Counties	Component—modified Method II: 1. Elementary and high school enrollment used 2. Constant relationship assumed between migration rates for school ages and total population.	Published annually in *Arkansas Business Bulletin*.
California	Population Research Unit* Department of Finance Sacramento, California 95814	Counties and cities, total population only. State by age and sex.	*Counties:* Component—Grade-progression. *Cities:* Various methods, including dwelling unit, sample surveys, and enumeration.	Published annually in *California Population*.
Colorado	Colorado State Division of Accounts and Control* Room 1-10 State Capitol Building Denver, Colorado 80203	Counties and cities	Component—modified Method II	Published annually in *The Colorado Year Book*.
Connecticut	Public Health Statistics Section State Department of Health 79 Elm Street Hartford, Connecticut 06115	Counties and towns, total population only. State by age and sex.	Component—Age-progression using school census data.	Published annually as a special issue of *Weekly Health Bulletin*.
Delaware	Bureau of Vital Statistics State Board of Health Dover, Delaware 19901	Counties and Wilmington city, by color.	Arithmetic extrapolation	Unpublished; prepared annually and available upon request.
	Delaware State Planning Office 45 The Green Dover, Delaware 19901	Counties and municipalities	Censal ratio—based on housing unit count obtained in State-wide field survey.	Estimates now in preparation, will be revised and published at irregular intervals and available upon request.
District of Columbia.	Biostatistics Section Planning, Research, and Statistics Division D.C. Department of Public Health 300 Indiana Avenue, N.W. Washington, D.C. 20001	District of Columbia by age, sex, and color. Statistical areas by age and color. Census tracts, total population only.	Composite—Bogue-Duncan method.	Published annually in *Vital Statistics Summary*.
Florida	Bureau of Economic and Business Research College of Business Administration 221 Matherly Hall University of Florida Gainesville, Florida 32603	Counties	Average of two methods: 1. Component—Method II 2. Censal ratio—based on vital statistics.	Published annually in *Population Bulletin Series*, *Economic Leaflets*, and *Business and Economic Dimensions*.
	Division of Public Health Statistics Florida State Board of Health Jacksonville, Florida 32201	Counties by color	Component—migration based on past trends, adjusted to county totals of University of Florida (above).	Published annually in *Florida Vital Statistics*.
Georgia	Biostatistics Service* Georgia Department of Public Health 47 Trinity Avenue, S.W. Atlanta, Georgia 30334	Counties and cities of 2,500 and over by color. State by age, sex, and color.	Component—adaptation of Method II.	Published annually as a separate release.
	Demographic Laboratory * Social Science Research Institute University of Georgia Athens, Georgia 30601	Counties and cities of 5,000 and over by color and sex.	Component—Method II	Estimates now in preparation and will be published by the Bureau of Business Research, University of Georgia.

TABLE A-4.—*Population estimates prepared by State agencies: Survey of 1965*—Continued

State	Name and address of agency preparing estimates	Areas and detail available	Method or types of data used	Source and frequency
Hawaii	Research, Planning, and Statistics Office State Department of Health 1250 Punch Bowl Street Honolulu, Hawaii 96801	Civilian population, counties, islands, and cities of Hilo and Honolulu. Honolulu by census tract.	Component—migration based on passenger statistics.	Published semiannually in *Estimated Civilian Population of the State of Hawaii by Geographic Area.*
	Department of Planning and Economic Development* 426 Queen Street Honolulu, Hawaii 96813	Resident population for above areas.	Estimates based on above plus data obtained on Armed Forces.	Published semiannually in *Statistical Report*, "The Population of Hawaii."
Idaho	Bureau of Vital Statistics Idaho Department of Health State House Boise, Idaho 83701	Counties	Natural increase method	Published annually in *Annual Report.*
Illinois	Bureau of Statistics* Department of Public Health Springfield, Illinois 62706	Counties and cities of 10,000 and over.	Component—natural increase with migration based on 1950-60 trend.	Published annually in *Vital Statistics—Special Report Series.*
Indiana	Public Health Statistics* State Board of Health 1330 West Michigan Street Indianapolis, Indiana 46202	Counties and cities of 5,000 and over.	Average of Method II and censal ratio based on vital statistics.	Published annually as a separate release.
Iowa	Division of Vital Statistics State Department of Health Des Moines, Iowa 50319	Counties and cities of 10,000 and over.	Component—modified Method II.	Published annually in "Statistical Supplement" of the *Biennial Report.*
Kansas	Kansas State Board of Agriculture State Office Building Topeka, Kansas 66612	Counties, cities, and townships.	Annual enumeration by county assessors.	Published annually in *Kansas Directory of Government Officials*, and by State Department of Health in *Annual Summary of Vital Statistics.*
	Division of Vital Statistics State Department of Health State Office Building Topeka, Kansas 66612	State by age, sex, and color. Counties and selected cities by age and sex.	Totals from State Board of Agriculture (see above) age, sex, and color based on a combination involving the use of school enrollment, age-specific death rates, and an estimate of net migration.	Unpublished; prepared at irregular intervals and available upon request.
Kentucky	Department of Sociology* University of Kentucky Lexington, Kentucky 40506	Counties	Average of four methods: 1. Censal ratio—based on vital statistics. 2. Component—Method II. 3. Ratio correlation—based on school census, vehicle registrations, and vital statistics. 4. Hamilton-Perry ratio projection.	Published annually as a special report of the Kentucky Agricultural Experiment Station.
	Division of Planning, Research, and Statistics Kentucky Department of Health 275 East Main Street Frankfort, Kentucky 40601	Counties and cities of 10,000 and over, by color.	Censal ratio—based on vital statistics.	Published annually in *Vital Statistics Report.*
Louisiana	Tabulation and Analysis Section State Board of Health New Orleans, Louisiana 70160	State by age and color. Parishes and cities of 10,000 and over by color	Natural increase method	Published annually as a statistical report of the Division of Public Health Statistics.
Maine	Division of Research and Vital Records Department of Health and Welfare State House Augusta, Maine 04330	Counties	Average of two methods: 1. Censal ratio—based on vital statistics. 2. Component—natural increase with migration implicitly assumed in adjustment to State total.	Published annually in *Vital Statistical Report.*
Maryland	Division of Statistical Research and Records* State Department of Health 301 West Preston Street Baltimore, Maryland 21201	Counties by age and color	Composite—Bogue-Duncan method.	Published annually in *Final Vital Statistics Tables, Maryland.*
Massachusetts	Office of the Secretary of the Commonwealth State House Boston, Massachusetts 02133	Counties, cities, and towns by age and sex.	State census on January 1, 1965	State census taken decennially in years ending in "5." Latest census published in *The Decennial Census, 1965.*
	Bureau of Research and Statistics State Department of Commerce and Development 150 Causeway Street Boston, Massachusetts 02114	Counties and cities	Data on migration, natural increase, and new family accommodations used.	Unpublished; prepared at irregular intervals and available upon request.

TABLE A-4.—*Population estimates prepared by State agencies: Survey of 1965*—Continued

State	Name and address of agency preparing estimates	Areas and detail available	Method or types of data used	Source and frequency
Michigan	Population Studies Center* University of Michigan 527 East Liberty Street Ann Arbor, Michigan 48103	Counties	Ratio correlation--multiple regression equation using births, auto registrations, sales tax, school census data, etc.	Published annually as a separate release by the State Department of Health and available upon request.
	Health Statistics and Evaluation Center Michigan Department of Health 3500 North Logan Lansing, Michigan 48914	Cities of 2,500 and over	Component—County net migration (per above) apportioned to cities and "balance of county."	Same as above.
Minnesota	Section of Vital Statistics* Department of Health 350 State Office Building St. Paul, Minnesota 55101	Counties	Composite—Bogue-Duncan method.	Published annually as a separate release.
Mississippi	Division of Sociology and Rural Life* Mississippi State University State College, Mississippi 39762	Counties by color	Censal ratio—based on vital statistics.	Published annually as special bulletins of the Agricultural Experiment Station, Mississippi State University.
Missouri	Statistical Services State Department of Health and Welfare State Office Building Jefferson City, Missouri 65102	Counties and selected cities	Component—natural increase with net migration based on 1950-60 trend.	Published annually in a special issue of monthly *Health*.
	Research Center* School of Business and Public Administration University of Missouri Columbia, Missouri 65202	Counties and selected cities	Natural increase method	Unpublished; prepared annually and available upon request.
Montana	Division of Records and Statistics State Board of Health Helena, Montana 59601	Counties	Component—Method II	Published annually in *Statistical Supplement* and as a separate release.
Nebraska	Bureau of Business Research 310 Social Science Building University of Nebraska Lincoln, Nebraska 68503	Counties and selected cities	Ratio-correlation—based on school census, number of votes cast, drivers licenses, vital statistics, and head tax.	Published annually in *Business in Nebraska*.
Nevada	Bureau of Business and Economic Research* College of Business Administration University of Nevada Reno, Nevada 89507	Counties	Censal ratio—based on school enrollment.	Unpublished; prepared at irregular intervals and available upon request.
New Hampshire	Division of Economic Security Department of Resources and Economic Development State House Annex Concord, New Hampshire 03301	Counties, cities, and towns	Composite—based on school census and State head tax data.	Prepared at irregular intervals and available upon request.
	Department of Employment Security* 32 South Main Street Concord, New Hampshire 03301	Counties	Based on work-force statistics	Prepared and published at irregular intervals. Estimates for 1964 published in *Economic Changes in New Hampshire Counties in 1957 to 1963*.
New Jersey	Division of Resource Development* Department of Conservation and Economic Development Bureau of Commerce Research and Statistics Section P.O. Box 1889 Trenton, New Jersey 08625	Counties and all incorporated municipalities.	Ratio-correlation—based on vital statistics, dwelling units, newspapers, and migration trends.	Published annually in research report, *New Jersey Population Estimates 196-*.
	Bureau of Public Health Statistics Department of Health Health-Agriculture Building John Fitch Plaza—P.O. Box 1540 Trenton, New Jersey 08625	Counties and cities of 50,000 and over.	Arithmetic extrapolation	Published annually in *New Jersey Health Statistics*.
New Mexico	Bureau of Business Research* University of New Mexico 1821 Roma Street, N.E. Albuquerque, New Mexico 87106	Counties	Censal ratio—based on school data, vital statistics, and other data.	Published annually in special issue of *Business Information Series*.

TABLE A-4.—*Population estimates prepared by State agencies: Survey of 1965*—Continued

State	Name and address of agency preparing estimates	Areas and detail available	Method or types of data used	Source and frequency
New York	Office of Biostatistics New York State Department of Health 84 Holland Avenue Albany, New York 12208	Counties and cities of 10,000 and over. State and New York City by age and sex.	New York City: See table 2. *Nassau, Suffolk, and Westchester Counties:* Censal ratio—based on utility data. *All other counties and cities:* Component—natural increase with net migration based on 1950-60 trends. Age and sex: Cohort survival.	Published annually in *Monthly Vital Statistics Review* and *Annual Statistical Report.*
North Carolina	Public Health Statistics Section State Board of Health P.O. Box 2091 Raleigh, North Carolina 27602	Counties and cities of 10,000 and over by color.	Arithmetic extrapolation	Published annually in a separate *Vital Statistics Report.*
	Department of Rural Sociology* University of North Carolina P.O. Box 5428 Raleigh, North Carolina 27607	Counties	Component—Method II	Estimates now in preparation.
North Dakota				No estimates reported.
Ohio	Economic Research Division* State Development Department Box 1001 Columbus, Ohio 43216	Counties, cities, villages, standard metropolitan statistical areas, and economic regions.	Component—natural increase with net migration based on 1950-60 trend and other factors for evaluating change in trend.	Estimates for counties and cities published semiannually; for villages, annually in *Population Estimates for Ohio,* Series A and B.
Oklahoma	Bureau of Business Research* University of Oklahoma Norman, Oklahoma 73069	Counties and cities of 2,500 and over.	*Counties:* Component—Method II. *Cities:* Censal ratio—based on gas meter connections.	Published annually in the *Statistical Abstract of Oklahoma.*
Oregon	Center for Population Research and Census* Portland State College P.O. Box 751 Portland, Oregon 97207 (Replaced State Board of Census)	Counties and cities	*Counties:* Component—Method II. *Cities:* Censal ratio—based on dwelling units and enumeration of some areas.	Published annually in *Population Estimates of Counties and Incorporated Cities of Oregon.*
Pennsylvania	Research Section* State Planning Board Governor's Office Harrisburg, Pennsylvania 17120	Counties and selected cities and boroughs.	Censal ratio—based on vital statistics.	Unpublished; prepared annually and available upon request.
Rhode Island [1]	Rhode Island Development Council* Roger Williams Building Hayes Street Providence, Rhode Island 02908	Counties, cities, and towns	Basically a nonstandardized, composite method.	Prepared at irregular intervals and available upon request.
South Carolina	Bureau of Vital Statistics State Board of Health Columbia, South Carolina 29201	State by age, sex, and color. Counties and cities of 10,000 and over by color.	Arithmetic extrapolation—adjusted by percent change in births since preceding year.	Published annually in the statistical supplement of *The Annual Report of the State Board of Health.*
South Dakota	Division of Public Health Statistics* State Department of Health Pierre, South Dakota 57501	Counties	Component—Method II	Published annually in *Special Report Series,* "South Dakota Population Changes by County."
Tennessee	Bureau of Business and Economic Research* College of Business Administration University of Tennessee Knoxville, Tennessee 37916	Counties by sex and color	Component—Method II	Prepared and published at irregular intervals.
Texas	Population Research Center* Department of Sociology University of Texas Austin, Texas 78712	Counties and SMSA's	Basic methods: 1. Component—variation of Method II. 2. Censal ratio—based on vital statistics and passenger car registration.	Published annually by the Bureau of Business Research, University of Texas, in *Texas Business Reviews.*
	Department of Agricultural Economics and Sociology* Texas A and M University College Station, Texas 77843	Counties and cities	Component—Method I	Unpublished; prepared at irregular intervals and available upon request.
Utah	Utah Population Committee* Department of Employment Security 174 Social Hall Avenue Salt Lake City, Utah 84110	Counties	Component—Grade-progression	Published annually by the Bureau of Business Research, University of Utah, in the April issue of *Utah Economic and Business Review.*

[1] A special census of the entire State was taken by the Bureau of the Census under contract with the State. Results are published by the Bureau of the Census in *Current Population Reports,* Series P-28, No. 1393.

TABLE A-4.—*Population estimates prepared by State agencies: Survey of 1965*—Continued

State	Name and address of agency preparing estimates	Areas and detail available	Method or types of data used	Source and frequency
Vermont	Division of Public Health Statistics, Department of Health, 115 Colchester Avenue, Burlington, Vermont 05401	Counties, total population only. State by age.	Composite—based on vital statistics, school enrollment, and 1940-60, 1950-60 trends.	Unpublished; prepared annually and available upon request.
Virginia	Bureau of Population and Economic Research*, University of Virginia, Charlottesville, Virginia 22903	Counties, cities, and towns of 2,500 and over.	Component—migration based on school enrollment, State income tax returns, and 1950-60 trend.	Published annually as a separate release.
	Virginia Employment Commission, P.O. Box 1358, Richmond, Virginia 23211	Standard metropolitan statistical areas, counties, and cities.		Estimates now in preparation.
Washington	State Census Board*, 102 Guthrie Hall, University of Washington, Seattle, Washington 98105	Incorporated towns and cities	Annual enumeration of approximately 100 cities and towns; remainder estimated using dwelling unit data.	Published annually in *Population Trends—Cities and Towns—State of Washington—1960 to 196-*.
	Public Health Statistics Section, State Department of Health, Public Health Building, Olympia, Washington 98502	Counties	Estimates for counties based on city trends shown by above agency.	Unpublished; prepared annually and available upon request.
West Virginia	Department of Sociology, Agricultural Experiment Station, West Virginia University, Morgantown, West Virginia 26506	Counties	Component—Age-progression	Released annually by the State Department of Health.
Wisconsin	Bureau of Vital Statistics, State Board of Health, State Office Building, Madison, Wisconsin 53702	Counties and selected cities by age and sex.	Component—Method II	Published annually in *Public Health Report*.
	Department of Rural Sociology, University of Wisconsin, Madison, Wisconsin 53706	Counties	Various methods including arithmetic extrapolation, Hamilton-Perry method, and ratio-correlation.	Prepared at irregular intervals and released as special departmental publications.
Wyoming	Division of Business and Economic Research, College of Commerce and Industry, University of Wyoming, Laramie, Wyoming 82071	Counties	Censal ratio—based on vital statistics, school enrollment data, auto license registration, and drivers license registration.	Prepared and published at irregular intervals.

Methodology for State Projections[3]

General

The methodology and underlying assumptions used to develop these State projections are similar to those used in developing the earlier State projections published in report No. 326. Each of the components of population change—births, deaths, and interstate and international migration—was projected separately. A single set of projected mortality rates was used for all States, but alternative assumptions relating to national fertility and interstate migration were introduced in order to provide a reasonable range of projected population for each State. A fundamental characteristic of the projections is the separate computation of gross out- and in-migration to derive net migration, as opposed to the more common practice of working directly with net migration.

Basically, the projections start with the estimates of the total population of States for July 1, 1965, published in report No. 348 of this series. Since the 1965 State figures were not available by detailed age-sex-color groups, operationally, the computations start with the April 1, 1960, Census data for each State, by age, sex, and color, and are carried forward to 1965 on the basis of separate projections of each of the components of change, also by age, sex, and color. At this point, the projections are forced into agreement with the estimates of the population of States, by broad age groups, for July 1, 1965, given in report No. 354. The adjusted projections for July 1, 1965 (which are now consistent with the current estimates), are then carried forward by 5-year periods to each projection date on the basis of

[3] Source: U.S. Bureau of the Census, *Population Estimates*, "Revised Projections of the Population of States, 1970 to 1985," Series P-25, No. 375, Oct. 3, 1967, pp. 8-13. (*Table numbers ours.*)

the assumptions chosen concerning future births, deaths, and migration.

All told, four main series of projections are shown—that is, two assumptions concerning future interstate migration combined with two levels of fertility, labelled Series I–B, I–D, II–B, and II–D. The Roman numerals relate to the migration assumption; the letters designate the national fertility series incorporated in the projections. The underlying assumptions for each series can be broadly summarized as follows:

Series	Interstate migration assumption	National fertility assumption
I–B	Migration rates will continue within the range observed in 1955–60 and 1960–65.	Moderate increase from present levels.
I–D	Same as I–B	Continued decline from present levels.
II–B	Migration rates will change from recent levels so as to result in no net migration among States in 50 years.	Same as I–B.
I–D	Same as II–B	Same as I–D.

The methods and assumptions used to derive the various projection series follow:

Projections of Births

Even at the national level, the number of births for future years cannot be projected with a high degree of certainty. Because of the wide range of reasonable possibilities in the future course of fertility, the national population projections report (Series P–25, No. 359), includes a number of alternative fertility levels based on different assumptions. Four principal series were developed to describe the future course of fertility. In the projection of births for a given State for future years, the elements of uncertainty existing at the national level still prevail; nonetheless, in terms of overall fertility, the factors determining changes in future national fertility are not believed to have an appreciable impact on the fertility differentials already existing among the States. Consequently, the approach here for State projections of fertility is not one of determining the future course of fertility in each specific State, but, rather, of distributing the previously projected national number of births to the State on the basis of some reasonable criteria. The main elements used to distribute the national projections of the number of births are: (a) the size of the female population of each State in the childbearing ages, and (b) each State's recent level of overall fertility (births per 1,000 women 15 to 44 years old) in relation to national levels.

For present purposes, two of the national series of fertility projections, Series B and Series D, were chosen. Fertility rates (defined here as the number of births per 1,000 women in the ages 15 to 44 years) for each State were computed for a 3-year period centered on April 1, 1960. The corresponding national rate for that period was also computed. Using the 1960 relationship of the State rates to the national rates (referred to hereafter as the State-national ratios) as starting points, it was assumed that the State-national ratios would reach unity in 50 years. It is assumed, in effect, that the factors producing State differences in fertility will gradually disappear and that, in approximately 50 years, the fertility rates for all States will be equal to the national rate.

The State-national ratios for the years intermediate between 1960 and 2010 were obtained by linear interpolation. State fertility rates for each projection period, 1960–65 to 1980–85, were derived by applying the projected State-national ratios to the previously computed national fertility rates for these periods (Series B and Series D, separately). These projected fertility rates for States, multiplied by the projected number of females 15 to 44 years of age, yield projections of the number of births for each State for each period. The projected number of females 15 to 44 years old for each State had been derived as part of another stage of the projections, by carrying forward the 1960 population using age-specific mortality and gross interstate migration rates. Births projected for each State for each 5-year period were then summed, and adjusted to add to total births from the national projections, Series P–25, No. 359. The computations described were carried through for the white and nonwhite population separately.

The national fertility rates by color used here as bases in computing the State rates are shown in table A–5.

TABLE A–5.—*Projected fertility rates, by color: 1960–65 to 1980–85*

Average annual number of births per 1,000 females 15 to 44 years of age, for middate of each period)

Period	Series B			Series D		
	All classes	White	Non-white	All classes	White	Non-white
1960–65	111.5	106.0	151.9	111.5	106.0	151.9
1965–70	100.4	95.2	136.5	88.4	84.2	118.2
1970–75	111.3	105.9	147.5	83.6	80.1	107.1
1975–80	115.4	109.9	150.7	85.4	82.2	106.4
1980–85	113.9	108.5	147.5	86.4	84.3	106.2

Projections of Death

One set of age-sex-color-specific mortality rates was used for all States for all series in the projections. The projected rates are consistent with those developed for and used in the new national population projections presented in Series P–25, No. 359. Rates for each 5-year projection period were obtained by linear interpolation between the rates by age, sex, and color observed in 1962, and those projected for the period 2000–2005, also by age, sex, and color, for the national projections. To assure exact agreement between these projections and the new national projections, the projected numbers of deaths for States for each 5-year period were summed and adjusted to agree with the number of deaths developed in the national projections also by age, sex, and color. The rates for the period 2000-2005 are consistent with the "high" mortality rates for the year 2000 developed in 1957 by the Social Security Administration.[4] For a general discussion of the underlying logic behind the mortality assumptions, see report No. 359; report No. 286, the previous detailed report of national projections, contains a more detailed discussion which is still generally applicable.

The use of only one set of age-sex-color—specific mortality rates is not intended to deny that State differences in mortality exist. It is believed, however, that allowing for the actual State differences in mortality would have very little impact on the present population projections.

Projections of Migration

General.—Interstate migration was projected by treating gross out- and gross in-migration separately, with net migration obtained as the difference between these components. (Separate computations were made for net immigration from abroad.) This is the same procedure used in the previous set of State projections published in Series P–25, No. 326. The more conventional approach is to project rates of net migration. The use of gross migration data is more logical in that at the outset total interstate in-migration is dependent upon and equal to total interstate out-migration; by contrast, the use of net migration rates quite often results in serious imbalances between total net in-migration and net out-migration, which are difficult to resolve.

The conventional procedure in using net migration rates for projections is to assume the continuation of past trends, and to multiply the projected rates by the population at the beginning of each projection period to determine the amount of projected net migration. Under these conditions the in-migration States automatically receive larger and larger numbers of in-migrants, while the remaining States are forced to provide unreasonable numbers of out-migrants, since the base population in the latter States becomes a smaller and smaller proportion of the national population as a result of out-migration. Thus the sums of net in-migration and net out-migration become seriously unbalanced, and the computed net migrations require progressively larger adjustments to balance them out to zero (or to a national control total representing net immigration from abroad).

These considerations suggested the use of gross migration rates. Pertinent migration statistics are available by States for only three periods, the most recent being the 1955–60 period. In the 1960 Census, a specific question was asked concerning the State of residence in 1955, thus providing information for all States on surviving in-migrants and out-migrants, by age, sex, and color.

In this report the rates of gross out-migration observed during the 1955–60 period (by 5-year age groups, sex, and color) were multiplied by the population at the beginning of each quinquennial period to determine the total number of out-migrants for each State for each 5-year period. These were summed to obtain a national total for each 5-year period, and then allocated to the States as in-migrants (again by age, sex, and color) according to the proportion of national interstate in-migration each State received during the 1955–60 period. Under this procedure, the sum total of net interstate migration for all States is zero. The difference between a State's contribution to the gross number of out-migrants and the number it receives as in-migrants represents net interstate migration for the State.

Unlike assumptions of continuing *net* migration rates, this procedure avoids automatic increases in the number of net migrants of gaining States. As the population base of a State grows in any age-sex-color group, it contributes more persons to the migration pool. The number of in-migrants it receives, however, represents a constant proportion of migration pool and is unaffected, or

[4] Social Security Administration, *Illustrative United States Population Projections*, by T.N.E. Greville, Actuarial Study No. 46, May 1957.

affected very little, by the changing size of its population. As its out-migration share grows, its net migration tends to become smaller.

The overall annual gross migration rate implied by the projections is relatively constant at about 3.3 percent throughout the projection period, consistent with the general stability of this rate as observed annually over the past 17 years in the Current Population Survey.[5] The consistency of this rate during the projection period is to be expected for Series I—discussed in paragraph 2 below—since this series assumes a continuation of the age-sex-color migration rates of the recent past. Even, however, under Series II—discussed in paragraph 3 below—where the state migration rates are assumed to change, the overall interstate migration rate remains similar to that of Series I.

Migration Series I.—For this series the computation of gross migration described above is carried out by age, sex, and color assuming that the gross out-migration rates and the in-migration distributions of the 1955–60 period will remain constant throughout the projection period. However, an adjustment is introduced (see paragraph b. (page 77)) to allow for net migration observed during the period 1960–65 as reflected in current estimates of State population.

Migration Series II.—In Series II, it is assumed that the gross out-migration rates will converge toward national levels. At the same time it is assumed that the in-migration distribution will converge toward the State population distribution. In effect, then, whereas Series I holds the 1955–60 rates constant, Series II assumes a trend in the rates. In seeking an alternative assumption, consideration was given to the possible ways in which out-migration rates and in-migration distributions could be modified. In reviewing the various possibilities, it was concluded from the historical evidence, as indicated earlier, that there is little prospect that the national rate of interstate migration will disappear in the future. Some equilibrium in *net* migration may be reached, however, if economic opportunities and other social and economic differentials among the States tend toward equality.

Thus, the assumption adopted for Series II is that State migration differentials will gradually be reduced, and that at some point in time (approximately 50 years), the number of persons migrating from a State will be offset by an equal number of persons moving into the State, thus providing zero net migration for each State. This alternative migration assumption requires some change from current gross migration rates during the projection period.

To achieve this equilibrium of migration, it was assumed that at the terminal point, 50 years hence, the total number of out-migrants and in-migrants will each be distributed in proportion to the population of the States. Thus, a State with 10 percent of the population of the country will contribute 10 percent of the total interstate migration pool (out-migrants) and receive in turn, 10 percent of the migration pool as in-migrants.

Operationally, the rates for Series II were obtained by interpolating between (1) the 1955–60 out-migration rates by age, sex, and color for each State; and (2) national interstate rates by age, sex, and color, so that by the period 2005–2010, the out-migration rates for all States are equal to the national averages. Interpolated values for intervening periods are then a function of time. For example, convergence becomes halfway complete in half the time, in other words after 25 years.

Symbolically, the series for a specific age-sex-color group is derived as follows:

M_i = out-migration rates in State i
P_i = population at beginning of period in State i
ΣP_i = total population of the United States (sum of States) at beginning of period
$M_i P_i$ = number of out-migrants from State i during specific quinquennium.

Then:

$\Sigma M_i P_i$ = sum of out-migrants from all States for specific period

$\frac{\Sigma M_i P_i}{\Sigma P_i}$ = national interstate migration rate for period.

The assumption under Series II is that in 50 years, for each State,

$$\frac{M_i P_i}{\Sigma M_i P_i} = \frac{P_i}{\Sigma P_i}$$

Thus, $M_i = \frac{\Sigma M_i P_i}{\Sigma P_i}$

For any given 5-year period, the out-migration rate is derived from $k(M_i) + (1-k)\frac{\Sigma M_i P_i}{\Sigma P_i}$,

[5] *Current Population Reports*, Series P-20, No. 156.

where k is computed proportionate to time. To illustrate, for the 1960–65 period, (k) equals 0.95; in 1980–85, (k) equals 0.55. Thus, by the year 2010, (k) equals 0 and the State out-migration rate is equal to the national rate.[6]

Similar computations were carried through for the in-migration distributions; that is, for each period of time, the interpolation was made between the percentage of in-migrants that a State received in the 1955–60 period and the percentage of migrants it would receive proportionate to its population. Here, too, the interpolations were made proportionate to time, so that in 50 years the percentage it would receive as in-migrants from the interstate migration pool would be the same as its share of the national population.

Migration assumption II, as compared to assumption I, has a more moderating effect on the projected redistribution of population. States growing well above the national average are slowed down in their rates of growth as gains through migration exchanges with other States are reduced. Correspondingly, States with heavy out-migration begin to receive migrants at a faster pace because of reduction in their contribution of out-migrants and of increases in their share of in-migrants. Although under migration assumption I such changes in population redistribution also occur, the effect under Series II is substantially greater. Table A-6 illustrates this effect for three States, one projected to have net in-migration, one with moderate net out-migration, and one with above-average net out-migration.

TABLE A-6.—*Rate of total net migration (in percent) for selected States: 1955–65, 1965–75, 1975–85*

State	Estimated, 1955–65	Projected			
		Series I-B		Series II-B	
		1965–75	1975–85	1965–75	1975–85
Colorado	+ 8.28	+4.46	+3.16	+4.05	+2.70
Alabama	− 2.79	−2.24	−1.58	−1.89	−0.96
North Dakota	−11.22	−9.51	−7.68	−7.98	−5.23

Net immigration from abroad.—Net immigration was allocated to the States separately, using as an overall total the level established for the national population projections—that is 400,000 per year. The distribution to States is made on the basis of the 1960 State of residence of the foreign-born population reported in the 1960 Census as living abroad in 1955.

Adjustment of migration rates.—As stated, data from the 1960 Census on interstate migration of the 1955–60 period were used to derive estimates of migration for the projection period. The gross migration rates of the 1955–60 period, however, were modified in two important respects, as follows:

a. Adjustment for military movement.—The basic 1955–60 migration data include both civilian and military interstate migration. Because the migration rates around military ages are unusually high and States with large military installations gain migrants at the expense of States with little or no military personnel, an assumption of the continuation of such migration rates for an extended period leads to unreasonable results in some instances. Specifically, the number of males becomes increasingly larger (compared to the number of females) in those States with the large military installations. Such results are inconsistent with the assumption underlying the basic projections, that is, the assumption of no significant changes in the size and distribution of the Armed Forces in the United States over the projection period.

In order to reduce the impact of such movements, the migration data have been modified to exclude, to as large an extent as possible, gross movements of military personnel. In effect, the 1955–60 rates were modified to reflect only civilian migration, with estimates of the Armed Forces movements being handled separately in the procedure.[7] Although this procedure had, in general, little impact on the overall total population of States, it does have significant impact on the age composition of males, particularly around the military ages, in those States with large military installations. Although the conversion of the migration rates of the total population to the civilian population is somewhat imperfect because of the lack of necessary information required to make such adjustments,

[6] $K(M_i)+(1-k)\frac{\Sigma M_i P_i}{\Sigma P_i}=\frac{\Sigma M_i P_i}{\Sigma P_i}$ when $k=0$.

[7] It was assumed that the size of the Armed Forces, at home and abroad, would remain constant at approximately the 1966 level. Persons entering the Armed Forces were distributed, by age, to States of preservice residence on the basis of the 1966 State population distribution. They were assigned to the State of their duty station using the 1966 States of station distribution of the Armed Forces. This procedure leads to a net movement of zero for each State between military and civilian population, as assumed in the projections. Within each State, however, there is substantial differential movement by age since most entries into the Armed Forces occur at ages 18 to 24 years and most returns to civilian life occur at later ages.

it is believed that the modification significantly improved the projections for those ages and States where such adjustments were important. Because the age group 18 to 24 years is particularly affected by this adjustment and because migration rates are very high for this group, the projected values should be interpreted with caution.

b. Adjustments for net migration, 1960 to 1965.—It was also necessary to adjust the 1955–60 rates in order to take account of population changes, mainly due to net migration, that have occurred since 1960. The resulting adjustment applies with equal effect to both migration Series I and II. The latest current population estimates, by age available at the time these projections were undertaken were those for 1965 published in report No. 354 of this series. Estimates of net migration by age consistent with the 1965 population estimates were developed for the period 1960–65. The 1955–60 gross migration rates were adjusted in such a manner that, when used for the period 1960–65, they produced estimates of net migration by age for each State about the same as that derived from the independent current estimates.

The adjustment was accomplished by first computing "projections" of net interstate migration for the 1960–65 period using the 1955–60 migration rates and then comparing these estimates with the independent current estimates for the 1960–65 period. The difference between the initial "projections" of net migration for 1960–65 for each State, and the net migration developed consistent with the current estimates for 1960–65 was then used to adjust the 1955–60 out-migration rates and the distribution of in-migrants. The amount of adjustment assigned to each flow, for each age-sex-color group, was determined by the ratios of gross out-migrants and gross in-migrants, respectively, to their sum. A second computation beginning with April 1, 1960, using the 1955–60 rates as adjusted, now yielded figures which closely approximated the 1965 independent current estimates.

Part of the adjustment was retained during the projection period, thus assigning weight to the 1960–65 migration experience in the subsequent derivation of the projections. From 1965 to 1970, three-quarters of the adjustment was used; from 1970 to 1975, one-half; and for the period 1975–80, one-quarter. From 1980 to 1985, the original 1955–60 values were employed. In effect, then, for the bulk of the projection period, the migration assumptions represent a blending of the 1955–60 gross migration experience with the more recent 1960–65 net migration estimates. This procedure yields projections which can be generally described as "based on the assumption that recent migration rates would prevail during the projection period." This system of combining the migration experience of the two most recent periods recognizes, and takes advantage of, the large amount of detailed data available from the census for the 1955–60 period, while broadening the base period of the migration projections.

Other Series Developed by the U.S. Bureau of the Census

The projections described above result in four main series—that is, two assumptions concerning future interstate migration combined with two levels of fertility (labelled Series I–B, I–D, II–B, and II–D). A number of other series based on alternative interstate migration assumptions were developed to provide background data for selecting the projection series to be used for specific projects.

One alternative series, designated as Series III, assumes "no net migration" for each State during the projection period. In this series, it has been assumed that, regardless of gross population movement, net interstate migration for each State in each period after 1965 will balance out to zero (for each age-sex-color group as well as for the total). This series is useful for measuring the impact on the population projections of alternative assumptions of future interstate migration. Net immigration from abroad at the rate of 400,000 per year is assumed to continue for this series.

Another alternative set of projections affects only the nonwhite population in States. Generally speaking, the rates and pattern of nonwhite migration differ to an appreciable degree from those of the white population. This alternative projection series has been derived to illustrate the effect of interstate migration on the population distribution of the nonwhite population if interstate migration rates for whites and nonwhites should become equal. For this series, it was assumed that for each State, the nonwhite out-migration rates of the 1955–60 period will converge toward those of the white population, so that 50 years hence, the out-migration rates of the nonwhites, State by State, would be equal to those of the whites. It was further assumed that the distribution by State of destination would, in 50 years, also be the same as for the white population.

A special set of projections was also derived for the District of Columbia. The District of Columbia is a small area and exclusively urban. Its population composition, the city's position as the core of a large metropolitan area, and past suburbanization, which has involved substantial interstate movement, all contribute to unusual gross interstate migration patterns. Furthermore, it is quite possible, because of present and planned land use, that the current population is not very far below the practical maximum. Consequently, migration assumptions I and II may not be at all appropriate for this area.

School Enrollment Projections[8]

The projections of school enrollment were made by the participation rate method. Projections were made of enrollment rates by age and sex, and these rates were applied to the projections of total population to obtain the number enrolled. Four series are shown, designated as B-1, B-2, D-1, and D-2. The two series designated as "B" are consistent with the series of population projections designated as "B" series. Hence these projections imply a moderate rise in fertility. The two series designated as "D" are consistent with the projections designated as "D" series. They imply a sharp drop in fertility.

[8] Source: U.S. Bureau of the Census, *Population Estimates*, "Summary of Demographic Projections," Series P-25, No. 388, March 14, 1968, pp. 26-27.

The two series designated as "1" imply marked increases in the percent of the population enrolled in school at the older ages. The two series designated as "2" imply that the increase at these ages will be less rapid. There is little difference between the two assumptions at the younger ages, for enrollment is already near 100 percent. There are important differences at the older ages, however.

Actually, it was the age-specific rates for those not enrolled in school which were projected. Series 1 assumes that the decline in the "nonenrollment rates" will continue at the annual rate observed for the period of 1950–1952 to 1963–1965. The complements of these rates are the enrollment rates. The Series 2 enrollment rates are the average of the Series 1 rates and the rates achieved in 1965. They imply that the increase in enrollment will be half that assumed for Series 1. The projections distribute enrollment among three levels of school: elementary (including (kindergarten), high school, and college and professional school. This step was also made by the participation rate method. Projections were made of the percentage distribution among the three levels of school for each age-sex group, and these percentages were applied to the projections of the enrolled population by age and sex. Series 1 implies that the proportion of enrollment at a given level of school would shift between 1964 and 1990 by the same overall percent as between 1951 and 1964. In effect, since the projection period was twice as long as the base period, the proportions were assumed to change about one-half as rapidly in the future as in the recent past. The series 2 proportions are the average of the Series 1 proportions and the proportions in 1965.

APPENDIX B

SMOOTHING AGE DISTRIBUTIONS[1]

Definitions, Usages and Limitations

One method often used consists of applying sets of constant multipliers to the enumerated 5-year age groups in order to obtain a smoothed distribution by single years of age. These multipliers also are of special interest for making population estimates when the census does not present data by single years of age. There are many different kinds of interpolation procedures—that is, interpolating within a 5-year age group so as to obtain estimates for single years; the procedure presented below is often used by the U.S. Bureau of the Census. These constant multipliers, or weights, are based on Sprague's osculatory fifth difference formula.[2] Another set of weights, somewhat similar to those derived from the Sprague formula, is that of Beers.[3]

The Sprague multipliers smooth (or graduate) the data reported into halves, fifths, or tenths. A 10-year period can be divided into single years or two 5-year periods; for example, a 2-year period can be divided into single years; a 5-year period can be divided into single years; and so forth. In the following paragraphs we show the detailed procedures for using the fifths; brief comments on the use of the halves and tenths follow.

The Sprague Multipliers

Dividing into Fifths

The Sprague fifths multipliers simply redistribute the number within the 5-year age period into estimated numbers for single years of age, without in any way altering the total reported for the 5-year age period. Thus in applying it, the data are first compiled into 5-year age periods; original tabulations by single years of age are not used. The same procedures, with but very little modification, can be used to smooth sets of rates by age, as for example, the proportion of the population of single marital status by 1-year age periods; also, age-specific birth rates for women by single years of age can be estimated from data on rates for 5-year age groups.

The limitations of this method are as follows: If any particular 5-year age group is greatly in error due to under-/or over-enumeration, this method will not correct such deficiencies; they must be corrected by graphic interpolation or by calculating the expected number of survivors from the preceding census.

If the original data curve very rapidly from one single year to the next, as is the case with the proportion of single marital status during the teen ages, this formula is somewhat inadequate. In such cases graphic smoothing may be the best. Thus, for example, in smoothing the proportion single for each year of age from age 15 to age 100, it may be best to use graphic smoothing for the age group 15 to 19 and for the older age groups. The specific older ones to which it should be applied will depend on how "regular" or "irregular" the data appear; this "irregularity" may represent respondents' biases which should be smoothed out by actually altering the reported numbers or percentages for given 5-year age groups. For all intermediate 5-year age groups the formula should be adequate.

This formula may also be inadequate for the very youngest age groups—under 5, and 5 to 9 years old. If birth and death statistics are available, it is preferable to calculate these ages by single years on the basis of the vital statistics. Only if complete and accurate birth and death statistics are not available, should these smoothing procedures be applied to the 5-year groups as enumerated by the census.

[1] Taken in part from *Handbook of Statistical Methods for Demographers*, by A.J. Jaffe, U.S. Bureau of the Census, Washington, D.C., third printing, 1960, pp. 94–100.

[2] For a derivation of Sprague's formula, see J.W. Glover, *United States Life Tables*, U.S. Bureau of the Census, Washington, 1931, pp. 344 and 345.

[3] Henry S. Beers, "Modified-Interpolation Formulas that Minimize Fourth Differences," *The Record*, American Institute of Actuaries, Vol. XXXIV, Part I, No. 69, June 1945, pp. 14-20. NOTE.—*Table numbers ours.*

In general the Sprague multipliers are very flexible and will fit most distributions of data by age. Certain very unusual distributions, perhaps cannot be smoothed adequately by this method. In such cases other formulas are available (see M. D. Miller and Beers) which can be used; these other methods may not reproduce the five-year total exactly, but may approximate such, and are preferable for smoothing death rates and for other distributions when one does not particularly care about keeping the reported 5-year totals unchanged.

These multipliers can be used on a great variety of data besides age distributions, provided that the data are continuous, and quantitative variables are being studied. Thus, income distributions can be graduated. Hours worked per week, on the other hand, can be considered as discrete—or semidiscrete—data since there are known peaks at 40 hours and 48 hours—peaks which must not be smoothed out if the results are to have any meaning. These multipliers are purely mathematical in their approach, and do not contain within themselves any means of taking into consideration, irregular but true (nonerror) fluctuations in the basic data.

Detailed Procedures.—Five sets of multipliers[4] are needed, one for smoothing the "mid-panels," one for each of the "end-panels," and one for each of the "next-to-end-panels." In terms of an ordinary age distribution ranging from age 0 to, say, age 99 by 5-year intervals, the above terminology has the following meaning:

Age:	
0 to 4	First end-panel.
5 to 9	First next-to-end-panel.
10 to 14 etc. to 85 to 89	Mid-panels.
90 to 94	Last next-to-end-panel.
95 to 99	Last end-panel.

It will be noted that the last end-panel is determined by the nature of the available frequency distribution. In the event that the end-panel is an open end class interval, specific 5-year age classes must first be estimated; this can be done by graphic smoothing. In tabulating census returns, of course, it will always be possible to tabulate the data into 5-year age groups and open end classes omitted. The group aged 100 and over, can generally be treated as ranging from 100 to 104 years.

[4] The specific panels of multipliers presented below were calculated by Wilson H. Grabill of the U.S. Bureau of the Census.

The *notation* used is as follows:

N_x=number of persons enumerated by the census in any 5-year age group.

n_x=estimated number of persons at the calculated single year of age.

The *mid-panel* multipliers are as follows:

	N_1	N_2	N_3	N_4	N_5	Sum
n_1	−.0128	+.0848	+.1504	−.0240	+.0016	+.2000
n_2	−.0016	+.0144	+.2224	−.0416	+.0064	+.2000
n_3	+.0064	−.0336	+.2544	−.0336	+.0064	+.2000
n_4	+.0064	−.0416	+.2224	+.0144	−.0016	+.2000
n_5	+.0016	−.0240	+.1504	+.0848	−.0128	+.2000

N_3 always is the 5-year age group which is being graduated into single years of age. N_2 is the first younger 5-year age group, and N_1 is the second younger group. N_4 is the first older class, and N_5 the second older. Thus, for example, in estimating single years of age for the class 25 to 29 years:

N_1=15 to 19 years
N_2=20 to 24 years
N_3=25 to 29 years
N_4=30 to 34 years
N_5=35 to 39 years

These multipliers are then applied as in table B–1.

TABLE B–1.—*Estimating single years of age, using mid-panel of the Sprague multipliers, age 20 to 24 years taken as an example*

(Data for native white males, United States, 1940; in thousands)

	Line No.	N_1 (age 10 to 14)	N_2 (age 15 to 19)	N_3 (age 20 to 24)	N_4 (age 25 to 29)	N_5 (age 30 to 34)	Sum
1	Reported number	1,441	1,518	1,439	1,317	1,157	....
2	Estimating age:						
3	20 (line 1×line 9)	−18.4	+128.7	+216.4	−31.6	+1.9	297.0
4	21 (line 1×line 10)	−2.3	+21.9	+320.0	−54.8	+7.4	292.2
5	22 (line 1×line 11)	+9.2	−51.0	+366.1	−44.3	+7.4	287.4
6	23 (line 1×line 12)	+9.2	−63.1	+320.0	+19.0	−1.9	283.2
7	24 (line 1×line 13)	+2.3	−36.4	+216.4	+111.7	−14.8	279.2
8	Sum, age 20 to 24						1,439.0
		Multipliers					
9	n_1 (age 20)	−.0128	+.0848	+.1504	−.0240	+.0016	+.2000
10	n_2 (age 21)	−.0016	+.0144	+.2224	−.0416	+.0064	+.2000
11	n_3 (age 22)	+.0064	−.0336	+.2544	−.0336	+.0064	+.2000
12	n_4 (age 23)	+.0064	−.0416	+.2224	+.0144	−.0016	+.2000
13	N_5 (age 24)	+.0016	−.0240	+.1504	+.0848	−.0128	+.2000

The *end-panel* multipliers are as follows:

First end-panel

	N_1	N_2	N_3	N_4	Sum
n_1	+.3616	−.2768	+.1488	−.0336	+.2000
n_2	+.2640	−.0960	+.0400	−.0080	+.2000
n_3	+.1840	+.0400	−.0320	+.0080	+.2000
n_4	+.1200	+.1360	−.0720	+.0160	+.2000
n_5	+.0704	+.1968	−.0848	+.0176	+.2000

Last end-panel

	N_1	N_2	N_3	N_4	Sum
n_1	+.0176	−.0848	+.1968	+.0704	+.2000
n_2	+.0160	−.0720	+.1360	+.1200	+.2000
n_3	+.0080	−.0320	+.0400	+.1840	+.2000
n_4	−.0080	+.0400	−.0960	+.2640	+.2000
n_5	−.0336	+.1488	−.2768	+.3616	+.2000

The N notation is as follows, assuming that ages 0 to 4 and 95 to 99 are respectively, the first and last end panels (if other classes are at the ends of the frequency distribution make the appropriate substitutions):

$N_1=0$ to 4 years and 80 to 84 years
$N_2=5$ to 9 years and 85 to 89 years
$N_3=10$ to 14 years and 90 to 94 years
$N_4=15$ to 19 years and 95 to 99 years

These multipliers are then applied as in table B–2.

The next-to-the-end-panel multipliers are:

First next-to-the-end-panel

	N_1	N_2	N_3	N_4	Sum
n_1	+.0336	+.2272	−.0752	+.0144	+.2000
n_2	+.0080	+.2320	−.0480	+.0080	+.2000
n_3	−.0080	+.2160	−.0080	+.0000	+.2000
n_4	−.0160	+.1840	+.0400	−.0080	+.2000
n_5	−.0176	+.1408	+.0912	−.0144	+.2000

Last next-to-the-end-panel

	N_1	N_2	N_3	N_4	Sum
n_1	−.0144	+.0912	+.1408	−.0176	+.2000
n_2	−.0080	+.0400	+.1840	−.0160	+.2000
n_3	+.0000	−.0080	+.2160	−.0080	+.2000
n_4	+.0080	−.0480	+.2320	+.0080	+.2000
n_5	+.0144	−.0752	+.2272	+.0336	+.2000

The N notation is as follows, assuming that ages 5 to 9 and 90 to 94 are, respectively the first and last next-to-the-end-panels:

$N_1=0$ to 4 years and 80 to 84 years
$N_2=5$ to 9 years and 85 to 89 years
$N_3=10$ to 14 years and 90 to 94 years
$N_4=15$ to 19 years and 95 to 99 years

These multipliers are then applied as in table B–3.

It should be noted that the multipliers for the end and next-to-the-end-panels are based on osculatory fourth difference interpolation. This method ties in smoothly with the procedures based on the fifth difference osculatory formula (the mid-panels multipliers), but does not present quite as satisfactory results as the latter procedures. This results from the fact that, in processing the mid-panels, the data for the two younger and two older age groups are taken into account;

TABLE B–2.—*Estimating single years of age using end-panels of the Sprague multipliers*

(Data for native white males, United States, 1940; in thousands)

Line No.		First end panel				Sum
		N_1 (age 0 to 4)	N_2 (age 5 to 9)	N_3 (age 10 to 14)	N_4 (age 15 to 19)	
1	Reported numbers	1,165	1,235	1,441	1,518	....
2	Estimating age:					
3	0 (line 1 × line 9)	+421.3	−341.8	+214.4	−51.0	242.9
4	1 (line 1 × line 10)	+307.6	−118.6	+57.6	−12.1	234.5
5	2 (line 1 × line 11)	+214.4	+49.4	−46.1	+12.1	229.8
6	3 (line 1 × line 12)	+139.8	+168.0	−103.8	+24.3	228.3
7	4 (line 1 × line 13)	+82.0	+243.0	−122.2	+26.7	229.5
8	Sum, age 0 to 4					1,165.0
		Multipliers				
9	n_1 (age 0)	+.3616	−.2768	+.1488	−.0336	+.2000
10	n_2 (age 1)	+.2640	−.0960	+.0400	−.0080	+.2000
11	n_3 (age 2)	+.1840	+.0400	−.0320	+.0080	+.2000
12	n_4 (age 3)	+.1200	+.1360	−.0720	+.0160	+.2000
13	n_5 (age 4)	+.0704	+.1968	−.0848	+.0176	+.2000

Line No.		Last end panel				Sum
		N_1 (age 0 to 4)	N_2 (age 5 to 9)	N_3 (age 10 to 14)	N_4 (age 15 to 19)	
1	Reported numbers	59.4	19.6	4.0	.7	....
2	Estimating age:					
3	95 (line 1 × line 9)	+1.05	−1.66	+.79	+.05	.23
4	96 (line 1 × line 10)	+.95	−1.41	+.54	+.08	.16
5	97 (line 1 × line 11)	+.48	−.63	+.16	+.13	.14
6	98 (line 1 × line 12)	−.48	+.78	−.38	+.18	.10
7	99 (line 1 × line 13)	−2.00	+2.92	−1.11	+.25	.06
8	Sum, age 95 to 99					.69
		Multipliers				
9	n_1 (age 95)	+.0176	−.0848	+.1968	+.0704	+.2000
10	n_2 (age 96)	+.0160	−.0720	+.1360	+.1200	+.2000
11	n_3 (age 97)	+.0080	−.0320	+.0400	+.1840	+.2000
12	n_4 (age 98)	−.0080	+.0400	−.0960	+.2640	+.2000
13	n_5 (age 99)	−.0336	+.1488	−.2768	+.3616	+.2000

this provides an equal amount of information on each side of the 5-year age group being graduated into 1-year age groups.

When smoothing the end and next-to-the-end-panels, however, equal amounts of information on each side are not available—only "lopsided" information. In the case of the end-panels data can be had only for the following or preceding classes, never for both. In the case of the next-to-the-end-panels information can be had only for one age class on one side (prior to or following the panel), and several classes on the other side. Because of this "lop-sidedness" it is sometimes preferable to use a graduation formula other than Sprague's multipliers for these two types of end-panels.

TABLE B-3.—*Estimating single years of age using next-to-end panels of the Sprague multipliers*

(Data for native white males, United States, 1940; in thousands)

Line No.	First next-to end panel				Sum
	N_1 (age 0 to 4)	N_2 (age 5 to 9)	N_3 (age 10 to 14	N_4 (age 15 to 19)	
1 Reported numbers	1,165	1,235	1,441	1,518	....
2 Estimating age:					
3 5 (line 1 × line 9)	+39.1	+280.6	−108.4	+21.9	233.2
4 6. (line 1 × line 10)	+9.3	+286.5	−69.2	+12.1	238.7
5 7. (line 1 × line 11)	−9.3	+266.8	−11.5	+00.0	246.0
6 8. (line 1 × line 12)	−18.6	+277.2	+57.6	−12.1	254.1
7 9. (line 1 × line 13)	−20.5	+173.9	+131.4	−21.9	262.9
8 Sum, age 5 to 9					1,234.9
	Multipliers				
9 n_1 (age 5)	+.0336	+.2272	−.0752	+.0144	+.2000
10 n_2 (age 6)	+.0080	+.2320	−.0480	+.0080	+.2000
11 n_3 (age 7)	−.0080	+.2160	−.0080	+.0000	+.2000
12 n_4 (age 8)	−.0160	+.1840	+.0400	−.0080	+.2000
13 n_5 (age 9)	−.0176	+.1408	+.0912	−.0144	+.2000

Line No.	Last next-to-end panel				Sum
	N_1 (age 80 to 84	N_2 (age 85 to 89)	N_3 (age 90 to 94)	N_4 (age 95 to 99)	
1 Reported numbers	59.4	19.6	4.0	.7	....
2 Estimating age:					
3 90 (line 1 × line 9)	−.86	+1.79	+.56	−.01	1.48
4 91 (line 1 × line 10)	−.48	+.78	+.74	−.01	1.03
5 92 (line 1 × line 11)	+.00	−.16	+.86	−.01	.69
6 93 (line 1 × line 12)	+.48	−.94	+.93	−.01	.48
7 94 (line 1 × line 13)	+.86	−1.47	+.91	+.02	.32
8 Sum, age 90 to 94					4.00
	Multipliers				
9 n_1 (Age 90)	−.0144	+.0912	+.1408	−.0176	+.2000
10 n_2 (Age 91)	−.0080	+.0400	+.1840	−.0160	+.2000
11 n_3 (Age 92)	+.0000	−.0080	+.2160	−.0080	+.2000
12 n_4 (Age 93)	+.0080	−.0480	+.2320	+.0080	+.2000
13 n_5 (Age 94)	+.0144	−.0752	+.2272	+.0336	+.2000

Calculating Rates for Single Years of Age.—Sometimes reported rates—birth and death, marriage, labor force participation, etc.—whether reported by 5-year groups, or single years of

TABLE B-4.—*Estimating percent of single marital status, by single years of age, using Sprague mid-panel multipliers, ages 15 to 19 and 20 to 24 years taken as examples*

(Data for white males, United States; 1940)

Line No.	Age 15 to 19					Sum
	N_1 (age 5 to 9)	N_2 (age 10 to 14)	N_3 (age 15 to 19)	N_4 (age 20 to 24)	N_5 (age 25 to 29)	
1 Reported percentages single	100.0	100.0	98.4	73.5	36.7	....
2 Times 5 (line 1 × 5)	500.0	500.0	492.0	367.5	183.5	....
3 Estimating percent single for age:						
4 15 (line 2 × line 11)	−6.4	+42.4	+74.0	−8.8	+.3	101.5
5 16 (line 2 × line 12)	−.8	+7.2	+109.4	−15.3	+1.2	101.7
6 17 (line 2 × line 13)	+3.2	−16.8	+125.2	−12.3	+1.2	100.5
7 18 (line 2 × line 14)	+3.2	−20.8	+109.4	+5.3	−.3	96.8
8 19 (line 2 × line 15)	+.8	−12.0	+74.0	+31.2	−2.3	91.7
9 Sum, age 15 to 19						492.2
	Multipliers					
10 n_1	−.0128	+.0848	+.1504	−.0240	+.0016	+.2000
11 n_2	−.0016	+.0144	+.2224	−.0416	+.0064	+.2000
12 n_3	+.0064	−.0336	+.2544	−.0336	+.0064	+.2000
13 n_4	+.0064	−.0416	+.2224	+.0144	−.0016	+.2000
14 n_5	+.0016	−.0240	+.1504	+.0848	−.0128	+.2000

Line No.	Age 20 to 24					Sum
	N_1 (age 10 to 14)	N_2 (age 15 to 19)	N_3 (age 20 to 24)	N_4 (age 25 to 29)	N_5 (age 30 to 34)	
15 Reported percentages single	100.0	98.4	73.5	36.7	20.7	....
16 Times 5 (line 15 × 5)	500.0	492.0	367.5	183.5	103.5	....
17 Estimating percent single for age:						
18 20 (line 16 × line 10)	−6.4	+41.7	+55.3	−4.4	+.2	86.4
19 21 (line 16 × line 11)	−.8	+7.1	+81.7	−7.6	+.7	81.1
20 22 (line 16 × line 12)	+3.2	−16.5	+93.5	−6.2	+.7	74.7
21 23 (line 16 × line 13)	+3.2	−20.5	+81.7	+2.6	−.2	66.8
22 24 (line 16 × line 14)	+.8	−11.8	+55.3	+15.6	−1.3	58.6
23 Sum, age 20 to 24						367.6

age, are too inaccurate to be employed for analytical purposes, without smoothing. Death rates to be employed in calculating life tables should be graduated by a refined formulae (see M. D. Miller); the Sprague multipliers can be used for most other smoothing or estimating purposes.

In calculating rates by single years of age, the Sprague multipliers are used exactly as described previously. The different sets for "mid-panels," "end-panels," and "next-to-end-panels," are all employed in the same manner as for smoothing the population. The only variation introduced is that the original rate for a 5-year group is multiplied by 5, after which the standard multipliers are applied. This procedure using a "mid-panel" is illustrated in table B–4.

It will be noted (from table B–4) that these multipliers fail in attempting to estimate the percent single in the teen ages, for individual years. Obviously, over 100 percent of the population in each of the ages 15, 16, and 17 cannot be single. The ages 20 to 24, by contrast, appear both smooth and plausible. Applying the multipliers for the first end panel (as is done in table B–5) does not appear to produce any better graduation for the age 15 to 19. Accordingly, this age group must be smoothed by some other method. Graphic smoothing can be used and this is shown in chart B–1. Age 15 is plotted as 100 percent and age 20 as 86.4 percent (this value is obtained from table B–4). A French curve is passed through these two points and the following values read off for ages.

15	100.0
16	99.9
17	99.3
18	97.8
19	95.0
Total	492.0

Use of the Halves and Tenths Multipliers.

The several sets of multipliers are given in table B–7. These are used in exactly the same manner as are the fifths. We may illustrate the use of the halves, mid-panel, as in table B–6.

TABLE B–5.—*Estimating percent of single marital status by single years of age, using Sprague first end-panel multipliers, age 15 to 19 years taken as example*

Data for white males, United States; 1940)

Line No.		N_1 (age 15 to 19)	N_2 (age 20 to 24)	N_3 (age 25 to 29)	N_4 (age 30 to 34)	Sum
1	Reported percentages single	98.4	73.5	36.7	20.7	....
2	Times 5 (line 1×5)	492.0	367.5	183.5	103.5	....
3	Estimating percent single for age:					
4	15 (line 2×line 10)	+177.9	−101.7	+27.3	−3.5	100.0
5	16 (line 2×line 11)	+120.9	−35.3	+7.3	−.8	101.1
6	17 (line 2×line 12)	+90.5	+14.7	−5.9	+.8	100.1
7	18 (line 2×line 13)	+59.0	+50.0	−13.2	+1.7	97.5
8	19 (line 2×line 14)	+34.6	+72.3	−15.6	+1.8	93.1
9	Sum, age 15 to 19					491.8
		Multipliers				
10	n_1	+.3016	−.2768	+.1488	−.0336	+.2000
11	n_2	+.2640	−.0960	+.0400	−.0080	+.2000
12	n_3	+.1840	+.0400	−.0320	+.0080	+.2000
13	n_4	+.1200	+.1360	−.0720	+.0160	+.2000
14	n_5	+.0704	+.1068	−.0348	+.0176	+.2000

TABLE B–6.—*Estimating 5-year age periods, using mid-panel of the Sprague multipliers, males age 45 to 54 years taken as an example*

(Data for unemployed males, United States, September 1966, in thousands)

Line No.		N_1 (age 25 to 34)	N_2 (age 35 to 44)	N_3 (age 45 to 54)	N_4 (age 55 to 64)	N_5 (age 65 to 74)	Sum
1	Reported number	172	156	178	134	62	....
2	Estimating age:						
3	45 to 49 (line 1 × line 6)	−2.01563	+13.40625	+89.00000	−11.51563	+0.72656	89.60155
4	50 to 54 (line 1 × line 7)	+2.01563	−13.40625	+89.00000	+11.51563	−0.72656	88.39845
5	Sum, age 45 to 54						178.00000
		Multipliers					
6	n_1 (age 45 to 49)	−.01171875	−.08593750	+.50000000	−.08593750	+.01171875	+.50000000
7	n_2 (age 50 to 54)	+.01171875	+.08593750	+.50000000	+.08593750	−.01171875	+.50000000

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CHART B-1.—Graphic Method for Estimating Percent Single, Ages 15 to 19 by Single Years, White Males, United States, 1940

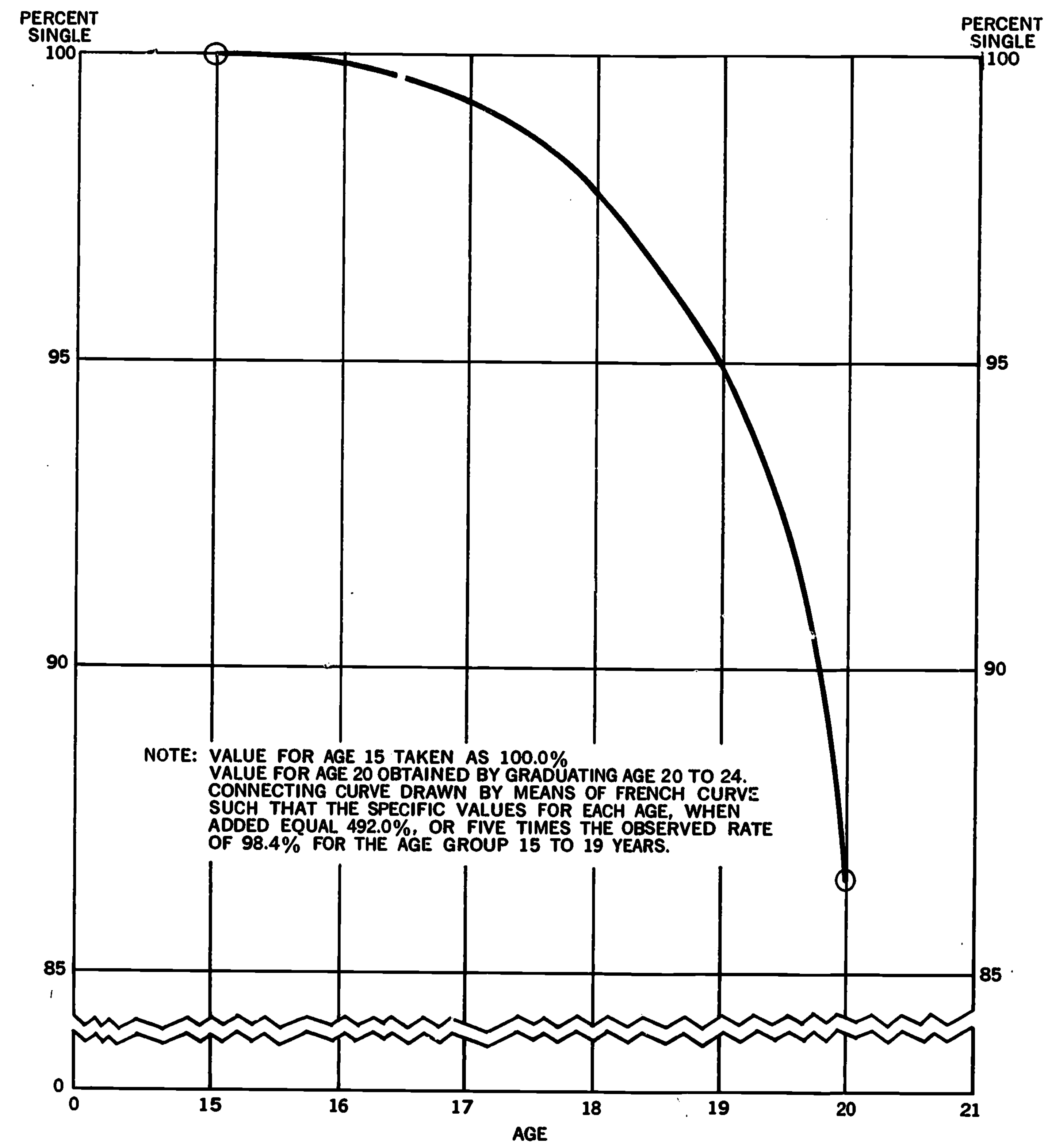

TABLE B-7.—*Coefficients for 3rd degree polynomial area interpolation*

Interval pattern: ⑥—5—5—5

(N_0 always is the category being subdivided)

Areas resulting from the subdivision of the interval containing N_0 into:	N_0	N_1	N_2	N_3
Tenths:				
n_1	+.19416250	−.16398750	+.09048750	−.02066250
n_2	+.16743750	−.11281250	+.05831250	−.01293750
n_3	+.14306250	−.06768750	+.03118750	−.00656250
n_4	+.12093750	−.02831250	+.00881250	−.00143750
n_5	+.10096250	+.00561250	−.00911250	+.00253750
n_6	+.08303750	+.03438750	−.02288750	+.00540250
n_7	+.06706250	+.05831250	−.03281250	+.00743750
n_8	+.05203750	+.07768750	−.03918750	+.00856250
n_9	+.04056250	+.09281250	−.04231250	+.00893750
n_{10}	+.02983750	+.10398750	−.04248750	+.00866250
Total	1	0	0	0
Halves:				
n_1	+.72656250	−.36718750	+.17968750	−.03906250
n_2	+.27343750	+.36718750	−.17968750	+.03906250

Interval pattern: 5—⑥—5—5

Areas resulting from the subdivision of the interval containing N_0 into:	N_{-1}	N_0	N_1	N_2
Tenths:				
n_1	+.02066250	+.11151250	−.04001250	+.00783750
n_2	+.01293750	+.11568750	−.03518750	+.00656250
n_3	+.00656250	+.11681250	−.02831250	+.00493750
n_4	+.00143750	+.11518750	−.01968750	+.00306250
n_5	−.00253750	+.11111250	−.00961250	+.00103750
n_6	−.00546250	+.10488750	+.00161250	−.00103750
n_7	−.00743750	+.09681250	+.01368750	−.00306250
n_8	−.00856250	+.08718750	+.02631250	−.00493750
n_9	−.00893750	+.07631250	+.03918750	−.00656250
n_{10}	−.00866250	+.06448750	+.05201250	−.00783750
Total	0	1	0	0
Halves:				
n_1	+.03906250	+.57031250	−.13281250	+.02343750
n_2	−.03906250	+.42968750	+.13281250	−.02343750

Interval pattern: 5—5—⑥—5

Areas resulting from the subdivision of the interval containing N_0 into:	N_{-2}	N_{-1}	N_0	N_1
Tenths:				
n_1	−.00783750	+.05201250	+.06448750	−.00866250
n_2	−.00656250	+.03918750	+.07631250	−.00893750
n_3	−.00493750	+.02631250	+.08718750	−.00856250
n_4	−.00306250	+.01368750	+.09681250	−.00743750
n_5	−.00103750	+.00161250	+.10488750	−.00546250
n_7	+.00103750	−.00961250	+.11111250	−.00253750
n_8	+.00306250	−.01968750	+.11518750	+.00143750
n_8	+.00493750	−.02831250	+.11681250	+.00656250
n_9	+.00656250	−.03518750	+.11568750	+.01293750
n_{10}	+.00783750	−.04001250	+.11151250	+.02066350
Total	0	0	1	0
Halves:				
n_1	−.02343750	+.13281250	+.42968750	−.03906250
n_2	+.02343750	−.13281250	+.57031250	+.03906250

Interval pattern: 5—5—5—⑥

Areas resulting from the subdivision of the interval containing N_0 into:	N_{-3}	N_{-2}	N_{-1}	N_0
Tenths:				
n_1	+.00866250	−.04248750	+.10398750	+.02983750
n_2	+.00893750	−.04231250	+.09281250	+.04056250
n_3	+.00856250	−.03918750	+.07768750	+.05203750
n_4	+.00743750	−.03281250	+.05831250	+.06706250
n_5	+.00540250	−.02288750	+.03438750	+.08303750
n_6	+.00253750	−.00911250	+.00561250	+.10096250
n_7	−.00143750	+.00881250	−.02831250	+.12093750
n_8	−.00656250	+.03118750	−.06768750	+.14306250
n_9	−.01293750	+.05831250	−.11281250	+.16743750
n_{10}	−.02066250	+.09048750	−.16398750	+.19416250
Total	0	0	0	1
Halves:				
n_1	+.03906250	−.17968750	+.36718750	+.27343750
n_2	−.03906250	+.17968750	−.36718750	+.72656250

Interval pattern: 5—5—⑥—5—5

Areas resulting from the subdivision of the interval containing N_0 into:	N_{-2}	N_{-1}	N_0	N_1	N_2
Tenths:					
n_1	−.00759375	+.05103750	+.06595000	−.00963750	+.00024375
n_2	−.00520625	+.03376250	+.08445000	−.01436250	+.00135625
n_3	−.00220625	+.01538750	+.10357500	−.01948750	+.00273125
n_4	+.00060625	−.00098750	+.11882500	−.02211250	+.00366875
n_5	+.00268125	−.01326250	+.12720000	−.02033750	+.00371875
n_6	+.00371875	−.02033750	+.12720000	−.01326250	+.00268125
n_7	+.00366875	−.02211250	+.11882500	−.00098750	+.00060625
n_8	+.00273125	−.01948750	+.10357500	+.01538750	−.00220625
n_9	+.00135625	−.01436250	+.08445000	+.03376250	−.00520625
n_{10}	+.00024375	−.00963750	+.06595000	+.05103750	−.00759375
Total	0	0	1	0	0
Halves:					
n_1	−.01171875	+.08593750	+.50000000	−.08593750	+.01171875
n_2	+.01171875	−.08593750	+.50000000	+.08593750	−.01171875

APPENDIX C

FITTING LINES

Least Squares

The method of least squares may be the most frequently used procedures for fitting a line to a time series. Both straight and curvilinear lines can be fitted by these procedures. We shall review the simplest procedures for fitting both types of lines here.

Straight Line

The basic formula is: $Y=a+bX$.
X is the year and Y is the number enrolled that year. We need two equations to fit the line:

$$\text{Sum } Y=na+b\text{Sum}X$$

$$\text{Sum } XY=a\text{Sum}X+b\text{Sum}X^2$$

Let us illustrate the procedures with the data in table C–1. Since the X values are all large, 1956, 1957, etc., we can simplify the calculations very much by adopting an Arbitrary Origin. We designate the middle year of the series, 1962 in this case, as zero. The year 1961 then becomes −1, 1960 becomes −2, etc.; 1963 is +1, 1964 is +2, etc. These values are shown in col. b of table C–1.

The observed numbers of elementary school students are shown in col. c. We compute cols. d and e as shown in table C–1. We then have the following equations:

$$398.2=13a+0\,b$$

$$108.0=0\,a+182\,b$$

a=398.2 divided by 13=30.63

b=108.0 divided by 182=0.593

We can now use these values to compute the calculated (as contrasted to the observed) Y. Thus, 1956 equals: 30.63+0.593 (−6)=27.07.

We can use these same values for projection purposes. Assume that we wish to project to 1975. This year is 13 years after the arbitrary origin of 1962. Therefore: 1975 equals: 30.63+0.593(13) =38.34.

TABLE C–1.—*Procedures for fitting a straight line to elementary school enrollment data: United States, 1956 to 1968*

(a)	(b)	(c)	(d)=(b×c)	(e)	(f)	(g)=(c–f)	
Year	X	Y observed (elementary school enrollment, in millions)	XY	$(X)^2$	Y calculated	Y observed minus Y calculated	
						Difference	Difference squared
1956	−6	26.2	−157.2	36	27.07	−0.87	.7569
1957	−5	27.2	−136.0	25	27.67	−0.47	.2209
1958	−4	28.2	−112.8	16	28.26	−0.06	.0036
1959	−3	29.4	−88.2	9	28.85	+0.55	.3025
1960	−2	30.3	−60.6	4	29.44	+0.86	.7396
1961	−1	30.7	−30.7	1	30.04	+0.66	.4356
1962	0	30.7	00.0	0	30.63	+0.07	.0049
1963	1	31.2	+31.2	1	31.22	−0.02	.0004
1964	2	31.7	+63.4	4	31.82	−0.12	.0144
1965	3	32.5	+97.5	9	32.41	+0.09	.0081
1966	4	32.9	+131.6	16	33.00	−0.10	.0100
1967	5	33.4	+167.0	25	33.60	−0.20	.0400
1968	6	33.8	+202.8	36	34.19	−0.39	.1521
Sum	0	398.2	+108.0	182	398.2	0.00	2.6890

This simplified approach is based on an odd number of years, 13 in this example. If we had an even number of years, the same procedure can be used with the following modification. Suppose we have 10 years; then:

Year:	X	Year:	X
1959	−4.5	1964	+0.5
1960	−3.5	1965	+1.5
1961	−2.5	1966	+2.5
1962	−1.5	1967	+3.5
1963	−0.5	1968	+4.5

Curvilinear Line [1]

The basic formula is $Y=a+bX+cX^2$

The procedures for fitting a curvilinear line are shown in table C–2. Note that columns a

[1] There are other versions of the curvilinear line but this simple one will suffice for present purposes. For further information see statistical texts such as: Mordecai J. B. Ezekiel and Karl Fox, *Methods of Correlation and Regression Analysis*, John Wiley and Sons, New York, p. 77 ff; Samuel B. Richmond, *Statistical Analysis*, 2nd Ed., The Ronald Press, New York. pp. 358-366, 1964. F. E. Croxton and D. J. Cowden, *Practical Business Statistics* (various editions).

Table C-2.—*Procedures for fitting a curvilinear line to elementary school enrollment data: United States, 1956 to 1968*

(a) Year	(b) X	(c) Y observed (elementary school enrollment, in millions)	(d)=(b×c) XY	(e) X^3	(f)=(c×e) X^2Y	(g) X^4	(h) Y calculated	(i)=(c−h) Y observed minus Y calculated: Difference	Y observed minus Y calculated: Difference squared
1956	−6	26.2	−157.2	36	943.2	1296	26.47	−0.27	.0729
1957	−5	27.2	−136.0	25	680.0	625	27.36	−0.16	.0256
1958	−4	28.2	−112.8	16	451.2	256	28.20	0.00	.0000
1959	−3	29.4	−88.2	9	264.6	81	29.00	+0.40	.1600
1960	−2	30.3	−60.6	4	121.2	16	29.71	+0.59	.3481
1961	−1	30.7	−30.7	1	30.7	1	30.39	+0.31	.0961
1962	0	30.7	00.0	0	0.0	0	31.01	−0.31	.0961
1963	1	31.2	+31.2	1	31.7	1	31.58	−0.38	.1444
1964	2	31.7	+63.4	4	126.8	16	32.09	−0.39	.1521
1965	3	32.5	+97.5	9	292.5	81	32.54	−0.04	.0016
1966	4	32.9	+131.6	16	526.4	256	32.95	−0.05	.0025
1967	5	33.4	+167.0	25	835.0	625	33.29	+0.11	.0121
1968	6	33.8	+202.8	36	1216.8	1296	33.59	+0.21	.0441
Sum		398.2	+108.0	192	5519.6	4550	398.20	0.00	1.1556

through e are identical to those in table C–1. We then compute columns f and g and have;

Sum $Y=na+b$ Sum $X+c$ Sum X^2
$398.2=13\,a+0\,b+182\,c$ (1)

Sum $XY=a$ Sum $X+b$ Sum X^2+c Sum X^3
$108.0=0\ a+182\ b+0c$ (2)

Sum $X^2Y=a$ Sum X^2+b Sum $X^3+\ c$ Sum X^4
$5519.6=182\ a+0\ b+4550c$ (3)

We then solve the equations as follows:

$b=108.0$ divided by $182=0.593$

Divide equation 1 by 13: $30.63=a+14.00\ c$

Divide equation 3 by 182: $30.33=a+25.00\ c$

Subtracting the second line from the first we have:

$$0.30=-11.0\ c$$

$$c=-0.0273$$

a then equals: $30.63-14(-0.0273)=31.01$

We can now use these values to compute the calculated Y. 1956 thus equals:

$31.01+[0.593(-6)]+[-0.0273(36)]=26.47$

We can project to 1975 as follows:

$31.01+[0.593(13)]+[-0.0273(169)]=34.11$

Comparison of the Two Lines

One way of estimating the goodness of fit of a line is to take the difference between the observed and calculated values, square, and sum. For the straight line the sum is 2.6890 and for the curvilinear line, 1.1556. Dividing each by the number of years (13) and taking the square root we have 0.455 for the straight line and 0.298 for the curvilinear. Obviously the curvilinear line gives a better fit.

When we compare the projections for 1975, however, the two lines give quite different figures. The straight line projects an enrollment of 38.34 millions whereas the curvilinear line projects to 34.11 millions of pupils enrolled in elementary school. Which may be more nearly correct? For a possible answer we can turn to the U.S. Census Bureau projections for 1975 (Series P–25, No. 388) where we find: there were 40.2 millions of children aged 5 to 14 in 1966; in 1975 there is expected to be a maximum of 42.2 millions, and a minimum of 38.5 millions in this age. In light of the anticipated small population growth or possibly even declino, we think that the lower enrollment projection supplied by the curvilinear line may be more nearly correct.

Triple Exponential Smoothing

Introduction

In chapter 7 a number of projections of local area shares of total State enrollment for California were made through the use of triple exponential smoothing. In this appendix we shall illustrate both the manual and computer mechanics for fitting this type of equation.

The mathematics for multiple exponential smoothing is given in R. G. Brown, *Smoothing, Forecast-*

ing, and Prediction of Discreet Time Series, Prentice Hall, New Jersey, 1963.

This type of equation can be fitted by a computer or manually. Considering the large amount of work involved, the former is preferable.

The Computer Program

The computer program shown is an IBM Application Program, System 1360 Scientific Subroutine Package (360A-CM-03X) Version III, Programmer's Manual, IBM, Technical Publications Department, 112 East Post Road, White Plains, New York, 10601. The reader who may wish further information may write to the Bureau of Applied Social Research, Columbia University; every effort will be made to supply such information applicable on an IBM 360 computer. Following is the IBM description:

Problem Description.—Given a time series X, a smoothing constant, and three coefficients of the prediction equation, this sample program finds the triple exponentially smoothed series S of the time series X.

Program Description.—The sample program for triple exponential smoothing consists of the main program named EXPON, and one subroutine, EXSMO, from the Scientific Subroutine Package

Capacity.—The capacity of the sample prog.ram and the format required for data input have been set up as follows:

1. Up to 1,000 data points in a given time series
2. (12F 6.0) format for input data cards

Therefore, if a problem satisfies the above conditions, the sample program need not be modified. However, if there are more than 1,000 data points, the dimension statement in the sample main program must be modified to handle this particular problem. Similarly, if input data cards are prepared using a different format, the input format in the sample main program must be modified. The general rules for program modification are described later.

Input.— Control Card.—One control card is required for each problem and is read by the main program, EXPON. This card is prepared as follows:

Columns	Contents	For sample problem
1- 6	Problem number (may be alphameric)	Sample
7-10	Number of data points in a given time series	0038
11-15	Smoothing constant, α ($0.0<\alpha<1.0$)	0.1
16-25	First coefficient (A) of the prediction equation	0.0
26-35	Second coefficient (B) of the prediction equation	0.0
36-45	Third coefficient (C) of the prediction equation	0.0

Smoothing constant and three coefficients must be keypunched with decimal points.

Leading zeros are not required to be keypunched.

Data Card.—Time series data are keypunched using the format (12F6.0). This format assumes that each data point is keypunched in a 6-column field and 12 fields per card.

Deck Setup.—The deck setup is shown in Figure 62. (see p. 89)[1]

Sample.—The listing of input cards for the sample problem is presented in figure 63, p. 89.[1]

Output

Description.—The output of the sample program for triple exponential smoothing includes:

1. Original and updated coefficients
2. Time series as input and triple exponentially smoothed time series

Example.—The output listing for the sample problem is shown in figure 64, page 91.[1]

Program Modification.—Program capacity can be increased or decreased by making changes in the dimension statement. Input data in a different format can also be handled by providing a specific format statement. In order to familiarize the user with the program modification, the following general rules are supplied in terms of the sample problem.

1. Changes in the dimension statement of the main program, EXPON:

The dimension of arrays X and S must be greater than or equal to the number of data points in time series, NX. Since there are 38 data points in the sample problem, the value of NX is 38.

2. Changes in the input format statement of the main program, EXPON:

Only the format statement for input data may be changed. Since sample data are three-digit numbers, rather than using six-column fields as in the sample problem, each data point may be keypunched in a three-column field and 24 fields per card. If so, the format is changed to (24F3.0).

Operating Instructions.—The sample program for triple exponential smoothing is a standard FORTRAN program. Special operating instructions are not required. Data set 5 is used for input, and data set 6 is used for output.

Timing.—The execution time of this sample program on a System/360, Model 30, using an IBM 2540 Card Reader as input and an IBM 1403, Model 3 as output, is 12 seconds.

[1] The Nos. 62, 63, and 64 used here reflect the numbering of figures in the original IBM document.

Subroutine EXSMO.—This subroutine calculates a smoothed series $S_1, S_2, \ldots, S_{NX}$, given time series $X_1, X_2, \ldots, X_{NX}$ and a smoothing constant α. Also, at the end of the computation, the coefficients A, B, and C are given for the expression $A+B(T)+C(T)^2/2$. This expression can be used to find estimates of the smoothed series for a given number of time periods, T, ahead.

The subroutine has the following two stages for i=1, 2, . . . , NX, starting with A, B, and C either given by the user or provided automatically by the subroutine which follows:

(*a*) Find S_t for one period ahead:

$$S_t=A+B+0.5C \quad (1)$$

(*b*) Update coefficients A, B, and C:

$$A=X_t+(1-\alpha)^3\,(S_t-X_t) \quad (2)$$

$$B=B+C-1.5(\alpha^2)(2-\alpha)(S_t-X_t) \quad (3)$$

$$C=C-(\alpha^3)(S_t-X_t) \quad (4)$$

where α=smoothing constant specified by the user

$$(0.0<\alpha<1.0)$$

If coefficients A, B, and C are not all zero (0.0), take given values as initial values. However, if $A=B=C=0.0$, generate initial values of A, B, and C as follows:

$$C=X_1-2X_2+X_3 \quad (5)$$

$$B=X_2-X_1-1.5C \quad (6)$$

$$A=X_1-B-0.5C \quad (7)$$

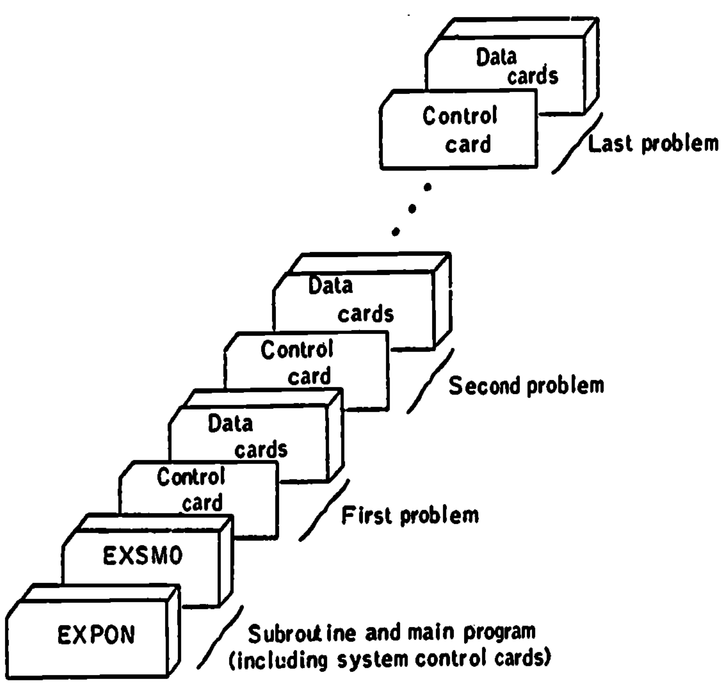

FIGURE 62

```
/DATA                                                                        10
SAMPLE  38  0.1      0.0           0.0          0.0                          20
   430  426   422  419   414  413   412  409   411  417  422  430            30
   438  441   447  455   461  453   448  449   454  463  470  472            40
   476  481   483  487   491  492   485  486   482  479  479  476            50
   472  470                                                                  60
```

FIGURE 63

Manual Application

The manual application technique is illustrated in the following tables C–3 to C–9 inclusive. The original data are given in table C–3. The four basic steps which follow are:

1. Solution of initial coefficients of the Triple Exponential Smoothing Prediction Equation. See table C–4.

2. Derivation of initial set of smoothed series using Triple Exponential Smoothing Prediction Equation. See table C–5.

3. Solution of updating equations to derive new sets of coefficients for Triple Exponential Prediction Equation. See table C–6 and C–7.

4. Projection of Triple Exponential Smoothing Prediction Equation from year to year. See tables C–8 and C–9.

Please note that to use the procedure, three initial data points are required. After solution of the initial sets of coefficient values, the procedure consists of resolving the updating equations to generate new values of A, B and C. When the last possible repetition of the procedure has been completed, the final updated set of coefficients are projected the required number of time periods ahead to derive the enrollment share forecasts. The example shown uses a 0.50 smoothing constant. Different smoothing constants can be used on a trial and error basis until, empirically, a best smoothing constant for projection purposes is derived for prediction purposes.

TABLE C–3.—*Input data for derivation of coefficients of triple exponential smoothing constants, example of Anne Arundel County, Md., 1956 to 1964: enrollment in public schools grades 1 to 4 as proportion of total State enrollment*

Year:	*Actual enrollment shares*
1956	.0665
1957	.0672
1958	.0695
1959	.0719
1960	.0738
1961	.0756
1962	.0762
1963	.0771
1964	.0811

Source: Table E-1.

TABLE C–4.—*Solution of initial coefficients of triple exponential smoothing equation and first smoothed value, based on input data shown in table C–3.*

$C=X_1-2X_2+X_3$ $\quad X_1=.0665,\ X_2=.0672,\ X_3=.0695$

$B=X_2-X_1-1.5C$

$A=X_1-B-0.5C$

$C=.0665-2(.0672)+(.0695)$

$C=.0665-.1344+.0695$

$C=.0016$

$B=.0672-.0665-1.5(.0016)$

$B=.0672-.0665-.0024$

$B=-.0017$

$A=X_1-B-0.5C$

$A=.0665-(-.0017)-(.0008)$

$A=.0674$

$S_1=A+B+0.5C$

$S_1=.0674-.0017+.0008$

$S_1=.0665$

TABLE C–5.—*Derivation of initial set of smoothed series, based on data shown in tables C–3 and C–4*

$$E=A+BT+\frac{CT^2}{2}$$

where:

E=estimates of the smoothed series for a given number of time periods T_1 ahead

Example:

$E_0=.0674+(-.0017)(0)+\frac{.0016(0)^2}{2}$

$E_0=.0674$

$E_1=.0674+(-.0017)(1)+\frac{.0016(1)^2}{2}$

$E_1=.0674-.0017+.0008$

$E_1=.0665$

$E_2=.0674+(-.0017)(2)+\frac{.0016(2)^2}{2}$

$E_2=.0674-.0034+.0032$

$E_2=.0672$

TABLE C–6.—*Updating equations of triple exponential smoothing prediction equation, based on preceding tables*

(1) $A=X_i+\beta^3(S_i-X_i)$

(2) $B=B+C-1.5\alpha^2(2-\alpha)(S_i-X_i)$

(3) $C=C-\alpha^3(S_i-X_i)$

where α is the smoothing constant specified by the user

$(0.0<\alpha<1.0)$

S_i is the smoothed X_i

and $\beta=1-\alpha$

TABLE C–7.—*Solution of updating equations of triple exponential smoothing equation, based on preceding tables*

	X_1 (1956)	X_2 (1957)	X_3 (1958)
Anne Arundel County	.0665	.0672	.0695
	E_1	E_2	E_3
Anne Arundel County	.0674	.0665	.0672

$\alpha=.50$

$\beta=.50$

$\beta^3=.125$

$\alpha^3=.125$

$A=X_1+\beta^3(S_1-X_1)$

$A_0=.0665+(.125)(.0000)$

$A_0=.0665$

$B_0=B+C-1.5\alpha^2(2-\alpha)(S_1-X_1)$

$B_0=(-.0017)+(.0016)-1.5(.25)(.0000)$

$B_0=-.0001$

$C_0=C-\alpha^3(S_1-X_1)$

$C_0=.0016-(.125)(.0000)$

$C_0=.0016$

$S_0=.0665-.0002T+\frac{.0016T^2}{2}$

$S_0=.0665,\ S_1=.0671,\ S_2=.0693$

TABLE C-8.—*Actual and smoothed values, based on tables C-3 to C-7 inclusive, example of Anne Arundel County, Md., 1956 to 1962, enrollment in public schools grades 1 to 4 as proportion of total State enrollment*

Year	Actual enrollment shares	Smoothed enrollment shares	Percent error
1956	.0665	.0665	.0000
1957	.0672	.0672	.0000
1958	.0695	.0695	.0000
1959	.0719	.0736	.0017
1960	.0738	.0769	.0031
1961	.0756	.0786	.0030
1962	.0762	.0791	.0029

TABLE C-9.—*Updated triple exponential prediction equation coefficients, example of Anne Arundel County, Md., 1956 to 1962, enrollment in public schools grades 1 to 4 as proportion of total State enrollment*

Year	A	B	C
1956	.0665	-.0001	.0016
1957	.0672	.0015	.0016
1958	.0695	.0034	.0016
1959	.0721	.0041	.0014
1960	.0742	.0039	.0010
1961	.0760	.0028	.0006
1962	.0765	.0016	.0002

FIGURE 64.—Output listing (triple exponential smoothing).

```
TRIPLE EXPONENTIAL SMOOTHING......SAMPLE

NUMBER OF DATA POINTS    38
SMOOTHING CONSTANT     0.100

COEFFICIENTS              A              B              C

ORIGINAL                0.0            0.0            0.0

UPDATED              484.80176       1.71309        0.04166

                           SMOOTHED DATA
        INPUT DATA          (FORECAST)
        430.00000           430.00000
        426.00000           426.00000
        422.00000           422.00000
        419.00000           418.00000
        414.00000           414.29980
        413.00000           410.23950
        412.00000           407.08960
        409.00000           404.66797
        411.00000           402.22363
        417.00000           401.25049
        422.00000           402.64575
        430.00000           405.61621
        438.00000           410.71338
        441.00000           417.46948
        447.00000           423.99829
        455.00000           431.18286
        461.00000           439.43359
        453.00000           447.87866
        448.00000           452.21558
        449.00000           454.10522
        454.00000           455.80713
        463.00000           458.54614
        470.00000           463.30518
        472.00000           469.06445
        476.00000           474.09521
        481.00000           479.11035
        483.00000           484.38623
        487.00000           488.94629
        491.00000           493.50854
        492.00000           498.05444
        485.00000           501.66992
        486.00000           502.12549
        482.00000           502.44434
        479.00000           501.16724
        479.00000           498.92749
        476.00000           496.84155
        472.00000           494.00806
        470.00000           490.30420
```

FIGURE 64—Continued

```
C
C     ...............................................................................
C
C        SAMPLE MAIN PROGRAM FOR TRIPLE EXPONENTIAL SMOOTHING—EXPON
C
C        PURPOSE
C           (1) READ THE PROBLEM PARAMETER CARD AND A TIME SERIES.
C           (2) CALL THE SUBROUTINE EXSMO TO SMOOTH THE TIME SERIES.
C           AND (3) PRINT THE RESULT.
C
C        REMARKS
C           A SMOOTHING CONSTANT SPECIFIED IN THE PROBLEM PARAMETER CARD MUST BE
C           GREATER THAN ZERO BUT LESS THAN ONE IN ORDER TO OBTAIN REASONABLE
C           RESULTS.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           EXSMO
C
C        METHOD
C           REFER TO R. G. BROWN, "SMOOTHING, FORECASTING AND PREDICTION OF DISCRETE
C           TIME SERIES," PRENTICE-HALL, N.J., 1963, PP. 140 TO 144.
C
C
C     ...............................................................................
C
C     THE FOLLOWING DIMENSION MUST BE GREATER THAN OR EQUAL TO THE NUMBER OF DATA
C     POINTS IN A GIVEN TIME SERIES.
C
         DIMENSION X(1000),S(1000)
C
C     ...............................................................................
C
    1 FORMAT (A4, A2, I4, F5.0, 3F10.0)
    2 FORMAT (12F6.0)
    3 FORMAT (34H: TRIPLE EXPONENTIAL SMOOTHING...., A4, A2//22 H NUMBER OF 1 DATA
     POINTS, 16/19H SMOOTHING CONSTANT, F9.3/)
    4 FORMAT (13HOCOEFFICIENTS, 9X, 1HA, 14X, 1HB, 14X, 1HC)
    5 FORMAT (9HOORIGINAL, F19.5, 2F15.5)
    6 FORMAT (8HOUPDATED, F20.5, 2F15.5/)
    7 FORMAT (1HO, 27X, 13HSMOOTHED DATA/7X, 10H INPUT DATA, 12X, 10H (FORECAST) 1)

    8 FORMAT (F17.5, 8X, F15.5)
C
C     ...............................................................................
C
C     READ PROBLEM PARAMETER CARD
C
  100 READ (5,1) PR,PR1,NX,AL,A,B,C
C        PR.......PROBLEM NUMBER (MAY BE ALPHAMERIC)
C        PR1.....PROBLEM NUMBER (CONTINUED)
C        NX.......NUMBER OF DATA POINTS IN TIME SERIES
C        AL.......SMOOTHING CONSTANT
C        A,B,C....COEFFICIENTS OF THE PREDICTION EQUATION
C
      WRITE (6.3) PR,PR1,NX,AL
C
C     PRINT ORIGINAL COEFFICIENTS
C
      WRITE (6,4)
      WRITE (6,5) A,B,C
```

FIGURE 64—Continued

```
C
C ..............................................................................
C
C    SUBROUTINE EXSMJ
C
C    PURPOSE
C       TO FIND THE TRIPLE EXPONENTIAL SMOOTHED SERIES S OF
C       THE GIVEN SERIES X.
C
C    USAGE
C       CALL EXSMO (X,NX,AL,A,B,C,S)
C
C    DESCRIPTION OF PARAMETERS
C       X     —INPUT VECTOR OF LENGTH NX CONTAINING TIME SERIES DATA WHICH IS TO BE
C               EXPONENTIALLY SMOOTHED.
C       NX    —THE NUMBER OF ELEMENTS IN X.
C       AL    —SMOOTHING CONSTANT, ALPHA. AL MUST BE GREATER THAN ZERO AND LESS
C               THAN ONE.
C       A,B,C—COEFFICIENTS OF THE PREDICTION EQUATION WHERE S IS PREDICTED T PERIODS
C               HENCE BY
C                                     A+B*T+C*T*T/2.
C               AS INPUT—IF A=B=C=0, PROGRAM WILL PROVIDE INITIAL VALUES. IF AT LEAST
C               ONE OF A,B,C IS NOT ZERO, PROGRAM WILL TAKE GIVEN VALUES AS INITIAL
C               VALUES. AS OUTPUT—A,B,C CONTAIN LATEST, UPDATED COEFFICIENTS OF
C               PREDICTION.
C
C       S     —OUTPUT VECTOR OF LENGTH NX CONTAINING TRIPLE EXPONENTIALLY
C               SMOOTHED TIME SERIES.
C
C    REMARKS
C       NONE
C
C    SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C       NONE
C
C    METHOD
C       REFER TO R. G. BROWN, 'SMOOTHING, FORECASTING AND PREDICTION OF
C       DISCRETE TIME SERIES', PRENTICE-HALL, N.J., 1963,
C       PP. 140 TO 144.
C
C ..............................................................................
C
      SUBROUTINE EXSMO (X,NX,AL,A,B,C,S)
      DIMENSION X(1),S(1)
C
C     IF A=B=C=0.0, GENERATE INITIAL VALUES OF A, B, AND C
C
      IF (A) 140, 110, 140
  110 IF (B) 140, 120, 140
  120 IF (C) 140, 130, 140
  130 C=X(1)-2.0*X(2)+X(3)
      B=X(2)-X(1)-1.5*C
      A=X(1)-B-0.5*C
C
  140 BE=1.0-AL
      BECUB=BE*BE*BE
      ALCUB=AL*AL*AL
C
C     DO THE FOLLOWING FOR I=1 TO NX
C
      DO 150 I=1,NX
C
C        FIND S(1) FOR ONE PERIOD AHEAD
C
      S(1)=A+B+0.5*C
C
C        UPDATE COEFFICIENTS A, B, AND C
C
      DIF=S(1)-X(1)
      A=X(1)+BECUB*DIF
      B=B+C-1.5*AL*AL*(2.0-AL)*DIF
  15C C=C-ALCUB*DIF
      RETURN
      END
```

Appendix D

ESTIMATE OF FUTURE POPULATION GROWTH BY SCHOOL DISTRICT, BUCKS COUNTY, PA.[1]

Introduction

This June, 1967 report on prospective population development of Bucks County, Pennsylvania, is the first of a series of working papers prepared for Bucks County Board of School Directors to assist in the analysis of post high school educational needs in the County. The data here will be used in conjunction with sample survey results concerning the present desires and aspirations of the County's high school seniors and adults who may be interested in furthering their education beyond the high school level. The data will also be used to help make estimates of the costs and suitability of alternative methods of meeting the needs indicated in the sample survey results. It is anticipated that this planning effort will continue and that changing desires and aspirations will be combined with revised estimates of future growth periodically as additional information is revealed by the passage of time.

In the preparation of this study, the Government Studies Center of the Fels Institute of Local and State Government at the University of Pennsylvania has served as consultants to Bucks County Board of School Directors. Government Studies Center Personnel participating in the development of this research are John K. Parker, Manager of Systems Division, project supervisor; Boyd Z. Palmer, in charge of research design, and Arnold R. Post, who has developed these estimates of the County's population growth.

Summary of Expectations

It is estimated that Bucks County's population will increase to about 575,000 as of 1980 or by 85 percent as compared to 1960's population of 309,000. An acceleration in growth is expected in the 1970's, which will be relatively intense in Middle Bucks County.

Growth at the present time is less intensive than it was in the 1950's so that the present era is one of relative lull. The lull is associated with the present general shortage of young adults in the population, who were born in the 1930's. The more intensive stages of growth in the 1950's and in the 1970's and 1980's are associated with the post war baby-booms, reflecting their maturity.

The geographic pattern is an extension of existing trends. In the 1950's, County development was most intensive along Route 1 between Trenton and Philadelphia in Lower Bucks. During the 1960's active development has tended to move out along Old York Road through Montgomery County and on to the Townships bounded by the Neshaminy in Middle Bucks. In the latter part of the 1970's land for additional residential development will become scarce in Lower Bucks County; and the intensity of development will shift gradually towards the Bethlehem Pike by the 1980's and 1990's in Upper Bucks.

The figures in table D-1 (*table numbers ours*), represent preferred estimates which are related to expected trends in building development. The detailed tables (D-14, D-15, D-16) indicate high and low estimates which, by 1980, have a range of about plus or minus 10% of these figures.

Overall, the County gained about 42,000 households in the 1950's, and it appears that the gain will be about 30,000 households in the 1960's. In the 1970's, with housing demand increasing rapidly and with less space for it in the central portions of the Metropolitan Area, accelerated development is in prospect for near-by areas which still have space available. The gain in households for the 1970's is assumed to be 60,000, which is

[1] Prepared by Government Studies Center, Fels Institute of Local and State Government, University of Pennsylvania, June 1967.

TABLE D-1.—*Estimated total population 1960-1980*

[Thousands]

Area	1960	1965	1970	1975	1980
Upper Bucks	47.2	51.9	50.4	64.7	74.2
Middle Bucks	70.4	91.5	104.2	137.2	177.5
Lower Bucks	191.2	202.7	232.7	274.3	322.8
County total	308.8	346.1	393.3	476.2	574.5
School districts:					
1. Palisades	9.3	10.2	10.8	11.4	12.1
2. Quakertown	16.4	17.2	18.6	21.3	24.0
3. Pennridge	21.5	24.5	27.0	32.0	38.1
4. Central Bucks	28.6	35.0	38.2	48.3	60.0
5. New Hope-Solebury	4.0	4.3	5.0	7.2	9.5
6. Council Rock	13.5	18.2	20.1	26.5	34.7
7. Centennial	24.3	34.0	40.9	55.2	73.3
8. Neshaminy	45.7	49.2	61.9	75.5	94.9
9. Pennsbury	42.5	47.6	53.8	65.0	78.0
10. Morrisville	7.8	9.6	9.4	8.9	8.5
11. Bristol Township	59.3	58.3	63.8	69.9	74.7
12. Bristol Borough	12.4	12.6	12.2	12.7	12.6
13. Bensalem	23.5	25.4	31.6	42.3	54.1

TABLE D-2.—*Estimate of senior class enrollments*

(Public and Private)

Area	1960	1965	1970	1975	1980
Upper Bucks	640	890	1,050	1,100	1,110
Middle Bucks	735	1,380	1,685	2,320	2,060
Lower Bucks	1,990	3,035	4,125	4,660	4,840
County total	3,365	5,305	6,860	8,080	8,010
School districts:					
1. Palisades	100	150	195	195	180
2. Quakertown	240	290	335	360	360
3. Pennridge	300	450	520	545	570
4. Central Bucks	300	525	570	810	900
5. New Hope-Solebury	40	65	90	120	140
6. Council Rock	150	280	375	450	520
7. Centennial	245	510	650	940	1,100
8. Neshaminy	455	735	1,070	1,280	1,420
9. Pennsbury	500	695	945	1,105	1,170
10. Morrisville	80	140	165	150	125
11. Bristol Township	595	895	1,170	1,190	1,125
12. Bristol Borough	125	190	230	215	190
13. Bensalem	235	380	545	720	810

equivalent to a full decade's development at the peak rate established in 1966 when 5,969 units were authorized. A major difference, however, is that, whereas 1966 saw authorization for many apartment units, single-family housing is expected to predominate again in the 1970's, as it did in the 1950's.

Tables D–2 and D–3 show estimates of senior class enrollments (in public and nonpublic school systems) and estimates of adults over 15 years of age and not enrolled in grades 1–12 for each of the school districts. To develop these estimates, estimates of the age distribution of the total County population were developed, as tabulated in the appendix, according to which estimates of County total senior class enrollments and adults over 15 were prepared. The estimates of these categories by school district were then derived in proportion to the estimates of district total populations. Again, preferred estimates are shown in the tables which follow.

The above methodology does not take into account differences in age distributions among the district populations; and it is assumed that the allowance for uncertainty in district totals is sufficient to provide an adequate range in the estimates of the seniors and the adults over 15. On the assumption that public senior enrollments in 1970 will amount to 90 percent of 1967's ninth grade enrollments as reported through the office of the County Superintendent of Schools, preliminary figures for the districts were examined to see that the minimum growth allowance was more than sufficient to accommodate such a condition. With the assistance now being given to potential high school dropouts coupled with the intense publicity placing a high economic value on a high school diploma, increases in holding power of the high schools are anticipated. In 1967, senior enrollment is about 80 percent of 1963–64's ninth grade enrollment in the public schools.

One of the constraints on a small area's population growth is the amount of land available for future residential development. Another is the intensity of residential development permitted by local regulations on this land. In 1959, the Bureau of Economic and Business Research of the School

TABLE D-3.—*Adults over 15 not enrolled in grades 1–12*

[Thousands]

Area	1960	1965	1970	1975	1980
Upper Bucks	27.9	31.1	34.4	40.1	46.1
Middle Bucks	41.6	54.9	63.6	85.2	111.8
Lower Bucks	112.9	121.7	141.9	170.1	203.3
County total	182.4	207.7	239.9	295.4	361.2
School district:					
1. Palisades	5.5	6.1	6.6	7.1	7.6
2. Quakertown	9.7	10.3	11.3	13.2	15.1
3. Pennridge	12.7	14.7	16.5	19.8	23.4
4. Central Bucks	16.9	21.0	23.3	30.0	37.8
5. New Hope-Solebury	2.4	2.6	3.0	4.5	6.0
6. Council Rock	8.0	10.9	12.3	16.4	21.8
7. Centennial	14.3	20.4	25.0	34.3	46.2
8. Neshaminy	27.0	29.5	37.8	46.8	59.8
9. Pennsbury	25.1	28.6	32.8	40.3	49.2
10. Morrisville	4.6	5.8	5.7	5.5	5.4
11. Bristol Township	35.0	35.0	38.9	43.3	47.0
12. Bristol Borough	7.3	7.6	7.4	7.9	7.9
13. Bensalem	13.9	15.2	19.3	26.3	34.0

of Business and Public Administration at Temple University prepared an estimate of future population growth for Bucks County Planning Commission "Bucks County population estimates for the years 1965, 1980, 2010." Part of this study was devoted to an analysis of available land capacity as controlled by the zoning ordinances then in effect. These findings have been adopted in this study, at least, as an indication that the growth anticipated is feasible. There are three exceptions to this general statement. In Morrisville Borough, authorizations since 1960 have exceeded the capacity of the 1959 zoning ordinance; and allowance for 25 additional dwellings has been made arbitrarily. No allowance for apartment development is evident for Bristol Township in the 1959 study and apartment development has occurred there since 1960 at a significant level. The capacity in 1959 was for about 5,000 additional units; an expectation of 7,600 has been incorporated here. In the summary tables of the 1959 report, no indication was found of dwelling unit capacity in Newtown Township and an arbitrary allowance for 15,000 units has been made, comparable to the allowance indicated for Wrightstown Township. (Table D-4.)

It should be noted that dwelling unit capacity figures are dependent on a certain "faith in princes" yet to rule; and it is not uncommon for the capacities implied by early zoning ordinances to be lower than capacities allowed under later ordinances when patterns of development have become more clearly defined. It may also be noted that the 1959 estimates of total County growth and Metropolitan Area growth as of 1980 are in substantial agreement with the estimates developed for this study. A population of 5.8 million is expected in the Metropolitan Area in both cases. A County population of 558,000 is indicated for Bucks County in the Temple University Study, which is within the range of uncertainty about the estimate of 574,500 preferred as the result of this analysis. The County Planning Commission's current estimate for 1980 population is 539,650, also within the range of uncertainty given here but closer to its lower limit of 515,000.

TABLE D-4.—*Utilization of land capacity 1960–1980*

[Thousands of dwellings]

Area	Available capacity 1960 [1]	Housing supply increments 1960–64	Housing supply increments 1960–80	Percent of capacity utilized	Capacity remaining
Upper Bucks	120.0	2.0	11.0	7.8	115.0
Middle Bucks	150.5	6.6	37.7	22.6	112.8
Lower Bucks	59.1	5.7	50.3	95.0	(2)
County total	335.6	14.3	99.0	30	(2)
School districts:					
1. Palisades	43.	0.4	1.4	3	41.6
2. Quakertown	34.	.5	3.2	8	30.8
3. Pennridge	49.	1.1	6.4	12	42.6
4. Central Bucks	59.	2.0	11.1	17	47.9
5. New Hope-Solebury.	6.5	.1	1.9	26	4.6
6. Council Rock	63.	1.4	7.5	9	55.5
7. Centennial	22.	3.1	17.2	71	4.8
8. Neshaminy	20.	1.6	17.0	82	3.0
9. Pennsbury	15	1.8	13.5	81	1.5
10. Morrisville	.5	.6	[2].7	(2)	(2)
11. Bristol Township	5.	.6	[2] 7.6	(2)	(2)
12. Bristol Borough	.6	.2	.6	100	0
13. Bensalem	18.	.9	10.9	65	7.1

[1] School of Business and Public Administration, Temple University, 1959.
[2] Not available. Apartment development in Bristol Township and Morrisville Borough has made these 1959 capacity figures obsolete.

The distribution of expected housing increments is shown in table D-5 both in absolute numbers and as a percentage of total County development.

TABLE D-5.—*Housing increments, 1950–1980*

[Thousands]

School district	Units 1950–60 Number	Units 1950–60 Percent	Units 1960–70 Number	Units 1960–70 Percent	Units 1970–80 Number	Units 1970–80 Percent
Palisades	490	1	700	2	700	1
Quakertown	980	2	1,100	3	2,100	3
Pennridge	1,340	3	2,200	6	4,200	6
Upper Bucks County	2,810	6	4,000	12	7,000	10
Central Bucks	2,720	6	3,300	10	7,800	12
New Hope-Solebury	370	1	400	1	1,500	2
Council Rock	1,770	4	2,300	7	5,200	8
Centennial	4,060	9	5,500	17	11,700	18
Middle Bucks	8,920	20	11,500	35	26,200	40
Neshaminy	8,570	19	6,000	18	11,000	17
Pennsbury	8,870	20	4,500	14	9,000	14
Morrisville	600	1	700	2	0	......
Bristol Township	11,620	26	3,000	9	4,600	7
Bristol Borough	280	1	300	1	300	......
Bensalem	3,300	7	3,000	9	7,900	12
Lower Bucks	33,240	74	17,500	53	32,800	50
County total	44,970	100	33,000	100	66,000	100

Methodology

A Note on Method

This estimate of population growth by school district in Bucks County depends on a methodology which is still under development but is consistent with results produced by special censuses taken in Bucks County since 1960. The basic variable considered is the relative increase in households to be expected in the Metropolitan Area in the decades of the 1960's and 1970's. The principal hypotheses have to do with the statistical dependence of population growth on housing growth by small area. Research based on all the municipal areas in the Philadelphia Standard Metropolitan Statistical Area outside of Philadelphia proper has revealed a reasonably simple relation which was very accurate between 1950 and 1960. Continuing research indicates that the form of this relation may be stable and that its coefficients may be predictable according to variations in rates of household increase.

Customary analysis considers three components of population: (1) an initial population, (2) the natural increase or surplus of births over deaths associated with that population, and (3) a net migratory increase. In this study, the analysis concerns itself with two components: (1) initial population and (2) marginal increase per household. In neither analysis, is it possible to make a count of people or houses and assign the individuals uniquely to the analytical categories. A person moved away or a house demolished is replaceable by any (rather than some particular) person moving in or house newly built; and one must deal in both cases, with patterns and equivalents rather than the fate of individuals.

The methodology here employed is thought to mark an improvement over customary methods. Houses are simpler to anticipate than people primarily because they are precisely located, for the most part, in a permanent fashion, and they are not self generating. In addition, there are fewer analytic categories to deal with, and the results conform well with other findings.

Housing vs. Population

People need housing, but the economy is exacting enough so that builders cannot supply housing in careless abundance. Under conditions of adequate economic development, which are assumed, an existing housing supply will not become overcrowded; however, as new families emerge from the old housing supply, their choice of where next to live will be restricted to areas where housing is available to them. Although individual builders will make some mistakes in estimating prospective demand for the houses they build, the industry as a whole will not persist in providing houses in areas where builders' expectations are not realized and actual new housing goes unwanted.

A given five year age group will use its largest number of housing units when it is 45–49 years old. Past that age, increases in the death rate will more than make up for increases in the household headship rate. Table D-6 indicates approximately how many household heads (or households) are to be expected from an age group numbering 1,000 at age 15 to 19 and it will be noted that almost 95 percent of the peak demand is exerted when the age group is 30–34 years old. The estimates are based upon average survival rates and average percentages of household headship.

TABLE D-6.—*Prospective housing demand of 1,000 15- to 19-year-olds*

	Housing units	Increase	Percent of maximum
First demand (15-19)	10	10	2
After 5 years (20-24)	200	190	45
After 10 years (25-29)	369	169	83
After 15 years (30-34)	416	47	94
After 20 years (35-39)	434	28	98
After 25 years (40-44)	444	10	100
After 30 years (45-49)	445	1	100
After 35 years (50-54)	431	−14	97
After 40 years (55-59)	399	−32	90
After 100 years	0	−399	0

A population, of course, consists of people of all ages. It is clear from the above, however, that periods of rapid housing increase will coincide with periods when relatively large numbers of people are in their twenties and early thirties. For this reason, more persons are to be expected per added household in decades when households are being added rapidly than when net household increase is slow.

The age distribution of the United States population is very irregular and that of the Metropolitan Area is also irregular. These irregularities now have a long history dating back to the 1920's when large scale immigration to the country was brought to a halt and the baby-boom of that time created a high potential for housing demand in the 1950's. The low birth rates of the 1930's were partly due to the depression but also to the absence of young adult immigrants. Although there has been nothing comparable to the Great Depression since World War II, the scarcity of young adults in the 1960's has been sufficient to lower the rate of housing development and to induce an echo of the low birth rates of the 1930's.

an echo which has perhaps been amplified by the development of new means of family planning.

The baby-boom following the second world war persisted until about 1960; and those born at the beginning of the period will enter the traditional house-buying age-groups in 1971 and thereafter. The last of them will not leave these age groups until about 1995. In the latter part of the 1970's, the house-buying age groups will still be growing so that record levels of single family housing construction may well be expected in suburban areas. The alternative would be misery. Variations in the decennial rates of household increase can thus be anticipated with a good degree of confidence. After the present lull, the number of householders will increase more rapidly during the 1970's.

In the 1950's, households in the Metropolitan Area increased by about 24 percent. Housing construction so far in the present decade indicates a likely gain of about 19 or 20 percent; and a growth rate of 23 percent is anticipated in the 1970's. Since the provision of additional housing is becoming more and more of a suburban phenomenon, the impact of heightened building activity will be greater in suburban areas. In the 1950's, about 42,000 additional families took residence in Bucks County; and it seems likely, with the decade now two-thirds gone, that the County's increase will total about 30,000 families for the 1960's. An estimate of 60,000 additional householders in the County for the 1970's seems within the realm of likelihood.

On this basis, it is estimated that the County's population in 1980 will be about 575,000, a figure in substantial agreement with Delaware Valley Council's estimate of 611,000 for 1985 and in the upper part of the range of 442,400 to 671,500 estimated in 1959 for the County Planning Commission by Temple University's School of Business and Public Administration.

Small Area Considerations

Census tracts are small areas having an average population of about five or six thousand persons. They are defined by the Bureau of the Census to coincide with municipal boundaries. There are 86 census tracts in Bucks County which may be combined to conform with the boundaries of the County's 54 municipalities. On the basis of actual changes reported for the decade of the 1950's, 1960 census tract populations may be estimated in proportion to the size of their 1950 populations and to their changes in housing supply or households. In the estimating equation below, $^{P}60$ stands for a tract's 1960 household population, $^{P}50$ is the household population for the tract in 1950 and (dX) indicates the change in the number of occupied dwellings or households:

$$^{P}60 = .88\,^{P}50 + 4.1(dX) - 15$$

In 95 percent of the metropolitan area's 427 census tracts outside of Philadelphia, such estimates are accurate to within 350 persons and the coefficient of multiple correlation is better than 99 percent, overall. The above formula applied to data for Bucks County *as a whole* yields an estimate of 303,100 persons in the County's households as of 1960. The reported figure was 304,900, for an error of 1,800 persons, or about 0.6 percent.

Research based on national census returns since 1910 yields an indication that the comparable relationships appropriate for household growth rates of 20 and 23 percent, as estimated for the metropolitan area in the 1960's and 1970's, are consistent with the following formula:

$$^{P}70 = .97\,^{P}60 + 2.80\ (dX)$$
$$^{P}80 = .91\,^{P}70 + 3.75\ (dX)$$

Short Time Considerations

In order to estimate population growth over a part of a decade adjustment of the estimating formulas has to be made. The coefficient of initial population (.88 in the formula for 1960 populations) clearly is time dependent. If it were employed to estimate a 1961 population based on a 1960 census report, it would imply a decimation of the population as if the census takers had carried the plague. The coefficient should be very close to 100 percent for any estimate applying to a year after the most recent count. Assuming a steady rate for the emergence of population out of the census year housing supply, one can adjust the coefficient by taking the n/10th root of the value expected for 10 years later, i.e. for 1965 the ½ root or square root of .970 would be appropriate.

The coefficient of persons per added dwelling also should also be adjusted; and a linear interpolation between the decennial values (4.1 and 2.8) for the 1960's produces the following correspondence with the special censuses that have been taken in the County since 1960 as shown in table D–7.

TABLE D-7.—*Comparison of special census reports with populations estimated on the basis of building permit reports and 1960 census reports*

Year of Census	Municipality	Population: Census	Population: Estimate	Error: Number	Error: Percent	Estimating coefficient: $^{P}60$	Estimating coefficient: (dX)
1963....	Northhampton Township.	8,462	8,355	107	1.3	.991	3.71
1964....	Falls Township........	31,152	32,060	−908	−2.8	.988	3.58
	Lower Makefield Township.	10,635	10,520	115	1.1	......	......
	Warminster Township.	24,116	23,210	906	3.9	......	......
	Warrington Township.	4,907	4,825	82	1.7	......	......
1965....	Lower Southampton Township.	14,603	15,260	−657	4.3	.985	3.45
1966....	Northampton Township.	11,369	9,900	1,469	14.9	.982	3.32
	Solebury Township...	3,086	3,420	−334	−9.8	......	......
	Upper Southampton Township.	11,494	10,690	804	7.5	......	......
	Total......................	119,824	118,240	1,584	1.3	......	......

Except for Northampton Township (1966) the errors of estimate are tolerable (table D-8). Although the overall bias of 1.3 percent is small it will be noted that the percentage errors are noticeably larger at the end of the period. It is possible to adjust the estimating equation for the decade to correspond more accurately with the special census results and this has been done although there is some danger in this procedure since the communities seeking special census are anything but a random sample of all the municipalities in the county. Since there is no advantage to having a special census taken unless a population increase is thereby established, special census are a characteristic of intensively developing areas. It would also be possible to adjust the estimate of dwellings added to secure an improvement in correspondence; however, this information is more highly pertinent to the situation in the county than the general considerations based on analysis of the national census returns since 1910. In short, it seems more reasonable to depart from theory with respect to the national returns although the pairing of the coefficients is based on this research. The estimating equation for 1970 population has therefore been adjusted to conform with the special census returns and reads as follows:

$$^{P}70 = .90\,^{P}60 + 3.85\ (dX)$$

The adjustment has reduced the overall bias to less than one percent and the absolute value of the percentage deviations has become a more simple and less variable function of time. The difference in estimating equations affects the distribution of estimated population in the County more than it does the estimate of total County population. The original formula leads to an estimate of 341,000 for the County's 1965 population and 384,000 for 1970. The revised formula leads to estimates of 346,100 and 393,500, respectively.

Stability and Growth

These relations have certain important implications regarding the nature of patterns of development in small areas. The most striking, perhaps, is that in periods of rapid housing development, small areas which don't receive their "share" of development, will tend to lose population; while areas receiving more than their "share" will tend to grow more rapidly than the general County average of 3.4 persons per dwelling might indicate. As new houses are built and occupied, some of the population moving into them moves out of the old housing supply. The more rapidly new housing is occupied, the greater will be the proportion of population moving out of the old housing to take advantage of it and the younger will be this population so that, over the course of a decade, with children and all, the greater will be the number of persons per added dwelling.

The relative growth of the housing supply, however, has to be judged on a regional basis. If many houses are added to the supply of a particular township, they will bring in relatively

TABLE D-8.—*Errors of estimate, 1963–1966*

Special Census year and municipality	Population: Reported	Population: Estimated	Population: Error	Population: Percent error	Estimating coefficient: $^{P}60$	Estimating coefficient: (dX)
1963:						
Northampton Township.	8,462	8,440	22	0.26	.97	4.025
1964:						
Falls Township.........	31,152	31,620	−468	−1.48	.958	4.000
Lower Makefield Township.	10,635	10,510	125	1.19	......	......
Warminster Township..	24,116	23,570	546	2.32	......	......
Warrington Township..	4,907	4,790	117	2.44	......	......
1965:						
Lower Southampton Township.	14,603	15,210	−607	−3.99	.947	3.975
1966:						
Northampton Township.	11,369	10,400	969	9.32	.938	3.950
Solebury Township.....	3,086	3,380	−294	−8.69	......	......
Upper Southampton Township.	11,494	10,790	704	6.52	......	......
Total................	119,824	118,710	1,114	0.94	......	......

fewer people if household development is generally slow in the region. It is likely that in times of relatively slow growth, the units added are more apt to be apartments than single family units, as has been the case over the past several years.

It is also to be observed, that even though 12,000 units were added to the 84,000 units already in Bucks County between 1960 and 1964, the trend in school enrollments which was implied by the 1960 Census report has scarcely been altered for the County as a whole. The gain in enrollments 1960-65 can be quite adequately explained by the large number of 0–4-year-old county residents counted in 1960. Immigration to one district has been offset by outmigration from another. Even 6,000 units added in the 1950's would have made a substantial difference in the school enrollment trend. The school enrollment trends referred to include public and private school reports, at the elementary level.

It is an open question whether there is any such thing as a purely local trend in the development of a small area's population. The population growth of a small area appears to depend not only on the growth of its own housing supply but also on the provision of housing in many, many other small areas. It is also worth noting that the relation derived from the 1950 and 1960 data applied very uniformly by small area where the small areas comprised a region of roughly 50-mile radius and included slums, suburbs and farmlands with diverse racial and economic characteristics. The formula did not apply particularly well to some 25 of the 427 census tracts studied though it did apply well to areas ranging in population from 142 on the riverfront in the City of Chester, where a net loss was registered, to nearly 60,000 (Bristol Township).

There is also an implication for communities, large and small, which have reached the geographical limits of their potential housing development. Such communities stand to lose population in the next succeeding period of generally rapid housing growth simply for lack of additional building space within their own boundaries, though the rate of loss will depend on the rate of new development elsewhere. To illustrate, it is reasonable to expect that present housing construction in Middle Bucks County is attracting some population flow from Lower Bucks to Middle Bucks, even though Lower Bucks is still undergoing development at this time.

The population expected in the Metropolitan Area, in Philadelphia and in Bucks County as of 1970 is shown in table D-9 as compared with the reported populations for 1950 and 1960. It will be noted that the Metropolitan Area total has been gaining fairly steadily while the two Counties have shown separate surges, Bucks County in the 1950's and Philadelphia County in the 1960's. These patterns of growth are implicit in the estimating equations employed. The coefficient of initial population is of primary importance in areas with large population while the coefficient of added households is of primary importance where changes in the housing supply are of greater importance.

As long as Philadelphia represents an important source of population for the suburban counties growth, it is reasonable to expect the rates of gain for the two classes of counties to be out of phase. It is worth noting that the 1965 estimate of population for the Metropolitan area by this method comes to 4,604,000 as compared to a Census Bureau estimate of 4,667,000, a difference of about 1.5 percent.

Application of Method to Bucks County

In order to use this method of population estimation, it is necessary to develop information on the growth of households in each small area. These data are not reported directly; however, in Bucks County, all municipalities except for the rural townships in Upper Bucks County report to the State Department of Labor and Industry the number of dwelling units authorized by building permit each month. In the rural Townships, it is required that subdivision plans be submitted to the County Planning Commission which keeps a record of the number of lots in

TABLE D-9.—*Regional population growth*

Year	PSMSA[1]		Philadelphia County		Bucks County	
	Population	Percent decade increase	Population	Percent increase	Population	Percent increase
1950......	3,671,000	15	2,072,000	7	145,000	35
1960......	4,342,000	18	2,003,000	−3	309,000	113
1970......	4,930,000	14	[2] 2,091,000	+4	[3] 393,300	27

[1] Philadelphia Standard Metropolitan Statistical Area.

[2] An enrollment study for Philadelphia completed in April 1969 indicates that the City's gain in households is less than that derived from its building statistics in the early part of the decade. Accordingly, it is believed that the City's population as of 1970 will number about 1.9 million, rather than the 2.1 million shown above.

[3] Further research completed in 1968 for Bucks County, which allocated nonpublic school students by public school district of residence, revealed a substantial additional number of Bucks County pupils in nonpublic schools near but not within the County . It is accordingly estimated that the County's 1970 population will be in the neighborhood of 410,000 persons

approved subdivisions. These are the sources of information that have been used in this analysis.

Between 1950 and 1960, change in the number of dwelling units correlated positively with size of the 1960 household population, the correlation being higher between these variables than between the 1950 and 1960 populations themselves. During the 1950's the reports of the Department of Labor and Industry indicate that 61 percent of the State's gain in dwelling units (as reported by the Census) was reported as authorized units by a variable set of municipalities which had 64 percent of the State's population in 1960. From this, it is assumed that reports to the State may have an accuracy of 95 percent, and this estimate of accuracy has been applied to the reports from Bucks County municipalities since 1960. For the northern rural Townships, a rough and ready comparison of subdivision activity and dwelling unit authorizations has led to the rule of thumb that housing supply growth is 1.5 times the number of lots reported in approved subdivisions.

Gains in total housing supply tend to exceed gains in occupied housing. In the metropolitan area the gain in occupied dwellings was 88 percent of the gain in total dwellings between 1950 and 1960. In the suburban counties the percentage was about 95 percent, while in Philadelphia, the percentage was only 63 percent. Since 1960 in Bucks County, it has been assumed to be 90 percent.

These figures taken in conjunction with the 1960 Census reports provide the information necessary to make current estimates of population for the County's municipalities.

School Enrollments

It has been noted that the addition of some 12,000 housing units to the County's housing supply from 1960 to 1964 hasn't made much difference to the school enrollment trend since 1960. The preschoolers of 1960 were sufficient to account for the reported gains. This observation is based on the following analysis (table D–10).

TABLE D–10.—*Enrollment growth, grades 1–12, 1960 and 1965, actual and estimated under conditions of no migration, Bucks County*

Age groups (1960)	Population	Percent enrolled	Enrollment (1960)	Survivors (1965)	Percent enrolled	Enrollment (1965)
0–4	42,834	0	0	43,700	75	32,800
5–9	38,616	75	29,000	38,400	98	37,600
10–14	30,365	98	29,750	29,750	72	[1] 21,400
15–19	19,218	72	13,850	19,150	2	380
20–24	13,402	2	270	13,300	0	0
Total Enrollment						
Estimated			72,870			92,180
Actual (U.S. Census)			72,083	(School reports)		89,662

The above estimating percentages are derived from a cross tabulation published by the Census for the State as a whole indicating school enrollment by single grade and age distribution by single year. The enrollment shown for 1959–60 is that reported by the Census for Bucks County and is slightly higher than the comparable figure reported by the schools surveyed for this study. It is likely that some County residents attend schools not in the County, which would explain the difference. The increase shown over the 5 years in the youngest cohort (0–4, 1960) takes some account of census underenumeration of small children. The declines in the other cohorts are attributable to deaths. No allowance has been made for migration. The narrow difference between estimate and report for 1964–65 indicates that little allowance should be made for net migration in the school age groups at the county level.

The major trend 1960–65 in school enrollments has had to do with aging of the resident population. A matter of considerable secondary importance is an indicated attrition of enrollments in the parochial school system. Between 1964 and 1965, enrollments in second to eighth grades numbered about 1,100 pupils fewer than the preceding year's enrollments in first to seventh grades.

Parochial school enrollment data have not been made available past the academic year 1964–65 and only the County total is available, not the distribution by public school district. (Parochial school districts don't coincide with public school districts.) In addition demographic analysis of each school district would require an undue effort relative to the other work which has to be done in this study. However, by comparing the first, second, and third-grade enrollments in the public schools for 1961–62 with the sixth, seventh, and eighth-grade enrollments for 1966–67, one can obtain an impressionistic picture of the likely trends of migration within the County over the past 5 years. In a stable situation some small attrition is to be expected (table D–11).

Districts gaining noticeably faster than the county average may be assumed to be experiencing some in-migration. The nature of migratory experience in the other districts is not established by this analysis; small gains may indicate transfers

TABLE D-11.—*Public school enrollments, 1962 and 1967*

	Enrollment, 1962, grades 1, 2, and 3	Enrollment, 1967, grades 6, 7, and 8	Gain, 10% or more
1. Palisades	411	434	....
2. Quakertown Community	923	910	....
3. Pennridge	1,261	1,276	....
4. Central Bucks	1,664	1,992	20
5. New Hope-Solebury	201	193	....
6. Council Rock	891	1,222	37
7. Centennial	1,653	2,039	23
8. Neshaminy	2,439	2,821	16
9. Pennsbury	2,531	2,763	....
10. Morrisville	285	321	12
11. Bristol Township	3,490	3,224	....
12. Bristol Borough	422	449	....
13. Bensalem Township	917	1,035	13
Total	17,088	18,679	9

from the private school systems while small losses may be explained by the death rate.

The population enrolled in school lies in the age group 5 to 34. To estimate the number of adults over 16 not enrolled in the grades, reference was made to the State cross-tabulations referred to above to determine percentages which might be applied to the estimated age distributions of Bucks County's population. The resulting estimates of County total were then prorated among the school districts according to district population estimates. Estimated age distribution for the county are shown in the appendix. Adults, 16 years and over, not enrolled in the grades range between 59 and 62 percent of the County's population on a rising trend. The trend in senior class enrollments as a percentage of county population is tabulated in table D-12.

Uncertainty

The County's total population as of 1980 has been estimated at 575,000 plus or minus 60,000, roughly 10 percent of the total or 30 percent of the increment between 1970–1980. A range of 10,000 is warranted on the basis of statistical reasoning assuming the stated increase in households proves true and that the estimating equations are actually the most efficient that could be chosen. It will not be possible to ascertain the actual truth of these assumptions until after the 1980 census is published. If the equations are valid, the range in

TABLE D-12.—*High school seniors as percent of county population*

Year:	Percent
1960	1.0
1965	1.5
1967	1.6
1970	1.8
1975	1.7
1980	1.5

uncertainty about the population translates to a range of uncertainty in household growth. An average of 6,000 additional households per year has been assumed for the 1970's. A range of plus or minus 1,600 new households a year (about 27 percent of the increment) would account for the range in population, assuming the 1970 estimate is reasonably accurate. The estimate for 1970 is 393,500 which relates to a current (1967) estimate of about 380,000.

It will be noted that even the 1965 figures are given within a range of uncertainty; the preferred estimate here is for 346,100 plus or minus about 10,000, though the extreme values are thought to be unlikely. In 1965, the Government Consulting Service published an estimate of current County population equaling 347,000 persons; however, this estimate was only of peripheral interest and was made without reference to school enrollment trends. Other ex post facto estimates of the County's population are shown table D-13.

In 1965 and 1966, 10,500 dwellings were authorthorized by building permit, and 750 more are estimated to be in townships not reporting building permit data, yielding an estimate of 11,800 new units overall or 10,600 new households in the past 2 years, which account for the estimated gain of 37,000 persons since 1965. In the first quarter of 1967, reported building permit authorizations for the County were down by more than 50 percent as compared to the first quarter last year, 534 as compared to 1,101. The estimate for 1970 allows for the occupancy of approximately 7,000 more units by 1970.

The range of uncertainty has not been applied symmetrically in the various parts of the County. It has been assumed that if the preferred estimate turns out to be low, the errors will tend to be most important in Middle Bucks County. In other words, unexpected growth seems most likely in this part of the County. On the other hand, if the preferred estimate turns out to be high, it seems most likely that unexpected stability will occur in the heavily populated area of Lower Bucks County and in the rural areas of Upper Bucks.

TABLE D-13.—*Estimates of Bucks County population*

	1964	1965
U.S. Census Bureau	322,000	[2] 345,000
State planning board	323,800	[2] 327,500
County planning commission	[1]	356,850

[1] Not available.
[2] Provisional.

Supplementary Tables

TABLE D-14.—*Estimates of total population, adults, and seniors*

Year	Total population[1]			Adults 16+[1]			Seniors		
	Low	Preferred	High	Low	Preferred	High	Low	Preferred	High
1960		308.8			182.4			3,265	
1965	330.4	346.1	360.2	198.8	207.7	216.2	5,060	5,305	5,545
1970	363.0	393.3	417.6	221.5	230.0	254.7	6,420	6,860	7,320
1975	445.5	476.2	515.4	273.7	295.4	217.8	7,500	8,080	8,675
1980	515.6	574.5	634.1	324.6	361.2	399.2	7,715	8,610	9,575

[1] In thousands

TABLE D-15.—*Estimates of population, adults 16 and over not enrolled in grades 1-12, and senior class enrollments, by County region 1960 to 1980 by 5-year intervals*

Year and area	Total population			Adults			Seniors		
	Low	Preferred	High	Low	Preferred	High	Low	Preferred	High
Upper Bucks:									
1960		47,200			27,900			840	
1965	48,600	51,900	54,100	29,100	31,100	32,400	815	890	970
1970	49,900	56,400	61,200	30,400	34,400	37,300	970	1,050	1,150
1975	57,900	64,700	71,500	35,900	40,100	44,400	985	1,100	1,215
1980	68,000	74,200	79,700	42,800	46,100	50,200	990	1,110	1,200
Middle Bucks:									
1960		70,400			41,600			735	
1965	86,300	91,600	97,000	51,700	54,900	58,200	1,295	1,380	1,450
1970	94,400	104,200	113,200	57,700	63,600	69,100	1,515	1,685	1,810
1975	129,300	137,200	154,100	80,100	85,200	95,500	2,200	2,320	2,620
1980	154,300	177,800	217,100	97,200	111,800	136,500	2,320	2,660	3,310
Lower Bucks:									
1960		191,200			112,900			1,990	
1965	195,600	202,700	209,100	118,000	121,700	125,600	2,950	3,035	3,125
1970	218,700	232,700	243,200	133,400	141,900	148,300	3,935	4,125	4,360
1975	258,300	274,300	286,800	157,700	170,100	177,900	4,315	4,660	4,840
1980	293,300	322,800	337,300	184,600	203,300	212,500	4,405	4,840	5,065

TABLE D-16.—*Estimates of population, adults 16 and over and not enrolled in grades 1-12, and senior class enrollments, by school district 1960-1980 by 5-year intervals*

District year	Total population			Adults			Seniors		
	Low	Preferred	High	Low	Preferred	High	Low	Preferred	High
Palisades:									
1960		9,300			5,500			100	
1965	9,300	10,200	11,000	5,600	6,100	6,600	140	150	165
1970	9,100	10,800	12,500	5,600	6,600	7,600	165	195	225
1975	9,300	11,400	13,500	5,800	7,100	8,400	160	195	230
1980	10,000	12,100	13,800	6,300	7,600	8,700	150	180	210
Quakertown:									
1960		16,400			9,700			240	
1965	16,400	17,200	18,200	9,800	10,300	10,900	275	290	305
1970	16,800	18,600	20,500	10,200	11,300	12,500	305	335	375
1975	19,000	21,300	23,500	11,800	13,200	14,600	325	360	400
1980	22,000	24,000	25,900	13,800	15,100	16,300	330	360	390
Pennridge:									
1960		21,500			12,700			300	
1965	22,900	24,500	24,900	13,700	14,700	14,900	400	450	500
1970	24,000	27,000	28,200	14,600	16,500	17,200	500	520	550
1975	29,500	32,000	34,500	18,300	19,800	21,400	500	545	585
1980	36,000	38,100	40,000	22,700	23,400	25,200	510	570	600

TABLE D-16.—*Estimates of population, adults 16 and over and not enrolled in grades 1-12, and senior class enrollments, by school district 1960–1980 by 5-year intervals*—Continued

District year	Total population			Adults			Seniors		
	Low	Preferred	High	Low	Preferred	High	Low	Preferred	High
Central Bucks:									
1960		28,600			16,900			300	
1965	32,000	35,000	36,200	19,200	21,000	21,700	480	525	540
1970	35,700	38,200	40,100	21,800	23,300	24,500	550	570	600
1975	45,000	48,300	54,000	28,200	30,000	33,400	775	810	920
1980	51,800	60,000	65,000	32,600	37,800	40,800	780	900	975
New Hope-Solebury:									
1960		4,000			2,400			40	
1965	3,700	4,300	4,700	2,200	2,600	2,800	55	65	70
1970	3,900	5,000	5,900	2,400	3,000	3,600	70	90	105
1975	6,000	2,200	12,400	3,700	4,500	7,700	105	120	210
1980	8,500	9,500	19,500	5,400	6,000	12,300	130	140	330
Council Rock:									
1960		13,500			8,000			150	
1965	17,900	18,200	19,800	10,700	10,900	11,900	270	280	295
1970	19,100	20,100	22,500	11,700	12,300	13,700	345	375	405
1975	24,400	26,500	30,000	15,100	16,400	19,000	415	450	520
1980	27,300	34,700	52,100	17,200	21,800	32,800	410	520	780
Centennial:									
1960		24,300			14,300			245	
1965	32,700	34,000	36,300	19,600	20,400	21,800	490	510	545
1970	35,700	40,900	44,700	21,800	25,000	27,300	550	650	700
1975	53,300	55,200	57,100	33,100	34,300	35,400	905	940	970
1980	66,700	73,300	80,500	42,000	46,200	50,600	1,000	1,100	1,225
Neshaminy:									
1960		45,700			27,000			455	
1965	47,800	49,200	50,400	28,700	29,500	30,200	720	735	760
1970	56,900	61,900	64,000	34,700	37,800	39,000	1,025	1,070	1,150
1975	69,500	75,500	78,500	43,100	46,800	48,800	1,180	1,280	1,350
1980	86,300	94,900	98,500	54,400	59,800	62,000	1,295	1,420	1,475
Pennsbury:									
1960		42,600			25,100			500	
1965	45,300	47,600	49,000	27,200	28,600	29,400	680	695	715
1970	50,300	53,800	55,500	30,700	32,800	33,900	905	945	980
1975	59,400	65,000	67,600	36,800	40,300	41,800	1,010	1,105	1,150
1980	70,000	78,000	81,000	44,600	49,200	51,000	1,050	1,170	1,220
Morrisville:									
1960		7,800			4,600			80	
1965	8,000	8,600	9,600	4,800	5,500	5,800	120	140	145
1970	8,400	9,400	10,000	5,100	5,700	6,100	150	165	180
1975	8,000	8,900	9,800	5,000	5,500	6,100	135	150	165
1980	7,500	8,500	9,500	4,700	5,400	6,000	115	125	145
Bristol Township:									
1960		59,300			35,000			595	
1965	58,700	58,300	60,700	35,200	35,500	36,400	880	895	910
1970	62,900	63,800	67,100	38,400	38,900	40,900	1,130	1,170	1,205
1975	66,300	69,900	72,500	41,200	43,300	45,000	1,130	1,190	1,220
1980	66,700	74,700	77,700	41,900	47,000	49,000	1,000	1,125	1,165
Bristol Borough:									
1960		12,400			7,300			125	
1965	12,100	12,600	13,200	7,300	7,600	7,900	180	190	200
1970	11,600	12,200	13,600	7,100	7,400	8,300	210	230	245
1975	11,200	12,700	14,200	7,000	7,900	8,800	190	215	240
1980	10,900	12,600	14,300	6,900	7,900	9,000	165	190	215
Bensalem:									
1960		23.5			13,900			235	
1965	24,600	25,400	26.2	14,800	15,200	15,700	370	380	395
1970	28,600	31,600	33,000	17,400	19,300	20,100	515	545	600
1975	39,400	42,300	44,200	24,600	26,300	27,400	670	720	750
1980	51,900	54,100	56,300	32,100	34,000	35,500	780	810	845

TABLE D-17.—*Age distributions, Bucks County, 1950, 1960, 1970, 1980*

[Thousands]

Age group	1950	1960	1970	1980
0-4	15.3	42.8	39.5	64.5
5-9	13.2	38.6	43.2	57.4
10-14	10.1	30.4	48.0	53.6
15-19	9.4	19.2	39.9	48.4
20-24	10.0	13.4	26.4	50.2
25-29	12.2	19.9	21.0	49.0
30-34	12.4	26.5	16.9	43.8
35-39	11.2	27.5	24.3	35.2
40-44	9.8	22.8	30.7	25.6
45-49	8.2	17.2	28.6	29.4
50-54	7.6	13.0	23.2	32.3
55-59	7.1	10.0	16.3	28.1
60-64	5.9	8.3	11.2	21.8
65 on	12.0	18.9	24.1	35.2
Total	144.6	308.5	393.3	574.5

TABLE D-18.—*Dwelling units, 1950, 1960 and as authorized, 1960-67, Bucks County*

Year	Dwelling units[1] Occupied	Dwelling units[1] Total
1950	40,714	44,331
1960	83,327	89,483
Increase	42,613	45,152
Increase as percent of total	94.4	100
Annual average increase	4,261	4,515

Year	Authorized dwellings[2]
1960	1,693
1961	1,940
1962	2,068
1963	2,349
1964	3,843
Subtotal	11,893
1965	4,567
1966	5,969
1967 (4 months)	724
Total	23,153
Subtotal (1960-66)	22,429
Annual average 1960-66	3,210

[1] U.S. Census of Population and Housing, 1950 and 1960.
[2] Pennsylvania Department of Labor and Industry, Building Operations in Pennsylvania, monthly reports and annual summaries, 1960 to April 1967.

TABLE D-19.—*Approved lots—Bucks County Planning Commission final review, 1960-66*

	1960	1961	1962	1963	1964	1965	1966
Bedminster Township	66	30	18	17	9	7	30
Bridgeton Township					4	14	21
Durham Township	6	6					
E. Rockhill Township		12	6	7	31	7	32
Haycock Township	8	11			15	18	22
Hilltown Township	37	69	54	47	74	83	43
Milford Township	7			23	5	19	43
Nockamixon Township	5	4		7	74	6	16
Plumstead Township	59	80	39	13	20	15	28
Richland Township	11	3	2	47	4		20
Springfield Township	10	38	8			42	21
W. Rockhill Township		12	14	41	30	10	19

Source: Bucks County Planning Commission.

APPENDIX E

STATISTICAL TABLES

TABLE E–1.—*Total State and county shares of fall public school enrollment, State of Maryland, 1956–1966, grades 1 to 4*

County	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966
State total	208,251	209,631	212,490	219,182	227,720	235,315	242,393	253,415	253,938	261,017	268,859
Allegany	.0283	.0267	.0259	.0245	.0234	.0230	.0222	.0212	.0217	.0210	.0204
Anne Arundel	.0665	.0672	.0695	.0719	.0738	.0756	.0762	.0771	.0811	.0834	.0836
Baltimore City	.2910	.2919	.2868	.2787	.2706	.2656	.2618	.2586	.2265	.2118	.2148
Baltimore	.1405	.1405	.1424	.1438	.1451	.1466	.1482	.1480	.1511	.1510	.1491
Calvert	.0077	.0079	.0083	.0083	.0084	.0085	.0082	.0082	.0082	.0078	.0073
Caroline	.0079	.0077	.0076	.0077	.0076	.0073	.0073	.0068	.0069	.0068	.0064
Carroll	.0178	.0173	.0172	.0171	.0172	.0171	.0174	.0175	.0184	.0188	.0168
Cecil	.0184	.0174	.0165,	.0172	.0174	.0170	.0175	.0172	.0175	.0165	.0164
Charles	.0129	.0132	.0132	.0133	.0133	.0130	.0136	.0137	.0144	.0150	.0152
Dorchester	.0109	.0104	.0105	.0103	.0101	.0098	.0097	.0094	.0094	.0089	.0087
Frederick	.0242	.0237	.0233	.0228	.0224	.0221	.0224	.0224	.0231	.0228	.0221
Garrett	.0085	.0087	.0085	.0083	.0079	.0075	.0074	.0071	.0073	.0074	.0069
Harford	.0293	.0284	.0273	.0278	.0280	.0287	.0289	.0298	.0311	.0323	.0341
Howard	.0114	.0120	.0119	.0123	.0127	.0134	.0139	.0144	.0151	.0157	.0151
Kent	.0062	.0059	.0060	.0059	.0058	.0057	.0055	.0054	.0053	.0052	.0051
Montgomery	.1094	.1127	.1156	.1207	.1243	.1254	.1186	.1167	.1212	.1222	.1192
Prince Georges	.1138	.1146	.1169	.1189	.1221	.1286	.1328	.1395	.1525	.1650	.1735
Queen Annes	.0066	.0065	.0066	.0064	.0065	.0067	.0066	.0063	.0064	.0061	.0056
St. Marys	.0118	.0109	.0105	.0111	.0111	.0099	.0109	.0120	.0129	.0127	.0131
Somerset	.0082	.0080	.0079	.0076	.0074	.0072	.0070	.0065	.0064	.0062	.0037
Talbot	.0076	.0076	.0077	.0074	.0073	.0073	.0071	.0068	.0067	.0066	.0062
Washington	.0326	.0318	.0312	.0293	.0287	.0287	.0292	.0287	.0292	.0286	.0276
Wicomico	.0182	.0187	.0186	.0189	.0190	.0187	.0186	.0177	.0185	.0186	.0182
Worcester	.0102	.0102	.0103	.0096	.0096	.0091	.0090	.0088	.0090	.0088	.0087

Source: Annual reports of fall public school enrollments, State of Maryland, Department of Education.

TABLE E-2.—*Total State and county shares of fall public school enrollment, State of Maryland, 1956-1966, grades 5 to 8*

County	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966
State total	163,737	175,295	187,338	195,086	205,869	207,863	213,208	219,322	227,123	235,981	245,666
Allegany	.0313	.0311	.0308	.0299	.0281	.0273	.0264	.0249	.0239	.0223	.0213
Anne Arundel	.0681	.0689	.0696	.0725	.0717	.0715	.0738	.0757	.0776	.0794	.0805
Baltimore City	.2742	.2674	.2675	.2541	.2646	.2602	.2527	.2447	.2321	.2242	.2195
Baltimore	.1428	.1473	.1481	.1495	.1480	.1483	.1472	.1493	.1513	.1517	.1523
Calvert	.0075	.0071	.0071	.0072	.0071	.0075	.0076	.0073	.0074	.0074	.0072
Caroline	.0086	.0083	.0079	.0077	.0074	.0073	.0074	.0073	.0073	.0072	.0069
Carroll	.0210	.0202	.0197	.0194	.0188	.0182	.0177	.0177	.0177	.0178	.0181
Cecil	.0157	.0176	.0169	.0165	.0162	.0156	.0151	.0153	.0153	.0151	.0151
Charles	.0140	.0135	.0130	.0130	.0126	.0134	.0133	.0131	.0133	.0127	.0134
Dorchester	.0118	.0115	.0112	.0110	.0105	.0103	.0100	.0100	.0099	.0097	.0094
Frederick	.0275	.0273	.0264	.0266	.0253	.0244	.0241	.0239	.0238	.0234	.0231
Garrett	.0100	.0092	.0092	.0091	.0094	.0085	.0081	.0081	.0078	.0073	.0069
Harford	.0295	.0294	.0288	.0293	.0283	.0290	.0290	.0288	.0298	.0305	.0318
Howard	.0121	.0116	.0118	.0125	.0127	.0139	.0140	.0145	.0146	.0152	.0156
Kent	.0066	.0065	.0063	.0060	.0059	.0054	.0052	.0051	.0048	.0049	.0047
Montgomery	.1054	.1114	.1147	.1203	.1205	.1271	.1330	.1319	.1352	.1360	.1338
Prince Georges	.1110	.1140	.1154	.1198	.1204	.1237	.1287	.1348	.1411	.1492	.1562
Queen Annes	.0073	.0070	.0067	.0066	.0064	.0065	.0064	.0062	.0063	.0064	.0058
St. Marys	.0109	.0105	.0102	.0104	.0105	.0098	.0097	.0102	.0103	.0106	.0107
Somerset	.0090	.0084	.0082	.0079	.0075	.0074	.0072	.0071	.0070	.0069	.0064
Talbot	.0081	.0075	.0074	.0074	.0071	.0070	.0069	.0070	.0065	.0062	.0060
Washington	.0354	.0359	.0354	.0343	.0326	.0308	.0302	.0294	.0293	.0288	.0287
Wicomico	.0190	.0183	.0183	.0184	.0180	.0184	.0181	.0182	.0180	.0179	.0177
Worcester	.0103	.0100	.0095	.0096	.0093	.0093	.0091	.0092	.0092	.0091	.0088

Source: See table E-1.

TABLE E-3.—*Total State and county shares of fall public school enrollment, State of Maryland, 1956-1966, grades 9 to 12*

County	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966
State total	104,801	115,437	124,498	131,502	140,870	156,698	171,838	187,220	196,496	227,343	208,522
Allegany	.0465	.0454	.0376	.0349	.0362	.0330	.0329	.0318	.0302	.0276	.0272
Anne Arundel	.0607	.0722	.0664	.0679	.0748	.0711	.0713	.0721	.0728	.0685	.0748
Baltimore City	.2840	.2830	.2559	.2557	.2658	.2427	.2393	.2321	.2278	.2290	.2122
Baltimore	.1416	.1618	.1503	.1522	.1651	.1579	.1603	.1607	.1622	.1650	.1613
Calvert	.0069	.0073	.0064	.0062	.0066	.0062	.0059	.0061	.0062	.0056	.0063
Caroline	.0090	.0094	.0084	.0080	.0084	.0075	.0070	.0066	.0068	.0061	.0068
Carroll	.0227	.0246	.0209	.0201	.0214	.0191	.0189	.0186	.0180	.0161	.0173
Cecil	.0162	.0175	.0155	.0152	.0160	.0145	.0146	.0140	.0138	.0132	.0136
Charles	.0135	.0144	.0125	.0124	.0130	.0117	.0114	.0113	.0112	.0118	.0121
Dorchester	.0132	.0144	.0127	.0121	.0121	.0106	.0103	.0097	.0095	.0082	.0091
Frederick	.0307	.0327	.0285	.0274	.0293	.0268	.0258	.0352	.0246	.0233	.0244
Garrett	.0115	.0117	.0098	.0090	.0093	.0077	.0073	.0071	.0071	.0062	.0069
Harford	.0296	.0324	.0296	.0292	.0311	.0289	.0285	.0285	.0282	.0284	.0293
Howard	.0135	.0145	.0132	.0125	.0142	.0135	.0142	.0148	.0152	.0143	.0136
Kent	.0066	.0069	.0063	.0059	.0060	.0053	.0050	.0047	.0046	.0039	.0043
Montgomery	.1007	.1189	.1134	.1214	.1378	.1342	.1344	.1406	.1416	.1453	.1499
Prince Georges	.1085	.1233	.1154	.1165	.1265	.1216	.1268	.1318	.1363	.1399	.1490
Queen Annes	.0074	.0075	.0068	.0069	.0069	.0060	.0059	.0057	.0057	.0052	.0055
St. Marys	.0093	.0098	.0089	.0093	.0106	.0094	.0094	.0095	.0096	.0122	.0099
Somerset	.0085	.0095	.0083	.0078	.0080	.0069	.0068	.0064	.0062	.0053	.0057
Talbot	.0091	.0097	.0082	.0077	.0077	.0067	.0064	.0063	.0062	.0059	.0060
Washington	.0428	.0445	.0383	.0359	.0373	.0346	.0338	.0328	.0322	.0295	.0302
Wicomico	.0185	.0202	.0176	.0169	.0177	.0160	.0158	.0157	.0156	.0131	.0153
Worcester	.0101	.0104	.0092	.0089	.0092	.0081	.0080	.0079	.0082	.0073	.0083

Source: See table E-1.

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TABLE E-4.—*Shares of total public fall school enrollment, for California Statistical Areas, 1947–1966, grades 1 to 4*

Statistical area	1947	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966
North Coast	.0137	.0145	.0144	.0156	.0158	.0153	.0158	.0164	.0176	.0163	.0149	.0145	.0144	.0134	.0129	.0121	.0114	.0111	.0109	.0107
Sacramento Valley	.0502	.0507	.0516	.0521	.0526	.0520	.0524	.0533	.0528	.0536	.0528	.0533	.0544	.0556	.0564	.0561	.0557	.0550	.0553	.0552
Mountain	.0316	.0309	.0292	.0289	.0269	.0251	.0248	.0239	.0230	.0227	.0220	.0220	.0220	.0214	.0214	.0208	.0209	.0209	.0209	.0208
San Francisco Bay	.2180	.2164	.2207	.2249	.2272	.2305	.2303	.2306	.2201	.2271	.2262	.2258	.2243	.2241	.2240	.2241	.2242	.2231	.2228	.2247
Central Coast	.0281	.0268	.0269	.0270	.0257	.0255	.0246	.0240	.0234	.0239	.0233	.0237	.0239	.0240	.0242	.0242	.0241	.0241	.0244	.0247
San Joaquin Valley	.1585	.1535	.1546	.1471	.1431	.1368	.1301	.1252	.1200	.1177	.1156	.1141	.1094	.1087	.1060	.1027	.1005	.0991	.0992	.0991
Santa Barbara-Ventura	.0208	.0216	.0216	.0218	.0214	.0215	.0210	.0222	.0205	.0205	.0208	.0220	.0238	.0253	.0274	.0301	.0317	.0337	.0352	.0359
Los Angeles Metrop. Area	.3677	.3708	.3729	.3741	.3760	.3780	.3845	.3886	.3950	.3989	.4035	.4011	.4006	.4019	.4016	.4037	.4050	.4055	.4035	.4004
San Diego Metrop. Area	.0480	.0480	.0496	.0512	.0538	.0559	.0580	.0586	.0591	.0597	.0594	.0618	.0646	.0647	.0655	.0643	.0627	.0618	.0615	.0627
Southeast	.0623	.0589	.0594	.0576	.0571	.0577	.0580	.0583	.0588	.0594	.0609	.0618	.0613	.0606	.0605	.0617	.0638	.0656	.0662	.0659

Source: Bureau of Educational Research, "Enrollment in California Schools," *California Schools*, Various Issues 1947–1966.

NOTE.—Counties included in California State Statistical Areas are shown in Exhibit 9 on page 47.

TABLE E-5.—*Shares of total public school fall enrollment, for California Statistical Areas, 1947–1966, grades 5 to 8*

Statistical Area	1947	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966
North Coast	.0144	.0156	.0156	.0168	.0169	.0169	.0168	.0167	0166	.0156	.0148	.0144	.0144	.0142	.0133	.0126	.0122	.0118	.0116	.0113
Sacramento Valley	.0513	.0519	.0530	.0516	.0538	.0532	.0527	.0531	.0528	.0533	.0533	.0539	.0548	.0561	.0569	.0568	.0560	.0555	.0556	.0553
Mountain	.0315	.0326	.0315	.0318	.0309	.0290	.0279	.0266	.0250	.0240	.0233	.0233	.0230	.0229	.0229	.0227	.3227	.0228	.0226	.0223
San Francisco Bay	.2142	.2143	.2145	.2175	.2160	.2175	.2173	.2182	.2190	.2221	.2212	.2250	.2250	.2249	.2245	.2252	.2245	.2229	.2232	.2236
Central Coast	.0281	.0271	.0265	.0268	.0255	.0251	.0247	.0242	.0236	.0235	.0231	.0234	.0237	.0236	.0237	.0239	.0239	.0240	.0242	.0244
San Joaquin Valley	.1596	.1580	.1607	.1543	.1500	.1444	.1369	.1329	.1274	.1218	.1165	.1148	.1105	.1092	.1070	.1050	.1030	.1016	.1009	.0993
Santa Barbara-Ventura	.0214	.0219	.0217	.0218	.0214	.0213	.0208	.0209	.0206	.0208	.0209	.0222	.0235	.0251	.0267	.0283	.0298	.0318	.0330	.0340
Los Angeles Metropolitan Area	.3674	.0386	.3694	.3719	.3759	.3826	.3901	.3950	.4002	.4015	.4027	.4010	.4002	.4004	.4008	.4025	.4042	.4048	.4037	.4036
San Diego Metropolitan Area	.0489	.0490	.0470	.0482	.0490	.0502	.0513	.0524	.0541	.0569	.0598	.0613	.0636	.0636	.0633	.0620	.0607	.0605	.0603	.0613
Southeast	.0627	.0621	.0610	.0602	.0599	.0601	.0614	.0603	.0607	.0607	.0608	.0609	.0604	.0600	.0606	.0614	.0629	.0641	.0647	.0648

Source: See table E-4.

TABLE E-6.—*Shares of total public school fall enrollment, for California Statistical Areas, 1947–1966, grades 9 to 12*

Statistical Area	1947	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966
North Coast	.0137	.0140	.0142	.0151	.0157	.0162	.0166	.0165	.0166	.0161	.0154	.0146	.0144	.0135	.0133	.0127	.0123	.0124	.0120	.0118
Sacramento Valley	.0498	.0515	.0510	.0517	.0520	.0531	.0534	.0541	.0542	.0537	.0534	.0541	.0541	.0550	.0562	.0569	.0573	.0574	.0572	.0567
Mountain	.0276	.0289	.0298	.0297	.0295	.0297	.0291	.0283	.0281	.0276	.0262	.0249	.0244	.0237	.0237	.0232	.0235	.0237	.0238	.0237
San Francisco Bay	.2223	.2223	.2204	.2193	.2209	.2208	.2214	.2210	.2204	.2194	.2184	.2198	.2217	.2261	.2269	.2285	.2301	.2283	.2272	.2272
Central Coast	.0254	.0258	0260	.0263	.0250	.0247	.0243	.0238	.0233	.0227	.0225	.0224	.0225	.0225	.0225	.0226	.0226	.0227	.0230	.0235
San Joaquin Valley	.1370	.1326	.1354	.1357	.1329	.1321	.1296	.1287	.1251	.1203	.1156	.1130	.1092	.1054	.1027	.0995	.0972	.0967	.0967	.0968
Santa Barbara-Ventura	.0211	.0211	.0210	.0213	.0211	.0212	.0208	.0209	.0206	.0205	.0212	.0220	.0230	.0243	.0255	.0271	.0285	.0299	.0310	.0315
Los Angeles Metropolitan Area	.3909	.3938	.3966	.3942	.3946	.3929	.3948	.3957	.4007	.4089	.4119	.4112	.4096	.4070	.4052	.4055	.4053	.4054	.4080	.4054
San Diego Metropolitan Area	.0518	.0481	.0491	.0501	.0505	.0508	.0513	.0510	.0514	.0528	.0544	.0571	.0603	.0622	.0639	.0633	.0617	.0611	.0601	.0604
Southeast	.0554	.0569	.0565	.0566	.0578	.0585	.0587	.0600	.0596	.0600	.0610	.0609	.0608	.0601	.0600	.0606	.0614	.0623	.0629	.0630

Source: See table E-4.

TABLE E-7.—*Trend equations for county shares, State of Maryland, 1957 to 1962* [1]

County	Curve	A	B
		Grades 1 to 4	
Allegany	5	3.40243 E-3	1.61797 E-4
Anne Arundel	2	646.007	2.51456 E-2
Baltimore City	1	3004.	−55.8571
Baltimore	1	1384.43	13.5714
Calvert	4	84.9784	−8.42658
Caroline	1	79.5714	−.928571
Carroll	4	170.463	6.8496
Cecil	4	168.419	13.5257
Charles	4	133.945	−4.86471
Dorchester	1	102.857	−1.85714
Frederick	3	243.38	−4.73336 E-2
Garrett	5	1.09655 E-2	3.50647 E-4
Harford	2	283.15	0
Howard	5	9.02476 E-3	−2.50132 E-4
Kent	5	1.89627 E-2	2.82946 E-4
Montgomery	6	8.03422 E-4	1.21915 E-4
Prince Georges	5	9.12676 E-4	−2.06793 E-5
Queen Annes	2	65.1419	0
St. Marys	4	103.973	13.1856
Somerset	1	84.2857	−2.03571
Talbot	2	77.9335	−1.20734 E-2
Washington	3	329.478	−7.21078 E-2
Wicomico	4	189.156	−6.59321
Worcester	5	9.32148 E-3	2.49988 E-4
		Grades 5 to 8	
Allegany	1	328.429	−8.92857
Anne Arundel	5	1.49092 E-3	−1.74244 E-5
Baltimore City	1	2746.43	−29.2143
Baltimore	4	1405.17	−62.171
Calvert	3	72.4453	0
Caroline	3	86.8468	−8.96158 E-2
Carroll	5	4.6303 E-3	1.4266 E-4
Cecil	4	128.455	11.1137
Charles	3	188.42	−.101558
Dorchester	5	8.17494 E-3	2.5706 E-4
Frederick	5	3.49807 E-3	9.19445 E-5
Garrett	1	100.571	−2.82143
Harford	5	3.34379 E-3	3.37956 E-5
Howard	2	111.463	3.11723 E-2
Kent	5	1.40275 E-2	6.99347 E-4
Montgomery	1	1017.71	43.5714
Prince Georges	5	9.20086 E-4	−1.94735 E-5
Queen Annes	3	72.9384	−7.05974 E-2
St. Marys	1	109.857	−1.82143
Somerset	3	91.0132	−.114017
Talbot	3	80.5278	−7.68156 E-2
Washington	2	379.385	3.09091 E-2
Wicomico	4	180.179	9.15758
Worcester	3	103.335	−6.23629 E-2
		Grades 9 to 12	
Allegany	5	2.04075 E-3	1.58039 E-4
Anne Arundel	4	735.364	−117.842
Baltimore City	5	3.57151 E-4	7.86024 E-6
Baltimore	6	6.12809 E-4	8.47716 E-5
Calvert	5	1.37741 E-2	4.19876 E-4
Caroline	1	96.4286	−3.5
Carroll	1	242.286	−7.[illegible]43
Cecil	1	171.143	−3.67857
Charles	5	6.91952 E-3	2.49679 E-4
Dorchester	1	146.143	−6.03571
Frederick	1	324.143	−9.17857

[1] See Note end of Table E-9.

TABLE E-7.—*Trend equations for county shares, State of Maryland, 1957 to 1962*—Continued

County	Curve	A	B
Garrett	5	7.36552 E-3	8.7277 E-4
Harford	4	298.117	0
Howard	3	136.244	0
Kent	1	71.8571	−2.06429
Montgomery	3	1016.38	.15208
Prince Georges	3	1108.38	6.27316 E-2
Queen Annes	1	78.2857	−2.64286
St. Marys	4	96.5287	0
Somerset	1	94.8571	−3.78571
Talbot	2	102.402	−6.63886 E-2
Washington	2	452.198	−.043711
Wicomico	1	198.714	−5.85714
Worcester	5	9.15422 E-3	4.73496 E-4

TABLE E-8.—*Unadjusted sums of local area projected school enrollment shares, States of Maryland and California, 1965 and 1980* [1]

Grade groups	Maryland		California				
	1965	1980	1965		1980		
			.500	.750	.400	.500	.750
1-4	.9988	1.0034	.9738	.9762	.9809	1.0202	1.0039
5-8	1.0071	1.0277	1.0019	.9964	.9503	.9970	1.0098
9-12	.9768	1.0102	.9994	.9988	.9569	1.0003	1.0056

TABLE E-9.—*Trend equations for county shares, State of Maryland, 1956 to 1966* [1]

County	Curve	A	B
		Grades 1 to 4	
Allegany	5	3.50412 E-3	1.33199 E-4
Anne Arundel	1	642.873	17.9909
Baltimore City	1	3106.25	−84.6636
Baltimore	1	1393.4	11.1455
Calvert	4	81.6279	0
Caroline	1	81.0727	−1.39091
Carroll	5	5.92209 E-3	−4.37328 E-5
Cecil	4	167.82	14.5644
Charles	5	7.97052 E-3	−1.09092 E-4
Dorchester	1	110.545	−2.04545
Frederick	4	224.201	18.8039
Garrett	5	1.11726 E-2	2.94972 E-4
Harford	5	3.72123 E-3	−5.49042 E-5
Howard	1	108.109	4.39091
Kent	1	82.7455	−1.06364
Montgomery	6	8.14623 E-4	1.05451 E-4
Prince Georges	5	9.50872 E-4	−3.17405 E-5
Queen Annes	1	67.8908	−.663636
Saint Marys	5	9.60033 E-3	−1.44616 E-4
Somerset	1	89.2545	−3.34545
Talbot	1	79.5818	−1.4
Washington	3	327.545	−6.44151 E-2
Wicomico	3	186.056	0
Worcester	5	9.51737 E-3	1.95327 E-4

TABLE E-9.—*Trend equations for county shares, State of Maryland, 1956 to 1966*—Continued

	Curve	A	B
		Grades 5 to 8	
Allegany	1	334.415	−10.6909
Anne Arundel	5	1.49542 E-3	−2.26181 E-5
Baltimore City	1	2525.29	−53.0152
Baltimore	3	1438.44	2.08151 E-2
Calvert	4	72.9814	0
Caroline	3	86.6256	−8.57505 E-2
Carroll	3	211.991	−7.80668 E-2
Cecil	3	187.313	−9.48567 E-2
Charles	6	7.72185 E-3	−5.29295 E-4
Dorchester	5	8.32909 E-3	2.09862 E-4
Frederick	5	3.55632 E-3	7.44802 E-5
Garrett	1	100.327	−2.69091
Harford	3	287.293	0
Howard	2	111.68	3.07686 E-2
Kent	5	1.41492 E-2	6.70567 E-4
Montgomery	3	1034.6	.115035
Prince Georges	5	9.42074 E-4	−2.58557 E-5
Queen Annes	3	73.2928	−7.56578 E-2
Saint Marys	2	103.868	0
Somerset	5	1.10484 E-2	3.87285 E-4
Talbot	1	80.6727	−1.76364
Washington	5	2.65658 E-3	8.29482 E-5
Wicomico	3	188.392	−2.14817 E-2
Worcester	3	103.057	5.94027 E-2

TABLE E-9.—*Trend equations for county shares, State of Maryland, 1956 to 1966*—Continued

	Curve	A	B
		Grades 9 to 12	
Allegany	5	2.07312 E-3	1.46388 E-4
Anne Arundel	4	733.986	−115.846
Baltimore City	2	2909.94	−2.19373 E-2
Baltimore	4	1637.19	−207.009
Calvert	3	71.2238	7.50016 E-2
Caroline	5	1.02109 E-2	5.18925 E-4
Carroll	5	4.07612 E-3	1.74452 E-4
Cecil	2	171.398	−2.37018 E-2
Charles	3	140.575	−8.57705 E-2
Dorchester	2	148.204	−.050784
Frederick	5	3.07019 E-3	1.08265 E-4
Garrett	5	7.80904 E-3	7.25887 E-4
Harford	5	3.44421 E-3	0
Howard	5	7.63601 E-3	−8.989 E-5
Kent	2	73.8854	−5.45912 E-2
Montgomery	3	1012.03	.156797
Prince Georges	5	9.15604 E-4	−2.01624 E-5
Queen Annes	5	1.25684 E-2	5.81553 E-4
Saint Marys	5	1.00233 E-2	0
Somerset	2	97.2944	−5.24148 E-2
Talbot	5	1.00491 E-2	6.83268 E-4
Washington	5	2.21542 E-3	1.06162 E-4
Wicomico	5	5.05959 E-3	1.67245 E-4
Worcester	3	106.208	−.129454

NOTE.—The coefficients for the trend equations for each of the 24 counties and three grade group combinations displayed in tables E-7 and E-9 are printed in exponent or "E" form. The sign and number following the "E" in each case indicates the direction and placement of the decimal point for the alpha and beta values of the trend equation. A− (minus) sign immediately following the "E" indicates that the decimal point should be moved to the left of its original position. A+ (plus) sign immediately following the "E" would indicate that the decimal point should be moved to the right of its original position—actually no plus entries appear in these tables. A number value following the minus (or plus) sign gives the number of places the decimal point should be moved either to the left or to the right. For example, the notation "E−4" means move the decimal point four positions to the left of the original one.

The designations of the curves are as follows (see table 27): simple linear curvilinear, log linear, log log, hyperbolic, and exponential.

TABLE E-10.—*State of California, 1965 Comparison of Actual and Projections, Parts I, II, and III*

Statistical area	Actual, grade groups			Projected linear, grade groups			Projected curvilinear, grade groups			Percent error[1]—linear, grade groups			Percent error[1]—curvilinear, grade groups		
	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12
	Part I.—Comparison of actual and projected enrollment ratios														
North Coast	.0109	.0116	.0120	.0161	.0150	.0165	.0154	.0143	.0158	47.7	29.3	37.5	41.3	23.3	31.7
Sacramento Valley	.0553	.0556	.0573	.0560	.0555	.0563	.0563	.0555	.0559	1.3	−0.2	−1.8	1.8	−0.2	−2.5
Mountain	.0209	.0226	.0238	.0148	.0169	.0235	.0154	.0168	.0227	−29.2	−25.2	−1.3	−26.3	−25.7	−4.6
San Francisco Bay	.2228	.2233	.2273	.2314	.2312	.2200	.2294	.2318	.2201	4.3	3.6	−3.2	3.0	3.9	−3.2
Central Coast	.0244	.0242	.0230	.0204	.0202	.0206	.0206	.0205	.0205	−16.[illegible]	−16.5	−10.4	−15.6	−15.3	−10.9
San Joaquin Valley	.0992	.1009	.0967	.0800	.0824	.1011	.0811	.0815	.0985	−19.4	−18.3	4.6	−18.3	−19.2	1.3
Santa Barbara-Ventura	.0352	.0330	.0310	.0223	.0219	.0220	.0225	.0223	.0224	−36.7	−33.6	−29.0	−36.1	−32.4	−27.8
Los Angeles-Long Beach Metropolitan Area	.4036	.4038	.4059	.4264	.4293	.4154	.4262	.4274	.4182	5.6	6.3	2.3	5.6	5.8	3.0
San Diego Metropolitan Area	.0615	.0603	.0601	.0718	.0688	.0605	.0714	.0698	.0615	16.7	14.1	.7	16.1	15.8	2.3
Southeast	.0662	.0647	.0629	.0608	.0588	.0641	.0617	.0601	.0644	−8.2	−9.1	1.9	−6.8	−7.1	7.6
	Part II.—Comparison of actual and projected enrollments State of California, 1965 (numbers in thousands)														
State total	1387.7	1258.4	1110.5	1406.7	1232.9	1100.0	1406.7	1232.9	1100.0	1.4	−2.0	−1.0	1.4	−2.0	−1.0
North Coast	15.2	14.7	13.3	22.6	18.5	18.1	21.7	17.6	17.4	48.7	25.9	36.1	42.8	19.7	30.8
Sacramento Valley	76.7	70.0	63.7	78.8	68.4	61.9	79.2	68.4	61.5	2.7	−2.3	−2.8	3.3	−2.3	−3.5
Mountain	29.0	28.5	26.4	20.8	20.8	25.8	21.7	20.7	25.0	−28.3	−27.0	−2.2	−25.2	−27.4	−5.3
San Francisco Bay	309.2	280.9	252.3	325.5	285.0	242.0	322.7	285.8	242.1	5.3	1.5	−4.1	4.4	1.7	−4.1
Central Coast	33.9	30.5	25.5	28.7	24.9	22.7	29.0	25.2	22.5	−15.3	−18.4	−11.0	−14.5	−17.4	−11.8
San Joaquin Valley	137.7	127.0	107.4	112.5	101.6	111.2	114.1	100.5	108.4	−18.3	−20.0	3.5	−17.1	−20.9	0.9
Santa Barbara-Ventura	48.8	41.5	34.4	31.4	27.0	24.2	31.7	27.5	24.6	−35.7	−34.9	−29.7	−35.1	−33.7	−28.5
Los Angeles-Long Beach Metropolitan Area	560.0	508.0	450.9	600.0	529.3	456.9	599.5	526.9	460.0	7.1	4.2	1.3	7.1	3.7	2.0
San Diego Metropolitan Area	85.4	75.9	66.7	101.0	84.8	66.6	100.4	86.1	67.7	18.3	11.7	−0.2	17.6	13.4	1.5
Southeast	91.9	81.5	69.9	85.5	72.5	70.5	86.8	74.1	70.8	−7.0	−11.1	0.9	−5.5	−9.1	1.3

Part III.—Mean, standard deviations, and coefficients of variation in percent error of school enrollment projections

	Coefficient estimates						Enrollment estimates					
	Projected linear			Projected curvilinear			Projected linear			Projected curvilinear		
	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12
$\bar{X}$, percent error	18.6	15.6	9.3	17.1	14.9	9.6	18.7	15.7	9.2	17.3	14.9	9.0
S, percent error	14.4	10.6	12.4	13.1	9.9	10.5	14.1	10.9	12.3	12.8	10.3	10.8
V, coefficient of variation	.77	.68	1.33	.77	.66	1.09	.75	.69	1.34	.74	.69	1.20

[1] Percent calculated on basis of actual values.

1965 estimates obtained by applying the ratios shown in table E-10 part I, to the State totals shown in table 35, projection series D-1.

NOTES.—1965 projected enrollment ratios derived from actual ratios for the years 1947 to 1959 inclusive.

TABLE E-11.—*Exponential smoothing trend equations for Statistical Area shares of total State Enrollment, State of California, period 1947–1960*

Statistical area	.500			.750		
	A	B	C	A	B	C
	Grades 1 to 4					
North Coast	.01342	−.00098	−.00016	.01341	−.00108	−.00023
Sacramento Valley	.05570	.00083	.00011	.05577	.00119	.00034
Mountain	.02149	−.00039	.00001	.02140	−.00076	−.00020
San Francisco Bay	.22406	−.00244	−.00056	.22393	−.00242	−.00034
Central Coast	.02404	.00012	.00005	.02399	−.00007	−.00004
San Joaquin Valley	.10856	−.00131	.00052	.10965	−.00071	.00093
Santa Barbara-Ventura	.02531	.00166	.00027	.02530	.00158	.00021
Los Angeles-Long Beach Metropolitan Area	.40149	−.00369	−.00157	.40156	−.00217	−.00024
San Diego Metropolitan Area	.06496	.00091	−.00009	.06472	−.00021	−.00077
Southeast	.03079	−.00078	−.00030	.06056	−.00155	−.00067
	Grades 5 to 8					
North Coast	.01416	−.00024	.00003	.01421	−.00008	.00010
Sacramento Valley	.05603	.00120	.00020	.05610	.00145	.00034
Mountain	.02289	−.00004	.00015	.02290	−.00003	.00015
San Francisco Bay	.22543	.00044	−.00031	.22491	−.00158	−.00142
Central Coast	.02366	.00021	.00009	.02361	−.00004	−.00007
San Joaquin Valley	.10898	−.00197	.00057	.10916	−.00119	.00103
Santa Barbara-Ventura	.02509	.00170	.00029	.02511	.00171	.00027
Los Angeles-Long Beach Metropolitan Area	.40077	−.00146	−.00073	.40036	−.00005	.00014
San Diego Metropolitan Area	.06392	.00110	−.00019	.06363	−.00010	−.00090
Southeast	.06003	−.00040	−.00009	.06000	−.00052	−.00014
	Grades 9 to 12					
North Coast	.01351	−.00081	−.00011	.01351	−.00085	−.00015
Sacramento Valley	.05488	.00051	.00007	.05499	.00093	.00031
Mountain	.02363	−.00086	−.00003	.02370	−.00059	.00012
San Francisco Bay	.22571	.00348	.00075	.22608	.00495	.00156
Central Coast	.02250	.00010	.00008	.02250	.00007	.00006
San Joaquin Valley	.10536	−.00393	−.00013	.10540	−.00382	−.00009
Sante Barbara-Ventura	.02427	.00134	.00022	.02430	.00142	.00026
Los Angeles-Long Beach Metropliltan Area	.40736	−.00236	−.00102	.40698	−.00362	−.00162
San Diego Metropolitan Area	.06241	.00290	.00035	.06223	.00212	−.00017
Southeast	.06018	−.00053	−.00020	.06011	−.00084	−.00038

TABLE E-12.—*Actual and projected enrollment by State statistical area, State of California, 1965, by 4-4-4 organization*

[Numbers in thousands]

	Actual enrollment			Projected enrollment, population projection D-1[1]						Percent error					
				.500			.750			.500			.750		
	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12
North Coast	15.2	14.7	13.3	14.1	16.8	11.7	13.2	17.8	11.3	−7.2	14.4	−12.5	−12.7	21.1	−15.0
Sacramento Valley	76.7	70.0	63.7	84.8	74.5	62.4	87.8	76.7	65.2	10.6	6.4	−2.0	14.5	9.6	2.5
Mountain	29.0	28.5	26.4	29.4	28.7	23.0	26.2	29.1	24.8	1.3	0.8	−12.9	−9.9	2.1	−6.2
San Francisco Bay	309.2	280.9	252.3	309.5	277.3	263.6	310.0	264.5	273.1	0.1	−1.3	4.5	0.3	−5.8	8.3
Central Coast	33.9	30.5	25.5	35.6	30.5	25.5	34.0	28.7	25.3	5.1	−0.2	−0.1	0.5	−5.8	−0.9
San Joaquin Valley	137.7	127.0	107.4	154.6	130.1	102.4	159.5	136.4	103.0	12.3	2.4	−4.6	15.8	7.4	−4.1
Santa Barbara-Ventura	48.8	41.5	34.4	45.4	38.7	32.2	44.6	39.0	32.8	−7.0	−6.8	−6.4	−8.7	−6.2	−4.8
Los Angeles-Long Beach Metropolitan Area	560.0	508.0	450.9	553.7	482.8	435.5	567.7	496.0	428.3	−1.1	−5.0	−3.4	1.4	−2.4	−5.0
San Diego Metropolitan Area	85.4	75.9	66.7	97.2	81.7	80.3	87.4	73.4	74.7	13.8	7.7	20.3	2.3	−3.3	11.9
Southeast	91.9	81.5	69.9	82.4	71.9	63.5	76.2	71.5	61.6	−10.3	−11.8	−9.2	−17.0	−12.2	−11.9
Total	1387.7	1258.4	1110.5	1406.7	1232.9	1100.0	1406.7	1232.9	1100.0	1.4	−2.0	−0.9	1.4	−2.0	−0.9

	Projected enrollment, population projection I & II[1]						Percent error					
	.500			.750			.500			.750		
	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12	1-4	5-8	9-12
North Coast	14.2	16.8	11.7	13.3	17.8	11.3	−6.8	14.7	−12.6	−12.4	21.4	−15.0
Sacramento Valley	85.2	74.6	62.4	88.1	76.9	65.2	11.1	6.6	−2.1	14.9	9.8	2.4
Mountain	29.5	28.8	23.0	26.3	29.2	24.7	1.7	1.0	−12.9	−9.5	2.3	−6.3
San Francisco Bay	310.8	277.9	263.5	311.3	265.0	273.1	0.5	−1.1	4.4	0.7	−5.6	8.2
Central Coast	35.7	30.5	25.5	34.2	28.8	25.3	5.5	0.4	−0.1	0.9	−5.6	−1.0
San Joaquin Valley	155.2	130.4	102.4	160.2	136.7	102.9	12.7	2.6	−4.6	16.3	7.6	−4.1
Santa Barbara-Ventura	45.6	38.8	32.2	44.8	39.0	32.8	−6.6	−6.6	−6.4	−8.3	−6.0	−4.8
Los Angeles-Long Beach Metropolitan Area	556.0	483.9	435.4	570.1	497.1	428.1	−0.7	−4.8	−3.4	1.8	−2.1	−5.1
San Diego Metropolitan Area	97.6	81.9	80.3	87.7	73.5	74.7	14.3	8.0	20.3	2.7	−3.1	11.9
Southeast	82.8	72.0	63.5	76.6	71.7	61.6	−10.0	−11.6	−9.2	−16.7	−12.0	−12.0
Total	1412.6	1235.6	1099.7	1412.6	1235.6	1099.7	1.8	−1.8	−1.0	1.8	−1.8	−1.0

[1] California Population Projection Series

TABLE E-13.—*Summary comparisons of projected and observed shares and enrollment by grade groups and Statistical Areas, California, 1965*

	Shares					
	.500			.750		
	1-4	5-8	9-12	1-4	5-8	9-12
$\overline{X}$, mean percent error	6.94	7.16	7.26	8.29	7.53	6.75
S, standard deviation percent error	4.55	4.71	6.06	6.51	6.73	4.51
V, coefficient of variation	.66	.66	.83	.79	.89	.67
	Enrollment, Population Projection D-1					
	.500			.750		
	1-4	5-8	9-12	1-4	5-8	9-12
$\overline{X}$, mean percent error	6.87	5.66	7.59	8.3.	7.59	7.06
S, standard deviation percent error	4.65	4.52	5.85	6.34	5.42	4.34
V, coefficient of variation	.68	.80	.77	.76	.71	.61
	Enrollment, Population Projection I & II					
	.500			.750		
	1-4	5-8	9-12	1-4	5-8	9-12
$\overline{X}$, mean percent error	6.98	5.72	7.61	8.43	7.56	7.37
S, standard deviation percent error	4.72	4.52	5.84	6.19	5.51	3.88
V, coefficient of variation	.68	.79	.77	.73	.73	.55

TABLE E-14.—*Exponential smoothing trend equations for Statistical Area shares of total State enrollment, and 1980 projected shares, State of California, data period 1947–1966*

	.50 smoothing constant			.75 smoothing constant		
	A	B	C	A	B	C
	Grades 1 to 4					
North Coast	0.01069	−0.00016	0.00009	0.01070	−0.00012	0.00010
Sacramento Valley	.05515	−.00019	−.00004	.05521	.00002	.00007
Mountain	.02065	−.00009	.00002	.02061	−.00030	−.00011
San Francisco Bay	.22441	.00089	.00031	.22466	.00205	.00101
Central Coast	.02468	.00028	.00005	.02470	.00037	.00010
San Joaquin Valley	.09904	.00016	.00048	.09912	.00036	.00056
Santa Barbara-Ventura	.03606	.00117	−.00013	.03591	.00056	−.00048
Los Angeles-Long Beach Metropolitan Area	.40072	−.00248	−.00063	.40040	−.00378	−.00137
San Diego Metropolitan Area	.06237	.00015	.00012	.06268	.00143	.00086
Southeast	.06616	.00050	−.00009	.06591	−.00055	−.00071
	Grades 5 to 8					
North Coast	0.01130	−0.00025	0.00004	0.01130	−0.00025	0.00004
Sacramento Valley	.05526	−.00039	−.00009	.05531	−.00022	−.00000
Mountain	.02235	−.00017	−.00001	.02230	−.00035	−.00011
San Francisco Bay	.22341	−.00013	.00000	.22360	.00063	.00043
Central Coast	.02440	.00021	.00003	.02440	.00023	.00004
San Joaquin Valley	.09942	−.00094	.00023	.09932	−.00142	−.00006
Santa Barbara-Ventura	.03410	.00126	−.00005	.03400	.00088	−.00026
Los Angeles-Long Beach Metropolitan Area	.40362	−.00043	−.00024	.40357	−.00052	−.00025
San Diego Metropolitan Area	.06104	.00021	.00010	.06129	.00118	.00065
Southeast	.06496	.00058	−.00003	.06481	−.00004	−.00039
	Grades 9 to 12					
North Coast	0.01148	−.00052	−0.00004	0.01180	−0.00022	0.00001
Sacramento Valley	.05593	−.00098	−.00024	.05670	−.00065	−.00030
Mountain	.02313	−.00040	−.00003	.02370	−.00010	−.00009
San Francisco Bay	.22732	−.00043	−.00018	.22717	−.00025	.00014
Central Coast	.02313	.00009	.00001	.02350	.00056	.00016
San Joaquin Valley	.09647	−.00020	.00036	.09681	.00050	.00054
Santa Barbara-Ventura	.03199	.00142	.00007	.03151	.00040	−.00035
Los Angeles-Long Beach Metropolitan Area	.40453	−.00084	−.00017	.40542	−.00050	−.00028
San Diego Metropolitan Area	.06382	.00246	.00040	.06038	.00030	.00045
Southeast	.06184	−.00059	−.00018	.06301	.00005	−.00028

TABLE E-14.—*Exponential smoothing trend equations for Statistical Area shares of total State enrollment, and 1980 projected shares, State of California, data period 1947–1966*—Continued

	1980 projected shares					
	1 to 4		5 to 8		9 to 12	
	.500	.750	.500	.750	.500	.750
North Coast	0.0165	0.0175	0.0121	0.0117	0.0046	0.0100
Sacramento Valley	.0474	.0589	.0407	.0515	.0183	.0185
Mountain	.0210	.0057	.0190	.0064	.0142	.0138
San Francisco Bay	.2620	.3337	.2226	.2723	.2027	.2365
Central Coast	.0332	.0377	.0301	.0309	.0254	.0407
San Joaquin Valley	.1458	.1511	.1096	.0730	.1283	.1560
Santa Barbara-Ventura	.0386	.0148	.0467	.0203	.0588	.0027
Los Angeles-Long Beach Metropolitan Area	.2984	.2023	.3750	.3682	.3752	.3690
San Diego Metropolitan Area	.0744	.1581	.0739	.1398	.1370	.1083
Southeast	.0627	.0202	.0703	.0259	.0355	.0385

TABLE E-15.—*Variation among grade groups, projected 1980 shares, State of California, 0.500 smoothing constant*

	Shares				Proportion of average		
	1-4 (a)	5-8 (b)	9-12 (c)	Average (d)	1-4 (e)	5-8 (f)	9-12 (g)
North Coast	0.0165	0.0121	0.0046	0.0111	1.486	1.090	0.414
Sacramento Valley	.0474	.0407	.0183	.0355	1.335	1.146	.515
Mountain	.0210	.0190	.0142	.0181	1.160	1.050	.785
San Francisco Bay	.2620	.2226	.2027	.2291	1.144	.971	.885
Central Coast	.0332	.0301	.0254	.0296	1.122	1.017	.858
San Joaquin Valley	.1458	.1096	.1283	.1279	1.140	.857	1.003
Santa Barbara-Ventura	.0386	.0467	.0588	.0480	.804	.973	1.225
Los Angeles-Long Beach Metropolitan Area	.2984	.3750	.3752	.3495	.854	1.073	1.074
San Diego Metropolitan Area	.0744	.0739	.1370	.0951	.782	.777	1.441
Southeast	.0627	.0703	.0355	.0562	1.116	1.250	.632

Appendix F

THE MULTIPLE REGRESSION APPROACH

The idea behind this approach is simple enough; we seek out a number of factors in an effort to explain changes in the proportion of the State's enrollment found in a given county. For example, one county might have a continuously higher birth rate and thus have a higher proportion of children enrolled in public school. Accordingly, this approach was tried for Maryland. Unfortunately, it did not provide projections more accurate than those obtained from the procedures discussed in chapters 5 to 8. Furthermore, the amount of preparation involved in using it is very considerable. Nevertheless, with the thought that it might prove to be useful in some other States, we are presenting it here. Only empirical testing beyond the scope of this study will determine how useful it may turn out to be.

The multiple regression equation or "cohort-ratio" is derived from the observed relationship of the changes in a number of different symptomatic data series to changes in shares of total school enrollment accounted for by a local political unit. In the case of the Maryland tests, there were three dependent variables for each of the 24 counties representing the three grade groups—the "4–4–4" organization of enrollment. Each dependent or "Y" variable in the regression equation represents the ratio of the county's share of the total State enrollment in a given year; for example, 1956 to its share in the following year, 1957. Thus for each grade-grouping, and each county in the experiments, there was a total of 10 dependent variables representing cohort-ratios for the years 1956 to 1966. The independent or "X" variables are expressed in the same way. The "X" variables in the experiments were:

Variable:	*Symbol*
Resident Births by County	X_1
Resident Deaths by County	X_2
Retail Sales by County	X_3
Number of Households by County	X_4
Effective Buying Income by County	X_5
Resident Income Tax Returns filed by County	X_6
Registered Private Vehicles by County	X_7
Resident Income Paid by County	X_9

A total of 75 multiple regression equations were derived from the basic data. Each of the 24 counties had three multiple regression equations fitted to the basic data—one for each of the three grade groupings. In addition, all the data points for all the counties were used to derive three separate multiple regression equations for the State of Maryland. The object of the 25th set of estimating equations was to test the forecasting efficiency of the "State" versus the individually tailored county forecasting enrollment ratios. As an example of the application of the technique, the results of the regression analyses are shown in tables F–1 to F–8 and cover the years 1956 to 1962, inclusive, for Calvert County, Md.

For making projections of school enrollment, it is only necessary to project each of the independent variables to the target date. Then, utilizing the multiple regression equation constants, the county's share of the total State enrollment is ultimately obtained.

Specific Steps in the Approach

There are a total of seven steps in the use of the multiple-regression or cohort-ratio approach.

Step 1. The compilation and computation of the shares of State annual series on school enrollment, the dependent or "Y" variables, and the independent or "X" variables described next.

Step 2. Computation of the cohort ratios of the dependent and independent variables.

Step 3. Development of multiple regression estimating relationships of the cohort ratio of county share of total State school enrollment, using the annual series of data for the independent or "X" variables developed in Step 2. This was accomplished for the three grade groupings for each county and for the "State."

Step 4. Fitting statistical trend functions to the significant independent or "X" variables derived in Step 3.

Step 5. Extrapolation of trend functions of significant independent or "X" variables derived in Step 3.

Step 6. Solution of multiple regression equation derived in Step 3, using projected values for significant independent or "X" variables derived in Step 5.

Step 7. Application of projected county enrollment shares of total State enrollment to independently derived total State enrollment projections.

Data Base Development and Description

In order to implement the school enrollment ratio, cohort-ratio multiple regression approach for Maryland, several historical series on demographic and economic changes in the counties had to be developed. The historical period covered in the tests was from 1955 to 1966.

The several demographic and economic historical series were derived from a variety of public and private published and unpublished sources.

County school enrollment ratios for grade groups 1–4, 5–8 and 9–12 were derived from *Statistics on Enrollment and Number of Schools, Public and Nonpublic, State of Maryland*, published annually by the Division of Research and Development, Maryland State Department of Education.

Resident births and deaths by county in Maryland, covering the period from 1955 to 1966, were derived from unpublished vital statistics series provided the project and prepared by the Division of Biostatistics, Department of Health, State of Maryland.

Statistics on the number of privately registered automobiles by county for the period 1955 to 1966 were obtained from the Director of Public Information, Department of Motor Vehicles, State of Maryland. The tallies for each county represented the total number of privately owned motor vehicles registered as of April 30 in each year.

Historical series on the number of households, volume of retail sales, and effective buying income per household for counties in Maryland, were derived from data published annually in the "Survey of Buying Power" published by *Sales Management* magazine about the second week in June. The data published cover each State, county and State of Maryland Statistical Areas (SMSA).

Data on the annual number of resident income tax returns filed by county in Maryland, as well as the residential income tax take per county, were derived from *Resident Income Tax Returns for the Year* ——, published by the Comptroller of the Treasury, Income Tax Division, State of Maryland. The summary report on the volume of resident income tax returns filed by county covered the period from 1959 to 1965. To fill in the gaps, trend lines were fitted to the available data for each county and estimates derived for the years 1956 to 1958 and 1966.

Two additional historical series were considered originally for the analysis, but were dropped subsequently because of significant lacunae in the data. U.S. Bureau of the Census, *Construction Reports, Building Permits*, Series C–40, contains monthly and annual summaries of building permits let by permit-issuing places in the United States. Out of the 24 counties in Maryland, only 15 were effectively covered under the Census series. Collection of building permit data from the counties was considered impractical and time consuming. U.S. Bureau of the Census, *County Business Patterns*, contains fairly detailed data on the industrial affiliation of the labor force of each county in the United States and would have proved of some assistance in the Maryland tests. However, the series has been published continuously and annually only since 1964. Prior to that time, the series was published every 3 years. A check with the State of Maryland, Bureau of Employment Security, revealed that continuous historical data on labor force composition by county was available only for the counties comprising the Baltimore SMSA and for Allegany and Washington, the latter designated as economically distressed areas in the Appalachia Program of the U.S. Department of Commerce, Economic Development Administration. At some future date, the U.S. Census *County Business Patterns* labor force estimates by county should be considered as a substitute or additional independent variable for possible inclusion in the county level school enrollment cohort-ratio multiple regression estimating relationships.

Data on utility installations by county were not considered feasible for the purposes of the tests. The costs of collecting the data annually, with the exception of telephone installations, by county in Maryland would be prohibitive.

TABLE F-1.—*Shares of total State enrollment, demographic and economic characteristics, Calvert County, Md., 1956 to 1965*

	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965
Grades 1-4	0.0077	0.0079	0.0083	0.0083	0.0084	0.0085	0.0082	0.0082	0.0082	0.0078
Grades 5-8	.0075	.0071	.0071	.0072	.0071	.0075	.0076	.0073	.0074	.0074
Grades 9-12	.0069	.0073	.0064	.0062	.0066	.0062	.0059	.0061	.0062	.0056
Births	.0057	.0062	.0055	.0059	.0059	.0058	.0065	.0065	.0059	.0063
Deaths	.0053	.0054	.0060	.0049	.0054	.0048	.0052	.0055	.0048	.0059
Private registered vehicles	.0046	.0045	.0046	.0035	.0047	.0047	.0047	.0048	.0047	.0047
Number of households	.0041	.0044	.0043	.0041	.0043	.0044	.0045	.0045	.0047	.0047
Private income taxes filed	.0022	.0026	.0030	.0031	.0032	.0032	.0035	.0036	.0038	.0038

Sources: See text.

TABLE F-2.—*Cohort ratios of shares of total State enrollment, demographic and economic characteristics, Calvert County, Md., 1956 to 1965*

	1957/1956	1958/1956	1959/1958	1960/1959	1961/1960	1962/1961	1963/1962	1964/1963	1965/1964	1966/1965
Grades 1-4	1.0260	1.0506	1.0000	1.0120	1.0119	0.9647	1.0000	1.0000	0.9512	0.9358
Grades 5-8	.9467	1.0000	1.0141	.9861	1.0563	1.0133	.9605	1.0137	1.0000	.9730
Grades 9-12	1.0580	.8767	.9688	1.0645	.9394	.9516	1.0339	1.0164	.9032	1.1250
Births	1.0877	.8871	1.0727	1.0000	.9831	1.1207	1.0000	.9077	1.0678	1.0159
Deaths	1.0189	1.1111	.8167	1.1020	.8889	1.0833	1.0577	.8727	1.2292	1.0000
Private registered vehicles	.9782	1.0222	1.0000	.7608	1.3428	1.0000	1.0212	.9791	1.0000	.9787
Number of households	1.0732	.9773	.9535	1.0488	1.0232	1.0227	1.0000	1.0444	1.0000	.9574
Private income taxes filed	1.1818	1.1538	1.0333	1.0322	1.0000	1.0937	1.0286	1.0555	1.0000	1.0000

Source: Table F-1.

TABLE F-3.—*Partial regression and correlation coefficients, Calvert County, Md., grades 1 to 4, 1956 to 1962*

Variables	Partial regression coefficients	Partial correlation coefficients	Beta values	t-values	Point elasticities	Contribution to R^2
X_4	−0.3448	−0.8304	−1.0324	−2.1076	−0.3490	−0.6668
X_5	−.1028	−.4802	−.4463	−.7741	−.1030	−.0900
X_6	−.0466	−.3944	−.3012	−.6070	−.0470	−.0553
X_7	.2613	.5162	.4067	.8525	.2626	−.1091

Intercept; 1.2479.
Multiple correlation coefficient—R; 0.8365.
Coefficient of determination—R^2; 0.6098.
Standard error of estimate—Se; 0.0250.
F-value; 1.1654.

TABLE F-4.—*Partial regression and correlation coefficients Calvert County, Md., grades 5 to 8, 1956 to 1962*

Variables	Partial regression coefficients	Partial correlation coefficients	Beta values	t-values	Point elasticities	Contribution to R^2
X_4	−0.0217	−0.0506	−0.0467	−0.0716	−0.0222	−0.0014
X_5	−.0525	−.1493	−.1639	−.2136	−.0533	−0.0121
X_6	.1066	.4682	.4948	.7494	.1088	−.1493
X_7	−.1790	−.2186	−.2011	−.3168	−.1830	−.2011

Intercept; 1.1460.
Multiple correlation coefficient—R; 0.6844.
Coefficient of determination—R^2; 0.4684.
Standard error of estimate—Se; 0.0463.
F-value; 0.4406.

TABLE F-5.—*Partial regression and correlation coefficients, Calvert County, Md., grades 9 to 12, 1956 to 1962*

Variables	Partial regression coefficients	Partial correlation coefficients	Beta values	t-values	Point elasticities	Contribution to R^2
X_4	−0.0025	−0.0041	−0.0028	−0.0058	−0.0025	−0.0000
X_5	−.2933	−.5150	−.4805	−.8497	−.3012	−.1043
X_6	−.2677	−.6883	−.6529	−1.3417	−.2767	−.2599
X_7	1.1640	.7185	.6836	1.4008	1.1087	−.3081

Intercept; 0.3759.
Multiple correlation coefficient—R 0.8433.
Coefficient of determination—R^2 0.7112.
Standard error of estimate—Se 0.0650.
F-value; 1.2315.

TABLE F-6.—*Trend equations*[1] *and estimated values for shares of total State demographic and economic characteristics, Calvert County, Md.*

		Estimated values		
		shares		Cohort ratios[2]
		1962	1965	
Births	$y=0.0058+0.00007x$	0.0060	0.0062	1.0327
Deaths	$y=0.0053-0.00008x$	.0051	.0048	.9412
Private registered vehicles.	$y=0.0045-0.00001x$	.0045	.0044	.9777
Number of households	$y=0.0043+0.00004x$	.0044	.0045	1.0227
Private income taxes filed	$y=0.0030+0.0002x$	.0036	.0042	1.1666

[1] Of the formula $y=a+bx$

where y=share of total State characteristic
a=constant
b=slope
x=number of years

Trend line was fitted to share data over period from 1957 to 1962.

[2] Computed by dividing 1965 estimated value by 1962 estimated value.

TABLE F-7.—*Derivation of 1956 to 1962, enrollment group cohort-ratio multiple regression, Calvert County, Md.*

Coefficients	(1) Estimated 65/62 cohort ratios	(2) Grades 1–4, estimating relations	(3) ((1) × (2))	(4) Grades 5-8, estimating relations	(5) ((1) × (4))	(6) Grades 9–12, estimating relations	(7) ((1) × (6))
a/Constant		1.2470	1.2479	1.1460	1.1460	0.3750	0.3750
X_4/Births	1.0327	−.3448	−.3561	−.0217	−.0224	−.0025	−.0026
X_5/Private registered vehicles	.9412	−.1028	−.0087	−.0525	−.0494	−.2933	−.2760
X_6/Number of households	.9777	−.0466	−.0456	.1066	.1042	−.2677	−.2617
X_7/Private income taxes filed	1.0227	.2613	.2672	−.1799	−.1840	1.1640	1.1904
Sum		1.0130		0.9985			1.0260

TABLE F-8.—*Projection of 1965 enrollment shares, by cohort ratios, "4-4-4" grade organization of enrollment, Calvert County, Md.*

Grade groups	(1) 1962 enrollment share	(2) Estimation, 1965/1962 cohort ratio	(3) ((1) × (2))	(4) Actual 1965 enrollment ratio
1-4	0.0082	1.0130	0.0083	0.0078
5-8	.0076	.9985	.0076	.0074
9-12	.0059	1.0260	.0061	.0056

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