DOCUMENT RESUME

ED 056 986 SP 005 384

AUTHOR Unkel, Esther R.
TITLE A Study of the Laboratory Approach and Guided Discovery in the Teaching Learning of Mathematics by Children and Prospective Teachers.
INSTITUTION Florida Atlantic Univ., Boca Raton.
SPONS AGENCY National Center for Educational Research and Development (DHEW/CE), Washington, D.C.
BUREAU NO BR-9-D-033
PUB DATE Sep 71
GRANT OEG-4-0-190033-0051-057
NOTE 15p.

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS *Discovery Learning; *Elementary School Mathematics; Elementary School Teachers; *Manipulative Materials; Mathematics Teachers; *Methods Courses; *Preservice Education; Teacher Education; Tutoring; Underachievers

ABSTRACT

This study tested 1) whether instruction based on guided discovery, the use of expensive manipulative aids, and maximum pupil participation increases achievement in mathematics for pupils in grades 1-6; 2) whether a program based largely on learning through activity and guided discovery is effective in the training of prospective teachers enrolled in the methods course in mathematics for the elementary teacher. The pupils were selected by classroom teachers as being underachievers in mathematics on the basis of standardized tests and observations. These pupils were tutored twice weekly by undergraduates enrolled in the methods class. Twenty-nine of the 66 pupils in grades 1 through 6 completed the full 9-month period, the others missing one quarter of instruction because they could not attend at a time when it was possible for an undergraduate to tutor them. A total of 72 undergraduates were included. Each was given a pretest to determine his basic mathematical knowledge, followed by a posttest at the end of the quarter. The t test was used to determine the exact degree of improvement. Results showed that there was a significant difference in achievement for the children in grades 1, 2, 3, 5, and 6. There was no difference in grade 4. There was a statistically significant improvement in knowledge of basic mathematical concepts for undergraduates enrolled in the methods course. (Author/RT)

ABSTRACT

A STUDY OF THE LABORATORY APPROACH AND GUIDED DISCOVERY IN THE LEARNING OF MATHEMATICS BY CHILDREN AND PROSPECTIVE TEACHERS

The aim of this study was twofold: (1) to test whether instruction based on guided discovery, the use of inexpensive manipulative aids and maximum pupil participation will increase achievement in mathematics for pupils in grades 1-6; and (2) to test whether a program based largely on learning through activity and guided discovery is effective in the training of prospective teachers enrolled in the method's course in mathematics for the elementary teacher.

The pupils were selected by classroom teachers as being underachievers in mathematics in view of their potential on the basis of standardized tests and observations. These pupils were tutored twice weekly by undergraduates enrolled in the method's class. Twenty-nine of the 66 pupils in grades 1-6 completed the full nine month period, the others missing one quarter of instruction because they could not attend at a time when it was possible for an undergraduate to be free to tutor them.

A total of 72 undergraduates were included in this study. Each undergraduate was given a pretest to determine his basic mathematical knowledge and followed by a post-test at the end of the quarter.

The t test was used to determine the exact degree of improvement.

1. Results showed that there was a statistically significant difference in achievement for the children, grades 1-6, as a result of the nine-month program in guided discovery as follows: (a) arithmetic reasoning--at the .05 level in grade 1 and at the .01 level in grades 2 and 6, (b) arithmetic fundamentals--at the .01 level in grades 1, 2, 5, and 6, and at the .05 level in grade 3, and (c) arithmetic totals--at the .01 level in grades 1, 2, 3, and 6, and at the .05 level in grade 5. There was no difference in grade 4 in any of the areas.

2. There was a statistically significant difference in achievement in basic mathematical concepts at the .001 level for undergraduates enrolled in the method's course.

Final Report

Project No. 9-D-033
Grant or Contract No. OEG-4-9-190033-0051-057

A Study of the Laboratory Approach and Guided Discovery in the Learning of Mathematics By Children and Prospective Teachers

Dr. Esther Unkel

Florida Atlantic University

Boca Raton, Florida

September 1971

The research reported herein was performed pursuant to a grant with the Office of Education, U.S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent offical Office of Education position or policy.

U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

Office of Education
National Center for Educational Research and Development

TABLE OF CONTENTS

LIST OF TABLES

A STUDY OF THE LABORATORY APPROACH AND GUIDED DISCOVERY IN THE LEARNING OF MATHEMATICS BY CHILDREN AND PROSPECTIVE TEACHERS

Despite increased amounts of money spent on education in the last ten years through federal grants, the use of teacher aids, reduction of pupils in classes from the previous average of forty pupils per classroom prior to 1960, and greatly increased teacher salaries, the achievement of elementary pupils in mathematics is too often below that of their potential. Underachievement in mathematics becomes a serious problem for children of all socioeconomic levels as early as the fifth grade, Unkel (11), when scores plunge below previous achievement and below the pupils' potential to achieve.

In light of this, it seems that (1) the training of teachers in method courses should be studied in the possibility that training could be more effective, and (2) to try a method of pupil instruction involving understanding rather than rote, varied practice in place of meaningless drill, and the use of simple manipulative aids and pupil participation to replace the use of sheer memory.

The aim of this study was twofold: (1) to test whether instruction based on guided discovery, the use of inexpensive manipulative aids, and maximum pupil participation will increase achievement in mathematics for pupils in grades one through six; and (2) to test whether a program based largely on learning through activity and guided discovery is effective in the training of prospective teachers in mathematics for the elementary teacher.

Related Research

The transfer of learning in problem solving, other than routine problems, is facilitated by learning through discovery and guided discovery as found by Scandura (10) and the methods became increasingly efficient as the problem increased in difficulty. Ray (8) and Rowlett (9) found that a discovery approach is more effective over a longer period of time.

Paradowski (7) found that making use of a child's curiosity resulted in improved learning of both the intentional task and incidental learning.

Much research substantiates the fact that pupils learn more then working with a peer as his partner. Fox and Lorge (2); Hall, Mouton, and Blake (4); Lorge and Solomon (6).

Research has determined that intermittent practice is preferable to long periods of concentrated practice. Duncan (1); Lorge (5).

These research findings were incorporated in this study. Instruction was given in the form of task cards whereby a child was directed to perform a task, usually working with a peer since research showed this to be effective. The task cards would allow for the transfer of learning in problem solving and give varied practice. An example of this is guided discovery in finding how to determine volume by first filling various boxes with inch cubes, keeping a record of the length, width, height of the box, and the number of cubic inches it took to fill each box. Estimation is important and the pupil is asked to record the number of inch cubes he thinks it will take to fill the box. After several short tasks using cubic blocks, he is then asked to get two odd shaped boxes, estimate the number of cubic inches or blocks it would take to fill the box, and then find the volume of the box. This is challenging to children and piques their curiosity to the point where they took odd shaped bottles as well as boxes to find the volume. They were much faster than the undergraduates in seeing that they could fill the boxes with sand or rice and then pour the contents into a box of rectangular or square shape and find the volume quickly by noting what they had by now discovered, that volume is found by knowing the number of inches in the width, length, and height of the receptacle and multiplying these to find the volume. They then made a cubic foot from tagboard to find the number of cubic feet of air in the classroom. They made use of varied practice in finding the volume of swimming pools, buildings, and many other things inside and outside the classroom which helped to reinforce the learning.

Procedure
(for the Elementary Pupils)

The pupils involved in this study came from one school in Boca Raton, Florida, sufficiently close so the

children could walk to the mathematics laboratory on the campus of Florida Atlantic University to be tutored by undergraduates enrolled in the method's course in the teaching of mathematics to elementary children. The school is integrated and the children come from low and middle socioeconomic levels. A total of twenty-nine children were instructed twice weekly throughout the nine-month period. A total of sixty-six children were tutored most of the time, but various pupils were unable to come one or part of one of the quarters at a time when an undergraduate was free to work with them. A few children moved away but no child left the program of his own choice. Only those who had been tutored throughout the nine-month period were used in the study.

The tutoring took place at the end of the school day, each undergraduate having a maximum of four children. In addition to the task cards which were corrected by the instructor before being used with the children, emphasis was on the use of inexpensive manipulative aids. The children worked a maximum of fifteen minutes before taking a break of three to five minutes. Virtually no writing was done by the pupils who spent their time actively engaged in weighing, measuring, using counters, a number line with moveable pieces, and numerous other aids.

The children were selected by the classroom teachers as being underachievers in mathematics in view of their potential as shown by standardized tests given at the beginning of the school year. The teachers' observation of the child's work and his general ability were used in selecting the children. Three children were not recommended by the teachers but came on their own initiative and continued to benefit from the program.

The pupils were given two pretests the second week in October, administered individually or in small groups of two to six children. The California Mental Maturity Test, Short Form, 1966 norms, and the arithmetic section of the California Achievement Test, Form X, 1966 norms, were used in October. Form W of the California Achievement Test, the arithmetic section, was given to each child as a post test the third week in May.

Procedure
(for the Undergraduates in Method's Class)

A total of seventy-two undergraduates were included in this study, which comprised all the students taking the method's course in mathematics for the winter and spring

quarter and tutoring underachievers in mathematics in the sections taught by Dr. Unkel. The pupils in grades one through three were tutored twice weekly for a thirty-minute period which included a short break; children in grades four through six received fifty minutes of tutoring biweekly.

Each undergraduate was given the Glennon Basic Test of Mathematical Knowledge (3) at the beginning of the quarter, followed by a post test at the end of the quarter an adaptation of the Glennon test, covering the same number and type of items.

A weekly seminar was attended by each undergraduate immediately following one of the two tutoring periods, with the undergraduates freely discussing the work, asking questions, sharing the humor which evolved as the children worked, and planning future work.

Each undergraduate worked with a peer during class periods, performing the work suggested by the task cards and getting the manipulative aids he chose from the shelves along the walls. The regular classroom chairs were removed and tables and straight chairs were used. Six undergraduates worked at a table, the instructor moving about among the tables to discuss the work with the students. Lectures were limited to less than an hour of a three-hour class period and learnings were crystallized only after the students had an opportunity to work, discuss, and think about the content.

Statistical Analysis
(of the Results Achieved by Elementary Pupils)

The powerful t test was used to determine the exact degree of improvement which the elementary pupils had made. The bias of systematic error was avoided because we took all the underachievers in mathematics as designated by the classroom teachers.

The statistical analysis of pupil achievement for the nine-month period was divided into three sections: arithmetic reasoning, arithmetic fundamentals, and arithmetic totals.

Table 1 shows the means, the mean differences, and the t ratios in each of the three sections, grades one through six.

Table 1

Comparative Data and t Ratios for Each Subtest

Number Level	Number Tested	Subtest	Mean 1	Mean 2	Mean of Differences	t
1	4	A. Reasoning	1.30	1.65	.34	2.916*
		A. Fundamentals	1.15	1.75	-.60	5.263**
		A. Totals	1.28	1.65	-.38	-5.723**
2	13	A. Reasoning	1.83	2.36	.61	5.295**
		A. Fundamentals	1.90	2.60	-.71	7.585**
		A. Totals	1.82	2.48	-.65	-7.611**
3	7	A. Reasoning	3.47	-.4	-.41	1.696
		A. Fundamentals	3.40	4.74	1.34	3.566**
		A. Totals	3.40	4.58	-1.14	4.19**
4	4	A. Reasoning	4.75	5.50	-.75	1.584
		A. Fundamentals	4.95	6.82	-.68	2.697
		A. Totals	4.92	5.62	-.70	1.980
5	5	A. Reasoning	5.63	6.56	-.93	-2.604
		A. Fundamentals	5.60	6.80	-1.20	10.3896**
		A. Totals	5.66	6.73	-1.07	4.743*
6	6	A. Reasoning	6.01	6.54	-.53	-6.170**
		A. Fundamentals	6.17	7.68	-1.49	4.76**
		A. Totals	6.25	7.42	-1.06	4.755**

*Significant at .05
**Significant at .01

Statistical Analysis
(of the Results Achieved by University Undergraduates)

The t test was used to determine the exact degree of improvement in achievement in mathematical understanding for the undergraduates. The number of subjects exceeded thirty but the critical ratio was not used because Underwood's direct difference method gives the same result as does the critical ratio with a correction for correlations.

Table 2 shows the means, the mean differences, and the t ratios for growth in mathematical understanding for the seventy-two undergraduates enrolled in the method's course in mathematics for the elementary teachers.

Table 2

Comparative Data of the Increase in Mathematical Understanding of Prospective Teachers

Pretest Mean 1	Post-test Mean 2	Mean of Differences	t
31.47	17.64	13.83	14.198***

***Significant at .001 level

Discussion

Table 1 related to the data for the elementary pupils. The fourth grade level showed the least statistically significant difference in achievement. The small sample of four children might be thought to be responsible but grade one also had a sample of four subjects while grades three, five, and six had samples small in number. It seems possible that a cumulation of content in grade four could be a major contributing factor. Many facts are crowded into this grade, including the multiplication and division facts, and the new topics of rational numbers and word problems.

More time should have been devoted to arithmetic reasoning and although no effort was made to work on topics

studied currently by the child in school, much time was spent on developing an understanding of mathematical principles concerning the four processes and the use of laws in helping the pupil learn the facts associated with addition, subtraction, multiplication, and division.

Table 2 related to the improvement in achievement of undergraduates in understanding basic mathematical principles. The degree of improvement was unexpected and surprising. A great many of the undergraduates missed only half as many items on the post-test as they missed on the pretest. Certainly the combination of tutoring and a weekly seminar succeeding one of the two weekly tutoring periods had an influence, but comments made by numerous students testified to the fact that they had never understood much of the basic mathematics originally until they "saw" what they were doing. Use of manipulative aids, task cards, discussions with a peer in performing the tasks and particularly the constant use of estimation seemed to be largely responsible for their improvement in understanding.

Conclusions

1. There was a statistically significant difference in achievement in arithmetic reasoning at the .05 level in grade one and at the .01 level in grades two and six as a result of a nine-month training program in guided discovery.

2. There was a statistically significant difference in achievement in arithmetic fundamentals at the .01 level in grades one, two, five, and six, and at the .05 level in grade three as a result of a nine-month training program in guided discovery.

3. There was a statistically significant difference in achievement in arithmetic totals in all grades except grade four as a result of a nine-month training program in guided discovery. The increase in achievement was significant at the .01 level except in grade five where it was significant at the .05 level.

4. There was a statistically significant difference in achievement in basic mathematical concepts at the .001 level for undergraduates enrolled in the method's course in mathematics for the elementary teacher as a result of one quarter's instruction in guided discovery combined with tutoring biweekly and a seminar each week.

The increase in achievement through the use of guided discovery and the interest which the children had in the program points to this method as having promise. The attention given to the individuals or to a small group of children was a factor but it must be remembered that these children came to the mathematics laboratory following a full day at school. It is impossible to determine the effect that the normal school work had on the total achievement of the children.

Previous rote learning was a factor in hindering some of the work in grades one and two in particular. It was difficult to get the children to stop counting to find addition and subtraction facts and to use the parts of a given number in finding the facts, i.e., in finding the fact 8 – 2, the child would show eight fingers, turn two fingers down, and count the remainder while we sought to establish all the facts associated with a given number such as 8 (6,2 5,3 4,4 and 7,1 with the reverses) before having the child write the answers to drill exercises. A great variety of practice was used to do this. He might use the floor number line to "walk" the part of 8, fusing the processes of addition and subtraction; he might use the trundle wheel to measure 8 yards down the hall, then use a tape measure to find 6 yards and then find how many yards he was short of 8 yards; he might bowl with 8 tenpins, quickly finding the number of pins he had knocked down by noting the number standing; two pins standing meant that 6 were down, the parts which make up 8. Unless this work is accomplished first, the pupils will use primitive methods, chiefly counting, to find the answers to exercises.

Guided discovery through the use of task cards and manipulative aids shows promise as a method of instruction as evidenced by the great improvement in scores for undergraduates. The writing of task cards and the use of them in tutoring elementary pupils was probably a factor but most of the improvement came as a result of increased understanding of decimal fractions and little work was done in this with elementary children due to the time limit. Most of the work in decimal fractions resulted from a thorough understanding of basic structure of the decimal system and work with odometers. Seventy-five percent of the instructional time was devoted to simple work in decimals with sheets divided into tenths on one side and hundredths on the other, the use of odometers, dollar bills, ten dollar bills, dimes and pennies, and work with place value charts. The task cards included a variety of different situations involving decimals: placing money on

place value charts, using the odometer in finding distances--distances in field day events and other sport events--using fraction sets in combination with coins and the tenth and hundredth sheets, and becoming thoroughly familiar with decimal fractions before advancing to multiplication and division of decimal fractions.

REFERENCES

1. Duncan, C. P. "The Effect of Unequal Amounts of Practice on Motor Learning Before and After Rest," Journal of Experimental Psychology, 42:257-264. 1951

2. Fox, D. J., and Lorge, I. "The Relative Quality of Decisions Written By Individuals and By Groups as the Available Time for Problem Solving Is Increased," Journal of Social Psychology, 57:227-242. 1962

3. Glennon, Vincent J. "Basic Test of Mathematical Knowledge." Storrs: University of Connecticut, 1948. (Mimeographed.)

4. Hall, E. J., Mouton, Jane, and Blake, R. R. "Group Problem Solving Effectiveness Under Conditions of Pooling Versus Interaction," Journal of Social Psychology, 59:147-157. 1963

5. Lorge, I. "Influence of Regularly Interpolated Time Intervals Upon Subsequent Learning," Teacher College Contributions to Education, No. 438. 1930

6. Lorge, I., and Solomon, H. "Group and Individual Performance in Problem Solving Related to Previous Exposure to Problem Level of Aspiration and Group Size," Behavioral Science, 5:28-38. 1960

7. Paradowski, William. "Effect of Curiosity on Incidental Learning," Journal of Educational Psychology, 58, No. 1:50-55. 1967

8. Ray, W. E. "Pupil Discovery Versus Direct Instruction," Journal of Experimental Education, 29:271-280. 1961

9. Rowlett, J. D. "An Experimental Comparison of Direct-detailed and Directed-discovery Methods of Presenting Tape-recorded Instruction," Status of Research in Industrial Arts, Fifteenth Yearbook of the American Council on Industrial Arts Teacher Education, Illinois. 1966

10. Scandura, J. M. "An Analysis of Exposition and Discovery Modes of Problem Solving Instruction," Journal of Experimental Education, 33:149-159. 1964

11. Unkel, Esther. "A Study of the Interaction of Socio-economic Groups and Sex Factors with the Discrepancy between Anticipated Achievement and Actual Achievement in Elementary School Mathematics," The Arithmetic Teacher, 13:662-670. 1966