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ABSTRACT

This paper traces the growth of the concept of a mathematics laboratory and reviews recent research and developments in this field. The first section quotes several interpretations of the term and discusses some of the activities advocated by its proponents. The second section quotes extensively from E. H. Moore (1902) and McLennan and Dewey (1895) to show that the idea is older than the present influence of Piaget, Bruner, Gattegno, etc. A section of quotations from more recent advocates of mathematics laboratories is followed by a review of research on the use of manipulative materials, desk-calculators, and science-linked courses: the correlation of motivation with achievement; and the practical difficulties of implementing a laboratory approach in a school. The final sections discuss laboratory materials and the use of laboratory methods in teacher training. (MM)

About Mathematics Laboratories

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The concept of a mathematics laboratory has become very popular in recent years. The phase has blossomed into popularity so fast that the variety of meanings which have been attributed to it all are struggling to co-exist. This paper will attempt to describe in detail some of the meanings of the phrase, summarize the results of research related to mathematics laboratories, and list some of the recent developments which have occurred and are relative to the paper.

It should be mentioned that the writer has spent the past few years promoting the concept of mathematics laboratories when working with both pre-and-in-service teachers. While we have attempted to remain objective, it is natural that the selection of content which we deem pertinent will be affected. At times our biases will show through in a blatent manner.

Various Meanings of Mathematics Laboratories

From the literature one can gather many different interpretations of the meaning of a mathematics laboratory. One of the more influential books to be published in the recent years is Freedom to Learn by Biggs and MacLean (1). In their forward they state:

"The phrases active learning, discovery method and laboratory approach have become part of our educational jargon the past few years. What do these phrases mean?

For children, these phrases mean an approach to learning that presents a wide variety of opportunities; an approach that encourages them to ask questions and find the answers; an approach that fosters the use of physical materials; an approach that gives the experience designed to help them analize and abstract; and an approach that provides a chance to develop their individual potential.

For teachers, these phrases mean an opportunity to explore and discover new and better ways of teaching mathematics; an opportunity to develop an awareness of mathematical possibilities; and an opportunity to use a highly motivated approach for more efficient education".

Another interpretation of what is meant by a mathematics laboratory is provided by the Nuffield Project booklet entitled How to Build a Pond (2). The booklet is an account of how one class of 9 and 10 year old children built a duck pond for a dozen tiny ducklings that were given to them. In the process of building the pond the children studied the various mathematical aspects inherent in the project, including measurement, graphing, drawing to scale, concepts of area and volume, and all of the arithmatic associated with these activities.

Still another example is provided by the Nuffield Project film and booklet entitled I Do and I Understand (3). The film describes the procedure for another mathematics classroom of 9 and 10 year olds in England. The children work in small groups of two or three on specific projects which are described on assignment cards in the classroom. All of the projects appeared to begin by manipulating physical materials and abstracting mathematical concept from the manupulation. The tasks include measuring on maps and globes, weighing things, measuring distances, making graphs, making estimates, outdoor work, and a variety of other activities.

Still another example of a mathematical laboratory is portrayed in the Cuisenaire Company film, "Numbers in Color", in which Caleb Gattegno is working with an entire class of children. Each of the children has a set of Cuisenaire Rods to work with individually, but all of the children in the classroom are working on the same mathematical task at the same time.

It's conceivable that a child sitting at a typewriter terminal which is hooked to a computer assisted instructional program could be thought of as working in a mathematics laboratory.

Another example of a mathematics laboratory would be kinds of outdoo measuring activities such as surveying and map making. This kind of activity has been used by teachers for many years. The National Councel of Teachers of Mathematics published a yearbook dealing with the topic in 1947 (4).

In 1954 the NCTM published another yearbook entitled Emerging in Mathematics Education (5). One of the major sections of this yearbook was entitled "Laboratory Teaching in Mathematics". In the first section, we find the statement

> "Laboratory Techniques has long been used in public schools in such areas as science, dramatics, home economics, and shop. Teachers have long been urged to use laboratory techniques in the teaching of mathematics. Enough teachers are doing that, so that we may well consider laboratory teaching as one of the emerging practices in teaching mathematics".

In a later section, Myers suggested several activities involving both physical and statistical measurements, representing data graphically, and making geometric models. Beyond that the chapter strays into a discussion of the use of radio, television, and slide presentations and the use of simple models such as painted spools and flannel boards as demonstration devices. The rationale for the use of such materials is given by Grossnickle:

> "It is not the manipulation of materials as such, but the use of the material which vitalizes instruction in arithmetic. If a child is able to make discoveries and generalizations in quantitative situations by use of symbols, he should not use manipulative materials. On the other hand, if he cannot deal understandingly with quantitative situations by use of symbols, he should use objective materials to discover relationships among quantities. The pupil should be encouraged at all times to operate at the highest level of abstraction at which he understands the work".

Still another example of a mathematics laboratory would be the Encyclopedia Brittanica Workshop. (6) These materials present paper and pencil experiences for children in which the children are presented patterns. Discovering patterns is also the object of the work presented by Walter in [illegible], Squares and Other Things (7). In this book, Walter describes how she presented the ideas to children in elementary classes. The work contains a combination of manipulation of physical materials with abstracting and generalizing. For example, the children are asked to visualize what a milk carton would look like if they cut it along the edges and flattened it out. Then, they actually cut the carton and verified the results.

Because of the popularity of the phrase, the commercial companies are jumping onto the band wagon. A blatently obvious example of this is the "mathematics laboratory" published by McCormick-Mathers Publishing Company. This "mathematics laboratory" consists of a cardboard block which contains 573 cards with each card presenting from 6 to 40 exercises intended for children from grades 3-6. These cards are intended;

> "to provide adequate practice for the development of speed and accuracy in mathematical computation for all students. It is aimed at tracking down and eliminating some of the students major difficulties with arithmetical operations".

Most mathematics laboratories employ some element of self-selection by the student. The effects of self-selection on learning mathematics were studied over a 3-year period by Ebeid, Fitzgerald and Snyder at the University of Michigan Lab school from 1962-1965 (8,9,10). At that time many of the concrete manipulative materials which are now available were not yet developed. At the conclusion of the three years of study, Snyder drew the following conclusions:

> "Students and teachers who have been involved in self-selection programs agree that some degree of student choice should be provided. While not all students are capable of full-time independent work, all students could work independently to varying degrees and, when self-selection was practiced to a limited extent, students achievement did not stop when measured by traditional objectives.
>
> While materials suitable for independent study are becoming available, there is a great need for more. Careful planning, preparation, and material selection will be necessary if the self-selection principle is to be given a thorough test".

Origins of and Motivations for Mathematics Laboratories

Current writers generally attribute the forces behind the development of mathematics laboratories to the work of Piaget, Bruner, Gattegno, etc. While it is true that this relatively recent work has done much to stimulate developments that are now taking place, we can find the seeds of the ideas in the literature of several decades ago.

One of the more influential proponents of mathematics laboratories was E.H. Moore (11). In his presidential address before the American Mathematical Society in 1902, he discussed the state of abstract and applied mathematics, then went on to discuss the state of the teaching of mathematics. In discussing elementary mathematics he said.

> "The fundamental problem is that of the unification of pure and applied mathematics. If we recognize the branching implied by the very terms 'pure', 'applied', we have to do with a special case of the correlation of different subjects of the curriculum, a central problem in the domain of pedagogy from the time of Herbart on. In this case, however, the fundamental solution is to be found rather by way of indirection--by arranging the curriculum so that throughout the domain of elementary mathematics the branching will not be recognized.
>
> Would it not be possible for the children in the grades to be trained in power of observation and experiment and reflection and deduction so that always their mathematics would be directly connected with matters of thoroughly concrete character? The response is immediate that this is being done today in the kindergartens and the better elementary schools. I understand that serious difficulties arise with children of nine to twelve years of age, who are no longer contented with the simple, concrete method of earlier years and who, nevertheless, are unable to appreciate the more abstract methods of the later years. These difficulties only implicitly in connection with the other subjects of the curriculum.

But rather the materials and methods of the mathematics should be enriched and vitalized. In particular, a grade teachers must make wiser use of the foundations furnished by the kindergarten. The drawing and the paper folding must lead on directly to a systematic study of intuitional geometry, including the construction of models and the elements of mechanical drawing with simple exercises in geometrical reasoning. The geometry must be closely connected with the numerical and literal arithmetic. The cross-grooved tables of the kindegarten furnish an especially important type of connection, viz , a conventional graphical depiction of any phenomena in which one magnitude depends upon another. These tables and the similar cross-section blackboards and paper must enter largely into all the mathematics of the grades. The children are to be taught to represent, according to the usual conventions, various familiar and interesting phenomena and to study the properties of the phenomena in the pictures: to know, for example, what concrete meaning attaches to the fact that a graph curve at a certain point is going down or going up or is horizontal. Thus, the problem of percentage-interest, etc.--have their depiction in straight or broken line graphs".

It is interesting to compare the description of the program as Moore would like to see it with some of the work represented by the Nuffield Foundation booklet entitled Pictorial Representations.

Moore also referred to activities taking place at that time in England when he quotes:

"Perry is quite right in insisting that it is scientifically legitimate in the pedagogy of elementary mathematics to take a large body of basal principles instead of a small body and to build the edifice upon the larger body for the earlier years, reserving for the later years the philosophic criticism of the basis itself and the reduction of the basal system".

Moore went on to describe his proposed program in more detail:

This program of reform calls for the development of a thorough-going laboratory system of instruction in mathematics and physics, a principle purpose being as far as possible to develop on the part of every student the true spirit of research, and an appreciation practical as well as theoretic, of the fundamental methods of science".

"As the world of phenomena received attention by the individual, the phenomena are described both graphically and in terms of number and measure: the number and measure relations of the phenomena enter fundamentally into the graphical depiction, and furthermore the graphical depiction of the phenomena serves powerfully to illuminate relations of number and measure. This is the fundamental scientific point of view. Here under the terms of graphical depiction I include representations by models.

"In the development of the individual in his relation to the world there is no initial separation of science into constituent parts, while there is ultimately a branching into the many distinct sciences. The troublesome problem of the closer relation of pure mathematics to its application: can it not be solved by indirection, in that through the whole course of elementary mathematics, including the introduction to the calculus, there be recognized in the organization of the curriculum no distinction between the various branches of pure mathematics, and likewise no distinction between pure mathematics and principle applications? Further, from the standpoint of pure mathematics: will not the twentieth century find it possible to give to young students during their impressionable years, in thoroughly concrete and captivating form, the wonderful new notions of the seventeenth century?

"By way of suggestion these questions have been answered in the affirmative, on condition that there be established a thorough-going laboratory system of instruction in primary schools, secondary schools, and junior colleges--a laboratory system involving a synthesis and development of the best pedagogic methods at present in use in mathematics and the physical sciences".

Seven years before Moore delivered his remarkable address, we find a description of the balance between the manipulation of concrete objects and the symbolization of concepts which is necessary for effective learning provided for us by McLennan and Dewey (12). It would be difficult to find a more contamporary description of the psychological basis of a mathematics laboratory so we quote at length:

"THE TWO METHODS: Things; Symbols.--The principle corresponding with the psychological law--the translation of the psychological theory into educational practice--may be most clearly brought out by contrasting it with two methods of teaching, opposed to each other, and yet both at variance with normal psychological growth. These two methods consist, the one in teaching number merely as a set of symbols; the other in treating it as a direct property of objects. The former method, that of symbols, is illustrated in the old-fashioned ways--not yet quite obsolete--of teaching addition, subtraction, etc., as something to be done with "figures", and giving elaborate rules which might guide the doer to certain results called "answers".

It is little more than a blind manipulation of number symbols. The child simply takes, for example, the figures 3 and 12, and performs certain "operations" with them, which are dignified by the names addition, subtraction, multiplication, etc.; he knows very little of what the figures signify, and less of the meaning of the operations. The second method, the simple perception or observation method, depends almost wholly upon physical operations with things. Objects of various kinds--beans, shoe-pegs, splints, chairs, blocks--are separated and combined in various ways, and true ideas of number and of numerical operations are supposed necessarily to arise.

Both of these methods are vitiated by the same fundamental psychological error; they do not take account of the fact that number arises in and through the activity of mind in dealing with objects. The first method leaves out the objects entirely, or at least makes no reflective and systematic use of them; it lays the emphasis on symbols, never showing clearly what they symbolize, but leaving it to the chances of future experience to put some meaning into empty abstractions. The second method brings in the objects, but so far as it emphasizes the objects to the neglect of the mental activity which uses them, it also makes number meaningless; it subordinates thought (i.e., mathematical abstraction) to things. Practically it may be considered an improvement on the first method, because it is not possible to suppress entirely the activity which uses the things for the realization of some end; but whenever this activity is made incidental and not important, the method comes far short of the intelligence and skill that should be had from instruction based on psychological principles.

While the method of symbols is still far too widely used in practice, no educationist defends it; all condemn it. It is not, then, necessary to dwell upon it longer than to point out in the light of the previous discussion why it should be condemned. It treats number as an independent entity--as something apart from the mental activity which produces it; the natural genesis and use of number are ignored, and as a result, the method is mechanical and artificial. It subordinates sense to symbol.

The method of things--of observing objects and taking vague percepts for definite numerical concepts--treats number as if it were an inherent property of things in themselves, simply waiting for the mind to grasp it, to "abstract" it from the things. But we have seen that number is in reality a mode of measuring value, and that it does not belong to things in themselves, but arises in the economical adaptation of things to some use or purpose. Number is not (psychologically) got from things, it is put into them.

It is then almost equally absurd to attempt to teach numerical ideas and process without things, and to teach them simply by things. Numerical ideas can be normally acquired, and numerical operations fully mastered only by arrangements of things--that is, by certain acts of mental construction, which are aided, of course, by acts of physical construction; it is not the mere perception of the things which gives us the idea, but the employing of the things in a constructive way.

The method of symbols supposes that number arises wholly as a matter of abstract reasoning; the method of objects supposes that it arises from mere observation by the senses--that it is a property of things, an external energy just waiting for a chance to seize upon consciousness. In reality, it arises from constructive (physical) activity, from the actual use of certain things in reaching a certain end. This method of constructive use unites in itself the principles of both abstract reasoning and of definite sense observation."

One can move back even further into the past to find the the beginnings of the idea of a mathematics laboratory. In the May, 1970 issue of The Arithmetic Teacher, Kristina Leeb-Lundberg describes the original kindergartens as they developed under the influence of Froebel in Germany in the early 1800's (13). The kindergarten she described includes grids on the tables and chalkboards as Moore asked for, studies of shapes, sand for measuring volume, early versions of attribute blocks, multibase arithmetic blocks, pattern blocks, linkages, and even lattice boards which could be used as geoboards are today.

Another item from the past illustrates that some teachers have felt a reaction to the highly rigid curriculum. In his paper of 1927, Austin proposed a laboratory approach to high school geometry. He stated:

> "The keynote of the laboratory idea is discovery by means of experimentation. Pupils should be permited to observe the laws of geometry operating in concrete form before they are required to do logical thinking".

Thus, it appears that at times through the years, there have been collections of voices asking for a more intuitive and less deductive approach to the teaching of mathematics at all levels. We feel the emphasis upon mathematics laboratories represents one of those crescendoes in response to the recent emphasis upon rigor and logic and the age-old emphasis upon meaningless manipulation.

Present Day Voices for Mathematics Laboratories

The literature of mathematics education at the present time is literally packed with discussion about mathematics laboratories. Two issues of The Arithmetic Teacher (Oct., 1968 and Jan., 1970) have had the topic as its central theme. In addition, nearly every issue of that journal as well as the English journal, Mathematics Teaching, contains pertinent articles.

In the newly published fifth edition of the Teaching of Secondary Mathematics (15) we find:

> "As the name implies, the underlying idea of the mathematics laboratory is that students will develop new concepts and understandings particularly well through experimental activities dealing with concrete situations such as measuring and drawing; counting, weighing, averaging, and estimating: taking reading from instruments; recording, comparing, analyzing, classifying, seeking patterns, and checking data; and that interest will be stimulated and understanding will be clarified through obtaining original data or impressions from concrete physical situations and working with such data. Most work of this nature will involve use of the various kinds of physical equipment and will entail such activities as those listed here. Some of this work can be done in the classroom that is suitably arranged and equipped; some can take the form of elementary field work, such as determination of angles and distances and the mapping of small areas. Most students find such work highly interesting, and it is doubtless true that through it they can develop many mathematical concepts and insights with an interest and clarity often not obtained through a strictly intellectual approach. It is also likely that these concepts and principles become more enduring and more functional and meaningful when they are seen in relation to actual application".

Much of the pressure to create laboratories comes from attempts to provide successful programs for unsuccessful and unmotivated students. For example, the L.A.M.P. (Low Achievement Motivational Project) in the Des Moines Public Schools describes its intentions in this way (16).

> "Primarily, a mathematics laboratory is a state of mind. It is characterized by a questioning atmosphere and a continuous involvement with problem solving situations. Emphasis is placed upon discovery resulting from student experimentation. A teacher acts as a catalyst in the activity between students and knowledge.
>
> Secondarily, a mathematics laboratory is a physical plant equipped with such material objects as calculators, overhead and opaque projectors, filmstrips, movies, tape recorder, measuring devices, geoboards, solids, graphboards, tachistoscope, construction devices, etc.

Since a student learns by doing, the lab is designed to give him the objects with which he can do and learn.

The primary goal of the lab approach is to change the student's attitude toward mathematics. Most students have become so embittered by habitual failure that they hate mathematics and everything connected with it. There is little possibility of this student learning mathematics until an attitude change can be affected. It is because of this goal that our approach is different. Some would label our approach as 'fun and games' but I am sure that close examination will bring realization that everything in the program is oriented toward the twin goals of attitude change and mathematics improvement.

In an article written for high school principals, Hoffman examined several programs being developed for slow learners (17). She observed:

"The directions that projects for the slow learner in mathematics share can be related to two aspects of the so-called 'modern mathematics' movement: (1) the use of mathematics laboratories, with all its ramifications including the calculators, remote terminals for computers, and flow-charting for problem analysis. (2) Emphasis on the structure of the number system and the beauty and interest of patterns in mathematics, and inclusion of selected topics of number theory, intuitive geometry, and informal topology".

She found most programs for slow learners shared the following characteristics:

1) a mathematics laboratory--whether it is a center for the school, a formal laboratory for the use of a few selective classes, or a classroom laboratory.

2) the use of calculators to help the student find his pattern of error in computation and to enable him to get past simple computational blocks to basic mathematical understanding.

3) A regulated program, with a pattern of activities for security but with a change of activities to accomodate the short attention span of the slow learner and the unit-a-day pattern for the satisfaction of a task completed and evaluated on the spot.

4) Provision for reinforcement of early basic concepts, which may be weak, utilizing the methods and techniques of the more modern programs in mathematics-- the exploration of structure of the number system, experimentation and discovery of patterns and their utilization.

5) The use of many manipulative devices, such as the abacus, cuisenaire rods, geoboards, etc.

6) The proper and controlled use of games, puzzles, and other motivational techniques.

7) Use, where possible, of remote terminals tied into computers for computer aided instruction units.

In a more general view, the question of the relative emphasis on the mathematics and its structure as opposed to the intellectual characteristic of the children is discussed by Travers (18). He feels that some of our recent efforts are misguided.

"Another observation to be made about the mathematics laboratory movement, concerns the influence of Piaget. This famous psychologist's emphasis on studying the child's patterns of thought and the development of ability as the child grows, has given rise to attempts to devise learning experiences in mathematics (such as the use of physical models) which will best account for the child's pattern of thoughts at his particular developmental level. But the curriculum reform movement on this side of the Atlantic seems to have gone in quite the opposite direction-- looking first at the mathematics that is to be taught, and then devising learning experiences that are dictated by the subject matter at hand with little regard for the learning patterns of the child.

The success of the curriculum reform movement in bringing about the needed improvement depends upon the extent to which the new materials are implemented in every classroom. If it ever was true that 'simply anyone' could teach the old-fashioned arithmetic, this certainly does not hold for the new mathematics. The teaching profession has come of age, and has met in the new program a challenge which demands its best efforts. The effect of teacher of the new mathematics is indeed a professional.

Morris Kline has long been a critic of developments in the mathematics curriculum. In his paper entitled 'Logic vs. Pedagogy', he makes a strong plea for the approach to mathematics which is more intuitive and less rigorous (19). It is interesting to compare his statement with those of E.H. Moore.

> "It is the contention of this paper that understanding is achieved intuitively and that the logical presentation is at best a subordinate and supplementary aid to learning and at worst a decided obstacle. Intuition should fly the student to the conclusion, make a landing, and then perhaps call upon plodding logic to show the overland route to the same goal. If this contention is correct, then the intuitive approach should be the primary one in introducing new subject matter at all levels".

Donald Cohen, who is the Madison Project representative to the New York City public schools, has written a book entitled Inquiry in Mathematics via the Geo-board (20). In response to the question, 'Why use the Geo-board'? Cohen states:

> "Children enjoy learning when they are actively involved, when they are not always being lectured to or told how to do something. Working with the Geo-board enables them to do things and discuss their work with their classmates: they learn from each other this way.
>
> During free play the children see patterns which they will use in problem solving situations later on. A strong intuitive grasp of what area is should certainly precede the learning or development of formulas for finding the area within different quadrilaterals, for example.
>
> They enjoy the challenging problems which, when possible, are presented in such a way as to allow them to decide, without recourse to the teacher, whether their answer works. The children should be encouraged to find different ways of solving a problem, not necessarily 'the teacher's method'. They should be encouraged to ask questions and to make up new problems and variations on other problems. Their discoveries, questions and results should be praised and displayed. Perhaps a bulletin board should be set up for the discovery of the week."

It is interesting to observe that many of the physical materials that have been developed for teaching mathematics in a laboratory setting during the last few years having come from curriculum study groups who were developing elementary science programs. The Elementary Science Study for example, has produced mathematics laboratory materials which include geoblocks, mapping, pattern blocks, attribute games and problems, and tangrams. Apparently, they felt a need to develop basic mathematical concept before students could be successful with the science concepts in the elementary school.

All of the voices regarding mathematics laboratories are not in full favor of them. Wilkinson is now in the process of completing a doctoral dissertation relating to the effects of laboratories in sixth grade classes (21). In his discussion he relates the source of some of the opposition to the use of laboratory techniques.

> "Two distinct points of view seem apparent regarding activity programs and laboratory methods of teaching mathematics.
>
> The Piaget-Bruner point of view places primary emphasis on the <u>process</u> of learning, the importance of discovery, and the need for laboratory experience in the concrete operations.
>
> The Gagne-Ausubel position is that the <u>product</u> of learning are of high importance, learning should be developed mental and highly structured, and the teacher-textbook method is a better teaching strategy than the use of laboratory type experiences when the objective is to have the learner become knowledgeable about the content in mathematics.

He then quotes Ausubel as saying,

> "Students waste many valuable hours in the laboratory collecting and manipulating emperical data which, at the very best helps them rediscover or exemplify principles that the instructor could present

verbally and demonstrate visually in a matter of minutes. Hence, although laboratory work can be invaluable in giving students some appreciation of the spirit and methods of scientific inquiry, and of promoting problem solving, analytic, and generalizing ability, it is a very time consuming and inefficient practice for routine purposes of teaching subject matter content for illustrating principles when didactic exposition or simple demonstration are perfectly adequate".

Research Related to Mathematics Laboratories

While we find many people in many places promoting the concept of mathematics laboratories, there are at this point very few research results which are available to guide in making decisions.

Kieren provides the most extensive review of the research literature dealing with both discovery learning, and manipulative learning in mathematics during the period 1964-1968 in the Review of Educational Research, October 1969 (22). Kieren says:

"The quality of the actual research and its attention to the questions raised above are questionable. Many of the studies were pilot studies in some sense, and questions were often asked that were not sufficiently complex to deal with the theoretical and practical issues. Careful definition of the problem was frequently lacking. Nevertheless, the research cited in the following pages represents an effort to establish the contribution of more play-like approaches to mathematics learning."

Most of the research activity has been aimed at determining the effectiveness of the use of specific materials on the achievement of children. The use of the cuisenaire rods has been studied in several studies with mixed results. Callahan (23), Crowder (24), Hollis (25), Locow (26), and Nasca (27) all report that the use of the rods promoted more learning by children while Brownell (28), Fedon (29), and Haynes (30) report either mixed results or no significant differences. Passy (31) reports a negative effect from [illegible]e rods.

On the basis of such confusing results, one could not at this time substantiate the advantages or disadvantages of using Cuisenaire Rods in teaching mathematics if the objective of instruction is to be measured by standardized tests.

Another popular set of materials which were developed by the Elementary Science Study and also by Z.P. Dienes are the attribute blocks. These are wooden blocks which vary in color, shape and size and are intended to give children experience in classifying and categorizing. In what appears to be a careful study, Lucas (32) found that children in grade one who were given ten weeks of attribute block training conserve cardinality and conceptualize addition and subtraction better then children without training. However, they were not so good at computation.

In another research report Ellis and Corum tried to measure the effects of the use of a desk calculator on the arithmetic achievement, the attitude toward mathematics, and the motivation toward school of children who were already in the mathematics laboratory (33). The students in this study were low achieving high school students in Miami Springs Sr. High School. No advantages to using the calculator were apparent from the test results but the use of the calculator was recommended in mathematics classes in spite of the lack of results. Both the experimental and the control groups recorded gains in attitude toward mathematics during the experiment. But, both groups also recorded a loss in academic motivation during the experiment which the writers attributed to negative factors outside the mathematics laboratory.

In an extensive project of teaching of mathematics through science by SMSG more than 12,000 (seventh, eighth, and ninth) grade children studied special materials written by a team of mathematicians, scientists, and teachers. Careful study was made by Higgins of the students of 29 eighth grade mathematics teachers from junior high schools in Santa Clara County, California, as they taught a unit entitled 'Graphing, Equations, and Linear Functions' (34).

The five-week period immediately preceding spring vacation was used for the experimental teaching. An extensive battery of tests was given to the students before and after the experimental work. There were 853 students in the experiment. At the conclusion of the experiment the students were grouped in 8 'natural' attitude groups such that all of the children in a given group has similar attitudes toward mathematics. It was found that differences in attitude patterns among groups are not reflected in significant differences in either ability or achievement. It was concluded that attitudes change clusterings are not a major consideration if one is concerned with mathematics achievement during a unit taught via physical approaches. The study found about 6% of the children developing rather strong cohesive unfavorable attitude toward the content. At the other end they found 8% of the children developing attitudes shifts favorable toward mathematics but in general, most students changed attitudes very little. Some liked the unit; some liked it, but found it harder; others found it easier, but less interesting; and a few disliked it quite strongly. None of these groups are large enough to represent a major fraction.

In Wilkinson's careful study he measured the effect of laboratory experiences on both the geometry achievement and the attitude of children in 6th grade when compared with children who were in a regular teacher-textbook classroom setting (21). He was able to control the teacher variable by having each of three teachers in the study teaching one control class and two experimental classrooms the children were assigned to work the geometry material contained in 18 shoe boxes; one each day for 18 days. They could work either alone or in groups of three or four, as they preferred. In the other experimental classrooms the children had, in addition to the shoe boxes, cassette tapes which gave verbal directions and posed verbal questions to the children. The teachers in each of the experimental classes were asked to use nondirective techniques and to serve as a resourced person asking and answering questions and providing direction when called upon to do so. Wilkinson found no differences in the geometry achievement of children from the various groups, but found that children from middle and slow IQ levels in the laboratory treatment had a greater gain in attitude toward mathematics than the high IQ group. The laboratory setting seemed to have a positive effect upon the average and slower children, while the brighter children seemed to be "turned off" by the laboratory method.

Woodby reported the development of the mathematics laboratory in Cleveland in the annual report from Merel in Sept. of 1967 (35). The basic purpose of the laboratory in Cleveland was to provide an intensive inservice training for two junior high school teachers in a laboratory for low achieving students in mathematics and to study the results of this training. A second objective was to develop instructional materials for use in the mathematical laboratory that are effective with low achievers. In the report are listed ten desireable behaviors one might expect of a teacher in a laboratory:

(1) The teachers asks questions that cause expl ration and inquiry by the student.

(2) The teacher devises and uses tasks that relate to fundamental mathematical concepts and techniques. A good example is Rosenbloom's simulated computer in which the student discovers the distributive principle.

(3) The teacher uses materials other than the textbook.

(4) The teacher provides individual and small group activities of an exploratory nature that results in the student trying something, gathering data, analyzing data, and testing conclusions.

(5) The teacher uses cues that come from the student in making teacher decisions about questions asked or tasks assigned.

(6) The teacher plans for and uses the basic strategy of student discovery.

(7) The teacher employs the strategy of asking the student to make decisions on the basis of observation of events.

(8) The teacher provides situations for the student to play an active role in learning, rather than a passive one.

(9) The teacher creates a new problem or task that is easier or more familiar to the student when difficulty occurs, then allows the student to return to their original problem.

(10) The teacher provides the student with a means for determining whether an answer is right or wrong, independently of the teacher or the textbook.

The mathematics laboratory was established in David Jr. High School in Cleveland in 1966-67 school year. Two groups of 20 seventh-grade students met in the laboratory for a 90-minute period each day.

"The teachers received notice of their participation in the project only a short time before the project began. They were selected in late January by the administration of the Cleveland school system as capable teachers who were willing to try and experiment in laboratory-type teaching. It should be noticed that neither teacher was teaching mathematics in the manner in which she had been. One was sufficiently disenchanted to be considering seriously withdrawal from the teaching profession. Although both teachers had previously heard of laboratory teaching the methods courses, each stated the technique had only been rarely used in their classrooms. Thus, the teachers were as unfamiliar with laboratory teaching techniques as the students were with laboratory learning techniques. Few guidelines for teacher behavior were established. In short, initially the teachers were in a position of learning laboratory teaching by the laboratory method".

The teachers remaining teaching load were reduced considerably and they were given a great deal of outside expert advice from a variety of consultants. At the end of the year the project director listed these summary statements.

(1) Organization for small group instruction or individualized instruction is difficult. Discipline was a major concern.

(2) The goals of mathematics instruction often get lost in the mechanics of the laboratory activity.

(3) Belief of the teacher in a discovery approach is not sufficient to accomplish the appropriate behavior. Teachers will revert to telling, explaining, and showing students. There is a wide gap between teacher belief and teacher behavior.

(4) The teachers expected too much of the materials; it was assumed that the material would provide the motivation.

(5) Rewards for the teachers were different from their expectations. They had anticipated great increases in the achievement by the group as a whole. The rewards turned out to be unusual accomplishments by individuals and these were infrequent and unpredictable.

(6) The teachers became more concerned with how students learned than with the achievement; questions asked by students became more important to the teacher later in the semester than they were at the beginning.

(7) The teachers worked longer and more intensively than they did before the project. Even if they had much more time for preparation they stayed late and usually took work home to (a) organize for instruction, (b) devise activities and write instructions, and (c) evaluate results.

(8) Teachers in this learning situation need someone to talk to. Supervisors and consultants are important to the teacher in this situation.

(9) From the teacher point of view, the learning was more nearly guided discovery than true discovery.

(10) The teachers became better teachers because of what they learned about students learning. For example, they talked less and listened more at the end then they had at the beginning.

During the summer following the project the two participating teachers reviews by the Cleveland Public Schools to train fourteen additional Cleveland teachers. Ten children were brought in each day for an hour and a half to serve as a demonstration class working in a laboratory setting. After the children left, the fourteen teachers worked with the materials in the laboratory and discussed their ...ible use in the classroom.

Mathematics Laboratory Materials

Throughout the paper we have discussed varieties of materials as though the reader was familiar with the materials. In any mathematics laboratory one is likely to find some of the more popular of the materials which are currently available and seemingly effective.

Two excellent bibliographies have appeared recently which serve as a guide to those who are interested in obtaining materials. The bibliographies are by Davidson (36) and Hillman (37).

While it should be clear that many of the materials which can be used in a laboratory are inexpensive and are readily available, more concern generally arises over those that are commercially available and sometimes expensive. Many items are available from several different sources.

As an illustration of some of the materials one might find in a laboratory, below is a basic listing of materials which was purchased recently at one major university which was developing a new lab.

> 20 Geoboards (a geoboard is a wooden board with nails driven in a lattice points usually in a 5x5 array. There are also circular geoboards).
>
> 4 spinners (common spinners with multicolored areas underneath to use for studying probability).
>
> 2 Sets of Madison Project Shoebox kits (a shoebox contains a manipulative device and a sequence of task cards leading the student to a concept).
>
> 1 Set of Multi-Base Arithemtic Blocks (unit cubes one dimensional strings of cubic (longs), two dimensional arrays of cubes (flats), and three dimensional cubes of cubes (blocks). They come in basis 2,3,4,5,6, and 10, and are used to study numeration systems).

20 Sets of colored rods (Cuisenaire Rods are unit centimeter lengths from one to ten and each length a different color. Used for a wide variety of concepts).

7 Equations games (A game by WFF-N-PROOF to provide experience with the system of real numbers)

2 Sets of Mirror cards (Plane mirrors and cards to place the mirrors on to reproduce patterns-- provides experience with symmetry).

3 Sets of Attribute Games and Problems (Attribute blocks, color cubes, people pieces and creature cards-- designed to give experience in patterns, classification and categorisation).

5 Sets of tangrams (traditional tangram pieces which are triangles, squares and parallelograms which, when put together, make different configurations).

Davidson and Fair provide a good description of the establishment of a laboratory in Oakhill Elementary School in Newton, Massachusetts (38). Among the materials they describe are a number of other commercial games and items such as paper, pencils, oak tag, construction paper, scissors, string, tongue depressors, egg cartons, beans, sticks, etc. On one table under the heading "Guess How Many" were jars of peas, beans, macaroni and rice. A few study carrels were constructed out of Tri-Wall which is a thick cardboard which is easy to work with.

The Training of Teachers

Many who have been active in the development of laboratories feel the greatest problem in promoting the ideas is training teachers to work in a laboratory setting.

For the second consecutive summer, the Department of Mathematical Education of Teachers College, Columbia is holding a summer study program on laboratory teaching of mathematics intended for teacher training personnel and for the third consecutive summer, Michigan State University is conducting a three-week conference for leaders in elementary mathematics. A major emphasis in this program is the use of mathematics laboratory.

Several institutions have built experience in a mathematics laboratory into the undergraduate teacher-training program, in some schools as part of a mathematics course and in others as part of a methods course. Among the institutions which already have such programs are Michigan State, Eastern Carolina, Georgia, Purdue (Calumet), and Oregon State.

There is considerable agreement that a teacher needs to have the experience of learning in a laboratory setting if she is going to be effective in directing a laboratory. The ideas is expressed by Johnson and Kipps in the introduction of their book, Geometry for Teachers (39).

> "Can teachers capture for themselves the excited enthusiasm shown by children in classes sponsored by such curriculum groups as the Madison Project or the Nuffield Project? Can a teacher raised on lectured-drill-homework classes feel and show the drama inherent in 'I do and I understand' activities, in peer group discussions, and in concept such as the concrete-iconic foundation of abstraction? This text focuses on these dynamic factors so that a teacher may learn their value from his own personal experiences and feelings".

"It appears that informal geometry is best learned through experience with physical objects, making guesses, trial and error comparisons and fittings, exploratory discussion of observations, and tentative conclusions made with seers who are similarly searching. An active learning approach of this kind emphasizes new and more rewarding type of teacher behavior. Small groups (four seems to be an optional size) are searching and discovering together. As the teacher moves from group to group, listening to the dialogue, he must consider when to ask a question, when to be silent, and when to withdraw altogether. The teacher recognizes that a pupil asks questions as steps in developing his thinking. Hence, only rarely will the teacher answer a pupil's question. Instead, he will encourage the pupil's effort or he will ask another question if some direction is needed".

One laboratory manual has been published which is designed to give teachers experience with many of the commercially available materials (40). In the preface of the Laboratory Manual to Elementary Mathematics by Fitzgerald et. al. is stated:

"The essence of the laboratory concept in learning mathematics is the fostering of inquiry and internal motivation to seek answers to questions. It is a fair generalization that teachers, at all levels, talk too much. A laboratory instructor must be wary of the temptations to provide excessive direction for the student, and thus rob the student of the experiences of finding answers for themselves. And some students will demand excessive direction, which may be evidence of the lack of the use of laboratory techniques in schools in the past".

Boonstra attempted to study the effects of laboratory experiences on the behavior of perspective elementary teachers in the classroom (41). Unfortunately he was working with student teachers and was only able to give them two laboratory experiences and observe their teaching once. He had the students working with laboratory materials related to the concepts of function and mathematical relations, then asked student teachers to present a lesson dealing with function in their elemantary classroom.

Boonstra concluded that two laboratory experiences were not sufficient to cause student teachers to adopt a student-centered approach to teaching nor were they sufficient to cause student teachers to adopt a teaching technique in which children learn through the use of manipulative material.

Summary

Schools as institutions tend to develop systems and routines for carrying out their assigned tasks. Through time these create rigidity and stability in the curriculum. The teaching of mathematics has shown many signs of these characteristics in the past. More than in most other subjects, the mathematics curriculum is thought of (needlessly) as being necessarily highly sequenced and lock-step.

Some fresh air was obtained by the curriculum reform movements of the past decade when working scientists and mathematicians provided an infusion of new and more appropriate content. Unfortunately, the mode of instruction was not changed sufficiently to eliminate the persistant problems which plague mathematics instruction.

The widespread efforts to teach mathematics in a more activity-oriented approach represent an attempt to provide experience for individual children to enhance curiosity and inquiry, to provide meaning to mathematical concepts, to make reasonable their applications, to build the intuitions which make abstractions possible, and to nurture the healthy natural development of intellect in children.

There is at this point no empirical evidence which will convince the unconvinced that mathematics laboratories are the best way to accomplish these aims. Many have found through experience that the school as a system makes the implimentation of a laboratory a difficult task to accomplish.

This writer, however, has never seen a teacher who, after shifting from a teacher-dominated, total-class approach to an individualized, activity-oriented approach, has chosen to shift back again.

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