DOCUMENT RESUME

ED 055 114 TM 000 845

AUTHOR Harris, Richard J.

TITLE An Interval-Scale Classification System for All 2 x 2 Games.

PUB DATE 71

NOTE 28p.; Paper presented at the Annual Meeting of the American Psychological Association, Washington, D.C., September 1971

EDRS PRICE MF-$0.65 HC-$3.29

DESCRIPTORS *Classification; Decision Making; *Games; *Game Theory; Mathematical Applications; *Mathematics; Models; Operations Research; Probability Theory; Problem Solving; Risk; *Transformations (Mathematics)

ABSTRACT

This extension of Harris' classification system permits classification of the interval-scale properties of any 2 x 2 (two person, two choice) game. The system is compared to taxonomies developed by various previous researchers. It is shown to permit compact description of any 2 x 2 game as an octet of numbers. It also permits rapid classification of comparisons among game matrices with respect to the strength of measurement which must be assumed for the comparison to be meaningful. (Author/CK)

ED055114

AN INTERVAL-SCALE CLASSIFICATION SYSTEM FOR ALL 2 x 2 GAMES

Richard J. Harris
University of New Mexico

An extension of Harris' (1969) classification system permits classification of the interval-scale properties of any 2 x 2 (two person, two choice) game. The system is compared to taxonomies developed by Rapoport and Guyer (1966), Hamburger (1969), Harris (1969), and Wolf (1969). It is shown to permit compact description of any 2 x 2 game as an octet of numbers. It also permits rapid classification of comparisons among game matrices with respect to the strength of measurement which must be assumed for the comparison to be meaningful.

AN INTERVAL-SCALE CLASSIFICATION SYSTEM FOR ALL 2 x 2 GAMES

Richard J. Harris

University of New Mexico

The present paper reports an extension of Harris' (1969) classification system for interval-symmetric games (i.e., games which look the same, up to a linear transformation of either or both players' payoffs, to both players) to encompass all 2 x 2 (two person, two choice) games. First, the classification procedure is outlined. Next, it is compared with the taxonomies of Rapoport and Guyer (1966), Hamburger (1969), Harris (1969), and Wolf (1969). Finally, possible applications are discussed.

The Classification System

General rationale. The payoff matrix of any 2 x 2 game can, by interchanging its rows and/or columns (equivalent to a re-labelling of one or both players' available strategies) be put into one of the four forms listed in Figure 1.

Insert Figure 1 about here

If one of the entries for each player (say t_i, $i = 1,2$) can be unambiguously identified (as, say, the largest of player i's four payoffs), the others can be identified in relation to it. In particular, Rapoport and Chammah (1965) have proved that the interval-scale properties of any set of four numbers can be completely described by any two independent ratios of differences between pairs of the numbers. Harris (1969) suggested a particular pair of difference-ratios, $(r_3,r_4) = ((p_i-s_i)/(t_i-s_i), (t_i-r_i)/(t_i-s_i))$, because of their relevance to the Prisoner's Dilemma game (a form D game in which $t_i > r_i > p_i > s_i$ for both players) and to the existence of dominant strategies. (If both difference-ratios are positive, the strategy making it possible for play^^ i to receive t_i dominates his other strategy.) These same difference-ratios are adopted by the present classification system, but are re-labelled as r_5 and r_6 in order to avoid confusion with the Harris (1969) system, which used a different criterion for identifying t_i.

Procedure. To classify any 2 x 2 game:

(1) Identify the cell in which row-player receives his highest payoff. Label this largest payoff as t_1. Label his payoff in the diagonally opposite cell as s_1; his payoff in the same row as t_1, as p_1; and his payoff in the same column as t_1, as r_1.

(2) Identify the cell in which the column-player receives his largest payoff and label this largest payoff as t_2. Label his payoff in the diagonally opposite cell as s_2; his payoff in the same column as t_2, as p_2; and his payoff in the same row as t_2, as r_2.

(3) Calculate for each player $\underline{i}$ the pair of difference ratios, $(r_5, r_6)_i = ((p_i-s_i)/(t_i-s_i), (t_i-r_i)/(t_i-s_i)), i = 1,2$.

(4) Plot each pair of difference-ratios as a point on the (r_5, r_6) plane illustrated in Figure 2, and enclose these two points in an ellipse. Write near this ellipse an "S" if t_1 and t_2 appear in the same cell of the payoff matrix; "R" if t_1 and t_2 appear in different cells of the same row; "C" if they appear in different cells, but the same column; and "D" if they appear in diagonally opposite cells.

Insert Figure 2 about here

The "configuration" of two points on the (r_5, r_6) plane together with the designation of the relative positions in the payoff matrix of the two players' largest payoffs uniquely defines any 2 x 2 game in the sense that two games with the same configuration have payoff matrices which are identical except possibly for a difference in the units in which the two players' payoffs are defined, or a constant difference between the corresponding payoffs.

Ties for largest payoff. The above classification procedure becomes ambiguous when one or both players have no uniquely largest payoff, since which of the tied largest payoffs to label as t_i becomes an arbitrary decision. Moreover, if the tie for largest payoff is between two diagonally placed payoffs, the "uniqueness theorem" stated above breaks down, since any such game, regardless of the values of r_i and p_i, will have an $(r_5, r_6)_i$ of $(-\infty, \infty)$.

This ambiguity may be handled by listing the alternative classifications obtained when each payoff involved in the tie for largest payoff is taken in turn as t_i, regarding the game as a hybrid cross of the various games whose ordinal regions "touch" at that point. The classification of a game in which diagonal entries are tied for largest payoff -- and for which $(r_5, r_6)_i$ thus = $(-\infty, \infty)$ -- can be made more nearly unique by adopting the convention that a truly zero value of $(t_i - s_i)$ be replaced by .001.

Relations to Other Classification Systems

Harris (1969). If the two points plotted on the (r_5, r_6) plane coincide -- i.e., if the two players have identical values of r_5 and of r_6 -- and t_1 and t_2 occur in the same or in diagonally placed cells, the game is interval-symmetric, i.e., looks the same, up to a linear transformation, to both players. Harris (1969) presented a classification system for interval-symmetric games and discussed its properties in more detail than is possible in the present paper.

Comparison of Figure 2 above with Figure 1 of Harris' (1969) reveals an inelegance in the present classification system as compared to that earlier one. The present system requires that r_5 always be ≤ 1.0 and r_6 always be ≥ 0.0, the equalities holding only when there is a tie for largest payoff. Thus the full (r_5, r_6) plane cannot be employed. Moreover, only six ordinal regions can be represented on this portion of the plane, thereby necessitating a parameter (relative location of t_1 and t_2, or the "form" of the game) in addition to the two players' (r_5, r_6) values to encompass all of the 78 games included in Rapoport and Guyer's (1966) ordinal taxonomy. The restrictions arise because of the requirement that t_i be larger than any of the other three payoffs for player i. The earlier system required only that t_i be larger than s_i, and thus permitted full use of the parameter plane and the representation of 12 ordinal regions. This less stringent restriction on t_i was made possible by the unique ordinal properties of interval-symmetric games, which always have two diagonally placed unequal-outcome cells and which thus permit identification of t_i as the largest payoff for player i in an unequal-outcome cell -- an unworkable definition in the more general case.

The fact that 12 ordinal regions can be paired in exactly 12(13)/2 = 78 ways -- the same as the number of ordinally unique games when ties are not considered -- continues to bedevil the present author, who has in fact constructed an alternative, single-plane classification system in which t_i is defined as the maximum of player i's outcome if he unilaterally switches from the game's "natural" outcome (that outcome which results if each player "goes for" his largest possible payoff) and player i's outcome if his partner unilaterally switches from the natural outcome. However, this alternative system, while permitting unique classification of any 2 x 2 game in terms of the location of two points on a single plane, suffers from two drawbacks: (1) the present system's separation of the problem of specifying the relationships among each player's four payoffs from that of specifying the relative location of the two players' largest payoffs is a more natural one than the rather involved definition of t_i in the alternative, single-plane system; and, more importantly (2) the alternative system is not internally consistent, in that combining row player's payoff set from an $[(r_5, r_6)_1;(r_5, r_6)]$ game with column player's payoff set from an $[(r_5, r_6)_1';(r_5, r_6)_2']$ game does not in general produce an $[(r_5, r_6)_1;(r_5,r_5)_2']$ game, whereas this property does hold for the present system.

Conversion of a game for which $(r_5, r_6)_1 = (r_5, r_6)_2$ and whose form is S or D, to Harris' (1969) (r_3, r_4) system can be accomplished via Equation I:

$$(r_3, r_4) = \begin{cases} (r_5, r_6) & \text{for form D games;} \\ (-r_5/d, -r_6/d) & \text{for form S games in which } d > 0; \\ (r_5/w, r_6/w) & \text{for form S games in which } d < 0; \end{cases} \qquad (1)$$

where $d = 1 - r_5 - r_6$ and $w = -d$.

Rapoport and Guyer (1966). These two authors provide a complete listing of all non-equivalent 2x2 games in which a strict preference ordering of the four possible outcomes can be constructed for each player. To determine the classification of a given game, a reader need only rank-order separately the payoffs for each of the players and then compare the resulting matrix of ranks with each of the 78 games until a match is obtained. The search process is facilitated for readers familiar with game-theoretic terminology by the authors' organization of the display of the 78 matrices on the bases of number, stability, and desirability of their equilibria. It is impeded by the frequent necessity of interchanging rows, columns, or players before the matrix being classified can be matched with any of the games listed by Rapoport and Guyer.

As hinted at already, ordinal relationships among player *i*'s payoffs can be represented quite easily within the present classification system in terms of restrictions on the values of r_5 and r_6. Specifically,

$$\begin{array}{lll} r > p & \text{iff} & r_5 + r_6 < 1.0; \\ r > s & \text{iff} & r_6 < 1.0; \\ p > s & \text{iff} & r_5 > 0.0; \\ t > r & \text{iff} & r_6 > 0.0; \\ t > p & \text{iff} & r_6 < 1.0; \\ t > s & \text{by definition.} & \end{array} \qquad (2)$$

Always true in present system.

Using these equivalences, any game classified in the following system can be related to the Rapoport-Guyer ordinal taxonomy by referring to Table 1, whose major subdivisions are based on the ordinal regions within which the two (r_5,r_6) points fall. Within each combination of a pair of ordinal regions, the upper left-hand entry applies if t_1 and t_2 lie in the same cell; the upper right-hand entry, if t_1 and t_2 lie in different cells but the same row of the payoff matrix; the lower left-hand cell, if this is a form C game; and the lower right-hand call if this is a form D game. Each entry in Table 1 includes the number assigned that game within the Rapoport-Guyer taxonomy, together with an alphabetic abbreviation of the ordinal properties of that game and the "name of the game" if such a name has been suggested.

Insert Table 1 about here

Examination of Table 1 reveals that the games which lie along the main diagonal of Table 1 are ordinally symmetric and that all games for which t_1 and t_2 fall in the same cell are "No Conflict" games. Further, in any game for which r_5 is positive for player i, that player has a dominant response. Finally, any game in which $r_5 = r_6$ for both players is a separable game (Hamburger, 1969). In a separable game, the choices can be presented to each player in the form "Give me x and give him y."

Wolf (1969). Wolf (1969) presents a classification system which is, to the present author's knowledge, the only system other than the present one which is applicable to all 2x2 games. Wolf treats the eight payoffs of the 2x2 game as the "dependent variables" in a standard, linear, analysis of variance model in which the factors are role (row vs. column player), own choice, and other's choice. Rather than dealing with sums of squares, Wolf uses the estimates of the parameters (grand mean, "treatment" main effects, interactions, and error components) to classify the game. Wolf's classification system has the enormous advantage of being directly and readily generalizable to games involving more than two players and more than two alternatives per player. It has the serious drawback, however, of requiring 24 parameters to completely specify a 2x2 game. It has the additional property (whether drawback or disadvantage is debatable) that no guidelines are provided to relate a game's classification within the Wolf system to the properties generally considered important to game theorists (dominance of strategies, ordinal relationships among payoffs, etc.). Wolf does present interpretations of some of his parameters in terms of a theory of social power. It is interesting that almost all of these parameters involve interpersonal comparison of the two players' payoffs--a type of comparison which game theorists generally avoid because of the enormous problems involved in attempting to establish the intersubjective comparability of utility measurements. It is clear that various functions of Wolf's 24 parameters could be defined which would require only interval-scale measurement of the payoffs (e.g., the sums of squares of various parameters corresponding to the usual terms in an analysis of variance, since the ratios of these sums of squares are known to be invariant under linear transformation of the dependent variables), but considerable work remains before the best such transformations can be determined.

Applications

Condensed matrix descriptions. The present classification system can be used to describe the outcome structure of a game without reproducing the payoff matrix in full. This procedure can be quite useful when studies employing a wide variety of games must be described. In addition to knowing r_5 and r_6 for each player and the relative positions of t_1 and t_2, a knowledge of (t_1-s_1)--which should include a designation of the units in which payoffs are expressed--of s_1, of $k_1 = (t_2-s_2)/(t_1-s_1)$, and of $k_2 = s_2-s_1$ is needed in order fully to reproduce the payoff matrix entries. Any 2x2 game can be identified by the ordered octet of numbers, $[(r_5,r_6)_1; (r_5,r_6)_2;(t_1-s_1),s_1,k_1,k_2]$, with a letter designating the relative positions of t_1 and t_2 appended by a hyphen to the end of the octet. This provides more than enough information to locate the game within the interval-scale classification system. If the reader wishes to reproduce the payoff matrix of the game in full, the entries may be calculated by noting that $t_i = (t_i-s_i) + s_i$; $p_i = (t_i-s_i)r_5 + s_i$; $r_i = (t_i-s_i)(1-r_6) + s_i$; $(t_2-s_2) = (t_1-s_1)k_1$; and $s_2 = k_2 + s_1$. Whenever (as will usually be the case) the values of k_1 and k_2 are equal to 1 and 0, respectively, they may be omitted from the octet. Whenever (as will usually be the case) the two players have identical values of r_5 and r_6, only one pair of (r_5,r_6) values need be included in the octet. Whenever (as will usually be the case), t_1 and t_2 are diagonally placed, the letter designating the form of the game may be omitted. ("Usually" in the preceding statements refers to the fact that form D games which are interval-symmetric and which provide identical payoff sets to the two players account for well over 90% of the empirical studies of games. Rapoport, 1967, has referred to such games as "psychologically interesting games," though SSE-7 and SSE-9 are excluded from this accolade, and Marwell & Schmitt, 1967, have pointed out that psychologically trivial games may be extremely interesting empirically.) While 9 parameters are thus needed to provide complete reproduction of the outcome matrix in the general case, most games will require only four of these numbers; even the 9 figures can be presented much more compactly and more economically than the 8 numbers constituting the original payoffs if they must be presented in bulky matrix form; and the parameters of the present system provide much more ready comparison of the properties of two games than examination of their payoff matrices does.

Perceived Games. As pointed out by Harris (1969) and by several other authors cited by him, considerable evidence exists that players do not seek merely to maximize their own individual payoffs, but rather seek to maximize some weighted average of own and other's payoffs. In other words,

$$U_i' = (1-g_i)U_i + g_iU_j ,$$

where U_i' is the utility (overall subjective satisfaction) to player i of a particular outcome; U_i is the direct payoff received by player i; U_j represents the other player's direct material rewards; and g_i is a coefficient of masochism-sadism which = 0 for a completely individualistic player, ½ for a player who seeks to maximize joint outcomes, and $-\infty$ for a player who seeks to earn as much more than the other player as possible.

One of the strengths of Harris' (1969) classification system was its provision of a very speedy procedure for determining what a given experimenter-defined game (i.e., a game matrix containing the points, pennies, or whatever E actually delivered to the players) might look like to players each of whom had some value of g_i other than 0. For interval-symmetric games, the loci of the "perceived games" which might arise from a given experimenter-defined game consist simply of two straight lines in the (r_3,r_4) plane, one of which passes through the point representing the experimenter-defined game; both of which end arbitrarily close to (but not on) the point $(\frac{1}{2},\frac{1}{2})$; and the second of which is the "mirror image," with respect to the line, $r_3+r_4 = 1$, of the first, i.e., passes through $(1-r_4, 1-r_3)$. It would be nice to be able to report that an equally simple situation pertains in the more general case covered by the present classification system. This, unfortunately, is not the case.

Perceived games do lie along straight lines in the (r_5, r_6) plane, with each player having, in general, a separate set of line segments along which his "perceived" values of (r_5, r_6) lie. Any combination of a point on row player's locus and a point on column player's locus represents a perceived game which arises from the original game for a particular pair of values of g_1 and g_2. The form of the resulting game requires special attention whenever one (or both) player's point lies far enough along his perceived game locus (in either a high g or a low g direction) to have crossed one of the three ordinal boundaries involving t_i (which usually entails a "jump" to a line segment other than the one passing through the original defining point where $g = 0$), since the form of such a game will be in general different from the form of the objectively defined game from which it arose.

To be more specific requires that we consider each game form separately. Equations (3) give the general form of the segment passing through the objectively defined point, $(r_5, r_6)_i$ and thus representing those payoff sets for which t_i' is the largest subjectively defined payoff, for any form D game.

Form D: $(r_{5i}, r_{6i})' =$

$$\left[r_{5i} - \frac{g_i k_1 (1-r_{5i}-r_{5j})}{1 - g_i - g_i k_1}, \; r_{6i} - \frac{g_i k_1 (1-r_{6i}-r_{6j})}{1 - g_i - g_i k_1} \right] \; ;$$

whence

$$r'_{6i} = r'_{5i} \frac{(1-r_{6i}-r_{6j})}{1-r_{5i}-r_{5j})} + \frac{(r_{6i}-r_{5i}) + (r_{5i} r_{6j} - r_{5j} r_{6i})}{1 - r_{5i} - r_{5j}} \; ; \quad (3)$$

where $(r_{5i}, r_{6i}) = (r_5, r_6)_i$ is player i's objectively defined point, and primed symbols -- r'_{5i}, r'_{6j}, etc. -- represent perceived difference ratios, based on the subjective utility to the designated player of each outcome.

The corresponding expressions for an objectively defined form S game are

Form S: $(r_{5i}, r_{6i})' =$

$$\left[r_{5i} + \frac{g_i k_1 (1-r_{5i}-r_{6j})}{1 - g_i + g_i k_1} ,\quad r_{6i} + \frac{g_i k_1 (1-r_{5j}-r_{6i})}{1 - g_i + g_i k_1} \right] ;$$

whence (4)

$r'_{6i} =$

$$r'_{5i} \frac{(1-r_{5i}-r_{6j})}{1-r_{5j}-r_{6i}} + \frac{(r_{6i}-r_{5i})(1-r_{5i}-r_{6i}) + (r_{5i}r_{6j}-r_{5j}r_{6i})}{1 - r_{5i} - r_{6j}} .$$

When the g = 0 game is of form C, then the segment of player i's perceived game locus which passes through $(r_5 r_6)_i$ is given by

Form C: $(r_{51}, r_{61})' =$

$$\left\{ r_{51} - \frac{g_1 k_1 [r_{51} r_{52} + (1-r_{62})(1-r_{51})]}{1 - g_1 - g_1 k_1 (1-r_{52}-r_{62})} ,\quad r_{61} - \frac{g_1 k_1 [r_{61} r_{62} + (1-r_{52})(1-r_{61})]}{1 - g_1 - g_1 k_1 (1-r_{52}-r_{62})} \right\}$$

and $(r_5, r_6)'_2 =$

$$\left\{ r_{52} + \frac{g_2 k_1 [(1-r_{51}) - r_{52}(1-r_{51}-r_{61})]}{1 - g_2 + g_2 k_1 (1-r_{51}-r_{61})} ,\quad r_{62} + \frac{g_2 k_1 [(1-r_{61}) - r_{62}(1-r_{51}-r_{61})]}{1 - g_2 + g_2 k_1 (1-r_{51}-r_{61})} \right\} ;$$

whence (5)

$$r'_{61} = r'_{51} \frac{(1-r_{52}) - r_{61}(1-r_{52}-r_{62})}{(1-r_{62}) - r_{51}(1-r_{52}-r_{62})} + \frac{r_{61}(1-r_{62}) - r_{51}(1-r_{52})}{(1-r_{62}) - r_{51}(1-r_{52}-r_{62})}, \text{ and}$$

$$r'_{62} = r'_{52} \frac{(1-r_{61}) - r_{62}(1-r_{51}-r_{61})}{(1-r_{51}) - r_{52}(1-r_{51}-r_{61})} + \frac{r_{62}(1-r_{51}) - r_{52}(1-r_{61})}{(1-r_{51}) - r_{52}(1-r_{51}-r_{61})} ;$$

where player 1 selects the rows and player 2, the columns.

The expressions for form R games are obtained from those for form C games by replacing each subscript "1" with a subscript "2" and vice versa/games.

Figure 3 illustrates the kind of loci which arise by considering the specific example of an objectively defined [(.1,.3); (-.2,.6); 20, 10, 2, 0] - R game, i.e., an instance of game 55 of the

Rapoport-Guyer ordinal taxonomy. Typically, the locus of perceived

Insert Figure 3 about here

points for each player consists of three line segments. For row player, the first segment "starts" at (.875,.192) for $g_1 = -\infty$ and reaches the boundary of the (r_5,r_6) plane at (.593, 0) for $g_1 = -.6$; the second segment extends from (-1.45,0) for $g_1 = -.6$ through (.1,.3) for $g_1 = 0$ to (1,.474) for $g_1 = .429$; and the third segment extends from (1, 2.107) for $g_1 = .429$ to (.192,.875) for $g_1 = \infty$. Column player's first segment extends from (.116,.808) for $g_2 = -\infty$ through (-.2,.6) for $g_2 = 0$ to (-1.107,0) for $g_2 = .571$; the second segment runs from (.525,0) for $g_2 = .571$ to (1,2.451) for $g_2 = 1.6$; and the final segment extends from (1,.407) for $g_2 = 1.6$ to (.808,.116) for $g_2 = \infty$. Table 2, which was arrived at by recognizing that crossing the $r_6 = 0$ boundary amounts to interchanging the roles of t' and r' in the classification, while crossing the $r_5 = 1$ boundary interchanges t' and p', lists the form of the game for every possible combination of a point from row player's perceived game locus with a point from column player's locus. Consideration of Figure 3 together with Tables 1 and 2 shows that this single objectively defined game might give rise to any of 29 ordinally

Insert Table 2 about here

distinct perceived games: 1-4, 6-14, 17-23, 26, 35, 39, 45, 46, 48, 50, 55, and 72 of the Rapoport-Guyer taxonomy.

The procedure used to determine the loci for the above example made use of the property that any perceived game on player i's locus is obtainable from any other game by treating that other game as if it were objectively defined and selecting an appropriate g_i. But the end point of one segment can be readily computed from the end point of the "adjacent" segment (adjacent in the sense of arising from the same value of g which brings the first segment to the boundary of the plane) as follows:

(1) If (c,0) is a point on i's perceived-game locus, so is $(\frac{c}{c-1},0)$.

(2) If (1,d) is a point on i's perceived game locus, so is (1,1/d).

(3) If $(-\infty, \infty)$ and (a,b) are points on the same segment of player i's perceived game locus, then $(-\infty, \infty)$ and (b,a) are points on an adjacent segment of i's locus.

Thus, to obtain the equation representing a segment adjacent to one whose equation is already known, use this known expression to obtain an end point of the adjacent segment, and then treat this end point, together with the other player's objectively defined (r_5,r_6), as an objectively defined game, plugging into the appropriate one of Equations (3)-(5) to compute the slope and intercept of this adjacent segment. The only catch comes in knowing which of the three equations to use. Table 3 specifies which expression (the one for form D, S, C, or R) to use in computing the adjacent segment as a function of the expression used in the present segment and of the boundary this segment and the adjacent one share in common.

Insert Table 3 about here

Strength of Measurement Scale. The present system, like Harris' (1969) system, can be used in determining the strength of the measurement process (interval, ordinal, ratio, interpersonal comparison of utilities required) needed to make various statements about experimental games meaningful. Statements involving comparisons of payoffs have only to be converted to corresponding statements involving r_5, r_6, k_1, and k_2. If only r_5 and r_6 appear in the statement, only interval-scale properties are required. If k_1 cannot be eliminated from the statement, interpersonal comparison of utilities is required. If the statement can be reduced to a statement about the ordinal regions of the (r_5,r_6) plane, with no need to specify relative location within that ordinal region, then only ordinal measurement is needed.

Summarizing Information About Optimal Policies

Probably the most valuable use of the present classification system is its most fundamental one, that of "storing" information about the properties of large sets of games. An example is provided by Harris' (1969b) algebraic expressions for the long-run expected payoffs of the 16 policies of play in iterated Prisoner's Dilemma which were first defined and discussed by Amnon Rapoport (1967). It can be shown that identification of the policy having the highest long-run expected payoff is unaffected by linear transformation of the player's payoffs. Thus each expression in Table 1 of Harris (1969b) can be reduced without loss of information to a form involving only (r_5,r_6) for the player trying to select a policy, and the other player's conditional probabilities of responding cooperatively to each of the four possible outcomes of a given trial. With those four conditional probabilities fixed, the difference in expected payoffs of any two policies becomes a function only of r_5 and r_6, with the boundary between games for which policy i is better than policy j and games for which the reverse is true being a straight line in the (r_5,r_6) plane. Combining the information contained in these pairwise policy maps for all pairs of policies leads in a straightforward way to partitioning of the (r_5,r_6) plane into the regions (usually only 2 or 3 for a given set of conditional response probabilities) in which different policies are optimal, each region being bounded by one or more straight line segments. Note that the other player's payoffs -- as represented by his point on the (r_5, r_6) plane -- and the form of the game are both irrelevant to determining which policy is optimal, _except_ as they influence the other player's selection of conditional response probabilities (i.e., the way in which he reacts to our actions). Thus the present classification system provides a highly compact summary of the games for which various policies of play are optimal against a particular other-player strategy.

References

Hamburger, H. Separable games. Behavioral Science, 1969, 14, 121-132.

Harris, R. J. A geometric classification system for 2 x 2 interval-symmetric games. Behavioral Science, 1969, 14, 138-146.

Harris, R. J. A note on 'Optimal policies for the Prisoner's Dilemma.' Psychological Review, 1969, 76, 363-375.

Marwell, G., & Schmitt, D. R. Are trivial games the most interesting psychologically? Behavioral Science, 1968, 13, 125-128.

Rapoport, A. Optimal policies for the Prisoner's Dilemma. Psychological Review, 1967, 74, 136-148.

Rapoport, A. Exploiter, leader, hero, and martyr: The four archetypes of the 2 x 2 game. Behavioral Science, 1967, 12, 81-84.

Rapoport, A. & Chammah, A. M. Prisoner's Dilemma. Ann Arbor: University of Michigan Press, 1965.

Rapoport, A. & Guyer, M. A taxonomy of 2 x 2 games. General Systems, 1966, 11, 203-214.

Wolf, G. Interpersonal relationships and choices as linear bi-matrix properties. Report no. 24, October 1969. Department of Administrative Sciences, Yale University.

Footnote

1. An 1800-word summary of the present paper appeared in _Proceedings, American Psychological Association_, 1971, _6_. The paper was discussed at the session on Bargaining and Game Theory, 79th annual convention of the APA, Washington, D. C. September 3, 1971.

Table 1

The 78 Ordinally Defined 2x2 Games and their Definitions within the Interval-Scale Classification System

Ordinal Regions of Figure 2

	PD		SSE-9		SSE-7		Chicken		Battle of Sexes		Apology	
PD-12 ($0<r_5, r_6<1$; $r_5+r_6<1$)	6 (NC)	21 (TVE) 12 (PD)	1 (NC) 19 (TVE)	17 (SE) 11 (SSE)	2 (NC) 20 (TVE)	18 (SE) 10 (SSE)	26 (NC) 48 (FVE)	55 (USE) 39 (TVE)	27 (NC) 47 (FVE)	56 (USE) 37 (TVE)	30 (NC) 57 (USE)	44 (FVE) 38 (TVE)
SSE-9 ($0<r_5, r_6<1$; $r_5+r_6>1$)			3 (NC)	14 (SE) 9 (SSE)	4 (NC) 13 (SE)	16 (SE) 8 (SSE)	22 (NC) 46 (FVE)	50 (USE) 35 (SE)	24 (NC) 43 (FVE)	52 (USE) 31 (SE)	28 (NC) 54 (USE)	41 (FVE) 33 (SE)
SSE-7 ($r_5>0; r_6>1$)					5 (NC)	15 (SE) 7 (SSE)	23 (NC) 45 (FVE)	49 (USE) 36 (SE)	25 (NC) 42 (FVE)	51 (USE) 32 (SE)	29 (NC) 53 (USE)	40 (FVE) 34 (SE)
Chicken-66 ($r_5<0; r_6<1$)							61 (NC)	72 (GWE) 66 Chicken	62 (NC) 78 (GWE)	71 (GWE) 67 (TENO)	58 (NC) 70 (GWE)	77 (GWE) 65 (TEEO)
BS-68 ($r_5<0; r_6>1$; $r_5+r_6<1$)									63 (NC)	76 (GWE) 68 (BS)	59 (NC) 74 (GWE)	75 (GWE) 64 (TEEO)
Apology-69 ($r_5<0$; $r_5+r_6>1$)											60 (NC)	73 (GWE) 69 Apology

Notes: (see next page)

Notes: (a) The table is entered with three pieces of information: the ordinal region of Figure 2 within which row-player's (r_5, r_6) point falls; the ordinal region within which column-player's point falls; and the relative position of the two players' largest payoffs (t_1 and t_2).

(b) Within each 2x2 sub-matrix of the table, the upper left-hand entry represents the game's classification if t_1 and t_2 fall in the same cell of the outcome matrix; the upper right-hand entry represents the game's classification if t_2 is in a different cell but the same row as t_1; the lower left-hand entry, if they are in different cells but the same column; and the lower right-hand entry, if t_1 and t_2 are diagonally placed in the outcome matrix of the game.

(c) The table is symmetrical. Thus, for instance, a (PD,BS)-R game is the same as a (BS,PD)-C game. The below-diagonal entries have therefore not been included in the present table.

(d) The abbreviation in parenthesis refers to Rapoport & Guyer's (1966) description of the game in terms of the number and stability of its equilibria and its likely ("natural") outcome. NC = "No Conflict"; SE = "Stable Equilibrium"; SSE = "Strongly Stable Equilibrium"; USE = "Unstable Equilibrium"; FVE = "Force-Vulnerable Equilibrium"; TVE = "Threat-Vulnerable Equilibrium"; TEEO = "Two-Equilibria with Equilibrium Outcome"; TENO = "Two Equilibria with Non-Equilibrium Outcome"; GWE = "Game w/out Equilibrium".

Table 2

FORM OF THE GAME WHICH RESULTS FROM VARIOUS COMBINATIONS OF g_1 AND g_2

Row Player's Segment	Segment on Which Column Player's (r_5, r_6) Lies: 1 $(-\infty \leq g_2 \leq .571)$	2 $(.571 \leq g_2 \leq 1.6)$	3 $(1.6 \leq g_2 \leq \infty)$
1 $(-\infty \leq g_1 \leq -.6)$	Form D	Form C	Form R
2 $(-.6 \leq g_1 \leq .429)$	Form R	Form S	Form D
3 $(.429 \leq g_1 \leq \infty)$	Form C	Form D	Form S

Table 3

EXPRESSION TO BE USED FOR ADJACENT SEGMENT OF PERCEIVED GAME LOCUS AS A FUNCTION OF EXPRESSION USED FOR THIS SEGMENT AND BOUNDARY CROSSED

Expression Used for this Segment	Boundary Crossed Getting to Adjacent Segment		
	$r_6 = 0$	$r_5 = 1$	$(-\infty, \infty)$
Form D[a]	Form R	Form C	Form S
Form S[b]	Form C	Form R	Form D
Form R[c]	Form D	Form S	Form C
Form C[d]	Form S	Form D	Form R

[a]Equations (3).

[b]Equations (4).

[c]Equations (5) with subscripts "1" & "2" interchanged. Note, however, that this table is presented from row-player's point of view. If the form R expression was used for column player's locus, the appropriate expression to be used for the adjacent segment is found in the "form C" row of this table.

[d]Equations (5). Note, however, that if the form C expression was used for column player's locus, the expression to be used for the adjacent segment is to be found in the "Form R" row of the present table.

Figure Captions

Fig. 1. The Four Possible Forms of the 2 x 2 Game. By definition, t_i is the largest payoff available to player i; s_i is the entry diagonal to t_i; r_i is the other payoff player i might receive if he tried to get s_i (i.e., it is the row player's payoff in the same row as s_1, and column player's payoff in the same column as s_2); and p_i is the other payoff player i might receive if he tried to get t_i. In Form S games, t_1 and t_2 occur in the same cell. In Form D games, t_2 is diagonally opposite t_1. In Form R, t_1 and t_2 are in different cells of the same row; and in Form C, they are in different cells of the same column.

Fig. 2. The (r_5, r_6) Plane. Note--The identifying information within each area bounded by a solid line refers to the classification applied to a Form D game both of whose (r_5, r_6) points fall within that ordinal area, and includes: the rank order of the matrix entries, the commonly applied name for such games, and the games' number within the Rapoport & Guyer (1966) taxonomy.

Fig. 3. Perceived Game Loci for a [(.1,.3); (-.2,.6); 20, 10, 2, 0] -R Game.

r_1, p_2	s_1, s_2
t_1, t_2	p_1, r_2

Form S

r_1, r_2	s_1, t_2
t_1, s_2	p_1, p_2

Form D

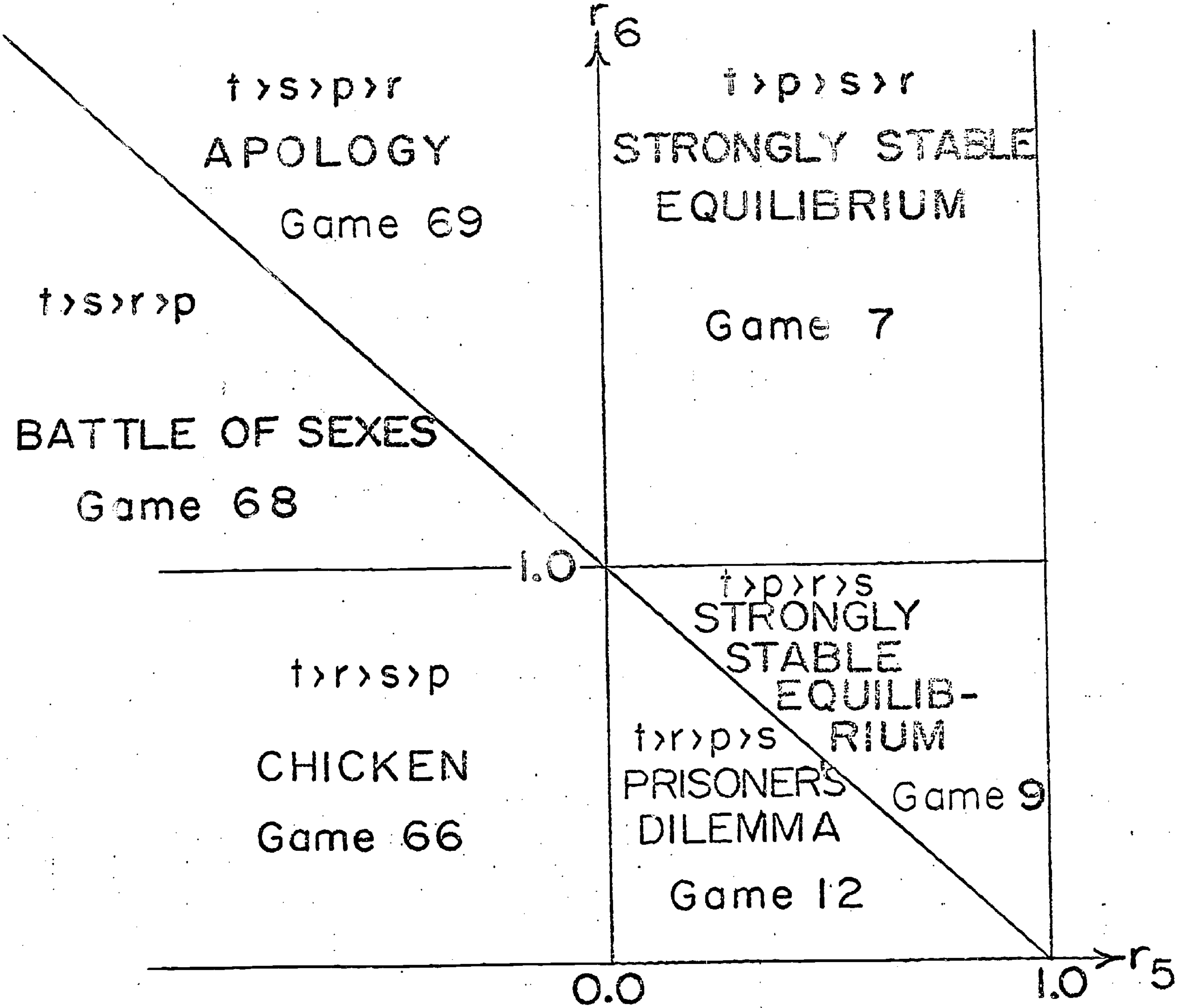
r_1, s_2	s_1, p_2
t_1, r_2	p_1, t_2

Form R

r_1, t_2	s_1, r_2
t_1, p_2	p_1, s_2

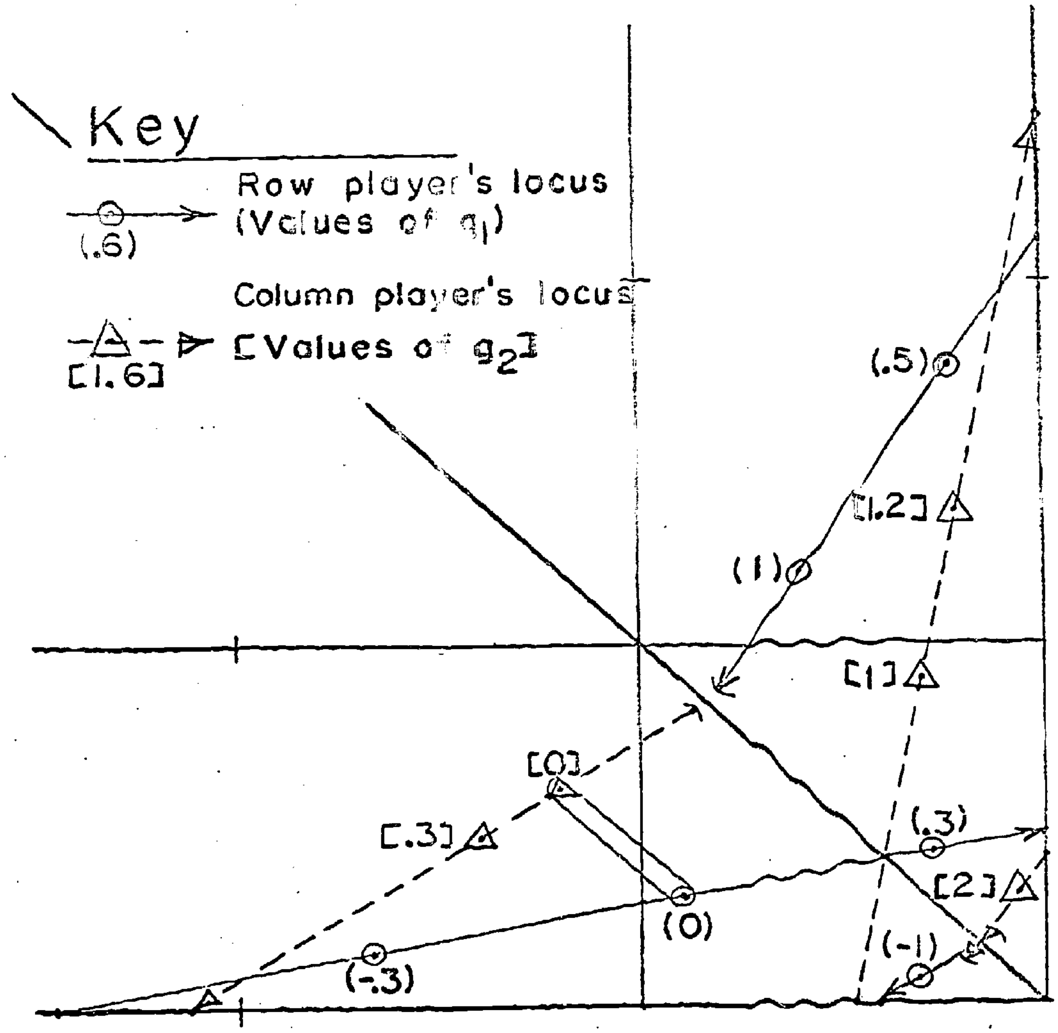
Form C

Fig. 1



r6
t>s>p>r
APOLOGY
Game 69
t>p>s>r
STRONGLY STABLE
EQUILIBRIUM
Game 7
t>s>r>p
BATTLE OF SEXES
Game 68
1.0
t>p>r>s
STRONGLY
STABLE
EQUILIB-
RIUM
Game 9
t>r>s>p
CHICKEN
Game 66
t>r>p>s
PRISONER'S
DILEMMA
Game 12
0.0
1.0
r5

Fig. 2



Key
Row player's locus
(Values of q_1)
(.6)
Column player's locus
[Values of g_2]
[1.6]
(.5)
[1.2]
(1)
[1]
[0]
[.3]
(.3)
[2]
(0)
(-1)
(-.3)

Fig. 3