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ABSTRACT

The personnel decision principally treated in this paper is that of remedial placement. The decision made with respect to candidates is to accept or reject, and performances by candidates are evaluated as pass or fail. A criterion and a selection variable were assumed, and the problem is to choose a cutting score on the selection variable such that the value of acceptance decision-outcome combinations is at an optimum. Optimization conditions are given in general functional notation and for normal surfaces. Numerical examples are given for three philosophies of evaluating pass and fail outcomes and for three levels of college course difficulties. (Author)

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Abstract

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A FOUR VALUE PERSONNEL DECISION MODEL: REMEDIAL PLACEMENT[1]

Robert F. Boldt

Educational Testing Service

In contrast to the highly selective institutions which catered to the elite of another era, many institutions exist today which "meet the student at his level." Rather than being academically oriented with highly selective admissions standards, these institutions minimize admissions requirements, choosing to guide the less able student into curricula which are appropriate for his skills, abilities, and interests, and to remedy background deficiencies he may have when such seems desirable in view of the path the student has planned for himself. Thus the guidance counselor and the student are faced with the decision as to whether some experience is needed which is aimed at remedying the effects of perhaps inadequate focus on academic skills, particularly in English and mathematics. The decision as to whether some remedial training is needed may be made in a variety of ways, and one common one is on the basis of scores obtained from orientation testing.

The problem of reaching this type of decision is not new. Many authors have discussed the assessment of personnel processes which depend on psychometric evaluation. The discussions and evaluations may be complicated by the need for externally generated information about the worth of a personnel action. Though most authors have of necessity made their evaluations in terms of criteria which are immediately observable, even if not based on ultimate values, some have been concerned with more remote criteria (Brogden, 1949; Brogden & Taylor, 1950; Cronbach & Gleser, 1965). Certainly, for the last three decades the systems analysts have been aware that the relative excellence of performance of some particular task by personnel is not to be equated to a contribution to the performance of a total system. In fact, it is probably only the simplest of systems in which it is possible to make a believable and direct connection between individual performance and system performance, contribution to society, or whatever the purpose of the system is within which the personnel process exists. Indeed, the most complex models of human performance have been constructed in the context of man-machine interactions, the most complicated of the complex being developed for control systems such as aircraft or fire control systems (e.g., Machol, Tanner, & Alexander, 1965, Chs 32-34, pp. 31-36), where the connection between human response and system performance is most directly and easily comprehended. In extremely complex systems such as the large military, industrial, or educational organizations, the tie between individual and system performance is quite often not apparent. In fact, an organizational action may be complex and multivalued and even may be partly evaluated on the basis of the effects on the individuals who perform within it, as in educational systems.

The problem becomes confusing beyond measure and yet one acquires the notion, and the experience to back it up, that some individual performances are better than others in terms of organizational output, or at least that some are clearly satisfactory and others are clearly not. One might, then, approach the problem of evaluating performances in complex situations by using only a few values reflecting a limited number of states of the personnel system. Such an approach has precedent (Berkson, 1947; Brown, 1950; Duncan et al., 1953; Taylor & Russell, 1939). For some purposes it is sufficient, and in some sense more meaningful, to think about situations where personnel decisions are of an accept-reject sort and criterion performances are of a pass-fail sort. Hence, outcomes are evaluated for four situations only, accept-pass, accept-fail, reject-pass, and reject-fail. High passes would be evaluated the same as near failures and low failures would be evaluated the same as near passes (if such distinctions exist).

In the remedial situation, a large group of incoming students is tested to determine who is prepared for academic competition at the college level and who should receive special treatment. It is taken for granted that the testing will occur and hence the cost of testing is not a factor. This paper will not consider differential costs of remedial as opposed to regular course work except insofar as they might be incorporated in evaluating the four outcomes listed above. Rather, the problem here is to set a cut score for determining who gets remedial treatment and who goes on to regular collegiate work.

In the theoretical scheme, it is assumed that one is dealing with a "candidate population" each member of which is to be "accepted" or "rejected." Decisions to "accept" or "reject" are, in the context of decisions about remedial training, to allow entry into a regular academic curriculum or to require makeup work of some kind. They are to be based on a "selection variable," x. Evaluation of the outcome of a decision is based on a "criterion variable" which is evaluated as "pass" or "fail." Here, passing and failing refer to the quality of performance in a regular college course.

Development of the Cut Score

It is assumed that the joint distribution of the selection variable, x, and the criterion variable, y, is known and is denoted $f(x,y)$.[2] Let c_y be a cut score on the criterion variable such that a pass occurs if $y \geq c_y$, failure otherwise. Let c_x be the cut score sought such that if $x \geq c_x$, acceptance occurs, rejection (route to remedial) otherwise. In Figure 1 the

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Insert Figure 1 about here
---

selection variable, x, is represented by the abscissa and a criterion variable, y, is represented by the ordinate. Then the dotted line whose equation is $x = c_x$ divides acceptees to the right from rejectees to the left. The dotted line whose equation is $y = c_y$ divides passers above from failures below. Greek symbols in Figure 1 are included to clarify the regions of integration in the definitions immediately below.

Then

$$\alpha(c_x) = \int_{c_x}^{\infty}\int_{c_y}^{\infty} f(x,y)dydx \text{ , is the proportion of selected passers,}$$

$$\beta(c_x) = \int_{c_x}^{\infty}\int_{-\infty}^{c_y} f(x,y)dydx \text{ , is the proportion of selected failures,}$$

$$\gamma(c_x) = \int_{-\infty}^{c_x}\int_{c_y}^{\infty} f(x,y)dydx \text{ , is the proportion of rejected passers,}$$

and

$$\delta(c_x) = \int_{-\infty}^{c_x}\int_{-\infty}^{c_y} f(x,y)dydx \text{ , is the proportion of rejected failures.}$$

Since the proper choice of the cut score on the selection variable, c_x, is the concern of this discussion, notation is used which indicates that α, β, γ, and δ are all functions of c_x. Note also that in Figure 1 both Roman and Greek letters appear. The Roman letter is the symbolic representation of the gain or cost of a case which falls in the area associated with a Greek letter. Let

a be the gain from selecting a passer,

b be the cost of selecting a failure,

g be the cost of rejecting a passer, and

d be the gain from rejecting a failure.

It is assumed that a , b , g , and d are real, nonnegative numbers, constants of the system and hence not functions of the cutting score. The numerical values of these constants, or certain relations among them, must somehow be determined if an optimal cut score is to be set.

Neglecting a constant for the number of people involved,

$$V(c_x) = a\alpha(c_x) - b\beta(c_x) - g\gamma(c_x) + d\delta(c_x) \tag{1}$$

is the value of a set of acceptance decisions using c_x as the cut score on the selection variable. One may note from the definition of γ and δ that if $\gamma + \delta$, the proportion of persons to receive remedial course work, is fixed, c_x is fixed and hence V does not enter into choice of c_x . Rather, with a fixed quota for remedial courses one merely rejects cases on x starting from the bottom and going up. If one does not have a fixed quota system, one chooses c_x by maximizing V . To do this, differentiate equation (1) with respect to c_x to obtain

$$dV(c_x)/dc_x = -a\int_{c_y}^{\infty} f(c_x,y)dy + b\int_{-\infty}^{c_y} f(c_x,y)dy - g\int_{c_y}^{\infty} f(c_x,y)dy + d\int_{-\infty}^{c_y} f(c_x,y)dy$$

$$= -a\int_{c_y}^{\infty} f(c_x,y)dy + b\left(\int_{-\infty}^{\infty} f(c_x,y)dy - \int_{c_y}^{\infty} f(c_x,y)dy\right)$$

$$- g\int_{c_y}^{\infty} f(c_x,y)dy + d\left(\int_{-\infty}^{\infty} f(c_x,y)dy - \int_{c_y}^{\infty} f(c_x,y)dy\right)$$

$$= (b + d)\int_{-\infty}^{\infty} f(c_x,y)dy - (a + b + g + d)\int_{c_y}^{\infty} f(c_x,y)dy \quad .$$

Equating $dV(c_x)/dc_x$ to zero yields

(2) $$(b + d)\int_{-\infty}^{\infty} f(c_x,y)dy = (a + b + g + d)\int_{c_y}^{\infty} f(c_x,y)dy \quad .$$

Thus a maximum of $V(c_x)$ exists at that array of x where the relationship (2) obtains between the proportion in the array that pass and the costs and values (provided such an array exists). Note that the integral on the left-hand side of equation (2) yields the marginal density of x at c_x . If one divides (2) by this marginal density and solves for the resulting conditional density, one obtains

(3) $$P(\text{pass}\,|\,c_x) = (b + d)/(a + b + g + d)$$

or

(4) $$P(\text{fail}\,|\,c_x) = (a + g)/(a + b + g + d)$$

as the condition required to maximize $V(c_x)$.

Application to the Normal Case

Suppose that in the candidate population x and y are jointly normally distributed. Then $P(\text{pass}\,|\,c_x)$ is the integral of the normal probability function from a lower bound

(5) $$B = \frac{((c_y - \mu_y)/\sigma_y) - ((c_x - \mu_x)/\sigma_x)\rho}{\sqrt{1 - \rho^2}} \quad ,$$

where the μ 's, σ 's, and ρ are the usual parameters of the bivariate normal distribution. Knowing the right-hand side of equation (3) or (4), the required value of B can be gotten from the normal tables. The value

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of c_y must be estimated, at least in standard score form, from one's knowledge of the difficulty of the college course. Then, knowing the values of the parameters of the joint distribution of x and y, the optimal value of c_x can be found by substituting all the known values into equation (5) and solving for c_x.

For illustration, three evaluations of the outcome of placement are considered. The first evaluation, in which the emphasis is on avoiding failure, would suggest the assumption that $2a = d$ and $2g = b$. In this evaluation, the left side of equation (3) becomes

$$(b + d)/(a + b + g + d) = (2a + 2g)/(a + 2g + g + 2a) = 2/3 \quad ,$$

and $B = -.43$. The second evaluation, one in which neither the costs of incorrect decisions nor the costs of correct decisions can be discriminated, would suggest assumptions of indifference, that is, $a = d$ and $g = b$. In this evaluation, the left side of equation (3) becomes

$$(b + d)/(a + b + g + d) = (b + d)/(b + d + b + d) = 1/2$$

and $B = 0$. The third evaluation, one in which the emphasis is on detecting potential passers, would suggest the assumptions that $a = 2d$ and $g = 2b$. In this evaluation, the left side of equation (3) becomes

$$(b + d)/(a + b + g + d) = (b + d)/(2d + b + 2b + d) = 1/3$$

and $B = .43$.

To further the illustration, three levels of difficulty will be assumed for the criterion (college level course): An easy one such that 80% of the candidates lie above c_y (would pass), one of middle difficulty such that half the students would pass the course, and a hard course where only 20%

of the candidates would pass. With the normality assumptions that have been made, c_y expressed in standard score form equals -.84 for the easy course, zero for the course of middle difficulty, and .84 for the hard course.

Using the three evaluations of outcomes of placement, the three difficulties of the college level course, assuming c_x is in standard score form, and that $\rho = .6$, six sets of values can be substituted in equation (5) to obtain six cut scores. The cut score for the situation where the desire to avoid failure predominates and the college course is easy is gotten by substituting into equation (5) to obtain

$$(6) \qquad -.43 = (-.84 - .6\, c_x)/\sqrt{1 - .6^2}$$

where c_x is assumed to be in standard score form. Solving for c_x yields about -.83 which, using normal tables, suggests that about 80% of the students should be let in the college course. However, if one emphasizes the detection of passers, the substitution in equation (3) yields

$$(7) \qquad .43 = (-.84 - .6\, c_x)/\sqrt{1 - .6^2}$$

which when solved for c_x yields about -1.97 and implies an acceptance rate of about 98%. If the hard college course were being used, the -.84 in equations (6) and (7) would be replaced by .84; if the intermediate position on evaluation were adopted the left-hand side of equations (6) and (7) would be zero.

The six sets of results are summarized in Table 1. Note that the cut score or the percent accepted are strongly related to the difficulty of the college course. It is believed that the remedial placement situation is one in which most people would pass the college course and hence line items (1),

(4) and (7) are the ones of concern here. Examination of these line items suggests that under the present assumptions very few students would be given remedial training unless the desire to avoid failures is much stronger than that to detect passers. Certainly, the results of Table 1 emphasize the effect of course difficulty and the evaluation of outcomes on remediation policy.

Insert Table 1 about here

Use of the Cumulative Distribution

Equations (3) and (4) indicate that the cut score, c_x, should be chosen using the arrays of y for given levels of x. In practice these arrays may not be usable since only a few scores may be near enough to x to use in an array. One would prefer to use the more stable cumulative distributions. Such a use may be justified considering that it is usually reasonable to believe that $P(\text{pass} | c_x)$ is monotonically increasing with c_x, i.e., if $v \leq c_x$, then

$$P(\text{pass} | v) \leq P(\text{pass} | c_x) \quad .$$

Then if $Q(\text{pass} | c_x)$ is the proportion of all cases at c_x <u>or below</u> that would pass,

$$Q(\text{pass} | c_x) \leq P(\text{pass} | c_x) \quad .$$

Hence, using equation (3) the value of x for which Q equals $(b + d)/(a + b + g + d)$ is an upper bound for c_x. Given admittedly grossly approximate estimates of a, b, g, and d one might well be willing to use Q to find c_x using some judgmental adjustment.

Summary

The present paper deals with a personnel situation characterized by a candidate pool of fixed size, each member of which is to be accepted or rejected on the basis of a selection variable. The outcome of performance if accepted is either pass or fail. All accepted passers are considered equivalent as are accepted failures, rejected failures, and rejected passers though the value of any one of the outcomes may be different from any other. An objective function is developed for evaluating cut scores and conditions for choosing or approximating an optimum cut score are presented. Examples are given under conditions of joint normality and it is found that the rejection rate is large only when one is highly concerned about failure. The model was developed with remedial placement in English and mathematics in mind.

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Footnotes

[1]This research was supported in part by the College Entrance Examination Board.

[2]Note that $f(x,y)$ is the joint distribution of the selection variable and the criterion variable <u>in the candidate pool</u>. Such a distribution cannot be observed in practice unless the means of reaching acceptance decisions at the time of a study are unrelated to the selection variable used in the study. Possibly some appeal to range restriction theory (Gulliksen, 1950, Ch. 13) could be used. One should be cautious about naive acceptance of statistics relating selectors and performances as experienced in a given situation.

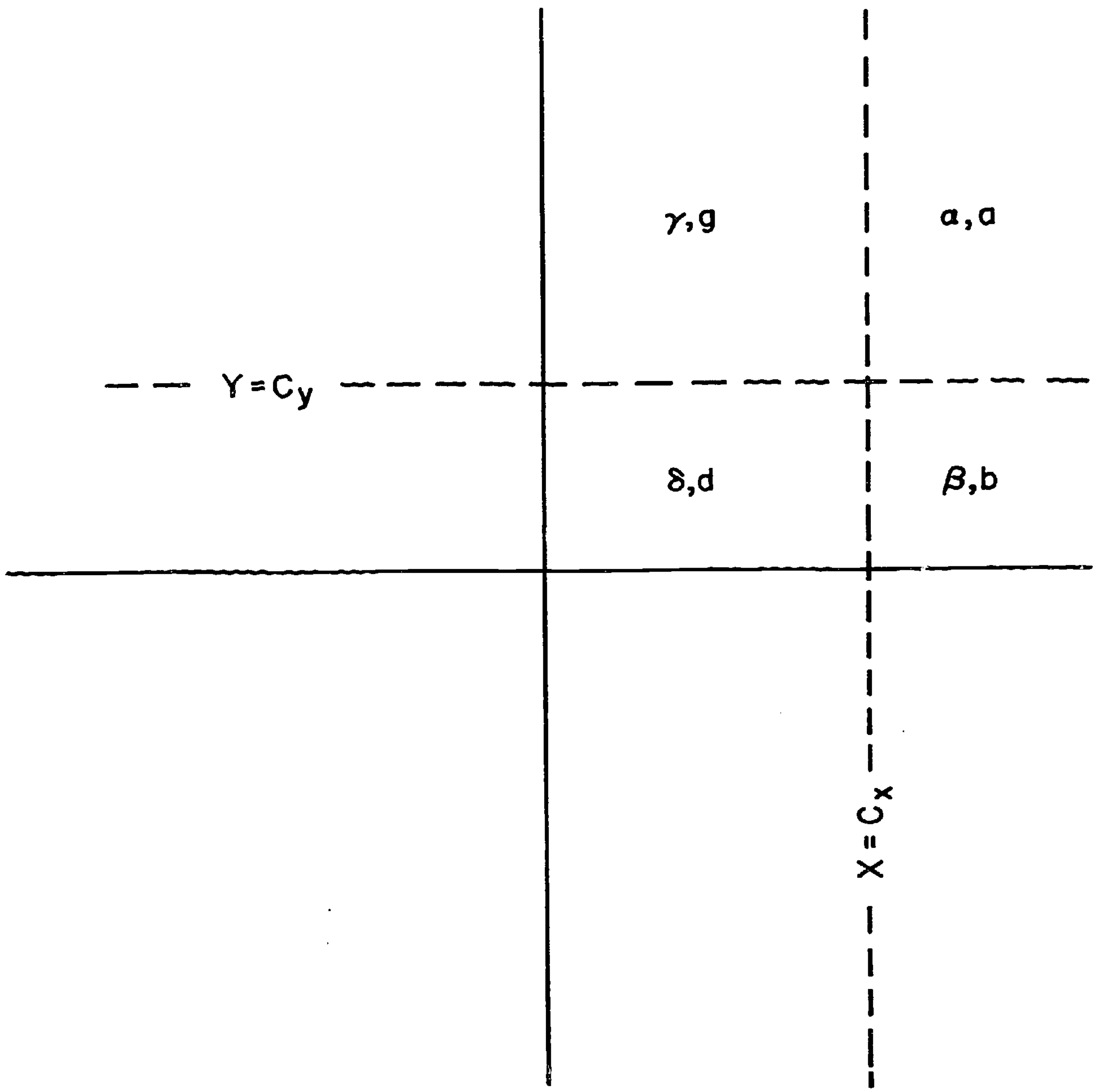
Table 1

Cut Score and Percent Accepted for Different Course Difficulties and Evaluations of Outcomes

Line No.	Evaluation	Difficulty of College Course	c_x	% Accepted
(1)	Avoid Failure	Easy	-.83	80
(2)	Avoid Failure	Intermediate	.57	28
(3)	Avoid Failure	Hard	1.97	2
(4)	Indifference	Easy	-1.4	92
(5)	Indifference	Intermediate	0	50
(6)	Indifference	Hard	1.4	8
(7)	Detect Passer	Easy	-1.97	98
(8)	Detect Passer	Intermediate	-.57	72
(9)	Detect Passer	Hard	.83	20

Figure Caption

Fig. 1. Four gain (cost) regions for given c_x and c_y .


γ,g
α,a
$Y=C_y$
δ,d
β,b
$X=C_x$