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ABSTRACT

Reported are the results of an investigation of the effects of small-group discussions in mathematics classes for preservice elementary school teachers. The content of the course included modern algebraic and geometric concepts. Both classes were taught by the same instructor and studied from the same set of understanding of the interdependence of all segments of the total environment. allotment: review lecture (5 minutes), class discussion (10 minutes), new material lecture (10 minutes), and class discussion (25 minutes). The "small-group" class used this allotment: review and new material lecture (10 minutes), small-group discussion (30 minutes), and class discussion (10 minutes). The size of the discussion groups alternated weekly between three and four students. Group composition was randomly determined. Group leadership was rotated daily. The following measures were analyzed: (1) computational skills, (2) achievement on examinations of independent reading ability in mathematics, (3) attitude toward mathematics, and (4) overall course achievement. The differences which were analyzed and found to be significant included: (1) the post-test measure of computational skills for the small-group class was better (.05) than the pretest measure, (2) the independent reading measure for the upper half of the small-group was better (.01) than that of the upper half of the lecture class. Other differences found between criterion measures were not significant. (Author/RS)

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The Wisconsin State Universities Consortium of Research Development

Research Report

A PILOT STUDY ON THE USE OF SMALL-GROUP DISCUSSION IN A MATHEMATICS COURSE FOR PRESERVICE ELEMENTARY SCHOOL TEACHERS

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Cooperative Research

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ABSTRACT

A PILOT STUDY ON THE USE OF SMALL-GROUP DISCUSSION IN A MATHEMATICS COURSE FOR PRESERVICE ELEMENTARY SCHOOL TEACHERS

H ry Howard Thoyre

Under the supervision of Professor John Grover Harvey

The Problem

The basic question considered in this study was "Can a mathematics class of preservice elementary teachers working in small groups examine all of the course content normally included in a class taught by lecture-class discussion technique without sacrificing overall course achievement?" Three related questions also considered dealt with computational skills, attitude toward mathematics, and the ability to read unfamiliar mathematical material independently possessed by participants in the two classes.

The Procedure

Two classes of preservice elementary school teachers enrolled, during the spring semester of 1969, in the four-credit mathematics course required of all elementary education majors at Wisconsin State University - Stevens Point participated in the study. There were 23 students in one section and 27 in the other. Both classes were taught by the same instructor and studied from the same set of instructor-prepared notes.

Approximately 30% of the 50 minute class period in the lecture-class discussion (LCD) section was devoted to instructor-dominated activities including review of previously discussed material and introduction of new topics. The remainder of the class period was an instructor-led class discussion of topics introduced in the lecture and development of new concepts.

In the lecture-small group discussion (LSGD) section, the initial time segment of ten minutes was an instructor-dominated lecture period. The following 30 minutes was devoted to small-group discussion of concepts related to the lecture and new material. The students were given no direct assistance by the instructor. The last ten minutes of the period was spent in a class discussion of topics and problems encountered by the individual discussion groups.

The sizes of the discussion groups alternated weekly between three and four students. Group composition was randomly determined. Group leadership was on a rotating basis daily.

Mean scores on the following criterion measures were analyzed by an analysis of covariance: (1) computational skills, (2) achievement on examinations of independent reading ability in mathematics, (3) attitude toward mathematics, and (4) overall course achievement.

A t-test was used to compare within-group mean scores on pre- and posttest computational skills and attitude toward mathematics.

Findings and Conclusions

The following are among the results and conclusions reported in the dissertation. Conclusions are based on data collected, analysis of the data, and an analysis of a questionnaire on the student's opinion of the course.

1. There was a significant difference, at the .05 level, between pretest and posttest scores of the LSGD section on the computational skills criterion measure.
2. Although posttest mean score exceeded pretest mean score on the attitude scale in both sections, neither was significantly greater at the .05 level of confidence.
3. Differences between mean scores of the two groups on each of the criterion measures failed to be significant at the .05 level, although in each case the mean score of the LSGD section was greater than that of the LCD group.
4. The mean score of the upper one-half abi[illegible] of the LSGD section was significantly greater, at the .01 level of confidence, than the mean score of the upper one-half ability level of the LCD section on the criterion measure "achievement on tests of independent reading ability in mathematics."
5. The LSGD group examined the same amount of mathematical material as the LCD section without sacrificing course achievement.
6. Students in the LSGD section were more inclined to believe their attitude toward mathematics had been enhanced over the semester than did students of the LCD group.

7. A group size of three and a group size of four with random pairing within the group were equally favored by students of the LSGD section.

ACKNOWLEDGMENTS

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TABLE OF CONTENTS

LIST OF TABLES

Table		Page

Table		Page

APPENDICES

CHAPTER I

THE PROBLEM

The two basic questions to be explored in this study are:

1. In a mathematics course for preservice elementary school teachers, can as much mathematical material be covered by a class in which the students read and discuss almost all of the course content within small work-groups as is covered by a class taught by a conventional lecture-class discussion technique?
2. In terms of overall course achievement, will the small-group discussion class compare favorably with a class taught by a lecture-class discussion technique?

In essence, the two basic questions ask whether a mathematics class for preservice elementary teachers can be taught using a small-group discussion approach without sacrificing content or understanding of basic concepts.

In anticipation of affirmative answers to the two basic questions, three secondary questions will be considered:

1. Will the students in the small-group discussion class perform as well as the students in the lecture-class discussion group on a test of arithmetic computational skills?
2. Will participation in the small-group discussion class have a more favorable effect on the students' attitude

toward mathematics than participation in the lecture-class discussion group?

3. Will the participants of the small-group discussion class perform significantly better than the students of the lecture-class discussion group on examinations requiring the ability to read unfami iar mathematical material independently?

The relevance of secondary question (1) to the undergraduate preparation of elementary school teachers of mathematics is obvious and the significance of a positive attitude toward mathematics will be discussed in Chapter II. The importance of the ability of elementary teachers to read unfamiliar mathematical material independently and with understanding can be argued by considering the recommendations of the Committee on the Undergraduate Program in Mathematics; the recommendations and projections of the Cambridge Conference on School Mathematics; the recommendations of the Cambridge Conference on Teacher Training, and then looking at the success achieved in the implementation of these recommendations.

The work of SMSG and UICSM in the latter part of the 1950's and early 1960's has had a profound effect on both the elementary and secondary school mathematics curriculum. There has been a substantial change in the topics previously thought to be the domain of the elementary school program, but an even greater change has occurred in the method and spirit of instruction. It is no longer sufficient to be just a good drill-master in the teaching of elementary school mathematics. Today's elementary teacher must not only have a firm grasp of the computational skills and algorithms of arithmetic, but must clearly understand the basic mathematical concepts of arithmetic.

The report of the Cambridge Conference on School Mathematics, Goals for School Mathematics (1963), suggests the need for an even greater knowledge of mathematics in the foreseeable future. Their curriculum for the elementary school, grades K through six, includes a study of 2 x 2 matrices, finite field, elementary Diophantine problems, density of the rational numbers, conic sections, polar coordinates, vectors, elementary logic, mathematical induction, isomorphisms, linear transformations, trigonometric and logarithmic functions. Adler (1966, pp. 210-17) cites four reasons for concluding that the report of the Cambridge Conference is indeed realistic:

1. The children can learn more than we think they can.
2. The transition from one stage of learning to the next can be accelerated by a better curriculum and better teaching.
3. The early use of the concepts of mathematical structure accelerates learning by simplifying the subject matter.
4. Changes like those proposed by the report have already been tried successfully.

The report of the Committee on the Undergraduate Program in Mathematics (CUPM) entitled Recommendations for the Training of Teachers of Mathematics (1961) recommended four three-semester-hour mathematics courses for prospective elementary teachers. These Level I recommendations are:

(A) A two-course sequence devoted to the structure of the real number system and its subsystems.

(B) A course devoted to the basic concepts of algebra.

(C) A course in informal geometry.

This recommendation presumes at least a two-year high school mathematics sequence, exclusive of general mathematics.

The recommendations of the Cambridge Conference on Teacher Training (1967) are somewhat more ambitious. Although the conference did not consider their recommendations to be in conflict with those of CUPM, they do concede that their proposals ". . . are aimed (hopefully) at a time when the CUPM recommendations will have been strongly implemented and the new generation of potential teachers, . . . , will have profited by that implementation" (Cambridge Conference on Teacher Training, 1967, p. 15).

Unfortunately there is a sizeable gap between the recommended undergraduate preparation of elementary teachers and that which is presently being offered. Fisher (1967, pp. 194-197), in a survey of 78 randomly selected teacher training institutions in the United States, found that in 1965 the average number of semester hours required of elementary teachers was only 4.15. A survey by the CUPM Panel on Teacher Training (Committee on the Undergraduate Program in Mathematics, 1966, pp. 138-148) of 901 colleges and universities engaged in teacher training revealed that in 1966, approximately 38% of the schools required from 3 to 4 hours, 37% required 5 to 6 hours, and about 12% required more than 7 hours in mathematics courses for prospective elementary school teachers. These percentages do indicate an increase in the required hours from 1961 to 1966, however, they still fall far short of what is desired. The demands being made for additional time in the undergraduate program of elementary teachers by other disciplines--English and the physical sciences, particularly--makes the prospect of achieving a twelve semester-hour sequence in mathematics seem rather remote at this time.

In view of the projections for the introduction of an increasing number of new topics in the elementary school program and the dim prospect of increasing the amount of mathematics required by preservice elementary teachers much beyond eight semester-hours, an undergraduate mathematics program must provide the opportunity for the student to enhance his ability to read new mathematical content independently and with understanding. Therefore, in the opinion of this investigator, an undergraduate mathematics program for elementary teachers must not only provide an opportunity for the student to master the basic mathematical concepts underlying arithmetic, to master the computational skills of arithmetic, and to enhance his attitude toward mathematics, but also to improve his ability to work independently in mathematics. Answers to the two basic questions and the three secondary questions posed at the beginning of this chapter will indicate whether one teaching technique is to be preferred over the other in the achievement of these objectives.

To answer these questions, two groups of preservice elementary teachers were used in this study. One group was taught using a lecture-class discussion technique and the other by a lecture-small group discussion technique. The first basic question will be answered by comparing the amount of material covered by one section to that covered by the other section. The remaining four questions will be answered by a statistical analysis of the following hypothesis:

Hypothesis 1. There is no significant difference between the mean scores of the two groups with respect to

(a) overall course achievement as measured by three unit tests and a final examination,

(b) computational skills as measured by a standardized test on basic arithmetic computational skills,

(c) attitude toward mathematics as measured by an attitude scale,

(d) the ability to read unfamiliar mathematical material independently and with understanding as measured by four instructor-prepared examinations.

In addition to comparing the two groups in terms of the basic question of overall course achievement and the three secondary questions, the upper and lower one-half ability levels, as determined by A.C.T. mathematics percentile scores, of each group will be compared. Specifically, the following hypotheses will be investigated:

Hypothesis 2. There is no significant difference between the mean scores of the two upper one-half ability levels as determined by A.C.T. mathematics percentile scores with respect to

(a) overall course achievement as measured by three unit tests and a final examination,

(b) computational skills as measured by a standardized test on basic arithmetic computational skills,

(c) attitude toward mathematics as measured by an attitude scale,

(d) the ability to read unfamiliar mathematical material independently and with understanding as measured by four instructor-prepared examinations.

Hypothesis 3. There is no significant difference between the mean scores of the two lower one-half ability levels as determined by A.C.T. mathematics percentile scores with respect to

(a) overall course achievement as measured by three unit tests and a final examination,

(b) computational skills as measured by a standardized test on basic arithmetic computational skills,

(c) attitude toward mathematics as measured by an attitude scale.

(d) the ability to read unfamiliar mathematical material independently and with

understanding as measured by four
instructor-prepared examinations.

Previous studies related to the three hypotheses will be discussed in Chapter II. There are many studies related to the undergraduate preparation of elementary teachers of mathematics, but the predominant theme is one of determining their abilities, positive attitudes toward mathematics, or lack thereof. Very little effort has been channeled in the direction of determining methods of making optimum use of the four to eight hours available in their undergraduate program for mathematics instruction.

The design used in the study is outlined in Chapter III. Basic definitions, instructional methods, and statistical procedures are given. Since analysis of covariance is used, the criteria for the selection of the covariates are discussed. The chapter also includes a description of the population sample, instruments of measurement, and data obtained from the measurements.

In addition to the analysis of the specific questions to be investigated, Chapter IV includes an analysis of a questionnaire dealing with mechanics of the instructional techniques, examinations given during the semester, and amount of time spent in study outside of the classroom.

Conclusions and recommendations stemming from the study are given in Chapter V. The recommendations include suggestions for additional research related to the main problem.

CHAPTER II

LITERATURE RELATING TO THE PROBLEM

The literature as it pertains to the two basic questions and the three secondary questions advanced in Chapter I will be surveyed in this chapter. Since a majority of the studies surveyed dealt with all three of the variables "computational skills," "attitude toward mathematics," and "understanding of the basic concepts of mathematics," no attempt will be made to separate the literature as it relates to these three variables. Studies relating to the ability of students to read mathematical material independently will be considered separately, however. Finally, studies relating to small-group discussion classes in mathematics as they pertain to classes of preservice elementary school teachers will be reviewed.

Since the performance by the small-group discussion section on various criterion measures will be compared with the performance of the lecture-class discussion section, the literature relating to the effect of an undergraduate mathematics course for prospective elementary school teachers, taught by a lecture or lecture-class discussion approach, on the computational skills, attitudes toward mathematics, and understanding of basic mathematical concepts will be surveyed. The investigator is not aware of any studies done with preservice elementary school teachers in which the lecture-small group discussion technique has been employed. In the remainder of the chapter, unless specifically stated to the contrary, the teaching technique used in each study cited was of the lecture or lecture-class discussion type or was not described in the study.

Studies relative to the computational skills, knowledge of basic mathematical concepts underlying arithmetic, and attitudes toward mathematics possessed by preservice elementary teachers before and after completing a mathematics course for elementary teachers are fairly consistent in their conclusions. The majority of the studies indicate that prospective elementary school teachers improve their computational skills, advance their understanding of the basic mathematical concepts underlying arithmetic, and enhance their attitude toward mathematics after taking a mathematics course specifically designed for the elementary school teacher of mathematics.

In a study conducted at Brigham Young University with 186 students enrolled in a required mathematics course for elementary education majors, Gee (1966, p. 6528A) concluded that "There was a significant gain in basic mathematical understanding by prospective elementary teachers while taking this course." Gee used Glennon's "A Test of Basic Mathematical Understanding" to measure the students' initial and final understanding of basic mathematical concepts. To measure the students' attitude toward mathematics, Gee used Dutton's "Arithmetic Attitude Scale." He found that ". . . attitudes of prosepctive elementary school teachers toward mathematics were improved by taking this course."

Additional results cited by Gee relevant to the present study include his conclusion that there is a positive significant relationship between pretest scores on the attitude scale and final grades and between A.C.T. mathematics scores and pretest scores on mathematical understanding.

He concludes that ". . . A.C.T. math score is a good predictor of success in this course as measured by the final grade."

Todd (1966, pp. 198-201) found that a course similar to the CUPM number systems course given to 287 students ". . . produced significant changes in understanding of arithmetic concepts and in attitude toward arithmetic for students who completed the course." Glennon's "A Test of Basic Mathematical Understanding" and Dutton's "Arithmetic Attitude Scale" were used to measure initial and final understanding of mathematics and attitude toward mathematics, respectively.

In a study comparing the performance of inservice teachers on an investigator-prepared test of traditional and modern arithmetic concepts and symbols, Harper (1964, pp. 543-46) found that teachers who had had a mathematics course specifically designed for elementary school teachers scored significantly better than teachers who had had up to six credits of college mathematics, but had not had the mathematics course for elementary teachers.

To determine whether a course in modern mathematics is a factor which influences teacher attitude toward mathematics in general, Rice (1965, p. 1433A) constructed a 45 item Likert-type attitude scale and administered it to 608 inservice elementary teachers. Rice concluded that training in modern mathematical materials appears to foster more favorable attitudes toward mathematics in general.

Foley (1965, p. 4320), in a study investigating the effectiveness of large group instruction with small discussion groups of approximately 20 students, measured the students' initial and final understanding of basic concepts of arithmetic and attitudes toward mathematics. He found that the students improved their understanding of basic mathematical concepts, but that there was no significant gain in attitude toward mathematics. Foley also concluded that there was ". . . no substantial correlation between mathematics competency and attitudes toward mathematics." This result contradicts the previously cited conclusion by Gee. Unfortunately Foley does not indicate what instruments were used to measure either of the two criterion variables.

In an effort to devise an instrument to measure the attitude toward mathematics possessed by prospective elementary school teachers that did not give the immediate appearance of an instrument obviously designed to sample attitudes toward mathematics, Kane (1968, pp. 169-175) prepared a questionnaire in which students were asked to rank their preference for English, science, social studies, and mathematics relative to questions such as "It was most (least) enjoyable to me," "It was the area in which I learned the most (least)," etc. The questionnaire was given to elementary education majors that had completed two courses in mathematics for elementary teachers and a three-credit methods of teaching elementary school mathematics course. Kane concluded (p. 173) that ". . . the attitude of these prospective teachers toward mathematics is relatively high. Mathematics and English (language arts) consistently command more positive attitudes than social studies and science." Kane further concluded (p. 174) that ". . . prospective teachers who have relatively unfavorable attitudes toward mathematics tend to prefer teaching assignments in the primary grades, while those that have the most favorable attitudes toward mathematics tend to prefer assignments in the intermediate grades."

An elaborate study dealing with the effect of various lecture time-treatments and lecture techniques on computational skills, understanding of basic concepts of mathematics, attitude toward mathematics and course achievement was done by Northey (1967). The two variables were "time spent in lecture" and "lecture technique (either inductive or deductive)." The time treatments were 74% to 26%, 50% to 50%, and 26% to 74% for lecture versus class discussion, respectively. Northey found no significant difference between the mean scores on the criterion measures of the various time and lecture treatments, but he did conclude that computational skills attitude toward mathematics, and course achievement were best enhanced by 74% discussion and either inductive or deductive lecture.

Even though all of the previous studies indicate that computational skills and knowledge of basic mathematical concepts of elementary school mathematics teachers are improved upon taking a mathematics course for elementary teachers, Dutton (19[illegible], pp. 223-31) feels that the improvement can be even greater. Dutton used an arithmetic concepts test to diagnose difficulties in arithmetic and students were then given suggestions for correcting them. He concludes (p. 230) that the ". . . elementary teachers in this study made marked progress in the mastery of mathematical concepts when instruction was individualized and adjusted to their needs."

In an earlier study involving 127 preservice elementary school teachers in which he investigated the attitude changes of prospective elementary school teachers, Dutton (1962, pp. 418-24) found that "Attitudes toward arithmetic, once developed, are tenaciously held by prospective elementary school teachers." Dutton goes on to say that ". . . continued study should be made of changing negative attitudes toward arithmetic at the university level and through inservice instruction while doing regular classroom teaching."

Dutton gives some evidence to support a claim that positive attitudes may be positively correlated with achievement, but he has no strong conviction along these lines. This last conjecture by Dutton is supported by a conclusion drawn by Gilbert (1966, p. 981A) in an investigation into the effects of various backgrounds in high school and college mathematics on the understanding of arithmetic concepts possessed by prospective elementary school teachers. Basing her conclusion on data gathered through the administration of standardized arithmetics tests and a questionnaire, Gilbert concluded that "Students indicating a more positive attitude toward arithmetic also seemed to exhibit a fuller understanding." Gilbert recommends that the two major objectives of content courses for prospective elementary school teachers of mathematics should be the development of a fuller understanding of arithmetic as well as an improvement in attitude toward mathematics.

Two studies, one by Bassham, et. al. (1964, pp. 67-72) and the other by Lerch (1961, pp. 117-19), when considered together stress the importance of striving to improve the attitudes toward mathematics of preservice elementary school teachers of mathematics. Bassham, et. al. studied the attitude and achievement of a group of 159 fifth and sixth grade students. Dutton's "Scale for Measuring Attitudes Toward Arithmetic," and the "Iowa Tests of Basic Skills (Arithmetic Concepts)" were used to measure attitude and achievement, respectively. The authors concluded (p. 71) that "After weighting for individual differences in intelligence and reading comprehension, an important difference in mean scores of mastery in fundamental concepts of arithmetic was found to exist between those pupils classified as in the upper two-fifths and those classified as in the lower two-fifths of a distribution of attitude scale scores."

In fact, the authors found nearly three times as many high-attitude pupils over-achieved .65 grade as under-achieved by .65 grade.

The study by Lerch involved two fourth grade classes; one class ability grouped and re-grouped when new topics were discussed while the other was a non-grouped class. Lerch, upon investigating the attitudes of the two groups toward arithmetic, was able to conclude (p. 119) that "The child's attitudes toward arithmetic are more basically dependent upon his teachers' attitudes and the methods they employ than they are upon classroom organization."

Another study done at the fourth grade level that cites an association between attitude toward arithmetic and achievement is reported by Lyda and Morse (1963, pp. 136-138). The students were given Dutton's "Arithmetic Attitude Scale" and the "Stanford Arithmetic Achievement Test" before and after instruction. The authors strove to teach "to reveal concepts and rationale of a process and the relationship of processes to each other." The authors called this "meaningful teaching." They were able to conclude (p. 138), as a result of their investigation, that "Associated with meaningful methods of teaching arithmetic and changes in attitude are significant gains in arithmetic achievement, that is, in arithmetical computation and reasoning."

The literature relating to the last of the secondary questions posed in Chapter one, that is, the question dealing with the ability of prospective elementary teachers to read, independently and with understanding, unfamiliar mathematical material is virtually non-existent. However, in terms of long range benefit to the student, it seems apparent that this question is of the utmost importance. The report of the Cambridge Conference (1967, p. 39) holds that ". . . it is unlikely that there would again be such a freezing of the mathematics curriculum as took place in the last century and a half. It therefore appears quite hopeless and inappropriate to expect that preservice training could be extensive enough to equip a teacher for a lifetime." Teachers possessing the ability to read and understand mathematical material could derive great benefit from well-written teacher commentaries and short monographs aimed specifically at this audience. Moody (1966, pp. 30-31) writes "It would be anticipated that a student who completed a course of study in mathematics, especially designed for elementary teachers, would have sufficient background to be able to pick up a good teachers' commentary to a 'new' textbook series in arithmetic, and with its help design a lesson in arithmetic which would present the material in a meaningful fashion." There does not seem to be any reason to believe that successful completion of a mathematics course for preservice elementary teachers taught by a lecture or lecture-class discussion approach will enhance the students' ability to do independent work.

In a study on the ability of college freshmen to read a chapter on the hyperbola independently, Filano (1957, pp. 16-18) concludes ". . . practical equivalence, in terms of student achievement of the two methods (i.e., independent reading versus lecture-class discussion)." However, Filano indicates that the results could be misleading since the unit on the hyperbola was covered immediately after a discussion of the ellipse and guidance was given in the form of specific problem assignments.

There is also one other consideration in the applicability of Filano's study to preservice elementary school teachers. The study does not state so specifically, but one may assume that the subjects had similar backgrounds in terms of high school units in mathematics. This has not been the case in classes of prospective elementary school teachers over the past eight years at Wisconsin State University - Stevens Point. Furthermore in each of the studies previously cited involving preservice elementary school teachers where the authors gave a description of the population sample, homogeneity of high school background did not exist. It is not unusual for a class of prospective teachers to have a background range of from one to four years of high school mathematics and up to four or more college credits in mathematics.

Turner, et. al. (1966, pp. 768-70) conducted a study using all students in their Fundamentals of Mathematics and College Algebra classes at Mankato State College. The experimental groups were taught by a lecture-discussion method two or three days per week with the remaining days spent in smaller discussion groups. The discussion groups were handled in one of three ways: (a) students worked together in groups of three students with one of the students acting as leader (the instructor in charge was present but did very little talking to the groups); (b) students worked together in groups of 5 or 6 with a mathematics major leading the discussion; and (c) a graduate assistant was in charge and used a variety of methods of instruction. Unfortunately, these methods were not reported. The control groups consisted of about 50 students per class and were taught by a lecture-discussion method each day. Turner (p. 770) concludes that "The results of using the various treatments indicated no significant differences in achievement [as measured by a common instructor-prepared examination]." No results were reported or conjectures made on the effect of the three-student work group treatment on the ability of these students to work independently.

As with Filano's study, the applicability of this study to a class of preservice elementary school teachers can be questioned because of the probable uniformity of high school preparation of the students in Turner's study.

There are a number of studies tangentially related to the acquisition of the ability to read mathematical material independently and with understanding. McKeachie (1963, p. 1132) says of the student-centered group discussion instructional technique ". . . if students are to achieve application, critical thinking, or some other higher cognitive objective, it seems reasonable to assume that they should have an opportunity to practice application and critical thinking." McKeachie (p. 1140) goes on to cite 11 studies that show ". . . significant differences in ability to apply concepts, in attitudes, in motivation, or in group membership skills . . ." where the discussion techniques favored greater student participation. Finally, McKeachie claims (p. 1140) "The more highly one values outcomes going beyond acquisition of knowledge, the more likely that student-centered methods will be preferred."

Studies dealing with the most effective small-group size are inconclusive. For example, Hare (1952, pp. 261-67) states that if the group task is a technical one, a larger group may have a higher probability of solving the problem in a shorter time and South (1927, pp. 348-368) states that on abstract tasks, groups of three took longer to solve the problem than groups of six. On the other hand, on a "Twenty Questions" type concept formation problems, Taylor and Faust (1952, pp. 360-368) report that groups of two persons obtained the answer in shorter time but failed more often than did groups of four.

Schellenberg (1959, pp. 73-79) found that in 32 social science classes in which the discussion groups were of size 4, 6, 8, and 10, a higher degree of satisfaction among the students and higher instructor grading existed in the smaller groups.

Hare (1962, p. 224), in discussing discussion-group size, states that when the time allowed for discussion is limited ". . . the average member has fewer chances to speak and intermember communication becomes difficult. Morale declines, since the former intimate contact between members is no longer possible." Hare (p. 225) argues again in favor of a smaller group size when he states "Although the larger group size has in its membership a greater variety of resources for problem-solving, the average contribution of each member diminishes and it becomes more difficult to reach consensus on a group solution." The conjecture that a group with an odd number of members may be desirable is advanced by Hare (p. 24); the odd number of members making a deadlock on a decision unlikely.

Bales and Borgatta (1955, pp. 396-413) reported a decrease in the group exploration of different points of view and a more direct attempt to reach a solution to a group problem regardless of disagreement when group sizes were increased from two through seven. These group actions were associated with the increased restriction of time available to the participants.

Guidelines for establishing group composition are not universal. Cohen (1957, pp. 135-144) claims that groups which are highly cohesive tend to work harder regardless of outside supervision. However, McKeachie (1963, pp. 1133-48) reports studies that suggest a cohesive group, although effective in maintaining group standards, may accept either low or high standards of productivity. McKeachie (p. 1135) states ". . . in creating 'groupy' classes an instructor may sometimes help his students develop strength to set low standards of achievement and maintain them against instructor pressures . . .". Grouping according to personality was determined by Hoffman (1959, pp. 27-32) to be unsuccessful in an experiment in group problem solving. In a study using psychology classes, Longstaff (1932, pp. 131-166) found no significant differences between classes homogeneously grouped according to intelligence quotient and heterogeneous classes. Basing his decision on four studies in which the special abilities of homogeneous groups were exploited, McKeachie (p. 1143) writes ". . . it seems safe to conclude that homogeneous grouping by ability is profitable, if teaching makes use of the known characteristics of the groups." This conclusion, however, is not applicable to the present study since the achievement of two entire classes will be compared and the success of the lecture-small group discussion technique will rest on the ability of all students to profit by small-group discussion of the same body of mathematical content.

In summary, the survey of the literature related to this study indicates that mathematics courses for preservice elementary school teachers taught by a lecture or lecture-class discussion technique improve the students' computational ability, contribute to their understanding of the fundamental concepts of arithmetic, and generally enhance their attitude toward mathematics. The evidence supporting a conjecture that preservice elementary school teachers possess or develop as a result of a preservice mathematics course the ability to read mathematical material independently is not known. The studies along this line have involved groups of students with similar backgrounds in high school mathematics. No studies relating to the use of a small-group discussion instructional technique with classes of preservice elementary school teachers were found.

Studies dealing with small group study in general are not in agreement as to optimum group size. Thelen's (1949, pp. 139-148) "principle of the least group size"--the group should be just large enough to include individuals with all the relevant skills for problem solution--seems to be most appropriate. Homogeneous grouping by ability appears to be favored; however, in the present study this was not feasible.

CHAPTER III

DESIGN OF THE STUDY

The purpose of this chapter is to provide a detailed outline of the design of the study, give a description of the actual classes involved in the study, and to summarize data relating to the mechanics of the two teaching procedures collected during the semester.

Section one discusses the sample and general procedures and the next two sections describe the two teaching techniques employed. The fourth section contains a description of the instruments of measurement, while section five is devoted to the statistical procedures.

The last three sections provide a description of the two classes involved in the study, data on the lecture-class discussion teaching technique and data on the lecture-small group discussion teaching method.

The Sample and General Procedures

The study involves two sections of Mathematics 115, "Concepts of Modern Elementary Mathematics," at Wisconsin State University-Stevens Point during the second semester of the 1968-1969 academic year. Section one met at 10:45 A.M. and section two at 11:45 A.M. each day of the week, except Tuesday. The class periods were 50 minutes in length.

Mathematics 115, a four-credit course, is the only mathematics course required of prospective elementary teachers at this university. There is no prerequisite mathematics course and since it also satisfies the university requirement in mathematics, it is usually the only college-level mathematics course taken by elementary education majors at Wisconsin State University-Stevens Point.

The same instructor taught both sections of the course. Since the notes from which the students studied had to be supplemented by lecture in several areas and because overall course achievement was based entirely on the students' success on examinations (prepared by the investigator) covering material discussed in class, it was felt that the achievement would be most accurately measured if the same instructor taught both classes.

Because of conflicts with other required courses in the elementary teacher curriculum and because the teacher variable was controlled, making two sections at the same hour impossible, random selection of students for the two sections was not possible. However, no student knew prior to the first day of class that he would be involved in a research study. Furthermore, the type of teaching technique used in either section was determined by a flip of a coin one day before classes convened. No student transfers from one section to the other section involved in the study were to be allowed except in cases of extreme hardship. There was one other section of the course taught by a different instructor in the afternoon and any students having to transfer would be requested to enroll in that section. There did not appear to be any factor other than class scheduling problems that influenced a choice of one section over another.

During the first class meeting the teaching technique to be used in each group, pretests required, posttest required, and the purpose of the study was explained to both groups. The students learned that the same examinations would be given to both sections and that the two sections would be graded together. Since the results of the pretests were used in the statistical analysis of overall course achievement, it was important that the students gave their best possible effort on each pretest. Thus, the classes were told that their letter grade for the course would be based solely on the number of points accumulated from three one-hour examinations and a two-hour final examination and that their pretest and posttest scores would in no way influence their letter grade. They were urged, however, in the interest of the experiment, to do their best on each pretest and each posttest.

Only those students possessing A.C.T. scores were used in the statistical analysis since this variable was used as a covariate in the analysis of covariance. This information was not disclosed and data was kept for all students.

The bulletin Using A.C.T. on Your Campus (American College Testing Program, Inc., 1964-65, pp. 6-7) describes the nature and purpose of the mathematics section as follows:

> "This test samples the student's ability to understand and use the principles and techniques of mathematics. In this sense, it is a test of the student's ability to reason mathematically. Test items involve two kinds of problems: (a) quantitative problems based upon practical situations, and (b) problems presented in formal exercises in algebra, geometry, and advanced arithmetic."

A mathematics major who was able to be present at every class meeting was employed as a student assistant. (His role within the classroom will be explained in the next two sections.) Outside of the classroom, he assisted in the scoring of objective examinations, in the compilation of data, and in the key punching of computer cards.

Course Content

The mathematical content of the course was a proper subset of the CUPM recommendations for level I preparation. The students were issued Wren's Basic Mathematical Concepts as the text for the course (Wisconsin State University-Stevens Point is on a textbook rental system); however, with the exception of a short unit on geometry, it was used only as a reference text by the student. The material covered in the course was prepared by the instructor and was dittoed for distribution to the students prior to class lecture or discussion. The same notes were used by both sections involved in the study.

The topics considered included a brief study of the logic of propositions, elementary properties of sets, an intuitive development of the number systems through the real numbers, numeration systems, elementary number theory, relations and functions, and an introduction to basic geometric concepts. Table 1 lists chapter titles and indications of chapter content.

Lecture-Class Discussion Instructional Technique

Of the possible 60 class periods of the semester, ten periods were used for pretests, posttests, and unit examinations. Each 50 minute class period was split into four major categories; namely, review of the previous day's work, student questions on the previous day's work and homework assignments, lecture on new material, and class discussion of instructor-posed questions. The segments were not

TABLE 1

OUTLINE OF COURSE CONTENT

Chapter	Content
I	Propositions; connectives, truth tables; tautologies; logical equivalence; propositional forms; counterexamples.
II	Terminology of sets; operations; Venn Diagrams.
III	Whole numbers; addition, multiplication, and subtraction defined; properties of operations; algorithms, numeration systems.
IV	Prime and composite whole numbers; Fundamental Theorem of Arithmetic; divisibility theorems.
V	The integers; operations defined on the integers; properties of the operations.
VI	The rational numbers, operations defined on the rationals.
VII	The real numbers; decimals; characterization of rationals as repeating decimals; denseness of rationals in reals.
VIII	Relations and functions; graphing relations and functions.
Geometry	Basic terminology, including point, line, plane, ray, half-line, half-plane, polygons, congruence of triangles, similarity of triangles.

necessarily disjoint in the sense that the lecture couldn't be interrupted by a short question from a student, for example; however, the sequencing of activities normally followed this pattern. The student assistant recorded the amount of time spent on each activity. During the first three weeks of the semester, the assistant was to keep detailed notes of the class activities and after class the instructor and assistant were to compare the notes with the time sheet to verify that the assistant had correctly distinguished between lecture activities and class discussion activities. When the instructor was satisfied that the distinctions were being made correctly, detailed notes were no longer kept. Table 2 lists the number of minutes that were allotted to each class activity.

TABLE 2

TIME EXPENDITURE IN LECTURE-CLASS DISCUSSION GROUP

Activity	Type of Activity	Minutes Allotted
Review of Previous Work	Lecture	5
Student Questions	Class Discussion	10
New Material Introduced	Lecture	10
New Material Discussed	Class Discussion	25

The review of the previous day's material was done entirely by the instructor. The key theorems and generalizations were reviewed and new examples given. At the beginning of a new chapter, an overview of the chapter was given in place of the review of previously covered material. Five minutes were to be allotted to this review or overview.

Ten minutes of each period was allotted to student questions on previous work and on homework problems. Student questions were redirected to another student or were solved through instructor-led class discussion. Any questions remaining at or near the end of the allotted time were reassigned with hints toward a solution. No homework problem was ever solved entirely by the instructor. An effort was made to solicit responses from all students. During ten class periods randomly selected from the last 14 weeks of the course, the student assistant was to record the names of students responding to questions and indicate whether the response was voluntary or was requested by the instructor.

The new material for the day was introduced by the instructor. The students were not encouraged to participate actively, although brief questions were usually answered immediately. The students were told that they would have ample opportunity to ask questions during the next time segment. The major portion of the time was spent explaining and illustrating definitions and in stating, proving, and illustrating certain theorems. The deductive nature of mathematics was stressed almost exclusively; however, on occasions the material was developed via an inductive approach.

The activities in the last time segment varied according to the material; however, in all cases the students were actively involved.

Student questions about the new material was discussed first. If the question was clear to most students, a volunteer was asked to respond. If there were no volunteers, the instructor would rephrase the question and, if necessary, give a hint toward an answer.

After all student questions about the new material had been answered, the remaining time was spent solving problems related to the lecture, giving reasons for steps in the proofs of additional theorems, or applying the theorems to specific problems.

Lecture-Small Group Discussion Teaching Technique

This class met in an economics statistics laboratory equipped with long rectangular tables rather than conventional seating. This arrangement made small group study very convenient.

The class periods were split into three time segments. The first ten minutes was an instructor-dominated lecture period, the next 30 minutes was devoted to work by students in small discussion groups, and the last ten minutes was an instructor-led class discussion period. The time schedule was to be closely followed, with only a two-minute deviation allowed in either direction.

The activities of the initial time segment included a brief review of the previous day's work and an introduction to the topic for the day. Emphasis was placed solely on tying together the ideas covered in the previous day's work rather than on explanations of specific concepts or problems related to the concepts.

The introduction to the day's material included an explanation of new symbolism, proofs of theorems not in the notes, and if time remained, an explanation of the more difficult definitions.

The students were randomly assigned to groups of three or four students each week. On the first day of the semester each student was assigned a positive integer and the discussion groups were determined by randomly selecting numbered uniform discs.

The group assignments were made on a weekly basis, alternating between groups of three and groups of four from week to week. Since the number of students was not a multiple of three and four, one group worked with one or two fewer students than all other groups. The weekly group assignments were made to minimize the effect of a particularly dominant student on two or three other students of the class. It can be argued that changing group composition so frequently tends to decrease group productivity, but since the development of individual skills was of prime consideration here, no attempt was made to seek the best possible group structure in terms of group productivity.

A group leader was determined by alphabetical and reverse alphabetical order for each class period of the week. The responsibilities of the group leader as outlined by the instructor included a charge to (1) keep the discussion moving, that is, if a problem was too difficult, move to the next, (2) be sure that all group members participated in the discussion, and (3) be relatively certain that all members understood a concept before moving on to the next.

On alternating weeks with group size four, each day the individual groups were randomly paired within the group. The individual pairs of students worked on the exercises but were encouraged to communicate frequently with the other two students of their group. The group leader was instructed to see that both pairs were progressing at similar rates. At the end of the ninth week the students were asked their preference of group size and composition and the results of the poll dictated which type of group composition was to be used for the remainder of the semester.

At the beginning of the second time segment the students were told about how much material they ought to complete and the group leader was reminded of his responsibility to see that the work progressed toward that goal. If a group completed the desired work before the end of the time period, they were encouraged to review topics from previous material rather than proceed beyond the given assignment.

While the groups were working on the assigned material, the instructor and his assistant moved from group to group observing the progress, listening to the discussion, and providing assistance when requested. Prior to each class meeting the instructor and his assistant reviewed the topic for the day and agreed on the type of assistance allowed on each problem or question. The assistance given adhered closely to the following format. The initial request for assistance was answered by a request to have the students state exactly, and in their own words, what the problem was asking. If they were unable to do this, the problem was explained to them and the group was left to try to work through their difficulty. If the group still could not reach agreement on a solution, a second request for assistance could be made. The instructor or assistant listened to any arguments proposed and if a correct solution or solutions had been given, he would state which one(s) they were and then left the group. If no correct solution had been presented, the group was urged to go onto another part of the assignment. At no time was the instructor or his assistant permitted to work on or to solve a problem for a group.

The final time segment of ten minutes was spent in discussion of the solved exercises and unsolved problems or the students were asked to submit one written solution per group to one or two exercises of the day. The latter was done about once every two weeks. In either case the students remained seated with their respective groups.

If exercises were to be discussed, each group was asked to indicate which problem, if any, presented the greatest difficulty. The instructor then selected those problems to be discussed as time permitted. The problems were resolved by (1) requesting an individual to give a solution verbally or at the board, (2) requesting a specific group to give their collective solution, or (3) the entire class, led by the instructor would discuss a solution. If no group or individual was able to solve the problem it was carried over to the next period along with broad hints for solution. If the problem remained unsolved after the second day, the instructor outlined a solution.

The problems submitted in writing during the last ten minute period were read by the instructor, marked satisfactory or incorrect, and returned at the beginning of the next class period. During the last ten minutes of that period, the instructor presented the best written solution along with comments explaining why he felt the solution was a particularly good one. No records were kept of the students involved in the written solution, however, the students were not aware of this.

Data and Measuring Devices

During the first week of classes, the California Mathematics Test (Form W), 1957 edition, devised by Ernest W. Tiegs and Willis W.

Clark, the Cooperative Mathematics Test (Algebra II, Form A), 1963 edition published by Educational Testing Service, and an attitude toward mathematics scale written by Aiken and Dreger (1961, pp. 19-24) was administered. During the last week of classes Form X of the California Mathematics Test and the same attitude test was given. Also during the last week an instructor-prepared questionnaire designed to solicit student opinions on the mechanics of the course was given to the students. In addition, three unit-examinations and a final examination covering the entire semester's work were used to measure the students' overall course achievement.

The California Mathematics Test consists of two main parts, namely, the "Mathematics Reasoning Test" and the "Mathematics Fundamentals Test." Sections on meanings; symbols, rules and equations; and word problems constitute the reasoning test and the fundamentals test is separated into one section each on the four arithmetic operations. Each subsection of the examination is timed with a total of 31 minutes given to the "Mathematics Reasoning Test" and 41 minutes allowed for the "Mathematics Fundamentals Test." A maximum score of 60 can be achieved on the reasoning tests and a score of 80 is possible on the fundamentals portion.

The Manual of the California Mathematics Test (1961, p. 6) states that the twenty questions in the "meanings" section of the reasoning portion of the test ". . . are designed to measure the extent to which the student understands the meanings of numbers, money, percent, etc., and whether he has adequate concepts of fractions, decimals, exponents, roots, and abstract numbers." The "symbols, rules, and equations" section is designed to ". . . reveal the extent of the student's comprehension of symbols, rules, and formulas, as well as general mathematical terminology." The "problems" section includes word problems dealing with square and cubic measure, ratio and percentage, and budgeting.

Regarding the "Fundamentals Test," the Manual of the California Mathematics Test (p. 6) claims that "The test items . . . reveal whether or not the student has sufficient mastery of the fundamental processes to use them with proficiency." The "Fundamentals Test" was used to measure the computational skills of the students before and after taking the course.

The maximum score on the reasoning portion of the California Mathematics Test is 60 and on the fundamentals portion the highest possi score is 80. The Manual (p. 8) lists the reliability coefficient of the "Mathematics Reasoning Test" as .89 and of the "Mathematics Fundamentals Test" as .94, where both were computed using Kuder-Richardson formula 21.

The Cooperative Mathematics Test on algebra consists of 40 multiple choice questions covering topics such as radicals, exponents, linear equations, systems of linear equations, logarithms, complex numbers, and linear inequalities. This test was included because Northey (1967) suggests that overall course achievement in a mathematics course for preservice elementary teachers may be directly related to the students' knowledge of algebraic concepts.

The mathematics attitude scale consists of twenty statements, each describing an attitude toward mathematics. The students are asked to indicate whether they strongly disagree, disagree, are undecided, agree, or strongly agree with each statement. The responses are weighted either one to five or five to one, depending on the statement, and so the lowest possible score is 20 and the highest possible score is 100. A low score indicates a negative attitude toward mathematics and a high score indicates a positive attitude toward mathematics. A copy of the Aiken and Dreger attitude scale is given in Appendix E.

Each of the three instructor-prepared tests was composed of two parts. Part I tested the students' understanding of concepts discussed in class and part II attempted to measure their ability to read and understand the definition of an unfamiliar mathematical concept, along with several examples, and then to work several exercises directly related to this definition. If an exercise in part II of an examination contained a symbol or term from previously discussed material that could have been forgotten by the student, the symbol or term was explained or defined within the exercise. Part II of unit-test one considered a definition of subtraction of whole numbers in terms of set-difference (the concept was also discussed in class but about two weeks after the examination), while part II of test two treated the concept of an equivalence relation, and part II of test three contained the definition of two binary operations on the cross-product of two sets.

Part I of unit test one covered Chapters I and II, and the first one-third of Chapter III. The first part of test two contained topics from the remainder of Chapter III and all of Chapter IV. Topics from Chapter V and VI were examined in the last unit test. Each unit test counted 100 points, with 86, 88, and 86 points for the three course content parts of the three examinations. Appendix G contains copies of the three unit-tests and the final examination.

Like the three unit-tests, the final examination also contained two separate parts. The first part consisted of twenty-five multiple choice questions covering topics from the entire course. The second part of the final examination contained three exercises, each one similar in format to the independent reading parts of the three unit-tests. The reliability of the final examination was computed using the Kuder-Richardson formula 20 and was found to be .86.

On the last regular class day of the semester, a questionnaire was given relating to the mechanics of the course (length of examinations, homework assignments, relationship of homework to examinations, etc.), amount of time spent in study outside of the class period, and a conjecture by each student about his change in attitude towards mathematics since the beginning of the course. Before the questionnaires were distributed, the students were urged to consider each question carefully and to answer each question as accurately and honestly as possible. They were told that anonymity would be obtained by having them place their completed questionnaires in a large envelope to be passed about the room. Appendix F contains the questionnaire and the percentage of students from each group responding to each alternative within a given question.

Statistical Procedures

To compare pre-test and post-test performance within groups on the California Mathematics Test and the attitude scale, the difference between test scores was computed for each individual and a *t*-test applied. There was no reason to believe that the population sampled was normal nor that the population variances were equal. However, regarding the first assumption, Hays (1963, p. 322) states that ". . . this assumption (normality) may be violated with impunity provided that sample size is not extremely small." Regarding the homogeniety of variance, Hays goes on to say ". . . for samples of equal size relatively big differences in the population variances seem to have relatively small consequence for the conclusions derived from a t-test."

The t value obtained will be computed by the usual formula,

$$t = \frac{\bar{d}\sqrt{n}}{s_d} \text{, where}$$

$$\bar{d} = \sum_{i=1}^{n} d_i \; n \text{ and } s_d^2 = (n \sum_{i=1}^{n} d_i^2 - (\sum_{i=1}^{n} d_i)^2) / n(n-1),$$

with d_i = posttest score minus pretest score for individual i.

It was clear from the data gathered that individual differences on the initial measures existed thus to control these initial differences an analysis-of-covariance technique was used to test for a difference of means of the various criterion measures.

The analysis-of-covariance technique used compares the means of two samples using one or two associated independent variables (covariates). alysis of covariance uses linear regression to predict criterion means based on the initial measures selected as covariates.

Letting Y represent the criterion measure and X the covariate, the analysis is of the total sum of squares of the residuals, $Y - \bar{Y}_X$, where $\bar{Y}_X = \bar{Y} + b_T(X - \bar{X})$, with $\bar{Y}$ and $\bar{X}$ representing the sample means and b_T the slope of the regression line of the total sample. In the case of two covariates, say X an , the residuals become $Y' = Y - \bar{Y}_{XZ}$, where $\overline{Y}_{xz} = \overline{Y} + b_{xT}(X - \overline{X}) + b_{zT}(Z - \overline{Z})$, with b_{xT} and b_{zT} representing the regression coefficients $b_{yx \cdot z}$ and $b_{yz \cdot x}$ for the entire group.

Whenever the rejection region is "two-tailed," the F-ratio for the lower critical value will be found by the relationship

$$F_{(n,m)} = \frac{1}{F_{(m,n)}}$$

where m and n represent the degrees of freedom (Hays, 1963, p. 350). The reader will recall that for a two-tailed test and a fixed significance level, α, the tables for F must be entered at $\alpha/2$.

The computational formulas and tests of hypothesis used are those of Walker and Lev (1953, pp. 387-422).

To insure that the covariates chosen were those that correlated highly with the criterion measure and also exhibited reasonable differences, correlation coefficients between initial measures and criterion measures were computed. The correlation between initial measure and criterion measure had to exceed 0.3 before consideration of the initial measure as a covariate in the analysis of the means of the different criterion measures.

In addition to a comparison of the two groups in toto on the various criterion measures, the upper one-half ability levels and lower one-half ability levels of each group as determined by the individual A.C.T. mathematics percentile scores were compared.

Description of the Population Sample

There were 27 students enrolled in the 10:45 A.M. section and 23 students in the 11:45 A.M. class. The earlier section was taught by the lecture-class discussion technique and the lecture-small group discussion instructional technique was used in the 11:45 A.M. section. There were no student withdrawals nor transfers in or out of either section after the beginning of the semester.

Twenty students in each group had available A.C.T. scores and were the only ones involved in the statistical analysis. Appendix contains data gathered on the students not involved in the study.

Table 3 provides a summary of the class composition. Since most elementary education majors spend one full semester off-campus while student teaching, Mathematics 115 is usually taken during their junior year. This is evidenced by the ratio of about three to one in favor of juniors over seniors in the total sample involved in the statistical analysis.

TABLE 3

CLASS COMPOSITION BY YEAR IN SCHOOL

Variable	LCD	LSGD	Total
Number in Class	27	23	50
Number of Juniors	16	19	35
Number of Seniors	11	4	15
Number of Juniors in Statistical Analysis	14	16	30
Number of Seniors in Statistical Analysis	6	4	10
Total Number in Statistical Analysis	20	20	40

The mathematical preparation of the subjects ranged from one year of high school mathematics to four years of high school mathematics in both sections. One student in the LCD group had had two semesters of college mathematics in addition to four years of secondary school mathematics. Two other students in the LCD group had had one semester of college mathematics in addition to three years of high school mathematics. Three students in the LSGD group had taken a one-semester college mathematics course plus three years of secondary school mathematics. The mean number of years of secondary school mathematics was 2.5 for both groups. Inasmuch as secondary school mathematics courses vary rather widely in scope, depth, and general quality, no attempt was made to further categorize the high school units in mathematics.

The A.C.T percentile scores in mathematics ranged from 1 to 99 in the LCD group and from 14 to 81 in the LSGD section, with means of 50.95 and 51.95, respectively. The sum of the five A.C.T. percentile scores obtained--English, mathematics, science, natural science, and general--ranged from 40 to 466 in the LCD section and from 36 to 419 in the LSGD class. Mean totals in the LCD and LSGD groups were 257.10 and 277.90, with standard deviations of 116.43 and 102.24, respectively. Group data is summarized in Table 4 and individual student scores are given in Table 5. Appendix B contains individual student percentile on the English, science, natural science, and general portions of the A.C.T. examinations.

Students numbered 1, 2, 5, 7, 9, 10, 12, 14, 15, and 19 comprise the upper one-half ability level of the LCD section and students numbered 1, 4, 6, 8, 12, 14, 15, 16, 18, and 19 make up the upper one-half ability level of the LSGD group. The ranges of the upper and lower one-half A.C.T. mathematics percentile of the LCD section are 56 to 99 and 1 to 47, respectively. The corresponding ranges in the LSGD group are 61 to 81 and 14 to 46. Mean percentiles for the two halves are 73.1 and 28.8 in the LCD group and 72.5 and 31.0 in the LSGD class.

TABLE 4

YEARS HIGH SCHOOL MATHEMATICS, A.C.T. MATHEMATICS PERCENTILES, AND A.C.T. TOTAL SCORES BY CLASSES

Variable	LCD Group	LSGD Group
Years High School Mathematics	Mean = 2.50 *S.D.= .827	Mean = 2.50 S.D. = .827
Range of Years High School Mathematics	1 to 4	1 to 4
A.C.T. Mathematics Percentile	Mean = 50.95 S.D. = 27.81	Mean = 51.75 S.D. = 23.37
A.C.T. Total	Mean = 257.10 S.D. = 116.43	Mean = 277.90 S.D. = 102.24

*S.D. represents standard deviation.

Data on the Lecture-Class Discussion Instructional Technique

It will be recalled that the first five minutes of each non-testing class period was allotted to review of previous work or an over-view of new material. The average number of minutes actually spent in this activity was 4.2 minutes per period.

Student questions on previous work or on homework problems were considered in the next ten minute period. The absolute difference between time allotted and actual time spent exceeded two minutes on eigh occasions and exceeded five minutes twice during the semester. An avera of 11.1 minutes per period was actually spent on student questions.

An average of 9.5 minutes per period was actually spent on the instructor-dominated lecture. Ten minutes had been allotted to this

TABLE 5

YEARS HIGH SCHOOL MATHEMATICS, A.C.T. MATHEMATICS PERCENTILES, AND A.C.T. TOTAL SCORES BY STUDENT

LCD GROUP				LSGD GROUP			
Student Number	Yrs. H.S. Math	A.C.T. Math Percentile	A.C.T. Total	Student Number	Yrs. H.S. Math	A.C.T. Math Percentile	A.C.T. Total
1	3	56	331	1	3	77	396
2	3	56	223	2	2	40	253
3	1	24	192	3	2	46	267
4	2	47	271	4	2	77	323
5	4	85	229	5	3	46	281
6	2	34	220	6	3	61	364
7	4	99	465	7	3	28	190
8	2	14	206	8	3	61	305
9	2	56	201	9	1	22	177
10	2	56	145	10	2	40	382
11	2	46	275	11	2	22	202
12	3	88	466	12	3	81	419
13	2	40	297	13	1	18	88
14	2	72	359	14	3	66	358
15	4	91	423	15	3	72	286
16	2	46	205	16	3	77	337
17	2	22	40	17	3	34	223
18	2	1	67	18	4	72	285
19	3	72	345	19	3	81	386
20	3	14	182	20	1	14	36

activity. Twice during the semester the lecture consumed more than 12 minutes and on three occasions less than eight minutes were used.

The last time segment, scheduled for 25 minutes, devoted to class discussion of the new materials took an average of 25.2 minutes per period. The actual time used fell within two minutes of the allotted time in all but eight class periods. Table 6 summarizes the time allotments and expenditures.

TABLE 6

TIME EXPENDITURE OF CLASS PERIODS IN LECTURE-CLASS DISCUSSION SECTION

Activity	Minutes Allotted	Mean Spent	Frequency of *d > 2	Frequency of *d > 5
Review of Previous Work	5	4.2	3	0
Student Questions	10	11.1	8	2
New Material Introduced	10	9.5	5	0
New Material Discussed	25	25.2	6	2

*d = Absolute difference between time allotted and time spent per period.

Student responses to questions by other students or the instructor were answered by volunteers 86.6% of the time. An effort was made to solicit responses from all students, however the attempt was generally not successful. The ten students responding most frequently, responded to approximately three-fifths of all questions asked, while the next ten most frequent responders, responded 31.1% of the time.

Data on the Lecture-Small Group Discussion Teaching Technique

The time allotments for the three distinct activity segments--a ten minute instructor-dominated review of previous work and introduction of new material session, a thirty minute small-group discussion period, and a ten minute instructor-led class discussion segment--were kept within the allowed two-minute deviation on all but six occasions. In each of these six exceptions, the last ten-minute period was deleted and given to additional small-group discussion.

The review of previous material took an average of 2.6 minutes, with a range of from one to five minutes. The introduction of new material consumed an average of 6.3 minutes, with five minutes the least amount of time given and eight minutes the greatest amount of time devoted to this activity. The ten minute time allotment for the initial time segment was exceeded only twice during the semester, however, the deviation was within the two minute time limit both times.

Data was kept on the number of times assistance was requested of the instructor or his assistant during each class period. Table 7 lists the average frequency of requests for assistance per session by all groups. The figures include first and second requests by the same group. It will be noted that since there were either six or eight groups per day, the average number of requests per group per day was about one. The number of requests for assistance was much lower than anticipated, in fact, with few exceptions one person could have handled all requests without causing a great deal of delay to any one group. An accurate count of the number of first requests and the number of second requests was not kept. However, both the instructor and his assistant feel that the ratio was near two first requests to each second request.

TABLE 7

MEAN FREQUENCY OF REQUESTS FOR ASSISTANCE PER CLASS PERIOD FOR THE LSGD SECTION

Chapter	Mean	Minimum	Maximum
I	6.1	1	9
II	5.3	1	7
III	6.7	0	12
IV	3.2	0	5
V	4.0	0	7
VI	4.3	0	8
VII	4.6	1	7
VIII	6.1	1	10
Geometry	5.0	2	8

The poll taken at the end of the ninth week regarding preferred group size, revealed that ten students favored a group size of three, one student favored a group size of four without pairing within the group, and twelve indicated a preference for a group size of four with pairing within the group. Therefore, for the remainder of the semester, all groups of size four were paired within the group.

CHAPTER IV

ANALYSIS OF DATA

An analysis of the data collected within groups, between groups, within ability levels, and between ability levels of the two groups is given in this chapter. A t-test is used to analyze pretest and posttest scores on the reasoning test, fundamentals test, and the attitude scale within groups and the results are reported in section one of this chapter. The second section deals with the selection of the covariates and the third section of the chapter contains an analysis between the two groups on the final examination, independent reading examinations, and overall course achievement using an analysis of covariance technique. The chapter concludes with an analysis of the questionnaire on the mechanics and general opinions about the course which was given on the last regular class day of the semester.

Within Groups Comparison of California Reasoning, California Fundamentals, and Attitude Scale Pretest and Posttest Scores

Form W of the *California Mathematics Test* was given during the first week of classes and Form X was given during the last week of classes. Mean pretest scores for the LCD group were 44.70 and 67.15 and mean posttest scores were 48.55 and 70.20 on the reasoning and fundamentals sections, respectively. The high and low scores differed by 22 points on the reasoning pretest and by 13 points on the posttest. A smaller decrease in range for the LCD group appeared on the fundamentals portion of the examination. Scores on the pretest spanned 32 points and spanned 28 points on the posttest.

The results were quite similar in the LSGD class. Mean test scores on the reasoning section increased from 45.50 on the pretest to 49.20 on the posttest. The range of scores spanned 32 points on the pretest and 28 on the posttest. The fundamentals pretest average score was 65.45, while the posttest mean was 70.85. The range of scores dropped from a difference of 32 points on the pretest to a difference of 22 points on the posttest.

As shown in Table 8, the standard deviation decreased from pretest to posttest on each test in both classes. The largest decrease occurred in the LSGD group on the fundamentals test. Individual test scores and the difference between pretest and posttest score for all students involved in the statistical analysis on each portion of the test are listed in Tables 9 and 10 and Appendix C lists the scores of the

TABLE 8

MEAN, STANDARD DEVIATION, AND RANGE OF SCORES ON CALIFORNIA MATHEMATICS TEST BY CLASSES

Test	LCD Group			LSGD Group		
	Mean	S.D.	Range	Mean	S.D.	Range
Reasoning Pretest	44.70	5.89	33-55	45.50	8.52	28-60
Reasoning Posttest	48.55	4.93	43-56	49.20	6.73	45-79
Fundamentals Pretest	67.15	8.26	47-79	65.45	10.03	30-58
Fundamentals Posttest	70.20	7.28	50-78	70.85	6.42	57-79

remaining students in the two classes. It will be observed that eleven students in the LCD group increased their reasoning score at least five

TABLE 9

CALIFORNIA MATHEMATICS TEST SCORES BY STUDENT

LCD Group						
Student Number	*Reasoning Pretest	Reasoning Posttest	Difference Posttest Minus Pretest	**Fund. Pre-Test	Fund. Post-Test	Difference Posttest Minus Pretest
1	40	43	+3	58	67	+9
2	45	50	+5	79	74	-5
3	45	46	+1	66	70	+4
4	43	48	+5	66	75	+9
5	47	54	+7	77	77	0
6	40	48	+8	70	64	-6
7	52	56	+4	58	80	+22
8	40	47	+7	47	50	+3
9	49	45	-4	69	70	+1
10	33	43	+10	69	70	+1
11	45	43	-2	73	72	-1
12	52	60	+8	75	77	+2
13	44	49	+5	63	69	+6
14	55	53	-2	74	78	+4
15	55	56	+1	79	78	-1
16	48	43	-5	65	67	+2
17	44	44	0	57	58	+1
18	39	44	+5	68	65	-3
19	40	51	+11	70	76	+6
20	38	48	+10	60	65	+5

* Highest possible score is 60.

** Highest possible score is 80.

TABLE 10

CALIFORNIA MATHEMATICS TEST SCORES BY STUDENT

LSGD Group						
Student Number	*Reasoning Pretest	Reasoning Posttest	Difference Posttest Minus Pretest	**Fund. Pre-Test	Fund. Post-Test	Difference Posttest Minus Pretest
1	55	55	0	75	75	0
2	31	43	+8	59	68	+9
3	34	39	+5	53	57	+4
4	52	58	+6	67	74	+7
5	47	54	+7	73	77	+4
6	43	48	+5	63	65	+2
7	50	49	-1	60	70	+10
8	39	46	*7	49	70	+21
9	39	49	+10	66	67	+1
10	50	48	-2	70	75	+5
11	44	46	+2	67	66	-1
12	50	53		70	76	+6
13	28	30	[illegible]	45	66	+21
14	56	56	0	79	79	0
15	53	52	-1	75	75	0
16	50	54	+4	77	76	-1
17	46	48	+2	60	62	+2
18	54	56	+2	72	79	+7
19	54	57	+3	77	79	+2
20	35	43	+8	52	61	+9

* Highest possible score is 60.

** Highest possible score is 80.

points and three of them increased their score from pretest to posttest by more than ten points. In the LSGD class, eight students scored at least five points higher on the second reasoning test and two of these exceeded a ten point difference. On the fundamentals test, six students in the LCD class improved their score by at least five points and two of these increased their score by ten or more points. In the LSGD section, ten students scored at least five points higher from pretest to posttest and three of the ten achieved a score at least ten points better. Two students in the LCD section scored at least five points lower on the second reasoning test than on the first.

Table 11 lists the student scores for both groups on the attitude scale and gives the difference in score between pretest and posttest. The range of scores for the LCD group was 30 to 83 on the pretest and 40 to 85 on the posttest. The mean score from pretest to posttest increased from 64.70 to 68.75. Two students scored at least eleven points lower on the posttest than on the pretest and four students increased their score by at least eleven points from first to last administration of the scale.

Scores in the LSGD group ranged from 34 to 94 on the pretest and from 47 to 87 on the posttest. The mean score increased less than one point from initial to final measurement. The posttest score was eleven or more points lower than the pretest in three cases and three students increased their score by at least eleven points from the first week to the last week of the course. Table 12 gives a summary of the frequencies of differences in score from pretest to posttest as well as the mean and the standard deviation of each test for each group.

TABLE 11

ATTITUDE-TOWARD-MATHEMATICS SCORE BY STUDENT

LCD Group				LSGD Group			
Student Number	Pretest	Posttest	*D	Student Number	Pretest	Posttest	*D
1	73	60	-13	1	74	75	+1
2	83	83	0	2	62	51	-11
3	80	68	-12	3	49	56	+7
4	50	57	+7	4	80	81	+1
5	81	87	+6	5	71	69	-2
6	54	60	+6	6	70	78	+8
7	77	87	+10	7	58	70	+12
8	37	40	+3	8	53	72	+19
9	30	44	+14	9	57	62	+5
10	57	64	+13	10	69	75	+6
11	83	74	-9	11	70	61	-9
12	55	81	+26	12	72	77	+5
13	47	63	+16	13	34	47	+13
14	82	76	-6	14	84	78	-6
15	79	81	+2	15	94	83	-11
16	42	51	+9	16	72	71	-1
17	79	85	+6	17	60	57	-3
18	66	70	+4	18	81	87	+6
19	71	69	-2	19	77	77	0
20	68	75	+7	20	89	58	-31

*D = Posttest score minus pretest score

TABLE 12

SUMMARY OF ATTITUDE-TOWARD-MATHEMATICS SCORES FOR BOTH GROUPS

Statistic	LCD Section	LSGD Section
Pretest Mean	64.70	68.80
Pretest S.D.	16.912	14.333
Posttest Mean	68.75	69.25
Posttest S.D.	13.522	10.941
Difference Interval	Number of Students Scoring Within the Interval	
*D < -10	2	3
-10 ≤ D ≤ -5	2	2
-5 < D < 5	5	6
5 ≤ D ≤ 10	7	6
10 < D	4	3

*D = Posttest score minus pretest score

An examination of the pretest and posttest scores reveals that both groups improved their mean scores from initial to final measurement on all three variables. A *t*-test for difference of mean scores between pretest and posttest scores on the "California Reasoning," "California Fundamentals," and the attitude scale was made for each group. As shown in Table 13, the difference between pretest mean and posttest mean is significant at the .05 level of confidence for the LCD section on the reasoning test and for the LSGD section on the fundamentals test. All other differences fail to be significant at the .05 level of confidence; however, at the .10 confidence level, the difference

TABLE 13

PRETEST-POSTTEST COMPARISON OF REASONING, FUNDAMENTALS, AND ATTITUDES WITHIN GROUPS

Variable	*t LCD Section	*t LSGD Section
Pre-Reasoning/ Post-Reasoning	2.216	1.508
Pre-Fundamentals/ Post-Fundamentals	1.262	2.013
Pre-Attitude/ Post-Attitude	.828	.111

* degree of freedom = 18
$t_{.90} = 1.328$; $t_{.95} = 1.729$; $t_{.975} = 2.093$

TABLE 14

MEAN AND STANDARD DEVIATION OF SCORES ON REASONING, FUNDAMENTALS, AND ATTITUDE FOR UPPER ONE-HALF ABILITY LEVEL OF EACH GROUP

Measure	LCD Group		LSGD Group	
	Mean	Standard Deviation	Mean	Standard Deviation
Reasoning Pretest	46.80	7.301	50.60	5.501
Reasoning Posttest	51.10	5.858	53.50	3.894
Fundamentals Pretest	70.80	7.714	70.40	9.009
Fundamentals Posttest	74.90	4.095	74.80	4.417
Pre-Attitude Scale	68.80	16.844	75.70	10.698
Post-Attitude Scale	73.20	13.887	77.90	4.841

between the mean scores for the LSGD section on the reasoning test is significant.

As indicated in Chapter III, students numbered 1, 4, 6, 8, 12, 14, 15, 16, 18, and 19 make up the upper one-half ability level of the LSGD group and students numbered 1, 2, 5, 7, 9, 10, 12, 14, 15, and 19 comprise the upper one-half ability level of the LCD section. Tables 14 and 15 provide a list of the mean score and the standard deviation of the scores for the upper and lower one-half ability levels, respectively, of each group.

The mean score for the two upper one-half ability levels increased from pretest to posttest in each case, while the standard deviation decreased in every instance. Inasmuch as the mean scores do not differ significantly at the .05 level of confidence (Table 15) from pretest to posttest, it can be guessed that the scores of the posttest are relatively less dispersed about the mean than the scores of the pretest. Computation of the ratio $s/\bar{x}$ in each case, where s is the standard deviation and $\bar{x}$ is the sample mean, substantiates this claim.

TABLE 15

PRETEST-POSTTEST COMPARISON OF REASONING, FUNDAMENTALS, AND ATTITUDE MEAN SCORES FOR UPPER ONE-HALF ABILITY LEVEL

Measure	LCD Group *t	LSGD Group *t
Reasoning	1.453	1.361
Fundamentals	1.485	1.387
Attitude	.637	.592

* degrees of freedom = 9
$t_{.90} = 1.383$; $t_{.95} = 1.833$; $t_{.975} = 2.262$

The mean and standard deviation of each pretest and each posttest for the lower one-half ability levels are given in Table 16. The mean score increased from pretest to posttest in every case except for the attitude scale in the LSGD group. The mean score for the pre-attitude scale was 61.90 and the post-attitude mean score was 60.60. This decrease in mean score is due largely to student number 20, whose attitude score dropped 31 points--from 89 to 58. As in the upper one-half ability groups, computation of the ratio $s/\bar{X}$ indicates that the relative dispersion about the mean decreased in every case, however, the decrease was very slight in the LCD group on the fundamentals test.

TABLE 16

MEAN AND STANDARD DEVIATION OF SCORES ON REASONING, FUNDAMENTALS, AND ATTITUDE FOR LOWER ONE-HALF ABILITY LEVEL OF EACH GROUP

Measure	LCD Group		LSGD Group	
	Mean	Standard Deviation	Mean	Standard Deviation
Reasoning Pretest	42.60	3.204	40.40	8.072
Reasoning Posttest	46.00	2.309	44.90	6.674
Fundamentals Pretest	62.50	7.412	60.50	8.759
Fundamentals Posttest	65.70	7.424	66.90	6.118
Pre-Attitude Scale	60.60	16.814	61.90	14.609
Post-Attitude Scale	64.30	13.013	60.60	8.733

The results of a *t*-test performed on the difference between the pretest and posttest mean scores show the differences to be significant at the .05 level of confidence in the LCD lower one-half ability level on the reasoning test and in the lower-one half ability level of the LSGD group on the fundamentals portion of the California Mathematics Test. Table 17 lists the values of t computed.

TABLE 17

PRETEST-POSTTEST COMPARISON OF REASONING, FUNDAMENTALS AND ATTITUDE MEAN SCORES FOR THE LOWER ONE-HALF ABILITY LEVEL OF BOTH GROUPS

Measure	LCD Group *t	LSGD Group *t
Reasoning	2.722	1.359
Fundamentals	.663	1.894
Attitude	.550	-.242

* degrees of freedom = 9
$t_{.90} = 1.383$; $t_{.95} = 1.833$; $t_{.975} = 2.262$

Summary of Unit Tests and Final Examination Scores

Table 18 lists the mean score and the standard deviation of scores of both gr[illegible] f[illegible] the remaining five criterion measures; namely, "final exami[illegible]-p[illegible]," "final examination-part II," "final examination-total," "total points on independent reading examinations," and "total points in the course." Table 19 lists the corresponding means for the two upper one-half ability levels and Table 20 summarizes the same information for the two lower one-half ability levels. Appendix A contains individual student scores on each of the five measures.

Perusal of Tables 18, 19, and 20 cast doubt upon the possibility of finding significant differences between the two groups or between the two ability levels of the two groups on the various criterion measures in all but one or two instances. Even though the differences between the two groups on the initial measures were not great, there is the possibility that these differences account for the apparent lack of significant differences on the criterion measures. The analysis of covariance technique is designed to answer the question.

TABLE 18

MEAN AND STANDARD DEVIATION OF SCORES ON FIVE CRITERION MEASURES FOR LCD AND LSGD GROUPS

Criterion	LCD Section		LSGD Section	
	Mean	S.D.	Mean	S.D.
Final exam - part I	59.25	22.20	63.50	21.89
Final exam - part II	12.60	4.27	13.30	4.91
Final exam - total	71.85	24.94	77.10	25.95
Total points on indep. reading examinations	36.40	8.14	40.30	11.68
Total points in course	275.60	57.50	295.00	60.40

TABLE 19

MEAN AND STANDARD DEVIATION OF SCORES ON FIVE CRITERION MEASURES FOR UPPER ABILITY LEVELS OF BOTH GROUPS

Criterion	LCD Section		LSGD Section	
	Mean	S.D.	Mean	S.D.
Final exam - part I	72.50	22.61	77.00	17.06
Final exam - part II	14.40	3.58	16.80	2.40
Final exam - total	86.90	25.36	93.80	18.48
Total points on indep. reading examinations	40.00	6.00	49.40	3.95
Total points in course	312.60	47.41	336.10	27.97

TABLE 20

MEAN AND STANDARD DEVIATION OF SCORES ON FIVE CRITERION MEASURES FOR LOWER ABILITY LEVELS OF BOTH GROUPS

Criterion	LCD Section		LSGD Section	
	Mean	S.D.	Mean	S.D.
Final exam - part I	46.00	8.60	50.00	15.97
Final exam - part II	10.80	3.92	9.80	3.94
Final exam - total	56.80	9.26	60.30	19.90
Total points on indep. reading examinations	32.80	7.80	31.20	8.84
Total points in course	238.60	36.01	253.90	52.64

Selection of Covariates

Since the covariates were selected primarily on the basis of their correlation with the criterion measure, a table of correlations between initial measures and criterion measures was constructed. The correlations are given in Table 21. It should be noted that for sample size 40, the correlation coefficient must exceed .264 to reject the hypothesis that the population correlation coefficient differs from 0 at the .05 level of confidence and must exc [illegible] the .01 level (Dixon and Massey, 1969, p. 569). All correlations exceed the former figure and all but three of the correlations exceed the latter; namely, pre-attitude versus final examination, part I; pre-attitude versus final examination, total points; and pre-attitude versus total points in the course.

The second consideration in the selection of the covariates was the existence of a reasonable difference between the two groups

TABLE 21

CORRELATIONS BETWEEN INITIAL AND CRITERION MEASURES

Criterion Measure	Years H.S. Math	Reas. Pretest	Fund. Pretest	Algebra Test	Attitude Pretest	A.C.T. Math	A.C.T. Total
Final Exam (Part I)	.686	.684	.450	.661	.279	.763	.701
Final Exam (Part II)	.737	.626	.609	.569	.470	.682	.648
Final Exam (Total)	.728	.709	.501	.674	.328	.785	.727
Total Indep. Reading Exams	.676	.607	.624	.495	.515	.704	.702
Total Points in Course	.721	.610	.493	.613	.332	.761	.725
Post-Attitude Score	.615	.596	.547	.538	.757	.568	.433

on the pretests. The mean scores of the two groups on the "California Reasoning," "California Fundamentals," and the algebra test differed by at most two points; however, the standard deviations differed enough to warrant inclusion of all three as covariates. Attitude pretest mean scores of 64.70 and 68.80 were judged sufficiently different to justify inclusion as covariates. The initial measure "years of high school mathematics" exhibited no difference between the two groups and so was not retained as a covariate. For convenience, the mean score and standard deviation of scores for each of the seven initial measures is summarized in Table 22. The individual student scores on the algebra test are given in Table 23. The individual student scores on the other six initial measures were given in Tables 5, 9, 10, and 11.

TABLE 22

DESCRIPTIVE STATISTICS ON INITIAL MEASURES FOR BOTH GROUPS

Initial Measure	Statistic	LCD Section	ISGD Section
Years of High School Math.	Mean S.D.	2.50 .827	2.50 .827
Pre-reasoning	Mean S.D.	44.70 4.93	45.50 8.52
Pre-fundamentals	Mean S.D.	67.15 8.26	65.45 10.03
Pre-attitude	Mean S.D.	64.70 16.912	68.80 14.333
Algebra Test	Mean S.D.	14.55 7.330	14.10 4.327
A.C.T. Math.	Mean S.D.	50.95 27.81	51.75 23.37
A.C.T. Total	Mean S.D.	257.10 116.43	277.90 102.24

To determine whether the use of all six initial measures was practical in the sense of producing a significant increase in the accuracy of prediction of the regression equations, the multiple correlation coefficients of all six initial measures on each of the criterion measures was compared with the multiple correlation of subsets of one or more covariates with each of the dependent variables. The subsets were selected on the basis of their individual correlation with the criterion measures.

TABLE 23

COOPERATIVE MATHEMATICS TEST (FORM A, ALGEBRA II) INDIVIDUAL SCORES

LCD		LSGD	
Student Number	Test Score	Student Number	Test Score
1	11	1	20
2	16	2	12
3	16	3	12
4	14	4	13
5	26	5	13
6	11	6	15
7	30	7	17
8	10	8	13
9	7	9	8
10	16	10	9
11	14	11	10
12	29	12	18
13	11	13	7
14	10	14	20
15	24	15	17
16	10	16	15
17	10	17	19
18	7	18	21
19	19	19	15
20	10	20	8

McNemar (1949, p. 266) indicates that the F-ratio

$$F = \frac{R_1^2 - R_2^2}{1 - R_1^2} \quad \frac{N - m_1 - 1}{m_1 - m_2}$$

with $m_1 - m_2$ and $N - m_1 - 1$ degrees of freedom, where R_1 is the multiple correlation based on all of the independent variables, R_2 is the multiple correlation based on the selected subset, N is the number of individuals, m_1 is the total number of independent variables, and m_2 is the number of independent variables in the subset, may be used to test the statistical difference between R_1 and R_2. McNemar (p. 266) states that "If F falls beyond the .01 point, we can safely assume that the apparent gain in using the additional variable or variables possesses statistical significance."

The multiple correlation of the selected subsets of one covariate were the first to be compared with the multiple correlation of all six covariates. If the F-ratio indicated that no significant gain could be made with the inclusion of additional covariates, the analysis of covariance was completed. For each criterion measure for which the F-ratio indicated a significant gain could be made using additional covariates, the multiple correlation of the subsets of two covariates was compared with the multiple correlation for all six covariates. In every case, no more than two covariates were needed. Table 24 lists the criterion measures with their associated covariates, the multiple correlation coefficients, and the F-ratios. It will be observed that in four of the eight cases, the gain made by inclusion of covariates other than "A.C.T. mathematics" does not possess statistical significance.

TABLE 24

COMPARISON OF MULTIPLE CORRELATION WITH CRITERION MEASURES OF SELECTED COVARIATES AND ALL SIX COVARIATES FOR TOTAL GROUPS

Criterion Measure	Multiple Correlation with *Covariates I,II,III,IV,V,VI	Subset	Multiple Correlation of Subset with Criterion Measure	**F
Post-reas.	.881	I	.811	3.490
Post-fund.	.852	I,II	.772	3.681
Post-attitude	.846	III	.757	3.329
Final Exam. Part I	.827	V	.763	2.125
Final Exam. Part II	.783	V	.682	2.535
Final Exam. Total	.847	V	.785	2.363
Total Points Inde. Reading Exam.	.823	II,V	.759	2.590
Total Points in Course	.798	V	.761	1.648

* I = Pre-reasoning
II = Pre-fundamentals
III = Pre-attitude
IV = Algebra Test
V = A.C.T. Mathematics Percentile
VI = A.C.T. Total

$**F_{.99}(33,5) = 3.66; F_{.99}(33,4) = 3.97$

Analysis of Covariance

The existence of a difference in group means on the "California Reasoning," "California Fundamentals," and attitude scale posttests will first be investigated. Table 25 summarizes the statistical information for the full groups. In each case, Appendix D contains the raw score mean and the adjusted mean score.

The F-ratios given in Table 26 indicate that there is no significant difference at the .05 level of confidence between the mean

TABLE 25

MEAN AND STANDARD DEVIATION OF SCORES OF TOTAL GROUPS ON THREE CRITERION MEASURES

Criterion	LCD Group		LSGD	
	Mean	S.D.	Mean	S.D.
Reas. Posttest	48.55	4.93	49.20	6.73
Fund. Posttest	70.20	7.28	70.85	6.42
Attitude Posttest	68.75	13.52	69.25	10.94

TABLE 26

ANALYSIS OF COVARIANCE: TOTAL GROUP COMPARISON OF MEANS ON THREE CRITERION MEASURES

Criterion	Covariates		
	Pre-reasoning	Pre-fundamentals & Pre-reasoning	Pre-attitude
	*F-ratios		
Post-reas.	.010		
Post-fund.		.526	
Post-attitude			.580

$^*F_{.975}$ (1,37) = 5.47 and $F_{.025}$ (1,37) = .0009 scores of the two groups on post-reasoning, post-fundamentals, or post-attitude scale. That is, it is not safe to assume that regression lines, or regression plane in the case of the post-reasoning variable, for the separate groups are different from those of the combined groups.

The analysis of covariance of the difference of mean scores between the two total groups on the last five criterion measures, that is, on final examination - part I; final examination - part II; final examination - total; total points on independent reading examinations; and on total points in the course, indicates that there is no significant difference at the .05 level of confidence between the two groups in any instance. Table 27 lists the criterion measures, covariates used, and the F-ratios computed.

TABLE 27

ANALYSIS OF COVARIANCE: TOTAL GROUP COMPARISON OF MEAN SCORES ON FIVE CRITERION MEASURES

Criterion	Covariates	
	A.C.T. Math.	A.C.T. Math. and Pre-fund.
	*F-ratios	
Final Examination - Part I	.671	
Final Examination - Part II	.312	
Final Examination - Total	.848	
Total Points Independent Reading Examinations		4.651
Total Points in Course	2.221	

$^*F_{.975}(1,37) = 5.47$ and $F_{.025}(1,37) = .009$

In the analysis of covariance of the criterion means for the upper and lower one-half ability levels, both of the initial measures, "A.C.T. mathematics" and "A.C.T. total" was deleted from the list of covariates. This was done because "A.C.T. mathematics" was used to determine the ability levels and "A.C.T. total" is a composite of scores including "A.C.T. mathematics."

The selection of the covariates to be used in the analysis of covariance followed the same procedure as with the total groups. With one exception, it was found that for both ability levels a single covariate was sufficient for each of the criterion measures "post-reasoning," "post-fundamentals," and "post-attitude." Of the five remaining criterion measures, four required two covariates in the upper ability level and three required two covariates in the lower ability level. Since the same two covariates were found to be sufficient for all five criterion measures, in both ability levels, no analysis of covariance using only one covariate was performed. Tables 28 and 29 give the multiple correlation coefficients and the corresponding F-ratios for the upper and lower ability level, respectively.

The analysis of covariance of the posttest mean scores of the two ability levels on the criterion measures "post-reasoning," "post-fundamentals," and "post-attitude" indicated no significant difference at the .05 level between the two groups on any one of the three criterion measures. The F-ratios computed are given in Table 30.

TABLE 28

COMPARISON OF MULTIPLE CORRELATION WITH CRITERION MEASURES OF SELECTED COVARIATES AND ALL FOUR COVARIATES FOR THE UPPER ONE-HALF ABILITY LEVEL

Criterion	Multiple Correlation with *Covariates I, II, III, IV	Subset	Multiple Correlation of Subset with Criterion Measure	**F
Post-reas.	.896	I	.803	4.018
Post-fund.	.810	I,II	.705	3.470
Post-attitude	.858	III	.745	3.431
Final Exam. Part I	.821	I,IV	.802	.708
Final Exam. Part II	.737	I,IV	.688	1.145
Final Exam. Total	.831	I,IV	.791	.479
Total Points Indep. Read.	.646	I,IV	.578	1.072
Total Points in Course	.796	I,IV	.791	.164

* I = Pre-reasoning
II = Pre-fundamentals
III = Pre-attitude
IV = Algebra test

**$F_{.99}$ (15,2) = 6.36 and $F_{.99}$ (15,3) = 4.89

TABLE 29

COMPARISON OF MULTIPLE CORRELATION WITH CRITERION MEASURES OF ALL FOUR COVARIATES AND SELECTED COVARIATES FOR THE LOWER ONE-HALF ABILITY LEVEL

Criterion	Multiple Correlation with *Covariates I, II, III, IV	Subset	Multiple Correlation of Subset with Criterion Measure	**F
Post-reas.	.711	I	.666	1.275
Post-fund.	.728	II	.716	.184
Post-attitude	.783	III	.693	1.716
Final Exam. Part I	.518	I,IV	.491	1.780
Final Exam. Part II	.632	I,IV	.414	2.848
Final Exam. Total	.569	I,IV	.535	.416
Total Points Indep. Read.	.594	I,IV	.356	2.623
Total Points in Course	.379	I,IV	.336	2.021

* I = Pre-reasoning
II - Pre-fundamentals
III = Pre-attitude
IV = Algebra test

**$F_{.99}$ (15,2) = 6.36 and $F_{.99}$ (15,3) = 5.42

TABLE 30

ANALYSIS OF COVARIANCE: COMPARISON OF UPPER ONE-HALF ABILITY LEVELS AND LOWER ONE-HALF ABILITY LEVELS ON THREE CRITERION MEASURES

Criterion	Ability Level	Covariates			
		Pre-reas.	Pre-fund.	Pre-attitude	Pre-reas.& Pre-fund.
		*F-ratios			
Post-reas.	Upper Lower	.004 .002			
Post-fund.	Upper Lower		2.150		.963
Post-attitude	Upper Lower			.086 1.474	

$*F_{.975}\ (1,17) = 6.04$; $F_{.025}\ (1,17) = .001$; $F_{.975}\ (1,16) = 6.12$

$F_{.025}\ (1,16) = .001$.

Table 31 gives the results of the analysis of covariance performed on the mean scores of the five remaining criterion measures for the two ability levels. It will be recalled that there was no statistically significant difference, at the .01 level of confidence, between the use of all four covariates and the covariates "pre-reasoning" and "algebra test."

The analysis of covariance indicates that at the .05 level of confidence, there is no significant difference between the mean scores of the two groups at either ability level on the criterion measures "final examination-part I," "final examination-part II," "final examination-total," and "total points in course." Furthermore, there is no significant difference, at the .05 level of confidence, between

TABLE 31

ANALYSIS OF COVARIANCE: COMPARISON OF UPPER ONE-HALF AND LOWER ONE-HALF ABILITY LEVELS OF THE TWO GROUPS ON FIVE CRITERION MEASURES USING "PRE-REASONING" AND "ALGEBRA TEST" AS COVARIATES

Criterion Measure	Ability Level	*F
Final exam - part I	Upper	.020
	Lower	1.704
Final exam - part II	Upper	3.225
	Lower	.058
Final exam - total	Upper	.254
	Lower	1.268
Total points on indep. reading examinations	Upper	19.387
	Lower	.012
Total points in course	Upper	3.083
	Lower	1.198

$*F_{.975}(1,16) = 6.12$; $F_{.025}(1,16) = .001$

the mean scores of the two lower ability levels on the criterion variable "total points on independent reading examinations."

However, for the two upper ability levels, the analysis of covariance indicates that the mean scores of the two groups differ significantly, at the .05 level of confidence, on the criterion variable "total points on independent reading examinations."

Analysis of Questionnaire on Mechanics and Opinions of the Course

Since complete anonymity was desired and since the students without A.C.T. scores did not know they were excluded from the statistical analysis, the questionnaire was answered by all 27 students in the LCD group and all 23 students in the LSGD section. Appendix F contains the questionnaire and the percentage of students responding to each alternative.

To obtain some measure of the relative difficulty of the course, the correlation between the students' opinion as to difficulty and hours spent outside of class in study was computed (questions 2 and 5). The numbers 1, 2, and 3 were assigned to the possible alternative responses, with the number 3 indicating, in question two, that the course was more difficult than expected by the student and, in question five, that more than six hours per week were spent in study outside of class. The correlation between the response to the two questions by the LCD group was .293 and was .329 for the LSGD section. A sample correlation coefficient of about .325 is required to declare that the correlation differs significantly from 0 at the .05 level with a sample of size 27. The corresponding figure for a sample of size 23 is approximately .350; thus, there does not appear to be significant linear relationship between the students' opinion on the difficulty of the course and the time spent in study outside of the class period.

There was a significant relationship between the amount of time spent in study outside of class and the anticipated letter grade in the LCD section (the letter grades A, B, C, and D were assigned the numbers 1, 2, 3, and 4, respectively). However, in the LSGD group the sample correlation coefficient fails to be significantly different from 0 at the .05 level. That is, in the LCD group there is some reason to believe that the students who spend the least time in study outside of class also anticipated the higher letter grade, but in the LSGD section the evidence does not support the same conjecture. Table 32 contains the correlation coefficients computed.

TABLE 32

CORRELATIONS WITHIN GROUPS AMONG THREE SELECTED QUESTIONS FROM "MECHANICS AND OPINION OF COURSE" QUESTIONNAIRE

*Question	Section	2	5	11
2	LCD	1.0	.293	.525
	LSGD	1.0	.329	.107
5	LCD	.293	1.0	.375
	LSGD	.329	1.0	.375
11	LCD	.525	.375	1.0
	LSGD	.107	.337	1.0

*Question 2:

Check one of the following:

____ 1. The course was easier than I had expected.

____ 2. The course was about as difficult as I had expected.

____ 3. The course was more difficult than I had expected.

Question 5:

How much time did you have to spend studying the material outside of the class period?

____ 1. less than four hours per week

____ 2. between four to six hours per week

____ 3. more than six hours per week.

Question 11:

What letter grade do you expect to receive in this course?

____ 1. A; ____ 2. B; ____ 3. C; ____ 4. D.

Five students in each class indicated that the course was more difficult than expected. Of the five students in the LCD section, two of them indicated that they felt their attitude toward mathematics had changed for the better (question 8) and the remaining three indicated that no decided change had occurred. In the LSGD section, however, only one student of the five indicating that the course was more difficult than expected responded that no change in attitude had occurred. Three students felt that their attitude had improved and one student felt that his attitude toward mathematics had deteriorated during the course of the semester. The remaining students in each group, that is, those that felt that the course was either easier than expected, or was about as difficult as expected, indicated that they did not feel that their attitude had changed or they felt that it had changed for the better.

Other statistical analysis of the questionnaire could have been performed, however, for this study all other questions were answered by considering the percentage of students responding to each alternative within each question. Any analysis relating the questionnaire to the eight criterion variables previously discussed could be misleading since all students completed the questionnaire and only 20 students from each group were included in the other statistical analyses.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The three hypotheses given in Chapter I which served as a basis for this study will be considered in the light of the data collected and the analysis of the data presented in Chapter IV. Conclusions relating to the hypotheses will be given in the section entitled "Specific Conclusions." An answer to the question "Can a mathematics class of preservice elementary school teachers working in small groups examine all of the course content included in a class taught by a lecture-class discussion technique?" will be given in the "General Conclusions" section. Recommendations and suggestions for additional research related to this study conclude the chapter.

As stated in Chapter I, the purpose of this study was to compare the effectiveness of two teaching techniques used in two mathematics classes for prospective elementary school teachers. The effectiveness of the two techniques was evaluated by considering the students' performance on certain criterion measures. A comparison was made between the two groups on (a) the amount of mathematical content examined by each section; (b) the students' understanding of the basic mathematical concepts underlying arithmetic; (c) their mastery of the computational skills of arithmetic; (d) their ability to read mathematical material independently and with understanding; and (e) the attitude toward mathematics held by the students of each group.

The two teaching techniques were the lecture-class discussion (LCD) method and the lecture-small group discussion (LSGD) instructional procedure. In the LCD section, approximately thirty percent of the class period was devoted to lecture by the instructor, with the remaining seventy percent given to instructor-led class discussion. Lecture consumed twenty percent of the time in the LSGD section and instructor-led class discussion used another twenty percent of the class period. The remaining sixty percent of the time in the LSGD section was devoted to small-group (three or four students) work on assigned material. The instructor and one assistant were available to give limited assistance to the discussion groups.

Throughout the chapter references will be made to the instruments used in obtaining the criterion measures. For convenience, the criterion measures along with the instruments used to obtain them are given below.

1. Mathematics reasoning skills: Part I, "Mathematics Reasoning Text," of the California Mathematics Test (Form X), 1957 edition, revised by Tiegs and Clark.
2. Computational skills: Part II, "Mathematics Fundamentals Test," of the California Mathematics Test, (Form X), 1957 edition, revised by Tiegs and Clark.
3. Attitude toward mathematics: Attitude toward mathematics scale devised by Aiken and Dreger (1961, pp. 19-24).
4. Final examination - part I: An instructor prepared multiple-choice test covering the entire semester's work.
5. Final examination - part II: An instructor prepared

examination containing mathematical concepts not previously covered in class and exercises based on this material.

6. Final examination - total: Union of parts I and II.

7. Achievement on independent reading examinations: The union of the instructor prepared unit test I - part II, unit test II - part II, unit test III - part II, and final examination - part II.

8. Overall course achievement: The sum of all points earned on the three unit tests and the final examination.

The validity of any conclusions drawn from the results of this study must be determined within view of its limitations. The acknowledged limitations of this investigation are:

1. Even though there did not appear to be any factor other than normal scheduling problems influencing class composition, the samples were not randomly selected.

2. Just two classes with a total enrollment of 47 students were used in this study. Furthermore, the statistical analysis was performed on data collected on only 40 of the students.

3. Even though both classes studied from the same set of notes, the LSGD section may have been more dependent upon the dittoed material constituting the course content. The progress made by the LSGD section during any class period seemed to be related to the organization and clarity of the notes. In some cases a greater number of easier exercises may have been beneficial, while in other cases students could perhaps have worked fewer exercises and still have mastered the necessary concepts.

Notwithstanding the acknowledged limitations of this study, certain conclusions can be drawn based on the data collected, the analysis of the data, and personal observations. Conclusions based on the data and the analysis of the data are given in the following section. Conclusions drawn from personal observations and impressions are included in the general conclusions section.

Specific Conclusions

Hypothesis 1. There is no significant difference at the .05 level of significance between pretest and posttest scores within groups with respect to

(a) mathematics reasoning skills

(b) computational skills

(c) positive attitude toward mathematics scale.

Conclusion 1. The *t*-test computed in Chapter IV indicates that Hypothesis 1 may not be rejected with respect to criterion (a) in the LSGD section, criterion (b) in the LCD section and criterion (c) in both sections.

Conclusion 2. The *t*-test computed in Chapter IV suggests that Hypothesis 1 be rejected in favor of the alternative hypothesis "posttest mean greater than pretest mean" with respect to criterion (a) in the LCD group and criterion (b) in the LSGD section.

Hypothesis 2. There is no significant difference at the .05 level of confidence between pretest and posttest means within groups at the upper and lower ability levels with respect to

(a) mathematics reasoning skills

(b) computational skills

(c) positive attitude toward mathematics scale.

Conclusion 3. The t-test computed in Chapter IV does not suggest rejection of Hypothesis 2 for the upper ability level within each group on any of the three criterion measures.

Conclusion 4. The t-test indicates that Hypothesis 2 be rejected in favor of the alternate hypothesis "posttest mean greater than pretest mean" with respect to criterion (a) in the lower ability level of the LCD section and with respect to criterion (b) in the lower ability level of the LSGD section.

Hypothesis 3. There is no significant difference at the 5 percent level of confidence between the mean scores of the two groups with respect to

(a) mathematics reasoning skills

(b) computational skills

(c) positive attitude toward mathematics scale.

Conclusion 5. The analyses of covariance computed in Chapter IV suggest that Hypothesis 3 may not be rejected for any of the three criterion measures.

Hypothesis 4. There is no significant difference at the .05 level of confidence between the mean scores of the upper one-half ability levels of the two groups with respect to

(a) mathematics reasoning skills

(b) computational skills

(c) positive attitude toward mathematics scale.

Conclusion 6. The analyses of covariance do not indicate rejection of Hypothesis 4 for any of the three criterion measures.

Hypothesis 5. There is no significant difference at the .05 level of confidence between the mean scores of the lower one-half ability levels of the two groups with respect to

(a) mathematics reasoning skills

(b) computational skills

(c) positive attitude toward mathematics scale.

Conclusion 7. The analyses of covariance do not support rejection of Hypothesis 8 on any of the three criterion measures.

Hypothesis 6. There is no significant difference at the .05 level of confidence between the mean scores of the two groups with respect to

(a) final examination - part I

(b) final examination - part II

(c) final examination - total

(d) achievement on independent reading examinations

(e) overall course achievement.

Conclusion 8. The analyses of covariance computed in Chapter IV do not support rejection of Hypothesis 6 for any of the five criterion measures.

Hypothesis 7. There is no significant difference at the .05 level of confidence between the mean scores of the upper one-half ability levels of the two groups with respect to

(a) final examination - part I

(b) final examination - part II

(c) final examination - total

(d) achievement on independent reading examinations

(e) overall course achievement.

Conclusion 9. The analyses of covariance fail to give evidence to reject Hypothesis 7 with respect to criterion measures (a), (b), (c), and (e).

Conclusion 10. The analyses of covariance indicate that Hypothesis 7 be rejected with respect to criterion (d). In fact, the difference between mean scores is significant at the .01 level of confidence.

Conclusion 11. Since the adjusted mean of the independent reading score was greater for the upper one-half ability level of the LSGD section than for the corresponding level of the LCD section and since the difference of means was significant, the investigator concluded that the mean score of the LSGD section was significantly greater (at the .05 level) than the mean of the LCD section on this criterion measure.

The correlations among three selected questions and the percent of students responding to various alternative responses to each question on the student questionnaire dealing with the mechanics of the course, length of examinations, time spent in study outside of class, etc., led to the following conclusions:

Conclusion 12. The students in the LSGD section indicate a tendency toward the opinion that their attitude toward mathematics had changed for the better during the semester. Students in the LCD section tended to believe that their attitude toward mathematics had not changed during the course of the semester.

Conclusion 13. A majority of the students in each class believe that the course will definitely be of value to them as elementary school teachers of mathematics. The percentage of students with this belief was greatest in the section taught by the LSGD instructional technique.

Conclusion 14. In the LCD section there appears to be a linear relationship between the letter grade a student expects to receive and their response to whether the course was easier, about as difficult, or more difficult than they had expected. The students stating that the course was easier or about as difficult as expected also anticipated the higher letter grade for the course. The evidence does not support the same conclusion in the LSGD group.

Conclusion 15. In the LCD section, there appears to be a linear relationship between the letter grade a student expects to receive for the course and the amount of time spent in study outside of class. The students indicating that they anticipated one of the higher letter grades also indicated that they did not have to spend much time in study outside of class. The evidence does not support the same conclusion in the LSGD group.

Conclusion 16. The students in the LSGD section were more inclined to believe that their attitude toward mathematics had improved since the beginning of the semester than were the students of the LCD section.

Conclusion 17. It will be recalled that the teaching technique employed in each section was explained to both sections at the beginning of the semester. The students in the LSGD section generally agreed that they could not have learned more mathematics had they been enrolled in the LCD section.

Conclusion 18. The students in the LSGD section almost unanimously agreed that discussion groups of size four without pairing within the group were the least effective. Group size three and group size four with pairing within the group are about equally favored by the students of the LSGD section.

Conclusion 19. Students in the LSGD section overwhelmingly agreed that the ten minutes allotted to regular lecture activities at the beginning of the period was sufficient.

General Conclusions

Conclusion 20. A comparison of the mathematical content covered by the two sections during the semester and the results of the statistical analysis in chapter four led this investigator to conclude that a mathematics class for preservice elementary school teachers taught by the LSGD instructional technique can cover as much mathematical material as a class taught by the LCD approach without sacrificing understanding of the basic concepts presented in the course. In fact, as observed in conclusions one through eleven, the LSGD teaching technique is at least as effective as the LCD method on all criterion measures and in a few cases is clearly superior.

Conclusion 21. The observations given below led this investigator to conclude that the LSGD instructional technique is superior to the LCD technique in

1. stimulating the students' interest in mathematics,
2. promoting the idea that mathematics is not static,
3. enhancing the students' ability to communicate verbally in mathematics,
4. contributing to the students' ability to work effectively in a small-group situation, and
5. developing the habit to think critically

about a given statement or concept in mathematics before accepting its validity.

The instructor and his assistant made it a practice to arrive at the classroom at least five minutes before each class was to begin. This was done primarily to provide an opportunity for students to ask individual questions about the homework or on some topic covered during the previous class period. No tally was kept, but the instructor and his assistant agree that with few exceptions the questions asked in the LCD section were of the nature "I tried to solve the problem in this way, but I couldn't get a result. What am I doing wrong?" In all cases, the questions that were asked related directly to homework or material covered in class.

In the LSGD section the questions and remarks during the five minutes before class were markedly different. The questions were rarely about specific problems, but were rather of the nature "Why do we approach this concept as we do? Wouldn't it be easier to understand if we said . . . instead?" Undoubtedly, the questions they would ask during these pre-class periods were prompted by difficulties encountered during their group's discussion on the previous day. For example, in the second chapter of the notes, the operation "set difference" was defined in the usual way and then the complement of a set A was defined as the set difference of the universe U and set A. In the exercises, the students were asked to show that the set difference of a set A and a set B is the same as the intersection of set A and the complement of set B. The question then raised by several students was "I understand that the complement of a set A is everything in the universe except the elements of A and I understand the definition of set intersection, so why do we need set difference at all?"

Other pre-class conversations in the LSGD section centered around mathematical puzzles and games suggested by students. The instructor of the course frequently used (in both sections of the class) mathematical diversions to stress a point or to stimulate interest in a particular topic and so the students knew of his interest in puzzles or games with a mathematical flavor. It is likely that the students suggesting the first two or three mathematical "oddities" possessed an interest in this type of mental activity before enrolling in the class; however, the informality and congeniality of the small-group discussion class fostered, in the opinion of this investigator, a similar interest by others. Neither the instructor nor his assistant sensed a similar atmosphere in the LCD section.

It is the opinion of this investigator that the opportunities for exchanging ideas on particular problems and the opportunities to examine critically the ideas and suggestions of other students afforded by the small discussion groups, advanced the thought that very little ought to be accepted in mathematics without reasonable justification. Whereas students in the LCD group were inclined to say to the instructor "I don't understand that statement," students in the LSGD section were apt to state "I don't believe that's true." It is quite likely they didn't believe the statement because they didn't clearly understand it, but the fact remains that a substantial number of students in the LSGD section were eager to make known their reluctance to accept a statement as true simply because the instructor made the statement.

It is the opinion of this investigator that participation in the small work groups of the LSGD section enhanced the students' ability to verbally communicate ideas in mathematics. As will be recalled, the final ten minutes of each class period was allotted to an instructor-led class discussion. During this period, the students gave their solutions to solved problems either verbally or presented them at the board. Since the time period was short, students were encouraged, although not required, to present their argument verbally. Since both sections used the same notes, the same questions were answered by students of each group, thus it was possible to subjectively rate the quality of student response on specific questions. The instructor and his assistant agree that students of the LSGD group invariably gave a clearer and more concise argument and, furthermore, seemed more adept at defending their argument upon questioning by the instructor or a fellow student than did students in the LCD section.

The tenor of the small-group discussions began to change after the first few weeks. Early in the semester, students appeared reluctant to criticize the arguments of others in the group. However, as the semester progressed, good, healthy debate over proposed solutions was occurring with regularity. As the instructor and his assistant circulated about the room listening to group conversations and answering questions, they looked for signs of animosity building between or among students. They found none at any time. After the early weeks of the semester, the need for a designated group leader for the day was diminished. The students appeared to be readily accepting their individual responsibility toward the completion of the assigned task. It is the opinion of this investigator that participation in the LSGD class enhanced the students' ability to work effectively in a small-group situation.

Recommendations

Recommendation 1. Some of the students in each class had taken the required four-credit "Science-Social Studies-Mathematics" techniques course (approximately three-sixteenths of the time is devoted to the techniques of teaching arithmetic). Since the techniques course may have provided additional motivation, it is recommended that future studies of this nature consider this variable.

Recommendation 2. It is recommended that in any future investigations similar to this study, students in the LCD section be required to submit written homework assignments on a regular basis. Since most class discussion was on a voluntary basis, poorer students frequently did not take the opportunity to work on the more difficult exercises, perhaps decreasing their chances of doing well on the independent reading examinations.

Recommendation 3. It is recommended that in any future investigations similar to this study, students in the LSGD section be required, as a homework assignment, to individually submit once or twice a week, written solutions to exercises solved through group discussion. This procedure would provide the instructor with an early indication of individual misconceptions.

Recommendation 4. It is recommended that in future studies similar to this study, several different upper and lower one-half ability levels be determined. Specifically, A.C.T. total score, algebra test score, and California Mathematics Test total should all be used separately to determine ability levels.

Recommendation 5. It is obvious that in studies comparing two teaching methods that the teacher variable is an important one. Future studies similar to this investigation should take the teacher variable into consideration.

Recommendation 6. It is highly recommended that in any future investigation similar to this study, an attempt be made to measure the students' ability to read mathematical material independently and with understanding at the beginning of the course. Research should be aimed at the development of valid tests designed to measure the ability of preservice elementary teachers to read independently and with understanding, mathematical material similar to Topics in Mathematics for Elementary Teachers published by the National Council of Teachers of Mathematics.

Recommendation 7. It is highly recommended that future investigations similar to this study make provisions for a follow-up study to determine the technique of teaching mathematics used by the students in their first full-time teaching assignment.

Recommendation 8. It is recommended that future investigations similar to this study make provision for laboratory-type experience in mathematics for students of the LSGD section.

The physical facilities required for implementation of the LSGD teaching technique (i.e., adequate facilities for small-group work) would seem to be well suited for the incorporation of laboratory work within the regular class period. The experiments could be very short or could take up to 50 minutes. Incorporating the laboratory work into the regular class period would eliminate the need for the careful (but quite often unsuccessful) coordination of classroom activities with laboratory work that is required when the laboratory period is scheduled separately.

There are no laboratory manuals presently available that could be used successfully within the framework just described.

Recommendation 9. Traditionally the "methods of teaching elementary school arithmetic" courses have been taught separately from the content courses. In fact, the content course is usually taught by the mathematics department and the methods course by the education department. If personnel knowledgeable in both areas are available, this author believes that the two courses ought to be integrated. The integration suggested here is not one of spending four class periods on content followed by a fifth day devoted to methods of teaching the content--although this arrangement is also believed to be preferable to teaching separate courses--but rather a fusion of method and content. For example, after discussing and proving the theorem "If a, b, c, and d are whole numbers, with $b \leq a$ and $d \leq c$, then $(a+c) - (b+d) = (a-b) + (c-d)$," an immediate question could be "Where in the elementary school arithmetic sequence would you have need for this theorem and how would you lead students to discover the theorem." Another question might be "How would you argue the theorem's validity at the third or fourth level?"

In the opinion of this investigator, the spirit of constructive criticism, the informality, the free exchange of ideas, and the opportunity students have to discover mathematics "on their own" that appears to exist in a class taught by the LSGD technique would contribute significantly to successful integration of method and content.

It is recommended, therefore, that research be directed toward determining the advantages or disadvantages of fusing mathematics content courses and methods of teaching arithmetic courses.

APPENDICES

APPENDIX A

UNIT TEST SCORES, FINAL TEST SCORES, AND TOTAL POINTS IN COURSE FOR BOTH GROUPS

TABLE 33

UNIT AND FINAL TEST SCORES FOR LCD SECTION

STUDENT NUMBER	*TEST							
	U_{11}	U_{12}	U_{21}	U_{22}	U_{31}	U_{32}	F_1	F_2
	HIGHEST POSSIBLE SCORE							
	86	14	88	12	86	14	150	25
1	71	11	61	6	58	6	50	11
2	66	12	60	9	61	8	50	12
3	55	5	50	7	43	5	40	7
4	57	8	53	6	53	6	55	7
5	75	9	80	9	59	6	90	20
6	41	9	53	8	33	7	45	16
7	79	11	80	7	63	4	120	20
8	41	3	39	9	45	7	50	4
9	61	4	64	7	52	5	65	11
10	64	11	65	7	48	9	50	9
11	61	13	46	6	39	8	30	14
12	80	14	86	9	74	12	90	14
13	65	12	73	8	54	7	60	14
14	63	9	45	8	52	8	65	15
15	78	12	84	9	63	10	90	15
16	47	3	40	6	62	6	45	8
17	58	5	42	11	51	6	40	10

TABLE 33 - Continued

UNIT AND FINAL TEST SCORES FOR LCD SECTION

STUDENT NUMBER	*TEST							
	U_{11}	U_{12}	U_{21}	U_{22}	U_{31}	U_{32}	F_1	F_2
	HIGHEST POSSIBLE SCORE							
	86	14	88	12	86	14	150	25
18	69	4	73	8	43	7	40	13
19	66	6	74	8	69	10	55	17
20	82	13	76	8	54	9	55	15

*U_{ij} = Unit test i, part j.

F_i = Final test, part i.

TABLE 34

TOTAL POINTS ON EXAMINATIONS FOR LCD SECTION

STUDENT NUMBER	*TOTAL		
	T_1	T_2	T
	HIGHEST POSSIBLE TOTAL		
	410	65	475
1	240	34	274
2	237	41	278
3	188	24	212
4	218	27	245
5	304	44	348
6	172	40	212
7	342	42	384
8	175	23	198
9	242	27	269
10	127	36	263
11	176	41	217
12	330	49	379
13	252	41	293
14	225	40	265
15	315	46	361
16	194	23	217
17	191	32	223

TABLE 34 - Continued

TOTAL POINTS ON EXAMINATIONS FOR LCD SECTION

STUDENT NUMBER	*TOTAL		
	T_1	T_2	T
	HIGHEST POSSIBLE TOTAL		
	410	65	475
18	225	32	257
19	264	41	305
20	267	45	312

* $T_1 = U_{11} + U_{21} + U_{31} + F_1$ (See Table 33)

$T_2 = U_{12} + U_{22} + U_{32} + F_2$ (See Table 33)

$T = T_1 + T_2$

TABLE 35

UNIT AND FINAL TEST SCORES FOR LSGD SECTION

STUDENT NUMBER	*TEST							
	U_{11}	U_{12}	U_{21}	U_{22}	U_{31}	U_{32}	F_1	F_2
	HIGHEST POSSIBLE SCORE							
	86	14	88	12	86	14	150	25
1	79	11	68	12	68	8	105	19
2	66	9	57	7	64	10	40	10
3	40	4	49	7	49	6	30	9
4	79	14	88	12	60	10	70	13
5	68	14	73	9	65	8	85	16
6	74	13	60	8	62	6	60	17
7	69	5	66	8	65	6	55	15
8	64	8	76	12	64	12	55	13
9	48	8	67	6	47	4	35	9
10	82	12	77	9	73	10	70	14
11	35	5	38	7	48	7	55	6
12	76	13	67	12	70	10	90	18
13	56	2	64	6	61	6	45	4
14	73	12	62	12	62	8	60	20
15	66	12	60	7	69	11	70	16
16	79	12	75	9	71	12	75	16
17	59	6	41	8	62	6	45	10

TABLE 35 - Continued

UNIT AND FINAL TEST SCORES FOR LSGD SECTION

STUDENT NUMBER	*TEST							
	U_{11}	U_{12}	U_{21}	U_{22}	U_{31}	U_{32}	F_1	F_2
	HIGHEST POSSIBLE SCORE							
	86	14	88	12	86	14	150	25
18	63	13	71	7	66	12	80	16
19	80	14	63	12	78	12	105	20
20	33	8	55	6	48	5	40	5

*U_{ij} = Unit test i, part j.

F_i = Final test, part i.

TABLE 36

TOTAL POINTS ON EXAMINATIONS FOR LSGD SECTION

STUDENT NUMBER	*TOTAL		
	T_1	T_2	T
	HIGHEST POSSIBLE TOTAL		
	410	65	475
1	320	50	370
2	227	36	263
3	168	26	194
4	297	49	346
5	291	47	338
6	256	44	300
7	255	34	289
8	260	45	305
9	196	27	224
10	305	45	348
11	177	25	202
12	304	53	357
13	226	18	244
14	257	52	309
15	266	46	312
16	301	49	350
17	207	30	237

TABLE 36 - Continued

TOTAL POINTS ON EXAMINATIONS FOR LSGD SECTION

STUDENT NUMBER	*TOTAL		
	T_1	T_2	T
	HIGHEST POSSIBLE TOTAL		
	410	65	475
18	280	48	328
19	326	58	384
20	176	24	200

$T_1 = U_{11} + U_{21} + U_{31} + F_1$ (See Table 35)

$T_2 = U_{12} + U_{22} + U_{32} + F_2$ (See Table 35)

$T = T_1 + T_2$

APPENDIX B

A.C.T. ENGLISH, SOCIAL SCIENCE, NATURAL SCIENCE, AND GENERAL PERCENTILES

TABLE 37

A.C.T. PERCENTILE SCORES

Student Number	LCD SECTION				LSGD SECTION			
	*E	SS	NS	G	E	SS	NS	G
1	51	89	67	68	73	85	79	82
2	51	43	27	46	42	51	67	53
3	31	36	61	40	49	76	41	55
4	77	32	61	54	67	51	60	68
5	10	33	55	46	74	69	39	53
6	27	74	39	46	81	80	67	75
7	80	89	99	98	42	22	60	38
8	42	57	55	38	67	89	27	61
9	51	38	18	38	42	38	44	31
10	27	22	15	25	88	93	79	82
11	59	57	60	53	42	45	55	38
12	99	93	89	97	92	80	79	87
13	67	74	55	61	5	8	9	5
14	88	69	55	75	88	69	60	75
15	74	74	93	91	46	61	45	62
16	34	69	18	38	74	51	67	68
17	4	3	7	4	42	57	44	46
18	5	17	39	5	34	45	73	61
19	34	85	79	75	81	63	79	82
20	59	51	27	31	10	3	5	4

*E = English

NS = Natural Science

SS = Social Science

G = General

APPENDIX C

DATA ON STUDENTS NOT INCLUDED IN STATISTICAL ANALYSIS

TABLE 38

DATA ON STUDENTS NOT UNCLUDED IN STATISTICAL ANALYSIS: YEARS HIGH SCHOOL MATHEMATICS, PRETEST, AND POSTTEST SCORES

Student	*YHS	CR_1	CR_2	CF_1	CF_2	ATT_1	ATT_2	ALG
LCD SECTION								
a	0	46	53	61	68	68	78	10
b	3	50	54	74	74	69	78	21
c	2	37	41	61	64	36	42	13
d	2	46	45	57	70	63	77	15
e	2	45	48	75	72	68	78	10
f	2	40	46	63	76	77	83	8
g	1	32	37	70	68	26	39	8
LSGD SECTION								
a	4	50	50	72	73	88	92	25
b	3	43	51	63	73	52	55	10
c	3	51	52	79	73	70	57	18

*YHS = Years of high school mathematics

CR_i = "California Reasoning" pretest and posttest

CF_i = "California Fundamentals" pretest and posttest

ATT_i = Attitude scale pretest and posttest

ALG = Algebra test

TABLE 39

DATA ON STUDENTS NOT INCLUDED IN STATISTICAL ANALYSIS: SCORES ON FIVE CRITERION MEASURES

Student	Final-part I	Final-Part II	Final-Total	Indep. Read. Total	Total Pts. in course
	LCD SECTION				
a	55	11	66	28	242
b	65	18	83	47	326
c	40	13	53	35	210
d	65	11	76	38	270
e	90	15	105	46	348
f	80	9	89	34	337
g	40	5	45	22	212
	LSGD SECTION				
a	70	16	86	49	327
b	65	17	82	42	257
c	45	8	53	37	245

APPENDIX D

RAW AND ADJUSTED MEAN SCORES ON EIGHT CRITERION VARIABLES

The adjusted mean scores listed in the following three tables were determined, in the case of one covariate, by

$$\text{Adj } \overline{Y}_i = \overline{Y}_i - b_x (\overline{X}_i - \overline{X}),$$

where

$\overline{Y}_i$ = raw mean score of group i

$\overline{X}_i$ = mean score of covariate X for group i

$\overline{X}$ = $(\overline{X}_1 + \overline{X}_2)/2$

b_x = regression coefficient.

In the case of two covariates, say X and Z,

$$\text{Adj } \overline{Y}_i = \overline{Y}_i - b_x(\overline{X}_i - \overline{X}) - b_z(\overline{Z}_i - \overline{Z}),$$

where b_x and b_z are the partial regression coefficients and the other symbols are defined as above (Walker and Lev, 1953, p. 397 and p. 404).

TABLE 40

MEAN SCORES AND ADJUSTED MEAN SCORES ON ALL EIGHT CRITERION MEASURES FOR BOTH GROUPS

Criterion	LCD SECTION		LSGD SECTION	
	Mean	Adjusted Mean	Mean	Adjust. Mean
Post-reasoning	48.55	48.82	49.20	48.93
Post-fundamentals	70.20	69.95	70.85	71.10
Post-attitude	68.75	69.99	69.25	68.01
Final exam.- part I	59.25	59.51	63.50	63.24
Final exam.-part II	12.60	12.65	13.30	13.25
Final exam.- total	71.85	72.16	77.10	76.79
Total points indep. read.	36.40	36.16	40.30	40.54
Total points in course	275.60	276.31	295.00	294.29

TABLE 41

MEAN SCORES AND ADJUSTED MEAN SCORES ON ALL EIGHT CRITERION MEASURES FOR UPPER ABILITY LEVEL

Criterion	LCD SECTION		LSGD SECTION	
	Mean	Adjusted Mean	Mean	Adjusted Mean
Post-reasoning	51.10	52.26	53.50	52.34
Post-fundamentals	74.90	75.49	74.80	74.21
Post-attitude	73.20	75.09	77.90	76.01
Final exam.- Part I	72.50	73.68	77.00	76.81
Final exam.- Part II	14.40	14.60	16.80	16.60
Final exam.- Total	86.90	89.29	93.80	92.41
Total points indep. read.	40.00	40.67	49.40	48.73
Total points in course	312.60	313.47	336.10	335.23

TABLE 42

MEAN SCORES AND ADJUSTED MEAN SCORES ON ALL EIGHT CRITERION MEASURES FOR LOWER ABILITY LEVEL

Criterion	LCD SECTION		LSGD SECTION	
	Mean	Adjusted Mean	Mean	Adjusted Mean
Post-reasoning	46.00	45.41	44.90	45.49
Post-fundamentals	65.70	64.81	66.90	67.79
Post-attitude	64.30	64.62	60.60	60.28
Final exam.- Part I	46.00	44.22	50.00	51.78
Final exam.- Part II	10.80	10.53	9.80	10.07
Final exam.-total	56.80	54.69	60.30	62.42
Total points indep. read.	32.80	32.17	31.20	31.83
Total points in course	238.60	234.28	253.90	258.22

TABLE 43

REGRESSION COEFFICIENTS

Criterion	TOTAL GROUPS		UPPER ABILITY LEVEL		LOWER ABILITY LEVEL	
	Covariate(s)	Regress. Coeff.	Covariate(s)	Regress. Coeff.	Covariate(s)	Regress. Coeff.
Post-reasoning	Pre-reasoning	.6705	Pre-reasoning	.6088	Pre-reasoning	.5356
	Pre-fund.	.4199	Pre-fund.	.1432	Pre-fund.	.5919
Post-fund.	Pre-reasoning	.2664	Pre-reasoning	.3232		
Post-attitude	Pre-attitude	.6043	Pre-attitude	.5469	Pre-attitude	.4956
Final exam.- part I	A.C.T. Math	.6581	Pre-reasoning Algebra Test	1.4563 1.6629	Pre-reasoning Algebra Test	1.2118 -.8160
Final exam.- part II	A.C.T. Math	.1225	Pre-reasoning Algebra Test	.2147 .2211	Pre-reasoning Algebra Test	.2664 .0373
Final exam.- Total	A.C.T. Math	.7823	Pre-reasoning Algebra Test	1.6696 1.8772	Pre-reasoning Algebra Test	1.5045 -.8204
Total points indep. read.	Pre-fund. A.C.T. Math	.3774 .2066	Pre-reasoning Algebra Test	.4875 .2696	Pre-reasoning Algebra Test	.5292 -.0892
Total points in course	A.C.T. Math	1.7724	Pre-reasoning Algebra Test	2.3465 3.7779	Pre-reasoning Algebra Test	2.9248 -2.0091

APPENDIX E

AIKEN AND DREGER ATTITUDE TOWARD MATHEMATICS SCALE

OPINIONAIRE

Directions: Please write your name in the upper right hand corner. Each of the statements on this opinionaire expresses a feeling which a particular person has toward mathematics. You are to express on a five-point scale, the extent of agreement between the feeling expressed in each statement and your own personal feeling. The five points are: Strongly Disagree (SD), Disagree (D), Undecided (U), Agree (A), Strongly Agree (SA). You are to encircle the letter which best indicates how closely you agree or disagree with the feeling expressed in each statement as it concerns you.

1.	I do not like mathematics. I am always under a terrible strain in a mathematics class.	SD	D	U	A	SA
2.	I do not like mathematics and it scares me to have to take it.	SD	D	U	A	SA
3.	Mathematics is very interesting to me. I enjoy math courses.	SD	D	U	A	SA
4.	Mathematics is fascinating and fun.		D	U	A	SA
5.	Mathematics makes me feel insecure and at the same time it is stimulating.	SD	D	U	A	SA
6.	I do not like mathematics. My mind goes blank and I am unable to think when working math.	SD	D	U	A	SA
7.	I feel a sense of insecurity when attempting mathematics.	SD	D	U	A	SA

8.	Mathematics makes me feel uncomfortable, restless, irritable, and impatient.	SD	D	U	A	SA
9.	The feeling that I have toward mathematics is a good feeling.	SD	D	U	A	SA
10.	Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way out.	SD	D	U	A	SA
11.	Mathematics is something which I enjoy a great deal.	SD	D	U	A	SA
12.	When I hear the word Math, I have a feeling of dislike.	SD	D	U	A	SA
13.	I approach math with a feeling of hesitation--hesitation resulting from a fear of not being able to do math.	SD	D	U	A	SA
14.	I really like mathematics.	SD	D	U	A	SA
15.	Mathematics is a course in school which I have always liked and enjoyed studying.	SD	D	U	A	SA
16.	I don't like mathematics. It makes me nervous to even think about having to do a math problem.	SD	D	U	A	SA
17.	I have never liked math and it is my most dreaded subject.	SD	D	U	A	SA
18.	I love mathematics. I am happier in a math class than in any other class.	SD	D	U	A	SA

19. I feel at ease in mathematics and I like it very much.	SD	D	U	A	SA
20. I feel a definite positive reaction to mathematics; it's enjoyable.	SD	D	U	A	SA

APPENDIX F

QUESTIONAIRE ON MECHANICS OF COURSE

QUESTIONAIRE

	Percentage response Sec. 1	Sec. 2	Total
1. Section Number ________.			
2. Check one of the following:			
_____ The course was easier than I had expected.	18.5	34.8	26.0
_____ The course was about as difficult as I had expected.	63.0	43.5	54.0
_____ The course was more difficult than I had expected.	18.5	21.7	20.0
3. Regarding the homework assignments...			
a. Were they too theoretical?			
Yes _____	18.5	13.0	16
No _____	44.5	34.8	40
About right _____	37.0	52.2	44
b. Were they too long:			
Yes _____	0	0	0
No _____	70.3	86.9	78
About right _____	29.7	13.1	22
c. Were there enough challenging exercises?			
Yes _____	81.5	86.9	84
No _____	18.5	13.1	16
4. Regarding the examinations...			
a. Were they too theoretical?			
Yes _____	33.3	21.7	28
No _____	33.3	21.7	28

Questionaire continued

	Percentage response Sec. 1	Sec. 2	Total
About right ______	33.4	56.6	54
b. Were they too long?			
Yes ______	51.8	34.8	44
No ______	25.9	21.7	24
About right ______	22.3	43.5	32
c. Did the test questions relate closely enough to the class exercises?			
Yes ______	85.2	78.2	82
No ______	14.8	21.8	18
5. How much time did you have to spend studying the material outside of the class period?			
less than four hours per week	48.1	39.1	44
between four to six hours per week	44.4	52.2	48
more than six hours per week	7.5	8.7	8
6. Were you satisfied with the way class time was utilized?			
Yes ______	92.6	87	90
No ______	7.4	13	10
7. Do you think you could have learned more in the other section?			
Yes ______	11.1	13	12
No ______	33.3	60.8	46
Don't know ______	55.6	26.2	42

Questionaire contined

	Percentage response Sec. 1	Sec. 2	Total
8. Do you think your attitude towards mathematics has changed since the beginning of the semester?			
Yes ____	44.4	56.5	50
No ____	55.6	43.5	50
If your answer was yes, how has it changed?			
For the better ____	100	92.1	96
For the worse ____	0	7.9	4
9. Do you think that this course will be useful to you as an elementary school teacher of mathematics?			
Definitely ____	55.6	78.2	66
Perhaps ____	40.7	21.8	32
Definitely not ____	3.7	0	2
10. Would you be interested in electing a specially designed 15 semester-hour sequence in mathematics as your area of concentration?			
Yes ____	22.2	21.7	22
No ____	48.1	60.8	54
Perhaps ____	29.7	17.5	24
11. What letter grade do you expect to receive in this course?			
A ____	25.9	21.7	24
B ____	29.6	43.5	36
C ____	37.0	34.8	36
D ____	7.5	0	4

Questionaire continued

	Percentage response Sec. 1	Sec. 2	Total
FOR SECTION 2 ONLY:			
12. Which size work group do you think was most effective?			
3 students per group _____		43.5	
4 students per group _____		4.4	
4 students per group, but paired within the group ____		52.1	
13. The amount of time spent in regular lecture activities was...			
too much _____		0	
about right _____		82.6	
too little _____		17.4	

APPENDIX G

UNIT TESTS AND FINAL EXAMINATION

TEST I - PART I.

I. Let p and q be propositions. Construct a truth table for the proposition $[(p \rightarrow q) \wedge q] \rightarrow q$ and tell whether it is a tautology or explain why it is not a tautology.

II. Let p and q be propositions. Use the fact that $(p \leftrightarrow q) \Leftrightarrow (\neg p \leftrightarrow \neg q)$ to rewrite the following statement in a logically equivalent form. A and B are sets.

$$A \subseteq B \text{ if and only if } A \cap B = A$$

III. Let $U = \{a, b, c, d, e\}$, $A = \{a, b\}$, $B = \{b, c\}$, and $C = \{a, c, d\}$. Find

(1) $(A \cup B) \cap C$

(2) $A \square B$

(3) $\sim A$

(4) $B \times C$

IV. Use a Venn diagram to illustrate that if A and B are sets, then $A \cap (A \cup B) = A$.

V. <u>TRUE or FALSE</u>. If the statement is always true, print <u>TRUE</u> in the space below the statement. If the statement is not always true, print <u>FALSE</u> in the space below the statement and then give one counterexample.

(1) If A and B are sets and $A \subseteq B$, then $(A \times B) \subseteq B$.

(2) If D and E are sets, then $D \subseteq (D \cup E)$.

(3) If X and Y are sets, then $(X \wedge Y) \subseteq X$ and $(X \cap Y) \subseteq Y$.

(4) If A and B are sets, then $(A \square B) \subseteq A$.

(5) If K and M are sets and $K \cap M \neq \emptyset$, then $n(K) + n(M) < n(K \cup M)$.

(6) If K is a set, then $n(K) \in K$.

(7) If R and S are disjoint sets, then $R \subseteq \sim S$.

(8) If D and E are sets and $n(D \cup E) = n(D)$, then $E = \emptyset$.

(9) If H and G are nonempty sets and $H \subset G$, then $n(G \square H) < n(G)$.

(10) If p and q are propositions and $p \rightarrow q$ is true, then the converse of the inverse of $p \rightarrow q$ is also true.

VI. Let a, b, c, and d represent whole numbers. If <u>one</u> of the properties $A+$, $A_{\bullet}$, $C+$, $C_{\bullet}$, or D justifies use of the "equals" symbol, state which one. If <u>more than one</u> is needed to justify use of the equality symbol, write the phrase, "more than one needed" below the statement.

(1) $a + (b + c) = (b + c) + a$

(2) $a + (b + c) = b + (a + c)$

(3) $(a + b) \cdot c + (a + b) \cdot d = (a + b) \cdot (c + d)$

(4) $\big((a + b) + c\big) + d = (a + b) + (c + d)$.

(5) $(a + b) \cdot (c + d) = (c + d) \cdot (a + b)$

TEST I - PART II.

I. The following statement defines a binary operation, called subtraction, on the set $W = \{0, 1, 2, 3, 4, \ldots\}$.

<u>Definition</u>: Let A and B be two finite sets. The difference of $n(A)$ and $n(B)$, denoted $n(A) - n(B)$, is $n(A \square B)$ iff $B \subseteq A$. That is, $n(A) - n(B) = n(A \square B)$ iff $B \subseteq A$. [Recall that $A \square B = \{x \mid x \in A$ and $x \notin B\}$. That is, $A \square B$ is the set of all elements of A not found in B.]

<u>Exercise</u>:

(1) Use the definition to find the difference of 5 and 3. That is, find 5 - 3.

II. Print one of the words <u>TRUE</u> or <u>FALSE</u> after each of the following statements. No counterexamples are necessary. D and E represent finite sets.

(a) $(n(D) - n(E)) \in W$

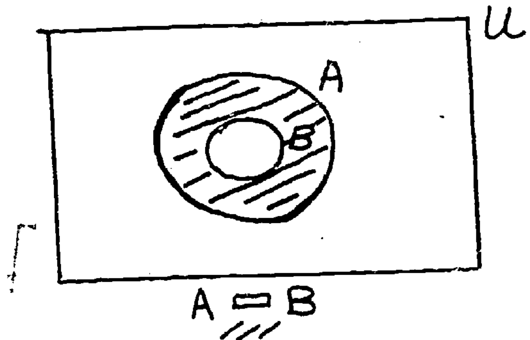
(b) If $D \subseteq E$, then $n(E) - n(D) = E \square D$.

(c) E - D is a set.

(d) If $n(D) - n(E) = n(D)$, then $E = \emptyset$.

(e) $n(D) - n(E) = n(E) - n(D)$.

III. We have verified that if A and B are two sets and $B \subseteq A$, then $B \cup (A - B) = A$ and $B \cap (A - B) = \emptyset$. The following diagram illustrates these two facts.



You may need one or both of these facts in the proof below.

Give a reason or reasons for each step in the following theorem.

Theorem: If $B \subseteq A$, then $(n(A) - n(B)) + n(B) = n(A)$.

Proof:	Statement	Reason
1.	$(n(A) - n(B)) + n(B) = n(A - B) + n(B)$	1.
2.	$= n((A - B) \cup B)$	2.
3.	$= n(A)$	3.

TEST II - PART I.

I. Give a reason justifying each step in the following computation. You are to assume that we know the basic 100 addition facts.

Statements		Reasons
1.	$46 + 73 = (4 \cdot 10 + 6) + (7 \cdot 10 + 3)$	1.
2.	$= [(4 \cdot 10 + 6) + 7 \cdot 10] + 3$	2.
3.	$= [7 \cdot 10 + (4 \cdot 10 + 6)] + 3$	3.
4.	$= [(7 \cdot 10 + 4 \cdot 10) + 6] + 3$	4.
5.	$= [(7 + 4) \cdot 10 + 6] + 3$	5.
6.	$= [11 \cdot 10 + 6] + 3$	6.
7.	$= [(1 \cdot 10 + 1) \cdot 10 + 6] + 3$	7.
8.	$= [(1 \cdot 10 \cdot 10 + 1 \cdot 10) + 6] + 3$	8.
9.	$= [1 \cdot 10 \cdot 10 + 1 \cdot 10] + (6 + 3)$	9.
10.	$= [1 \cdot 10 \cdot 10 + 1 \cdot 10] + 9$	10.
11.	$= (119)_{ten}$	11. B.T.N.S.

II. Write $(475)_{ten}$ as a base eight numeral.

III. Write $(2310)_{four}$ as a base ten numeral.

IV. (a) Find the prime factorization of 756.

(b) Find the prime factorization of 990.

(c) Use (a) and (b) to find the G.C.D. of 756 and 990.

(d) Use (a) and (b) to find the L.C.M. of 756 and 990.

V. Prove or disprove: 163 is a prime number.

VI. Use the theorems of Chapter IV to show that 45 divides 83,765,430.

VII. In each of the following statements, tell whether the statement is always true or not always true. If it is always true, write TRUE in the space below the statement. If it is not always true, write FALSE in the space below the statement and then give one counter-example.

(a) If a, b, and c are counting numbers and if a is a factor of c and b is a factor of c, then a•b is a factor of c.

(b) If k is the L.C.M. of two counting numbers x and y, then (x + y) is a factor of k.

(c) If d is the G.C.D. of x and y, then d divides (x + y).

(d) If r, s, and t are whole numbers such that r s is a multiple of t, then r is a multiple of t or s is a multiple of t.

TEST II - PART II.

<u>Definitions</u>. Let A be a nonempty finite set and let R be a subset of AxA, the cross product* of A with itself.

1. If (a,a) R, for every element a A, we say R is a <u>reflexive subset</u> of AxA.

2. If (b,a) R whenever (a,b) R, we say R is a <u>symmetric subset</u> of AxA.

3. If (a,c) R whenever (a,b) and (b,c) both belong to R, we say R is a <u>transitive subset</u> of AxA.

4. If R is a reflexive, symmetric, and transitive subset of AxA, let us say R is a <u>dandy subset</u> of AxA.

<u>Example</u>: Let A = {1,2,3}. Then AxA = {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}. The subset R = {(1,1),(2,2),(3,3),(1,2),(2,1)} is a dandy subset of AxA. The subset S = {(1,1),(2,2),(3,3),(1,2),(2,1),(2,3)} is <u>not</u> a symmetric subset of AxA because (2,3) $\in$ S and (3,2) $\notin$ S, hence S is <u>not</u> a dandy subset of AxA.

---

* Recall that the cross-product of two sets X and Y, denoted XxY, is the set of all possible ordered pairs of elements (x,y), where x $\in$ X and y $\in$

<u>Exercises</u>:

1. Let A = {1,2,3}. Is the subset K = {(1,1)(1,2),(2,1),(2,2)} of AxA
   (a) a reflexive subset of AxA? (yes or no?)
   (b) a symmetric subset of AxA? (yes or no?)
   (c) a transitive subset of AxA? (yes or no?)
   (d) a dandy subset of AxA? (yes or no?)

2. Let $A = \{1,2,3\}$ and let R be the set of all ordered pairs (x,y) of AxA such that x is smaller than y.

(a) List two elements of R.

(b) Is R a reflexive subset of AxA? (yes or no?)

(c) Is R a symmetric subset of AxA? (yes or no?)

(d) Is R a transitive subset of AxA? (yes or no?)

(e) Is R a dandy subset of AxA? (yes or no?)

TEST III - PART I.

I. If the statement is always true, print TRUE in the blank provided. If the statement is not always true, print FALSE in the blank provided. NO COUNTEREXAMPLES NECESSARY.

___ 1. If b is a nonzero integer, then (^{-}b) is less than zero.

___ 2. If $x \in Z$, $y \in Z$, and $y \neq 0$, then $^{-}\left(\frac{x}{y}\right) = \frac{^{-}x}{^{-}y}$.

___ 3. If $c \in Z$ and $d \in Z$ and $d \neq 0$, then $(c \div d) \in R$.

___ 4. If a, b, c, and d are all nonzero integers and $\frac{a}{b} = \frac{c}{d}$, then $\frac{a \cdot d}{b \cdot d} = \frac{c}{d}$.

___ 5. If a, b, c, d, e, and f are nonzero integers and $\frac{a}{b} \div \frac{c}{d} = \frac{e}{f}$, then $\frac{a}{b} = \frac{e}{f} \cdot \frac{d}{c}$.

___ 6. If r, s, and t are nonzero integers, then $(r \cdot s) \div t = (r \div t) \cdot s$.

___ 7. If p, q, and r are nonzero integers, then $r \div (p + q) = (r \div p) + (r \div q)$.

___ 8. If a, b, c, and d are elements of Z, and $c \neq 0$ and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = (a \div b) \div (c \div d)$.

___ 9. If m, n, and p are nonzero rational numbers, then $(m \div n) \div p = m \div (n \div p)$.

___ 10. If $r \in Ir$ and $s \in Ir$, then $r \cdot s \in Ir$.

___ 11. If a, b, and c are positive integers, then $\frac{a}{b} < \frac{a + c}{b + c}$.

___ 12. If $x \in R$, then $x \in Z$.

___ 13. If $x \in R$ and $y \in R$, then $(x + y) \in R$.

___ 14. If $x \notin R$ and $y \notin R$, then $(x \cdot y) \notin R$.

___15. If $x \in R$ but $x \notin Z$, then $x \in Ir$.

___16. If $x \in Z$, $y \in Z$, and $y \neq 0$, then $\frac{x}{y} \in Re$.

___17. If $x \in Z$ and $y \in Z$, then $(x - y) \in R$.

___18. If $x \in Ir$ and $y \in R$, then $x \cdot y \in Ir$.

II. Recall the following statements relative to the addition and/or subtraction of integers.

Statement A: If a and b are positive integers and $b < a$, then $a + (^{-}b) = a - b$.

Statement B: If a and b are positive integers and $a < b$, then $a + (^{-}b) = {}^{-}(b - a)$.

Statement C: If a and b are integers, then $a - b = a + (^{-}b)$.

Complete each of the following statements:

1. To compute the sum $(^{-}2) + 7$, we could use statement ____, with a = ____ and b = ____. The sum, according to the statement you intend to use, is ____.

2. To compute the difference $(^{-}2) - (^{-}7)$, we could use statement ____, with a = ____ and b = ____. The difference, according to the statement you intend to use, is ____.

III. Use the following theorem (and show your work) to find whole numbers q and r so that $58 = 8 \cdot q + r$, where $r < 8$. Theorem: Let a and b be positive integers with $b \leq a$. If $a - \underbrace{b - b - \ldots - b}_{q\text{ - b's}} = r$, then $a = b \cdot q + r$.

IV. Prove: The set of nonzero rational numbers is closed under division.

V. (a) Give a reason (or reasons) justifying each statement below.

1. If k and x are real numbers and $k \cdot x$ is a rational number, then $k \cdot x = \frac{c}{d}$ for some integer c and d.

   Reason

2. If $k \cdot x = \frac{c}{d}$, then $x = \frac{c}{d} \div k$.

   Reason

3. If k is a nonzero rational number, then $\frac{c}{d} \div k$ is a rational number.

   Reason

4. If $\frac{c}{d} \div k$ is a rational number, then x is a rational number.

   Reason

(b) Complete the following statement by giving the strongest possible conclusion based on the sequence of statements in part (a).

If k is a nonzero rational number and x is an irrational number, then $k \cdot x$ is a(n) ________________ number.

TEST III - PART II.

Recall that the <u>cross-product</u> of two sets A and B, denoted A x B, is the set of all ordered pairs of the form (x,y), where $x \in A$ and $y \in B$.

<u>Definition 1</u>: Let F represent the subset of all ordered pairs of Z x Z of the form (a,b), where $a \in Z$, $b \in Z$ <u>and</u> $b \neq 0$. That is, F is the set of all ordered pairs of integers where the <u>second</u> component is <u>not</u> zero.

Let $(r,s) \in F$ and $(t,w) \in F$. We will say that (r,s) and (t,w) are <u>kin</u>, denoted $(r,s) \approx (t,w)$, iff $r \cdot w = s \cdot t$.

<u>Exercises</u>:

I. Verify that (2,3) and (6,9) are kin.

II. Verify that $(3,7) \approx (4,8)$.

<u>Definition 2</u>: Define two binary operations, denoted $\oplus$ and $\odot$, on F as follows: If $(a,b) \in F$ and $(c,d) \in F$, then

$$(a,b) \oplus (c,d) = (a \cdot d + b \cdot c,\ b \cdot d) \text{ and}$$
$$(a,b) \odot (c,d) = (a \cdot c,\ b \cdot d).$$

<u>Examples</u>:

1. $(2,3) \oplus (4,5) = (2 \cdot 5 + 3 \cdot 4,\ 3 \cdot 5) = (22,\ 15)$

2. $(2,3) \odot (4,5) = (2 \cdot 4,\ 3 \cdot 5) = (8,\ 15)$

<u>Exercises</u>:

III. Is F closed under $\oplus$? Explain. (Recall that a set A is closed under an operation * defined on A iff $x*y \in A$ for all $x \in A$ and $y \in A$.

IV. Is F closed under ⊙ ? Explain.

V. Is ⊕ a commutative operation? Prove or disprove.

VI. Prove or disprove: If k is a nonzero integer, then

$(a, b) \approx (a \cdot k, b \cdot k)$.

VII. Find the ⊕ inverse of (2,3). (Note: If A is a set and * is an operation defined on A and a * Z = Z * a = a for all $a \in A$, we say Z is the *-identity element of A. Also, if $a \in A$ and $b \in A$ and a * b = b * a = Z, where Z is the *-identity element of A, we say b is the *-identity inverse of a).

FINAL EXAMINATION - PART I

<u>Directions</u>: In each of the following 25 exercises there is one <u>best</u> alternative. Print the letter of this best response in the blank provided to the left of the statement.

______ 1. If the universe $U = \{0,1,2,3,4,5,6,7\}$ and subsets A, B, and C of U are $A = \{0,1,2\}$, $B = \{2,4,6\}$, and $C = \{3,7\}$ then

a) $\sim(A \cup B) = C$

b) $\sim A \cap C = C$

c) $A - B = C$

d) a) and b) are both correct.

e) a), b), and c) are all correct.

______ 2. If the universe $U = \{1,2,3,4,5,6,7,8,9,10\}$ and if $A = \{1,2,3\}$ and $B = \{2,4,6\}$, then which one of the following statements is <u>false</u>?

a) $n(A \cap B) = 1$

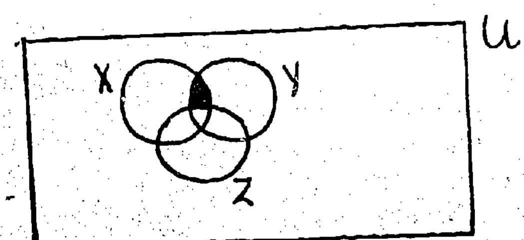
b) $n(A \cup B) = 5$

c) $n[\sim(A \cup B)] = 4$

d) $n(\sim A) = 7$

e) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

______ 3. The shaded region in the Venn Diagram below is



a) $(X \cup Y) \cap Z$

b) $(X \cap Z) \cup Y$

c) $(X \cap Y) \cap Z$

d) $(X \cap Y) - Z$

e) not equal to any of the previous answers.

____ 4. Let A and B be two <u>nonempty</u> sets. If $n(A) - n(B) = n(A \cap B)$, we can conclude

a) A and B are disjoint.

b) $n(A) + n(B) \neq n(A \cup B)$

c) $A \cap B = \emptyset$

d) A and B can be matched one-to-one.

e) none of the previous answers.

____ 5. If A and B are two nonempty finite sets, then we can conclude

a) $n(A \times B) = n(A) \cdot n(B)$

b) $n(A \cup B) = n(A) + n(B)$

c) $A \times B = B \times A$

d) $A \times B \subseteq A \cup B$

e) none of the previous alternatives.

____ 6. The prime factorization of 756 is

a) $2 \cdot 2 \cdot 3 \cdot 7 \cdot 9$

b) $2 \cdot 3 \cdot 7$

c) $2 \cdot 2 \cdot 3 \cdot 3 \cdot 21$

d) $7 \cdot 2 \cdot 2 \cdot 5 \cdot 5 + 5 \cdot 2 \cdot 5 + 2 \cdot 3$

e) none of the above are correct.

____ 7. The whole number represented by $(34)_{six}$ is relatively prime to

a) $(24)_{ten}$

b) $(24)_{nine}$

c) $(24)_{eight}$

d) $(24)_{seven}$

e) none of the above.

_____ 8. The whole number represented by $(102)_{three}$

a) can also be represented by $(92)_{ten}$

b) can also be represented by $(12)_{six}$

c) is divisible by $(2)_{three}$

d) is a prime number.

e) none of the above are correct.

_____ 9. If $420 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$ and $3465 = 3 \cdot 3 \cdot 5 \cdot 7 \cdot 11$, then

a) the G.C.D. of 420 and 3465 is $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$.

b) the L.C.M. of 420 and 3465 is $2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 11$.

c) the L.C.M. of 420 and 3465 is greater than $420 \cdot 3465$.

d) the G.C.D. of 420 and 3465 divides the L.C.M. of 420 and 3465.

_____ 10. If x, y, and z are whole numbers, then in order to prove that $x + (y + z) = y + (x + z)$, we would have to use

a) only the commutative property of addition of whole numbers.

b) only the associative property of addition of whole numbers.

c) both the commutative and associative properties of addition of whole numbers.

d) only the distributive property.

e) the commutative, associative, and distributive properties.

______ 11. The statement "If x and y are whole numbers, then x + y is a whole number," is logically equivalent to

a) If x is not a whole number or y is not a whole number, then x + y is not a whole number.

b) If x + y is not a whole number, then x is not a whole number and y is not a whole number.

c) If x + y is a whole number, then x and y are whole numbers.

d) If x + y is not a whole number, then x is not a whole number or y is not a whole number.

e) none of the above.

______ 12. If x, y, and z are counting numbers such that x•y divides z, then we can conclude

a) x and y are relatively prime.

b) x divides z and y divides z.

c) the G.C.D. of x and z is y.

d) there is a counting number k so that z•k = x•y.

e) none of the above.

______ 13. If r and s are counting numbers whose G.C.D. is r, then

a) s is their L.C.M.

b) r•s is their L.C.M.

c) r is a prime number.

d) there is a counting number k so that s•k = r

e) none of the previous answers are correct.

_____ 14. If r, s, and t are counting numbers such that r divides t and s divides t, then we can conclude

a) r and s are relatively prime.

b) r•s divides t.

c) (r + s) divides t.

d) t is the L.C.M. of r and s.

e) none of the above.

_____ 15. If x, y, and z are counting numbers such that x divides z, then we can conclude

a) if y divides z, then x•y divides z.

b) z is a multiple of x•y.

c) if z divides y, then x divides y.

d) if x divides y, then y divides z.

e) none of the above.

_____ 16. If p and q are two <u>different</u> prime numbers, then we can conclude

a) p•q is never a prime number.

b) p + q is never a prime number.

c) the L.C.M. of p and q is always less than p•q.

d) the G.C.D. of p and q is always either p or q.

e) none of the above.

_____ 17. If k is a counting number such that 2, 3, 4, and 9 all divide k, then we can definitely state that

a) 8 divides k.

b) 24 divides k.

c) 27 divides k.

d) 36 divides k.

e) 8, 24, 27, and 36 all divide k.

______ 18. If x, y, z, and w are all nonzero integers such that $x \cdot y = z \cdot w$, then we can conclude

a) the fractions x/z and w/y are equivalent.

b) the quotient $(x \cdot y) \div z$ is an integer.

c) z is a multiple of $x \cdot y$.

d) a) and b) are both true.

e) a), b), and c) are all true.

______ 19. Let x and y be nonzero numbers. If $x \cdot y$ is rational, then we can conclude

a) x is rational and y is rational.

b) the multiplicative inverse of $x \cdot y$ is rational.

c) $x \cdot y \div x$ is rational.

d) if x is irrational, then y is rational.

e) none of the above.

______ 20. If r is a nonzero rational number, then we can safely conclude

a) the product $r \cdot r$ is a positive real number.

b) the additive inverse of r is negative.

c) r can be expressed as a terminating decimal.

d) the square root of r is irrational.

e) none of the above.

_____ 21. If r, s, t, and u are nonzero integers, then

a) $(r \div s) \div (t \div s) = r \div t$.

b) $\frac{r}{s} \div \frac{t}{u} = \frac{r \div t}{s \div u}$.

c) $\frac{r}{s} \div \frac{t}{u} = \frac{r}{t} \div \frac{s}{u}$.

d) a) and b) are both true.

e) a), b), and c) are all true.

_____ 22. The rational number $\frac{2}{^{-}7}$

a) is less than the rational number $\frac{^{-}3}{8}$.

b) is the additive inverse of the rational number $\frac{^{-}2}{7}$.

c) is equal to the rational number $^{-}\left(\frac{2}{7}\right)$.

d) can be expressed as a terminating decimal.

e) is equal to $(^{-}2) \div \frac{1}{7}$.

_____ 23. If x and y are positive integers and $x < y$, then

a) $\frac{x}{y} = \frac{x + 7}{y + 7}$

b) $\frac{y}{x} = q + r$, for some integers q and r.

c) $\frac{x}{y} + \frac{y}{x} > 1$.

d) $\frac{x}{y} \div \frac{y}{x} = 1$.

e) none of the above answers are correct.

_____ 24. If s and t represent <u>two distinct</u> lines in space and $s \cap t = \emptyset$, then we can conclude

a) s and t are parallel.

b) s and t are skew lines.

c) if π_1 is a plane containing s and π_2 is a plane containing t, then $\pi_1 \cap \pi_2 = \emptyset$.

d) there is a plane containing s such that $\pi \cap t = \emptyset$.

e) none of the preceding.

_____ 25. If g is the relation from Re to Re defined by $g(x) = x \cdot x + 1$, for all $x \in Re$, then

a) g(x) is a positive real number, for all $x \in Re$.

b) $g(\sqrt{2})$ is a rational number.

c) g is a function from Re to Re.

d) a) and b) are both true.

e) a), b), and c) are all true.

FINAL EXAMINATION - PART II

QUESTION I.

Definition: Let a, b, and m be integers, where $m > 1$. We say a is congruent to b modulo m, denoted $a \equiv b \mod m$, iff m divides $(a - b)$.

Examples:
1. $11 \equiv 3 \mod 4$ because 4 divides $(11 - 3)$.
2. $13 \not\equiv 2 \mod 5$ because 5 does not divide $(13 - 2)$.

Exercises:

I. Is $29 \equiv 2 \mod 9$? (Just answer yes or no).

II. Find a whole number m so that $16 \equiv 4 \mod m$.

III. Find the smallest whole number b so that $31 \equiv b \mod 6$.

IV. Let x and y be whole numbers and suppose that x divides y. Find the smallest whole number k so that $y \equiv k \mod x$.

V. **The following five theorems will be given without proof.** The exercises below relate to the theorems and to the definition of congruence given above.

Theorem 1. If $a \equiv b \mod m$ and c is an integer, then $a \cdot c \equiv b \cdot c \mod m$.

Theorem 2. If $a \equiv b \mod m$ and c is an integer, then $a + c \equiv b + c \mod m$.

Theorem 3. If $a \equiv b \mod m$ and $b \equiv c \mod m$, then $a \equiv c \mod m$.

Theorem 4. If $a \equiv b \mod m$ and $a \equiv b \mod n$ and if the greatest common divisor of m and n is 1, then $a \equiv b \mod m \cdot n$.

Theorem 5. If $a \cdot c \equiv b \cdot c \mod m$ and if the greatest common divisor of c and m is d, then $a \equiv b \mod (m/d)$.

Exercises:

1. If $6 \cdot a \equiv 6 \cdot b \mod 14$, find m so that $a \equiv b \mod m$.

2. If $a \equiv b \mod m$, find n so that $(a - b) \equiv 0 \mod n$.

3. Find a whole number c so that if $a \cdot c \equiv b \cdot c \mod 9$, then $a \equiv b \mod 9$.

4. Find an integer m so that if $7 \equiv b \mod 6$ and $7 \equiv b \mod m$, then $7 \equiv b \mod 6 \cdot m$.

In each of the following exercises, if the statement is always true, write true in the blank provided. If the statement is not always true, write false in the blank to the left of the statement.

______ 5. If $7 \equiv b \mod 6$ and $7 \equiv b \mod 8$, then $7 \equiv b \mod 48$.

______ 6. If $32 \equiv 2 \mod m$, then $m \equiv 0 \mod 30$.

______ 7. If $5 \cdot a \equiv 1 \mod 4$, then $a = 1$.

______ 8. If $a \equiv b \mod m$ and $a \equiv c \mod m$, then $a \cdot b \equiv a \cdot c \mod m$.

______ 9. If $a \equiv 0 \mod m$, then $a + b \equiv b \mod m$.

FINAL EXAMINATION - PART II

QUESTION II.

Definition: A Boolean Algebra consists of a nonempty set B and two binary operations defined on B satisfying the four conditions given below. Let us denote the binary operations by o and *, read circle and star, respectively.

1. For all elements a and b in B, a o b and a * b are also in B. (We say B is closed under o and *, respectively).

2. For all elements a and b in B, a o b = b o a and a * b = b * a. (These are called the commutative properties of o and *).

3. There are two elements, denoted i_o and i_*, in B such that for each element a in B,

$a \circ i_o = a$ and

$a * i_* = a$.

(The two elements, i_o and i_*, are called the circle-identity and star-identity elements respectively).

4. For all elements a, b, and c in B,

$a \circ (b * c) = (a \circ b) * (a \circ c)$ and

$a * (b \circ c) = (a * b) \circ (a * c)$.

(These are called the distributive properties of o over * and * over o, respectively).

5. For each element a in B, there is an element, denoted by a', in B such that

$a \circ a' = i_*$ and $a * a' = i_o$.

(The element a' is called the inverse of a).

<u>Theorem</u>: The set $B = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$, together with the set operations union and intersection is a Boolean Algebra.

<u>Remarks</u>: To prove this theorem we must verify that all four conditions specified in the definition are satisfied. Early in the semester we verified that the union of two sets and the intersection of two sets is a set and that set union and set intersection are both commutative operations, hence, we need not check conditions 1 or 2. Also, we have shown that intersection distributes over union and union distributes over intersection, for arbitrary sets X, Y, and Z, hence, we need not check condition 5 with respect to the elements of B. To complete the proof, you are to consider the following two exercises.

<u>Exercise</u>: Identify the identity elements, specifying which is which. Illustrate the fact that you have probably found the correct identity elements by using the two elements $\{1,2\}$ and $\{1,3\}$ in B.

<u>Exercise</u>: Find the inverse of $\{1,2\}$.
Find the inverse of $\{2,3\}$.

FINAL EXAMINATION - PART II

QUESTION III.

Recall that in our brief study of plane geometry, we did not define what we meant by the terms point or line, but merely accepted them on an intuitive basis. We then accepted as true, a number of statements called postulates or axioms.

We are now going to do the same type of thing, except that our space will contain only a finite number of points, rather than an infinite number. Because we will only have a finite number of points, we can't really talk about lines in the usual sense, hence, we will discuss things called "lins" instead.

Undefined terms: Point, lin

Definition 1: Space is the set of all points in our present discussion.

Definition 2: Two lins are parallel when they have no point in common.

Postulates:

1. Two points determine a lin. That is, there is one and only one lin that contains two given points.
2. A lin contains exactly two points.
3. Given a point P and a lin k not containing P, there is exactly one lin that contains P and is parallel to k.
4. The space consists of at least three points.

Remarks: The postulates give us a method of visualizing a lin, viz., a lin is just a pair of points.

Exercises: Give a reason(s) wherever requested, for each step in the following proofs.

Theorem 1: The space consists of at least four points.

Proof:

1. The space consists of at least three points.

   Reason:

Call the points A, B, and C.

2. The pair of points A and B constitutes a lin.

   Reason:

Call the lin k.

3. C is not contained in k.

   Reason:

4. There is a lin, say m, containing C that is parallel to k.

   Reason:

5. m contains one additional point, say D.

   Reason:

6. ∴ There are at least four points in the space.

Theorem 2: The space consists of exactly four points.

Proof:

1. The space contains at least four points, say A, B, C, and D.

   Reason: Theorem 1.

2. If the space contains 5 points, say A, B, C, D, and E, then the lin containing C and D and the lin containing D and E are both parallel to the lin containing A and B.

Reason:

3. But these two lines cannot be parallel to the lin containing points A and B.

Reason:

4. ∴ There cannot be a fifth point.

R E F E R E N C E S

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