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ABSTRACT

The discovery method, according to the author, promotes logical thinking, is motivational in nature, makes learning more permanent, produces more usable knowledge, and gets students actively involved. A discovery lesson is characterized by the following: (1) the concept to be learned is not announced, (2) experiences and activities are developed which involve the concepts to be learned, (3) the ideas are given names, (4) definitions are generated, (5) pupils are actively involved, (6) practice experiences are provided and disguised to some extent, and (7) questions are raised to provoke additional thought. These points are illustrated with an example discovery lesson on prime numbers. (JG)

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"The Art of Questions and Discovery"

by

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After a brief discussion of the discovery approach and how questions contribute to students' learning mathematics, a lesson will be developed to exemplify this process.

Every elementary school mathematics program devotes considerable time to addition, subtraction, multiplication, and division within the set of whole numbers and fractional numbers. It is questionable if the development of logical reasoning receives the appropriate emphasis in each program.

What is mathematics? Mathematics is primarily concerned with ideas. Mathematics is a way of thinking and reasoning. Mathematics is a way of making predictions. Mathematics is a way of drawing conclusions and generalizing. Mathematics is a way of getting correct answers.

The reasoning aspect of the elementary mathematics program is as important as learning the basic facts and developing computational skills. After all, having facility in computational skills is not necessarily a saleable skill. One dare not compete with the calculator or the computer in computational endeavors. Not only are the machines quicker, but they do not get tired nor do they make mistakes as those of us who have skills will do in such efforts.

Do we learn elementary school mathematics in order to figure our taxes, check prices, determine salary deductions, balance checkbooks? Even though

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these are very valuable outcomes, I believe the motivation for learning elementary school mathematics is primarily an outgrowth of mathematics being presented in an interesting setting. The main thrust of the elementary school mathematics program should be to teach boys and girls the mathematics which they can use as tools to learn additional mathematics. This arrangement is very much akin to our efforts in teaching reading. Children entering formal educational programs are taught to read so that they in turn can read to learn. And so it is with the mathematics curriculum in the elementary school where the mathematics learned should be used to learn additional mathematics.

Certain strategies and procedures which are frequently identified with contemporary mathematics can assist us in reaching these goals.

The discovery approach which is frequently referred to as the inductive reasoning process is one of these. This approach may need some clarification. This might best be done by citing a situation which developed in a science class where the question was raised, "Why do plants grow upward?" The conjecture was made by a student that it was due to light. This response was challenged and brought forth a series of investigations whereby potted plants were placed on their sides in a dark closet. Several days later it was determined that the plants were growing upward, hence, indicating that some force other than light was causing this upward growth. This was the generalization of the students after several experiments were accomplished.

In a similar vein, considerable mathematics can be taught. A teacher writes 3 + 5 = 8 on the chalk board. Students identify the addends as odd numbers and the sum as an even number. Additional examples may be checked; and eventually, the generalization is forthcoming that an odd

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number plus an odd number yields an even number as a result. Although such conclusions can be rather convincing, we know that conclusions reached in this manner are tentative ones and do not constitute a proof.

A second dimension of the discovery approach might be illustrated by a discussion held with a second grade class. The first question raised in this discussion was, "What is the smallest counting number you know?" Several responses were forthcoming including zero and one. The class members began to interact and justify the various responses and finally drew the conclusion that one is the smallest counting number. One boy convinced the class of this by indicating that in determining the number of students in the class he would not start with zero in the counting process but with one.

A second question was raised, "What is the largest counting number you know?" Several responses were given including 9, 99, a million, and even a zillion. At this point, I was not able to let the class resolve this question in as much as the correct response was not given. The responsibility became mine to raise the appropriate questions to lead them to the correct generalization. This was accomplished by asking the boy who stated the largest number he knew was a zillion to imagine that he had a "zillion" marbles at home. I asked him, "If I gave you one more, how many would you have?" He replied, "A 'zillion' and one." I inquired, "If I gave you another marble, how many would you have?" His response, "A 'zillion' and two." Finally, the generalization became evident that you add one to the previous counting number to get the next one and that there is no last one. The inappropriate "zillion" was tactfully discounted after the idea had been grasped.

The main thrust of this procedure is identifying the right kind of situations and raising the right questions at the right time so that the desired conclusion will be generated by the class.

One might question the use of this procedure in as much as it requires considerable time and the teacher must have a good grasp of the mathematical background and concepts under consideration. Here are several important outcomes which support the use of this strategy:

1. It promotes the logical reasoning process which is a very important part of the elementary school program.
2. It is motivational in nature and tends to keep the interest of the children.
3. It makes the learning a little more permanent.
4. It makes the knowledge one acquires more usable in learning additional mathematics.
5. It gets pupils actively involved.

Mathematics is not a spectator subject but requires active participation on the part of the learner. It is like eating and sleeping; only those who participate get benefits from these activities. Sophocles, a Greek philosopher, once said, "One must learn by doing the thing for though you think you know it, you have no certainty until you try." And so it is with learning mathematics. One must have experiences with mathematics to assure himself that he has some knowledge of the activity and the concept.

A sharp contrast to the discovery approach is the show and tell procedure where a teacher shows and tells how to do the first example, then a second one, and the student is required to solve the third. The show and tell process is very much like giving a friend an arrangement of cut flowers. When the flowers arrive from the florist, they are a joy to behold. After a few days, however, the flowers begin to wither and fade and soon no beauty is left. Watching someone explain mathematics is also enjoyable; but as the flowers, ideas so gained generally wither and fade. Our job is not to give our children cut flowers but rather to help them grow their own. The discovery approach provides the "grow your own" idea.

Although the discovery approach is a very valuable process in the elementary school program, we cannot hope to have students discover all the mathematics concepts in the elementary school mathematics program. Such extensive efforts would not only require an excessive amount of time; but in some cases, teachers would be hardpressed to devise ways to discover some concepts. The discovery approach is not easily built into the mathematics textbook; hence, each teacher must generate his own discovery approach and strategies.

At this point, we will consider a teaching situation which is designed to exemplify the discovery approach. Pretend that you are a fifth grader and respond only in your thinking and not verbally. You should pay special attention to the questions--the kind and the number.

Today we will work with the universal sets U = {2, 3, 4, . . .} and are primarily interested in numbers which can be represented by rectangular arrays with at least two lines and two columns. What kind of rectangular array can you develop to represent the number two? You do yours while I do mine on the overhead projector. (The projector light is off). After a moment, the screen is lighted to show the two arrays:

1 x 2 xx   2 x 1 $\begin{matrix}x\\x\end{matrix}$

Can you develop rectangular arrays to represent the number three? You do yours while I do mine. Can you represent the number four with rectangular arrays? Have you found a rectangular array which meets our definition of having at least two lines and two columns? The number four can be represented by a 2 x 2 array so this number conforms to our definition. Let us classify our universal set into two subsets--those which meets our definition and those which do not. Under the "yes" column, we have the number four. At this point, what do we have under the "no" column? Where does the number five fit? The number six? Seven? Eight, etc.? If you are having trouble determining which column the numbers fit in, make rectangular arrays to assist you. After a reasonable pause, the teacher shows his work and students adjust their sets as necessary.

Will each member of the universal set belong to the "yes" or the "no" column? Is there a member which belongs to both? Does this represent an exhaustive and exclusive classification?

Let us check the factors of the "no" column. What are the factors of two? Three? Five? Seven, etc.? Is there a pattern here? Check the factors of the "yes" column. What differences do you see? Are there similarities? Let us give specific names to the two sets we have identified. The set containing 2, 3, 5, 7, . . . we will call prime numbers and the set containing 4, 6, 8, 9, . . . will be called composite numbers.

What is a prime number? What is a composite number? From the experiences, students should be able to draw out definitions of these two sets of numbers. Such generalizations are a natural outgrowth and represent an important part of the discovery approach.

At this point, it is well to raise questions which might cause a student to think beyond current experiences and activities and make conjectures. How many even primes are there? How many odd primes are there? How many odd composite numbers are there? How many even composite numbers are there? Do the prime numbers go on and on? Is this a finite or an infinite set? Are there more prime numbers between 1 and 100 or between 100 and 200? Can you find out? Can you find a prime number greater than 1,000?

Some students may wish to look up the sieve of Eratosthenes.

An interesting activity involving prime numbers could be brought in. Each student might write a three-digit number name and repeat the three-digits to create a six-digit number name such as 835,835. Ask each student to divide his number by 7 and casually suggest that the remainder will be zero. The quotient of this process then may be divided by 11. "Don't worry about the remainder. It will be zero." The quotient of this process should be divided by 13 and again the remainder will be zero. The quotient may be a surprise. Why is the final quotient the same as the three-digit number name written originally? If no solution or suggestion is forthcoming, suggest that the three divisors, all prime numbers, be multiplied together. An additional clue might be to multiply the three-digit number used originally by 1,001.

A prime game may then be employed which actively involves each student in the class. A description of this game may be found in The Arithmetic Teacher, February, 1969, issue. The article is entitled "Prime, A Drill in the Recognition of Prime and Composite Numbers" by Gregory Holdan, Indiana University of Pennsylvania.

What have we done in this lesson? (1) Initially the concept to be learned was not announced, (2) Experiences and activities were developed which involved the concepts to be learned, (3) The ideas were given appropriate names, (4) Definitions were generated from the experiences, (5) Pupils were actively involved, (6) Practice experiences were provided and disguised to some extent, and (7) Questions were raised to provoke additional thought, some of which were left unanswered.