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AUTHOR	Ravindran, Arunachalam
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ABSTRACT

The optimal storage of books by size in libraries is considered in this paper. It is shown that for a given collection of books of various sizes, the optimum number of shelf heights to use can be determined by finding the shortest path in an equivalent network. Applications of this model to inventory control, assortment and packaging problems are also given. An extension of the basic model which minimizes the wasted shelf space in libraries is also discussed (Author)

"ON COMPACT BOOK STORAGE IN LIBRARIES"*

Arunachalam Ravindran

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School of Industrial Engineering
Purdue University
Lafayette, Indiana 47907

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ABSTRACT

This paper considers the optimal storage of books by size in libraries. It is shown that for a given collection of books of various sizes, the optimum number of shelf heights to use can be determined by finding the shortest path in an equivalent network. Applications of this model to inventory control, assortment and packaging problems are also given. An extension of the basic model which minimizes the wasted shelf space in libraries is also discussed.

1. INTRODUCTION

Books may be stored in a library according to its subject or author classification, by order of accession, by size or by any other method which permits its orderly retrieval. The purpose of this paper is to present mathematical models for compact book storage or storage of books by their size. Faced with the problem of growing volume of acquisitions, most libraries for reasons of economy are forced into using some form of compact storage. A commonly prevalent practice is to store only the most recent acquisitions according to their subject which facilitates user-access and browsing; while the other volumes are kept in compact storage, not readily accessible to the user. In most libraries, oversized books are shelved separately so as to conserve shelf space in terms of height. The introduction of microform and other newer methods of storage devices preclude direct user access and hence models of compact storage by size may become useful in the future.

In this paper we shall present a network model which permits the selection of shelf heights and lengths such that the storage cost is minimized for a given collection of books. It will be shown that the above problem is equivalent to finding the shortest path in a directed network. Applications of this model to a wide variety of inventory and packaging problems will be indicated. Also an extension of the compact book storage problem will be shown equivalent to a linear programming problem.

2. A NETWORK MODEL FOR COMPACT BOOK STORAGE

Leimkuhler and Cox [7] presented an unconstrained compact book storage model for selecting the shelf heights and lengths which minimizes the presented shelf area for a given collection of books. They assumed that the dimensions of all books in a particular library are known, including the total length of shelving required for the collection. If the books are stored upright using only one shelf height (corresponding to the tallest book) for the whole collection, then the total shelf area presented is the product of the total length and the height of the tallest book. Instead, if the collection is divided by height into two or more groups, then the presented shelf area in each group will be the product of the length and the tallest book in that group; and the total shelf area will be less than that of the undivided collection. The Leimkuhler-Cox model then considered the problem of finding the particular set of shelf heights which minimizes the total presented shelf area for a given number of divisions of the collection and used dynamic programming for its solution. But the Leimkuhler-Cox model ignored the cost of dividing the collection and constructing stacks of different heights. Hence for their model the process of sub-dividing can be carried out to its limit and there will be finally as many shelf heights as there are book heights and the associated shelf area will be the minimum obtainable for the given height distribution. This simple solution may not be optimal if a fixed cost is associated with each division of the collection. Then it becomes a nontrivial problem to determine the optimum number of divisions for the collection and the set of shelf heights which will minimize the total presented shelf area. It should be remarked here that minimizing the total presented shelf area is equivalent to minimizing the wasted space between the book-tops and the shelf above.

Suppose the heights and thickness of all books in a collection are given. (Since only a two dimensional problem is considered, the widths of the books are ignored). Let the book heights be arranged in ascending order of its n known heights $H_1, H_2, \ldots, H_n$:

$$H_1 < H_2 < \ldots < H_n$$

Since the thickness of the books is known, the required length of shelving for each height class i can be computed and is denoted by L_i. For each shelf height H_i, associate a fixed cost K_i, independent of the shelf area, and a variable cost C_i per unit area. For example, let the collection be placed in two different shelves of heights H_m and H_n ($H_m < H_n$) (i.e. books of height H_m or less are placed in shelf H_m and the rest in shelf H_n). Then the cost of shelving the collection will be:

$$[K_m + C_m H_m \sum_{i=1}^{m} L_i] + [K_n + C_n H_n \sum_{i=m+1}^{n} L_i]$$

The problem is to determine the number of shelves and their respective heights which will minimize the total shelving cost. It can be easily seen that the optimal shelf heights will be chosen from the set of all known book heights in the collection.

We will show here that the compact book storage problem can be formulated as a network flow problem. Consider a directed network of $(n + 1)$ nodes $(0,1,2,\ldots,n)$ where the nodes correspond to the various book heights in the collection. (Here, node 0 corresponds to height zero and node n to the height of the tallest book). A distance function on the set of paths connecting node 0 to node n will be proposed in terms of the shelving cost, where each intermediate node in the path is a possible partition of the set of all shelf heights. To make the network model compatible with the storage problem, the following assumptions are made:

A1: $0 = H_0 < H_1 < H_2 < \ldots . < H_n$

A2: From every node i, there exists a connecting (directed) arc to node j, only if $j > i$. This corresponds to the situation in the book storage problem where having chosen a shelf of height H_i, the next shelf must be of height greater than H_i.

Because of this assumption, the number of arcs in the network is $n(n + 1)/2$.

A3: The distance function between node i and node j is of the form:

$$d_{ij} = K_j + C_j H_j \sum_{k=i+1}^{j} L_k \qquad \text{for } j > i$$

$$= +\infty \qquad \text{for } j \le i$$

The distance function between i and j represents the fixed partition cost K_j plus the cost of shelving books of height H_j or less (but greater than H_i) in the shelf of height H_j. (A value of $+\infty$ indicates that there exists no connecting arc between those two nodes).

It may be seen clearly that as a consequence of the assumptions A1, A2 and A3, finding a shortest path between the source node 0 and sink node n in the above network is equivalent to determining the number of different shelves and their respective heights which minimizes the shelving cost for a given collection. For example, a minimum path solution of the form (0,5,10,n) means, to go from node 0 to node n, the shortest route is to use the intermediate nodes 5 and 10. This says then; store all the books of height H_5 or less on the shelf of height H_5, books of height H_{10} or less (but over H_5) on the shelf of height H_{10} and the rest on the shelf of height H_n. The shortest distance between 0 and n in the network is the sum of $d_{0,5}$, $d_{5,10}$ and $d_{10,n}$. From assumption A3, it is equivalent to the total cost of shelving the collection using three shelves of height H_5, H_{10} and H_n.

Number of algorithms are available for finding the shortest path between two nodes in a network. An appraisal of the well known shortest path algorithms is given by Dreyfus [2]. Hitchner [6] discusses a comparative investigation of the computational efficiency of the shortest path algorighms. He concludes that for networks with 25% or more density (the ratio of existing arcs to the maximum possible arcs is termed as density), the algorithm by Dijkstra [1] is the most efficient one. In the network model of compact book storage, the number of existing arcs is $n(n + 1)/2$, while the maximum possible is n(n + 1). Hence the density of the network is 50% and Dijkstra's algorithm will be computationally more efficient to find the shortest path in this network. A brief description of Dijkstra's algorithm through an example is presented below. For further discussion on Dijkstra's algorithm, refer to Dijkstra [1] or Dreyfus [2].

3. ILLUSTRATION

Consider an example of a book storage problem with four different book heights: H_1 = 5", H_2 = 7", H_3 = 9", H_4 = 12". An equivalent network has five nodes 0,1,2,3,4 corresponding to each book height. (node 0 corresponds to height zero). We can calculate the direct distances (d_{ij}) between each pair of nodes (i,j) in terms of the shelving cost as explained in Section 2. Let the distance matrix be given by:

$[d_{ij}]$ =

nodes	0	1	2	3	4
0	∞	3	4	8	10
1	∞	∞	3	4	8
2	∞	∞	∞	2	4
3	∞	∞	∞	∞	3
4	∞	∞	∞	∞	∞

From the table we see that if four different shelves of heights 5", 7", 9", 12" were used then the total cost of storage is $(d_{01} + d_{12} + d_{23} + d_{34}) = 11$; while using only 12" high shelves, the cost of storage is $d_{04} = 10$. Thus, to find the optimum shelf-heights which minimizes the total storage cost, its sufficient to determine the shortest path between node 0 and node 4 using Dijkstra's algorithm.

Dijkstra's algorithm requires the elements of the distance matrix $[d_{ij}]$ to be nonnegative. It proceeds by assigning tentative node labels which are upper bounds on the shortest distace from node 0 to all nodes. When a node is labelled permanent, it represents the shortest distance from node 0. The algorithm terminates when node 4 is permanently labelled.

Initially, node 0 is labelled permanently as zero and all other nodes are given a tentative label equal to their direct distance from node 0. Thus, the node labels at step 0 are:

```
L(0) = [0,3,4,8,10]
        *
```

(An asterisk indicates a permanent label)

At step 1, the smallest of the tentative node labels is made permanent. Thus, node 1 gets a permanent label equal to 3 and it is the shortest distance from node 0. Hence, we get

```
L(1) = [0,3,4,8,10]
        * *
```

For each of the remaining node k(k=2,3,4), compute a number which is the sum of the permanent label of node 1 and the direct distance from node 1 to node k. Compare this number with the tentative label of node k. The smaller of the two values becomes the new tentative label for node k. For example, the new tentative label for node 2 = Minimum{4,3+3} = 4. Similarly, for nodes 3 and 4, the new tentative labels are 7 and 10 respectively. Again, the minimum of the new tentative labels is made permanent. Thus, at step 2:

```
L(2) = [0,3,4,7,10]
        * * *
```

Now, using the permanent label of node 2, the tentative labels of nodes 3 and 4 are modified as 6 and 8. The smaller of the two tentative labels is made permanent. Thus, at step 3:

$$L(3) = [0,\overset{}{\underset{*}{3}},\underset{*}{4},\underset{*}{6},\underset{*}{8}]$$

It should be emphasized here that at each step, only the node which has been recently labelled permanent is used in further calculations. Thus, using node 3, the new tentative label for node 4 remains unchanged and is made permanent. The algorithm now terminates and the shortest distance from node 0 to every other node is [0,3,4,6,8]. To compute the sequence of nodes in the shortest path, determine which nodes have permanent node labels that differ by exactly the length of the connecting arc. Then, by retracing the path backwards from node 4 to node 0, the minimal path solution may be reconstructed as (0,2,4). This means, books of height 5" and 7" will be placed in shelves of height 7", and the rest in shelves of height 12" so as to minimize the total storage cost.

4. OTHER APPLICATIONS OF BOOK STORAGE MODEL

A classic application of our basic model is an assortment problem in structural steel beams considered by Sadowski [8] and later extended by Frank [4]. Here one is given a set of amounts demanded $d_i > 0$ for $i=1,\ldots,n$ of structural steel of a given cross section but varying in lengths $L_i > 0$ for $i=1,\ldots,n$. $(L_i + 1 > L_i)$. If an order for a length L_i which is not carried in inventory is received, then a beam of longer length is cut down to the desired length which results in unusable steel as scrap. It may not be economical to carry all the demanded lengths in inventory because of the cost of setting up facilities to handle each length in inventory. Thus, the problem is to determine what are the optimal lengths to carry in inventory which minimizes the total steel lost as scrap. It can be easily seen that our basic model may be used to solve this problem. Similarly, the basic model may be applied to a variety of packaging problems where items of various sizes have to be stored efficiently in boxes.

The present model may also be applied to inventory problems where an optimal sequence of inventory levels is to be determined so as to satisfy a known demand over a finite number of planning periods. If the supply is instantaneous, setup costs vary with planning periods, carrying costs are proportional to time and no shortages are allowed, then the basic book storage model can be used to find the optimal sequence of setups to minimize the total cost.

5. AN EXTENSION OF THE BASIC MODEL

The basic model developed in sections 2 and 3, can be used to determine the optimal shelf heights to use so as to minimize the total storage cost. This in turn gives the length of the shelf required in various heights to permit storage of all books in a collection. But in practice, shelves of any arbitrary length are not available since they come only in standard sizes. Also, more and more, libraries use adjustable shelves where the shelf heights can be varied as needed. For example, in a shelf unit of 72" high, it is possible to get seven 10" high shelves, wasting 2" of the shelf space or any other combination of optimal shelf heights. By dividing the total length of space required for each shelf height by the standard length of the shelf, the number of shelf spaces for each shelf height can be computed. Then the problem is to optimally adjust each shelving unit to various shelf heights so as to create enough shelf spaces for the collection and minimize the wasted shelf space in all units. This can be shown to be equivalent to a linear programming problem. Eiseman [3] discusses this formulation in the context of a paper trim problem and Gilmore and Gomory [5] have given an extensive mathematical treatment of such problems.

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