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ABSTRACT

This booklet contains some 80 problems in mathematics suitable for computer solution. The problems range from very simple to complex. Problems have been included from most mathematic disciplines, including number theory, analytical geometry, probability and statistics, trigonometry, equations, and sequence and limits. For the most part these problems are suitable for solution using an interactive computer language such as BASIC, FOCAL, or APL. A batch language may be used for their solution, but because many problems encourage the student to explore a concept beyond one specific solution, an interactive language is more desirable. Many of the fundamental concepts which these problems explore are introduced in a very simple way (with solutions) in the companion volume (EM 009 064). (Author/JK)

DIGITAL EQUIPMENT CORPORATION

education

Problems for Computer Mathematics

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INTRODUCTION

This booklet contains some 80 problems in mathematics suitable for computer solution. The problems range from very simple to rather complex. Problems have been included from most mathematical disciplines.

For the most part these problems are suitable for solution using an interactive computer language such as BASIC, FOCAL, or APL. A batch language may be used for their solution. However, many problems encourage the student to explore a concept beyond one specific solution, hence an interactive language is highly desirable.

Many of the fundamental concepts which these problems explore are introduced in a very simple way (with solutions) in the companion volume "BASIC Applications Programs."

For additional problems in computer mathematics, the following books are recommended:

Danver, Jean H.
SUGGESTIONS FOR PROGRAMS
Kiewit Computation Center
Dartmouth College
Hanover, New Hampshire 03755

Gruenberger, F. J., and Jaffray, G.
PROBLEMS FOR COMPUTER SOLUTION
John Wiley and Sons, Inc.
605 Third Avenue
New York, New York 10016

Order additional copies from

Digital Equipment Corporation
Direct Mail Department
Maynard, Massachusetts 01754

Price: $1.25 per copy.

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GETTING STARTED

A. Write a program to print out your name.

B. Write a program to compute the sum of the first 10 integers. Modify your program to sum the first 10 even integers. Odd integers.

C. Write a program to compute the first ten integers, their squares and cubes. Label the columns.

D. Read a list of numbers and print out every other number.

E. Add up the squares of odd numbers from 101 to 201.

F. For the numbers .5 through 5.0 in steps of .5 inclusive, write a program that will center a three-column table on the paper with the first column containing the number, the second column containing the 5th power of the number, and the third column containing the 5th root. Label the columns.

G. Write a program to compute absolute value of any input number n (without using the ABS(x) function in BASIC or FABS(x) function in FOCAL).

H. Determine if any number n is evenly divisible by any other number d.

I. Write a program to determine the common factors of any two numbers a and b.

J. Change any fraction n/d to a decimal value.

K. Write a program to compute N^E where N = any number and E = any integer. Don't use the operator ↑. Can you do the program without using * either?

WEIGHTED AVERAGE

As a testing procedure for each marking period, 4 quizzes and 4 examinations are given. The average grade is to be an average of the 4 examinations with the 4 quizzes counting as a 5th exam score.

Write a program which will type out the average of a set of scores. Use an INPUT statement to read in first the four quiz scores followed by the 4 examination scores.

CREDIT PURCHASE

Liz Lark finds a $10 skirt that she simply must have. There is a slight problem--she hasn't got ten dollars. She does have credit at the store, however, and all the ads say "easy credit." Why not? The store charges Liz just $1 down which she must pay when she purchases the skirt. The remaining money will be paid at the rate of $1 per week for ten weeks. Compute the simple annual interest rate.

PROFIT EQUATION

A hockey league is starting to be in direct competition with the Eastern Hockey League. You purchase a franchise in the newly formed association, and you are told that your profit (in $1000. units) can be projected for the next 8 years by the formula

$$p=t^3-5t^2+10t-51$$

(p represents your profit, t the time in years.)

At t=0, the time of the purchase of the franchise, p=-51. Your cost of the franchise, therefore, is $51,000., a negative profit indicating a loss.

A. At the close of which year do you show a positive profit?

B. What is your total profit, or loss, for the cumulative 8 years?

INTEREST

$1000. was deposited with a bank that pays 5-1/4% interest, compounded quarterly. This deposit was made on January, 1960.

On January 1, 1961, an additional deposit was made in the amount of $500. This was repeated on January first of each year until the final deposit was made on January 1, 1969.

1. How much is in the account on January 1, 1970.

2. How much of this amount is interest?

CONVERSIONS

A. Write a program to convert linear measures in the metric system (meters and centimeters only) to equivalent measures in the English system (feet and inches only). INT(x) or FITR(x) may be useful here.

B. Write a program which will convert monetary measures in dollars and cents to measures in pounds, shillings, and pence. There are 12 pennies to the shilling and 20 shillings to the pound. The exchange rate was, before England went decimal, about $2.40 per pound.

PAYROLL

Write a program which computes a weekly payroll for a firm which employs 8 people.

1. Compute regular wages on a 40 hour basis.

2. Overtime is determined at time and a half.

3. Include standard deductions of 13% Federal Income Tax, 4% State Income Tax, and 4-3/4% FICA (Social Security).

4. Your output should include the employee number and should have the following format:

EMP No.	HOURS	RATE	GROSS PAY	FED TAX	ST TAX	FICA	NET PAY
1	50	$5.20					
2	40	4.60					
3	35	6.60					
4	40	3.50					
5	42	4.00					
6	48	3.00					
7	40	7.80					
8	34	4.25					

LIMITS

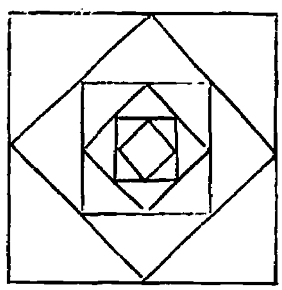
A. Find each sum:

1. $1 + 3 + 5$
2. $1 + 3 + 5 + 7$
3. $1 + 3 + 5 + 7 + 9$
4. $1 + 3 + 5 + 7 + 9 + 11$
5. $1 + 3 + 5 + 7 + 9 + 11 + 13$
6. $1 + 3 + 5 + 7 + \ldots + 289 + 291$
7. $1 + 3 + 5 + 7 + \ldots + R$

B. Find the value of

$$\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \cdots}}}}}$$

C. The figure at the right contains a succession of squares formed by connecting midpoints of the next larger square. Square #1 is the largest. One side of #3 measures 4". Write a program to find the area of #9. Number n?



D. Find the sum to infinity.

$$\frac{1}{7} + \frac{2}{7^2} + \frac{1}{7^3} + \frac{2}{7^4} + \frac{1}{7^5} + \ldots$$

E. Find K if

$$K = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \frac{1}{5\cdot 6} + \cdots + \frac{1}{(n-1)\cdot n} + \frac{1}{n\cdot(n+1)}$$

WAGES

Mr. Jones is offered employment by Shaft Company, Inc., and is afforded the opportunity of taking 2 different methods of payment. He can receive a monthly wage of $500 and a $5. raise each month, or he can receive a monthly wage of $500 with a yearly raise of $80.

Write a program which will determine the monthly wages for the next 8 years in each case. Determine the cumulative wages after each month, and from the information determine which is the better method of payment.

FAMINE

City A has 1,000 residents and is agriculturally self-sufficient (i.e., it cultivates enough food to feed itself). In fact, it produces enough food for 100,000 residents. However, every 10 years the population doubles and in that time enough food can be produced to feed 4,000 more people than in the previous 10 years.

A. Output a table of data in the following manner:

After Years	Population	Food Supply for
0	1,000	100,000
10	2,000	104,000
.	.	.
.		
.		

Have your data stop when the population outgrows the food supply.

B. Modify your program so that it is adaptable to any city with population N, food supply enough for S people, population increasing at rate R every Y years, and food supply increasing for additional I people every Y years.

BOUNCING BALL

A small ball is dropped from a height of 8 feet. It bounces back each time to a height one-half of the height of the last bounce. Write a program to determine approximately how far the ball will have traveled when it comes to rest.

Modify your program for a ball that rebounds to two-thirds of its previous height on each bounce.

LIMIT OF FUNCTIONS

Consider the function $F(x) = \frac{Q(x)}{x-1}$

(Each student will be given a different Q(x).)

F(1) does not exist but the function does exist for all other real values of x.

Write a program to evaluate F(x) for values of x closer and closer to 1. Take the open interval (0,2). Take values in increments of .05. What value does F(x) seem to approach as x gets closer and closer to 1? (Your output should be to six decimal places and when x is within .05 of 1, the increments should be .01.)

Here are some possible Q(x)'s for you to try:

1. $Q(x) = 1$
2. $Q(x) = -1$
3. $Q(x) = x$
4. $Q(x) = -x$
5. $Q(x) = x^2-1$
6. $Q(x) = x^2 - 2x + 1$

FIBONACCI SEQUENCE

A famous mathematical problem from medieval Italy describes the population growth of a hypothetical group of rabbits. A mixed pair of rabbits can produce a mixed pair each month, but only starting with their maturation during their second month of life. (Assume that any arbitrary pair of rabbits sired will be mixed.)

In the first month, therefore, there is one pair of rabbits. In the second month there is still one pair, but in the third month there are two. The following illustrates their population growth during the first six months:

Month	Pairs of Rabbits	No. of Pairs
1	P_1	1
2	P_1	1
3	P_1 P_2	2
4	P_1 P_3 P_2	3
5	P_1 P_4 P_3 P_2 P_5	5
6	P_1 P_6 P_4 P_7 P_3 P_2 P_8 P_5	8

This rabbit problem is attributed to the medieval Italian mathematician Leonardo of Pisa, or as he is commonly known, Fibonacci. Hence, the sequence 1,1,2,3,5,8,... is called the Fibonacci Sequence.

A. Write a program which will type out the first n terms of this sequence.

B. Consider the ratio of any two consecutive terms: i.e., $\frac{A_{n-1}}{A_n}$. Try 10 different cases; $A_5|A_6$, $A_{10}|A_{11}$, ..., $A_{50}|A_{51}$.

   What do you notice about the ratio?

C. Begin a Fibonacci-like Sequence using any two real numbers to start your generation of the sequence. (e.g.,7,3,10,13,111) What do you notice?

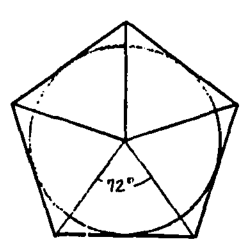
D. The "Golden Rectangle" is considered to be the rectangle which has the most esthetic eye-appeal. Its proportion has been given to conform to the following ratios:

$$\frac{\text{width}}{\text{length}} = \frac{\text{length}}{\text{width+length}}$$

Consider a rectangle whose width is one unit. Solve the resulting equation to 5 decimal places. What do you notice about the positive root and your conjectures in parts b and c?

POLYGON

Design a program to compute and print the area A and length of perimeter P of a polygon with n sides circumscribed about a circle of radius r. Input values for r and n and output the values of A and P. For example, if n=5, we have the following figure:



Modify your program to output A and P for a polygon <u>inscribed</u> within the circle.

APPROXIMATING π

Consider a regular hexagon, whose side measures 1 unit, inscribed in Circle O. The perimeter of the hexagon is 6. (The radius of an inscribed hexagon = the measure of the side.)

The area of the circle subtended by each chord are bisected and the points of bisection are joined in consecutive order with the existing vertices of the hexagon with line segments, so as to form a regular dodecagon. (12 sided polygon.) The perimeter of the dodecagon is then taken.

If we continue this bisecting process, we will produce n-gons of 24, 48, 96,...sides, and the perimeters of the n-gons will begin to approach the circumference of the circle. (See diagram)

Write a program which will determine the perimeters of n-gons of 1536 sides, (up to 1536), and output the result of each computation to 6 decimal places, thereby approximating the circumference of the circle.

Since the measure of the radius = 1, the measure of the diameter = 2, and c/d yields what result?

(As in the diagram, the first perimeter, $P_1=6*S_1$

$$\text{Apothem}_1 = \sqrt{1-(.5S_1)^2}$$

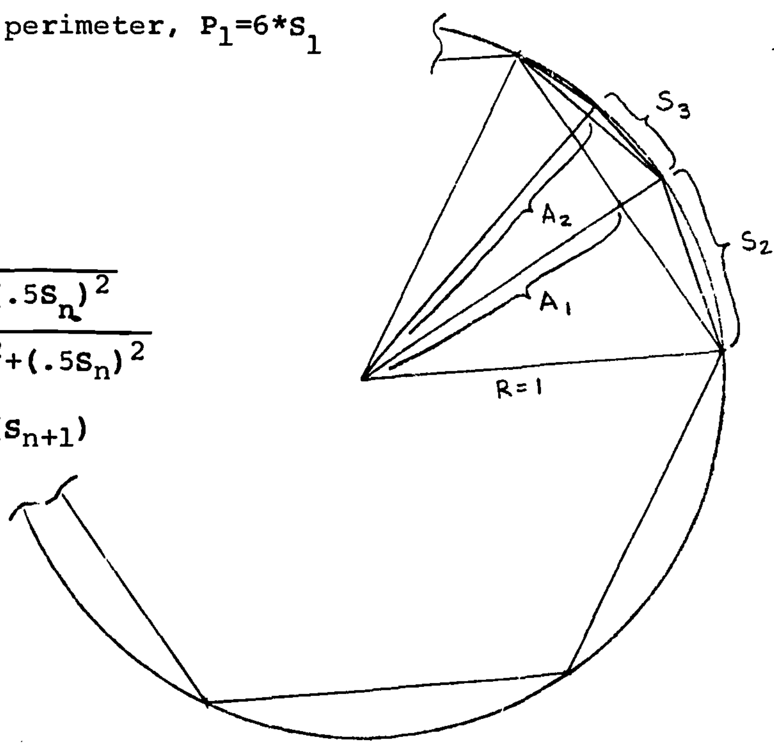
$$S_2 = \sqrt{(1-A_1)^2+(.5S_1)^2}$$

$$P_2 = 2*6*S_2$$

In general, $A_n = \sqrt{1-(.5S_n)^2}$

$$S_{n+1} = \sqrt{(1-A_n)^2+(.5S_n)^2}$$

$$P_{n+1} = (n)(6)(S_{n+1})$$



PRIME NUMBERS

(The following program deals only with the set of counting numbers.)

A PRIME NUMBER is a counting number that has exactly two distinct factors. All other numbers, except the number 1, are called composites. (The number 1 is neither prime nor composite.)

A. Write a program which will determine whether a number is or is not prime.

B. Mathematicians have always searched for some formula which will generate prime numbers. One 'simple' attempt was the formula, $f(n)=n^2-n+41$, for $n=1,2,3,\ldots$. Starting with n=1,

$$f(1)=41, \text{ a prime}$$
$$f(2)=43, \text{ a prime.}$$

However, the formula 'quickly' breaks down.

Write a program which will determine the value of n for which the formula first breaks down. Your output could look something like this:

n	$f(n)=n^2-n+41$	Prime or Composite
1	41	P
2	43	P
3	47	P
•	•	•
•	•	•
•	•	•
		C

C. Write a program which will determine the prime factors of a counting number.

PERFECT, ABUNDANT, & DEFICIENT NUMBERS

A positive integer n is a perfect number if the sum of its positive divisors is 2n. E.G., 28 is a perfect number since its positive integral divisors are 1,2,4,7,14,28, and 1+2+4+7+14+28=2(28).

The first perfect number is 6; i.e., 1+2+3+6=12.

If a positive integer is not perfect it is either abundant or deficient. It is abundant if the sum of its divisors is greater than twice itself, deficient if the sum is less than twice itself.

A. Write a program which will type out all the positive integral divisors of a positive integer.

B. For each integer 1,2,3,...,99, determine if it is abundant, deficient, or perfect.

C. Determine the first four perfect numbers.

PYTHAGOREAN TRIPLES

The Pythagorean Theorem, $a^2+b^2=c^2$, a & b being the legs of a right triangle and c being its hypotenuse, occurs often enough in mathematics so that it holds a certain fascination. One such interest is displayed in the question, "How many such ordered triples are there such that $a^2+b^2=c^2$, where a, b, and c are integers?".

The question is easily answered by knowing that (3,4,5) fulfills the requirements, and with the assistance of similar triangles, so does any integral multiple (e.g., (6,8,10)) of (3,4,5).

However, the response is more limiting if we consider the possibility that a, b, & c have no factors in common. (If any set of integers lack common integral factors, the numbers are called relatively prime.) If (a,b,c) satisfy $a^2+b^2=c2$, and a, b, & c are relatively prime integers, then the ordered triple (a,b,c) is called a PYTHAGOREAN TRIPLE.

A. Write a program which will determine the relative primeness of three numbers.

B. Pythagoras showed that for any odd integer m, if a=m, $b = \frac{m^2-1}{2}$, and $c = \frac{m^2+1}{2}$, the theorem is satisfied.

   Generate a table for ordered triples using m=3,5,7,...,19.

C. Plato showed that if a=2m, $b=m^2-1$, and $c=m^2+1$, it will produce values satisfying the Pythagorean Theorem and m is not restricted to being odd. However, it does not produce only relatively price triples. For example:

m	2m	m^2-1	m^2+1
2	4	3	5
3	6	8	10
4	8	15	17

   Generate a table for ordered triples using m=2,3,4,...,50, eliminating those sets of numbers which are not relatively prime.

TRIANGLE INEQUALITY

A basic geometry theorem deals with the possible measures of the 3 sides of a Δ. The theorem states that the sum of the measures of the sides of a Δ must be so arranged that the sum of the measures of any two sides is greater than the measure of the third.

A. Write a program which will determine if any three numbers can be the measures of the sides of a triangle.

B. Write a program which will generate the possible measure of a side of a triangle in increments of .01, if the measures of the other two sides are 1.15 and 1.37.

Watch out for round off error.

EQUATION OF A LINE

A. Defining a straight line using the linear equation $y = mx + b$, write a program which will determine the coefficients of the equation given the slope m and any point on the line (x,y).

B. Write a program which will determine the coefficients of the equation given any two points on the line (x_1, y_1, and x_2, y_2).

LIGHTHOUSE DISTANCE

A lighthouse is located at coordinates (7.64, 12.12). A boat initially located at (2.00, 0.35) is moving in a linear direction which will take it past the lighthouse. After one minute, the location of the boat is (3.37, 1.87).

A. Determine the coordinates (x,y) on the boat's path when the distance from the boat to the lighthouse is a minimum (to 2 decimal places).

B. Determine at what integral unit of time the boat will be at the minimum distance from the lighthouse (i.e., t must be an integer).

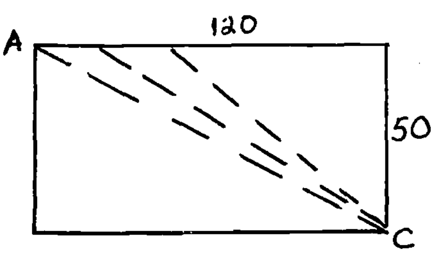
Draw a diagram before you start to help you visualize the problem.

CENTER OF CIRCLE

Given that a circle passes through (2.1, -.3), (.1, .5), and (1.02,-.03), find the coordinates of the center and the measure of the radius. Output your result to three decimal places.

POOL PROBLEM

A rectangular pool has dimensions 120 ft, by 50 ft.



A person standing at vertex A wishes to go to vertex C in the least possible time. He has the option of running the entire distance around (170 ft.), swimming diagonally across (130 ft.), or combining some running with some swimming. He determines that the fastest route is the combination.

His running rate is 3 ft./sec. and his swimming rate is 2 ft./sec.

(As an illustration, if he decides to run only 21 feet (at 3 ft./sec = 7 secs.) then his swimming distance $=\sqrt{99^2+50^2} = \sqrt{12301} \approx 110.91$, (at 2 ft./sec.=55.46 sec.) yielding a total time of 62.46 secs.

Write a program which determines the running distance and the swimming distance which determines the least required time to go from A to C. Output the running distance, the swimming distance, and the minimum time. Let your distances be incremented every .1 feet.

FACTORIAL TABLE

A set of numbers which appears often and significantly in probability and analysis is N FACTORIAL (N!).

N! is defined as $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdots (N-1) \cdot N$

$1! = 1$

$2! = 1 \cdot 2 = 2$

$3! = 1 \cdot 2 \cdot 3 = 6$

$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$

Write a program which will output a table of factorials up to and including any variable N.

N	N!
1	1
2	2
3	6

(Hint: Since $N! = 1 \cdot 2 \cdot 3 \cdots (N-1) \cdot N$

$N! = (N-1)!(N)$

PASCAL'S TRIANGLE

Consider the expansion of $(a+b)^n$ for n=0,1,2,3,4, and the coefficients obtained for each term of this expansion.

$(a+b)^0$	1	1
$(a+b)^1$	$a + b$	1 1
$(a+b)^2$	$a^2 + 2ab + b^2$	1 2 1
$(a+b)^3$	$a^3 + 3a^2b + 3ab^2 + b^3$	1 3 3 1
$(a+b)^4$	$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$	1 4 6 4 1

The triangular arrangement of these coefficients is referred to as Pascal's Triangle, after the celebrated 17th Century French mathematician, Blaise Pascal.

A. Determine the pattern involved in generating these coefficients and write a program which will output this triangle for the first 12 lines (n=0 through n=11).

B. Modify the program to output only the nth line of the triangle for any integral n.

C. How many even numbers are there in the 21st line? In any given line n?

D. Find the sum of all the numbers in the 15th line. In any given line n?

E. Find the sum of all the numbers up to and including the 12th line. Up to the nth line. Can you find a fairly simple formula which gives this sum?

F. (Not for computer solution.) The Fibonacci numbers are hidden in the triangle. Can you find them? (1, 1, 2, 3, 5, 8, 13, 21, 24,...)

GROUPING DATA - MEAN, MEDIAN

The following scores were recorded on the Math 11 Regents Exam:

78, 83, 61, 100, 91, 51, 93, 87, 82, 79, 68, 67, 72, 75
74, 97, 98, 85, 83, 81, 82, 79, 67, 100, 53, 92, 86

Write a program which will:

A. Group the scores into intervals and output this grouping in the following manner:

INTERVAL	NO. OF SCORES	% OF SCORES IN INTERVAL
100		
90-99		
80-89		
70-79		
65-69		
Failing		

B. Find the mean score.

C. Output the scores in descending order and output the median.

PROBABILITY PROBLEMS

A. What is the probability that a non-leap year will have 53 Fridays?

B. What is the probability of any one of the digits appearing five consecutive times in a table of 2000 random digits? In a table of n random digits?

C. Sam has 16 red socks and 16 blue socks in a drawer. He picks out two socks at random. What is the probability that he will get a matching pair? How about for three socks? Four? Five? ...

D. Six men are to be selected from among twelve applicants for positions. In how many different ways can this selection be made? Generalize your program to select m men from n applicants.

E. Kamo and Sert are both 50% marksman with the blowgun. They fight a duel where they exchange alternate shots. If Sert shoots first, what is the probability that he will win?

F. How many possible batting orders are there for a baseball team of 9 players?

TRIG TABLE

BASIC includes subroutines to evaluate functions which are used a great deal in mathematical computations. Two such functions are sine(x) and cosine(x), where x is a real number. These stored functions are called library functions.

Since Tangent(x)=sin(x)/cos(x), and sec(x), csc(x), and cotan(x) are reciprocals of cos(x), sin(x), and tan(x) respectively, the 6 standard trigonometric functions can be evaluated.

A. Write a program which will output in column form, the sin, cos, and tan of x, where x is in degree measure. Input your starting angle a, the increment i, and the final angle b. Your table should appear as follows:

X	Radian X	Sin(X)	Cos(X)	Tan(X)
a				
a+i				
A+2i				
.				
.				
.				
b-i				
b				

Use Pi≈3.14159 and make allowances for angles whose terminal ray is on the y-axis. For example, 90^{o} = Pi/2, but since Pi is being approximated, tan(90^{o}) will yield a very large number but will not be undefined.

LAW OF SINES AND COSINES

A. The Law of Cosines states that for any triangle ABC, $c^2=a^2+b^2-2ab\text{Cos}(c)$. Given the measures of sides a, b, and c, write a program which will output all the angles of a triangle in degrees.

B. The Law of Sines states that for any triangle ABC,

$\frac{a}{\text{Sin}(a)} = \frac{b}{\text{Sin}(b)} = \frac{c}{\text{Sin}(c)}$. Given the measures of the sides a and b and the measure of angle c, determine the measure of angle b, and using the Law of Cosines, determine the measure of c and angle c.

DIOPHANTINE EQUATIONS

The ancient Greek mathematician Diophantus is considered one of the earliest geniuses in number theory. However, Diophantine solutions to algebraic equations involved only positive rational numbers, and in fact, modern usage restricts solutions to Diophantine Equations to positive integers.

A. Find all positive integral solutions to the equation $17y-3x=558$, such that $x<50$.

B. An actual equation as found in the writing of Diphantus reads,

   Find three numbers such that their sum is a square and the sum of any two of them is a square.

   Write a program to determine three such numbers.

SIMULTANEOUS EQUATIONS

Consider the system of equations,

$$Ax+By+C=0$$
$$Dx+Ey+F=0$$

Write a program which will:

A. Determine if the lines have the same slope, and if they do, whether they are parallel or the same line, and

B. If the lines are independent, determine their point of intersection.

QUADRATIC EQUATION

Write a program to solve the quadratic equation

$$Ax^2 + Bx + C = 0$$

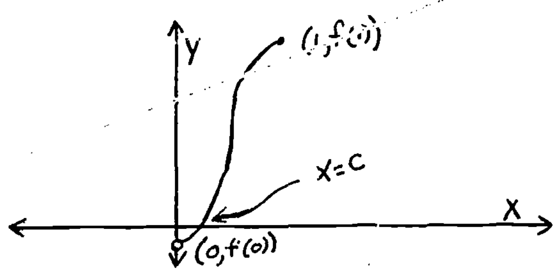
for any input value of A, B and C.

Determine if the roots are real or complex and output the roots.

BISECTION METHOD

Consider the equation $f(x)=6x^3+17x^2+15x-3=0$. Since the coefficients are real, if $f(x)=0$ is to have any imaginary roots the complex roots must be in conjugate pairs. Hence we are assured of at least one real root to the equation, and possibly three since $f(x)=0$ must have exactly three roots.

Two calculations indicate that $f(0)=-3$ and $f(1)=35$, and this information is graphed in the figure below.



If we assume the continuity of the function $y=f(x)$, then the curve must cross the x-axis at least once between 0 and 1, and at one point, say c, $0<c<1$, $f(c)=0$. c is therefore a root of $f(x)=0$.

A method for approximating this root is as follows:

Since $f(0)<0$ and $f(1)>0$, i.e., $f(0)$ and $f(1)$ have opposite signs, there is at least one root between 0 and 1. Make your first guess $c = \frac{0+1}{2}$. Calculate $f(c)$. If this is negative, then the root must be between c and 1; if positive, then between 0 and c. Continue this process of BISECTING each new interval to determine the new c. Evaluate $f(c)$ and when two consecutive $f(c)$ have a difference of less than .0001, output the desired approximation of the root.

Write a program which will find a solution, correct to 3 decimal places of each of the following equations:

1. $x^3+17^2+15x-3=0$

2. $x^3-1=0$

3. $\sin^2 x+\sin x = 1$

TRAPEZOIDAL RULE

Consider the curve $f(x)=x^2+1$, as given in figure 1. Consider the area under the curve, above the x-axis, and between the interval (0,1), as shown in figure 1.

Divide the interval 0 to 1 into n equal parts, $x_1, x_2, \ldots, x_n$. Erect perpendicular lines to the x-axis at the points $x_1, x_2, \ldots, x_n$, and extend these lines to the curve. Connect these consecutive points of intersection with line segments as shown in figure 2. The sum of the areas of the trapezoids thus formed approximates the area under the curve whose bounds are as previously defined.

As the number of equal divisions of the interval 0 to 1 is increased, the sum of the areas of the trapezoids begins to form closer and closer approximateions to the defined area.

The area of each trapezoid = $1/2h(b_1+b_2)$. (See figure 3). Since the intervals are equally divided, each of the heights of the trapezoids = $\frac{1-0}{n} = \frac{1}{n}$.

Suppose the interval is divided into 4 equal parts as in figure 4; i.e., n-4. The height of each trapezoid = $\frac{1-0}{4}$ = .25.

Consider the leftmost trapezoid first. $A_1=1/2(.25)(b_1+b_2)$. Since one endpoint of each base is on the x-axis, the length of each base = $f(x_n)-0 = f(x_n)$. $f(0)=1$, $f(.25)=1.0625$, etc. $A_1=(.5)(.25)(1+1.0625)$ or in general, $A_1=(.5)(\frac{1-0}{n})(f(x_0)+f(x_1))$.

Consider each of the following trapezoids in the following manner:

$A_2 = (.5)(.25)(f(x_1)+f(x_2))$

$A_3 = (.5)(.25)(f(x_2)+f(x_3))$

$A_4 = (.5)(.25)(f(x_3)+f(x_4))$

The area under the curve = $A_1+A_2+A_3+A_4$. In general,

$$\text{Area} = (.5)(\frac{1-0}{n})(f(0)+2f(x_1)+2f(x_2)+2f(x_3)+f(1))$$

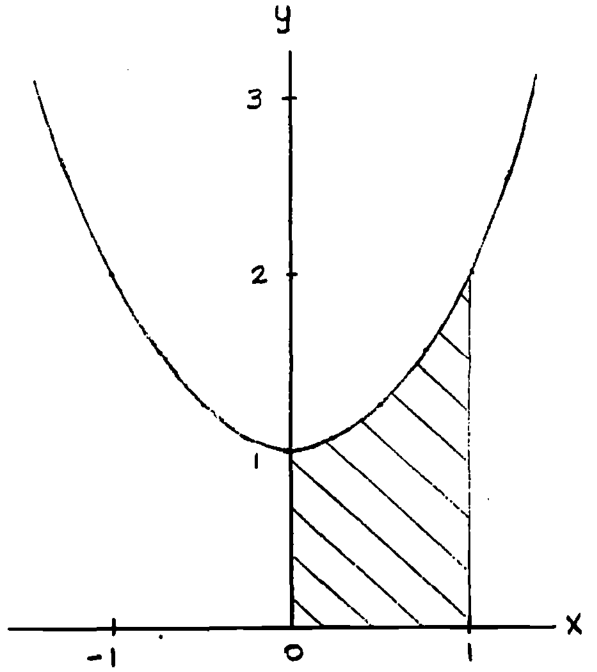


FIGURE 1

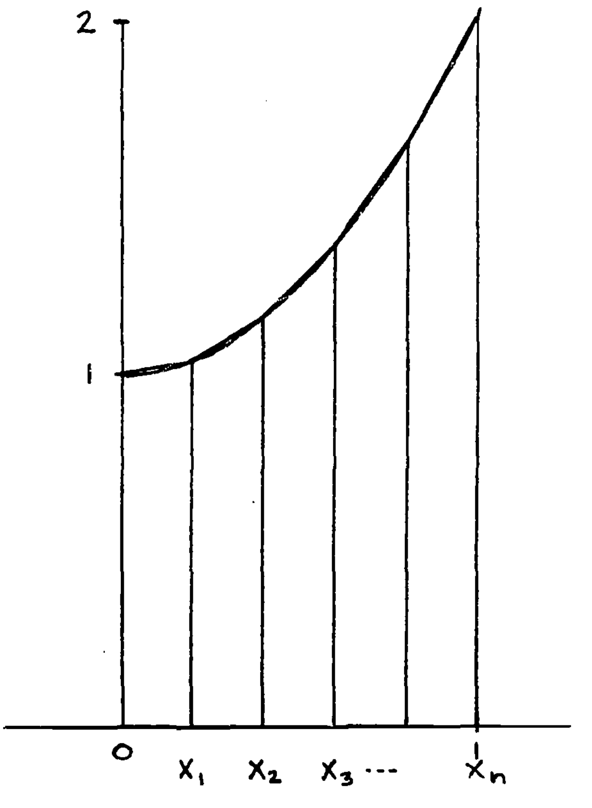


FIGURE 2

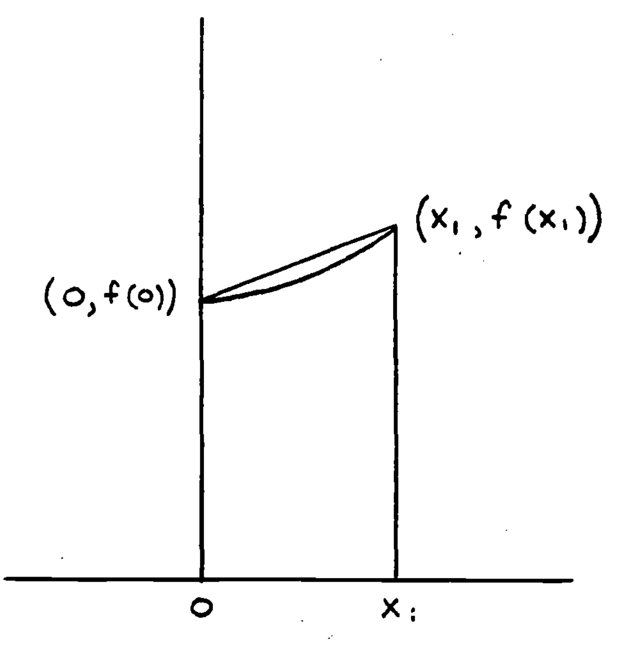


FIGURE 3

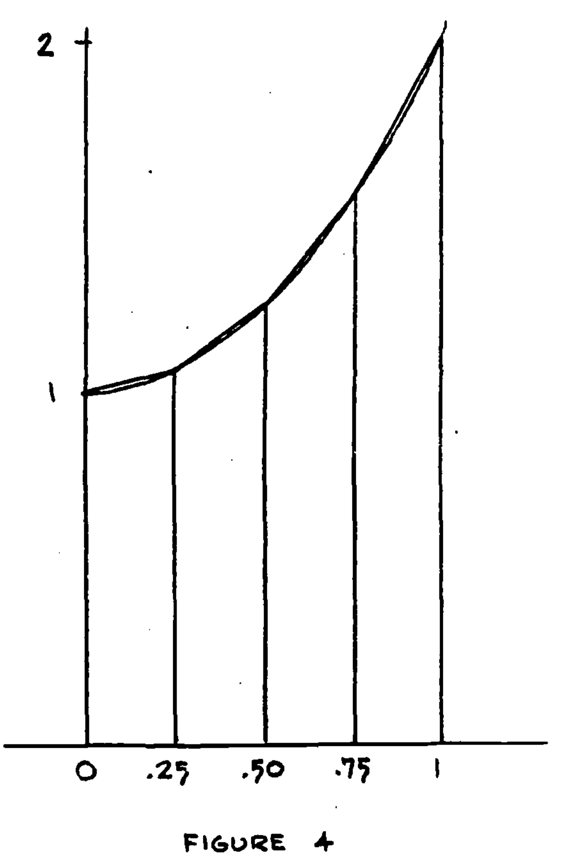


FIGURE 4

Increasing n and setting a=0 and b=1, area under curve over the interval (a,b), equals:

$$A=(.5)\left(\frac{b-a}{n}\right)(f(a)+f(b)+2f(x_1)+2f(x_2)+\cdots+2f(x_n))$$

A. Write a program to determine the area under the curve $f(x)=x^2+1$ over the interval (0,1) for n=4.

B. Modify the program using n-4, 6, 8, ... , 20, and output your results in the following manner:

N	Area
4	
6	
.	
.	
.	
20	

C. Modify the program to find the area under $f(x)=x^2+1$ for any interval (a,b).

D. Modify your program so that your area approximation is correct to 3 decimal places. Use the following general method:

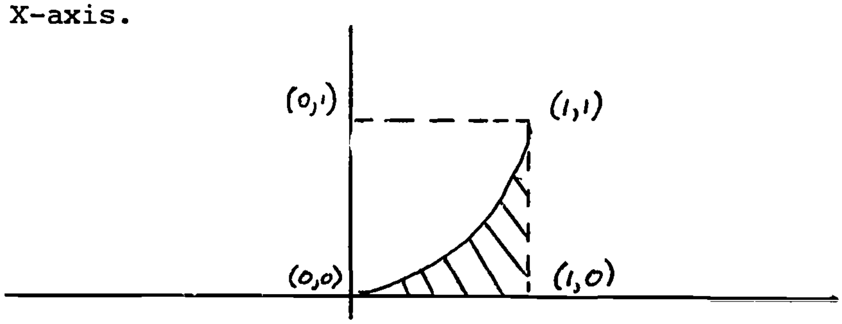
if n=8, $A=A_8$

n=16, $A=A_{16}$.

If $A_{2n}-A_n$ is less than the desired accuracy, then your desired result has been achieved.

AREA UNDER CURVE (MONTE CARLO METHOD)

Consider the area under the curve $f(x)=x^2$, for x between 0 and 1, and bounded by the X-axis.



Consider the square whose vertices are (0,0), (1,0), (1,1), and (0,1). Think of the coordinate plane as a cork board in which you are going to throw an unlimited number of darts, and each dart will stick in one point in the plane. The darts which are to be counted are those darts which land in the interior of the square. E.G., (2,5) and (0,1) would not be counted since they are not in the interior of the square.

If a sufficient number of darts is thrown, the area under the curve is approximated by the ratio of the number of darts under the curve but in the square to the number of darts in the square.

A. Write a program which will generate random numbers R, $0<R<1$; i.e., R is a four digit decimal.

B. Determine the area under the curve for 10, 20, 30, 50, 100 darts.

C. The exact area under a given curve may be obtained by use of integral calculus. In this case, the exact area is 1/3. Plot the accuracy of the Monte Carlo method versus the number of points used in the computer runs in Part B. Note: "accuracy" should be calculated as the percentage difference between the Monte Carlo answer and the "correct" answer.

D. Change your program to find the area under the curve $y = \sin(x)$ for x between 0 and π.

E. Modify your program to find the area under the curve $y = \sqrt{1-x^2}$. This is the equation for a quarter circle of radius 1 with its center at 0,0. $(x^2 + y^2 = r^2)$. Multiply your answer by 4; what do you notice?

MATRIX SUM AND PRODUCT

A. Let X and Y with elements $X_{i,j}$ and $Y_{i,j}$ respectively, be a 3 x 3 matrix. Define Z to be a 3 x 3 matrix such that $Z_{i,j}=X_{i,j}+Y_{i,j}$. Write a program which will input the elements of X and Y and output the elements of Z.

B. Let A and B be two 2 x 2 matrices such that

$$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \quad \text{and define}$$

$$AxB = \begin{pmatrix} a_1b_1+a_2b_3 & a_1b_2+a_2b_4 \\ a_3b_1+a_4b_3 & a_3b_2+a_4b_4 \end{pmatrix}$$

Write a program to input the elements of A and B and output the elements of AxB.

TRUCK AND DESERT

A truck when fully loaded can carry enough fuel to take it half-way across a barren desert. If the truck can return to the starting point as often as is necessary, write a program to determine the minimum amount of fuel required to take it all the way across. Assume that any amount of fuel can be taken from the truck at any point in the desert and cached and that this amount will remain undiminished until subsequently collected.

WORDS, SENTENCES and POEMS

A. Write a program that generates random similes, i.e. (adjective) AS A (noun). For example, SLOW AS A TURTLE.

B. Write a program that, given nouns and verbs, will generate two-word random sentences. Modify the program by adding adjectives and adverbs.

C. Write a program that generates random 4-word sentences. Modify it to generate random length sentences.

D. Write a program that generates different kinds of random poetry (Couplet, Haiku, Sonnet, etc.).

E. Write a program to generate random words out of letters. Try to dream up definitions for the words you like.

F. In English, certain letters appear with much higher frequency than others. This is why the "words" generated in Exercise E look like they have too many of the letters F, K, Q, U, V, X, Y, and Z. Count each of the letters on the page of a book and assign probabilities of occurance to each letter. Modify your program to generate random words using letters with these probabilities. Do they look more like real words? What can you do to make them look even more like English?

PROBLEMS JUST FOR FUN

1. *Paradoxically, certain fractions may be reduced by striking out a common integer in the numerator and denominator. For example:

   $$\frac{16}{64} = \frac{1}{4}, \quad \frac{19}{95} = \frac{1}{5}, \quad \frac{154}{253} = \frac{14}{23}, \quad \frac{682}{781} = \frac{62}{71}$$

   Write a program to find all such fractions up to $\frac{998}{999}$.

2. Write a program to simulate playing a dice game. Tic tac toe.

3. *Write programs to solve the following problems:

SEND	EIGHT	HALF	AB2DEF
+MORE	- FIVE	+HALF	x2
MONEY	FOUR	WHOLE	2DEFAB

4. $1^3 + 5^3 + 3^3 = 153$. Find all the other numbers (under 4 digits) in which this is true, i.e., sum of cubes = number.

5. Using standard American coins, in how many ways can you make change for 50 cents?

6. Jim can do 50% more work than Tom and 25% more work than Pete. Working together the three men need 15 days to lay a foundation. Find the time needed by each man to do the job alone.

7. Write a program to generate random music.

8. *Wishes are horses provided that horses cannot fly. Beggars will not ride, provided that wishes are not horses. If it cannot be the case that both beggars will ride and wishes are non-equine, then horses can fly. If the inability of horses to fly, and the non-riding of beggars, cannot be set up as valid alternatives, then beggars are not always rich. But beggars <u>will</u> ride. Are beggars rich?

*<u>Note</u>: Programs to solve these problems could take hours or even days of computer time. Think! Optimize your approach!

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