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ABSTRACT

This paper presents a proposed analytical method, based on linear programing techniques that can be used by school districts for salary evaluation of professional personnel. Linear programing techniques allow for school district objectives and constraints and provide an internally consistent, effective assessment of an employee's relative worth to the school district in terms of salary. A similar document is EA 003 593. (Author/RA)

SALARY SCHEMES FOR EDUCATIONAL PERSONNEL WHICH REFLECT SCHOOL-DISTRICT PRIORITIES

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SALARY SCHEMES FOR EDUCATIONAL PERSONNEL WHICH REFLECT SCHOOL-DISTRICT PRIORITIES

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ABSTRACT

Developing school district salary schedules involves solving problems which are both analytical and political in nature. In many cases the rigor and logic of the analytical techniques used in solving the problem can also aid in dealing with some of its political aspects. The purpose of this paper is to present an analytical method, which could be used by school districts for salary evaluation of professional personnel.

The method proposed in this study is based upon linear programming techniques. One of the more important benefits derived from utilizing linear programming is its ability to simultaneously consider available school district resources as well as establish priorities and objectives in the determination of a logical and internally consistent salary structure. Reported in this paper is the derivation of the linear programming salary evaluation model and its application to a school district salary structure. In this illustrative application the model yields optimal solutions (salary schedules) which (1) are consistent with both the imposed hierarchical and budgetary constraints of a school district, (2) consider nine factors in the salary evaluation, and (3) maximize the school district's desired criterion of effectiveness, i.e., maximization of the highest teacher salary. This design criterion would reflect a situation in which a school district desired a career-oriented, experienced staff.

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I. INTRODUCTION

Developing effective school district salary schedules involves solving problems of both an analytical and political nature. In many instances the logic and rigor of the analytical methods employed in dealing with the problem can also be used to overcome some of its political aspects. This study will concern itself almost exclusively with the use of an analytical technique for determining school district salary schedules. This study will also attempt to demonstrate, by direct application to a school district salary structure, how this technique could be used to overcome some of the severe political and logical shortcomings of traditionally used methods for school district salary evaluation.

The most widely used method of salary evaluation for school district personnel is based upon the fixed-step salary schedule. The fixed step in the salary schedule is generally taken to represent the increment in salary associated with years of experience in the school system. In general, school district personnel receive higher salaries as (1) their number of years of experience in the school system increase, and (2) the amount of formal training (college units, advanced degrees, etc.) becomes more extensive. Typically, the fixed-step salary schedule only considers these two factors in the evaluation of salary for school district personnel.

The rigidity of this approach for determining salaries has given rise to serious problems which are recently causing widespread concern among educators. In order to demonstrate the ineffectiveness of the fixed-step salary scheme, consider what factors most educators and economists consider essential in any effective salary evaluation scheme for school district personnel.

First and foremost, the salary schedule should be logical and internally consistent, that is, each member of the organization (school district) should receive a fair and adequate salary in relation to every other member of the organization.

The internal consistency of the salary evaluation scheme is essential for morale purposes. The fixed step is neither a logical nor internally consistent scheme. The minimum and maximum salaries are usually arbitrarily determined and, as Benson notes, the intermediate salaries are often calculated by simple rule-of-thumb procedures.(1)

Secondly, many economists seem to feel that the salary schedule for school district personnel should reflect the relative difficulties in the learning or instructional environment. In large metropolitan districts, all school personnel are usually on the same fixed-step salary schedules regardless of the difficulty of the assignment. The disparity in learning situations in the Los Angeles area and the lack of recognition of this disparity in teacher salaries was indicated in an article in *The Los Angeles Times*(10) quoting the economist Werner Hirsch:

> Teachers in Los Angeles are given no salary differential. Not so surprisingly, then, most of them choose to work in places like Westwood or Sherman Oaks. The result has been a serious decline in teacher quality in areas where teacher quality should be the highest. One answer would be incentives for teachers in ghetto schools. This would bring better talent into these classrooms and would work to the advantage of other problems brought on by a uniform salary schedule (e.g., shortages of teachers of English, mathematics and sciences and surpluses of teachers of art, social studies and physical education. A differential wage, tried to supply and demand conditions, could give the L.A. system a better balance of education.

Thirdly, school district resources should at least be considered in determining the final school district salary structure. The fixed-step salary schedule does not and cannot consider school district resources in the evaluation of salary. If an additional $50,000 were available to support salaries in a school district, this fixed-step evaluation method would be incapable of incorporating these additional resources into the school district's salary structure in a logical and consistent manner.

Fourth, overlaps should be permitted between the various salary hierarchies in a school district. It is now a common practice in many districts for the

salaries of school administrators to be linked with the teachers' salary schedule. Frequently, salaries for these administrators are expressed as a ratio to some figure in the classroom teachers' schedule.(7) In other situations a fixed or absolute dollar differential is maintained. As Benson notes, the effect of establishing administrative salaries by a relationship to teachers' pay, whether by using a percentage or a dollar differential, is to preserve the gap between the salaries for these two groups.(1) Most economists and educators would agree that there is no reason why a highly qualified teacher should receive less salary than a low-qualified administrator. Highly qualified and experienced teachers should be permitted to remain in the classroom and not suffer from the economic discrimination of the fixed-step salary schedule. While administrators might be justified in receiving additional salary because of additional workload and responsibility, there is no reason to consider these personnel as separate in the salary schedule. School districts should have one salary schedule for both administrators and teachers, with each salary internally consistent to every other salary.

In short, a major shortcoming of the fixed-step salary schedule is that it does not provide an economic incentive for good teaching. This lack of economic intentive is clearly demonstrated by noting that the earnings for school administrators usually exceed by a wide margin the expected lifetime earnings of the most highly trained and conscientious teachers. For example, in a recent study in Minneapolis it was found that the earnings of a teacher who succeeded to a junior high principalship after 10 years of teaching was $178,622, while the present value of earnings of a teacher (30 semester credits and a masters degree) who remained in the classroom was only $162,000. Relative to the earnings for 40 years of teaching with only a bachelors degree, the extra pay was nearly twice as great for the principal as for the well-trained teacher: $35,394 to $18,794.(3)

Fifth, an effective salary evaluation scheme should be able to incorporate salary differentials into the wage salary scheme in order to consider the economic supply versus demand laws for teachers and administrators with specialized skills. This would place the school district in an advantageous position of being able to compete, economically, for teachers and administrators possessing certain specified skills and knowledge. It is interesting to note that economists McKean and Kershaw attribute teacher shortages in certain areas to the inability of school districts to economically compete for highly trained personnel.(5)

According to one educator, the refusal to adjust pay to market scarcities and surpluses is to blame, at least in part, for the fact that we have an oversupply of intending teachers in the fields of social studies, speech, and physical education and a shortage in English, mathematics, and science.(8)

Sixth, an effective salary evaluation scheme should consider a large set of factors for salary evaluation, besides the two factors currently considered (experience and formal training) by the fixed-step salary schedule. These additional factors might include special awards or distinctions for good teaching or administration, inservice credits, subject matter priority, etc. Recent studies have reported that the characteristics of the teacher, especially the quality of his training, are the most important school variables, statistically speaking, in stimulating pupils to achieve.(2,4) If the findings of these studies can be generalized, then it seems logical to consider these additional characteristics in the evaluation of salary.

Finally, the seventh feature of an effective salary evaluation scheme concerns itself with the reflection of established school district priorities and objectives by means of the salary structure. This important aspect of the salary evaluation scheme insures that school district resources for support of the salary structure are expended in a manner which economically reflects the desired

performance goals of the school district. For example, if a school district desires to attract young, inexperienced teachers, or compete on the job market for recent college graduates, it might desire to pay the highest possible starting salaries. On the other hand, if the school district is desirous of continuity of programs, older, experienced teachers, and low faculty turnover rates, it might desire to pay the highest possible maximum salaries. The school district might also desire to place maximum salary weight to one or more than one factor which make up the salary evaluation. For example, it might wish to give the greatest possible salary weight to school district personnel teaching in difficult learning areas or those possessing special awards or distinctions, etc.

In summary, an "ideal" salary schedule should be able to consider all factors which both teacher groups and school boards consider important in the evaluation of salary and to the attainment of the school district objectives. Most important, the salary evaluation scheme should be internally consistent and, if desired, maintain the organizational salary hierarchy. It should be flexible enough, however, to permit overlaps in salary between the various hierarchies (e.g., it should permit a highly qualified teacher to receive more salary than a low qualified administrator). Finally, an effective salary schedule should be able to consider the financial constraints placed upon school district resources which support the entire salary structure. This latter feature could be extremely important in collective bargaining negotiations between teacher unions and school boards.

To develop a salary schedule which satisfies the above-mentioned specifications is obviously a very complex problem. Multiple regression analysis, common to many salary evaluation schemes in business and industry, is inappropriate for two reasons. First, all the factors to be considered in the salary evaluation cannot be correlated to salary; and secondly, the multiple regression approach is incapable of handling constraint conditions. There are some sophisticated

mathematical techniques of operations research which can aid in developing effective salary schedules. Recently, linear programming has been applied to certain wage salary schemes in business and industry with some success.[6,9]

The purpose of this paper is to derive a linear programming model for salary evaluation, then apply it to a school system to test its validity and effectiveness. The model should contain a set of factors which could be used for salary evaluation as well as a constraint set which establishes the salary hierarchy, permits overlaps in salary, and considers the budgetary amounts available to support the entire salary structure.

It might be helpful at this point to discuss some of the essential characteristics of linear programming for those readers not acquainted with this technique. This will also provide the reader with a means of better assessing the value of utilizing this technique for this particular problem in education.

BASIC FEATURES OF LINEAR PROGRAMMING

A linear programming model consists basically of two parts, a constraint set and an objective function.

In a linear programming model, the purpose of the constraint set is to place upper and lower bounds upon the resources available to the system, limit the consumption of these resources by any element or activity in the system, and finally, control the flow of resources to these activities.

The second and most valuable aspect of a linear programming model is known as its objective function. The purpose of the objective function is to insure that the specified criterion of effectiveness (what is desired from the system such as lowering costs, etc.) is achieved at its maximum or minimum value, consistent with all the constraints imposed on the system. Essentially, the objective function selects that one solution from an infinite set of feasible solutions which maximizes

or minimizes the particular criterion for effectiveness. This one solution is known as an "optimal" solution since it is the best solution consistent with the constraint set.

It should be especially noted that the following demonstration of the application of the model to a school district is only meant as an illustration of one possible model formulation from a large set of possible methods for formulating the model. The reader should, therefore, concern himself with the logical design and rigor of the model rather than the specifics which have arbitrarily been incorporated in the model for demonstrative purposes.

II. DEVELOPMENT OF THE SALARY EVALUATION MODEL

PRELIMINARY CONSIDERATIONS

Before developing an effective salary evaluation scheme for school-district personnel, the various criteria used in the schemes have to be discussed and agreed upon by school administrators, the PTA, the school board, and teacher groups. The essential questions to be answered would have to include the following:

1. What are the basic objectives of the school district (e.g., preparation for college, citizenship, social adjustment, etc.)?

2. What are the primary job functions or job classifications needed by the school district to attain these objectives (e.g., administrators, teachers, teacher aides)?

3. What factors--i.e., training and/or qualifications--are considered necessary for each of the job functions in the district (e.g., education, responsibility, subject-matter training)?

4. What is the hierarchical salary structure in the school district (e.g., will administrators be paid more than teachers, who will be paid more than teacher aides, etc.)?

5. What characteristics, in decreasing order of relative importance, constitute each of the factors in 3 above (e.g., education characteristics would be Ph.D., M.A., B.A.)?

Since the essential characteristic of any job-evaluation scheme is consistency, it should be stressed that all parties concerned with teacher salaries (school board, teacher groups, PTA, administration) be included in these preliminary discussions.

DEVELOPING THE MODEL

Note that the linear-programming salary evaluation scheme to be described later is not limited by the number of factors or functions which might be considered in classifying school district personnel. Essentially, the linear-programming approach calculates relative weights for each factor, such that the ranking of job functions (positions or job classifications) by salary corresponds to the ranking of that function in the school-district salary hierarchy. The resulting analysis and solution of the model then provides the school administration with some measure of the relative importance of each factor used in the salary evaluation scheme.

There are six phases in the development of an effective job-evaluation scheme by the use of linear programming:

1. Specification of the organizational salary hierarchy
2. Determination of the factors to be included or considered in the salary evaluation
3. Development of a relative rating system for the characteristics in each factor
4. Determination of the relative weights for each factor in the evaluation
5. Determination of the salaries for the miscellaneous positions in the hierarchy after the key positions have been determined
6. Evaluation of positions and salaries of individuals within the system

The application of these phases to the school-district job/salary-evaluation model is described below.

Specification of the Organizational Salary Hierarchy

The job functions or classifications which correspond to the school-district salary hierarchy and relate to the goals and objectives of the school district can be viewed in many ways. This paper will consider the salary hierarchy from the

framework of responsibility in the job classification. These classifications would include responsibilities which are district-wide, school-wide, department-wide, class-wide, student. This roughly corresponds to superintendents, principals, department heads, teachers, and teacher aides.

Determination of Factors to be Included

The factors to be included in the analysis can usually be determined by an initial survey of the personnel in the school district who are involved in the evaluation. This is followed by several meetings at which all the factors to be used are agreed upon. Some of the factors used will probably reflect compromise situations, but this is not a major problem to the evaluation scheme. One important result of this phase of the study will be the improvement of ambiguous or poorly written position descriptions.

Development of a Rating System for Characteristics in Each Factor

A relative rating for each characteristic of each function must be defined. For example, the characteristics in education might be ranked as follows:

Characteristics	Rating
Ph.D. or Ed.D.	5
M.A. or M.S.	4
M.Ed.	3
B.A. or B.S.	2
A.A.	1
H.S. Diploma	0

Education is a relative objective factor, since each characteristic is directly measurable by the completion of a college degree. A more subjective factor might be the difficulty of the learning situation of the school within the school district. A suggested rating scheme for this factor would be the following.

Characteristic	Rating
Difficult; disadvantaged area	3
Medium or average difficulty	2
Not difficult; culturally advantaged	1

This rating scheme might present problems for many school districts. Since the Federal Government has developed indices for determining whether a given school is culturally disadvantaged, this type of index could be used. Other partial measures of the difficulty of the learning situation could include percentage of dropouts, percentage of minority groups, discipline or police records of the students, number of teachers requesting transfer, etc. A dichotomous classification (difficult-not difficult) could also be used for this factor. An important concept to remember is that a factor with too few ratings or characteristics will not sufficiently discriminate among deficiencies in ability whereas one with too many will result in ambiguity. The establishment of the ratings for each characteristic of each function is usually based upon the job description and the mutually agreed-upon order of relative importance of the characteristics to the particular factor.

Determination of Relative Weights for Each Factor

The determination of the weights to be assigned to each factor is accomplished by means of the model. The model will yield a point system for a number of key positions in the school district. A key position might be considered to be one for which a pay differential has been established, such as between administrators and teachers. These key positions usually correspond to the different functions of the personnel in the school district and are determined by the bounds established by these key positions. One extremely important benefit of this scheme is that only the theoretically highest and theoretically lowest paid individuals, with their characteristics, have to be specified.

Determination of Relative Ranking of Miscellaneous Positions

The relative weights, determined by the solution of the model, are used to evaluate and place, in a logical and consistent manner, and according to the imposed hierarchy, the remaining positions in the school district. If the salaries for certain positions seem out of place with established school district policies in terms of salary, then either the mathematical formulation of the evaluation scheme must be revised, with more key positions included in the model, or else a significant factor (or factors) has been omitted in the analysis of the position.

Evaluation of Individual Positions and Salaries

The primary benefit of the proposed salary-evaluation scheme for school districts is the effective evaluation of the certified school district personnel--their positions in the hierarchy of the district, and their salaries. This approach offers the school administration a method for developing an internally consistent salary structure based upon a set of salary evaluation factors.

ADDITIONAL REFINEMENTS

The school district may desire to include the following additional refinements to its usage of the salary model:

1. Establishment of the relative importance of the factors themselves
2. Investigation of other environmental constraints, especially budgetary
3. Determination or development of objective functions or measures of effectiveness

A school district may wish to place heavier emphasis upon a factor such as the difficulty of the learning situation or subject-matter preparation. The establishment of the relative importance of factors or combinations of factors can be established by an ordering of them or by a relative rating scheme in the model.

This procedure would be very similar to the relative ordering of characteristics within a factor.

SUMMARY OF PROCEDURES FOR DEVELOPING THE MODEL

The procedures for developing the salary-evaluation model for a school district could be summarized as follows:

1. Identification of the school-district personnel by function or position in the school-district salary hierarchy
2. Identification of those factors which contribute to the performance of the personnel in each function or position in the school-district organization
3. Identification of the descriptors or characteristics within each factor, with an ordering by relative importance
4. Formulation of the mathematical equations which represent the theoretically lowest- and highest-paid school-district personnel for each function in the salary hierarchy
5. Inclusion in the model of other environmental constraints, such as those reflecting the "financial environment" of the school district and the various interposition and intraposition percentage spreads in salary
6. Determination of the objective functions or measures of effectiveness to be employed by the school district in the evaluation scheme

III. DERIVATION OF THE MODEL FOR A SCHOOL-DISTRICT SALARY SCHEDULE

IDENTIFICATION OF PERSONNEL BY FUNCTION

In this illustrative application of the model, five school-district functions (job classifications) were considered. These were in decreasing order of responsibility and salary:

1. Superintendent
2. Administrator (principals, vice principals, etc.)
3. Department heads
4. Teachers
5. Teacher aides and other teacher assistance personnel such as lab assistants and readers

Obviously, other positions such as school nurses, counselors, psychometristics, etc., could be included in any direct application of the model to a school district. For simplicity, however, this illustrative application will be limited to the five positions listed above.

DEFINITION OF RELEVANT FACTORS WITHIN FUNCTIONS

Each of the school-district functions included in the model is evaluated according to nine factors. These factors, although arbitrary, are considered reasonable for illustrative purposes:

1. The type of area in which the school is located, i.e., the relative difficulty of the educational situation as determined by the teaching environment (X_1)
2. The subject-matter area being taught, in terms of demand compared to available supply of teachers (X_2). For example, high-demand areas would be English, mathematics, and science; average-demand areas would be Latin

and French; low-demand areas would be physical education for men and social studies. For an administrator, a high-priority or skill area might be a systems analyst or computer specialist, etc.

3. Supervisory responsibilities of the personnel, in terms of the area of responsibility (X_3)
4. The highest academic degree attained by the individual at the time of the evaluation (X_4)
5. The total work experience of the individual within the district, plus years worked in previous organizations or school districts (X_5)
6. Special distinctions or awards earned by individuals, such as Phi Beta Kappa administrative or teaching awards (X_6)
7. The number of college units or semester hours completed beyond the highest degree attained (X_7)
8. The number of inservice-hours credits obtained per year by the teacher in programs of professional development (X_8). This factor will only be considered in the salary evaluation of department heads, teachers, and teacher aides
9. The relative additional workload corresponding to the job classification within the organization (X_9)

The above-mentioned set of factors should not be considered complete. Depending upon the objectives of the school district and compromise agreements with teacher unions, other factors might also be considered. The linear programming approach is unique because there is virtually no limit to the number of factors which may be considered for the evaluation of salaries. For illustrative purposes only, this study will limit itself to the nine above-mentioned factors.

SPECIFICATION OF HIERARCHICAL CONSTRAINTS

Once the factors, their characteristics, and the relative ratings of the characteristics are specified, it is then possible to describe each function in the organization hierarchy by means of two equations; one represents the theoretically most highly qualified person for the function and hence the highest salary, and the other represents the theoretically lowest qualifications, hence the lowest salary. In generalized mathematical terms, the equations representing the highest (λ_j) and lowest (σ_j) levels within each function j take the following form in the model:

$$\alpha_1 X_1 + \alpha_2 X_2 + \ldots + \alpha_n X_n \leq \lambda_j$$

$$\beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_n X_n \geq \sigma_j$$

where

α_i = the highest rated characteristics associated with the factors appropriate to function j in the school district

β_i = the lowest rated characteristics associated with the factors appropriate to function j in the school district

X_i = the factors associated with function j

λ_j = the theoretically highest or maximum salary to be paid within function j

σ_j = the theoretically lowest or minimum salary to be paid within function j

n = the number of factors considered in the evaluation scheme;

i = 1, ... n

Dollar differences between the highest salaries in each of the school districts' job classifications can be represented by means of the following equation:

$$\lambda_j - \lambda_{j+1} \leq \delta$$

where

λ_j = the theoretically highest salary in job classification j

λ_{j+1} = the theoretically highest salary in the next lower job classification j+1

δ = the specified dollar difference in salary

PERCENTAGE CONSTRAINTS CONTROLLING SALARY SPREADS

Percentage overlaps in salary between job classifications (e.g., the lowest salary of the higher classification to the highest salary of the next lower classification) can be controlled in the model by means of the following equation:

$$\sigma_j = \omega \lambda_{j+1}$$

where

λ_{j+1} = the theoretically highest salary in job classification j+1

σ_j = the theoretically lowest salary in job classification j

ω = the desired percentage overlap in salary between job classification j and j+1

The percentage relationships between the highest and lowest salary schedules in each job classification or function can be mathematically expressed as:

$$\sigma_j \geq \gamma \lambda_j$$

where

σ_j = the theoretically lowest salary for job

λ_j = the theoretically highest salary in job classification j

γ = the percentage spread in salary for job classification j

SPECIFICATION OF BUDGETARY CONSTRAINTS

Finally, the budgetary constraints which would affect the salary structure can be incorporated in the model by means of the following equation:

$$\Sigma n_{ik}\ \eta_{ik}\ X_i \leq \psi$$

where

n_{ij} = the number of certified employees having characteristic j of factor i

X_i = factor i used in the evaluation scheme

η_{ik} = the relative rating given to characteristic k of factor i

ψ = the total amount of school-district funds available for distribution for certified-personnel salaries

Determination of the Objective Function Used in the Model

The objective function in a linear programming model contains one or more (in linear combination) of the variables used in the constraint set and is the mathematical representation of the criterion of effectiveness desired by the school district. The essential feature of the linear programming technique is to insure that this criterion of effectiveness is maximized or minimized, subject to the constraints imposed on the system. This one unique solution, derived from a set of feasible solutions, is known as the optimal solution.

The criterion of effectiveness selected by the school district will, to a large extent, depend upon the established goals and objectives of the school district. The model presented in this paper offers the school district the flexibility of choosing among several options which might be used as effectiveness measures. For example, beginning teacher salaries, σ_5, could be maximized if the school district desires to attract young, inexperienced teaching personnel. If the district desires retention and continuity of its teaching staff, then the

maximization of the highest teacher salary, λ_5, could be used as an objective function. Suppose the school district desired to pay higher salaries to school-district personnel who undertake assignments in highly difficult learning areas, then the maximization of factor X in the salary evaluation scheme could be used as an objective function.

In this study, assume that the school district desires to rate each factor in descending order of relative importance. One such rating scheme might be as follows:

Table 1

Variable	Relative Rating	Factor
X_1	4	Difficulty of learning environment
X_6	4	Special awards or distinctions
X_2	4	Subject matter priority
X_4	3	Highest academic degree
X_5	3	Work experience
X_7	2	Additional college credits completed
X_8	2	Inservice units completed
X_3	1	Supervisory responsibility
X_9	1	Additional workload by hierarchy

This can be expressed mathematically as:

Maximize

$$4X_1 + 4X_6 + 4X_2 + 3X_4 + 3X_5 + 2X_7 + 2X_8 + X_3 + X_9$$

This would reflect a situation in which the school district places highest salary priorities on school-district personnel having special awards or distinctions, subject-matter priority, or those assigned to a difficult learning environment. Following these highest priorities would be consideration of highest academic

degree and work experience, followed by college and inservice credits completed. Finally, the position of the individual in the organizational hierarchy would assume least weight.

The above can be mathematically expressed in linear form as:

$$\text{maximize} \quad \Sigma W_i X_i$$

where

X_i = factor i

W_i = the relative rating given to factor i

With this objective function, greater financial "rewards" will be given to those certified personnel possessing the higher-rated characteristics in the higher-rated factors.

If all factors are to be weighted equally, then the objective function takes the following form:

$$\text{maximize} \quad \sum_{i=1}^{9} X_i$$

For illustrative purposes, the objective function used in the application of the model presented in this paper will reflect the priorities of maximizing the top teachers' salary, λ_5. This is expressed mathematically as:

$$\text{maximize} \quad \lambda_5$$

IV. ILLUSTRATIVE APPLICATION OF THE MODEL

INPUTS TO THE MODEL

In order to test the model's validity and effectiveness in meeting certain specified criteria for salary evaluation of educational personnel, it was applied to a typical school-district salary structure.

Table 2 lists the nine evaluation factors included in the model for determining the salary, the variables which represent these factors in the model, the descriptive characteristics in decreasing relative importance of each factor, the number of school-district personnel possessing each characteristic in each factor, the relative weight assigned to each characteristic of each factor, and finally, the total relative factor weight. Assume the school-district salary schedule applies to 1386 personnel including 6 superintendents, 30 administrators, 90 department heads, 1200 teachers, and 60 teacher aides. Also assume the school district has a total of $13 million to support its salary structure.

SUMMARY OF THE MODEL USED IN THE STUDY

Maximize: λ_5

Subject to:

Organizational Hierarchy Constraints:

Superintendent*	$3X_1 + 3X_2 + 7X_3 + 5X_4 + 7X_5 + 2X_6 + 5X_7 + \quad + 5X_9 \leq \lambda_1$	(1)
(Function 1)	$X_1 + X_2 + 6X_3 + 3X_4 + X_5 + X_6 + X_7 + \quad + 5X_9 \geq \sigma_1$	(2)
Administrator*	$3X_1 + 3X_2 + 5X_3 + 5X_4 + 7X_5 + 2X_6 + 5X_7 + \quad + 4X_9 \leq \lambda_2$	(3)
(Function 2)	$X_1 + X_2 + 4X_3 + 3X_4 + X_5 + X_6 + X_7 + \quad + 4X_9 \geq \sigma_2$	(4)
Department Head	$3X_1 + 3X_2 + 3X_3 + 5X_4 + 7X_5 + 2X_6 + 5X_7 + 5X_8 + 3X_9 \leq \lambda_3$	(5)
(Function 3)	$X_1 + X_2 + 3X_3 + 2X_4 + X_5 + X_6 + X_7 + X_8 + 3X_9 \geq \sigma_3$	(6)

*Notice the factor X_8, inservice credits, is not included in the salary evaluations for these personnel.

Teacher* $3X_1 + 3X_2 + 2X_3 + 5X_4 + 7X_5 + 2X_6 + 5X_7 + 5X_8 + 2X_9 \leq \lambda_4$ (7)

(Function 4) $X_1 + X_2 + 2X_3 + 2X_4 + X_5 + X_6 + X_7 + X_8 + 2X_9 \geq \sigma_4$ (8)

Teacher Aide* $3X_1 + 3X_2 + X_3 + 2X_4 + 7X_5 + 2X_6 + 5X_7 + 5X_8 + X_9 \leq \lambda_5$ (9)

(Function 5) $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 \geq \sigma_5$ (10)

Percentage Salary Spreads Within Each Job Classification

$$\sigma_1 \geq .80\ \lambda_1 \quad (11)$$

$$\sigma_2 \geq .75\ \lambda_2 \quad (12)$$

$$\sigma_3 \geq .60\ \lambda_3 \quad (13)$$

$$\sigma_4 \geq .55\ \lambda_4 \quad (14)$$

$$\sigma_5 \geq .50\ \lambda_5 \quad (15)$$

Percentage Salary Overlaps Between Job Classifications

$$\lambda_2 \geq .95\ \sigma_1 \quad (16)$$

$$\lambda_3 \geq .90\ \sigma_2 \quad (17)$$

$$\lambda_4 \geq .80\ \sigma_3 \quad (18)$$

$$\lambda_5 \geq .80\ \sigma_4 \quad (19)$$

Minimum Dollar Spreads Between the Highest Salaries for Each Job Classification

$$\lambda_1 - \lambda_2 \geq 3000 \quad (20)$$

$$\lambda_2 - \lambda_3 \geq 3000 \quad (21)$$

$$\lambda_3 - \lambda_4 \geq 1500 \quad (22)$$

$$\lambda_4 - \lambda_5 \geq 1500 \quad (23)$$

School-District Resources Available to Support the Salary Structure

$$2992X_1 + 2873X_2 + 2892X_3 + 6126X_4 + 5432X_5 + 1422X_6 + 5736X_7 + 5842X_8 + 2880X_9 = \psi \quad (24)$$

$$\psi = \$13,000,000 \quad (25)$$

*Notice the factor X_5, work experience, has its highest relative weight among these personnel. That is, in these two classifications you would expect to find individuals with the greatest experience.

Table 2

FACTORS INCLUDED IN THE MODEL WITH RELATIVE WEIGHT, CHARACTERISTICS, AND NUMBER OF SCHOOL-DISTRICT PERSONNEL POSSESSING EACH CHARACTERISTIC

Factor	Variables	Relative Weight and Characteristics	Number of Employees Possessing This Characteristic	Weighted Total for Each Characteristic
Learning environment	X_1	3 Difficult	220	660
		2 Medium	1166	2332
		1 Easy	0	0
				2992
Subject matter or special skills	X_2	3 High priority	236	708
		2 Medium	1015	2030
		1 Low priority	135	135
				2873
Supervisory responsibility	X_3	7 Single district-wide	2	7
		6 District-wide	5	30
		5 Simple school-wide	5	25
		4 School-wide	25	100
		3 Department-wide	90	270
		2 Class-wide	1200	2400
		1 Student	60	60
				2892
Highest academic degree attained	X_4	5 Ph.D. or Ed.D.	20	100
		4 MA	120	480
		3 M.Ed.	1100	3300
		2 BA	1100	2200
		1 AA	46	46
				6126
Work experience	X_5	7 12- years	16	112
		6 10-12 "	100	600
		5 8-10 "	300	1500
		4 6- 8 "	400	1600
		3 4- 6 "	500	1500
		2 2- 4 "	50	100
		1 0- 2 "	20	20
				5432
Special awards and distinctions	X_6	2 With	36	72
		1 Without	1350	1350
				1422
College credits completed in addition to degree	X_7	5 28- units	600	3000
		4 22-28 "	500	2000
		3 15-21 "	200	600
		2 8-14 "	50	100
		1 0- 7 "	36	36
				5736
Inservice units completed	X_8	5 41-	100	500
		4 31-40	200	800
		3 21-30	700	2100
		2 11-20	200	400
		1 0-10	150	150
				3950
Relative additional workload in the hierarchy	X_9	5 District-wide	6	30
		4 School-wide	30	120
		3 Department-wide	90	270
		2 Class-wide	1200	2400
		1 Student	60	60
				2880

Upper and Lower Bounds Placed Upon the Value of Each Factor in the Model

$$100 \leq X_1 \leq 1000 \qquad (26)$$

$$200 \leq X_2 \leq 1000 \qquad (27)$$

$$100 \leq X_3 \leq 2000 \qquad (28)$$

$$100 \leq X_4 \leq 2000 \qquad (29)$$

$$100 \leq X_5 \leq 500 \qquad (30)$$

$$250 \leq X_6 \leq 1000 \qquad (31)$$

$$100 \leq X_7 \leq 500 \qquad (32)$$

$$100 \leq X_8 \leq 500 \qquad (33)$$

$$100 \leq X_9 \leq 2000 \qquad (34)$$

In the above model, five hierarchies or job classifications are specified with Eqs. 1, 3, 5, 7, and 9 representing the theoretically highest salary in the school-district salary hierarchy, and Eqs. 2, 4, 6, 8, and 10 representing the theoretically lowest salaries. These equations can be thought of as representing salaries of the theoretically most highly qualified and the theoretically least qualified individual in each functional classification in the school district. The spread in salaries within each classification is controlled by the percentage relationships specified in Eqs. 16-19. Spreads between the highest salaries between classifications are controlled by means of Eqs. 20-23. Overlaps in salaries (highly qualified teachers to receive higher pay than department heads) is permitted. The amount of this salary overlap between classifications is controlled by the percentage relationships specified in Eqs. 16-19. The resources available in a school district for salaries determine the salary schedule. These resources are considered in the model by means of constraint Eqs. 24-25. Finally, upper and lower bounds are placed upon each of the factors constituting the salary schedule. This procedure insures that no single factor accounts for more than a certain part

of the individual's salary. These bounds on the salary weights for each factor are specified in Eqs. 26-34.

RESULTS OF THE STUDY

The optimal weights for each factor in the salary scheme were determined by solving the linear programming model. The following table summarizes these optimal weights.

Table 3

OPTIMAL WEIGHTS GIVEN TO EACH FACTOR

X_1	Difficulty of the learning situation	100
X_2	Subject matter priority	200
X_3	Supervisory responsibility	1145
X_4	Academic degree	100
X_5	Work experience	100
X_6	Special awards	654
X_7	Credits	100
X_8	Inservice	100
X_9	Additional workload by hierarchy	2000

The theoretically highest and lowest salaries in each job classification was then determined by substituting these weights in Eqs. 1-10 in the model. The following table summarizes the salaries for each of the job classifications, or the optimal salary schedule for the district.

Table 4

OPTIMAL SALARY SCHEDULE[a]

Function 1	Superintendents	(highest)	23,031
	Assistants	(lowest)	18,425
Function 2	Principals	(highest)	18,173
	Assistants	(lowest)	14,035
Function 3	Department head	(highest)	14,440
	" "	(lowest)	8,640
Function 4	Teacher	(highest)	10,800
	"	(lowest)	7,200
Function 5	Teacher Aide	(highest)	9,000
	" "	(lowest)	4,500

[a]Solving the model for different objective functions (criteria or effectiveness) would, of course, yield different values for the factory weights; hence, different salary schedules.

ALTERNATE SALARY SCHEDULES

Alternate "optimal" salary schedules can be determined by the parameterization of the amount of school-district resources which are available to support the district's salary structure. The following table lists these salary schedules which resulted from increasing the amount of school resources from 12.5 million to 15.5 million in increments of .5 million.

Table 5

ALTERNATE OPTIMAL SALARY SCHEDULE

Function 1	(highest)	20,905	23,031	23,625	25,543	26,411	27,184	27,500
	(lowest)	16,724	18,425	18,900	20,434	21,129	21,747	22,000
Function 2	(highest)	17,352	18,713	18,655	19,854	20,549	21,284	21,539
	(lowest	13,014	14,035	15,055	15,804	16,499	17,100	17,293
Function 3	(highest)	14,400	14,400	14,185	14,665	15,186	15,684	16,079
	(lowest)	8,640	8,640	11,570	12,115	12,636	13,083	13,206
Function 4	(highest)	10,800	10,800	10,800	11,077	11,424	11,784	12,199
	(lowest)	7,200	7,200	8,085	8,427	8,774	9,083	9,225
Function 5	(highest)	9,000	9,000	6,914	6,988	7,162	7,316	7,387
	(lowest)	4,500	4,500	4,500	4,500	4,500	4,500	4,500
School-District Resources Used (in millions)		12.5	13.0	13.5	14.0	14.5	15.0	15.5

Other alternate optimal salary schedules can also be derived by parameterizing other variables used in the model. These might include increasing the percentage overlaps in salary between job classifications in increments of 5 percent, or increasing the beginning teacher-aide salary in increments of $500, or increasing the lower limit on the salary evaluation factor X_1 (difficulty of the learning environment) in increments of $100, etc.

The school district now possesses a logical and internally consistent salary schedule. The schedule is consistent with the hierarchical salary organization of the district and is based upon school-district resources. It also evaluates the teacher or administrator along nine dimensions or factors.

Once the weights for each factor in the salary scheme are calculated by solving the linear programming model, all school-district personnel can be placed into the educational system's salary hierarchy.

DETERMINATION OF INDIVIDUAL SALARIES

To determine an individual salary, the school administrator merely multiplies the relative weight for the particular characteristic possessed by the individual times the factor weight calculated from the model.

Mathematically, this can be represented as:

$$\Sigma\ Y_{ij}\ X_i = S$$

where

S = the individual's salary

Y_{ij} = the relative weight of characteristic j found in factor i which is possessed by the individual

X_i = the weight for factor i by solving the linear programming

For example, a teacher ($2X_3$) with a masters degree ($4X_4$) teaching in a difficult learning area ($3X_1$) with five years' experience as a teacher ($3X_5$), with no special distinction ($1X_6$) or subject matter priority ($2X_2$), with 10 additional college units ($2X_7$) and 15 inservice units ($2X_8$) with normal additional workload ($2X_9$) would expect to earn the following salary:

$$3X_1 + 2X_2 + 2X_3 + 4X_4 + 3X_5 + 1X_6 + 2X_7 + 2X_8 + 2X_9 \geq \sigma_4$$

$$3(100) + 2(200) + 2(1145) + 4(100) + 3(100) + 1(654) + 2(100) + 2(100) + 2(2000) =$$

$$300 + 400 + 2290 + 400 + 300 + 654 + 200 + 200 + 4000 = 7889$$

Similarly, an administrator with single school-wide responsibility (principal) ($5X_4$), assigned to a difficult learning area school ($3X_1$), with five years' experience ($3X_5$) as an administrator, with no special distinctions ($1X_6$) or special skills ($2X_2$), with additional college credits ($1X_7$) with a normal workload ($4X_9$) for administrators in the district would expect to earn the following salary:

$$3X_1 + 2X_2 + 5X_3 + 5X_4 + 3X_5 + 1X_6 + 1X_7 + 0X_8 + 4X_9 =$$

$$3(100) + 2(200) + 5(1145) + 5(100) + 3(100) + 654 + 100 + 0(100) + 4(2000) =$$

$$300 + 400 + 5725 + 500 + 300 + 654 + 100 + 8000 = 15,379$$

In a similar manner, the salaries for each number in the school-district organization can be determined. Using the model, each individual would receive a salary which is consistent with the salary paid to every other member of the organization. In addition, the salary would be consistent with both the organizational salary hierarchy and school-district resources.

V. CONCLUDING REMARKS

The linear-programming approach to position/salary evaluation of school-district personnel allows a school district to calculate a system of relative weights, thereby establishing the relationship of one position to another, in quantitative terms. The approach proposed in this paper has several characteristics and advantages which distinguish it from previous salary-evaluation schemes in education:

1. It is an internally consistent evaluation scheme which is valid for all school-district functions considered in the model, and which takes into consideration all the agreed-upon factors which constitute those functions
2. The model presents to the school district a more effective assessment of each individual's relative worth to the school district, which can be reflected in terms of salary
3. The model could be used to justify salary increases for school-district personnel and could play an important role in wage-salary negotiations with teacher unions and associations
4. The model establishes a salary hierarchy consistent with the objectives of the school district, but allows for highly qualified personnel in one function to receive larger salaries than the lowest-qualified personnel in a higher function
5. The model allows the school district to establish salary priorities (e.g., a school district can pay higher salaries to teachers in difficult teaching situations or to those in high-demand, low-supply teaching areas or, as demonstrated in this paper, pay the highest maximum teacher salaries

6. The evaluation system encourages participation of teacher groups, administrators, PTA, and the school board in setting the objectives of the school, the functions or job descriptions of its personnel, the establishment of those factors necessary for performance of the particular function, and the rating of the characteristics which constitute each factor.

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