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ABSTRACT

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Abstract

Failure to consider errors of measurement when using partial correlation or analysis of covariance techniques can result in erroneous conclusions. Certain aspects of this problem are discussed and particular attention is given to issues raised in a recent article by Brewer, Campbell, and Crano.

ERRORS OF INFERENCE DUE TO ERRORS OF MEASUREMENT[1,2]

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Brewer, Campbell, and Crano (1970) have justifiably criticized the use of partial correlation procedures in hypothesis testing research where errors of measurement are not taken into consideration. Ignoring measurement errors is much more serious when dealing with partial correlations than when dealing with simple zero-order correlations. In the latter case we know that the effect of errors of measurement, that are mutually uncorrelated and uncorrelated with true scores, is to reduce the absolute value of the zero-order correlation between the fallible measures. As Lord (1963) has pointed out, however, we cannot ordinarily know the effect of such errors of measurement on a partial correlation. Errors of measurement can increase or decrease the magnitude of a partial correlation and may even result in a partial correlation of a different sign.

As an alternative, Brewer et al. (1970) have suggested that factor analytic techniques be used to test a single-factor model before drawing conclusions about the nature of underlying conceptual variables. The purpose of the present paper is to reconsider the issues raised by these authors and the reasoning that led to their conclusions. Attention also will be given to some related arguments that were made in a recent attack on some commonly used methods for the evaluation of compensatory educational programs (Campbell & Erlebacher, 1970). Our thesis is that the basic problem is a lack of relevant information--a problem that cannot be resolved by the choice of a statistical procedure.

Relationship between Factor and Partial Correlation Analyses

Ignoring errors of measurement, the relationship between the loadings on a single common factor and the partial correlations in the case of three variables is straightforward. The squared factor loadings on a single common factor can be expressed:

$$a_i^2 = \frac{\rho_{ij}\rho_{ik}}{\rho_{jk}} \quad (1)$$

for $i,j,k = 1,2,3$; $i \neq j \neq k$, where a_i is the factor loading on the single common factor for variable i and the ρ 's are the intercorrelations among the variables, i,j,k . When $\rho_{jk} = 0$, a_i^2 is undefined. Assuming none of the three zero-order correlations equal zero, the squared factor loading can be written as a function of the partial correlation, $\rho_{jk.i}$:

$$a_i^2 = 1 - C\rho_{jk.i} \quad , \quad (2)$$

where

$$C = \frac{\sqrt{1 - \rho_{ij}^2}\sqrt{1 - \rho_{ik}^2}}{\rho_{jk}} \quad .$$

Provided that C is positive, it may be seen from (2) that when $\rho_{jk.i} = 0$, $a_i^2 = 1.0$ and when $\rho_{jk.i} < 0$, $a_i^2 > 1.0$.

Frederic Lord (personal communication) suggested that the relationship between the factor and partial correlation analyses could be clarified by an example such as the one depicted in Figure 1. Given $\rho_{X_1X_2} = .50$, the possible values of $\rho_{X_1X_3}$ and $\rho_{X_2X_3}$ are contained in the ellipse in Figure 1.

Regions of the figure that contain negative partial correlations are indicated. Factor loadings are denoted by a_i and regions that contain imaginary loadings or squared loadings greater than 1.0 are indicated.

Insert Figure 1 about here

On line segment <u>ac</u> $\rho_{23.1} = 0$ and $a_1 = 1.0$, on line segment <u>bd</u> $\rho_{13.2} = 0$ and $a_2 = 1.0$, and on line segments <u>aeb</u> and <u>cfd</u> $\rho_{12.3} = 0$ and $a_3 = 1.0$. Imaginary values of the a 's occur when one of the three zero-order correlations is negative while the other two are positive.

Bias in Partial Correlation

Brewer et al. (1970) argue that errors of measurement introduce a systematic bias into partial correlations. More specifically, they state: ". . . the assumption is made that the variable being partialled out contains no unique components and is measured without error. Using partialling techniques when these assumptions are not met introduces systematic bias toward the unparsimonious conclusion that more conceptual factors are involved in a phenomenon than may actually be the case" (Brewer et al., 1970, pp. 1-2). Although it is true that this may be the effect of a violation of the assumption of an error free measure, the bias may be in the opposite direction. It is easy to construct an example where the direction of the bias is toward a more parsimonious conclusion that fewer conceptual factors are involved in a phenomenon than is actually the case. Suppose, for example, that three latent variables (T_1 , T_2 , and T_3) had the following intercorrelations in the population:

$$\rho_{T_1T_2} = .6 \quad ,$$

$$\rho_{T_1T_3} = .6 \quad ,$$

and $$\rho_{T_2T_3} = .18 \quad .$$

The correlation between T_2 and T_3 with T_1 partialed out is -.28125 and the corresponding conclusion is that more than one conceptual variable is involved in this phenomenon. Suppose, however, that only a fallible measure of the first variable, say X_1, was available, where

$$X_1 = T_1 + E_1$$

and E_1 is uncorrelated with T_1, T_2, or T_3. Further, assume that the variance of X_1 is equal to twice the variance of T_1 (i.e., the reliability of X_1 is .50). Under these conditions the resulting intercorrelations among T_2, T_3, and X_1 would be:

$$\rho_{X_1T_2} = .6\sqrt{.5} \doteq .424 \quad ,$$

$$\rho_{X_1T_3} = .6\sqrt{.5} \doteq .424 \quad ,$$

$$\rho_{T_2T_3} = .18 \quad .$$

The correlation between T_2 and T_3 with X_1 partialed out would be 0.0 which would result in the more parsimonious, but erroneous conclusion that a second conceptual variable is not required. There is no intention to imply by this illustration that the bias of errors of measurement is typically, or even frequently, in the direction of producing a partial correlation that is closer to zero. Rather the point is that the direction of the bias cannot be determined without imposing additional assumptions (e.g., all reliabilities and all zero-order and partial correlations among true scores are nonnegative) and/or obtaining additional information such as the reliabilities of the measures. Given classical test theory assumptions, an estimate of the partial correlation among underlying true scores may be obtained by simply applying standard corrections for attenuation to the zero-order correlations. As Lord (1963) has noted, the need to make corrections for attenuation "...poses somewhat of a dilemma, since, first, it is often hard to obtain the particular kind of reliability coefficients that are required for making the appropriate correction, and, further, the partial corrected for attenuation may be seriously affected by sampling errors. These obstacles can hardly justify the use of an uncorrected coefficient that may have the wrong sign, however" (Lord, 1963, p. 36).

The Single Factor Model vs. Partial Correlations

As noted above, Brewer et al. (1970) have suggested that a single-factor model be tested before conclusions are drawn about the nature of underlying conceptual variables from partial correlations. We shall argue that partial correlation analyses and factor analyses are based on different models and pose different questions. Knowing that a single factor can reproduce the intercorrelations among three observed fallible variables is not sufficient to draw conclusions about the partial correlations among the underlying conceptual variables or true scores that correspond to the observed scores.

Assuming that three infallible measures (T_1 , T_2 , and T_3) have a multivariate normal distribution, the partial correlation between T_2 and T_3 with T_1 partialed out has a very simple interpretation. It is equal to the zero-order correlation between T_2 and T_3 for any subpopulation defined by a particular value of T_1 . Thus, it provides a means of investigating the relationship between T_2 and T_3 with T_1 held constant in the above sense. The question of whether or not T_2 and T_3 are related when T_1 is held constant is not the same as the question answered by a test for single factoredness for the observed scores. This is, in principle, acknowledged by Brewer et al. (1970) in footnote number 3 where they discuss an example in which the control variable (I.Q.) has a factor loading of .43. They conclude that "...if one has 'factored out' a variable upon which I.Q. loads only .43, one has not in any meaningful sense 'factored out I.Q.'" (Brewer et al., 1970, p. 7). They go on to indicate that they are working on a technique of "focused factoring," wherein the control variables are used to define the factor. Hopefully this procedure would exclude from the communality of a control variable only that variance that properly might be considered error variance.

If the observed variables (X_i) are related to their underlying true scores (T_i) by the model,

$$X_i = T_i + E_i , \qquad i = 1,2,3 \quad ,$$

where the errors (E_i) are mutually uncorrelated and are uncorrelated with the true scores, then (1) may be expressed in terms of the correlations among the true scores, $\rho_{T_iT_j}$, and the reliabilities of the observed measures, ρ_{ii} , i.e., the variance of T_i divided by the variance of X_i . Thus

$$a_i^2 = \rho_{ii} \frac{\rho_{T_iT_j}\rho_{T_iT_k}}{\rho_{T_jT_k}} \quad . \tag{3}$$

The correlation between T_j and T_k with T_i partialed out is proportional to

$$\rho_{T_jT_k} - \rho_{T_iT_j}\rho_{T_iT_k} \quad ,$$

which, given equation (3), equals:

$$\rho_{T_iT_j}\rho_{T_iT_k}\left(\frac{\rho_{ii}}{a_i^2} - 1\right) \quad .$$

Considering cases where a single factor reproduces the intercorrelations among X_1 , X_2 , and X_3 and $0 < a_i^2 < 1$ $(i = 1,2,3)$, the above expression can be seen to have the following implications:

A. When $\rho_{T_iT_j}$ and $\rho_{T_iT_k}$ have the same sign,

1. $a_i^2 < \rho_{ii}$ implies $\rho_{T_jT_k \cdot T_i} > 0$,

2. $a_i^2 = \rho_{ii}$ implies $\rho_{T_jT_k \cdot T_i} = 0$,

3. $a_i^2 > \rho_{ii}$ implies $\rho_{T_jT_k \cdot T_i} < 0$;

B. When $\rho_{T_iT_j}$ and $\rho_{T_iT_k}$ have opposite signs,

1. $a_i^2 < \rho_{ii}$ implies $\rho_{T_jT_k \cdot T_i} < 0$,

2. $a_i^2 = \rho_{ii}$ implies $\rho_{T_jT_k \cdot T_i} = 0$,

3. $a_i^2 > \rho_{ii}$ implies $\rho_{T_jT_k \cdot T_i} > 0$.

These results show that when the correlations among the observed scores are reproduced by a single factor with squared loadings between 0 and 1, no conclusions are warranted regarding the partial correlations among the true scores. Given positive reliabilities and nonzero intercorrelations among observed scores, if the three observed variables do not fit the single factor model, then the three partial correlations among true scores may be positive or negative but not zero.

T.e relationship between the observed loadings on a single common factor, the partial correlations among observed scores, and the partial correlations among true scores may be clarified by the example depicted in Figure 2. For the case $\rho_{T_1T_2} = .50$ and $\rho_{11} = \rho_{22} = \rho_{33} = .50$, Figure 2 shows the possible values of $\rho_{T_1T_3}$ and $\rho_{T_2T_3}$. A set of regions is defined within which the factor loadings on a single common factor, the partial correlations among observed scores, and the partial correlations among true scores have specified characteristics. The ellipse in Figure 2 contains the values for

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Insert Figure 2 about here

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which the determinant of the matrix containing the intercorrelations of T_1 , T_2 , and T_3 is greater than or equal to zero. Larger values of $\rho_{T_1T_2}$ would define a thinner ellipse and smaller values a rounder ellipse. The numbers inside the ellipse identify the various regions of the ellipse, and the letters identify line segments separating regions. For the regions in Figure 2, the factor loadings (a_i) for a single common factor that will reproduce the intercorrelations among the observed scores, the partial correlations among the observed scores $(\rho_{jk.i})$, and the partial correlations among the true scores $(\rho_{T_jT_k.T_i})$ are shown in Table 1. The values of a_i , $\rho_{jk.i}$

and $\rho_{T_jT_k \cdot T_i}$ for values of $\rho_{T_iT_j}$ and $\rho_{T_iT_k}$ on the boundaries between regions are shown in Table 2.

Insert Tables 1 and 2 about here

As was stated in implications A.2 and B.2 above, $\rho_{T_jT_k \cdot T_i}$ equals zero when $a_i^2 = \rho_{ii}$. This occurs on line segments co, do, io, jo, cmd, and inj. When $a_i^2 = 1$ (line segments bo, eo, ho, and ko) the partial, $\rho_{jk.i}$, among observed scores is zero; however, $\rho_{T_jT_k \cdot T_i}$ is nonzero. The location of line boh and line eok depends on the magnitude of ρ_{11} and ρ_{22} : boh is defined by points where $\rho_{T_2T_3} = \rho_{11}\rho_{T_1T_2}\rho_{T_1T_3}$ and eok is defined by points where $\rho_{T_1T_3} = \rho_{22}\rho_{T_1T_2}\rho_{T_2T_3}$. A line where $a_3^2 = 1$ does not exist for this example because there are no possible values of $\rho_{T_1T_3}$ and $\rho_{T_2T_3}$ for which $\rho_{T_1T_2}$ equals $\rho_{33}\rho_{T_1T_3}\rho_{T_2T_3}$. Regions 2a, 3a, 4, 6a, 7a, and 8 are of interest since they define combinations of $\rho_{T_iT_j}$ and $\rho_{T_iT_k}$ for which a partial correlation for observed scores and a partial correlation for true scores have opposite signs. Regions 1, 2a, 3a, 4, 5, 6a, 7a, and 8 are where a satisfactory single-factor solution is obtained yet all three correlations between pairs of true scores with the third true score partialed out are nonzero. Different conclusions about the number of underlying conceptual variables involved in the phenomenon presumably would be drawn for instances in those regions.

This problem should not be dealt with by simply invoking the principle of parsimony and thereby concluding that the fit of a single factor model indicates that there is only one dimension underlying the phenomenon. Rather, the problem should be dealt with by obtaining the additional information that is necessary to make inferences within a given model. A brief discussion of the use of multiple measures to obtain the needed information is presented below in the section on needed additional information.

Errors of Measurement in the Analysis of Covariance

Campbell and Erlebacher (1970) have provided a much needed criticism of the common misuse of the analysis of covariance as a means of trying to adjust for preexisting differences between experimental and control groups for the evaluation of compensatory education programs. They argue that "error" and "uniqueness" in the covariate result in bias when the groups differ on the direction of underestimating the slope of the regression of the dependent variable, on the covariate (for a good discussion see Cochran, 1968). Porter (1967) has illustrated the nature of the resulting bias for various group differences in means on the covariate and on the dependent variable. When using the analysis of covariance, bias due to errors of measurement in the covariate might make a compensatory education program look bad (or good).

The effect of "uniqueness" depends on its sources. If uniqueness is due to errors of validity (e.g., a perfectly reliable symptom of the underlying variable), then bias will result in the same way that it does from unreliability. On the other hand, if uniqueness merely refers to unshared variance between the covariate and the dependent variable as in Campbell and Erlebacher's (1970) treatment of covariance adjustments, then the question of bias is ambiguous. Given independent errors, unshared variance may be due to unreliability, invalidity or a lack of perfect correlation between underlying variables. The latter is not a source of bias and should not be corrected for as is done by Campbell and Erlebacher's adjustment procedure.

This problem needs to be viewed from the perspective of Lord's (1967) paradox. Lord has shown that the comparison of preexisting groups by means of an analysis of covariance (statistician 2) and by means of an analysis of difference scores (statistician 1) can result in paradoxically different results, both of which are manifestly correct. In his hypothetical illustrative example, Lord depicted an experiment in which girls received one diet and boys another. For each group the mean and variance of the final weight was identical to the mean and variance of the initial weight. There were preexisting differences between the groups in mean weight, and for each group the within-group correlation between initial and final weight was .50. Assuming that the weight measures are error free, the above correlation would be the correlation between true initial weight and true final weight. In the absence of measurement errors the analysis of mean change would indicate no "treatment" effect, whereas the analysis of covariance would indicate a "treatment" effect.

Campbell and Erlebacher (1970) have suggested that in pretest-posttest designs a "common-factor coefficient" might be used to correct for errors of measurement and uniqueness in the covariate. Using the proper common factor coefficients for both pretest and posttest in the standard correction for attenuation formula would result in a "corrected" pretest-posttest correlation of 1.00. Assuming equal coefficients for the pretest and the posttest, the common factor coefficient for Lord's example would be .50. Applying this "correction" would increase the slope of the within-group regression lines to 1.00 and result in identical intercepts for the two groups. In essence, Campbell and Erlebacher have devised a roundabout way of siding with Lord's first statistician. However, they have not resolved Lord's paradox. Rather than impose a restriction, such as the one that the "corrected" correlation between pretest and posttest be 1.00 (which, in our opinion, is unjustified), it would seem far better to conclude with Lord (1967) that ". . . there simply is no logical or statistical procedure that can be counted on to make proper allowances for uncontrolled preexisting differences between groups" (p. 305).

Needed Additional Information for Fallible Measures

Dealing with fallible measures will generally require additional assumptions and additional information. In some instances, using parallel forms of one or more of the measures may provide the needed additional information. One difficulty with this procedure is that most observed measures are really symptoms or indirect measures of the variable or influence to be measured, which is to say that even if the symptoms were measured with perfect reliability, they would be imperfectly correlated with the "true" variable. The researcher must decide which symptoms are reflections of the relevant underlying variable. This question is crucial since different sets of symptoms will typically define different "true" factors depending on the particular statistical procedure employed. The multitrait-multimethod approach introduced by Campbell and Fiske (1959) attempts to deal with this validity problem by using different methods of measuring the same variable. Correlations between different method measures of the same trait typically will correlate less than equivalent measures, i.e., in this model the classical psychometric approach using parallel forms is apt to underestimate correlations among underlying conceptual variables. An alternative way of stating this problem is to assume that part of the correlation between the two measures X_1 and X_1^* of T_1 is due to correlated errors of measurement and that factors causing this correlation are uncorrelated with the true scores. In this case, the square root of the correlation between X_1 and X_1^* no longer provides a reasonable estimate of the correlation between X_1 and T_1. Assuming that the errors are positively correlated, the correlation between X_1 and X_1^* will overestimate the squared correlation between X_1 and T_1 and using this inflated coefficient to correct for attenuation will result in the kind of undercorrection that Brewer et al. (1970) warned against. Correlated errors may, in fact, be one of the reasons that Brewer et al. wanted to correct for "uniqueness." There are advantages, however, to formulating the problem in terms of correlated errors rather than simply saying that we should correct for uniqueness. The former makes it possible to devise procedures for estimating the desired coefficient (the correlation between X_1 and T_1) given the possibility of either positively or negatively correlated errors, whereas the latter only allows the conclusion that the correlation between X_1 and X_1^* overestimates the desired coefficient if the errors are in fact positively correlated.

Conclusion

From our perspective, "focusing on the conceptual problem of choosing a one-factor vs. a two-factor model" (Brewer et al., 1970, p. 3) distracts the researcher's attention from the task of constructing a model which is consistent with everything we know or hypothesize about the phenomena under study. Any inferences will necessarily be no more valid than the assumptions made about reality. For heuristic purposes we have assumed that the linear additive model was relevant; however, there is no rule of nature that effects are either linear or additive. No provision was made, e.g., for catalytic, feedback, or interactional type influences. It is important for the research design to be set up to study the question of which of the plausible alternative models more closely simulates reality. Rather than focus on the conceptual problem of choosing a one-factor vs. a two-factor model, it seems to us far more worthwhile to spend time in designing the study to explore the relevant alternate models, ensuring collection of the information necessary to test which is the best simulation of reality. Depending on the problem, the factor model may be one of the alternatives. The assumption that the factor model is a priori relevant appears to us to be unjustified given the current state of the art.

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Footnotes

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[2]We are grateful to Frederic M. Lord for suggesting the idea that was used for the illustrative example in Figure 1.

Table 1

Values of Factor Loadings and Partial Correlations

for Regions of Figure 2

Region	Factor Loadings			Partial Correlations Among Observed Scores			Partial Correlations Among True Scores		
	a_1	a_2	a_3	$\rho_{23.1}$	$\rho_{13.2}$	$\rho_{12.3}$	$\rho_{T_2T_3 \cdot T_1}$	$\rho_{T_1T_3 \cdot T_2}$	$\rho_{T_1T_2 \cdot T_3}$
1	+	+	+	+	+	+	+	+	+
2a	+	+	+	+	+	+	-	+	+
2b	G[a]	+	+	-	+	+	-	+	+
3a	+	+	+	+	+	+	+	-	+
3b	+	G	+	+	-	+	+	-	+
4	+	+	+	+	+	+	+	+	-
5	-	-	+	-	-	+	-	-	+
6a	-	-	+	-	-	+	+	-	+
6b	G	-	+	+	-	+	+	-	+
7a	-	-	+	-	-	+	-	+	+
7b	-	G	+	-	+	+	-	+	+
8	-	-	+	-	-	+	-	-	-
9	i	i	i	-	+	+	-	+	+
10	i	i	i	+	-	+	+	-	+

[a]G denotes that the factor loading is greater than 1.0 in absolute value.

Table 2

Values of Factor Loadings and Partial Correlations

for Lines Separating Regions in Figure 1

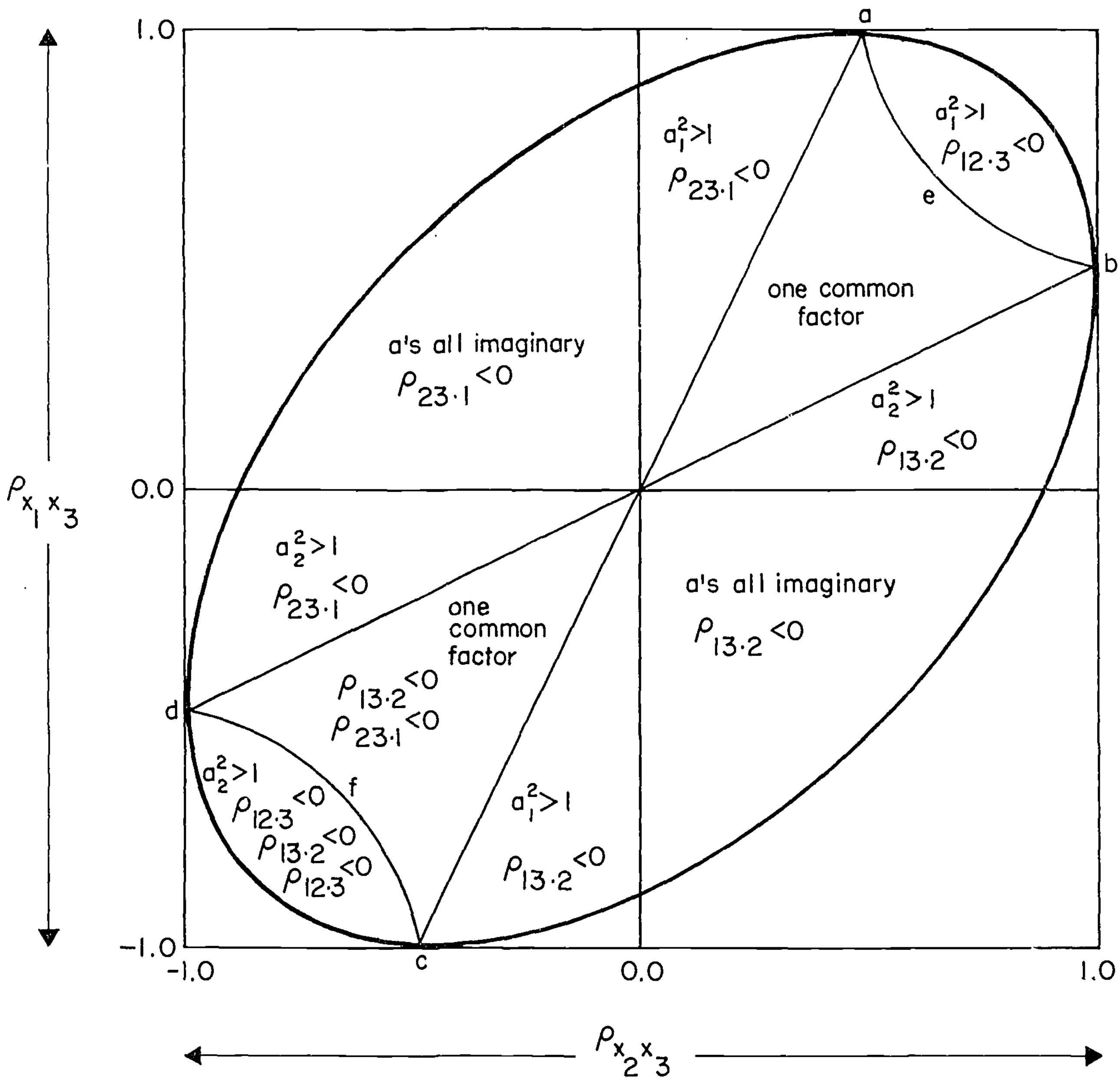
Line Segment	Factor Loadings			Partial Correlations Among Observed Scores			Partial Correlations Among True Scores		
	a_1	a_2	a_3	$\rho_{23.1}$	$\rho_{13.2}$	$\rho_{12.3}$	$\rho_{T_2T_3.T_1}$	$\rho_{T_1T_3.T_2}$	$\rho_{T_1T_2.T_3}$
ao	U[a]	0	0	-	+	+	-	+	+
bo	1	+	+	0	+	+	-	+	+
co	$\sqrt{\rho_{11}}$	+	+	+	+	+	0	+	+
do	+	$\sqrt{\rho_{22}}$	+	+	+	+	+	0	+
eo	+	1	+	+	0	+	+	-	+
fo	0	U	0	+	-	+	+	-	+
go	U	0	0	+	-	+	+	-	+
ho	-1	-	+	0	-	+	+	-	+
io	$-\sqrt{\rho_{11}}$	-	+	-	-	+	0	-	+
jo	-	$-\sqrt{\rho_{22}}$	+	-	-	+	-	0	+
ko	-	-1	+	-	0	+	-	+	+
eo	0	U	0	-	+	+	-	+	+
cmd	+	+	$\sqrt{\rho_{33}}$	+	+	+	+	+	0
inj	-	-	$\sqrt{\rho_{33}}$	-	-	+	-	-	0

[a]U denotes that the factor loading is undefined.

Figure Captions

Fig. 1. Regions which define values of factor loading and partial correlations for possible values of $\rho_{X_1X_3}$ and $\rho_{X_2X_3}$ given $\rho_{X_1X_2} = .50$.

Fig. 2. Regions which define values of factor loadings and partial correlations for possible values of $\rho_{T_1T_3}$ and $\rho_{T_2T_3}$ given $\rho_{T_1T_2} = .50$, and $\rho_{11} = \rho_{22} = \rho_{33} = .50$.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$\rho_{23\cdot1}<0$
$a_1^2>1$
$\rho_{12\cdot3}<0$
one common factor
a's all imaginary
$\rho_{23\cdot1}<0$
$a_2^2>1$
$\rho_{13\cdot2}<0$
$a_2^2>1$
$\rho_{23\cdot1}<0$
one common factor
$\rho_{13\cdot2}<0$
$\rho_{23\cdot1}<0$
a's all imaginary
$\rho_{13\cdot2}<0$
$a_2^2>1$
$\rho_{12\cdot3}<0$
$\rho_{13\cdot2}<0$
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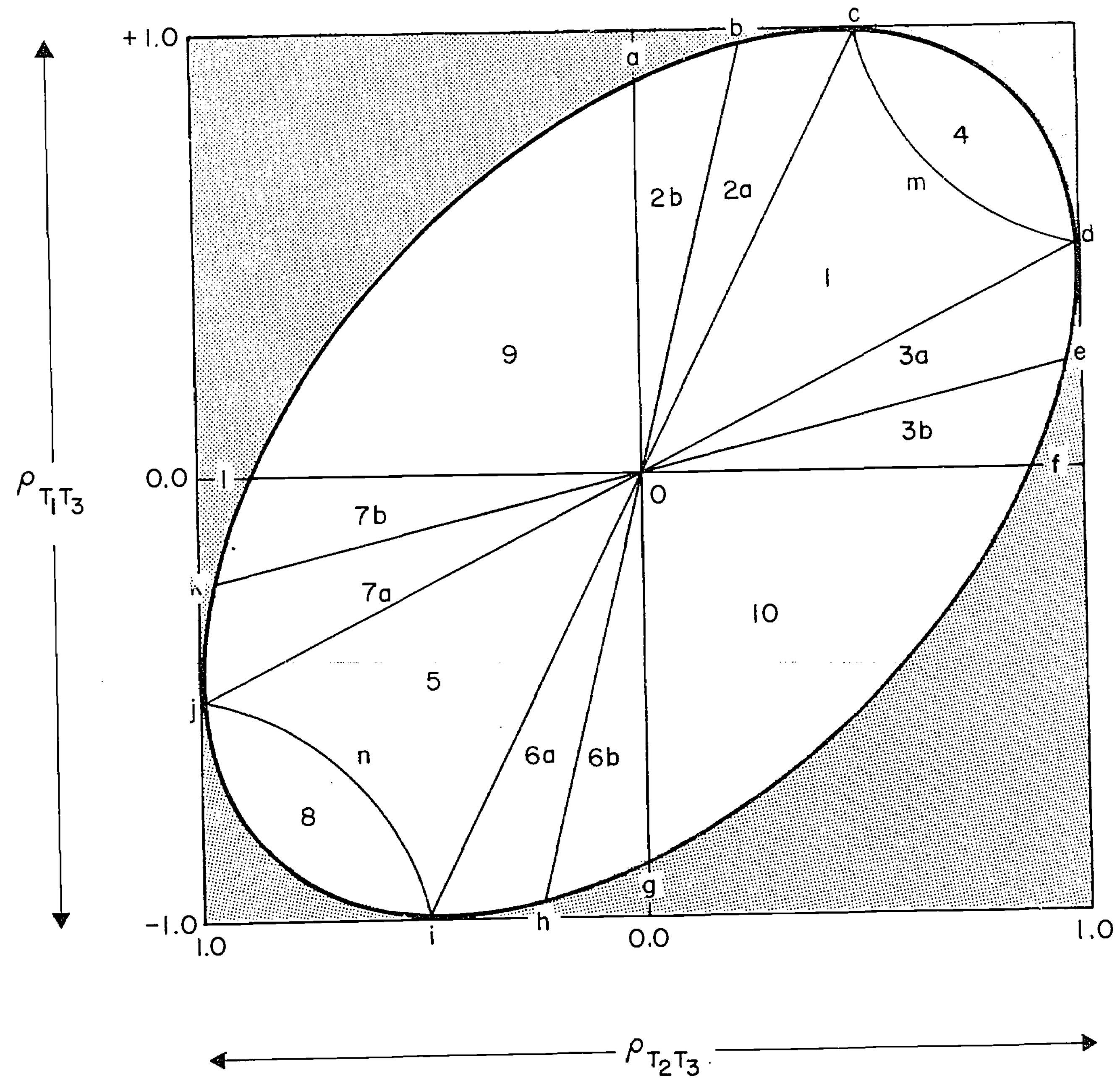
$\rho_{x_1x_3}$
$\rho_{x_2x_3}$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