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ABSTRACT

GRADES OR AGES: Grades 6-8. SUBJECT MATTER: Mathematics. ORGANIZATION AND PHYSICAL APPEARANCE: The guide is divided into three chapters. The central and longest chapter, which outlines course content, is further subdivided into 17 units, one for each of 17 content objectives. This chapter is in list form. The guide is offset printed and staple-bound with a paper cover. OBJECTIVES AND ACTIVITIES: The central chapter lists 17 mathematical concepts such as numeration systems, ratio and proportion, size and shape, measurement, and statistics and probability. A list of related behavioral objectives for each concept at each grade level is then presented. A "summary" list for grades K-5 is also included. No specific activities are mentioned, although one chapter gives general guidelines on developing problem-solving situations. INSTRUCTIONAL MATERIALS: No mention. STUDENT ASSESSMENT: Readers are referred to the guide for grades K-6 (SP 007 249). (RT)

Guidelines to Mathematics
6-8

Key Content Objectives, Student Behavioral Objectives, and Other Topics Related to Grade 6-8 Mathematics

Issued by

William C. Kahl, State Superintendent
Wisconsin Department of Public Instruction

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Foreword

In a dynamic society such as ours, change is constant. People accept this fact more readily in many fields than in the area of mathematics. And yet, in recent years new projects, new courses, and new materials remind us that a great potential exists for improved changes in the mathematics program.

The primary purpose of *Guidelines to Mathematics, 6-8* is to help those who have responsibility in local school districts for providing a well conceived mathematics program. Each of the sections of this guide has been designed to meet specific objectives. Although no section represents the final word on the topic being discussed, it is intended that all sections represent definitive statements which can serve as a sound foundation for further study and investigation.

The main section of this guide has been devoted to a careful development of major concepts and associated behavioral objectives. In addition, other sections have been devoted to issues concerning the effective teaching of mathematics.

The Wisconsin State Department of Public Instruction trusts that the efforts of all who helped make this publication possible will result in improved mathematical experiences for all Wisconsin youth.

WILLIAM C. KAHL
State Superintendent

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Mathematics Instruction: A point of View

The variety of mathematics programs now available for schools promises to benefit both teachers and students alike. Having been freed from the fixed form that characterized the traditional program, teachers now have a greater opportunity than ever to select the materials and teaching techniques best suited to a student's needs. The basis used for selecting methods and materials is important and is dependent upon the viewpoint of the teacher in regard to his instructional task. Just as mathematics programs have changed, so might certain aspects of the traditional view of instruction be changed.

It has been common practice heretofore to consider mathematics skills and concepts as being "all-or-nothing" attainments. That is, mastery of a certain body of facts or skills has been traditionally regarded as the proper province of the teacher at a particular grade level. The failure of some students to achieve the expected level of proficiency has forced the teacher of the next grade to "reteach" these areas, a task that has long been a source of irritation to many teachers. This type of compartmentalization of a course of study should be minimized, if not eliminated. Instead of expecting boys and girls at a given grade level to master in a rather final sense a certain portion of the mathematical training, educators should develop an approach which recognizes the existence of individual growth rates and stresses the continuity of the instructional process.

Teaching for Growth

Perhaps the one factor most essential to the success of the mathematics curriculum is an emphasis on understanding, that is, understanding of mathematics. This emphasis represents a marked shift in the focus of educators' attention from overt behavioral skills and social applications to understandings developed as a person organizes and codifies mathematical ideas. It is no longer deemed sufficient for a teacher to be satisfied with competent computational performance by a student in spite of the obvious necessity for such skill. The need to find a more efficient, a more enjoyable, and a more illuminating method of instruction has led to the consensus that clear and penetrating understanding of certain essential mathematics must precede, but certainly not supplant the traditional point of emphasis, computation. How is this to be achieved?

Experiencing the Physical World

Certain principles of instruction deserve renewed emphasis. It is agreed that one should traverse from the known to the unknown; from the particular to the general; from the concrete to the abstract. Clearly, students should be encouraged to develop many concepts of mathematics from their experiences with physical objects.

This approach implies, in many cases, that prior to the introduction of a concept, time should be provided for each student to experiment with physical objects appropriate to the objective. For instance, every student should have the experience of comparing volumes of various three-dimensional figures such as cones, cylinders, spheres and prisms by using sand or liquids as a prelude to volume problems; he should work on making and/or comparing thermometer scales before using the formula to convert Celsius to Fahrenheit; he should have many experiences with locating cities on maps, and playing various games on coordinate charts, before graphing solution sets of linear equations.

By such purposeful "playing" important concepts that have been left largely to chance development will be given appropriate attention and will be established on a firm foundation.

Useful Unifying Concepts

Because understanding is an ambiguous term and because a teacher must make as clear as possible the relationship between former objectives and present points of emphasis, it may be well to consider just a few of the many concepts that are especially important to clarify and develop from the earliest stages.

For the foreseeable future, the study of number systems will continue to be the essence of the mathematics program. Involved in this study are the collections of various kinds of number ideas, such as natural numbers, *together with* the operation defined on these number ideas and the properties of these operations (for example, commutative, associative, distributive properties). Obviously, the development of an understanding of such ideas must take place over an extended period of time and at considerably different rates for different students. There is no such thing as complete understanding of any particular number system by a student. Rather, teachers at each level should ascertain that the instruction is directed toward deepening and extending the broad mathematical streams cited in later sections of this report. How can this be done?

As a student's grasp of mathematics grows, he must be guided toward the acquisition of the broad and essential concepts which tie together seemingly disconnected particulars into a coherent general structure. The same operations are encountered in several peculiar and different contexts to succeeding grades. Each particular use really represents a different operation.

For instance, the multiplication of two natural numbers such as 3 x 5 may be properly regarded as the process of starting with nothing and adding 5 objects to a set 3 times. But can ½ x ⅓ be similarly interpreted as the process of repeatedly adding ⅓ objects for ½ times? Or what meaning can be ascribed to (—3) (—5)? Certainly not —5 things repeatedly added for —3 times! Such statements require different interpretations because the operation called *multiplication* has several meanings and definitions depending upon the kinds of numbers or set elements to which it is applied.

Natural numbers, integers, and rational numbers each combine somewhat differently under a given operation. But rather than invent new symbols to represent the changing definitions of an operation as it applies to the various number systems, one uses the same symbol throughout. In this situation, as in many similar ones, students must be taught to interpret symbols in context. Only in this way will mathematics shorthand serve the important purpose of clarifying and improving communication. Furthermore, the student should be led to understand that although the specific interpretation of an operation differs from one system to the next, a structural unity still remains which is founded on the commutative, associative, and distributive properties as they pertain to the operations. Because of this structural unity, one can rely on the context in which a symbol is used to determine its meaning, rather than create new symbols for each particular variation of a concept.

Another matter often treated too lightly involves the freedoms or restrictions which may or may not exist for a given operation. When one matches the elements of a set with the word names of the ordinal numbers, he must know that the order in which the objects are matched to the numerals will not affect the count. On the other hand, cases exist in which the order of presentation of information is important. One cannot mix up numerator and denominator with impunity or name the coordinates of a point by whim. The basis for free choice, where it exists, and the reasons for restrictions, as they occur, must be made clear if understanding is to be achieved.

To sum up, students should come to realize that mathematics is a logical system *which exists as a human invention*, formulated, enriched, extended, and revised in response to the twin needs to perfect it as a logical structure and to use it as a convenient method of describing certain aspects of nature as seen by man.

Undue Emphasis on Particulars

Educators should avoid giving undue emphasis to any single aspect of the total mathematics program. The idea of sets, for instance, should not be glorified beyond its usefulness in contributing to the attainment of the broad goals of the program. Similarly, it is unwise to go to the extreme of downgrading the importance of computational proficiency to the point where long range goals are placed in jeopardy. Furthermore, big words or impressive terms and symbols must never interfere with a student's understanding of the concepts which the words or symbols represent. Clarity of communication is the chief purpose of mathematical language and symbolism. If a particular term or symbol does not serve the cause of clarity, then it should not be used. Students, of course, must eventually learn how to use the language effectively and understand it in context. However, teachers should exercise great care in working toward such a goal and should avoid any proliferation of symbols or premature verbalizations that will hinder goal attainment.

Problem Solving

One of the greatest challenges for the teacher is the development of appropriate, real-life problems which meaningfully involve learners. Classroom teachers recognize the limitations of textbooks in providing such verbal problems. Many problems fail to challenge students; they fail to represent a world which is real to young people and too often they fail to contribute to the student's skill in problem solving situations outside the textbook. Notwithstanding these limitations, textbook verbal problems will continue to be prominent in most classrooms until individual teachers identify other ways of meeting the problem solving obligation.

There is increasing evidence that through experimental programs and through the efforts of individual classroom teachers improved ways of meeting this obligation are being developed. Much is being heard about the importance of relating mathematics to other areas, of the use of computers in mathematics instruction, of units of study in mathematics, of enrichment activities in mathematics, of the study of the history of mathematics, and of students creating their own mathematics problems. Instead of stressing the social development of the student these efforts emphasize the structure of mathematics through problem solving experiences. These activities are hopeful signs pointing to a greater stress being placed, in the near future, on the importance of meaningfully-structured problem solving experiences as a part of the mathematics program.

Individual Differences

If mathematics instruction is viewed as a process of initiating understanding and of carefully nurturing this understanding as the student matures, it will then be necessary to discover effective techniques for accommodating the widely differing rates at which young people develop. Implicit in this statement is the need for schools and teachers to recognize that some students may not be able to grow substantially or may seem to terminate their potential for growth at some point along the way. In such cases, provision should be made for experiences most appropriate to the individuals welfare.

To date, no satisfactory method has been developed to cope with the vast range of individual differences. Flexible grouping procedures, nongraded classrooms, individualized instruction, team-teaching, computer-assisted instruction, and television teaching are receiving extensive testing and evaluation. Perhaps the experiments currently underway will yield effective techniques once more is learned about the problem. In the meantime, each teacher must exercise professional judgment and common sense in adopting an optimal arrangement that is compatible with his own abilities, with the characteristics of students in his class, and with the physical facilities and administrative policies of the school.

In summary, mathematics instruction should be viewed as a continuous effort to develop in the individual a knowledge of mathematics that is characterized by its depth and connectedness. To the extent possible, the student should be encouraged to experiment with the objects of his environment. Thus prepared, he may be led to the invention or discovery of those ideas which provide both a broad basis for further exploration and a sense of delight in a well-founded mastery of the subject. The task is a challenging one and deserves much effort. To this end, the teacher should do everything possible to see that instruction is well-planned and is provided on a regular basis. As an additional, but essential measure, cooperative action such as inservice education should be taken by school personnel to develop in the teaching staff a view of instruction appropriate to present day needs and opportunities.

Key Mathematical Content Topics

and

Related Student Behavioral Objectives

In the following outline, an attempt has been made to point out where key content topics might be introduced and developed in the school mathematics program, including a "summary" for grades K-5, and specific student behavioral objectives for grades 6, 7 and 8. The behavioral objectives suggest a sequence of development of mathematical ideas which provides for their reinforcement and continuity from time of introduction through grade 8. By means of such a sequence of development, key ideas can be extended to conform with the maturity and background experiences of students.

The content topics have been organized under seventeen main topics: Sets and Numbers; Numeration Systems; Order; Number Systems; Ratio and Proportion; Computation; Size and Shape; Sets of Points; Symmetry; Congruence; Similarity; Coordinate Systems and Graphs; Constructions; Measurement; Mathematical Sentences; Ordered Pairs, Relations and Functions; and Statistics and Probability. The first fifteen topics are indentical to those in *Guidelines to Mathematics, K-6*, published in 1967 by the Wisconsin Department of Public Instruction.

It is not intended that the placement of topics in this outline be considered as the only correct arrangement or that all of the topics necessarily be taught at every grade level as presented here. The ideas listed for each grade level should be regarded as a *suggested* guide for introducing various topics; the outline is not intended to be all-inclusive. Teachers will find it necessary to alter the order of topics to meet the needs of students or the needs of particular groups of students.

Furthermore, no fixed amount of time or emphasis has been suggested for any objective in this outline. A disproportionate amount of space has been devoted to some topics for purposes of clarity, and the amount of space devoted should not be considered an indication of the relative importance of a topic.

This outline extends for two more grades the outline originally presented in *Guidelines to Mathematics, K-6*. Grade 6 objectives are *repeated* in this outline without modification.

School administrators and supervisors, mathematics curriculum committees, teachers, and university instructors should find this outline useful for one or more of the following purposes:

- As an orientation to the key topics and behavioral objectives of the school mathematics curriculum, grades 6-8.
- As a source of knowledge of the related objectives of introductory secondary mathematics.
- As a "yardstick" to compare against present mathematics programs.
- As a guide in determining the concepts that need to be highlighted in pre-service and in-service education programs for teachers.
- As an aid to program evaluation.

Throughout this outline, an asterisk(*) appearing under any behavioral objective, or in lieu of objectives, indicates that the understanding and skills listed for previous grades are to be expanded and reviewed.

Mathematical Topics

Sets and Numbers

Any clearly defined collection of distinguishable objects is called a set. The objects in this set are called members or elements. A set may be identified by a description (possibly containing a rule), a listing of the set's members, a Venn diagram, or a graph. Basic set operations include union, intersection and complementation.

In grades 7 and 8 the emphasis is on the study of rational numbers, negative as well as non-negative. A brief introduction is also included to the irrational numbers which leads the students to the set of real numbers. A more complete development of the set of irrational numbers and the set of real numbers is instroduced in grades 9 through 12.

Remarks

Set concepts can assist in the understanding of numerical and geometric ideas. The use of the Venn diagram might be employed to clarify definitions of such terms as prime and composite numbers, odd or even numbers, universal sets, subsets, overlapping sets, and disjoint sets. Special attention should be given in the 7th and 8th grades to the clarification of finite, infinite, equal and equivalent sets.

Students should be able to:

Numeration Systems

The understanding of base and place value of a numeration system is extended in 7th and 8th grades. Expanded notation, scientific notation using positive, zero and negative integral exponents, and the relationship between fractional and decimal representations of rational numbers are emphasized.

Remarks

Comparison of base ten with other numeration systems, ancient and modern, strengthens understanding of base and place, but computational facility should be encouraged only in base ten.

Students should be able to:

Order

The concept of order is extended from generalizations about cardinal numbers of sets to order in the set of real numbers based on the following two principles.

(1) If a and b are any two real numbers, then one and only one of the following is true: $a = b$, $a < b$, or $a > b$.

(2) Given three distinct real numbers, one is between the other two.

In grades 7 and 8, emphasis is placed on the order of rational numbers.

Remarks

Practice in ordering numbers should include inspection of digits in decimals, ($3.21089 < 3.21098$), application of skills with fractions and decimals, ($3/8 < 5/9$ and $.375 < .5$), and comparison of relative locations on the number line.

Students should be able to:

Behavioral Objectives

Grades K-5

Sets and Numbers

K Identify two equivalent sets by placing the members of the set in one-to-one correspondence.

K Use such terms as **more than, as many as, fewer than** when comparing sets of objects.

1 Count the members of a set containing one hundred or fewer members.

1 Use "0" as the symbol for the number of elements in the empty set.

2 Identify ½, ¼, ⅓ of a whole by using physical objects.

3 Determine the cardinality of a set to 10,000 through appropriate experiences.

3 Use ordinal numbers beyond tenth.

4 Determine the factors of a counting number (a whole number other than zero).

5 Find the prime factors . numbers through 100.

5 Construct sets of equivalent fractions through working with sets of objects. An example of such a set is {2/3, 4/6, 6/9, 8/12 . . .}.

Numeration Systems

K Identify the numerals 0 through 9.

1 Give different numerals for a given number such as 6 + 2, 10 — 2, and 8 for eight.

1 Interpret the place-value concept for writing whole numbers to one hundred; such as, 89 is the same as 8 tens, 9 ones.

2 Write three-digit numerals in expanded notation; for example, 765 = 700 + 60 + 5.

3 Recognize that numerals such as 57 can be expressed as 40 + 17.

4 Interpret place value for large numbers.

5 Write many names for the same rational number.

5 Work with bases, such as 3, 4, 5, 6, and 7, to demonstrate an understanding of the base of a numeration system.

Order*

K Determine whether two sets are equivalent (can be matched or placed in a one-to-one correspondence).

1 Determine that 8 is greater than 5 and that 5 is less than 8 by comparing appropriate sets of objects and do this for any two numbers less than 10.

2 Use symbols >, <, and = in mathematical sentences.

3 Determine betweenness, greater than, or less than for numbers through 999.

3 Recognize greater than or less than for the fractions ¼, ⅓, ½ with physical objects.

5 Determine greater than, less than, and betweenness for rational numbers.

Behavioral Objectives

Sixth Grade

Sets and Numbers

1. Use negative numbers in many different situations.

Numeration Systems

1. Represent rational numbers by decimals and fractions.
2. Express large numbers by using scientific notation, such as the distance from earth to the sun as 9.3×10^7 miles.
3. Use exponential notation in representing numbers; for example, $2345 = 2 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 5$.
4. Demonstrate an understanding of the relationship between decimals and common fractions.

Order*

1. Determine greater than, less than, and betweenness for (positive, negative, and zero) integers.

Behavioral Objectives

Seventh Grade

Sets and Numbers

1. Given a universal set describe or list the members of a given set. For example, in the universal set of counting numbers the set of multiples of 3 is {3, 6, 9, 12 . . .}.
2. Determine whether an element is a member of a given set.
3. Determine whether a set, including the empty set, is a subset of a given set.
4. Determine whether two sets are equivalent.
5. Determine whether two sets are equal.
6. Identify the elements in the intersection of two sets.
7. Identify the elements in the union of two sets.
8. Given a subset of a universal set identify the members of its complement.
9. Express a counting number in prime factor form. For example, $24 = 2 \times 2 \times 2 \times 3 = (2^3)(3)$.
10. Interpret a fraction as a part of a whole, as expressing a ratio, as part of a group, or as an indicated division.
11. Given a set of numerals, classify them as representing whole numbers, and/or integers, and/or rational numbers.

    Examples: 8/4 represents a whole number and an integer and a rational number .333 . . . . represents a rational number; —4 represents an integer and a rational number.

Numeration Systems

1. Use positive integral exponents to express the power of a positive rational number.
2. Use expanded notation in representing whole numbers.

   (Examples: $314_{seven} = 3 \times 7^2 + 1 \times 7 + 4 \times 1$, and $314_{ten} = 3 \times 10^2 + 1 \times 10 + 4 \times 1$.)
3. Use scientific notation to express numbers greater than ten.
4. Recognize that rational numbers can be expressed in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.
5. Write any positive rational number in decimal notation.

Order*

1. Recognize that one and only one of the following statements is true when a and b are any two rational numbers: $a < b$, $a = b$, $a > b$.
2. Determine, given two rational numbers, which is greater.
3. Determine, given three rational numbers, which one is between the other two.

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Behavioral Objectives

Eighth Grade

Sets and Numbers

1. Show that a rational number can be expressed as a repeating decimal, and that a repeating decimal can be expressed in fractional form.
2. Identify non-repeating decimals representing irrational numbers. For example, .232332333233332 . . . .
3. Determine the number of subsets in a set containing N elements. For example, a set containing 5 elements has 32 or 2^5 subsets.

Numeration Systems

1. Use positive, zero, and negative exponents correctly.
2. Use scientific notation to express positive rational numbers; for example, the length of a microwave is 2.2×10^{-2} meters.
3. Use the symbol $\sqrt{a}$ to indicate one of two equal factors whose product is a.
4. Use expanded notation to express positive rational numbers.

   (Example: $21.37 = 2 \times 10^1 + 1 \times 10^0 + 3 \times 10^{-1} + 7 \times 10^{-2}$)

Order*

1. Determine, given a rational number and an irrational number, which is greater.

   For example: which is greater 2 or $\sqrt{3}$?

Mathematical Topics

Number Systems

A number system consists of a set of numbers, two operations defined on the numbers, and properties of the operations.

The whole number system and the system of positive rational numbers are studied in the elementary grades and the system of integers is introduced. The rational number system is studied in more detail in grades 7 and 8. Subsequent understanding of the real number system depends on careful and complete development of the systems of integers and rational numbers.

The important and useful properties of the operations on the rational numbers include:

Addition	Multiplication
Closure	Closure
Commutativity	Commutativity
Associativity	Associativity
Identity Element	Identity Element
Inverse Elements	Inverse Elements

Distributivity

Remarks

Emphasis should be placed on how the properties of number systems facilitate computation. Finite systems can be used as models to help students understand the concept of number system.

Students should be able to:

Ratio and Proportion

A ratio is a pair of numbers used to compare quantities or to express a rate.

Symbols commonly used for a ratio are (a,b), a:b, and a/b. Note that the symbol (a,b) is also used to denote the coordinates of a point in a plane and that the symbol a/b is generally used to name a rational number, when a and b are integers and b is not zero.

A proportion is a statement that two ratios are equivalent (that two pairs of numbers express the same rate). A proportion is written in the form $a/b = c/d$, when $a \times d = c \times b$.

Students should be able to:

Computation

Computation can be described as the application of systematic (algorithmic) procedures to the process of renaming numbers.

Remarks

Understanding and appreciation of many mathematical concepts are facilitated by computational proficiency. It is expected that in grades 7 and 8 the computational skill developed in earlier grades will be maintained and extended.

Students should be able to:

Behavioral Objectives

Grades K-5

Number Systems

K Rearrange sets of objects to demonstrate the joining and separating of sets, and thereby develop a readiness for addition and subtraction.

1 Recognize examples of the commutative property for addition in the set of whole numbers. Demonstrate with sets of objects the relationship between such sentences as $4 + 2 = 6$, $6 - 2 = 4$, and $6 - 4 = 2$.

2 Use the associative property of addition in the set of whole numbers; for example, $(3 + 4) + 5 = 3 + (4 + 5)$.

2 Recognize zero as the identity element for addition in the set of whole numbers and its special role in subtraction.

3 Recognize the role of 1 as the identity element for multiplication in the set of whole numbers.

3 Recognize the distributive property of multiplication over addition in the set of whole numbers.

4 Recognize the inverse relation between addition sentences and two subtraction sentences, such as $725 + 342 = 1067$ and $1067 - 725 = 342$ and $1067 - 342 = 725$.

4 Use parentheses to show order of operation; for example, $2 + 4 \times 3 = 2 + (4 \times 3) = 14$ and $(2 + 4) \times 3 = 6 \times 3 = 18$.

5 Recognize that subtraction is not always possible in the set of positive rational numbers and in the set of whole numbers.

Ratio and Proportion

3 Interpret simple ratio situations, such as 2 apples for 15¢, written $\frac{2 \text{ (apples)}}{15 \text{ cents}}$.

4 Determine if two ratios are equivalent by using the property of proportions commonly called **cross multiplication.** For example, $3/4 = 9/12$ because $3 \times 12 = 4 \times 9$, whereas $6/7 \neq 7/8$ because $6 \times 8 \neq 7 \times 7$.

4 Find the missing whole number in two equivalent ratios like $2/3 = \square/9$ or $5/\square = 25/70$.

5 Use the ideas of ratio and equivalent ratio with problems that include fractions as terms. For example, find the missing number in $\frac{2/3}{5} = \square/20$.

5 Use members of sets of equivalent ratios with the same first term or the same second term to compare different ratios.

Computation

1 Use the addition facts through the sum of 10 and the corresponding subtraction facts.

2 Use the multiplication facts through the product 18.

3 Use the vertical algorithm in addition and subtraction with two- and three-place numerals when regrouping may be necessary.

3 Estimate the sum of two numbers. For example, $287 + 520$ is approximately $300 + 500$ or 800.

4 Do column addition with several four-place or five-place addends.

4 Use the subtractive division algorithm with two-place divisors ending in 1, 2, 3, 4.

5 Add and subtract rational numbers.

5 Express the quotient of integers as a mixed numeral; for example, $24 \div 5 = 4\ 4/5$.

Behavioral Objectives

Sixth Grade

Number Systems

1. Recognize that 1/1 or 1 is an identity element for multiplication in the set of rational numbers.
2. Recognize the multiplicative inverse (reciprocal) for every positive rational number except zero and use it in the division of rational numbers. For example, $1/2 \div 3/4 = 1/2 \times 4/3$.
3. Recognize that the operation of division is the inverse of multiplication in the set of positive rational numbers. For example, the sentences $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$, $\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$, and $\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$ have this relationship.
4. Recognize that there is no smallest or largest rational number between two positive integers.
5. Recognize that the integers (positive and negative whole numbers and zero) are an extension of the whole numbers.
6. Find the additive inverse (opposite) for each integer by using the number line.
7. Recognize that the rational numbers (positive and negative whole numbers, positive and negative fractions, and zero) are an extension of the integers.
8. Recognize that finding an integral power of a number involves repeated multiplication of the same number. For example, $(\frac{2}{3})^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$.
9. Use the commutative and associative properties of multiplication for rational numbers.
10. Use the distributive property of multiplication with respect to addition of rational numbers.
11. Use the commutative property of addition for integers.
12. Recognize that the rational number system is dense; that is, between each two different rational numbers, there is a rational number.

Ratio and Proportion

1. Interpret percent as a ratio in which the second number is always 100.
2. Solve all three cases of percentage problems as problems in which they find the missing term of two equivalent ratios. For example, 20% of 30 and: $20/100 = \square/30$; 30 is what percent of 55 and: $\square/100 = 30/55$; 25 is 40% of what number and: $40/100 = 25/\square$.
3. Use equivalent ratios to convert fractions to decimals and conversely; for example, to write 3/5 as hundredths, solve for n in $3/5 = n/100$; to write 44 hundredths as 25ths, solve for n in $n/25 = 44/100$.
4. Solve ratio problems where some or all of the terms of the ratios are written as decimals.
5. Use proportions in problems about the lengths of sides of similar triangles.

Computation

1. Multiply and divide non-negative rational numbers.
2. Use the conventional division algorithm.
3. Add integers.

Behavioral Objectives

Seventh Grade

Number Systems

1. Recognize and apply the addition and multiplication properties of the system of positive rational numbers. (See description of Number Systems.)
2. Recognize and apply the addition properties of the system of integers.
3. Recognize and apply the distributive property in the system of positive rational numbers.

Ratio and Proportion*

1. Recall that a ratio can be used to *compare quantities* as well as to *express a rate.*

Computation

1. Add and subtract integers by using the number line.
2. Express positive rational numbers in decimal form.
3. Add, subtract, multiply and divide nonnegative rational numbers in decimal form, recognizing that division by zero is undefined (impossible).
4. Average (compute the *mean)* sets of nonnegative rational numbers.
5. Compute positive integral powers of nonnegative rational numbers. (Examples: $3^4 = ?$; $\frac{1}{2}^3 = ?$).
6. Solve all types of percentage problems as problems in which they find the missing term in a proportion. (Including problems involving percents less than one and greater than one hundred.)
7. Given any two rational numbers, find a rational number that is *between* them.
8. Apply divisibility tests for 2, 3, 4, 5, 6, 8, and 9. Example: 135 is divisible by 3 because the sum of its digits is divisible by 3, by 5 because the last digit is 5, and by 9 because the sum of its digits is divisible by 9.
9. Using prime factorization find the least common multiple (LCM) and greatest common divisor (GCD) of two whole numbers and use them in computation with non-negative rational numbers.

Behavioral Objectives

Eighth Grade

Number Systems

1. Recognize and apply the addition and multiplication properties of the rational number system.
2. Recognize and apply the multiplication properties of the system of integers.
3. Define subtraction in terms of addition in the systems of integers and rational numbers. (Example: $a - b$ means $a + (-b)$.)

Ratio and Proportion*

1. Use proportions to solve problems involving similar triangles.

Computation

1. Add, subtract, multiply and divide integers and rational numbers.
2. Approximate square roots of positive integers.
3. Express repeating decimals in fractional form.
4. Compute the *mean* of a set of rational numbers.
5. Compute products and quotients of numbers expressed in exponential notation (including scientific notation).
   Examples:
   $3^7 \cdot 3^5 = 3^{12}$; $a^5 \div a^2 = a^3$;
   $(3 \times 10^2)\ (2.3 \times 10^3) = 6.9 \times 10^5$.
6. Perform a series of operations in proper order when grouping symbols are omitted, i.e., multiplications and divisions are performed first, in left-to-right order, then additions and substractions are performed in left-to-right order.

Mathematical Topics

Size and Shape

The classification of and distinguishing characteristics of two- and three-dimensional geometric figures are determined by their size and shape. The work of earlier grades is extended to include a more careful and detailed examination of these figures.

Remarks

Sufficient time should be allowed for students to examine models, determine their own classification schemes, and to make and test conjectures about properties of plane and solid figures.

Students should be able to: ⇨

Sets of Points

A point is represented by a location in space; lines, planes and space are sets of points with certain properties.

Remarks

These concepts should be developed intuitively using the real world as a model, emphasizing the subset relationships involved.

Students should be able to: ⇨

Symmetry

Many geometric figures have a kind of balance called symmetry. If a figure can be folded so that corresponding parts coincide, it is said to have a line of symmetry; in much the same way, a figure can be said to have a plane of symmetry.

Remarks

Ideas developed in earlier grades should be expanded to include the symmetry of positive and negative numbers illustrated by the number line, the symmetry with respect to diagonals in addition and multiplication tables, and symmetries with respect to lines in the coordinate plane.

Students should be able to: ⇨

Congruence

Intuitively, geometric figures are congruent if they "fit" each other exactly — that is, if they have the same size and shape. More precisely, two sets of points are congruent if there is a one-to-one correspondence between the two which preserve distance — that is, if two points are one inch apart, the corresponding points of a congruent set are one inch apart.

Remarks

Students should see the necessity for the more precise definition because of congruent sets of points (two- and three-dimensional) which cannot be made to "fit."

Students should be able to: ⇨

Behavioral Objectives

Grades K-5

Size and Shape

K Recognize squares, rectangles, circles, and triangles.
1 Observe distinguishing features of spheres, rectangular prisms (boxes), cylinders, and other objects.
4 Recognize isosceles and equilateral triangles and parallelograms.
5 Recognize common polyhedra, such as a tetrahedron, a cube, a rectangular prism.
5 Identify faces, edges, vertices, and diagonals of common polyhedra.

Sets of Points

1 Recognize that squares, rectangles, triangles, and circles are closed curves and tell whether a point is inside, outside, or on such a curve.
2 Recognize a straight line as a set of points with no beginning and no end.
2 Recognize a simple curve (in a plane) as one that does not cross itself.
3 Recognize rays and angles.
3 Recognize that there is only one line through two points and that two lines can intersect at only one point.
4 Describe lines as intersections of planes.
4 Interpret a circle as the set of all points in a plane that are at the same distance from a fixed point.
5 Recognize parallel planes.
5 Recognize perpendicular lines.
5 Recognize that a plane is determined by three points not all on one line.

Symmetry

3 Recognize symmetry with respect to a line by folding paper containing symmetrical figures.
4 Recognize that some figures have two or more axes of symmetry through paper folding.
5 Recognize symmetry with respect to a point by folding a paper along a line through the center of such geometric figures as a circle and a square.

Congruence

3 Recognize congruent angles.
5 Recognize that triangles are congruent if corresponding sides are congruent and corresponding angles are congruent.

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Behavioral Objectives

Sixth Grade

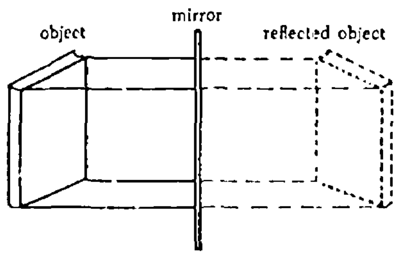
Size and Shape*

Sets of Points

1. Recognize the properties of isosceles triangles, equilateral triangles, and scalene triangles, such as the fact that the longest side of a triangle is opposite the angle of greatest measure.
2. Recognize that a line (one-dimensional space) is a subset of a plane (two-dimensional space) and that both are subsets of space (three-dimensional space).
3. Recognize the relationship between the circumference and the diameter of a circle.

Symmetry

1. Recognize the reflection of a plane figure in a mirror and draw diagrams such as the figure at right.



Congruence*

Behavioral Objectives

Seventh Grade

Size and Shape

1. Classify sets of polygons (quadrilaterals, rectangles, squares, rhombuses, parallelograms, trapezoids, pentagons, hexagons, etc.).
2. Classify angles (right, acute, obtuse); recognize supplementary and complementary angles.
3. Classify prisms and pyramids according to their bases.
4. Identify regular polygons.

Sets of Points

1. Describe, in terms of set, subset, union and/or intersection of sets of points, the following:
   Half plane
   Angle
   Triangle
   Polygon
   Intersecting lines
   Parallel lines
   Interior regions
   Etc.

Symmetry

1. Identify symmetry with respect to a line.

Congruence

1. Identify corresponding parts of congruent plane figures.
2. Recognize that corresponding parts of congruent plane figures are congruent, ($\cong$).
3. Recognize that circles are congruent if they have congruent radii.
4. Recognize that radii, diameters, corresponding altitudes and corresponding diagonals of congruent plane figures are congruent.
5. Recognize that triangles are congruent if corresponding sides are congruent and corresponding angles are congruent.

Behavioral Objectives

Eighth Grade

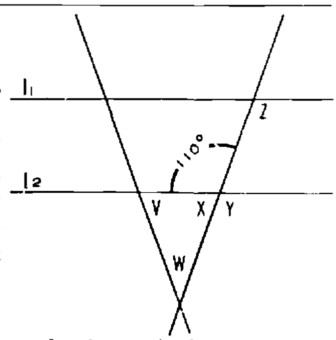
Size and Shape

1. Identify spheres, circular cylinders, and circular cones.
2. Recognize convex and nonconvex polygons
3. Identify regular solids.

Sets of Points

1. Describe, in terms of set, subset, union and/or intersection of sets of points, the following:
   - Skew lines
   - Tangent, secant
   - Central angle
   - Arc
   - Diagonal
2. Recognize the various plane figures formed when a plane intersects a given figure such as a square pyramid or a cube.
3. Use relationships such as those involved in finding the solution of the following:

l_1 is parallel to l_2
Find the number of degrees in <x, <y, <z and the sum of the measures of <v and <w.



(Relationships involved: vertical angles, supplementary angles, alternate interior angles, exterior angle to remote interior angles of a triangle)

Symmetry

1. Identify symmetry with respect to a plane.

Congruence

1. Recognize that two triangles are congruent if two sides and the included angle of one are congruent to the corresponding parts of the second (SAS).
2. Recognize that two triangles are congruent if two angles and the included side of one are congruent to the corresponding parts of the second (ASA).
3. Recognize that two triangles are congruent if the three sides of one are congruent to the corresponding three sides of the second (SSS).

Mathematical Topics

Similarity

Two geometric figures that have the same shape, though not necessarily the same size, are said to be similar.

Students should be able to:

Coordinate Systems and Graphs

A line is called a number line if a one-to-one correspondence exists between a given set of numbers and a subset of points on the line and if, by means of this correspondence, the points are kept in the same order as their corresponding numbers. The number corresponding to a point is the coordinate of that point. The idea of assigning numbers to points can be extended to points in a plane, that is, a one-to-one correspondence between ordered pairs of numbers and points in a plane.

Students should be able to:

Constructions

Remarks

In grades K-6 the emphasis was on using instruments to do "constructions." In grades 7 and 8 the emphasis should be on using identified properties and relationships of figures to do "constructions." For example, constructing a triangle given two sides and the included angle utilizes the relationship of *congruence* to insure the uniqueness of the "construction."

Students should be able to:

Behavioral Objectives
Grades K-5

Similarity

3 Recognize that figures are similar if they have the same shape. For example, all squares are similar.

4 Recognize that all congruent figures are similar, but not all similar figures are congruent.

5 Recognize the similarity of maps made with different scales.

Coordinate Systems and Graphs

1 Use the number line to illustrate addition and subtraction problems.

3 Recognize that a point on a line can be described by a number (coordinate).

3 Use the number line to illustrate multiplication problems.

4 Recognize that points in a plane (the first quadrant) can be represented by (ordered) pairs of numbers (coordinates).

5 Construct simple picture, bar, and line graphs.

5 Use the number line to represent negative integers.

Constructions

4 Demonstrate through paper folding an understanding of a line as an intersection of two planes.

4 Bisect a line segment by using a compass and straight edge.

5 Demonstrate an understanding of various polyhedra by making appropriate paper models.

5 Bisect an angle. (Students may discover several different constructions.)

5 Reconstruct an angle and a triangle by using a compass and a straight edge.

Behavioral Objectives

Sixth Grade

Similarity*

Coordinate Systems and Graphs*

Constructions

1. Construct a line perpendicular to a given line.
2. Construct parallel lines.
3. Make models of various prisms and find their surface areas.

Behavioral Objectives

Seventh Grade

Similarity

1. Identify corresponding parts of similar plane figures.
2. Recognize that corresponding angles of similar plane figures are congruent.
3. Recall that measures of corresponding sides of similar plane figures are proportional.

Coordinate Systems and Graphs*

(See Ordered Pairs, Relations and Functions and Mathematical Sentences)

Constructions

1. Construct a rhombus and a square given one side.
2. Construct an equilateral triangle given one side.
3. Construct the perpendicular bisector of a line segment.
4. Make models of pyramids.

Behavioral Objectives
Eighth Grade

Similarity

1. Recall that the measures of radii, diameters, corresponding altitudes, and corresponding diagonals of similar plane figures are proportional.
2. Recognize that triangles are similar if corresponding angles are congruent.
3. Recognize that triangles are similar if the measures of corresponding sides are proportional.
4. Recall that the ratio of the measures of two sides of a triangle is the same as the ratio of the measures of the corresponding two sides of a similar triangle.
5. Recall that sine, cosine and tangent ratios are independent of the measures of the sides of the triangles involved.

Coordinate Systems and Graphs*

(See Ordered Pairs, Relations and Functions and Mathematical Sentences)

Constructions

1. Given three sides, or two sides and the included angle, or two angles and the included side, construct the triangle.
2. Determine whether you can construct none, one, two, or many triangles given three angles or given two sides and an angle not included between them.
3. Construct representations of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, etc.

Mathematical Topics

Measurement

The process of measuring associates a number with a property of an object. Measuring an object is done either directly or indirectly. In direct measurement the number assigned to an object is determined by its direct comparison to a selected *unit* of measure of the same nature as the object being measured (a unit segment to measure segments; a unit angle to measure angles; a unit closed region to measure closed regions; and a unit solid to measure solids). When a measuring instrument cannot be applied directly to the object to be measured, indirect measurement is employed. In either case, measure of physical objects is approximate.

In grades 7 and 8, basic mensuration formulas for calculating areas and volumes are developed and applied to increasingly complex figures.

Remarks

The accuracy of the measure obtained is restricted by the unevenness of the object measured, by the limitations of the measuring instrument used, and by human inabilities. Students should be encouraged to devise their own units for measuring various objects and then led to appreciate standard units of measurement. Indirect measurement of things such as popularity, I.Q., humor, relative humidity, etc., can be discussed.

Students should be able to: ⇨

Mathematical Sentences

Mathematical sentences and ordinary linguistic sentences have the following common characteristics:

1. Both types of sentences use symbols to communicate ideas.
2. The construction of both types of sentences follows a predetermined set of rules.
3. Both types of sentences may express true statements or false statements depending upon the symbols used and the contexts in which they are used.

It is quite common in mathematics, however, to use sentences that do not possess characteristic 3. Such sentences are called *open* sentences. For example, the sentence $\square + 3 = 7$ expresses neither a true statement nor a false statement until a meaningful replacement for $\square$ has been supplied. The set of all replacements for $\square$ which produce a true statement is called the *solution set* for the given sentence. Consider the sentence $12 - X > 5$. The set of all allowable replacements for X is sometimes called the universe. Thus, if the universe is the set of positive integers, the solution set is $\{1,2,3,4,5,6\}$; but, if the universe is the set of rational numbers, the solution set consists of all rational numbers less than 7, an infinite set.

Students should be able to: ⇨

Behavioral Objectives

Grades K-5

Measurement

K Use appropriately such words as *longer, shorter, heavier, lighter, higher, lower, larger, smaller.*

1 Recognize the comparative value of coins (pennies, nickels, dimes) and use them in making change.

1 Identify various instruments of measurement of time, temperature, weight, and length, such as clocks, thermometers, scales, rulers.

2 Make a ruler with divisions showing half units.

2 Tell time to the nearest quarter hour.

3 Find the perimeter of a rectangle or parallelogram.

4 Find areas of simple regions informally. For example, a rectangular region with dimensions 2″ by 3″ can be covered by six one-inch squares (regions).

5 Measure an angle by using a protractor.

5 Estimate distances to the nearest unit.

5 Recognize that all measurement involves approximation.

Mathematical Sentences

1 Find solutions for sentences like $\square + \triangle = 7$ in which many correct solutions are possible.

2 Use equivalent sentences like $3 + \square = 7$ and $7 - \square = 3$ to show subtraction as the inverse of addition.

3 Place the correct symbol ($<$, $>$, $=$) in the place holder in sentences such as $3 \times 5 \square 7 + 8$, $25 + 42 \square 87 - 28$, and $65 - 39 \square 5 \times 7$.

3 Find solutions for sentences like $\square + 239 = 239 + \square$ and $1987 + (\square + 548) = (1987 + \square) + 548$ to generalize the idea of the commutative and associative properties for addition.

4 Use sentences like $\square \times 5 = 45$ and $45 \div 5 = \square$ to show division as the inverse of multiplication.

4 Recognize that $3 \times \square = 7$ has no whole number solution. Find solutions for mathematical sentences involving more than one operation such as $(2 \times 5) + 4 = \square$ and $(3 \times 2) + \square = 10$.

5 Write sentences using fractions to represent physical situations.

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Behavioral Objectives

Sixth Grade

Measurement

1. Find the volume of a rectangular prism.
2. Estimate and compare perimeters of polygons, such as rectangles, triangles, and parallelograms.
3. Estimate the area of an irregular plane region by use of a grid where an approximation to the area is the average of the inner and outer areas.
4. Use formulas for the areas of rectangles, parallelograms, and triangles.
5. Use the formula for the circumference of a circle.
6. Use the metric system of measure for length.
7. Use formulas of volume for common solids.
8. Work with approximate numbers. For example, know that the area of a square whose sides measure 6.5 and 3.6 inches to the nearest tenth of an inch has an area between 6.4×3.5 and 6.6×3.7 square inches.
9. Solve problems involving the measurement of inaccessible heights and distances indirectly by using the properties of similar triangles.

Mathematical Sentences

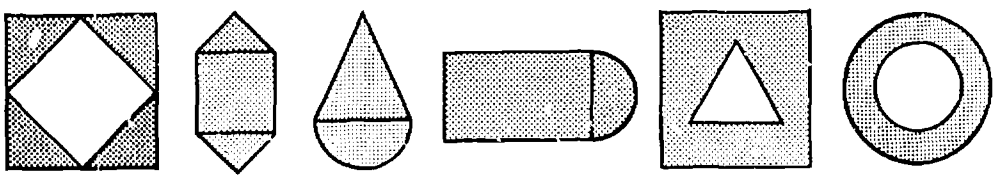
1. Use all of the previously introduced sentence forms with decimal numerals.
2. Write sentences using decimal numerals to represent physical situations.
3. Use previously described sentence forms to generalize the commutative property of multiplication, the associative property of multiplication, and the distributive property of multiplication over addition for rational numbers in any form.

Behavioral Objectives

Seventh Grade

Measurement

1. Use various measuring instruments (such as rulers having English and Metric scales, protracters, calipers) and record results to the nearer smallest unit of the scales. Example: Use rulers calibrated in tenths of an inch, in sixteenths of an inch, in centimeters and millimeters, etc.
2. Determine the perimeters and areas of regular polygons.
3. Use the formula for the area of a circle.
4. Determine the areas of figures such as shown below:



5. Determine the surface area and volume of prisms and pyramids.
6. Recall that the sum of the measures of the angles of a triangle is one hundred eighty degrees.
7. Determine the sum of the measures of the angles of a quadrilateral.
8. Determine greatest possible error for any calibration of a given measuring instrument.
9. Recognize the relationship between *precision* and *greatest possible error.*
10. Estimate linear measurements without using measuring instruments. **Example:** The length of a building, the width of a street, the height of a flagpole.

Mathematical Sentences

1. Establish the truth value of simple mathematical sentences. (Examples: $3 + 4 = 8$ is *false;* $35 \times 24 < 30 \times 20$ is *false;* $13 - 7 \neq 5$ is *true.*)
2. Find the solution sets of simple open sentences using as a universe the whole numbers. (Example: $3X + 4 = 16$.)
3. Graph using the number line solution sets for equalities and inequalities where the replacement set is the set of integers, such as $X > -2$; $X + 3 < 12$; $X\ 3$; $X > -3$ and $X < 5$; $X > 5$ and $X < 4$; $X > -2$ or $X < 3$; $X < 5$ or $X < 3$; $X > 4$ or $X < -2$.

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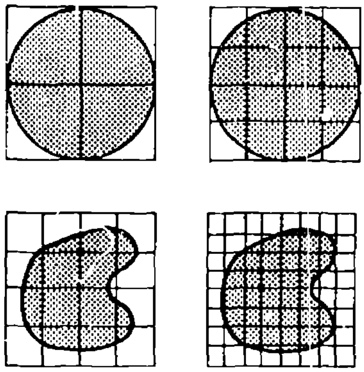
Behavioral Objectives

Eighth Grade

Measurement

1. Use formulas to calculate volumes and surface areas of spheres, cylinders and cones.
2. Determine the relative error given a measuring instrument and an object to measure.
3. Determine the sum of the measures of the angles of regular polygons.
4. Recall the relationship between degrees, minutes and seconds as units of angular measurement.
5. Recall the relationships between units of the Metric System.
6. Do computation with measures (approximate numbers) and determine the greatest possible error and the relative error of each result. (Addition, subtraction, multiplication, division)
7. Use the Pythagorean relationship to determine the length of any side of a right triangle given the lengths of the other two sides.
8. Solve problems involving indirect measurement using the sine, cosine and tangent ratios. (Including use of tables)
9. Improve results obtained by estimating the area of plane figures through the use of grids by using grids containing smaller units. Examples:



Mathematical Sentences

1. Find solution sets of simple open sentences using as a universe the integers. (Examples: $5X + 4 = -16$.)
2. Graph using the Cartesian plane $I \times I$ (integers) solution sets of inequalities and equalities such as $x + y \neq 19$; $x + y > 19$; $x + y < 19$; $x + y = 19$.
3. Define equivalent open sentences as sentences having the same solution set.
4. Find solution sets of open sentences by finding equivalent open sentences using the properties of equality.

Mathematical Topics

Ordered Pairs, Relations and Functions

An ordered pair of numbers is simply a pair of numbers such as (4,7). The order in which the numbers are named in the pair is important. Thus the ordered pair (4,7) is not the same as the ordered pair (7,4).

Ordered pairs may be used to represent rates, comparisons, integers, rational numbers, vectors, and coordinates of a point in a plane.

A *relation* is a set of ordered pairs. A *function* is a relation where the first element in each ordered pair is unique (not the same as any other first element).

Remarks

Students should become familiar with the basic properties of ordered pairs, relations and functions and should know how to represent relations and functions graphically. These ideas (ordered pair, relation, function) should be introduced using an intuitive approach.

Students should be able to: ▷

Statistics and Probability

Statistics refers to the ways of collecting, organizing, analyzing, interpreting and summarizing numerical data of all kinds. Inferential statistics refers to a means of using data from a relatively small representative sample to predict information about a total population. Since sample data generally does not give exact information about the population, the art of decision-making or inference from incomplete data involves some knowledge about the theory of probability. The level of probability that one assigns to an inference about a certain population indicates the degree of certainty with which one can expect the same result to occur if representative samples of the same size are repeatedly drawn from the same parent population.

The study of probability includes the study of experiments involving chance events, the outcomes of such experiments, and the likelihoods that particular outcomes will occur. The *probability* of an outcome for a particular experiment is a numerical measure of the likelihood that the outcome will occur. Thus the theory of probability can be considered to be the methods for assigning probabilities to outcomes of experiments and the study of the relationships among them.

Remarks

In grades 7 and 8 the study of statistics should include drawing inferences from data as well as the organization and representation of data. The study of probability should begin at the intuitive level, drawing upon the notions of chance which the students have already formulated based on their experiences. A substantial amount of experimentation and work with concrete materials should be included to "test" students' intuitive notions. Abstract symbolism should only be used to represent generalizations that are obtained through less formal laboratory activities.

Students should be able to: ▷

Behavioral Objectives
Grades K-5

Ordered Pairs, Relations and Functions

Statistics and Probability

Behavioral Objectives
Sixth Grade

Ordered Pairs, Relations and Functions

Statistics and Probability

Behavioral Objectives

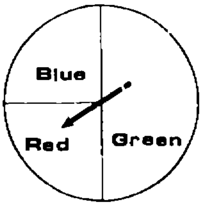
Seventh Grade

Ordered Pairs, Relations and Functions

1. Determine equality of two given ordered pairs of numbers. Example: $(2,3) \neq (3,2)$.
2. Use ordered pairs of real numbers as coordinates of points in a plane. Example: $(2,3)$, $(-5,7)$, $(-\frac{1}{2},-3)$, $(\sqrt{2},-8)$.
3. List the members of a Cartesian product such as A × B where Set A has elements 1, 2 and 3, and Set B has elements 2, 4, −6, and 0, and graph A × B. (A × B is read "A cross B").
4. Calculate the number of elements in a Cartesian product given the number of elements in each set.
5. Identify *relations* such as parallelism, perpendicularity, equality, not-equal-to, congruence and similarity.
6. Identify *relations* such as is-a-factor-of, less than, greater than, is-a-subset-of, is-an-element-of, is-the-square-of, etc., and formulate a set of ordered pairs of elements from each. Example: Using the relation is-a-factor-of in the following set of numbers, {1,2,3,6} formulate the set of ordered pairs identifying the relation. Answer: { (1,1), (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6), (6,6) }.
7. Graph relations (including functions) in the planes I × I (integers), Q × Q (rationals), and R × R (reals).

Statistics and Probability

1. Organize and present data using a frequency table and graphs.
2. Read and interpret the information about a set of data presented in the form of a frequency table or a graph.
3. Distinguish between certain and uncertain events.
4. Recognize when the outcomes for a given experiment are equally likely. For example, when flipping a fair coin the outcomes "head" and "tail" are equally likely.
5. List and/or count all the possible outcomes for an experiment for which the simple events are single elements. For example, spinning a spinner, tossing a die, drawing a lot, flipping a coin.
6. Assign probabilities to the outcomes of an experiment for which the simple events are equally likely. For example, the probability of tossing a "2" with one toss of a die is 1 chance out of 6 or 1/6.
7. Recognize that the probability of an event is a number p such that $0 \leq p \leq 1$, and recognize that the probability of a certain event is 1 and the probability of an impossible event is 0.
8. Assign probabilities to the outcomes of experiments for which the simple events are not equally likely. For example, the probability of the arrow stopping on red on the spinner at the right is 1 chance out of 4 or ¼.



9. Recognize that the probability of an event does not guarantee how often the event will occur. For example, the probability of obtaining a "2" when a fair die is tossed is 1/6; however, this does not mean that the "2" will occur once out of *every* six tosses.
10. Estimate the probability of an outcome of an experiment by empirical methods. For example, estimate the probability of a bottle cap landing with the cork side up when it is flipped (the student can flip a bottle cap 100 or more times, keeping a record of how the cap lands, and estimate the probability from his data).

Behavioral Objectives

Eighth Grade

Ordered Pairs, Relations and Functions

1. Given a domain (such as a subset of the integers) and given a description of a relation, determine the set of ordered pairs and graph the relation. Example: Domain is {—3,—2,—1,0,1,2,3} and the description of the relation is "double the number and add 3." The determined set of ordered pairs should be { (—3,—3), (—2,—1), (—1,1), (0,3), (1,5), (2,7), (3,9) }.

2. Determine whether a given relation is or is not a function.
   Examples: $\{ (a,b) \mid b=a^2, a,b \in R \}$ is a function but
   $\{ (a,b) \mid a=b^2, a,b \in R \}$ is not a function.

3. Given the graph of a linear function, determine a set of ordered pairs and describe the function.

Statistics and Probability

1. List and/or count all the possible outcomes for an experiment for which the simple events are ordered pairs, simple combinations, or simple permutations. For example, flipping two coins, tossing a pair of dice, rearranging the order of three letters of the alphabet, spinning a spinner and simultaneously tossing a coin, etc.
2. Assign probabilities to the outcomes of an experiment for which the simple events are ordered pairs, combinations or permutations. For example, the probability of getting 2 heads when 2 coins are flipped simultaneously is 1 out of 4 or ¼.
3. Recognize how the probability of a specific outcome will change from trial to trial when sampling is done without replacement. For example, from an urn containing 3 red marbles and 2 white marbles the probability of drawing a red marble is 3/5, but the probability of drawing a red marble if the first is not replaced then is either 2/4 or 1/4.
4. Estimate the probability of an event from previous data. For example, a basketball player made 50 free throws out of 75 tries; what is the probability that he will make his next free throw attempt?
5. Estimate information about a population by random sampling. For example, out of 100 flash bulbs selected at random 5 were defective.
   How many out of 3000 are likely to be defective?

Problem Solving

A situation becomes a problem for a student only if the following conditions exist:

1. A question is posed (either implicitly by the materials being studied or explicitly by the student) for which an answer is not immediately available.

2. The student is sufficiently interested to feel some intrinsic need to find a solution.

3. The student has enough confidence in himself to believe a solution is possible.

4. The student is required to use more than immediate recall or previously established patterns of action to find a method for arriving at a conclusion.

Not all pupils will consider a given situation a problem. If a student can see a "method of solution" immediately, then the situation is not a problem for him, but simply an exercise or an application of some process which he has already mastered. For example, an exercise such as 3×14 is not a problem for a student who has already mastered the idea of multiplication with two place numerals. However, this example could very well be a problem for a student who has studied only a few elementary multiplication combinations. In order for this student to obtain a correct solution, he would have to think of some method of renaming 14 such as $10 + 4$ and find some method of reducing the problem to simpler stages with which he could work, such as $3 \times 14 = 3 \times (10 + 4) = (3 \times 10) + (3 \times 4)$.

Real problem solving is not recalling a numerical fact or fitting a situation into a memorized pattern, but is discovering one's own method for extending previous learnings to new situations.

It is not the intent of this chapter to consider all of the general aspects of problem solving. Only some of the problem situations that involve the use of known mathematical ideas by students will be discussed. The verbal problems found in textbooks make up a large portion of the problem situations considered in mathematics programs. When such problems are presented, the mathematical ideas necessary to solve them are generally known to the student, but the contexts in which they are presented require the student to find the methods of solution or to decide which of the ideas he already knows will help him find the solution.

The solution of a typical textbook verbal problem involves a series of four interrelated steps. The complexity of each of the steps varies with the problem, but the method of solution follows this general pattern.

Step 1 — Recognizing the question being asked.

Step 2 — Translating the verbal problem into a mathematical sentence.

Step 3 — Finding a solution for the mathematical sentence.

Step 4 — Analyzing the solution for the sentence to see if it provides a reasonable solution for the original verbal problem.

Each step of the solution can involve many facets. Although all steps in the solution of a verbal problem are important, Steps 1 and 2, the abstraction or translation phase, can be considered the "heart" of the process.

The student may have to decide many things before he is able to write the mathematical sentence which represents the problem. If the question is not explicitly stated in the verbal problem, the student must formulate his own question or questions.

He will have to decide if all of the necessary data for the solution is given in the statement, and he will have to select the pertinent data from the given information. He will have to decide which mathematical operation is suggested by the "action" of the problem. He must determine the order in which the data of the problem will appear in the corresponding sentence. The student must also decide what relationships, if any, are involved that will enable him to use previously solved problems as "models" and what approach is the most efficient or easiest for him to use in obtaining a correct solution. Steps 1 and 2 could also involve some trial and error activity in which different sentences are tested as to which one best fits the given situation. These steps include problem solving techniques commonly referred to in many textbooks and teacher guides as "Reading and understanding the problem" and "Restating the problem in your own words."

Step 3 involves finding a solution to the mathematical sentence. This procedure can be very simple or very complex depending on the given situation. In general this step involves the application of knowledge that the student has already attained through practice work with similar sentence forms.

Step 4 is the familiar "check" in which the student determines if the solution is acceptable for the given problem situation. The student must decide if the solution is reasonable or "make sense." If it is apparent that the solution is not correct, the student must then "rework" his problem, check his computations, examine his sentence to see if it actually symbolizes the "story" of the problem, and look for careless errors or possible misinterpretations or misrepresentations of the data.

Some examples of typical verbal problems are given below.

In the first grade, simple "picture" or oral problems are presented which can be represented by sentences like 5 + 2 = □ where the sum does not exceed ten. The problem might be presented to the students in this form: "John has 5 toy cars. His mother gave him two more toy cars for his birthday. How many toy cars does he have now?" The "action" of the story is the *joining* of two sets of toy cars, and the mathematical sentence which represents this problem is 5 + 2 = □. Through such presentations, the student will learn to associate the notion of "joining" with the operation of addition. He will thus have a better understanding of what addition means and will develop techniques for solving similar problems.

In the second and third grades, problem situations are extended to include verbal problems represented by sentences like 3 + □ = 8, □ + 5 = 9, 7 − 2 = □, 8 − □ = 5, and □ − 2 = 6. A typical verbal problem at these grade levels is: "Mary had 15 doll dresses. After she gave some of the dresses to her sister, she found that she had 8 dresses left. How many doll dresses did she give to her sister?" This problem is represented by the sentence 15 − □ = 8. The "action" of the problem is "taking away"; therefore, the corresponding operation of the sentence must be subtraction. The 15 represents the number of dresses Mary had in the beginning, the □ represents the number of dresses she gave to her sister, and the 8 represents the number of dresses she had left. Students should realize that although 15 − □ = 8, 8 + □ = 15, and 15 − 8 = □ can be represented by different physical situations, the computation involved in solving each of the sentences is the same.

In the later elementary grades, problems are presented that involve larger numbers as in sentences such as 48 + □ = 327 or 239 − □ = 76 and that use the same principles as do the problems first introduced in the primary grades.

In the third and fourth grades, problem situations represented by sentences such as 3 × 7 = □, 5 × □ = 20, and □ × 6 = 30 are introduced. A sample problem for this grade level is: "Tom found that he needed sixteen small cartons to cover the bottom of a packing case. If he needed four layers of cartons to fill the case, how many small cartons were in the case?" The "action" of this problem is repeated addition or multiplication. Thus the sentence used to represent the story could be 16 + 16 + 16 + 16 = □ or 4 × 16 = □. The student should be allowed to use either sentence to represent the problem, but should be led to realize that the sentence 4 × 16 = □ is the shorter way of writing a sentence for this type of problem. In the latter sentence, the 4 represents the number of sets, the 16 represents the number of objects in each set, and the □ represents the total number of objects in all of the sets.

In the sentence $5 \times \square = 30$, the 5 represents the number of sets, the □ represents the number of objects in each set, and the 30, the number of objects in all. Likewise the sentence $\square \times 6 = 30$ represents a problem situation in which the total number of objects (30) is given, the number of objects in each set (6) is given, and the number of sets, represented by the □, is the solution to the problem. The sentences $5 \times \square = 30$ and $\square \times 6 = 30$ represent different physical situations, but both types of sentences can be solved by the same computational method of repeated subtraction if the multiplication fact that makes the sentence true is not known.

Problem situations introduced at the fourth and fifth grade levels are similar to the problem: "If John separates 12 marbles into small groups with 4 marbles in each group, how many groups of marbles will he have?" This problem can be represented by the sentence $\square \times 4 = 12$, for the question of the problem can be restated: "How many sets of 4 are there in 12?" The problem can also be interpreted as the division of a set of 12 objects into sets of 4 objects. With the latter interpretation, the problem can be represented by the division sentence $12 \div 4 = \square$. Both sentences, $\square \times 4 = 12$ and $12 \div 4 = \square$, represent the same physical situation and implicitly ask the same question, "How many sets of 4 are there in 12?" Both sentences are solved by repeated subtraction if the number which makes the sentences true is not known.

As fractions, decimal numerals, and integers are introduced and used in grades 6-8, problem situations involving their use in sentence forms similar to those considered previously are presented. It's important that students be able to write the sentence representing the "story" of a verbal problem.

The verbal "story" type problems are not the only source of problem situations for use in the school mathematics program. Many other mathematical problems arise from physical situations, from social or mathematical applications of mathematical ideas, or from situations made up by the teacher or the students. These varied situations can be real, imagined, or in the nature of a puzzle.

Many of these problems fall into a category which might be regarded as a "higher order" of problem solving than the typical textbook "story" problem. It is not possible to outline a regular progression of steps to be followed in solving all such problem situations as the process will vary with the nature of the problem presented and the ability and experience of the student. The four steps outlined for the solution of verbal problems do not necessarily apply in general problem solving situations, for it is not always possible to translate such a problem into a mathematical sentence.

However, it is possible to list a few of the activities that are essential in the general problem solving process. These activities include: translating the problem into a simpler form, critically examining the given data, forming hypotheses or conjectures, reasoning on a trial and error basis, analyzing or evaluating results on the basis of past experience, and forming generalizations from similar problem situations. These activities are not the only ones that may be involved, and not every problem situation will require the student to become engaged in all of the activities listed above. It is important to note that the order in which the student performs these activities may vary for different problem situations.

Teachers can do much to help students improve their problem solving abilities for all types of mathematical problems. The teacher's function should be to create a questioning, challenging atmosphere; to introduce problem situations; and to guide and encourage students to develop their own problem solving techniques.

A few suggestions that can help the teacher promote good problem solving techniques are listed below.

- The teacher should present problem situations that involve many basic mathematical principles. He should be certain that these situations are related to the kinds of experience the pupil has already had so that the pupil is able to apply the principles.
- The teacher should let his students use their own methods. Many problems have no single, best method of solution. Insistence on one "favorite" method of solution often destroys enthusiasm and original thinking.
- Whenever the problem permits, the teacher should emphasize the writing of a mathematical sentence that shows the "action" of the problem. If students are to acquire good problem solving habits, they must be able to describe the action of the problem in terms of mathematical symbols whenever applicable. Writing the mathematical sentence for a problem is as important mathematically as finding the solution for the sentence.
- The teacher should encourage his students to use diagrams, estimates, dramatizations, or other techniques that help them understand the problem.
- The teacher should suggest that students try various approaches to a problem when they are not certain of the correct method. Students can evaluate the method used by ascertaining that the solution is reasonable. Students should realize that the trial and error method can be an effective approach to difficult problems.
- The teacher should confront students with some situations in which they must formulate their own questions. This type of presentation is similar to the kinds of problems they are apt to face in their future vocations.

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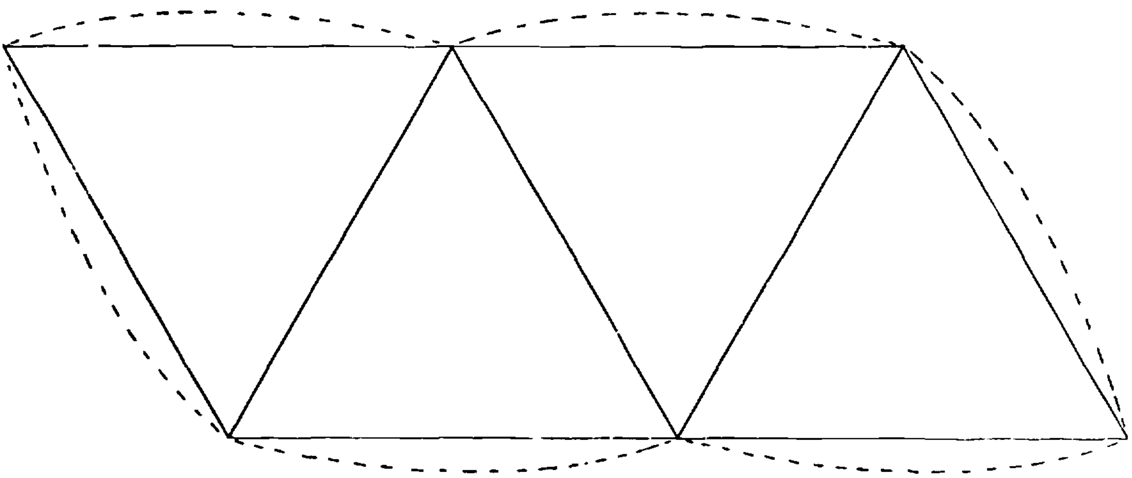

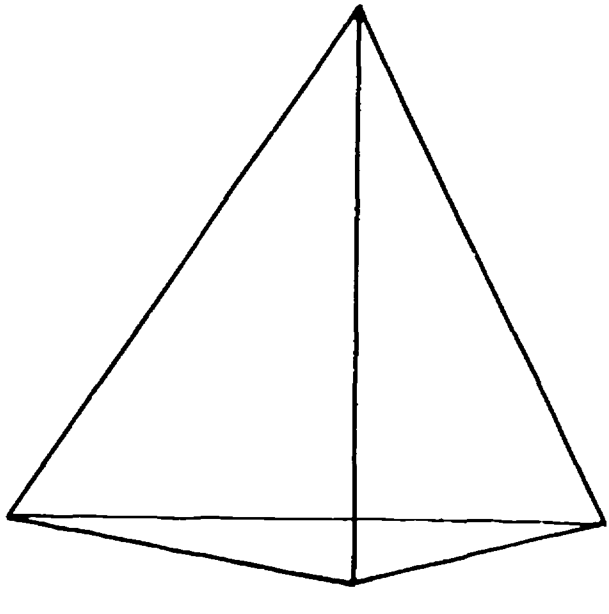

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