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ABSTRACT

This curriculum guide is specifically designed for the slow learning students in grade 8. It is one of a series of course guides for grades 6-11. The intent of the curricular designers was to outline mathematical experiences which would be appropriate for the characteristics of these students. The areas of mathematical content included are: 1) numbers, operations, and algorithms, 2) geometry, 3) measurement, 4) graphing, 5) probability and statistics, 6) algebra, 7) logic. Each content area contains: 1) master charts for grades 6-11, 2) grade level chart for grade 8, 3) behavioral objectives for the area, 4) teacher commentary sheets, 5) student worksheets. A collection of recreational activities is included for student motivation. (RS)

MATHEMATICS FOR BASIC EDUCATION

GRADE 8

Baltimore County Public Schools

BALTIMORE COUNTY PUBLIC SCHOOLS

Mathematics for Basic Education

Grade 8

A Tentative Guide

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FOREWORD

The last ten years have reflected a growing awareness and concern by mathematics educators for slow learners. Recent conferences sponsored by the National Council of Teachers of Mathematics and the School Mathematics Study Group have focused their attention upon the various aspects of the slow learner--economic, sociological, psychological, and pedagogical. Researchers are now able to present some of the preliminary findings of their work to curriculum specialists. Curriculum materials are being developed, but these are limited and have not yet been produced commercially.

The Office of Mathematics has been deeply involved in this pioneering stage. Staff members have participated in the recent NCTM conference, attended national conventions which have presented important addresses on the slow learner, and invited consultants for assistance and guidance. The summer workshops in 1963 and 1964 produced bulletins which presented guidelines and recommendations for the mathematics education of the slow learner. The summer workshop in 1966 produced a resource manual of activities--developmental, recreational, and computational--as a first stage in creating a mathematics program for the slow learner. The efforts of this 1967 summer workshop have been directed toward providing structure and continuity of the desired behavioral outcomes as they relate to mathematical concepts and skills

This publication is specifically designed for the slow learning students in Grade 8. It is one of a series of tentative course guides in Mathematics for Basic Education, Grades 6-11. It will presume teacher evaluation in terms of student reaction and behavioral responses. The teacher is urged to study carefully the philosophy of behavioral objectives as stated in action words and its implications for instruction and assessment. These ideas are stated in the following sections of the Introduction.

The Board of Education and Superintendent of Schools wish to express their appreciation to the curriculum committee and to all mathematics teachers of Baltimore County whose effort made possible the development of this curriculum publication. The staff wishes to acknowledge the valuable contributions of Dr. Henry H. Walbesser, University of Maryland, and Dr. Max A. Sobel, Montclair State College.

Special commendation is due Mrs. Betty V. Hagedorn, Mrs. Barbara S. Parks, and Miss Martha Ann Lynch for their careful and painstaking effort in the production of this bulletin.

William S. Sartorius
Superintendent of Schools

T son, Maryland
ember, 1967

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INTRODUCTION

A. PHILOSOPHY

The rapid trends toward greater automation, use of computers, and the increased technological skills demanded of workers have dramatically reduced the market for unskilled and semi-skilled laborers. Twenty percent of our population consists of persons, who, according to their academic talents, are termed "slow learners." It becomes apparent that young people of limited ability, who are potential unskilled and semi-skilled workers, must be prepared for a useful place in our society. Unless these youths are taught salable skills, they must be supported by tax money. This situation could lead to a society composed of one segment which is overworked to contribute tax dollars and services, and another segment which is unemployed and consumes the wealth, yet produces nothing.

The magnitude and urgency of this problem demand that schools develop appropriate educational opportunities for slow learning students throughout their school experiences. The schools are thus faced with the problem of training students for jobs and services which may be outmoded by the time they enter the business world. Equally disturbing is the fact that no one can foretell the many new jobs and products which will be created for which no training has been provided. It is generally conceded that the service occupations hold the greatest promise of employment for the slow learner. Functional competence in mathematics is essential for all persons entering these service occupations. Industry is retaining many semi-skilled and unskilled workers who have been displaced by automation through extensive retraining. Reports indicate that greater success is obtained in retraining workers who have more mathematics background than those who do not. Furthermore, training in mathematics provides youth with broader choices of vocational employment. It is imperative that the student be given a sound foundation in mathematics if he is to function effectively as a producer and consumer, and a citizen in his community.

It is axiomatic that the slow learner should be educated in his own right and to the maximum of his ability. Any adaptation of an academically oriented program must surely fail. A program of mathematics for the slow learner should be based upon the latest developments and research in learning theory, an appropriate selection and reorganization of mathematical topics, and the inclusion of new materials as well as new techniques for presenting mathematical concepts and developing skills. Proper pacing of these concepts and skills must underlie the entire structure. All the human resources of the educational system - the mathematics teacher, the principal, the mathematics supervisor, resource teachers, the guidance counselor, and other specialists - must be brought to bear on this problem.

Probably the most important factor in the success of a mathematics program for the slow learner is the teacher. Such a teacher should be prepared psychologically to teach students of limited ability. This implies an acceptance of the student for what he is, and an awareness of the operational level of the student. Furthermore, the teacher should have such characteristics as emotional maturity, a broad background of mathematics and a curiosity for more, a liking for young people, patience, and above all, a sense of humor. Such a teacher can do much to enhance the usually poor self-image of the slow learner, and convince the student that he is indeed a person worthy of dignity and respect in this society.

B. IDENTIFICATION OF THE SLOW LEARNER

The most obvious characteristic of the slow learner is his inability to keep pace with those students who are average in their rate of academic growth. However, other psychological, social, cultural, and physical factors may be considered in identifying these students. The following criteria, which are divided into two categories, may be used to form a basis for the selection of the students who may be classified as slow learners. The two criteria - measurable and traits - should receive equal consideration when the student is being identified.

Measurable Criteria

1. I.Q. Range 75 - 90 resulting from at least two group tests or an individual test.
2. Percentiles on group tests of mental ability and achievement ranging from 0 - 19 (approximately two or more years below grade level in reading comprehension and arithmetic.)
3. Teacher grades - consistently below average, as indicated by "ability" C's and D's as well as E's.

Traits Criteria

1. Limited academic interest
2. Difficulties in planning and in carrying out work without supervision
3. Limited creativity and intellectual curiosity
4. Indications of short attention span
5. Severe limitations in the ability to communicate orally or in writing.

C. BEHAVIORAL OBJECTIVES

An integral part of any collection of instructional materials is a statement of the objectives. This bulletin is no exception. The objectives stated here are stated in behavioral terms. That is, each objective is stated in terms of the desired student behaviors.

To clarify, consider the following example of a behavioral objective which is taken from the Grade 7 geometry section of this bulletin.

> The student should be able to construct a drawing of a quadrilateral using a straightedge.

The characteristics of this objective are that it tells who is to perform, how he is to perform, and what constitutes an acceptable performance.

To assess the acquisition of the above stated behavior it is only necessary to give a student paper, pencil, and straightedge and instruct him to make a drawing of a quadrilateral. In response, the student can either make such a drawing or he cannot. In any event, it is possible to decide whether or not the stated objective has been realized. Any well stated behavioral objective should point clearly to the type of performance task necessary to assess its attainment.

The clarity of a behavioral objective such as the one stated above is in clear contrast to the vagueness of comparable objectives which state that the student should "understand the concept of quadrilateral" or that the teacher should "develop the concept of quadrilateral." These and other objectives such as "developing appreciations and attitudes" do not lend themselves well to evaluation. Indeed, the assessment of these qualities have always posed difficulties for researchers.

Behavioral scientists such as Jean Piaget and Robert Gagne have asserted that true learning involves a change on the part of the learner so that he no longer reacts as he did before. His whole being views similar situations in a new light. If our instructional program is to effect such changes in slow learning students, then the objectives such be so constructed that they specifically state the desired behavioral responses which are observable and hence can be assessed.

D. ACTION WORDS USED IN STATING BEHAVIORAL OBJECTIVES

The action words which are used to construct behavorial objectives are:

1. IDENTIFY

The student selects by pointing to, touching, picking up, or circling the correct object or class name. This class of performances also includes identifying object properties such as rough, smooth, straight, curved.

e.g. The student should be able to identify the prime numbers from a given set containing prime and composite numbers.

2. DISTINGUISH

The student identifies objects or events which are potentially confusable. This is a more difficult identification.

e.g. The student should be able to distinguish between ordered pairs such as (a, b) and (b, a).

3. CONSTRUCT

The student generates a construction using instruments, a freehand drawing, or by building a model.

e.g. The student should be able to construct a copy of an angle given a straight edge and a compass.

The student constructs an answer or example. The teacher is concerned only with the student's ability to construct the answer or example, not the method or procedure he uses in arriving at the solution.

e.g. The student should be able to construct the product of a fraction and a whole number.

4. NAME

The student supplies the correct name for a class of objects or events orally or in written form.

e.g. The student should be able to name the associative property of addition in the set of whole numbers.

The student names the correct solution to a problem. This is different from construct in that an immediate response is expected. In this sense Name is used in relation to the basic arithmetic facts which students should commit to memory.

e.g. The student should be able to name the addition facts through 9.

5. ORDER

The student arranges or classifies two or more objects or events in proper order in accordance with a stated category. This word is used when the student arranges something from largest to smallest, most to least, or fastest to slowest.

e.g. The student should be able to order a set of whole numbers from largest to smallest.

6. DESCRIBE

The student states all the necessary categories or properties relevant to the description of a designated situation. The student's description must be stated so clearly that any other individual could use the description to do a task, identify an object, or perform an operation. The description is mostly verbal, however a model, hand motions, or a written example could be used to aid in the description.
The teacher must be willing to accept more than one response. For example, the student might describe something in terms of his surroundings, by using example or by stating a definition. The description may include color, size, shape, etc.

e.g. The student should be able to describe sample spaces as ordered arrangements, listing all possible outcomes.

7. STATE A PRINCIPLE OR RULE

The student makes a verbal statement which conveys a rule or principle. This is more limiting than describing in that only one basic response is acceptable. Students may use their own words in stating the rule. For example, when asked the question, "How do you find the area of a square?" A student may respond, "To find the area of a square, measure the length of a side and multiply this number by itself," or "$A = S^2$"-- both are acceptable answers. Any formula, theorem, or definition is a statement of a rule.

e. g. The student should be able to state the principle that the circumference of a circle equals pi times diameter, ($C = \pi D$).

8. APPLY THE RULE

The student uses a rule or principle to derive an answer to a question. The question is stated in such a way that the student must employ a rational process to arrive at the solution. Students might not be able to state the rule, however, he may still be able to apply it.

e. g. The student should be able to apply the principle of casting

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nines to check addition problems involving whole numbers.

9. DEMONSTRATE

The student shows a procedure or test for the application of a rule or principle. The teacher wants the student to show how he arrived at an answer, not just the answer alone. This usually involves some action, other than verbal, on the part of the student

.g. The student should be able to demonstrate a procedure for finding the least common multiple of a given pair of numbers.

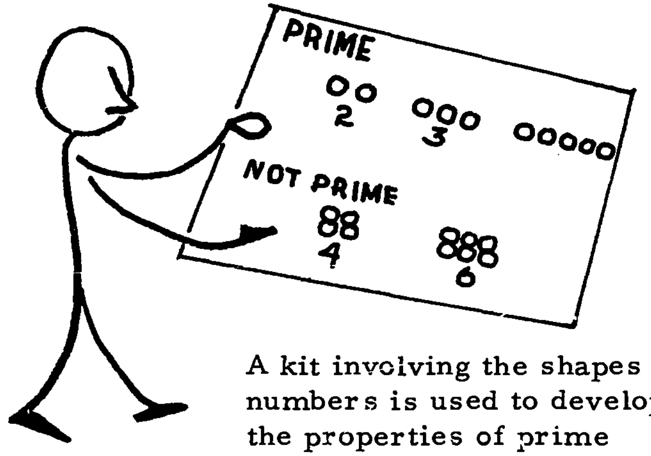
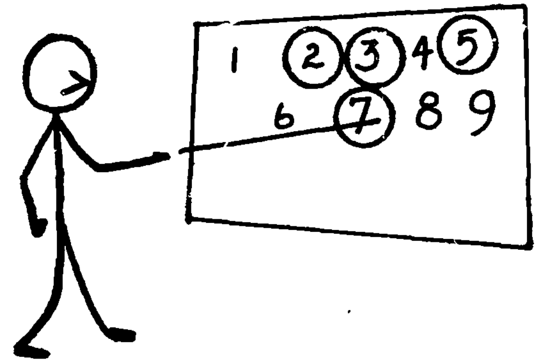
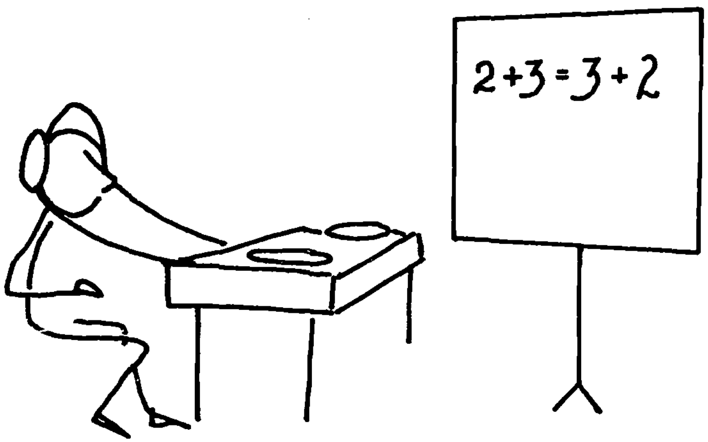
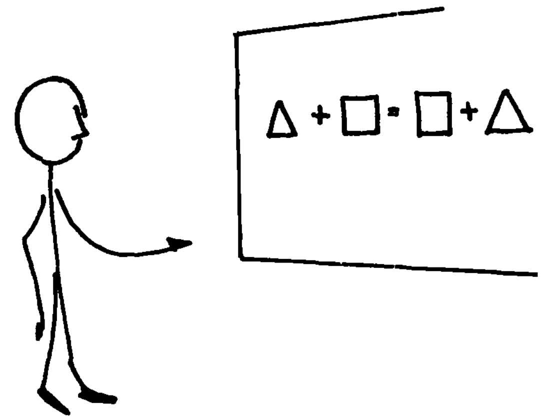
10. INTERPRET

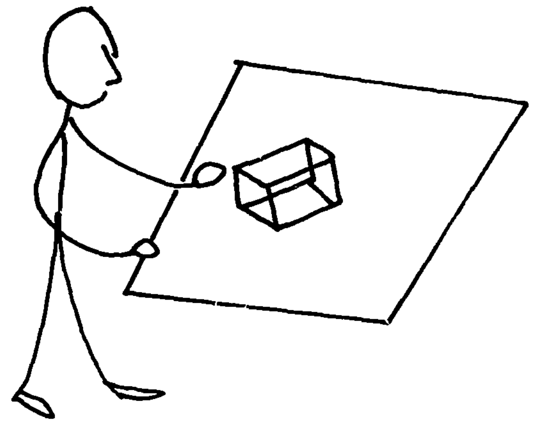
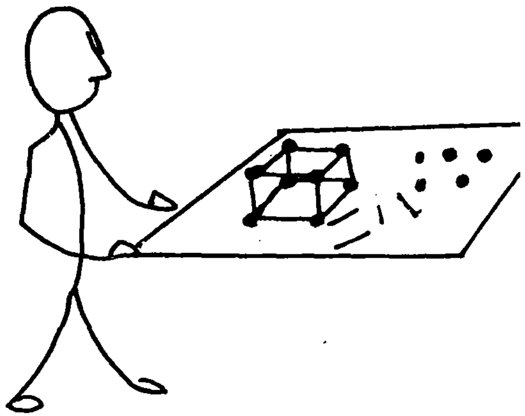
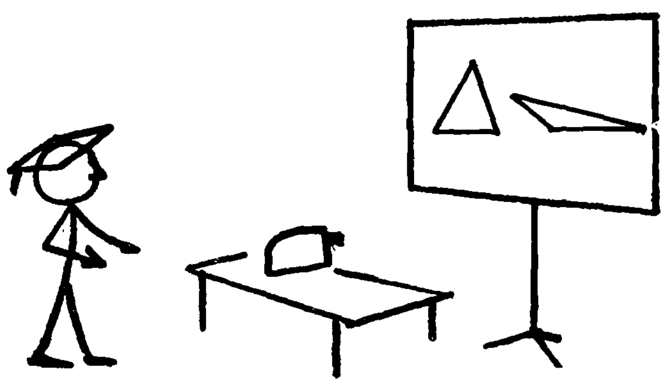
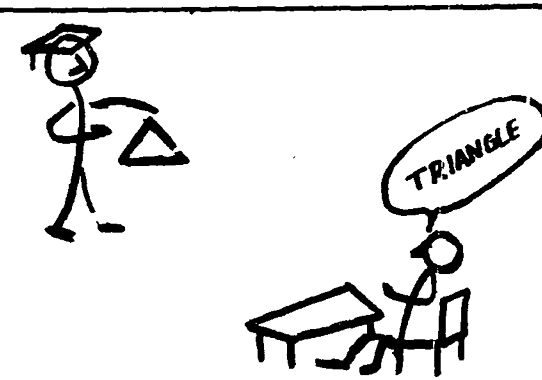
The student uses several rules or principles to draw a conclusion, or identifies objects and/or events in terms of their consequences. This constitutes a high level of learning since the student must see various relationships in order to arrive at the desired conclusion.

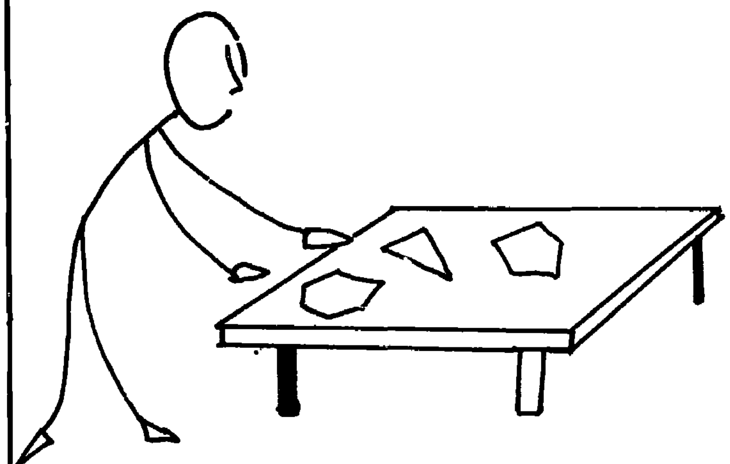
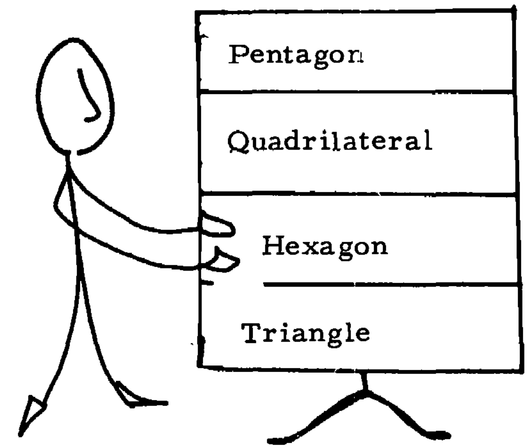
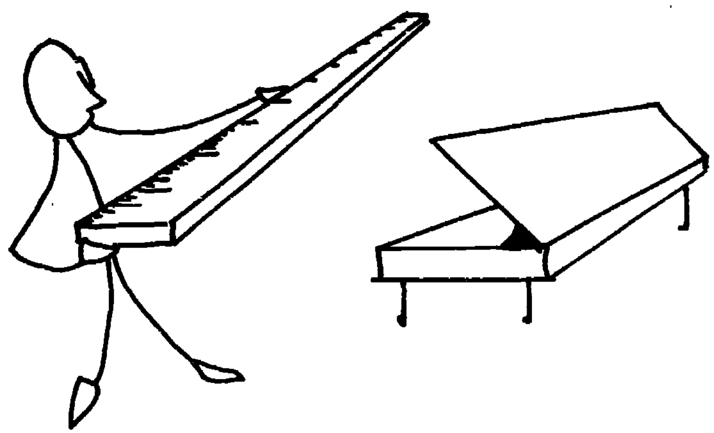
e.g. The student should be able to interpret the principles of angle measure by measuring and then classifying angles as right, acute, obtuse, straight, complementary and supplementary.

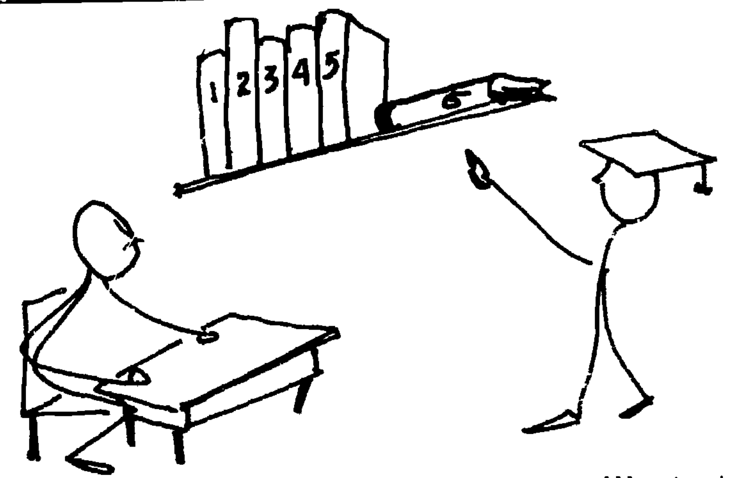
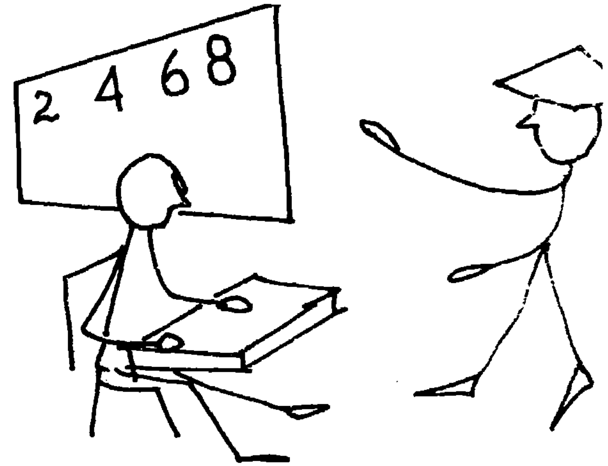
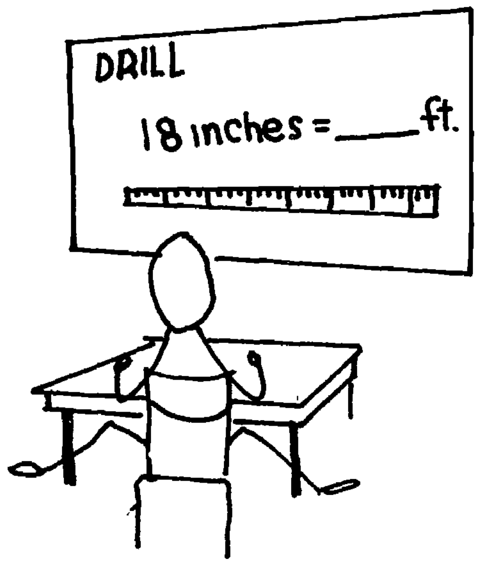
E. OBJECTIVES - INSTRUCTION - ASSESSMENT

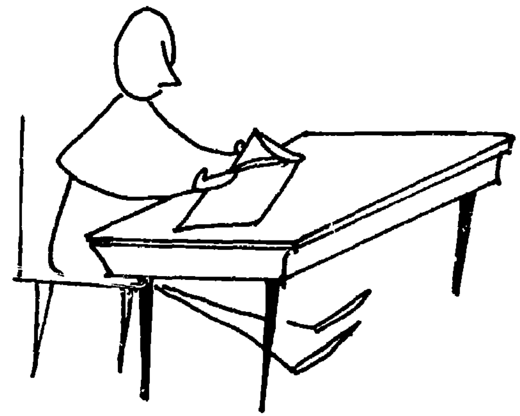
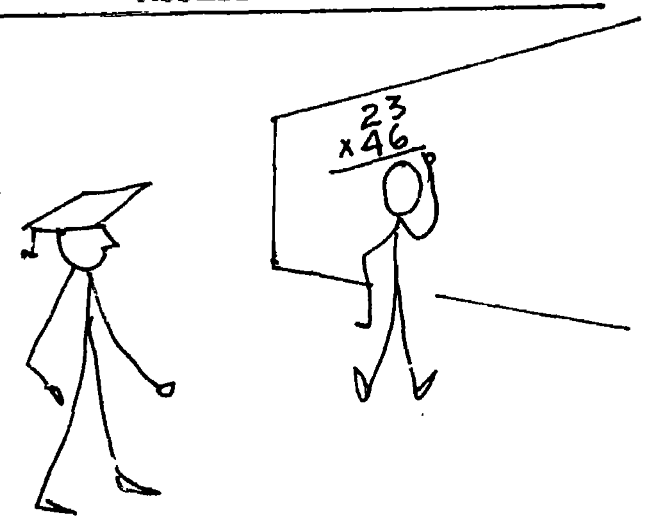
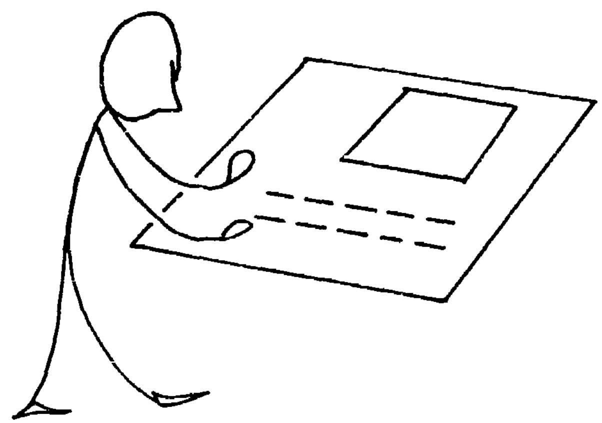
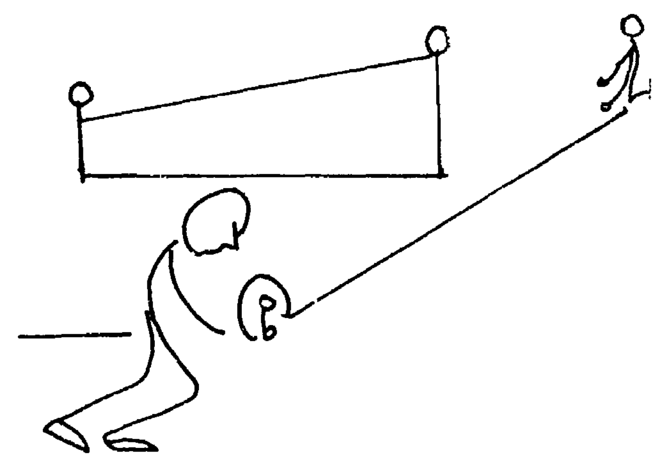
In each of the instructional activities included in this grade the focal point of most comments is the student. First the objectives of the activities are stated in terms of the desired behavioral outcomes on the part of the student. Secondly, the lessons are devoted primarily to student activities. Finally, the suggested assessment procedures indicate ways in which the student shows whether or not he has acquired the desired behavior. In effect, there should be a one-to-one correspondence between the set of objectives, the set of learning activities, and the set of assessment items. The following examples should clarify the relationship.

OBJECTIVE	INSTRUCTION	ASSESSMENT
1. The student should be able to <u>identify</u> a prime number less than 25.	 A kit involving the shapes of numbers is used to develop the properties of prime numbers.	 Students are to circle the numbers which are prime.
2. The student should be able to <u>distinguish</u> among the commutative, associative and distributive properties of addition.	 A tape and filmstrip are used to present the properties.	 Students are given examples on the board and instructed to identify each property.

OBJECTIVE	INSTRUCTION	ASSESSMENT
3. The student should be able to construct a model of a cube given appropriate materials.	 Given straws and string make a model of a cube.	 Use toothpicks and gumdrops to build a model in the shape of a cube.
4. The student should be able to <u>name</u> a triangle.	 Filmstrip is used initially to present the name triangle.	 A model is used to assess the student's ability to name a triangle.

OBJECTIVE	INSTRUCTION	ASSESSMENT
5. The student should be able to order polygons in terms of increasing number of sides.	 Use models to order polygons in terms of increasing number of sides.	 Use the flannel board to order polyg in terms of increasing number of sid
6. The student should be able to describe a foot.	 Using kits students are introduced to the foot as a unit of measure.	 Student may describe a foot by a hand motion.

OBJECTIVE	INSTRUCTION	ASSESSMENT
7. The student should be able to state the principle that events which are certain to happen have a probability of 1.	 Six math books on a shelf are used to illustrate what is the probability of picking a math book if blindfolded.	 Student states "The probability of selecting an even number is certain to happen, so it has a probability of 1."
8. The student should be able to apply the principle that 12 inches equals 1 foot to convert a measure expressed in one unit to the other unit.	 A drill on converting inches to feet is used to introduce the topic.	 A written test is used for assessment purposes.

OBJECTIVE	INSTRUCTION	ASSESSMENT
9. The student should be able to demon-strate a procedure for constructing the product of two whole numbers.	 Student uses book to work problems at his seat.	 Students show work at the board.
10. The student should be able to interpret the rules for finding the areas of rectangles and computing in order to solve related problems.	 Students are asked to find the number of tiles necessary to cover the sheet of paper, and then compute to check their results.	 Students measure the dimensions of the tennis court, then compute to find its area.

F. THE BANDED APPROACH

Teachers who participated in the experimental program last year developed a method of teaching which seems to be effective with the slow learner. The key to the success of this method was that a variety of mathematical topics were incorporated into each lesson. Naturally, small group instruction, individual lab work, extensive use of audio-visual aids, games and the like provided the variety of activities within the lesson which is necessary to change the pace when working with this ability student. Thus, the student was exposed to a program of instruction which provided a variety of activities as well as a variety of mathematical content within a given period. This method of teaching will be referred to as the "banded approach."

To elaborate further the "banded approach" is a flexible way of organizing instructional activities in the class period. Normally the lesson is divided into three bands, although sometimes it may be divided into one, two, or even four bands depending on the nature of the activity. All bands are not necessarily concerned with the same mathematical topic. For example, a unit on Geometry might be taught along with related activities on Fundamental Operations. Thus, the unit on Geometry is split into smaller parcels and presented over a longer period of time rather than being presented as a two week concentrated unit. Since basic students typically have short attention spans, the material must be presented within a smaller time interval. Thus, the major portion of the lesson might be presented during a 25 minute segment since this seems to be about the maximum length of time these students can concentrate on any one activity.

Description of Bands

Band I is usually a short activity of about 5 - 10 minutes duration. For example, students may review their addition facts using the Math Builder, have an oral number puzzle, or complete a number pattern. The variety of activities which might be used is numerous.

Band II usually contains the major topic for the day. It is about 25 minutes in length. For this activity, specific behavioral objectives are stated. Students are exposed to instructional activities which are designed to enable them to acquire the desired behaviors. Assessment procedures might also be employed here to determine whether students have acquired some of the behaviors specified in the objectives. Remaining objectives may be assessed in other bands of subsequent lessons.

Band III is usually a short activity of about 5 - 10 minutes. This band can be handled two ways. First, all the students might begin work at the same time on a class activity. Secondly, as each student completes his work in Band II he begins some planned individual or small group activity. For example, after a student has completed his work from Band II, he may go to a specified place in the room and pick up an interesting puzzle or game, work on one of the SRA kits, or listen to a tape at the listening post. This approach keeps students actively involved in learning activities rather than just waiting for the class to finish an assignment. Thus, a more efficient use of the student's time is made.

The teacher should realize that the above descriptions indicate a general outline of what constitutes a banded approach. Flexibility is the key. Teachers should vary the number of bands as well as the length of time devoted to each depending upon what is being presented.

To illustrate how this approach could be implemented with your students, a sample two week unit is included in section H at the end of the Introduction. This unit contains:

1. A block plan indicating the topics to be presented each day.
2. Detailed lesson plans indicating the materials to be used, the behavioral objectives, suggested methods for presentation, student work sheets, and assessment items.
3. A series of inventory tests designed to indicate areas of difficulty.

This two week unit should be taught near the beginning of the school year. It is hoped that this unit will provide a model from which the teacher can create other units utilizing the same approach.

G. HOW TO USE

This guide provides a structured program of instruction for the slow learner in mathematics grades 6 - 11. Suggestions for implementing the program are included. The teacher is urged to read this section carefully. Familiarity with the materials included and suggestions for their use should be of great assistance in determining the best program of instruction for basic students.

The guide is divided into the following major areas of mathematical competency: Fundamental Operations, Geometry, Measurement, Graphing, Algebra, Probability and Statistics, and Logic. Recreation is the last section of the book.

Each of the areas of mathematical competency contains the following items:

MASTER CHARTS

These charts give an overview of the mathematical content and the behaviors students are to acquire in grades 6 - 11. The teacher can use these charts to get a picture of the total mathematics program for the slow learning student. Furthermore, the teacher can see which behaviors the students should have acquired prior to entering this grade, which behaviors will be developed during this grade, as well as those to be developed later.

GRADE LEVEL CHART

These charts are identical to the master charts except they contain only the information for a specific grade. They can be used to get an overview of those behaviors which should be acquired by the student during the school year.

LIST OF BEHAVIORAL OBJECTIVES

A list of behavioral objectives for this grade should enable the teacher to interpret the details omitted in the chart. The teacher can use these objectives when planning lessons, since they state precisely what is expected of the student. The teacher should realize that these objectives should not necessarily be taught in the order they are presented, rather objectives from several areas might be used in order to present a logical development of the topic. However, by the end of the year the students should be able to exhibit all the behaviors mentioned.

Also included in this section are references to the student activities which have been developed. These activities have been specifically designed to bring about the desired behavioral changes indicated in the objectives. This should assist the teacher in identifying the type of activity which might be used when developing a particular topic.

STUDENT ACTIVITIES

This section contains a series of suggested activities.

For each activity a Teacher Commentary printed on yellow paper is included. This commentary indicates the title of the activity, the unit, the behavioral objectives, necessary materials, a procedure for implementation and suggested assessment items. Student work sheets are printed on white paper and immediately follow the Teacher Commentary. The teacher can reproduce these work sheets by taking the master copy out of the guide and making a thermal spirit master. The spirit master can then be used to run off copies for the students. Be sure to place the original copy back in the guide so it can be used again at a later date.

If color is desired it may be added by using colored masters before duplication.

The Student Activities section also includes references to:

1. Kits. These kits are effective devices for use in small groups or with individual students. Students perform various experiments and as a result of this experimentation, are lead to generalizations. The teacher is supplied with all the necessary instructions for constructing the kit as well as the accompanying student work sheets. It is suggested that the teacher use student help in the construction of the kits.

2. Tapes. Some tapes and their related student work sheets are included. These tapes cover a variety of topics on each grade level. It is suggested that these tapes be used with small groups of students using listening posts, rather than as a class activity.
3. Programed Instruction. Several programed booklets are included in the guide. These can be used with individual students or small groups for remedial purposes or when the teacher feels additional development might be necessary. The programed booklets can be reproduced using the same procedures outlined for the student work sheets. Again, it is suggested that student help be employed in assembling the programed booklets.
4. Films. Several films are included in the Teacher Commentary. These films can be obtained from the Baltimore County Central Film Library. They can be used with the banded approach since the average running time is between 10 - 15 minutes.

RECREATIONAL ACTIVITIES

The Recreation section of the guide is significantly different from the sections dealing with mathematical competencies. The activities described in this section are designed to develop a positive attitude towards mathematics. There are no behavioral objectives specified in this section. Games and puzzles play an important role in the teaching of mathematics. These activities are to be used throughout the year for motivational purposes. When using the banded approach, the recreational activities are used extensively since they help provide the variety which is necessary to the success of this method of teaching.

In the period of time allocated to produce this guide it was impossible to create activities in each of the areas. Therefore, provisions were made to supplement this guide as other activities are developed. Teachers are requested to send activities which they have found to be successful to The Office of Mathematics so that they may be added to this guide.

H. A SAMPLE UNIT USING THE BANDED APPROACH

TWO WEEK UNIT OF BANDED LESSONS - Grade 8

OUTLINE OF TOPICS

LESSON	BAND I	BAND II	BAND III
1	Drill-addition and subtraction of whole numbers	Introduction to graphing	Puzzle-addition and subtraction
2	Drill-multiplication and division of whole numbers	Pattern for "RaP" cubes	Cross Number Puzzle-four operations with whole numbers
3	Test-Part I, SRA	Results of SRA Test checked	Review of graphing
4	Drill-addition and subtraction of fractions, G. C. F.	Construction of cubes for "RaP"	Rules for "RaP"
5	Drill-multiplication and division of fractions, L. C. M.	Cross Number Puzzle-four operations with fractions	"RaP" played in pairs
6	SRA Survey Test-Part II	Results of SRA Test checked	Tape-verbal problems
7	Drill-multiplication by eleven	Puzzle-money	"RaP" played in pairs
8	Drill-solution sets	Pictures drawn using ordered pairs	Puzzle-hidden messages
9	Drill-addition and subtraction of whole numbers	Evaluation exercise on graphing	Pictures drawn using ordered pairs
10	Math Mate	Rolling Stone Teen Club Gets Started	Drill-time

LESSON	BAND I	BAND II	BAND III
11	Magic Circle	Rolling Stone Teen Club Gets Started	Pictures drawn using ordered pairs
12	Diagnostic Test	Diagnostic Test checked	Group work on exercise cards

LESSON 1

I. Unit: Graphing

II. Objectives: The student should be able to:

A. Name and identify a pair of axes

B. Demonstrate how to plot points in the first quadrant given the ordered pairs

C. Distinguish between ordered pairs such as (a, b) and (b, a)

D. Name and identify the coordinates of a point in the first quadrant

E. Construct the ordered pair given a point in the first quadrant

III. Materials:

A. Band I - None

B. Band II

1. A pair of cubes with sides numbered 1 through 6. Each cube should be a different color, e.g., one red, one white

2. Transparency with the game board on it

3. Overhead projector

4. Student work sheet entitled, "What is Your Graphing I.Q. ?"

C. Band III - None

IV. Procedure:

A. Band I - Written drill involving addition and subtraction of whole numbers

1. Have this drill written on the board for students to do as they come into your class:

What comes next?

a. 3, 7, 11, 15, ____, ____, ____. (Answers: 19, 23, 27)

b. 19, 16, 13, 10, ____, ____, ____. (Answers: 7, 4, 1)

c. 2, 4, 3, 5, 4, ____, ____, ____, ____. (Answers: 6, 5, 7, 6)

d. 5, 4, 8, 7, 11, ____, ____, ____, ____. (Answers: 10, 14, 13, 17)

2. Discuss patterns and answers with students.

3. The student should put the number of patterns correct out of four $\left(\frac{n}{4}\right)$ in the margin of his paper.

4. Have students put their drill paper in their notebook until tomorrow at which time they will use it to do a similar exercise as they come into your classroom.

B. Band II - Introduction to Graphing

1. Students will soon pair up to challenge each other in a game called RaP, (Roll and Plot). To play this game they will need a pair of cubes which they will construct and a game board. Have the game board made on a transparency. Show them what the game board looks like.

2. Use the game board on the transparency to introduce the term axis, (plural - axes). Write this word on the board and have the class pronounce it. Label the axes with a small letter to identify the color of the cube, e.g., r (red) and w (white). (Let us assume that one cube is red and the other white.) Note that the scale goes up to six, the largest number on each cube, and the lines, not spaces, are numbered. The axes are calibrated like a ruler.

3. Have a student come to the front of the room and roll the two cubes. Ask him for the number showing on each cube. Show where it is to be plotted on the grid. Do this with several other students. Emphasize the need for order.

4. Show how the result can be written as an ordered pair, e.g., (4, 3) shows a 4 on the red cube and a 3 on the white cube. To emphasize the order involved, have the student arrange the cubes in order (red, white) after they have been rolled. In plotting the point, go right the number of spaces shown on the red cube, then go up the number of spaces shown on the white cube.

5. Have about three or four other students come up to roll the cubes and put them in order (red, white). Have them write the results on the board as an ordered pair, then demonstrate how to plot the point on the game board.

6. Introduce the name coordinates. Write the word on the board and have students pronounce it. Ordered pairs such as (6, 3) which give the location of a point are called the coordinates of the point.

7. Now the student is ready to construct the ordered pair which corresponds to a given point on the grid. Roll the two cubes and plot the point which corresponds to your result. Tell

students to write the ordered pair which corresponds to the point on their paper. Ask a student to come to the board and construct the ordered pair which corresponds to your result.

8. Summary - Pass out the work sheet entitled, "What is Your Graphing I. Q. ?" Allow time for students to answer the ten questions. Use the transparency with the game board to help students check their answers.

C. Band III - Oral puzzle involving reversing digits, difference, subtraction, units digit, addition, sum

	Examples
1. Have each student write a 3-digit number on his paper. Each number should be different.	1. 729
2. Have them reverse the digits in the number.	2. 927
3. Find the difference of the two numbers. This is the answer.	3. 927 - 729 = 198

4. Have a student (one at a time) tell you the units digit in his answer. You can then tell him his answer.

   Solution:

   a. The sum of the units digit and the hundreds digit will be 9, the tens digit is always 9.

   b. From the example problem: Since the last digit is 8, the answer must be 198.

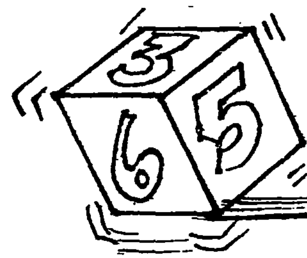
5. If time allows, you may go one step further with this puzzle.

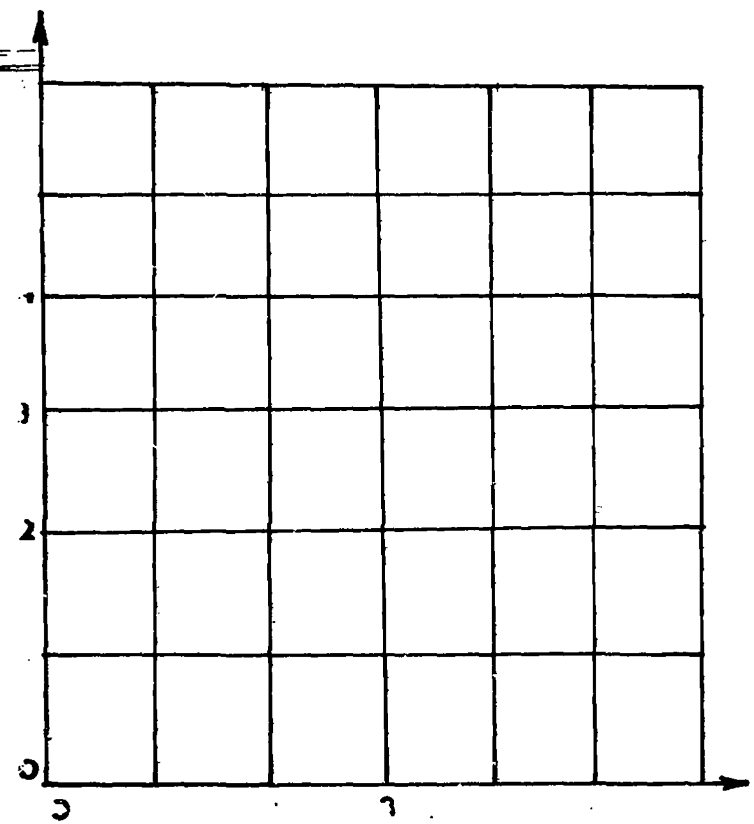
6. Reverse the digits in the difference. If the difference is a two-digit number, put a zero in the hundreds place then reverse the digits.	6. 891
7. Add this new number to the difference.	7. 891 + 198 = 1089

   Solution: Everyone should get the same sum, 1089.

8. Try the puzzle again. This time have students choose a different number.

ROLL PLOT





WHAT IS YOUR GRAPHING I.Q.? ______________ (Score)

Part I: Plot these five points on the grid. Name each point by its letter.

A. (4, 1)

B. (0, 2)

C. (3, 4)

D. (6, 1)

E. (2, 5)

Part II: Write the coordinates of the points which have been named on the grid.

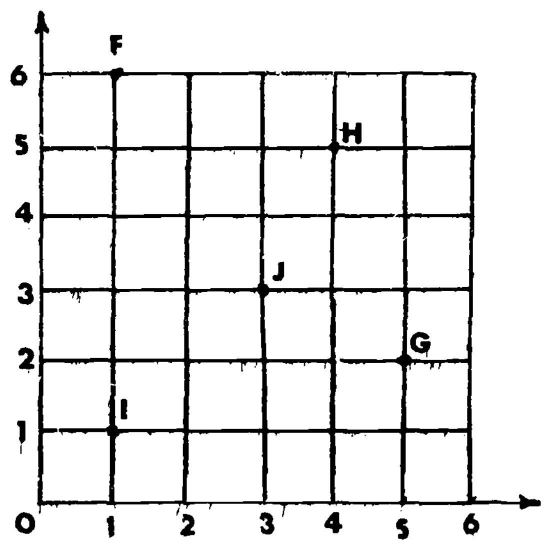
6. ____________

7. ____________

8. ____________

9. ____________

10. ____________



LESSON 2

I. Unit: Graphing

II. Objectives: The student should be able to:

Construct a model of a cube

III. Materials:

A. Band I - None

B. Band II

1. Pencil and ruler
2. Pattern of a cube for each student to copy
3. Two small pieces of tag board (4 in. x 5 in.) for each student
4. One straight pin for each student
5. Scissors
6. Felt pen or colored pencil

C. Band III - Cross Number Puzzle - Review of Whole Numbers

IV. Procedure:

A. Band I - Written drill involving multiplication and division

1. Have this drill written on the board for students to do when they come into your class.

   What comes next?

   a. 3, 9, 27, 81, ___, ___, ___. (Answers: 243, 729, 2187)

   b. 128, 64, 32, 16, ___, ___, ___, ___. (Answers: 8, 4, 2, 1)

   c. 2, 8, 4, 16, 8, ___, ___, ___, ___. (Answers: 32, 16, 64, 32)

   d. 1, 2, 4, 8, ___, ___, ___, ___. (Answers: 16, 32, 64, 128)

2. Discuss patterns and answers with students.
3. Have each student place his score in the margin of his paper (same as yesterday).
4. Students put drill paper into notebook.

B. Band II - Constructing the Pattern for the RaP Cube

1. Be sure each student has a pencil and ruler.
2. Distribute tag board, pattern and a straight pin to each student.

3. Center the pattern over one piece of tag board. Make sure that it doesn't move. Pierce the paper at each of the 14 vertices. Repeat this using the other piece of tag board.

4. Circle the pin marks lightly on the tag board.

5. Collect the straight pins.

6. Explain - Using the ruler, connect points 1 to 2, 3 to 6, 7 to 10, 11 to 12, 13 to 14, then 6 to 10, 2 to 14, 1 to 13, and 3 to 7.

7. Pass out scissors and have the student cut out his two patterns.

8. Use the pattern to number each face of the cube.

9. Make sure each student has written his name on the outside (front) of his two patterns. Collect scissors, pattern, work sheet, and tag board patterns.

10. Homework for teacher: The models of cubes will be neater if you will score along the dotted lines with a knife.

C. Band III - Cross Number Puzzle - Review of Whole Numbers

1. Give students a cross number puzzle and help them begin.

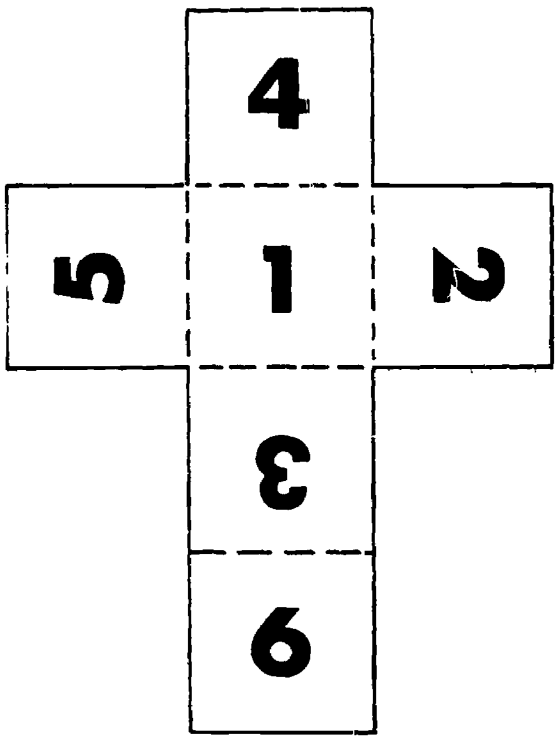
2. They should be able to finish this for homework. This should help prepare them for the Survey Test tomorrow.

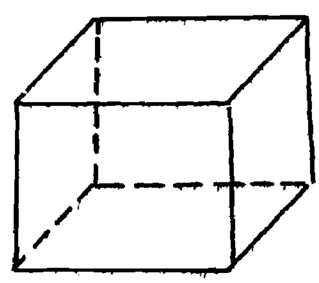
3. Solution:

a 5	b 1	X	c 9	d 2	X	e 8	f 2
g 9	0	X	h 1	7	i 3	X	9
X	j 9	k 1	X	X	l 1	m 2	9
n 9	X	o 1	7	p 7	X	q 7	9
r 1	s 4	6	X	t 6	u 3	X	X
X	4	X	X	X	v 1	w 4	x 2
y 8	X	z 1	0	A 1	X	B 4	0
C 1	0	0	X	D 1	0	0	0

PATTERN FOR THE RaP CUBE

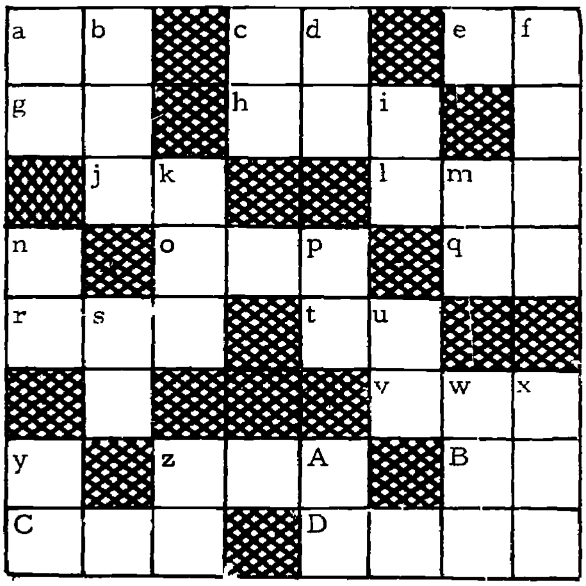
Directions: Listen carefully to your teacher.





CROSS NUMBER PUZZLE - REVIEW OF W[illegible]ERS

Directions: Fill in the blanks.



Across		Down	
a.	17 + 34	a.	34 + 25
c.	56 + 36	b.	54 + 55
e.	41 + 2	c.	(10 x 9) + 1
g.	270 ÷ 3	d.	52 - 25
h.	200 - 27	f.	3000 - 1
j.	20 + 60 + 11	i.	76 - 45
l.	43 x 3	k.	250 - 134
o.	286 - 109	m.	18 + 9
q.	25 + 31 + 23	n.	63 + 28
r.	2 x 73	p.	37 + 39
t.	21 x 3	s.	11 x 4
v.	260 - 118	u.	93 ÷ 3
z.	200 - 99	w.	880 ÷ 2
B.	320 ÷ 8	x.	10 x 20
C.	10 x 10	y.	440 - 359
D.	100 x 10	z.	100 - 90
		A.	6 + 5

LESSON 3

I. Unit: Graphing

II. Materials:

A. Band I

1. SRA Student Record Book
2. SRA Teacher's Guide
3. Pencil

B. Band II

1. Checking pencil
2. Pencil

C. Band III - None

III. Procedure:

A. Band I - SRA Computational Skills Development Kit Survey Test, Part I: Exercises 1 - 12.

1. Prior to this time the teacher should acquaint himself with the kit. The teacher's guide offers many helpful ideas. Pay particular attention to pp. 14 - 15.

2. Inform students that the purpose of the survey test is for them to locate skills in which they need practice. Encourage them to take their time and work carfully. Explain that if they have difficulty with the test it will not affect their grade as long as they work hard. If this tone is established at the outset you should find this program successful.

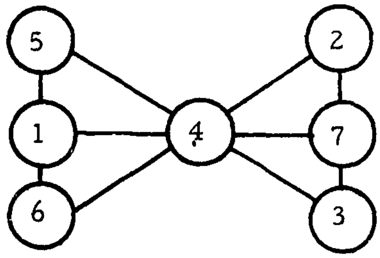
3. Pass out a Student Record Book to each pupil. Have each student write his name on his book.

4. Students take Survey Test, p. 8, Part I: Exercises 1 - 12 on whole numbers. After each student has checked over his test, have him close his book and place it on the corner of his desk.

5. As each student finishes the Survey Test, give him the Brain Teasers work sheet. He may finish the puzzle for homework or for extra credit.

6. Brain Teaser Solutions:

1)



2)

8	1	6
3	5	7
4	9	2

B. Band II - Check and record the results of the Survey Test

1. You should find p. 14, step II of the Teacher's Guide helpful for this work.
2. After each student has finished his test, allow him to score his own test with a checking pencil.
3. Record results of the test on the student progress record in the Student Record Book, pp. 28 - 29.
4. Explain to the class how to determine which operation to begin with.
5. Tell students they will complete Survey Test Part I in a few days.

C. Band III - Tic-Tac-Toe Game to assess objectives of Lesson I.

1. Have Tic-Tac-Toe diagram on board.
2. On the paper have the diagram which has each space filled with questions, pertaining to Lesson 1 - Introduction to Graphing.
3. Divide class into two teams - x's and o's.
4. Number the players on each team and question them in that order.
5. Teacher questions x's and o's alternately. The first player chooses a space and is asked the question from that space on the teachers paper.
6. If the question is answered correctly an "x" or "o" is placed in the space.
7. The game is over when one team has three in a row horizontally, vertically, or diagonally.

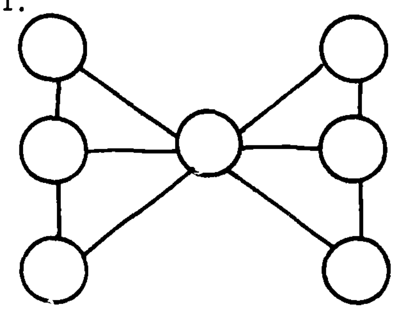
8. Some sample questions.

What do we call these lines on the game board? (Teacher points to Axes.)	What is the highest number you can roll with a RaP cube?	What do we call ordered pairs which give us the location of a point on the grid?
What is wrong with this statement? After I rolled my RaP cubes, I found the coordinates of my point were (2, 7).	Ordered pairs give us the location of a ________ on the grid.	If I rolled a red 4 and a white 2 where would I plot the point?
If I rolled a red 5 and a white 6 how would I express the result as an ordered pair?	(Teacher places finger on a point on the grid.) Name the coordinates of this point.	My result may be written (3, 4). What number did I roll with the red cube?

BRAIN TEASERS

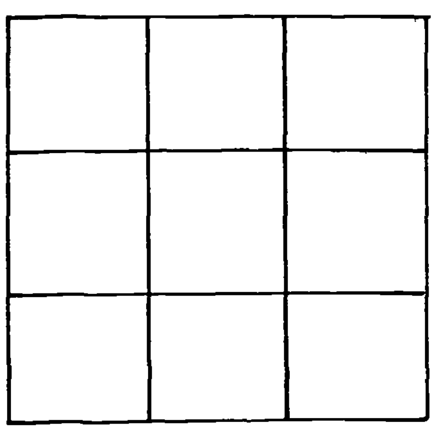
Try these puzzles!

1.



Can you place the numerals 1, 2, 3, 4, 5, 6, and 7 in the seven circles, so that the sum in each of the five rows is 12?

2.



Can you place the numerals from 1 to 9 in these nine squares so that the sum of the three numbers in each row, each column, and each diagonal is 15?

44

LESSON 4

I. Unit: Graphing

II. Objectives: The student should be able to:

A. Construct a model of a cube

B. Describe how to play and score Roll and Plot

III. Materials:

A. Band I - None

B. Band II

1. Scored tag board patterns

2. Scissors

3. Masking tape

C. Band III

1. RaP cubes

2. Transparency with game board

3. Overhead projector

IV. Procedures:

A. Band I - Written drill involving greatest common factor, addition, and subtraction of fractions.

1. Have this drill written on the board.

a. $\frac{1}{4} + \frac{1}{8} =$

b. $\frac{11}{12} - \frac{1}{3} =$

c. What is the largest number which will divide both 6 and 12?

d. What is the largest factor of 6 and 12?

e. What is the Greatest Common Factor of 6 and 12?

2. Discuss the drill problems.

B. Band II - Completing the construction of the RaP cubes

1. Pass out tag board patterns which have been scored.

2. Give each student 6-7 inches of masking tape.

3. Have students cut tape and place it on the appropriate sides, as shown on the pattern.

4. Fold tape lengthwise along edge of pattern.

5. Tape edges together.

6. Have students keep their cubes to learn how to play the game in the next band.

C. Band III - Explain-The rules of Roll and Plot

1. The cubes have been constructed and the class will now learn the rules of the RaP game.

   a. Each student chooses a partner. (The teacher may have to fill in if the class has an odd number of students.)

   b. Each player chooses an X or an O.

   c. Each player in turn rolls his cubes, constructs an ordered pair, and places his symbol at the corresponding point on the grid.

   d. If the point associated with his result has been used, he loses his turn.

   e. The game ends when all results are exhausted.

2. Explain-The scoring of Roll and Plot

   a. If a player has three or more symbols on consecutive points in a row, column, or either diagonal labeled with dashed lines he scores points. The points are scored as follows:

      (1) 3 consecutive symbols = 1 point

      (2) 4 consecutive symbols = 2 points

      (3) 5 consecutive symbols = 3 points

      (4) 6 consecutive symbols = 4 points

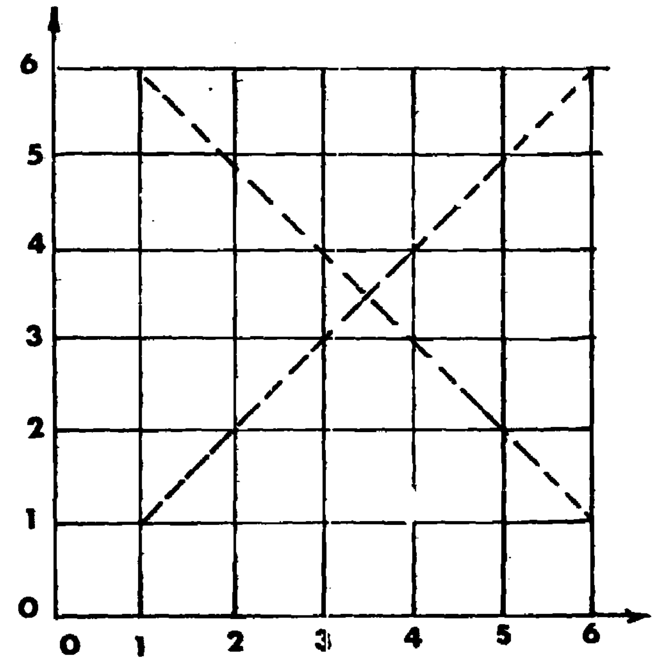
3. To clarify the rules before students pair up, divide the class into two teams, O's and X's. Let each student, on each team, roll his cubes in turn and record his marks. If the teams are not even, one student can record all the marks on the game board.

4. Another way to clarify the rules is for the teacher to challenge the class to a game. Students roll in turn for the class and the score is kept on the transparency.

ROLL and PLOT

RaP SCORE

X	O



LESSON 5

I. Unit: Graphing

II. Materials:

A. Band I - None

B. Band II - Cross Number Puzzle - Review of Fractions

C. Band III

1. RaP cubes

2. Game sheet for each pair of students

III. Procedure:

A. Band I - Mental Drill involving Least Common Multiple; multiplication and division of fractions

1. Ask the questions below and have the students record just their answers on paper.

a. What are the first 4 multiples of 2?

b. What are the first 4 multiples of 3?

c. What is the smallest number that is a multiple of both 2 and 3?

d. How many $\frac{1}{2}$'s are there in 4?

e. What is $\frac{1}{2} \times \frac{1}{4}$?

2. Check this together. In c, bring out the term Least Common Multiple.

B. Band II - Cross Number Puzzle reviewing operations with fractions

1. Give each student a cross number puzzle.

2. Help students begin the puzzle, showing them where to place the answers.

3. Allow students to work on this until the last 20 minutes of the period.

4. Have students finish the puzzle for homework.

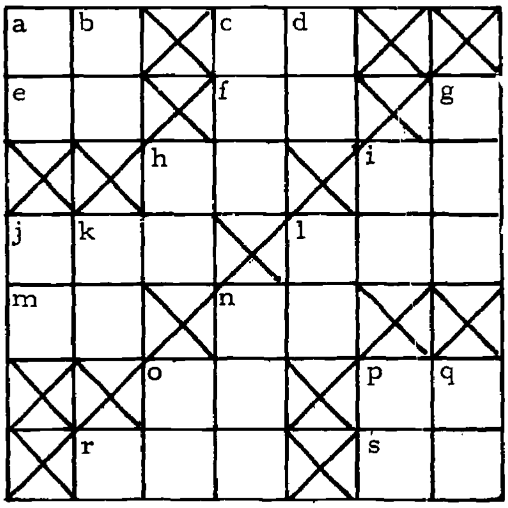
5. Cross Number Puzzle: <u>Solution</u>

Across	Down
a. 42	a. 44
c. 31	b. $2\frac{1}{2}$
e. $4\frac{1}{2}$	c. $36\frac{5}{6}$
f. $6\frac{1}{3}$	d. $1\frac{1}{3}$
h. $2\frac{5}{6}$	g. $19\frac{1}{2}$
i. 69	h. $2\frac{1}{6}$
j. $24\frac{1}{6}$	i. 62
l. $22\frac{1}{2}$	j. 27
m. $7\frac{1}{2}$	k. $4\frac{1}{2}$
n. $5\frac{9}{10}$	l. $2\frac{9}{10}$
o. 49	n. $59\frac{3}{8}$
p. 24	o. 49
r. $39\frac{3}{8}$	p. 26
s. $6\frac{2}{3}$	q. $4\frac{2}{3}$

C. Band III - Students play RaP game in pairs

1. Have each student choose a partne-.
2. Give each pair of students a game sheet and their RaP cubes. Students should then play the game according to the established rules.
3. You may want to use this game to determine a class champion. The teacher then plays the class champion.

CROSS NUMBER PUZZLE



Across

a. $12 \times \frac{7}{2}$

c. $34\frac{1}{2} - 3\frac{1}{2}$

e. $3 + 1\frac{1}{2}$

f. $4\frac{2}{3} + 1\frac{2}{3}$

h. $3 - \frac{1}{6}$

i. $21 \times \frac{23}{7}$

j. $\frac{5}{6} \times 29$

l. $10 \div \frac{4}{9}$

m. $4\frac{1}{4} + 3\frac{1}{4}$

n. $6 - \frac{1}{10}$

o. $21 \div \frac{3}{7}$

p. $8 \div \frac{1}{3}$

r. $\frac{9}{2} \times \frac{35}{4}$

s. $\frac{2}{3}$ of 10

Down

a. $\frac{4}{5} \times 55$

b. $1\frac{1}{8} + 1\frac{3}{8}$

c. $30 + 6\frac{5}{6}$

d. $\frac{2}{3} \div \frac{1}{2}$

g. $\frac{13}{2} \times 3$

h. $1 + 1\frac{1}{6}$

i. $31 \div \frac{1}{2}$

j. $19\frac{1}{3} + 7\frac{2}{3}$

k. $2 \times \frac{9}{4}$

l. $2\frac{4}{10} + \frac{5}{10}$

n. $\frac{19}{2} \times \frac{25}{4}$

o. $\frac{1}{2} \times 98$

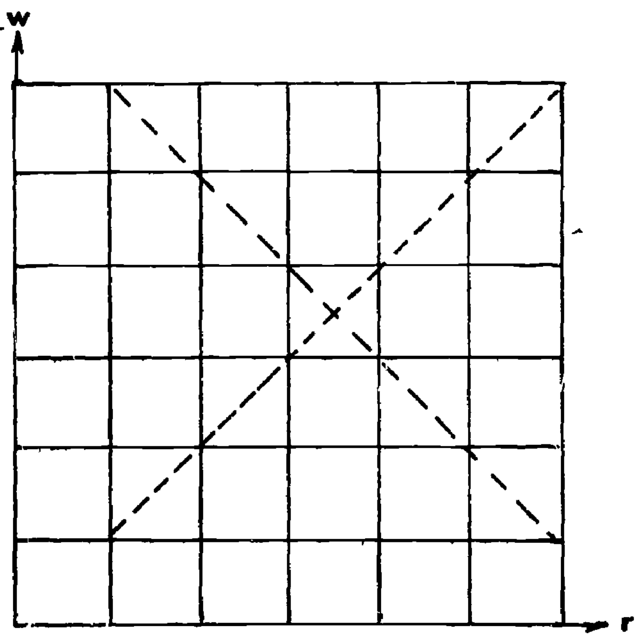
p. $13 \div \frac{1}{2}$

q. $1\frac{1}{3} + 3\frac{1}{3}$

ROLL and PLOT

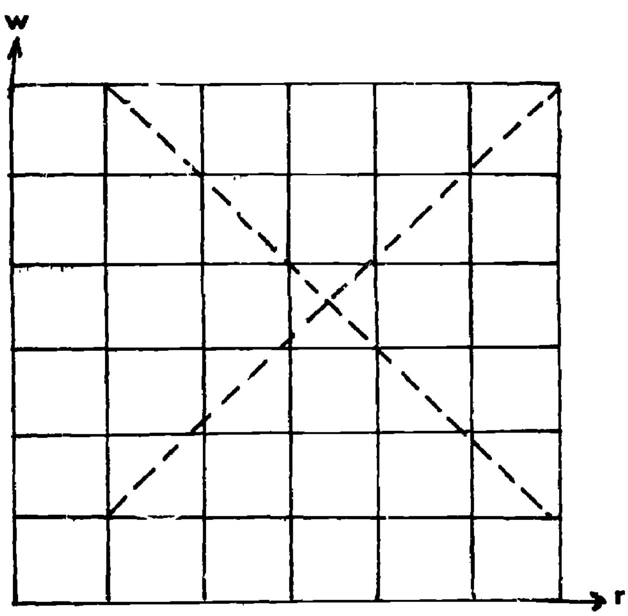
RaP SCORE

X	O



RaP SCORE

X	O



LESSON 6

I. Unit: Graphing

II. Materials:

A. Band I

1. SRA Student Record Books and pencil
2. Student work sheet entitled, "Scramble"

B. Band II

1. Pencil
2. Checking pencil

C. Band III - None (see Band III)

III. Procedure:

A. Band I - SRA Survey Test, Part I: Exercises 13 to 25

1. Distribute Student Record Books.
2. Student's work page 9, exercises 13 to 25.
3. After each student has finished checking over his test he should close his book and place it in the corner of his desk.
4. Explain - As students complete the test they should obtain the puzzle sheet entitled, "Scramble" which is placed on the window sill.
5. Have students finish the puzzle for homework or extra credit.

B. Band II - Check and record the results of the Survey Test

1. After each student has finished his test, allow him to score his own test with a checking pencil.
2. Record the results of the test on the student progress record in the Student Record Book, pp. 28-29.

C. Band III - Tape on Verbal Problems or Oral Puzzle

1. If 15 minutes remain in the period, do the following:

   a. Obtain a tape recorder and tape entitled, "Verbal Problems - 2."

   b. Preview the tape.

   c. The students should have on their desks a pencil and a piece of paper.

   d. The tape is composed of ten verbal problems. Complete instructions to the student which are on the tape. Answers to each problem are also on the tape.

2. If 15 minutes do not remain in the period, do the following:

   Puzzle involving doubling, the tens digit, product, multiplication, sum.

	Example:
a. Have each student write a two digit number. Each digit must be less than nine.	a. 26
b. Double the tens digit.	b. $2 \times 2 = 4$
c. Add one to this product.	c. $4 + 1 = 5$
d. Multiply by five.	d. $5 \times 5 = 25$
e. To this product add the ones digit of the original number.	e. $25 + 6 = 31$
f. Add 106 to this sum. This is the final answer.	f. $31 + 106 = 137$

   g. Ask each student, one at a time, to give you his final answer. Then you can tell him his original number.

   Solution:

   1. To get the original number, subtract 111 from their final answer.

   2. From the example problem:

   $$\begin{array}{lcr} \text{Final answer} & = & 137 \\ & & \underline{-111} \\ \text{Original No.} & = & 26 \end{array}$$

   h. Try the puzzle again. This time have students choose a different number.

SCRAMBLE

I. The letters in the words below have been moved around. Can you put the letters in order so they spell words used in mathematics?

dad	net	eon	hlfa
ezor	wot	ums	trocaf

II. Make as many words as you can by using only the letters found in fraction. You may use the letters in any order.

Example: ran, tin.

54

LESSON 7

I. Unit: Graphing

II. Objectives: The student should be able to:

Apply the principle for multiplying by 11 in order to construct products of two digit numbers, one of which is 11.

III. Materials:

A. Band I - None

B. Band II - Student work sheet entitled, "Money Funny"

C. Band III

1. RaP cubes

2. Game sheet for each pair of students (same as Lesson 5)

IV. Procedure:

A. Band I - Written drill involving multiplying two digit numbers by 11

1. Have these examples on the board

$$\begin{array}{r} 34 \\ \times 11 \\ \hline \end{array} \quad \begin{array}{r} 52 \\ \times 11 \\ \hline \end{array} \quad \begin{array}{r} 26 \\ \times 11 \\ \hline \end{array} \quad \begin{array}{r} 31 \\ \times 11 \\ \hline \end{array} \quad \begin{array}{r} 81 \\ \times 11 \\ \hline \end{array} \quad \begin{array}{r} 45 \\ \times 11 \\ \hline \end{array}$$

2. Have the students complete the first 3 examples.

3. Explain -

a. Compare the factors and products.

b. Think of a short way to solve such examples.

4. Have the students complete the remaining examples, using their short method if they have one.

5. Check examples together.

6. Have a few students explain and illustrate what they have observed about multiplying by 11.

B. Band II - Money Puzzle

1. Students should be given the puzzle work sheet and allowed about 15 minutes to work on it before playing the RaP Game.

2. Solution:

	$1.00	50¢	25¢	10¢	5¢	1¢
1.58	1	1			1	3
3.48	3		1	1	1	3
.99		1	1	2		4
1.39	1		1		1	4
2.86	2	1	1	1		1
2.07	2				1	2
.76		1	1			1

C. Band III - Students play the RaP game in pairs

Students choose a partner and play the RaP game.

MONEY FUNNY

Look at the sum of money in column A. Express this sum using the fewest pieces of money possible. Number one is completed for you.

A	$1.00	50¢	25¢	10¢	5¢	1¢	Fewest Number of Coins
$1.24	1			2		4	7
$1.58							
$3.48							
$ .99							
$1.39							
$2.86							
$2.07							
$ .76							

LESSON 8

I. Unit: Graphing

II. Objectives: The student should be able to:

A. Demonstrate how to construct a coordinate axis for the first quadrant using a straight edge

B. Describe the procedure for plotting points in the first quadrant

C. State the principle that the first number in an ordered pair indicates the number of spaces to go to the right and the second the number of spaces to go up.

D. Apply the principle by plotting points to make a graph

III. Materials:

A. Band I - Teacher-made flash cards

B. Band II

1. Pencil and ruler
2. Graph paper
3. Work sheet with directions for the picture

C. Band III - Student work sheet

IV. Procedure:

A. Band I - Oral drill with flash cards.

1. Tell the students you are going to see how well they recall the trick they learned about multiplying by 11.
2. Have flash cards made with the following examples.

1. 24 x11	5. 54 x11	9. 35 x11
2. 43 x11	6. 11 x33	10. 44 x11
3. 11 x31	7. 62 x11	11. 27 x11

4. 72 x11	8. 81 x11	12. 18 x11

3. Have students answer orally. Make sure each person has at least one turn.

4. Have students state the rule as a summary.

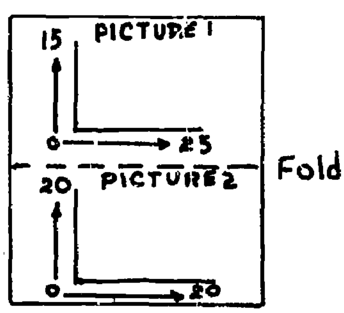
B. Band II - Plotting points to make a picture

1. Make sure each student has a pencil, ruler, and graph paper.

2. Explain - Fold the graph paper in half so one picture will be on each half.

3. Explain - Locate axes so picture will be centered.

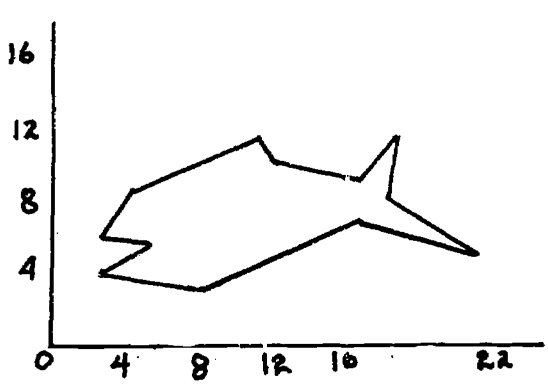
4. The paper should be prepared as follows:



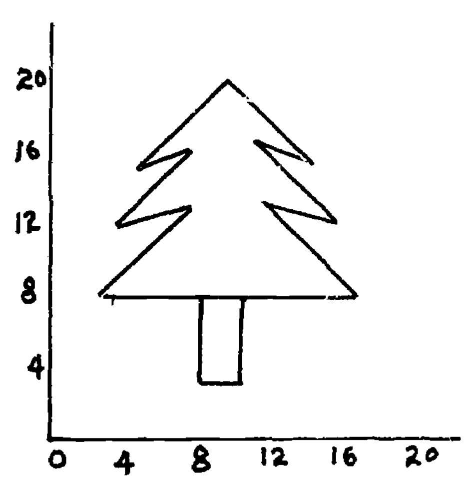
5. Be sure students number lines and not spaces when they construct the scale.

6. Pass out work sheet entitled, "Make My Picture."

7. Solution:



FISH



PINE TREE

C. Band III

1. If you did not use the tape recording entitled, "Verbal Problems - 2," in Band III of Lesson 6, and if 15 minutes remain in this period, then follow the procedure outlined in Lesson 6, Part IV, Item C. 1.

2. If you did use the tape recording entitled, "Verbal Problems - 2," in Band III of Lesson 6, or if 15 minutes does not remain in this period, then do the following:

   Hidden message puzzle involving addition

   a. Explain - As students complete the graphing picture they should obtain the puzzle sheet entitled, "Hidden Message" which is placed on the window sill.

   b. Solution: Job Well Done

A HIDDEN MESSAGE

Secret messages are sometimes in code. Often the code will have numbers or letters in it. Today you will figure out a message by solving some examples.

I. The answers to each example stand for a letter.

1.	7 + 3	10
2.	3 more than 12	________
3.	1 more than 1	________
4.	4 + 19	________
5.	3 more than 2	________
6.	3 + 8 + 1	________
7.	8 + 3 + 1	________
8.	2 more than 1 + 1	________
9.	6 + 4 + 5	________
10.	5 more than 6 + 3	________
11.	2 more than 1 + 2	________

II. Look at your first answer,(10), and find that number on the decoding chart. Right under this number you will find the letter it stands for (J). Write this letter (J) in space 1 in the hidden message. Do the same thing for all of your answers, and you will see the message.

DECODER

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

HIDDEN MESSAGE

___ ___ ___   ___ ___ ___ ___   ___ ___ ___ ___

1 2 3   4 5 6 7   8 9 10 11

MAKE MY PICTURE

Directions:

1. Plot the following points.
2. Number each point as you go.
3. Use your ruler to connect the points in order.
4. The first and last points are to be connected.

I. Picture 1

1. (18, 9)
2. (20, 12)
3. (19, 8)
4. (22, 5)
5. (18, 7)
6. (9, 3)
7. (3, 4)
8. (6, 6)
9. (3, 6)
10. (5, 9)
11. (12, 12)
12. (13, 10)

II. Picture 2

1. (10, 20)
2. (15, 15)
3. (12, 16)
4. (16, 12)
5. (12, 13)
6. (17, 8)
7. (11, 8)
8. (11, 3)
9. (9, 3)
10. (9, 8)
11. (3, 8)
12. (8, 13)
13. (4, 12)
14. (8, 16)
15. (5, 15)

LESSON 9

I. Unit: Graphing

II. Objectives: The student should be able to:

A. Construct a pair of coordinate axes for the first quadrant

B. Demonstrate how to plot points

III. Materials:

A. Band I - None

B. Band II - Student work sheet entitled, "Graphing Exercises"

C. Band III

1. Pencil and ruler

2. Graph paper

3. Work sheet entitled, "What Am I?" with directions for the picture

IV. Procedure:

A. Band I - Oral drill involving addition and subtraction of whole numbers

The exercise below should be given orally while the students record only their answers on paper.

1. One addend is 7. The sum is 16. What is the other addend? (9)

2. What is 30 minus 7? (23)

3. What number should I add to 36 if my sum is 56? (20)

4. The difference between two numbers is 3. One of the numbers is 5. What is the other number? (8 or 2)

5. Take 7 from 25. What is your answer? (18)

B. Band II - Graphing Exercises

1. This should be an individual exercise used for assessment purposes.

2. Be sure each student understands the directions.

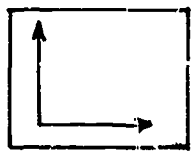
3. Papers should be collected to check students' progress.

C. Band III - Graphing Pictures

1. Make sure students have all materials.

2. Explain - Locate axes so picture will be centered. It may be helpful to have students fold their papers lightly in order to center the grid on the graph paper.

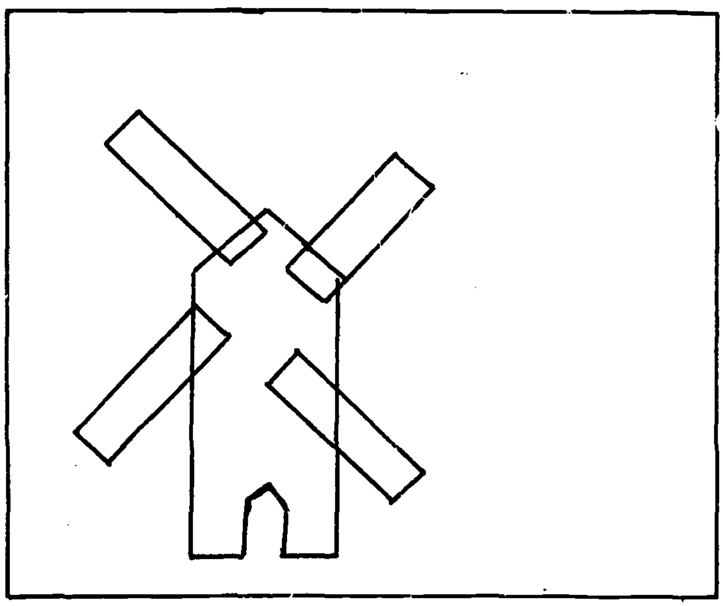
3. The paper should be prepared as follows



4. Explain - Number the lines, not spaces.

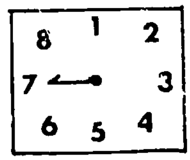
5. Pass out work sheets for pictures.

6. Solution:



GRAPHING EXERCISES

1. Mr. Brown's math students want to play a game called Spin and Plot, SaP. To play the game, they need two spinners which look like this,



and a game board.

One spinner is red, the other is white. Construct a pair of axes and a grid for the SaP Game.

2. Which of the following represent the coordinates of a point?

   a. 2 - 5

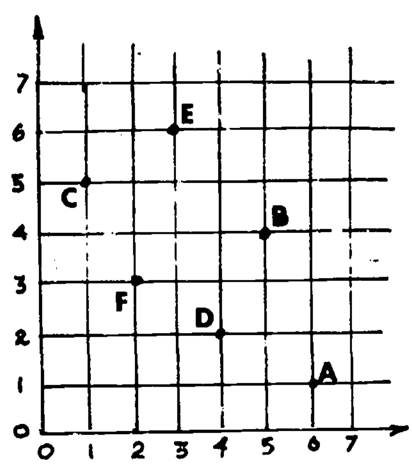
   b. 2, 5

   c. (2, 5)

   d. 2.5

3. Following is a list of coordinates. Select the letter which names the point corresponding to each pair of coordinates. Place the letter in the appropriate blank.

(1, 5) ______	(3, 6) ______
(2, 3) ______	(4, 2) ______
(5, 4) ______	(6, 1) ______



66

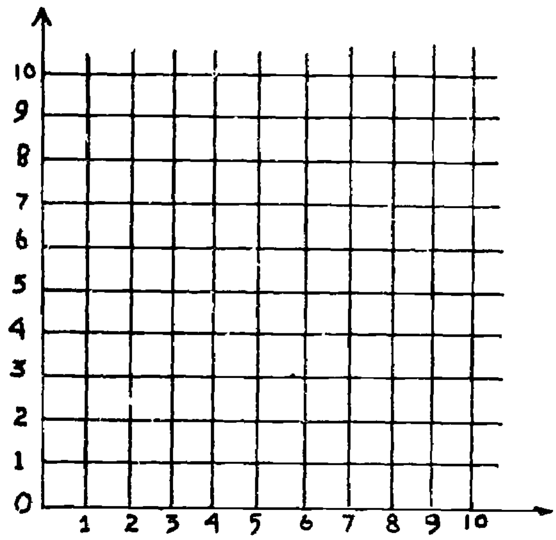
4. Listed below are coordinates for several points. Plot and label these points on the graph.

A = (3, 5) E = (5, 3)

B = (9, 2) F = (2, 9)

C = (0, 7) G = (7, 0)

D = (0, 0) H = (6, 6)



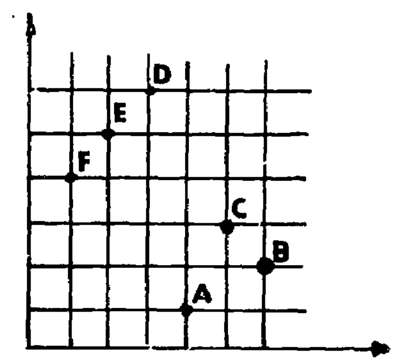
5. Another name for an ordered pair of numbers associated with a point is ______________________.

6. On the graph the results of 6 rolls have been recorded. These results have been labeled by a capital letter. Tell the result for each letter.

A = (____, ____) D = (____, ____)

B = (____, ____) E = (____, ____)

C = (____, ____) F = (____, ____)



7. Does the ordered pair (9, 2) name the same point as (2, 9)? __________

Why? ____

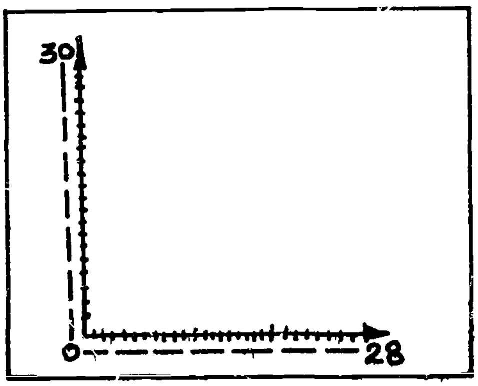
8. The first number in an ordered pair tells the number of spaces to go to the ______________, and the second number tells the number of spaces to go ______________.

WHAT AM I?

In Holland I am often found,
When the wind comes I turn around.

A. Directions

1. Prepare your graph paper so that the completed picture will be centered. Place the grid so the axes can be numbered as follows:



2. Plot the following points. Number each point as you go. Use your ruler to connect the points in order. When you see the instruction "lift pencil," you have completed one part of your picture.

B. Picture

1. (12, 14)
2. (10, 16)
3. (3, 9)
4. (5, 7)
5. (23, 25)
6. (25, 23)
7. (18, 16)
8. (16, 18)

LIFT PENCIL

9. (12, 18)
10. (14, 20)
11. (7, 27)
12. (5, 25)
13. (23, 7)
14. (21, 5)
15. (14, 12)
16. (16, 14)

LIFT PENCIL

17. (15, 2)
18. (15, 5)
19. (14, 6)
20. (13, 5)
21. (13, 2)
22. (10, 2)
23. (10, 18)
24. (12, 20)
25. (14, 21)
26. (16, 20)
27. (18, 18)
28. (18, 2)
29. (13, 2)

LIFT PENCIL

LESSON 10

I. Unit: Graphing

II. Objectives:

A. Name and identify a table of values

B. Construct a table of values from a given rule

C. Construct a set of coordinates from a table of values

D. Construct a graph using ordered pairs from the table of values

E. Apply the given rule to answer related questions

III. Materials:

A. Band I - Math Builder and filmstrip #ML-AR-3

B. Band II - Student work sheet entitled, "Rolling Stone Teen Club Gets Started"

C. Band III - None

IV. Procedure:

A. Band I

1. Obtain the Math Builder and filmstrip 3 from the set ML-AR.

2. Preview the filmstrip.

3. Have the students do the first 3 or 4 problems from Exercise A using the left-to-right scanning device.

4. Complete Exercise A using the full line frame and masking the answer.

5. The speed should be set at approximately 15 or 20.

6. Have the students letter on a piece of paper from a through j. Using the full line frame and masking the answer, have the students do Exercise B. They should respond by writing the missing term. The speed should be as slow as possible.

B. Band II - Discussion Questions - "Rolling Stone Teen Club Gets Started"

1. Tell students they will read a short story about a teen club starting in Trogg City. Ask. How many students have heard of Trogg City?

2. Have students read the story.

3. Discussion questions. Have students read the first few questions orally, one at a time. Discuss each question before going on.

4. You may want to put the table on the board and have students come up and fill it in. Do the same with the graph.

5. Continue discussion questions in a similar manner.

Band II - Solutions for "Rolling Stone Teen Club Gets Started"

1. 8

2. 16

3. 10

4.

boys (b)	0	2	4	6	8	10
girls (g)	0	2	4	6	8	10

5.

6.

7. a straight line

8. C

9. a. 37

   b. 61

   c. 105; 105

C. Band III - Puzzle involving addition, subtraction, and multiplication of whole numbers

Read the puzzle aloud.

	Example
1. Have each student write a number (any number of digits).	1. 23
2. Add 2 to this number.	2. $23 + 2 = 25$
3. Multiply the sum by 3.	3. $25 \times 3 = 75$
4. Subtract 4 from the product.	4. $75 - 4 = 71$
5. Multiply the result by 3.	5. $71 \times 3 = 213$
6. Add the original number to this product.	6. $213 + 23 = 236$
7. You can tell each student his original number.	

Solution:

a. The original number is found by crossing out the one's digit.

b. From the example problem:
Since the last sum is 236 the original number must be $23\cancel{6} = 23$.

8. Try the puzzle again.

This time have students choose a different number.

ROLLING STONE TEEN CLUB GETS STARTED

A new teen center is being formed for the students of Rolling Stone Junior High School in Trogg City. Jan and Terry are in charge of the membership drive. They wanted the same number of boys as girls in the new Rolling Stone teen club. After one day of the membership drive there were 8 boys.

Discussion Questions

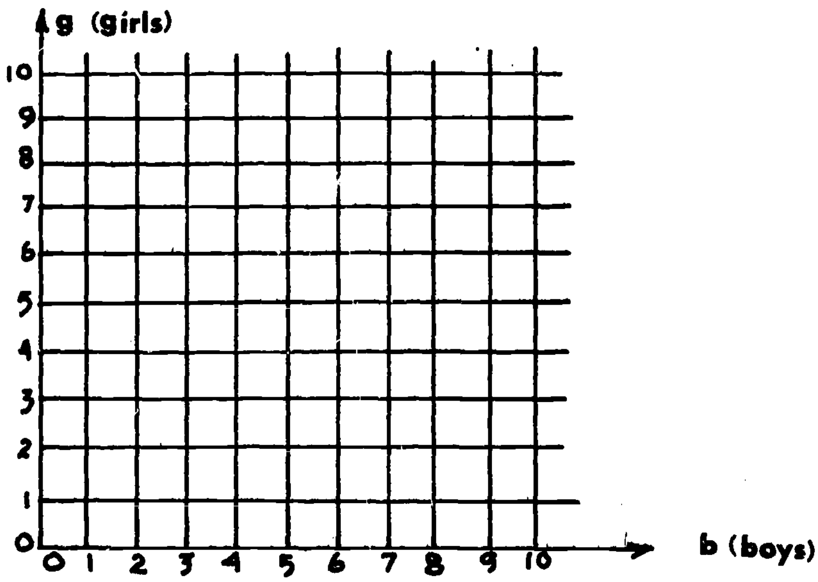
1. How many girls should there be after one day? ____________
2. What should the total membership be at the end of the first day? ____________
3. Suppose there were 10 girls, how many boys should there be? ____________
4. Below is a Table of Values. This table of values is an organized way to show the number of girls and the number of boys as the membership grew.

TABLE OF VALUES

boys (b)	0	2	4			10
girls (g)				6	8	

5. Use the Table of Values to construct a set of coordinates of the form (b, g) where "b" represents the number of boys and "g" represents the number of girls. { (0, 0) , ______ }

6. Use the coordinates to plot the information on a graph



7. What pattern is suggested by the points that you plotted on the graph?

____

8. Which of the following rules states the relationship between the number of boys (b) and girls (g)? ______________

   a. b = 4   b. b = 2g   c. b = g   d. g = b - 1

9. Use your rule to answer the following questions.
   a. How many girls belong if there are 37 boys? __________
   b. How many boys are members if there are 61 girls? __________
   c. At the first "T" night, all of the members were there. Suppose the total attendance was 210. How many boys were there? ______ How many girls were there? ______

LESSON 11

I. Unit: Graphing

II. Materials:

A. Band I - None

B. Band II - Student work sheet entitled, "Rolling Stone Teen Club Gets Started"

C. Band III

1. Work sheet with directions for pictures

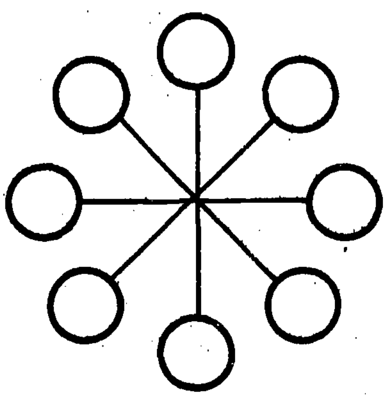
2. Graph paper

3. Pencil and ruler

III. Procedure:

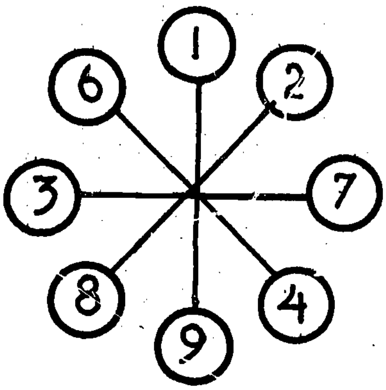
A. Band I - Written drill involving addition

1. Write the following problem on the board. Fill in its circles using the numerals, 1, 2, 3, 4, 5, 6, 7, 8, 9, so that each line adds up to 15.



2. Discuss the answers with the students.

3. Possible solution.



B. Band II - Assessment exercise on objectives of Lesson 10 entitled, "Rolling Stone Teen Club Get Started"

1. This should be an individual project used for assessment. Don't call this a quiz.

2. Be sure students understand the problem before they begin working on their own.

3. Collect the exercises for checking pupil progress.

C. Band III - Plotting points to make pictures

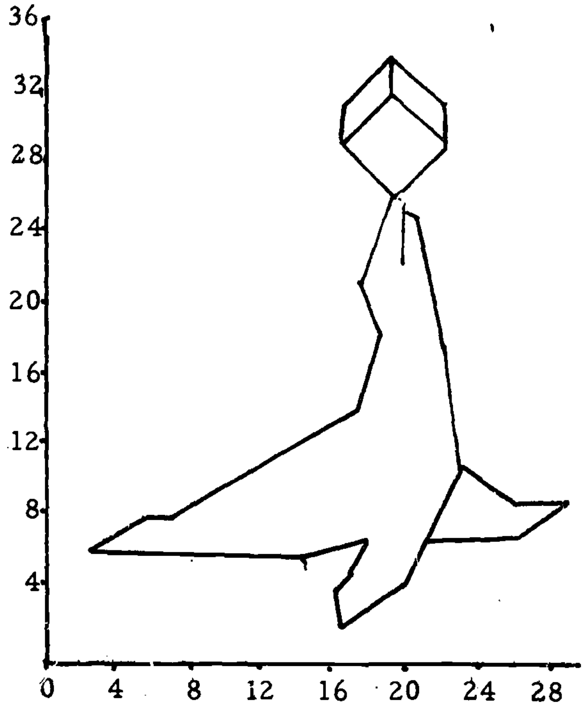
1. Make sure students have all materials.

2. Explain - Locate axes so the picture will be centered.

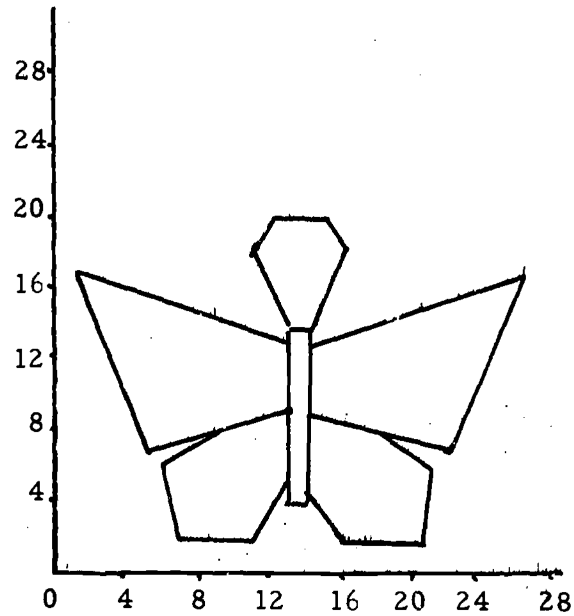
3. Explain - Number the lines, not the spaces.

4. Pass out work sheets for pictures entitled "What Am I?" Note - Since there are several different pictures for this activity, allow the student to select one of his choice. This could be a continuing activity until each student has completed all the pictures. See next page for solutions.

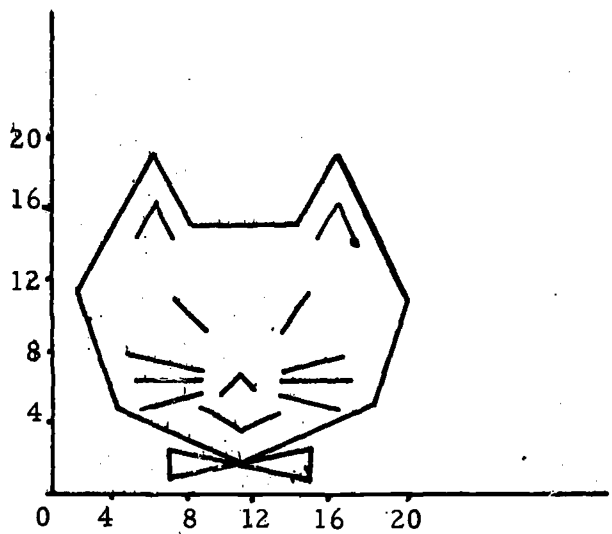
Solution: "What Am I?"



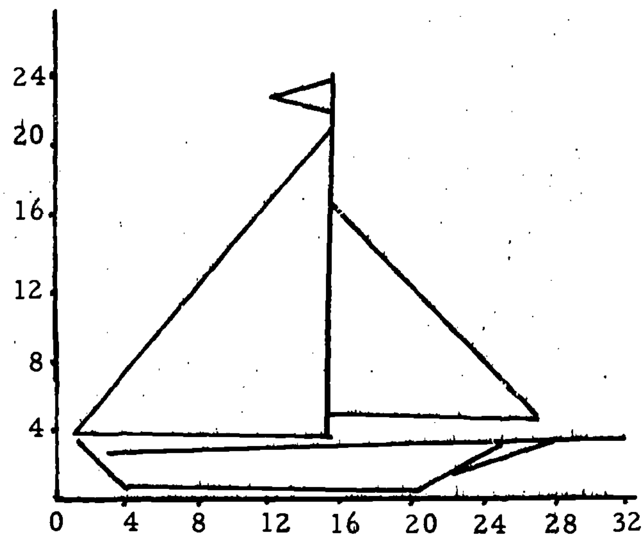
Seal



Butterfly



Cat



Sail Boat

ROLLING STONE TEEN CLUB GETS STARTED

Suppose that the membership rule has been changed. The new rule states that there must be twice as many girls as boys.

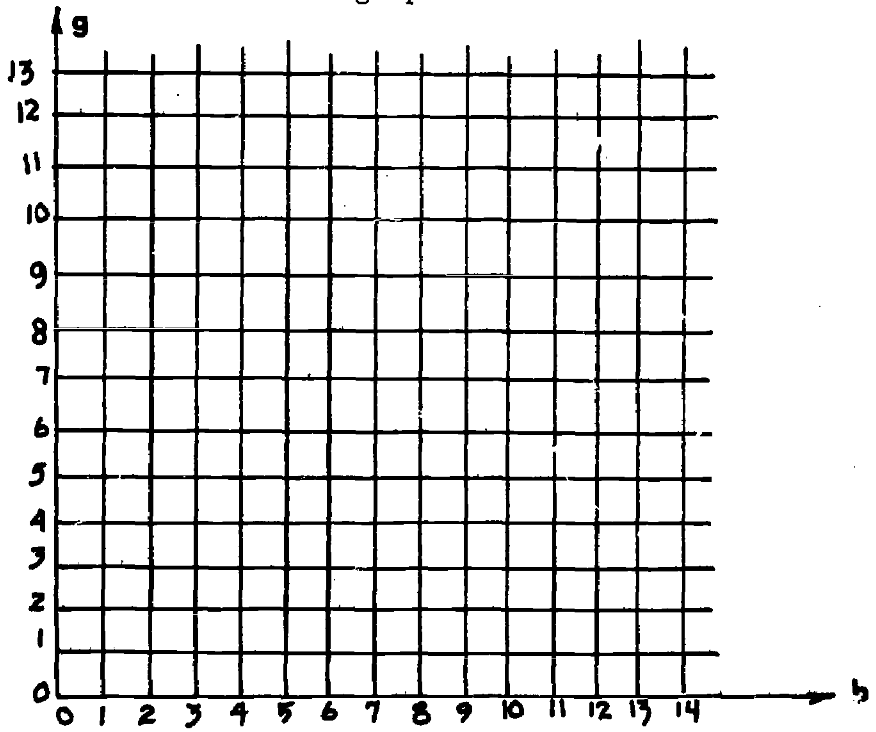
1. Complete a table of values which shows the relationship between the number of boys (h) and girls (g) under the new rule.

Table of Values

b	0	1		3		5	6	
g			4		8			14

2. Use the Table of Values to construct a set of coordinates of the form (b, g).

3. Plot the coordinates on a graph.



4. What pattern is suggested by the points plotted on the graph? __________

5. Which of the following rules states this relationship? __________

   a. b = 2g b. g = 2b c. b = g d. g = b

6. Use your rule to answer the following questions.

   a. If there are 12 boys, how many girls are there? __________

   b. If there are 44 girls, how many boys are there? __________

   c. Suppose the total attendance is 90.

      How many girls are present? __________

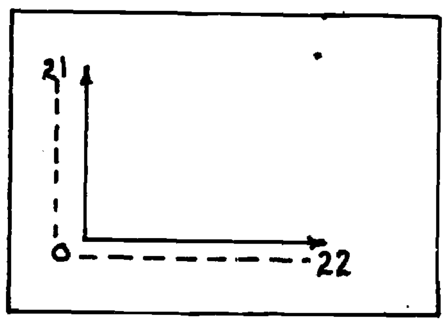
      How many boys are present? __________

WHAT AM I?

I like mice
With sugar and spice.

A. Direction:

1. Prepare your graph paper so that the completed picture will be centered. Place the grid so the axes can be numbered as follows:



2. Plot the following points.

   Number each point as you go. Use your ruler to connect the points in order. When you see "lift pencil," you have completed a part of your picture.

B. Picture

1. (7, 11)
2. (9, 9)

LIFT PENCIL

3. (15, 11)
4. (13, 9)

LIFT PENCIL

5. (13, 7)
6. (17, 8)

LIFT PENCIL

7. (13, $6\frac{1}{2}$)
8. (17, $6\frac{1}{2}$)

LIFT PENCIL

9. (13, 6)
10. (17, 5)

LIFT PENCIL

11. (5, 8)
12. (9, 7)

LIFT PENCIL

13. (5, $6\frac{1}{2}$)
14. (9, $6\frac{1}{2}$)

LIFT PENCIL

15. (5, 5)
16. (9, 6)

LIFT PENCIL

17. (10, 6)
18. (11, 7)
19. (12, 6)

LIFT PENCIL

20. (9, 5)
21. (11, 4)
22. (13, 5)

LIFT PENCIL

23. (5, 14)
24. (6, 16)
25. (7, 14)

LIFT PENCIL

26. (15, 14)
27. (16, 16)
28. (17, 14)

LIFT PENCIL

29. (11, 2)
30. (7, 3)
31. (7, 1)
32. (15, 3)
33. (15, 1)
34. (11, 2)

LIFT PENCIL

35. (8, 15)
36. (6, 19)
37. (2, 11)
38. (4, 5)
39. (11, 2)
40. (18, 5)
41. (20, 11)
42. (16, 19)
43. (14, 15)
44. (7, 15)

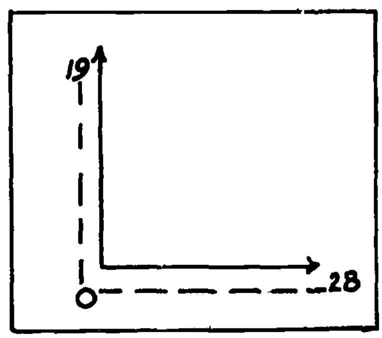
LIFT PENCIL

WHAT AM I?

Fly I do for all to see,
But I''m seldon in a tree.

A. Directions:

1. Prep- 'e your graph paper so that the completed picture will be centered. Place the grid so the axes can be numbered as follows:



2. Plot the following points.
Number each point as you go. Use your ruler to connect the points in order. When you see "lift pencil," you have completed one part of your picture.

B. Picture

1. (13, 13)
2. (1, 17)
3. (5, 7)
4. (13, 9)

   LIFT PENCIL

5. (9, 8)
6. (6, 6)
7. (7, 2)
8. (11, 2)
9. (13, 6)

   LIFT PENCIL

10. (12, 20)
11. (11, 18)
12. (13, 14)
13. (14, 14)
14. (16, 18)
15. (15, 20)

   LIFT PENCIL

16. (13, 14)
17. (13, 4)
18. (14, 4)
19. (14, 14)

   LIFT PENCIL

20. (14, 13)
21. (26, 17)
22. (22, 7)
23. (14, 9)

   LIFT PENCIL

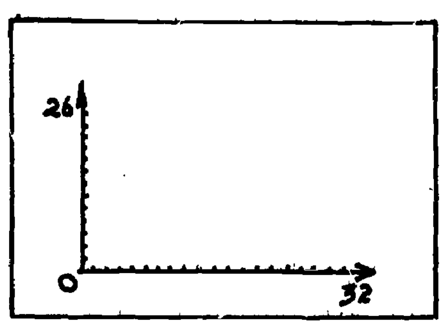
24. (14, 6)
25. (16, 2)
26. (20, 2)
27. (21, 6)
28. (18, 8)

WHAT AM I?

If you're on the high seas,
You need one of these.
But if the wind is not blowing,
I will not be going.
Make my pictureto see what I am.

A. Directions:

1. Prepare your graph paper so that the completed picture will be centered. Place the grid so the axes can be numbered as follows:



2. Plot the following points.
Number each point as you go.
Use your ruler to connect the points in order. When you see "lift pencil," you have completed one part of your picture.

B. Picture

1. $(15, 3\frac{1}{2})$
2. (15, 4)
3. (1, 4)
4. (3, 3)
5. (25, 4)
6. (30, 4)

   LIFT PENCIL

7. (3, 3)
8. (4, 1)
9. (20, 1)
10. (25, 4)

    LIFT PENCIL

11. $(21, 1\frac{3}{5})$
12. (28, 4)

    LIFT PENCIL

13. (15, 5)
14. (27, 5)

    LIFT PENCIL

15. (15, 17)
16. (28, 4)

    LIFT PENCIL

17. (1, 4)
18. (15, 21)

    LIFT PENCIL

19. (15, 22)
20. (11, 23)
21. (15, 24)
22. (15, 4)

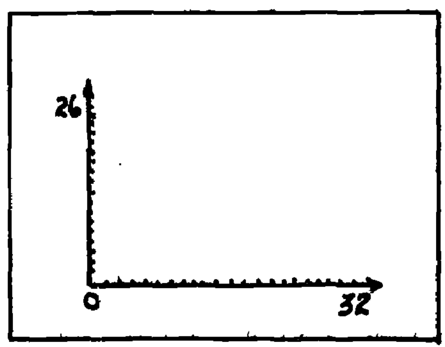
    LIFT PENCIL

WHAT AM I?

In the circus you'll find me posed,
Balancing an object upon my nose.

A. Directions:

1. Prepare your graph paper so that the completed picture will be centered. Place the grid so the axes can be numbered as follows:



2. Plot the following points.
   Number each point as you go.
   Use your ruler to connect the points in order. When you see "lift pencil," you have completed one part of your picture.

B. Picture

1. (19, 32)
2. (16, 29)
3. (16, 31)
4. (19, 34)
5. (22, 31)
6. (22, 29)
7. (19, 32)
8. (19, 34)

LIFT PENCIL

9. ($19\frac{1}{2}$, $25\frac{1}{2}$)
10. ($19\frac{1}{2}$, $23\frac{1}{2}$)

LIFT PENCIL

11. (23, 11)
12. (26, 9)
13. (29, 9)
14. (26, 7)
15. (21, 7)
16. (22, 18)
17. (23, 11)
18. (20, 5)
19. (16, 2)
20. (16, 4)
21. (17, 5)
22. (18, 7)
23. (13, 6)
24. (2, 6)
25. (5, 8)
26. (7, 8)
27. (17, 14)
28. (18, 18)
29. (17, 21)
30. (19, 26)
31. (20, 25)
32. (22, 18)

LIFT PENCIL

33. (16, 29)
34. (19, 26)
35. (22, 29)

LIFT PENCIL

LESSON 12

I. Unit: Graphing

II. Materials:

A. Band I

Diagnostic test and pencil

B. Band II

1. Diagnostic test
2. Student Record Book (SRA)
3. Checking pencil
4. Pencil

C. Band III

1. Pencil and paper
2. Student Record Book
3. SRA Exercise cards

III. Procedure:

A. Band I - Administer First Diagnostic Test in SRA Kit

1. Important - See Teacher's Guide for SRA Computational Skills Development Kit, page 15, Step III, parts A and B.
2. You will probably find that you will have some students who will have to begin by taking the addition test, while others may be ready to begin with subtraction or multiplication. Group your students according to the diagnostic test which they are taking. You will find that this makes it easier to keep track of the operation each student is doing.
3. Pass out proper diagnostic test to each student.
4. Students take test.

B. Band II - Checking Diagnostic Test

1. See Teacher's Guide for SRA Kit, page 15, Step III, C, D, E.
2. As students complete their diagnostic test you may want to work with groups explaining how to check the test and record the results.

C. Band III - Independent Work with Exercise Cards

1. See Teacher's Guide for SRA Kit, page 15, step IV.
2. Make sure students understand how to tell which exercise card he needs and how to find it.
3. Allow students to work until the last five minutes. At this time you may want to assign group leaders - one from addition, one from subtraction, etc., to collect exercise cards and put them in order. One person could be in charge of filing the cards correctly. Another student could collect Student Record Books.

RESOURCE MATERIALS

A. Books

Adler, Irving. *The Giant Golden Book of Mathematics.* New York: Golden Press. 1958

Bergamini, David. *Mathematics, Life Science Library.* New York, N.Y.: Time, Inc. 1963

Heddens, James M. *Today's Mathematics.* Chicago: Science Research Associates, Inc. 1963

Highland, H. J. *The How and Why Wonder Book of Mathematics.* New York: Wonder Books. 1963

Hughes, Toni. *How to Make Shapes in Space.* New York: E. P. Dutton and Co., Inc. 1955

Johnson, Pauline. *Creating With Paper.* Washington: University of Washington Press. 1958

Morris, Dennis and Topfer, Henry. *Advancing in Mathematics, Grade 7.* Chicago: Science Research Associates, Inc. 1963

Morris, Dennis and Topfer, Henry. *Advancing in Mathematics, Grade 8.* Chicago: Science Research Associates, Inc. 1963

Northrop, Eugene P. *Riddles in Mathematics.* New York: D. Van Nostrand Co., Inc. 1944

Wirtz, Robert; Botel, Morton and Nunley, B. G. *Discovery in Elementary School Mathematics.* Chicago: Encyclopaedia Britannica Press, Inc. 1963

Young, Mary. *Singing Windows.* New York: Abingdon Press. 1962

B. Pamphlets and Periodicals

Amir-Mo-Az. *Ruler, Compasses and Fun.* New York: Ginn and Co. 1966

Bazdon, Jack and Murtin, Mark. *Cross Number Puzzle Boxes.* Chicago: Science Research Associates, Inc. 1966

Criflinski, Henry. *Modern Mathematics, Ditto Workbooks.* Washington, D.C.: Hayes School Publishing Co. 1964

Herrick, Marian C. Modern Mathematics for Achievement. New York: Houghton Mifflin Co. 1966

Johnson, Donovan. Paper Folding for the Mathematics Class. Washington, D. C.: National Council of Teachers of Mathematics. 1957

Johnson, Donovan A. and Glenn, William H. Topology: The Rubber-Sheet Geometry. Atlanta: Webster Publishing Co. 1960

Larson, Harold. Enrichment Program for Arithmetic Grades 3-8: Elmsford, New York: Harper and Row Publishers. 1963

Murray, William D. and Rigney, Francis. Paper Folding for Beginners. New York: Dover Publications, Inc. 1960

Potter, Mary and Mallory, Virgil. Education in Mathematics for the Slow Learner. Washington, D. C.: National Council of Teachers of Mathematics. 1958

Proctor, Charles and Johnson, Patricia. Computational Developmental Skills Kit. Chicago: Science Research Associates, Inc. 1965

School Mathematics Study Group. Conference on Mathematics Education for Below Average Achiever. Pasadena, California: Vroman Co. 1964

Topics in Mathematics for Elementary School Teachers. Washington, D. C.: National Council of Teachers of Mathematics. 1964

Wirtz, Robert and Botel, Morton. Math Workshop, Levels A-F. Chicago: Encyclopaedia Britannica Press, Inc. 1961

Woodby, Lauren. The Low Achiever in Mathematics. Washington, D. C.: U. S. Office of Education. 1964

C. Games

Milton Bradley Co., Springfield, Mass.
"Primary Peg Board #474X"
Pegs #472X or #475X

Edmund Scientific, Barrington, New Jersey 08007
"Dr. Nim" ($2.98)
"Probability Kit" ($4.00)
"Soma" ($2.00)

Ideal Supply Co., 11315 Watertown Plank Road, Milwaukee, Wis. 53201
"Geometric Wire Forms and Patterns #794" ($3.00)

Kohner Bros., Inc., 155 Wooster Street, New York, N.Y. 10012
"Euclid" ($1.00)
"Hexed" ($1.00)
"Hi-Q" ($1.00)
"Kwazy Quilt" ($1.00)
"Pythagoras" ($1.00)
"Tormentor" ($1.00)
"Voodoo" ($1.00)

Krypto Corporation, 2 Pine Street, San Francisco, California 94111
"Krypto" ($3.95)

Parker Bros., Inc., P. O. Box 900, Salem, Mass.
"Take Twelve" ($3.00)

Science Research Associates, 259 East Erie Street, Chicago, Illinois 60611
"Equations" ($3.00)
"Cross Number Puzzles" ($22.75)

D. Films

Baltimore County Central Film Library

<u>Probability</u>, McGraw Hill Book Co.

<u>Mean, Median and Mode,</u> McGraw Hill Book Co.

NUMBERS, OPERATIONS AND ALGORITHMS

NUMBERS, OPERATIONS AND ALGORITHMS

I. Master Chart - Grades Six through Eleven

II. Grade Eight Chart and Behavioral Objectives

   A. Whole Numbers

   B. Fractional Numbers - Preliminary Topics

   C. Multiplication of Fractional Numbers

   D. Division of Fractional Numbers

   E. Addition of Fractional Numbers

   F. Subtraction of Fractional Numbers

   G. Decimal Numerals

   H. Percent by Ratio and Proportion

   I. Square Root

III. Activities

UNIT WHOLE NUMBERS GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Numerals to Millions	6	6	6						6	6
Numerals to Billions	7	7	7						7	7
Rounding to Millions	6	6	6				6			6
Rounding to Billions	7	7	7				7			7
Expanded Notation to Millions	6	6	6			8				
Expanded Notation to Billions	7	7	7			8	7			
Renaming Numbers	6	6	6			8	6			6
Vocabulary	6	6								
Place Value to Millions	6	6								
Place Value to Billions	7	7								
Denominate Numbers		6	6		6	7	6			
Verbal Problems.			6, 7, 8	6, 7, 8				6, 7, 8		
Betweenness			6, 7		6					
Symbol(s)	6, 7, 8	6, 7, 8		6, 7, 8						
Number Sentences	6	6	6		6		6			
Vertical Form Addition			6, 7, 8	6, 7, 8						
Using Number Line	6		6	6	6		6			
Basic Facts	6	6	6							

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UNIT WHOLE NUMBERS GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Number Patterns			6	6	6		6			
Estimation			7	7	7		7			
Casting Out Nines	8		8		8		8			
Closure	8				6	9				
Commutative Property	8	6	6		6	9	6			8
Associative Property	8	6	6		6	9	6			8
Identity Element	9	6	6		6	7	6			
Inverse Operations		7	6		7	10	6			
Vertical Form Subtraction			6, 7, 8	6, 7, 8						
Checking			6		6					
Role of Zero			6		6	7	6			
Order of Operations			6, 8		6, 8	10	6, 8			
Vertical Form Multiplication			6, 7, 8	6, 7, 8						
Factoring		6	6							
Divisors	6	6	6							
Rules for Divisibility						6, 7, 8	6, 7, 8			
Prime Numbers	7	7	7							7
Composite Numbers	7	7	7		7					

UNIT WHOLE NUMBERS GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Prime Factorization	7	7	7	7	7					7
Multiples	7	7		7	7					8
Power	7	7	7	7	7					
Base	7	7	7		7					
Exponent	7	7	7		7					
Distributive Property	9	9	9			10	6			
Division			6, 7, 8	6, 7, 8						
Remainders	6	6			6					
Role of One			6		6	7	6			

PRELIMINARY TOPICS OF

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning of Fractions	6	6	6		6	8				
Numerator and Denominator	6	6			6					
Symbols (Fraction Bar)	6	6			6	6				
Number Line	6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8					
Comparing Fractions			6	6					6	
Divisibility Rules		6, 7, 8	6, 7, 8			6, 7, 8	6, 7, 8, 9			6, 7, 8
Fractional Names for One	6	6	6	6	6	8	7			
Greatest Common Factor	7, 8, 9	7, 8, 9	7, 8, 9	7, 8	7, 8, 9					7
Simplifying Fractions	6, 7, 8	6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8			
Renaming Fractions in Higher Terms	6, 7, 8	6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8			
Mixed Form	6, 7	6, 7	6, 7	6, 7	6, 7					
Rename Mixed Form as Fractions			6, 7, 8	6, 7, 8	6, 7, 8	8	6, 7, 8			
Rename Fractions as Mixed Form			6, 7, 8	6, 7, 8	6, 7, 8	8	6, 7, 8			
Equivalent Fractions	6, 7	6, 7	6, 7	6, 7		6, 7	6, 7, 8			6, 7
Nonequivalent Fractions		6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8		6, 7, 8	

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UNIT FRACTIONAL NUMBERS GRADE(S) Six through Nine

Multiplication of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Expressing a Whole Number as a Fraction	6	6	6		6	6	6			
Fraction Times Whole Number, Meaning	6, 7	6, 7	6	7	7					
Fraction Times Whole Number	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Fraction Times Fraction, Meaning	6, 7	6, 7	6, 7	7						
Fraction Times Fraction	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Mixed Form Times Whole Number	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Mixed Form Times Fraction	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Mixed Form Times Mixed Form	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Closure Property					7	8, 9				
Commutative Property	7	6, 7	7	6, 7, 8, 9		7, 8				
Associative Property	8	8	8	8		8, 9	9			8, 9
Distributive Property	9	9	9			9	9			9
Identity Element	6	6	6	6	6	6	6			
Estimation	6	6	6	6, 7, 8, 9						
Translating Verbal Problems into Number Sentences	6, 7 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9			6, 7, 8, 9		

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Nine

Division of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning	6	6	6, 7	6, 7	7					
Reciprocals	7	7	7, 8	8	8		7			
Complex Fractions	7	7	7, 8, 9	7, 8, 9	8, 9	9	7, 8, 9			
Fraction Divided by a Whole Number	7	7	7, 8, 9	7, 8, 9	8, 9	9	7, 8, 9			
Whole Number Divided by a Fraction	7	7	7, 8, 9	7, 8, 9	8, 9		7, 8, 9			
Fraction Divided by a Fraction	7	7	7, 8, 9	7, 8, 9	8, 9	9	7, 8, 9			
Mixed Form Divided by a Whole Number	8	8	8, 9	8, 9	9		8, 9			
Whole Number Divided by a Mixed Form	8	8	8, 9	8, 9	9		8, 9			
Mixed Form Divided by a Fraction	8	8	8, 9	8, 9	9		8, 9			
Fraction Divided by a Mixed Form	8	8	8, 9	8, 9	9		8, 9			
Mixed Form Divided by a Mixed Form	9	9	9	9	9	9	9			
Closure Property					8	8				
Non-Commutativity		7, 8	7, 8	8, 9						
Non-Associativity	9	9	9							
Identity Element	8	8		8						
Inverse Operation	8	8	8	8	9		9			
Translating Verbal Problems into Number Sentences	7, 8, 9	7, 8, 9	7, 8, 9	7, 8, 9	7, 8, 9			7, 8, 9		

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Nine

Addition of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning of Addition of Fractions	6	6	6	6	7					
Addition of Fractions, Like Denominators	6	6	6	6		6	6, 7			
Least Common Multiple	6	6	6, 7, 8, 9	6, 7, 8, 9	7	6, 7, 8, 9	6, 7, 8, 9			6, 7, 8, 9
Addition of Fractions, Unlike Denominators	7, 8	7, 8	7, 8	7, 8, 9			7, 8, 9			
Whole Number Plus Fraction	6	6	6	6	6, 7		6, 7			
Whole Number Plus Mixed Form	6	6	6	6	6, 7		6, 7			
Fraction Plus Mixed Form, Like Denominators	6	6	6	6	6, 7	7	6, 7			
Mixed Form Plus Fraction, Unlike Denominators	7	7	7	7			7			
Mixed Form Plus Mixed Form, Like Denominators	6	6	6, 7	6, 7	6, 7	7	6, 7			
Mixed Form Plus Mixed Form, Unlike Denominators	7	7	7	7	7		7			
Miscellaneous Problems of Adding Fraction Expressions			8	8, 9		9	8, 9			
Closure Property	7	7				6, 7, 8, 9				
Commutative Property	7	6, 7	6, 7	6, 8, 9		7, 8	6, 7, 8			8, 9
Associative Property	8	8	8	8, 9		8	8, 9			8, 9
Identity Element	6	6	6	6	6	6	6			
Estimation	6	6	6	6, 7, 8, 9	6		6, 8, 9			
Translating Verbal Problems into Number Sentences	6, 7 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9			6, 7, 8, 9		

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Ten

Subtraction of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Relationship of Addition and Subtraction, Meaning	6	6	6	6	6					
Subtraction, Like Denominators	6	6	6	6	6	7	6, 7			
Subtraction, Unlike Denominators	7	7	7	7, 8	7	8	7, 8			
Fractions from Mixed Form, Like Denom., No Renaming	6	6	6	6, 7	6	7	6, 7			
Fractions from Whole Numbers	6	6	6	6, 7	6	7	6, 7			
Renaming Whole Numbers as Mixed Forms	6	6	6	6	6	7	6, 7			
Fractions from Mixed Forms, Like Denom., Renaming	6	6	6	6, 7, 8	7	8	6, 7, 8			
Fractions from Mixed Forms, Unlike Denom., No Renaming	7	7	7	7	7	8	7, 8			
Fractions from Mixed Forms, Unlike Denom., Renaming	7	7	7	7	7	8	7, 8			7, 8
Whole Number from Mixed Forms	6	6	6	6	6	7	6, 7			
Mixed Forms from Mixed Forms, No Renaming	6	6	6	6, 7	6	7	6, 7			
Mixed Form from Whole Number	6	6	6	6	6	7	6, 7			
Mixed Forms from Mixed Form, Like Denom., Renaming	6	6	6	6, 7, 8	7	8	6, 7, 8			6, 7, 8
Mixed Form from Mixed Form, Unlike Denom., No Renaming	7	7	7	7, 8	7	8	7, 8			
Mixed Form from Mixed Form, Unlike Denom., Renaming	7	7	7	7, 8	7	8	7, 8			7, 8
Closure Property		6	7	6, 7, 8		7, 8				
Non-Commutativity	7	6	7, 8							
Non-Associativity	7	6		6, 7, 8						

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Ten

Subtraction of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Identity Element		6	6		6	6,7,8	6			
Inverse Operation	8	8		8	9	10	9			
Translating Verbal Problem to Number Sentences	6,7, 8,9	6,7,8,9	6,7,8,9	6,7,8,9	6,7,8,9					

UNIT DECIMAL NUMERALS GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Place Value	6, 7, 8	6, 7, 8				6, 7, 8				6, 7, 8
Numbers to a Million	6	6	6						6	
Numbers to a Billion	7, 8	7, 8	7, 8						7, 8	
Number Patterns				6, 7, 8		6, 7, 8	6, 7, 8			
Equivalent Powers of Ten						6, 7, 8				
Expanded Notation			6, 7, 8							
Decimal Equivalence	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8						
Annexing Zeros	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9							
Number Line	6, 7	6, 7	6, 7			6, 7			6, 7	
Comparing Numbers			6, 7, 8						6, 7, 8	
Betweenness	6, 7, 8, 9		6, 7, 8, 9							
Rounded Numbers	6, 7, 8, 9	6, 7, 8, 9					6, 7, 8, 9			
Decimal Simplifying Numerals	6, 7, 8	6, 7, 8		6, 7, 8						
Equivalence Decimal Charts			8	8						
Column Form Addition			6, 7, 8, 9	6, 7, 8, 9						
Arrange Addends in Column Form			6, 7, 8, 9			6, 7, 8, 9				
Estimating Sums			8	8	8					
Subtraction			6, 7, 8, 9	6, 7, 8, 9						

UNIT DECIMAL NUMERALS GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON- STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN- GUISHING
Division of a Decimal by a Decimal			10	10						
Quotients Rounded			10	10	10					
Decimal Equivalents	10	10	10	10						

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UNIT PERCENT BY RATIO AND PROPORTION GRADE(S) Eight through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning of Ratio	8	8	8	8	8					
Meaning of Rate Pair	8	8	8	8	8					
Simplifying a Rate Pair	8	8	8			8	8			
Translating Verbal Problems into Rate Pairs		8	8	8	8					
Meaning of Percent by Rate Pairs		8	8	8	8					
Changing Percents in Verbal Problems to Rate Pairs		8		8						
Equivalent Rate Pairs						9	9			
Proportion	9	9	9	9						
Solving Multiplication Equation	9	9	9			9	9, 10			
Translating Verbal Problems into Equivalent Rate Pairs		9	9	9	9					
Solving Proportions		9	9			9	9, 10			
Finding What Percent One Number is of Another		9	9				9, 10			
Translating Verbal Problems into Equivalent Rate Pairs		9	9	9	9					
Finding a Percent of a Number		10	10			10	10			10
Finding a Number When a Percent of it is Known		10	10			10	10			10
Translating Verbal Problems into Equal Rate Pairs		10	10	10	10					10

UNIT SQUARE ROOT GRADE(S) Seven through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Square Numbers	7	7	7	7	7					7
Vocabulary	7, 8	7, 8								8
Symbol	8	8		8						8
Expansion of Powers	8	8	8	8	8					8
Use of Square Root Table	9	9	9	9	9					
Extracting Square Roots			10, 11	10, 11	10, 11		10, 11			

UNIT COMPUTING DEVICES GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Vocabulary	10	10								10
Calculator			10	10	10					
Addition, Subtraction, Multiplication, Division			10	10						
Accuracy			10		10					
Combined Operations			10	10				10		

FO-15

UNIT SLIDE RULE GRADE(S) Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Vocabulary	11	11								11
Construction of Slide Rules			11	11	11					
Reading Scales			11		11					
Multiplication			11	11						
Division			11	11						
Combined Operations			11	11	11			11		
Estimating Solutions			11	11			11			
Square Root			11	11						
Squaring			11	11						

UNIT WHOLE NUMBERS GRADE(S) Eight

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Expanded Notation to Millions						8				
Expanded Notation to Billions						8				
Renaming Numbers						8				
Verbal Problems			8	8				8		
Symbol (s)	8	8		8						
Vertical Form Addition			8	8						
Casting Out Nines	8		8		8		8			
Closure	8									
Commutative Property	8									8
Associative Property	8									8
Vertical Form Subtraction			8	8						
Order of Operations			8		8		8			
Vertical Form Multiplication			8	8						
Rules for Divisibility						8	8			
Multiples										8
Power									8	

NUMBERS, OPERATIONS AND ALGORITHMS
WHOLE NUMBERS - Grade 8

Note: In the following list of objectives, we shall agree to use phrases such as "a three digit number" to mean "a number named by a numeral containing three digit symbols."

Page

Expanded Notation

The student should be able to:

1. Apply the principle of writing numerals in expanded notation to billions without exponents

Renaming Numbers

The student should be able to:

1. State the principle that the name of any number may be replaced by another name using basic facts

ADDITION

Vertical Form Addition

The student should be able to:

1. Demonstrate how to add to four four digit addends
2. Construct the sum of four four digit addends

Properties

The student should be able to:

1. Name the properties of closure, commutativity and associativity for addition FO-8
2. Distinguish between commutativity and associativity

Casting of Nines

The student should be able to:

1. Name the process of casting out nines in addition
2. Demonstrate how to cast out nines to check addition problems
3. Describe the process of casting out nines as a check in addition
4. Apply the principle of casting out nines to solve related problems

SUBTRACTION

Page

Vertical Form Subtraction

The student should be able to:

1. Demonstrate how to subtract two four digit numbers with renaming
2. Construct the difference of two four digit numbers with renaming

Properties

The student should be able to:

1. Name non-closure, non-commutativity, and non-associativity of subtraction
2. Distinguish between non-commutative and non-associative

Casting of Nines

The student should be able to:

1. Name the process of casting out nines in subtraction
2. Demonstrate how to cast out nines to check subtraction problems
3. Describe the process of casting out nines as a check in subtraction
4. Apply the principle of casting out nines to solve related problems

MULTIPLICATION

Symbols and Vocabulary

The student should be able to:

1. Name, identify, and draw the symbol "( )" in multiplication
2. Distinguish between factor and multiple

Vertical Form Multiplication

The student should be able to:

1. Demonstrate how to multiply three digit numbers by four digit numbers
2. Construct the products of a three digit number by a four digit number

Casting of Nines

The student should be able to:

1. Name the process of casting out nines
2. Demonstrate how to cast out nines to check multiplication problems
3. Describe the process of casting out nines as a check in multiplication
4. Apply the principle of casting out nines to solve related problems

Power

The student should be able to:

1. Order powers of numbers using less than, more than or the number line

Multiplies

The student should be able to:

1. Distinguish between multiple and factor

Rules for Divisibility

The student should be able to:

1. State the rules for divisibility by four, six, and eight
2. Apply the rules for divisibility by four, six, and eight to solve related problems

Properties

The student should be able to:

1. Name the properties of closure, commutativity, and associativity in multiplication

Page

FO-7[illegible]
R-3
R-11

Page

2. Distinguish between commutativity and associativity

Order of Operations

The student should be able to:

1. Demonstrate the order of operations involving both addition and multiplication
2. Describe the order of addition and multiplication
3. Apply the principle that multiplications are done in order from left to right before additions in expressions involving both addition and multiplication

DIVISION

Division

The student should be able to:

1. Demonstrate how to find quotients with remainders using three digit divisors FO-81 R-3
2. Construct quotients with remainders using three digit divisors

Casting of Nines

The student should be able to:

1. Name the process of casting out nines in division
2. Demonstrate how to cast out nines to check division problems
3. Describe the process of casting out nines as a check in division
4. Apply the principle of ca~ting out nines to check related problems

Properties

Page

The student should be able to:

1. Name non-closure of whole, and non-commutative, and non-associative properties of division
2. Distinguish between non-commutative and non-associative properties of division

COMBINED OPERATIONS

Order of Operations

The student should be able to:

1. Demonstrate how to simplify expressions involving all four fundamental operations
2. Describe simplifying expressions involving all four fundamental operations
3. Apply the principle that terms inside parentheses are combined first, with multiplication and division done in order, followed by addition and subtraction in order

Verbal Problems

The student should be able to:

1. Demonstrate how to solve a verbal problem involving two or more operations
2. Construct the solution of a verbal problem involving two or more operations FO-91
3. Interpret the steps involved in the solution of a verbal problem involving two or more operations

UNIT FRACTIONAL NUMBERS GRADE(S) Eight

Preliminary Topics

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning of Fractions						8				
Number Line	8	8	8	8	8					
Divisibility Rules		8	8			8	8			8
Fractional Names for One						8				
Greatest Common Factor	8	8	8	8	8					
Simplifying Fractions	8	8	8	8	8	8	8			
Renaming Fractions in Higher Terms	8	8	8	8	8	8	8			
Rename Mixed Form as Fractions			8	8	8	8	8			
Rename Fractions as Mixed Form			8	8	8	8	8			
Equivalent Fractions							8			
Nonequivalent Fractions		8	8	8	8	8	8		8	

NUMBERS, OPERATIONS AND ALGORITHMS
FRACTIONAL NUMBERS - Grade 8

Preliminary Topics

Page

Meaning of Fractions

The student should be able to:

1. State the principle that when zero appears in the denominator and a whole number appears in the numerator, the resulting fraction is meaningless

Number Line

(Use fractions whose denominators are 2, 4, 8, 16, 3, 6, 5, and 10)

The student should be able to:

1. Name and identify fractions by associating them with points on the number line
2. Demonstrate how to find the point on the number line which is associated with a given fraction
3. Construct a number line, dividing it into different units as specified by the restriction above
4. Describe a procedure for dividing a number line into units specified by the restriction above

Divisibility Rules

(Use rules for dividing by 4, 6, and 8)

The student should be able to:

1. Identify numbers which are divisible by 4, 6, and 8
2. Demonstrate a procedure for testing whether or not a number is divisible by 4, 6, and 8
3. State the principle that:
    a. A number is divisible by 4 if the number represented by the last two digits is divisible by 4
    b. A number is divisible by 6 only if it is an even number and the sum of its digits is divisible by 3

Page

c. A number is divisible by 8 only if it is an even number and the number represented by the last three digits is divisible by 8

4. Apply the principles of the divisibility rules to solve related problems

5. Distinguish among the divisibility rules for 4, 6, and 8

Fractional Names for One

The student should be able to.

1. State the principles that:

   a. A fraction is equivalent to one if the numerator is the same as the denominator

   b. A fraction cannot be expressed as $\frac{0}{0}$ or $\frac{7}{0}$

Greatest Common Factor

(Use multiples of 2, 3, 4, 5, 6, 8, 10 of a set of numbers)

The student should be able to:

1. Name and identify the greatest common factor of a set of numbers

2. Demonstrate how to find the greatest common factor by expressing each number in exponential notation

3. Construct the greatest common factor of a set of numbers

4. Describe a procedure for finding the greatest common factor of a set of numbers

Simplifying Fractions

(Listed below are some representative examples. Note that both numerator and denominator are divisible by 2, 3, 4, 5, 6, 8, or 10)

$$\frac{36}{60}, \frac{12}{16}, \frac{8}{20}, \frac{18}{24}, \frac{24}{32}, \frac{20}{36}, \frac{80}{100}$$

Page

The student should be able to:

1. Name and identify fractions which are expressed in lowest terms
2. Demonstrate how to simplify fractions using the divisibility rules or the greatest common factor
3. Construct the simplest form of a given fraction
4. Describe a procedure for simplifying fractions
5. State the principle that a fraction is simplified when the numerator and denominator have no common factor except one
6. Apply this principle for simplifying fractions to solve related problems

Renaming Fractions in Higher Terms

(Listed below are some representative examples)

$$\frac{5}{8} = \frac{\square}{24}, \quad \frac{13}{16} = \frac{\square}{32}, \quad \frac{17}{24} = \frac{\square}{48}, \quad \frac{2}{25} = \frac{\square}{100}$$

$$\frac{7}{9} = \frac{\square}{45}, \quad \frac{7}{12} = \frac{\square}{48}, \quad \frac{7}{12} = \frac{\square}{36}, \quad \frac{43}{50} = \frac{\square}{100}$$

The student should be able to:

1. Name and identify fractions in higher terms
2. Demonstrate how to rename fractions by multiplying by a fractional name for one
3. Construct a fraction in higher terms equivalent to a given fraction
4. Describe a procedure for renaming a fraction in higher terms using a specific example
5. State the principle for renaming fractions in higher terms
6. Apply this principle for renaming fractions in higher terms to solve related problems

Renaming Mixed Form as a Fraction

(Use whole numbers less than 100, and fractions whose denominators are 2, 4, 8, 16, 3, 6, 5, and 10)

Page

The student should be able to:

1. Demonstrate a procedure for renaming mixed forms as a fraction
2. Construct a fraction given a number written in mixed form
3. Describe a procedure for renaming mixed forms as a fraction using specific examples
4. State the principle for renaming mixed forms as a fraction
5. Apply the principle for renaming mixed forms as a fraction to solve related problems

Renaming a Fraction as a Mixed Form

(Listed below are some representative examples)

$$\frac{45}{20}, \frac{46}{16}, \frac{33}{9}, \frac{100}{32}, \frac{50}{12}, \frac{80}{24}$$

The student should be able to:

1. Demonstrate a procedure for renaming a fraction as a mixed form
2. Construct a mixed form given a fraction
3. Describe a procedure for renaming a fraction as a mixed form
4. State the principle for renaming a fraction as a mixed form
5. Apply this principle for renaming a fraction as a mixed form to solve related problems

Equivalent Fractions

The student should be able to:

1. Apply the principle for finding sets of equivalent fractions to solve related problems

Nonequivalent Fractions

(Listed below are some representative examples)

Page

$$\frac{4}{5} < \frac{7}{8}, \quad \frac{19}{25} > \frac{3}{4}, \quad \frac{5}{6} < \frac{11}{12}, \quad \frac{7}{16} > \frac{3}{10}$$

The student should be able to:

1. Identify pairs of nonequivalent fractions
2. Demonstrate a test to determine whether or not two fractions are nonequivalent
3. Construct an example of a pair of nonequivalent fractions
4. Describe nonequivalent fractions in terms of cross products or as being less than or greater than one another
5. State the principle that a pair of fractions are nonequivalent if the corss products are not equal
6. Apply this principle to solve related problems
7. Order nonequivalent fractions

UNIT FRACTIONAL NUMBERS GRADE(S) Eight

Multiplication of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Fraction Times Whole Number			8	8	8	8	8			
Fraction Times Fraction			8	8	8	8	8			
Mixed Form Times Whole Number			8	8	8	8	8			
Mixed Form Times Fraction			8	8	8	8	8			
Mixed Form Times Mixed Form			8	8	8	8	8			
Closure Property						8				
Commutative Property				8		8				
Associative Property	8	8	8	8		8				8
Estimation				8						
Translating Verbal Problems into Number Sentences	8	8	8	8	8			8		

NUMBERS, OPERATIONS AND ALGORITHMS
FRACTIONAL NUMBERS - Grade 8

Multiplication of Fractions

<u>The Product of a Fraction and a Whole Number</u> | Page

(Some representative examples will be found in Chart I)

The student should be able to:

1. Demonstrate how to construct the product of a fraction and a whole number, by using the multiplicative and renaming principles of fractions, simplifying the final answer, and/or using the "short-cut" method

2. Construct the product of a fraction and a whole number

3. Describe how to find the product, in simplest form, of a fraction and a whole number using specific examples

4. State the principle of multiplying a fraction and a whole number

5. Apply the following principles for multiplying a fraction and a whole number:

   a. Rename the whole number as a fraction

   b. Use the "short-cut" method before multiplying or multiply the numerators and denominators, expressing the answer in simplest form

Page

CHART I

Fraction x Whole Number - Grade 8

$\frac{1}{3} \times \frac{4}{1}$	$\frac{5}{6} \times \frac{9}{1}$	$\frac{5}{6} \times \frac{15}{1}$
$\frac{2}{3} \times \frac{4}{1}$	$\frac{5}{9} \times \frac{6}{1}$	$\frac{7}{25} \times \frac{15}{1}$
$\frac{1}{4} \times \frac{4}{1}$	$\frac{2}{3} \times \frac{14}{1}$	$\frac{13}{15} \times \frac{14}{1}$
$\frac{3}{4} \times \frac{4}{1}$	$\frac{3}{14} \times \frac{14}{1}$	$\frac{13}{14} \times \frac{14}{1}$
$\frac{1}{2} \times \frac{4}{1}$	$\frac{3}{7} \times \frac{14}{1}$	$\frac{15}{16} \times \frac{32}{1}$
$\frac{3}{4} \times \frac{8}{1}$	$\frac{3}{24} \times \frac{12}{1}$	$\frac{13}{24} \times \frac{12}{1}$
$\frac{1}{4} \times \frac{2}{1}$		$\frac{15}{6} \times \frac{15}{1}$
$\frac{3}{4} \times \frac{2}{1}$		$\frac{17}{25} \times \frac{15}{1}$

The Product of Two Fractions

(Some representative examples will be found in Chart II)

The student should be able to:

1. Demonstrate how to construct the product of two fractions using the multiplicative principle of fractions, simplifying the final answer, and/or using the "short-cut" method
2. Construct the product of two fractions
3. Describe how to find the product, in simplest form, of two fractions using specific examples

Page

4. State the principle for multiplying two common fractions

5. Apply the following principles for multiplying two fractions: Use the "short-cut" method before multiplying or multiply the numerators and denominators, expressing the answer in simplest form

CHART II

Fraction x Fraction - Grade 8

$\frac{1}{3} \times \frac{1}{2}$	$\frac{4}{5} \times \frac{1}{17}$	$\frac{14}{15} \times \frac{1}{17}$
$\frac{1}{3} \times \frac{4}{5}$	$\frac{1}{3} \times \frac{14}{15}$	$\frac{1}{13} \times \frac{14}{15}$
$\frac{2}{3} \times \frac{4}{5}$	$\frac{12}{13} \times \frac{4}{5}$	$\frac{12}{13} \times \frac{14}{17}$
$\frac{1}{3} \times \frac{3}{5}$	$\frac{1}{4} \times \frac{4}{15}$	$\frac{1}{14} \times \frac{14}{15}$
$\frac{2}{3} \times \frac{3}{5}$	$\frac{8}{13} \times \frac{1}{8}$	$\frac{11}{13} \times \frac{13}{15}$
$\frac{2}{3} \times \frac{1}{2}$	$\frac{1}{8} \times \frac{16}{17}$	$\frac{18}{19} \times \frac{1}{18}$
$\frac{3}{5} \times \frac{2}{3}$	$\frac{3}{8} \times \frac{16}{17}$	$\frac{13}{15} \times \frac{2}{13}$
$\frac{1}{2} \times \frac{4}{5}$	$\frac{5}{6} \times \frac{14}{17}$	$\frac{1}{12} \times \frac{24}{25}$
$\frac{2}{3} \times \frac{6}{7}$	$\frac{1}{4} \times \frac{2}{13}$	$\frac{3}{16} \times \frac{32}{35}$
$\frac{5}{6} \times \frac{8}{9}$	$\frac{3}{4} \times \frac{2}{13}$	$\frac{15}{16} \times \frac{22}{24}$
$\frac{1}{4} \times \frac{2}{5}$	$\frac{5}{16} \times \frac{6}{7}$	$\frac{1}{22} \times \frac{11}{13}$
$\frac{3}{8} \times \frac{4}{5}$	$\frac{2}{9} \times \frac{9}{16}$	$\frac{5}{22} \times \frac{11}{13}$

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CHART II (Cont'd.)

Fraction x Fraction - Grade 8

$\frac{5}{9} \times \frac{6}{7}$	$\frac{6}{9} \times \frac{9}{21}$	$\frac{5}{16} \times \frac{6}{17}$
$\frac{2}{3} \times \frac{3}{4}$	$\frac{6}{16} \times \frac{8}{9}$	$\frac{2}{13} \times \frac{13}{16}$
$\frac{6}{7} \times \frac{7}{9}$	$\frac{12}{15} \times \frac{6}{9}$	$\frac{6}{13} \times \frac{13}{21}$
$\frac{6}{8} \times \frac{4}{9}$		$\frac{15}{16} \times \frac{8}{12}$
$\frac{6}{9} \times \frac{6}{9}$		$\frac{12}{25} \times \frac{15}{16}$

The Product of a Number in Mixed Form and a Whole Number
(Some representative examples are given in Chart III)

The student should be able to:

1. Demonstrate how to construct the product of a mixed form and a whole number, by using the multiplicative and renaming principles of fractions, simplifying the final answer, and/or using the "short-cut" method
2. Construct the product of a mixed form and a whole number
3. Describe how to find the product, in simplest form, of a mixed form and a whole number, using specific examples
4. State the principle for multiplying a mixed form and a whole number
5. Apply the following principles for multiplying a mixed form and a whole number:
   a. Rename the mixed form as a fraction
   b. Rename the whole number as a fraction
   c. Use the "short-cut" method before multiplying or multiply the numerators and denominators, expressing the answer in simplest form

CHART III

Mixed Form x Whole Number - Grade 8

$1\frac{2}{3} \times \frac{4}{1}$	$2\frac{3}{5} \times \frac{14}{1}$	$3\frac{1}{5} \times \frac{14}{1}$
$1\frac{3}{4} \times \frac{4}{1}$	$4\frac{1}{4} \times \frac{4}{1}$	$4\frac{1}{4} \times \frac{14}{1}$
$1\frac{1}{4} \times \frac{8}{1}$	$3\frac{3}{4} \times \frac{8}{1}$	$1\frac{6}{9} \times \frac{18}{1}$
$1\frac{1}{4} \times \frac{2}{1}$	$3\frac{3}{4} \times \frac{2}{1}$	$1\frac{3}{24} \times \frac{12}{1}$
$1\frac{1}{6} \times \frac{9}{1}$	$2\frac{5}{6} \times \frac{9}{1}$	$1\frac{7}{10} \times \frac{18}{1}$
$1\frac{1}{6} \times \frac{4}{1}$	$2\frac{5}{6} \times \frac{4}{1}$	$2\frac{5}{6} \times \frac{14}{1}$

The Product of a Number in Mixed Form and a Fraction

(Some representative examples will be found in Chart IV)

The student should be able to:

1. Demonstrate how to construct the product of a mixed form and a whole number, by using the multiplicative and renaming principles of fractions, simplifying the final answer, and/or using the "short-cut" method
2. Construct the product of a mixed form and a fraction
3. Describe how to find the product of a mixed form and a fraction, using specific examples. The final answer should be simplified.
4. State the principle for multiplying a mixed form and a fraction
5. Apply the following principles for multiplying a mixed form and a fraction:
    a. Rename the mixed form as an improper fraction
    b. Use the "short-cut" method before multiplying or multiply the numerators and denominators, expressing the answer in simplest form

Page

CHART IV

Mixed Form x Fraction - Grade 8

$2\frac{2}{3} \times \frac{1}{3}$	$3\frac{2}{3} \times \frac{1}{6}$	$4\frac{1}{3} \times \frac{11}{16}$
$1\frac{1}{2} \times \frac{5}{7}$	$3\frac{1}{4} \times \frac{6}{17}$	$1\frac{4}{9} \times \frac{21}{23}$
$1\frac{3}{5} \times \frac{1}{8}$	$5\frac{1}{5} \times \frac{1}{26}$	$3\frac{1}{5} \times \frac{13}{16}$
$1\frac{3}{5} \times \frac{3}{8}$	$3\frac{4}{5} \times \frac{3}{19}$	$1\frac{3}{16} \times \frac{13}{19}$
$1\frac{1}{5} \times \frac{1}{3}$	$3\frac{1}{5} \times \frac{1}{8}$	$2\frac{2}{15} \times \frac{1}{6}$
$1\frac{3}{5} \times \frac{3}{4}$	$4\frac{2}{3} \times \frac{2}{7}$	$2\frac{2}{7} \times \frac{13}{16}$
$1\frac{1}{6} \times \frac{2}{3}$	$1\frac{1}{16} \times \frac{2}{3}$	$4\frac{2}{4} \times \frac{7}{13}$
$1\frac{3}{5} \times \frac{5}{8}$	$2\frac{1}{7} \times \frac{7}{15}$	$1\frac{1}{13} \times \frac{13}{14}$
$1\frac{1}{2} \times \frac{4}{9}$	$5\frac{1}{3} \times \frac{9}{32}$	$1\frac{1}{15} \times \frac{30}{64}$
$1\frac{4}{5} \times \frac{5}{6}$	$2\frac{4}{7} \times \frac{7}{8}$	$2\frac{1}{17} \times \frac{17}{25}$

The Product of Two Numbers in Mixed Form

(Some representative examples will be found in Chart V)

The student should be able to:

1. Demonstrate how to construct the product of two mixed forms, by using the renaming and multiplicative principles of fractions, simplifying the final answer, and/or using the "short-cut" method
2. Construct the product of two mixed forms
3. Describe how to find the product, in simplest form, of two mixed forms, using specific examples
4. State the principle of multiplying two mixed forms

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5. Apply the following principles for multiplying two mixed forms:

   a. Rename the mixed forms as improper fractions

   b. Use the "short-cut" method before multiplying or multiply the numerators and denominators, expressing the answer in simplest form

CHART V

Mixed Form x Mixed Form - Grade 8

$1\frac{1}{2} \times 1\frac{1}{2}$	$1\frac{1}{2} \times 1\frac{3}{10}$	$6\frac{1}{2} \times 1\frac{3}{10}$
$1\frac{1}{2} \times 1\frac{2}{3}$	$1\frac{1}{2} \times 5\frac{2}{3}$	$6\frac{1}{2} \times 5\frac{2}{3}$
$1\frac{1}{2} \times 1\frac{1}{6}$	$1\frac{1}{2} \times 1\frac{1}{15}$	$1\frac{3}{12} \times 1\frac{18}{30}$
$1\frac{1}{3} \times 3\frac{1}{2}$	$1\frac{1}{5} \times 1\frac{1}{16}$	$3\frac{2}{5} \times 1\frac{1}{16}$
$1\frac{1}{5} \times 1\frac{3}{4}$	$6\frac{2}{3} \times 1\frac{4}{5}$	$6\frac{2}{3} \times 1\frac{7}{10}$
$1\frac{1}{2} \times 1\frac{1}{3}$	$1\frac{3}{15} \times 1\frac{3}{4}$	$1\frac{3}{15} \times 1\frac{1}{16}$
$1\frac{1}{6} \times 1\frac{1}{7}$	$1\frac{1}{2} \times 5\frac{1}{3}$	$6\frac{1}{2} \times 1\frac{3}{13}$
$2\frac{1}{4} \times 2\frac{2}{3}$	$1\frac{3}{4} \times 2\frac{4}{7}$	$4\frac{1}{4} \times 1\frac{1}{17}$
$2\frac{1}{4} \times 1\frac{2}{6}$	$2\frac{1}{4} \times 5\frac{1}{3}$	$3\frac{1}{8} \times 1\frac{1}{15}$
$1\frac{3}{6} \times 1\frac{3}{6}$	$3\frac{3}{4} \times 1\frac{2}{6}$	$1\frac{1}{14} \times 3\frac{1}{9}$
	$1\frac{1}{8} \times 1\frac{2}{12}$	$1\frac{17}{18} \times 2\frac{2}{15}$

Closure Property of Fractions

Page

The student should be able to:

1. State the principle that the set of fractions is closed under multiplication because the answers in every case are fractions

Commutative Property

(Use examples found in Chart I through Chart V)

The student should be able to:

1. Construct a number sentence to illustrate the commutative property of multiplication of fractions
2. State the principle of commutativity for multiplication of fractions

Associative Property

The student should be able to:

1. Name and identify number sentences which illustrate the associative property of multiplication of fractions
2. Demonstrate a procedure which tests number sentences for associativity
3. Construct number sentences which illustrate the associative property of multiplication of fractions
4. State the principle of associativity of multiplication of fractions
5. Distinguish between the associative and commutative properties of multiplication of fractions

Translating Verbal Problems Involving Multiplication of Fractions into Number Sentences

Page

(Choose only those problems whose solution can be determined in one or two steps)

The student should be able to:

1. Name and identify verbal problems whose solutions require multiplication of fractions
2. Demonstrate a procedure for translating verbal problems into number sentences
3. Construct a number sentence from the information contained in a verbal problem
4. Describe the steps needed to translate verbal problems into number sentences, using specific examples
5. Interpret the information contained in a verbal problem in order to construct a corresponding number sentence

UNIT FRACTIONAL NUMBERS GRADE(S) Eight

Division of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Reciprocals			8	8	8					
Complex Fractions			8	8	8		8			
Fraction Divided by a Whole Number			8	8	8		8			
Whole Number Divided by a Fraction			8	8	8		8			
Fraction Divided by a Fraction			8	8	8		8			
Mixed Form Divided by a Whole Number	8	8	8	8			8			
Whole Number Divided by a Mixed Form	8	8	8	8			8			
Mixed Form Divided by a Fraction	8	8	8	8			8			
Fraction Divided by a Mixed Form	8	8	8	8			8			
Closure Property					8	8				
Non-Commutativity		8	8	8						
Identity Element	8	8		8						
Inverse Operation	8	8		8						
Translating Verbal Problems into Number Sentences	8	8	8	8	8			8		

Division of Fractions

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Reciprocals

(Listed below are some representative examples)

$$2, \quad 10\frac{1}{8}, \quad \frac{3}{5}, \quad 2\frac{1}{2}, \quad 4\frac{3}{4}$$

The student should be able to:

1. Demonstrate how to write the reciprocal of a whole number, a fraction, and a mixed form
2. Construct the reciprocal of a whole number, a fraction, and a mixed form
3. Describe how to find the reciprocal of a whole number, a fraction, and a mixed form by using specific examples

Complex Fractions

(Listed below are some representative examples)

$$\frac{\frac{1}{3}}{6}, \quad \frac{\frac{8}{1}}{\frac{1}{4}}, \quad \frac{\frac{1}{2}}{\frac{1}{3}}, \quad \frac{2\frac{1}{2}}{4}, \quad \frac{3}{4\frac{1}{3}}, \quad \frac{3\frac{2}{3}}{\frac{1}{2}}, \quad \frac{\frac{3}{4}}{1\frac{1}{3}}$$

The student should be able to:

1. Demonstrate how to write a complex fraction given a division problem of the form $\frac{1}{3} \div 6$, etc.
2. Construct complex fractions given a division problem of the form $8 \div \frac{1}{4}$, etc.
3. Describe how to rename a division example as a complex fraction by using specific examples
4. Apply the principle to rename a division example as a complex fraction

Fraction Divided by a Whole Number

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(Some representative examples will be found in Chart I below)

The student should be able to:

1. Demonstrate how to construct the quotient of a fraction and a whole number by using complex fractions or the definition of division of fractions
2. Construct the quotient of a fraction and a whole number
3. Describe how to divide a fraction by a whole number by using specific examples
4. Apply the following principles for dividing a fraction by a whole number:
   a. Rename the division problem as a multiplication problem
   b. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answers in simplest form

CHART I

Fraction Divided by a Whole Number - Grade 8

$\frac{1}{2} \div 8$	$\frac{1}{3} \div 12$
$\frac{3}{5} \div 8$	$\frac{4}{5} \div 21$
$\frac{7}{8} \div 7$	$\frac{9}{15} \div 9$
$\frac{4}{5} \div 2$	$\frac{18}{21} \div 2$
$\frac{2}{3} \div 4$	$\frac{6}{10} \div 12$
$\frac{9}{10} \div 6$	$\frac{12}{15} \div 8$
$\frac{6}{10} \div 9$	$\frac{8}{15} \div 12$

Whole Number Divided by a Fraction

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(Some representative examples will be found in Chart II below)

The student should be able to:

1. Demonstrate how to construct the quotient of a whole number and a fraction by using complex fractions or the definition of division of fractions

2. Construct the quotient of a whole number and a fraction

3. Describe how to divide a whole number by a fraction by using specific examples

4. Apply the following principles for dividing a whole number by a fraction:

   a. Rename the division problem as a multiplication problem

   b. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest form

CHART II

Whole Number Divided by a Fraction - Grade 8

$6 \div \frac{1}{3}$	$16 \div \frac{1}{3}$
$7 \div \frac{2}{3}$	$17 \div \frac{2}{3}$
$4 \div \frac{4}{5}$	$4 \div \frac{4}{15}$
$8 \div \frac{4}{5}$	$10 \div \frac{5}{6}$
$2 \div \frac{4}{5}$	$6 \div \frac{12}{13}$
$2 \div \frac{4}{6}$	$2 \div \frac{14}{16}$
$6 \div \frac{9}{10}$	$8 \div \frac{12}{15}$
$9 \div \frac{6}{10}$	$12 \div \frac{8}{15}$

Fraction Divided by a Fraction

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(Some representative examples will be found in Chart III)

The student should be able to:

1. Demonstrate how to construct the quotient of fractions by using complex fractions or the definition of division of fractions

2. Construct the quotient of two fractions

3. Describe how to divide pairs of fractions by using specific examples

4. Apply the following principles for dividing pairs of fractions:

    a. Rename the division problem as a multiplication problem

    b. Use the "short-cut" method before multiplying, or multiply the fractions, expressing the answer in simplest form

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CHART III

Fraction Divided by a Fraction - Grade 8

$\frac{1}{4} \div \frac{1}{3}$	$\frac{1}{4} \div \frac{1}{13}$
$\frac{3}{4} \div \frac{1}{3}$	$\frac{3}{14} \div \frac{1}{3}$
$\frac{3}{4} \div \frac{2}{3}$	$\frac{3}{14} \div \frac{2}{3}$
$\frac{1}{4} \div \frac{3}{4}$	$\frac{1}{14} \div \frac{3}{14}$
$\frac{5}{8} \div \frac{5}{8}$	$\frac{5}{18} \div \frac{5}{18}$
$\frac{4}{5} \div \frac{4}{7}$	$\frac{4}{15} \div \frac{4}{17}$
$\frac{5}{6} \div \frac{5}{12}$	$\frac{5}{12} \div \frac{5}{24}$
$\frac{4}{7} \div \frac{2}{7}$	$\frac{4}{17} \div \frac{2}{17}$
$\frac{2}{5} \div \frac{3}{10}$	$\frac{3}{10} \div \frac{16}{20}$
$\frac{3}{4} \div \frac{1}{2}$	$\frac{3}{20} \div \frac{4}{10}$
$\frac{2}{3} \div \frac{4}{9}$	$\frac{2}{3} \div \frac{14}{27}$
$\frac{6}{8} \div \frac{9}{12}$	$\frac{6}{8} \div \frac{12}{18}$

Mixed Form Divided by a Whole Number

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(Some representative examples will be found in Chart IV below)

The student should be able to:

1. Name and identify a division problem whose parts are a mixed form and a whole number
2. Demonstrate how to construct the quotient of a mixed form and a whole number by using complex fractions or the definition of division of fractions
3. Construct the quotient of mixed form and a whole number
4. Apply the following principles for dividing a mixed form by a whole number:
   a. Rename the division problem as a multiplication problem
   b. Rename the mixed form as a fraction
   c. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest form

CHART IV

Mixed Form Divided by a Whole Number - Grade 8

$2\frac{1}{2} \div 2$	$8\frac{1}{2} \div 14$
$1\frac{1}{2} \div 3$	$6\frac{1}{2} \div 13$
$2\frac{2}{3} \div 4$	$6\frac{2}{3} \div 10$
$1\frac{1}{3} \div 8$	$2\frac{2}{5} \div 24$
$1\frac{1}{5} \div 9$	$3\frac{1}{3} \div 14$
$1\frac{4}{5} \div 6$	$4\frac{2}{3} \div 10$

Whole Number Divided by a Mixed Form

(Some representative examples will be found in Chart V below)

Page

The student should be able to:

1. Name and identify a division problem whose parts are a whole number and a mixed form
2. Demonstrate how to construct the quotient of a whole number and a mixed form by using complex fractions or the definition of division of fractions
3. Construct the quotient of a whole number and a mixed form
4. Apply the following principles for dividing a whole number and a mixed form:
   a. Rename the mixed form as a fraction
   b. Rename the division problem as a multiplication problem
   c. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest form

CHART V

Whole Number Divided by a Mixed Form - Grade 8

$2 \div 1\frac{2}{3}$	$14 \div 7\frac{2}{3}$
$3 \div 1\frac{1}{2}$	$15 \div 3\frac{3}{4}$
$8 \div 1\frac{1}{3}$	$16 \div 1\frac{3}{5}$
$4 \div 2\frac{2}{3}$	$6 \div 2\frac{2}{5}$
$6 \div 2\frac{1}{4}$	$8 \div 2\frac{2}{5}$
$12 \div 1\frac{3}{5}$	$16 \div 1\frac{1}{11}$

Mixed Form Divided by a Fraction

(Some representative examples will be found in Chart VI)

Page

The student should be able to:

1. Name and identify a division problem whose parts are a mixed form and a fraction
2. Demonstrate how to construct the quotient of a mixed form and a fraction by using complex fractions or the definition of division of fractions
3. Construct the quotient of a mixed form and a fraction
4. Apply the following principles for dividing a mixed form by a fraction:
   a. Rename the division problem as a multiplication problem
   b. Rename the mixed form as a fraction
   c. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest form

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CHART VI

Mixed Form Divided by a Fraction - Grade 8

$1\frac{1}{2} \div \frac{2}{3}$	$1\frac{1}{2} \div \frac{10}{13}$
$1\frac{1}{2} \div \frac{3}{5}$	$1\frac{1}{2} \div \frac{3}{17}$
$1\frac{1}{2} \div \frac{6}{7}$	$1\frac{1}{2} \div \frac{15}{16}$
$1\frac{1}{3} \div \frac{2}{7}$	$1\frac{1}{5} \div \frac{16}{17}$
$1\frac{1}{5} \div \frac{4}{7}$	$6\frac{2}{3} \div \frac{5}{9}$
$1\frac{1}{2} \div \frac{3}{4}$	$1\frac{3}{15} \div \frac{4}{7}$
$1\frac{1}{6} \div \frac{7}{8}$	$1\frac{1}{2} \div \frac{3}{16}$
$2\frac{1}{4} \div \frac{3}{8}$	$1\frac{3}{4} \div \frac{7}{18}$
$2\frac{1}{4} \div \frac{6}{8}$	$2\frac{1}{4} \div \frac{3}{16}$
$1\frac{2}{3} \div \frac{1}{4}$	$3\frac{3}{4} \div \frac{6}{8}$
$1\frac{3}{4} \div \frac{1}{4}$	$2\frac{3}{5} \div \frac{1}{14}$
$1\frac{1}{4} \div \frac{1}{8}$	$4\frac{1}{4} \div \frac{1}{4}$
$1\frac{1}{4} \div \frac{1}{2}$	$3\frac{3}{4} \div \frac{1}{8}$
$1\frac{1}{6} \div \frac{1}{9}$	$3\frac{3}{4} \div \frac{1}{2}$
$1\frac{1}{6} \div \frac{1}{4}$	$2\frac{5}{6} \div \frac{1}{9}$
	$2\frac{5}{6} \div \frac{1}{4}$

Fraction Divided by a Mixed Form

(Some representative examples will be found in Chart VII)

Page

The student should be able to:

1. Name and identify a division problem whose parts are a fraction and a mixed form
2. Demonstrate how to construct the quotient of a fraction and a mixed form by using complex fractions or the definition of division of fractions
3. Construct the quotient of a fraction and a mixed form
4. Apply the following principles for dividing a fraction by a mixed form:
   a. Rename the mixed form as a fraction
   b. Rename the multiplication problem as a division problem
   c. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest form

CHART VII

Fraction Divided by a Mixed Form - Grade 8

$\frac{2}{3} \div 1\frac{1}{2}$	$\frac{2}{13} \div 1\frac{1}{2}$
$\frac{7}{8} \div 1\frac{4}{3}$	$\frac{7}{18} \div 1\frac{4}{3}$
$\frac{4}{5} \div 2\frac{2}{3}$	$\frac{4}{5} \div 1\frac{11}{17}$
$\frac{6}{7} \div 1\frac{1}{2}$	$\frac{6}{17} \div 1\frac{1}{2}$
$\frac{6}{9} \div 2\frac{1}{4}$	$\frac{6}{19} \div 2\frac{1}{4}$
$\frac{3}{4} \div 1\frac{4}{8}$	$\frac{3}{4} \div 2\frac{14}{16}$

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CHART VII (Cont'd.)

Fraction Divided by a Mixed Form - Grade 8

$\frac{7}{8} \div 1\frac{1}{6}$	$\frac{17}{18} \div 1\frac{1}{16}$
$\frac{6}{9} \div 1\frac{1}{3}$	$\frac{16}{21} \div 4\frac{2}{3}$
$\frac{1}{4} \div 2\frac{1}{3}$	$\frac{1}{14} \div 2\frac{1}{3}$
$\frac{1}{4} \div 1\frac{3}{4}$	$\frac{1}{14} \div 1\frac{3}{14}$
$\frac{1}{4} \div 1\frac{1}{8}$	$\frac{1}{4} \div 1\frac{5}{16}$
$\frac{1}{4} \div 1\frac{1}{2}$	$\frac{1}{14} \div 1\frac{1}{2}$
$\frac{1}{6} \div 1\frac{1}{9}$	$\frac{1}{16} \div 1\frac{1}{24}$

Closure Property

The student should be able to:

1. Describe how to determine closure for the set of fractions

2. State the principle of closure of fractions under division

Non-Commutative Property
(Use examples found in Chart I through Chart VII)

The student should be able to:

1. Identify number sentences which illustrate that division of fractions is not commutative

2. Demonstrate a procedure which shows that division of fractions is not commutative

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3. Construct number sentences which illustrate that division of fractions is not commutative

Identity Element

The student should be able to:

1. Name and identify fractions
2. Construct number sentences which illustrate that any number divided by one is that number
3. State the principle that any number divided by one is that number

Inverse Operations

The student should be able to:

1. Name and identify number sentences which illustrate that division is the inverse operation of multiplication
2. Demonstrate a procedure which shows that division is the inverse operation of multiplication
3. Construct number sentences which illustrate inverse operations given either a multiplication or a division example

Translating Verbal Problems Into Number Sentences

(Choose only those problems whose solution can be determined in two steps)

The student should be able to:

1. Name and identify verbal problems whose solutions require division of fractions
2. Demonstrate a procedure for translating verbal problems into number sentences
3. Construct a number sentence from the information contained in a verbal problem
4. Describe the steps needed to translate verbal problems to number sentences using specific examples
5. Interpret the information contained in a verbal problem in order to construct a corresponding number sentence

UNIT FRACTIONAL NUMBERS GRADE(S) Eight

Addition of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Least Common Multiples			8	8		8	8			8
Addition of Fractions, Unlike Denominators	8	8	8	8			8			
Miscellaneous Problems of Adding Fraction Expressions			8	8			8			
Closure Property						8				
Commutative Property			8			8	8			8
Associative Property	8	8	8	8		8	8			8
Estimation				8			8			
Translating Verbal Problems into Number Sentences	8	8	8	8	8			8		

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Addition of Fractions

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Least Common Multiple

(Listed below are some representative examples. Note that these are pairs and triplets of numbers with no number greater than 150.)

(4, 10) (65, 150) (140, 150) (4, 10, 12)

(27, 36, 54) (75, 25, 15) (100, 125, 150)

(26, 49, 39)

The student should be able to:

1. Demonstrate how to find the least common multiple of a pair or a triplet of numbers
2. Construct the least common multiple for a pair or a triplet of numbers
3. State the principle that the least common multiple is the smallest number divisible by the given number
4. Apply the principle for finding the least common multiple to solve related problems
5. Distinguish between the least common multiple for a pair of triplet of numbers and another multiple for the same numbers

Addition of Fractions with Unlike Denominations

(Listed below are some representative examples. Note the denominators are 2, 3, 4, 5, 6, 8, 10 and 16. Answers should be in simplified form.)

$$\begin{array}{r} \frac{3}{5} \\ + \frac{1}{8} \\ \hline \end{array} \qquad \begin{array}{r} \frac{1}{8} \\ \frac{3}{4} \\ + \frac{5}{16} \\ \hline \end{array}$$

$$\frac{7}{10} + \frac{1}{4} + \frac{1}{6}$$

$$\frac{19}{16} + \frac{3}{5} + \frac{1}{4}$$

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The student should be able to:

1. Name and identify addition problems when the addends are fractions with unlike denominators
2. Demonstrate a procedure for adding fractions with unlike denominators
3. Construct sums in simplest terms of fractions with unlike denominators
4. Describe how to add fractions with unlike denominators by using specific examples
5. Apply the principle for adding fractions with unlike denominators to solve related problems

Addition of Whole Numbers and Fractions with Unlike Denominators

(Listed below are some representative examples. Note the denominators are 2, 3, 4, 5, 6, 8, 10, 16. The results should be simplified.)

$$8 + \frac{2}{3} + \frac{5}{6}, \quad 9 + \frac{5}{6} + 2, \quad 8\frac{1}{2} + 9\frac{2}{3} + 16\frac{5}{8}$$

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$$18\frac{2}{5} + 9\frac{1}{3} + 6, \quad 25\frac{4}{5} + 11 + \frac{11}{16}, \quad 8 + 9\frac{1}{5} + 16$$

The student should be able to:

1. Demonstrate the procedure to find the sum of expressions containing whole numbers and fractions
2. Construct sums, in simplest terms, for addition of fractions and whole numbers
3. Apply the principle for adding fractions and whole numbers by writing the fractions with equal denominators

Closure Property of Fractions under Addition

The student should be able to:

1. State the principle that the set of fractions is closed under addition

Commutative Property of Addition of Fractions

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The student should be able to:

1. Construct a number sentence showing the commutative property
2. State the commutative property that a change in the order of addends does not change the sum
3. Apply this commutative property to solve related problems
4. Distinguish between the commutative and associative properties

Associative Property of Addition of Fractions

The student should be able to:

1. Name and identify a number sentence which shows the associative property
2. Demonstrate how to use the associative property in adding fractions
3. Construct a number sentence showing the associative property of adding fractions
4. State the principle of the associative property that regrouping the addends differently does not change the answer
5. Apply associative property to solve related problems
6. Distinguish between the associative and commutative properties

Estimation

The student should be able to:

1. Construct sums of fractions by estimation
2. Apply the principle to estimate sums of fractions by rounding fractions to solve related problems

Translating Verbal Problems into Number Sentences

(Choose only those problems whose solution can be determined in one or two steps)

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The student should be able to:

1. Name and identify verbal problems involving addition of fractions
2. Demonstrate a procedure to translate verbal statements involving addition of fractions into number sentences
3. Construct a number sentence from the information contained within the verbal problem
4. Describe how the number sentence was translated from the verbal problem
5. Interpret the information such that the sequence of steps will lead to the correct conclusion

UNIT FRACTIONAL NUMBERS GRADE(S) Eight

Subtraction of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Subtraction, Unlike Denominators				8		8	8			
Fractions from Mixed Forms, Like Denom., Renaming				8		8	8			
Fractions from Mixed Forms, Unlike Denom., No Renaming						8	8			
Fractions from Mixed Forms, Unlike Denom., Renaming						8	8			8
Mixed Form from Mixed Forms, Like Denom., Renaming				8		8	8			8
Mixed Form from Mixed Forms Unlike Denom., No Renaming				8		8	8			
Mixed Form from Mixed Forms, Unlike Denom., Renaming				8		8	8			8
Closure Property				8		8				
Non-Commutativity			8							
Non-Associativity				8						
Identity Element						8				
Translating Verbal Problems into Number Sentences	8	8	8	8	8					

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Subtraction of Fractions

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Subtracting Fractions With Unlike Denominators

(Listed below are some representative examples. Note that the denominators are 2, 3, 4, 5, 6, 8, 10, and 16. The answer should be in simplified terms.)

$$\frac{5}{6} - \frac{1}{3} \qquad \frac{9}{10} - \frac{1}{2} \qquad \frac{7}{8} - \frac{3}{16} \qquad \frac{3}{16} - \frac{1}{3} \qquad \frac{5}{16} - \frac{1}{4}$$

The student should be able to:

1. Construct the difference, in simplified terms, of two fractions with unlike denominators
2. State the principle for subtracting fractions with unlike denominators by renaming the fractions with equal denominators
3. Apply this principle for subtracting fractions with unlike denominators to solve related problems

Subtracting a Fraction From a Mixed Form With Like Denominators, Renaming Required

(Listed below are some representative examples. Note that the denominators are 2, 3, 4, 6, 8, 10, and 16. The answers should be in simplified form.)

$$8\frac{1}{5} - \frac{4}{5} \qquad 16\frac{7}{16} - \frac{15}{16} \qquad 21\frac{3}{8} - \frac{5}{8}$$

The student should be able to.

1. Construct the difference, in simplified terms, of a fraction and a mixed form with like denominators, renaming required
2. State the principle for subtracting a fraction from a mixed number with like denominators, renaming required as:

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a. Rename the minuend so that the fractional part is larger than the subtrahend

b. Subtract the fraction from the mixed form

3. Apply this principle for subtracting a fraction from a mixed number with like denominators, renaming required, to solve related problems

4. Distinguish between problems which require renaming and those which do not

<u>Subtracting a Fraction From a Mixed Form With Unlike Denominators, Renaming Required</u>

(Listed below are some representative examples.
Note that the denominators are 2, 3, 4, 6, 8, 10 and 16.)

$$\begin{array}{r} 18\frac{1}{3} \\ -\ 6\frac{5}{6} \\ \hline \end{array} \qquad \begin{array}{r} 21\frac{5}{8} \\ -\ 6\frac{15}{16} \\ \hline \end{array} \qquad \begin{array}{r} 70\frac{3}{5} \\ -\ 16\frac{5}{8} \\ \hline \end{array} \qquad 91\frac{1}{6} - 19\frac{3}{8}$$

The student should be able to:

1. Construct the difference, in simplified terms, of mixed forms with unlike denominators, renaming required

2. State the following principles for subtracting mixed forms with unlike denominators, renaming required

   a. Rewrite the fractional parts of the mixed forms so that the denominators are equal

   b. Rewrite the minuend so that the fractional part is larger than the fractional part of the denominator

   c. Subtract the mixed forms

3. Apply these principles for subtracting mixed forms with unlike denominators, renaming required, to solve related problems

4. Distinguish between problems which require renaming and those which do not

Non-Closure

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The student should be able to:

1. Construct an example to show that the set of fractions is not closed under subtraction
2. State the principle of non-closure for the set of fractions under subtraction

Non-Commutative Property of Subtraction for Fractions

The student should be able to:

1. Construct a number sentence to show non-commutativity for subtracting fractions

Non-Associative Property of Subtracting for Fractions

The student should be able to:

1. Construct a number sentence to show the non-associative property for subtracting fractions

Identity Element

The student should be able to:

1. State the principle that zero is a right identity element for subtraction of fractions

Translating Verbal Problems to Number Sentences
(Choose only those problems whose solution can be determined in two steps)

The student should be able to:

1. Name and identify verbal problems which contain subtractions of fractions
2. Demonstrate a procedure for writing a number sentence from the information contained in a verbal problem
3. Construct a number sentence from the information contained in a verbal problem
4. Describe how a verbal problem is translated into a number sentence by using specific examples

UNIT DECIMAL NUMERALS GR DE(S) Eight

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Place Value	8	8				8				8
Numbers to a Billion	8	8	8						8	
Number Patterns				8		8	8			
Equivalent Powers of Ten						8				
Expanded Notation			8							
Decimal Equivalence	8	8	8	8						
Annexing Zeros	8	8	8							
Comparing Numbers			8						8	
Betweenness	8		8							
Rounded Numbers	8	8					8			
Decimal Simplifying Numerals	8	8		8						
Equivalence Decimal Charts			8	8						
Column Form Addition			8	8						
Arrange Addends in Column Form			8			8				
Estimating Sums			8	8	8					
Subtraction			8	8						
Arrange a Subtraction Problem in Vertical Form			8			8				
Estimating Differences			8	8	8					

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UNIT DECIMAL NUMERALS GRADE(S) Eight

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Multiplication by Powers of Ten			8			8	8			
Products of Decimal and Whole Numbers			8	8						
Products of Two Decimals			8	8						
Estimating Products			8	8	8					
Division of a Decimal by a Whole Number			8	8						
Division of a Whole Number by a Decimal Fraction			8	8						
Division of a Decimal by a Decimal			8	8						
Quotients Rounded Off			8	8	8					
Decimal Equivalents	8	8	8	8						

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Place Value for Numbers Between Billions and Millionths

The student should be able to:

1. Name and identify the place value and face value for any digit symbol in a given numeral
2. State the principle that in a number of two or more digits written in decimal form:
   a. The place value of a digit is ten times the place value of the digit immediately to its right
   b. The place value of a digit is one-tenth the place value of the digit immediately to its left
3. Distinguish between the place value and the face value of a digit symbol in a numeral

Numbers Between Billions and Millionths

The student should be able to:

1. Name and identify any numeral
2. Demonstrate how to write any numeral
3. Order numbers by telling if one is greater than or less than another

Number Patterns Using Thousandths

The student should be able to:

1. State the rule necessary to complete a given pattern
2. Apply the rule to construct the next three numbers in the pattern
3. Construct their own number pattern either written or orally

Equivalent Powers of Ten

The student should be able to:

1. State the principle: $1/100,000 = .00001$
   $1/1,000,000 = .000001$

Page

Expanded Notation

The student should be able to:

1. Demonstrate how to write expanded numerals for any number between billions and millionths: e.g. $4.067325 = 4 \times 1 + 0 \times .1 + 6 \times .01 + 7 \times .001 + 3 \times .0001 + 2 \times .00001 + 5 \times .000001$

Decimal Equivalence

The student should be able to:

1. Name and identify the decimal equivalent of a fraction with denominator of 1000
2. Name and identify a fractional equivalent with denominator of 1000 given a decimal to thousandths place
3. Construct the decimal equivalent of a fraction with denominator of 1000
4. Construct a fractional equivalent with denominator of 1000 given a decimal to thousandths place
5. Name and identify any fractional numeral expressed by a whole number numerator and a denominator of eight as a decimal numeral
6. Demonstrate how to use the multiplicative identity to construct decimal names for the given fractional numeral

Annexing Zeros

The student should be able to:

1. Name and identify decimals having zeros annexed which are the same as a given decimal
2. Demonstrate how to rename decimal numerals by annexing zeros

Comparing Numbers

The student should be able to:

1. Demonstrate how to determine the largest number when some numbers are expressed in fractional form with denominators of 8 or 1,000 and others in decimal form to thousandths place

Page

2. Order numbers from largest to smallest or vice versa when some of the numbers are expressed as mixed decimals and others as fractions with denominators of either 8 or 1,000

Betweenness Using Thousandths

The student should be able to:

1. Name a decimal between two given numbers

2. Name two whole numbers between which a given decimal lies

3. Demonstrate betweenness on a number line

Rounding Decimal Numbers to Nearest Hundredth

The student should be able to:

1. Name and identify a number which has been rounded to the nearest hundredths place

2. Apply the following principles to construct a number rounded to the nearest hundredths place

   a. Consider the numbers named in the hundredths place and the thousandths place. If the number named in the thousandths place is less than 0.005, then replace the digits right of the hundredths place by 0

   b. If the number named in the thousandths place is 0.005 or greater, then replace the digits at the right of the hundredths place by 0 and increase the number named in the hundredths place by 0.01

Simplifying Decimal Numerals

The student should be able to:

1. Name and identify the simplest fractional numeral for a given decimal where the fractional numeral can be named as whole number/1000

2. Construct the simplest fractional numeral for a given decimal where the fractional numeral can be named as $\frac{\text{whole number}}{1000}$

Page

Decimal Equivalence Charts

The student should be able to:

1. Construct a decimal equivalence chart using fractional numerals with denominators of 2, 4, 5, 8, 10, 100, and 1,000

2. Demonstrate how to read such a chart

Column Form Addition

The student should be able to:

1. Demonstrate how to construct sums of up to and including four four digit addends, each of which contains a thousandths digit

2. Demonstrate how to construct sums of up to and including four four digit addends, some of which contain a thousandths digit

3. Construct sums of up to and including four four digit addends using thousandths

Arrange Addends in Column Form

The student should be able to:

1. Demonstrate how to arrange a given number of addends in column form to preserve place value

2. State the principle that the decimal point must remain in a straight line to preserve place value

Estimating Sums to Nearest Whole Number

The student should be able to:

1. Demonstrate how to construct the estimation of a sum to the nearest whole number by rounding each addend to the nearest whole number and then adding

2. Construct sum to the nearest whole number

3. Describe a procedure for constructing an estimate

Subtraction, No Renaming Required

Page

The student should be able to:

1. Demonstrate how to construct the difference of two five digit numbers each of which contains a thousandths digit

2. Construct the difference of two five digit numbers each of which contains a thousandths digit

Subtraction, Renaming Required

The student should be able to:

1. Demonstrate how to construct the difference of two five digit numbers each of which contains a thousandths digit

2. Construct the difference of two five digit numbers each of which contains a thousandths digit

Subtraction, No Renaming Required, Annexing Zeros

The student should be able to:

1. Demonstrate how to construct the difference of two numbers where the subtrahend contains a thousandths digit

2. Construct the difference of two numbers where the subtrahend contains a thousandths digit

Subtraction, Renaming Required, Annexing Zeros

The student should be able to:

1. Demonstrate how to construct the difference of two numbers where the subtrahend contains a thousandths digit

2. Construct the difference of two numbers where the subtrahend contains a thousandths digit

Arrange a Subtraction Problem in Vertical Form

The student should be able to:

1. Demonstrate how to arrange a subtraction problem in vertical form where the thousandths digit is used in some numbers

2. State the principle that the decimal points are in a straight line to preserve place value

Page

Estimating Differences to Nearest Whole Number

The student should be able to:

1. Demonstrate how to construct the estimation of a difference to the nearest whole number by rounding each number to the nearest whole number
2. Construct a difference to the nearest whole number
3. Describe a procedure for constructing an estimate

Multiplication by .001

The student should be able to:

1. Demonstrate how to construct products using the factor .001
2. State the principle that when .001 is used as a factor the product is found by "moving" the decimal point three places to the left
3. Apply the principle to construct products using a factor of .001

Products of Decimals and Whole Numbers

The student should be able to:

1. Demonstrate how to construct products using a three digit decimal factor and a three digit whole number factor FO-88
2. Demonstrate how to construct products using a three digit whole number factor and a three digit decimal factor, containing a hundredths digit
3. Construct products using two three digit factors, of which the decimal factor contains a hundredths or thousandths digit, and the other factor is a whole number

Products of Two Decimals

The student should be able to:

1. Demonstrate how to construct products using two two digit decimal factors
2. Construct products using two two digit decimal factors

Estimating Products to Nearest Whole Number	**Page**

The student should be able to:

1. Demonstrate how to construct the estimate of a product to the nearest whole number by rounding each factor to the nearest whole number, and then multiplying
2. Construct a product to the nearest whole number
3. Describe a procedure for constructing an estimate
4. Construct an estimate to the nearest whole number

Division of a Decimal by a Whole Number

The student should be able to:

1. Demonstrate how to construct quotients with no remainder using a three digit divisor and a decimal dividend containing a thousandths digit — FO-88
2. Demonstrate how to construct quotients with no remainder using a three digit divisor and a mixed decimal dividend which contains a thousandths digit
3. Construct a quotient using a three digit divisor and a decimal dividend which contains a thousandths digit

Division of a Whole Number by a Decimal Fraction

The student should be able to:

1. Demonstrate how to construct quotients with no remainder using a two digit divisor containing a hundredths digit and a whole number dividend
2. Construct quotients with no remainder using a two digit divisor containing a hundredths digit and a whole number dividend

Division of a Decimal by a Decimal

The student should be able to:

1. Demonstrate how to construct quotients with no remainder using a two digit divisor containing a hundredths digit and a dividend which contains a hundredths digit

Page

2. Construct quotients with no remainder using a two digit divisor containing a hundredths digit and a dividend which contains a hundredths digit

Quotients Rounded to Nearest Tenth

The student should be able to:

1. Demonstrate how to construct a quotient to the nearest tenth by getting a hundredths place in the quotient, and then rounding to nearest tenth
2. Construct a quotient to the nearest tenth
3. Describe how to find a quotient to the nearest tenth

Decimal Equivalents

The student should be able to:

1. Name and identify any fraction with a whole number as the numerator and a denominator of eight
2. Demonstrate how to use division to construct decimal numerals for fractions with a denominator of eight
3. Construct a decimal numeral when given a fraction with denominator of eight

UNIT PERCENT BY RATIO AND PROPORTION GRADE(S) Eight

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning of Ratio	8	8	8	8	8					
Meaning of Rate Pair	8	8	8	8	8					
Simplifying a Rate Pair	8	8	8			8	8			
Translating Verbal Problems into Rate Pairs		8	8	8	8					
Meaning of Percent by Rate Pairs		8	8	8	8					
Changing Percents in Verbal Problems to Rate Pairs		8		8						

FO-70

Page

Meaning of Ratio

The student should be able to:

1. Name and identify a ratio written in fractional form
2. Demonstrate how to write a ratio in fractional form
3. Construct a fraction given a ratio
4. Describe a ratio as meaning a comparison of two numbers by division

Meaning of a Rate Pair

The student should be able to:

1. Name and identify a pair of numbers written in the rate pair form
2. Demonstrate how to write a ratio as a rate pair
3. Construct a rate pair when given a ratio or fraction
4. Describe a rate pair as a pair of ordered numbers

Simplifying a Rate Pair

(Listed below are some representative examples. Note that the second numbers (or the denominator) of each pair are 2, 3, 4, 6, 8, 10, or 16.)

(6, 8)   (2, 4)   (8, 10)   (10, 16)

The student should be able to:

1. Name and identify a rate pair in simplified form
2. Demonstrate a procedure for simplifying rate pairs
3. State the principle that a rate pair is in simplest terms when the pair of numbers have no common factor other than one
4. Apply this principle for constructing rate pairs in simplified terms to solve related problems

Translating Verbal Problems Into Rate Pairs

The student should be able to:

1. Identify verbal problems which contain rate pairs

Page

2. Demonstrate how to write rate pairs contained in verbal problems

3. Construct a rate pair from the information contained in a verbal problem

4. Describe how to translate verbal problems into rate pairs

Meaning of Percent by Rate Pairs

The student should be able to:

1. Identify a rate pair which represents a percent

2. Demonstrate how to write a percent as a rate pair

3. Construct a rate pair given a percent

4. Describe a percent as a rate pair with a second number of 100

Translating Percents in Verbal Problems to Rate Pairs

(Use only those percents which in fractional form have a whole number in the numerator and 100 in the denominator)

The student should be able to:

1. Identify a verbal problem which contains percents

2. Construct a rate pair from the information contained in a verbal problem involving percent

UNIT SQUARE ROOT GRADE(S) Eight

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Vocabulary	8	8								8
Symbol	8	8		8						8
Expansion of Powers	8	8	8	8	8					8

FO-73

NUMBERS, OPERATIONS AND ALGORITHMS
SQUARE ROOT - Grade 8

Page

Vocabulary

The student should be able to:

1. Name and identify the terms square root, power and exponent
2. Distinguish between square and square root

Symbol

The student should be able to:

1. Name, identify and draw the symbol $\sqrt{}$
2. Distinguish $\sqrt{}$ from $\overline{)}$

Expansion of Powers

The student should be able to:

1. Name and identify powers FO-90
2. Demonstrate how to find the power of a number
3. Construct powers of numbers such that the power does not exceed 256
4. Describe a power as another name for a number which is a product of equal factors or a base with its exponent
5. Distinguish between powers and exponents

LATTICE METHOD OF MULTIPLICATION

Teacher Commentary

I. Unit: Fundamental Operation

II. Objectives: The student should be able to:

A. Construct the product of two whole numbers.

B. Demonstrate a method of finding products by the use of lattices

III. Materials: Work sheet entitled, "Lattice Multiplication"

IV. Procedure:

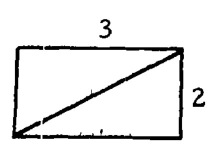
A. Introduction

Lattices were used many, many years ago before books were printed. A lattice is a framework of lines used to keep our answers in their proper places. Because of the way each cell is divided in two by a diagonal, no carrying is required in the multiplication. The teacher should explain this method of multiplication to the entire class by using the following 4 examples.

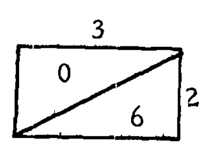
B. <u>Example 1</u> $\boxed{2 \times 3 = 6}$

1. Draw the lattice. The number of boxes in the lattice will increase as the numbers to be multiplied become larger.

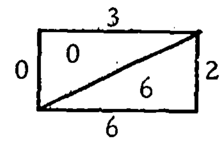
2. Place one of the numbers to be multiplied across the top and the other on the right side.



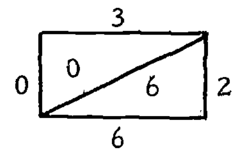
3. Multiply the numbers and place the ones digit in the lower triangle and the tens digit in the other.



4. Add the numbers in the diagonals and record them at the bottom and left.



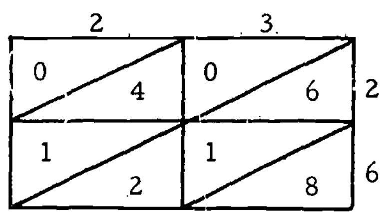
5. Read the answer going down the left side and across the bottom.



6. Thus, 2 x 3 = 6

C. <u>Example 2</u>

$\boxed{23 \times 26 = 598}$



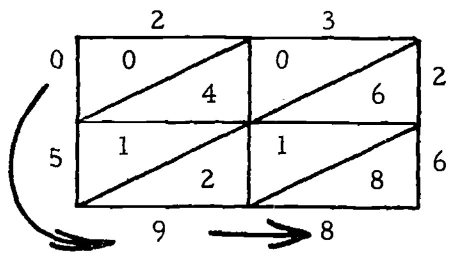
The eight digits within the lattice are arrived at as follows:

2 x 3 = 6 ones and no ten = 06

2 x 2 = 4 ones and no ten = 04

3 x 6 = 8 ones and 1 ten = 18

2 x 6 = 2 ones and 1 ten = 12

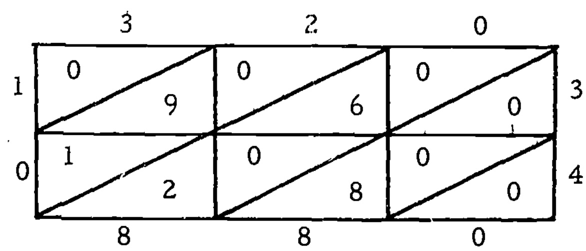


The answer is found by adding the numbers along the diagonal.

Answer: 0598 means there are no thousands, 5 hundreds, 9 tens, 8 ones since each diagonal is a place holder. Thus, the answer is 598.

D. Example 3

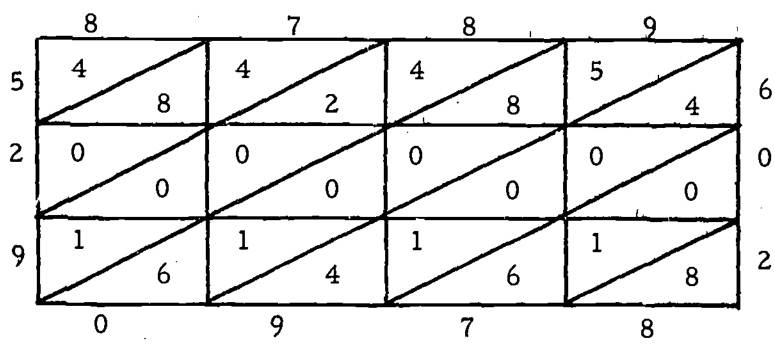
320 x 34 = 10, 880



In adding diagonally to obtain the answer, 1 + 9 + 0 of course equals 10. The zero is recorded and the 1 is added to the next diagonal. The answer is 10, 880.

E. Example 4

8, 789 x 602 = 5, 290, 978



F. Assessment - Distribute work sheet entitled, "Lattice Multiplication."

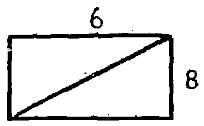
Solutions to work sheet:

1. 48	4. 3,978
2. 126	5. 139,104
3. 2,108	6. 7,318,192
	7. 74,160

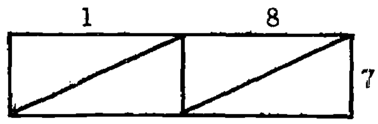
LATTICE MULTIPLICATION

See if you can find these answers by the lattice method. Place your answers in the box provided.

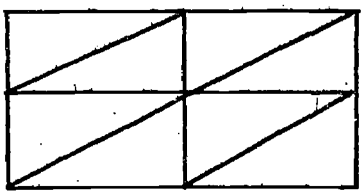
1. 6 x 8 =



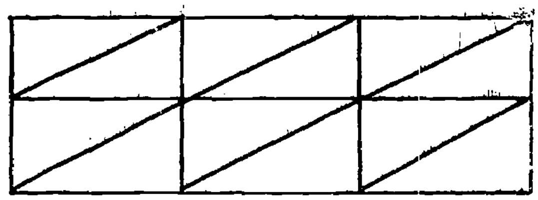
2. 18 x 7 =



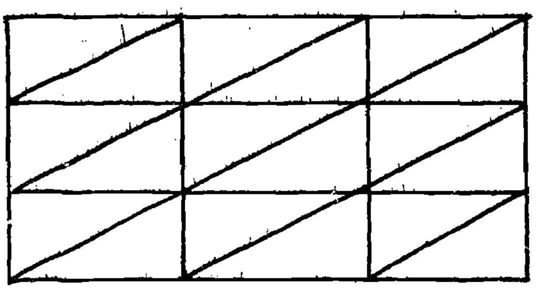
3. 34 x 62 =



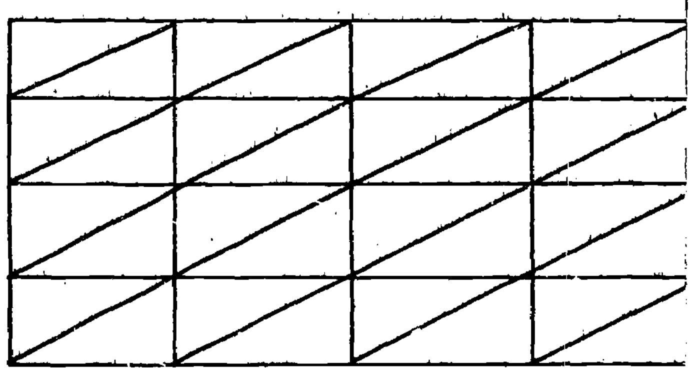
4. 221 x 18 =



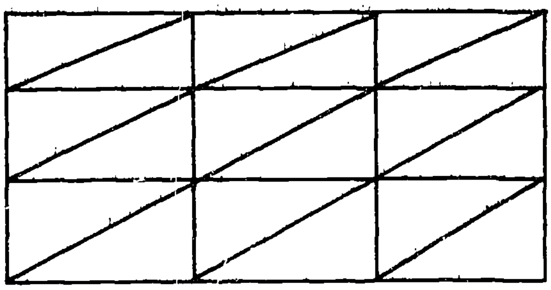
5. 336 x 414 =



6. 2164 x 3428 =



7. $\boxed{206 \times 360 =}$



NOMOGRAPH
MULTIPLICATION MIXED FORMS
Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

Demonstrate how to construct the product of two mixed forms using the nomograph

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribute the materials to each student.

B. Discuss the three scales A, B and C.

1. Scales A and B begin with one and end with ten. Each unit is divided into fourths.

2. Scale C begins with one and ends with one hundred. The first two units are divided into sixteenths. Units three thru six are divided into eighths. Units six thru ten are divided into fourths. Units ten thru twenty are divided in half.

3. Locate some points on the scales and have the students identify them. Students will have to name points which lie between the incurement marks.

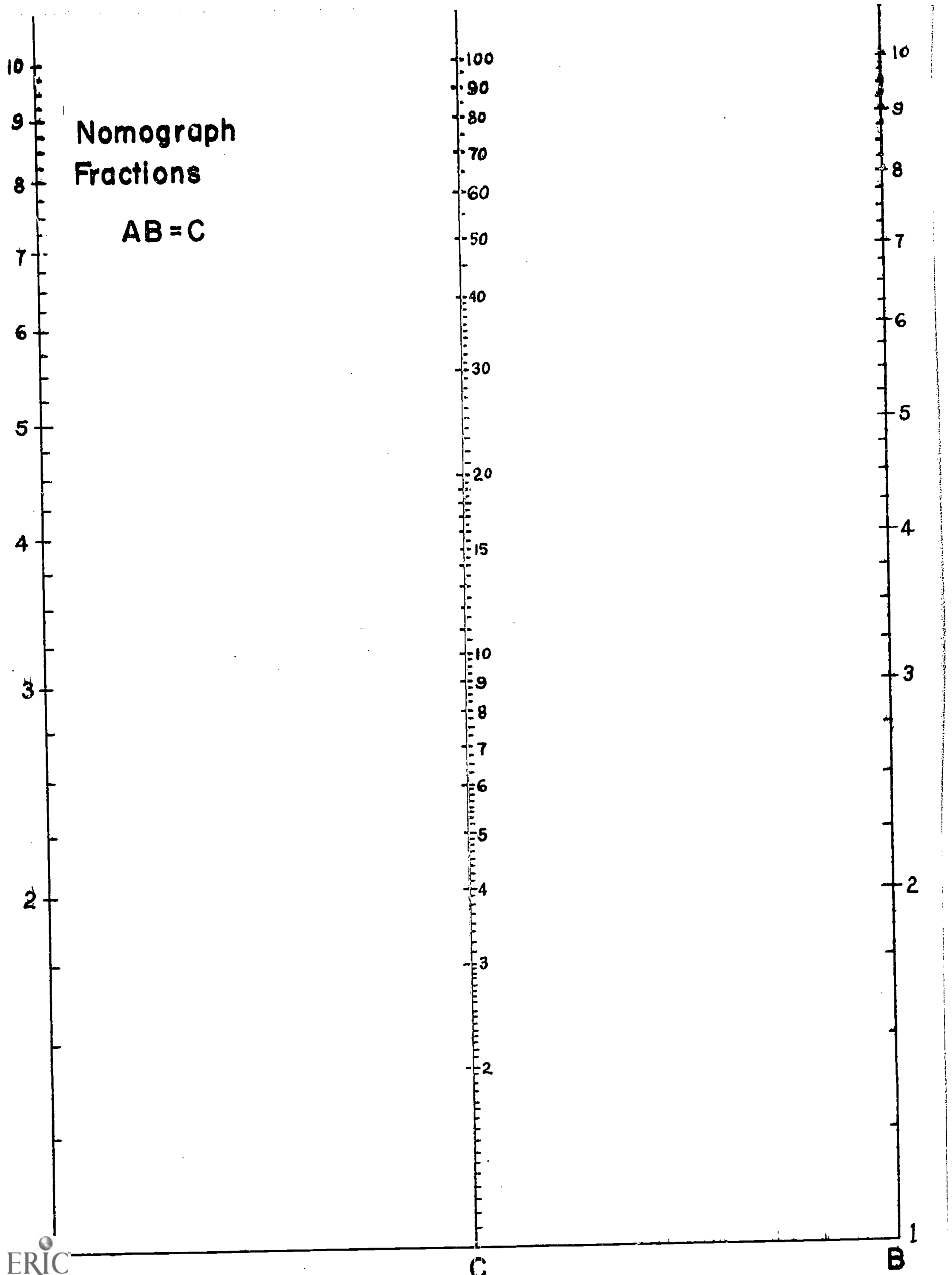
4. Have the students locate points on the scales. On scale C, make sure that the students can locate numbers like $20\frac{1}{2}$, $9\frac{1}{8}$, and $4\frac{1}{16}$.

C. In order to multiply any two mixed forms ($2\frac{1}{4} \times 3\frac{1}{2}$), locate $2\frac{1}{4}$ on scale A and $3\frac{1}{2}$ on scale B. The line joining these two points will cross scale C at a point that represents the product ($7\frac{7}{8}$). The students will have to estimate the answer since the line will be between $7\frac{3}{4}$ and 8.

D. This nomograph may also be used to find the area of a square, rectangle and parallelogram when the dimensions are given as mixed forms and/or whole numbers greater than or equal to one.

E. The nomograph may be used to illustrate the commutative property of multiplication of fractions.

F. After the students have completed some written exercises, they could use the nomograph to check their results.


Nomograph
Fractions
AB = C
10 9 8 7 6 5 4 3 2
100 90 80 70 60 50 40 30 20 15 10 9 8 7 6 5 4 3 2
C
10 9 8 7 6 5 4 3 2 1
B

DIVISION BY FLOW CHART

Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

A. Demonstrate how to find quotients with remainders using two digit divisors

B. Construct quotients with remainders using two digit divisors

III. Materials: Student work sheet entitled, "Division by Flow Chart"

IV. Procedure:

A. This is an activity to teach long division by a computer flow chart approach.

B. Distribute work sheet entitled, "Division by Flow Chart." Explain to students that this chart simulates a computer's method of problem solving following sequential directions. Tell students they are to be human computers.

C. To use the attached work sheet, propose a division problem such as 256 ÷ 7. Have students list 7 in the Division space and 256 under Dividend, next to the Start sign.

1. Students must then take a guess at what they think the quotient might be. This should be listed under Guess. Have the entire class use the same guess such as 20.

2. Have students find the product of their last guess with the Divisor. This should be listed in the Product chart. For the example given, this product would be 140.

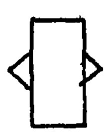
3. Have students compare their last product with their last dividend. As the arrows indicate, they proceed to one instruction if the product is greater than the dividend and to a different instruction if the product is less than or equal to the dividend.

If the product in step 2 above is greater than the dividend, students must return to the Dividend- Guess chart and take a smaller guess. This guess should be listed as the next guess in this chart, and the last dividend should be repeated. They then repeat steps 2 and 3 above.

5. When the product found in step 2 above is less than or equal to the dividend, have students follows the arrow to the chart instructing them to "List Last Guess Here."

6. From step 5 above, have students subtract the last product in the Product chart from the last dividend. This Difference should be listed in the space provided. For the example given this difference is 116.

7. Students next compare the difference found in step 6 with the Divisor. Have them follow the arrow to the appropriate next instruction.

8. If the difference found in step 6 is less than the Divisor have students add the guesses listed in the chart shaped  . This sum is their quotient, and the last difference in the Difference chart is their remainder.

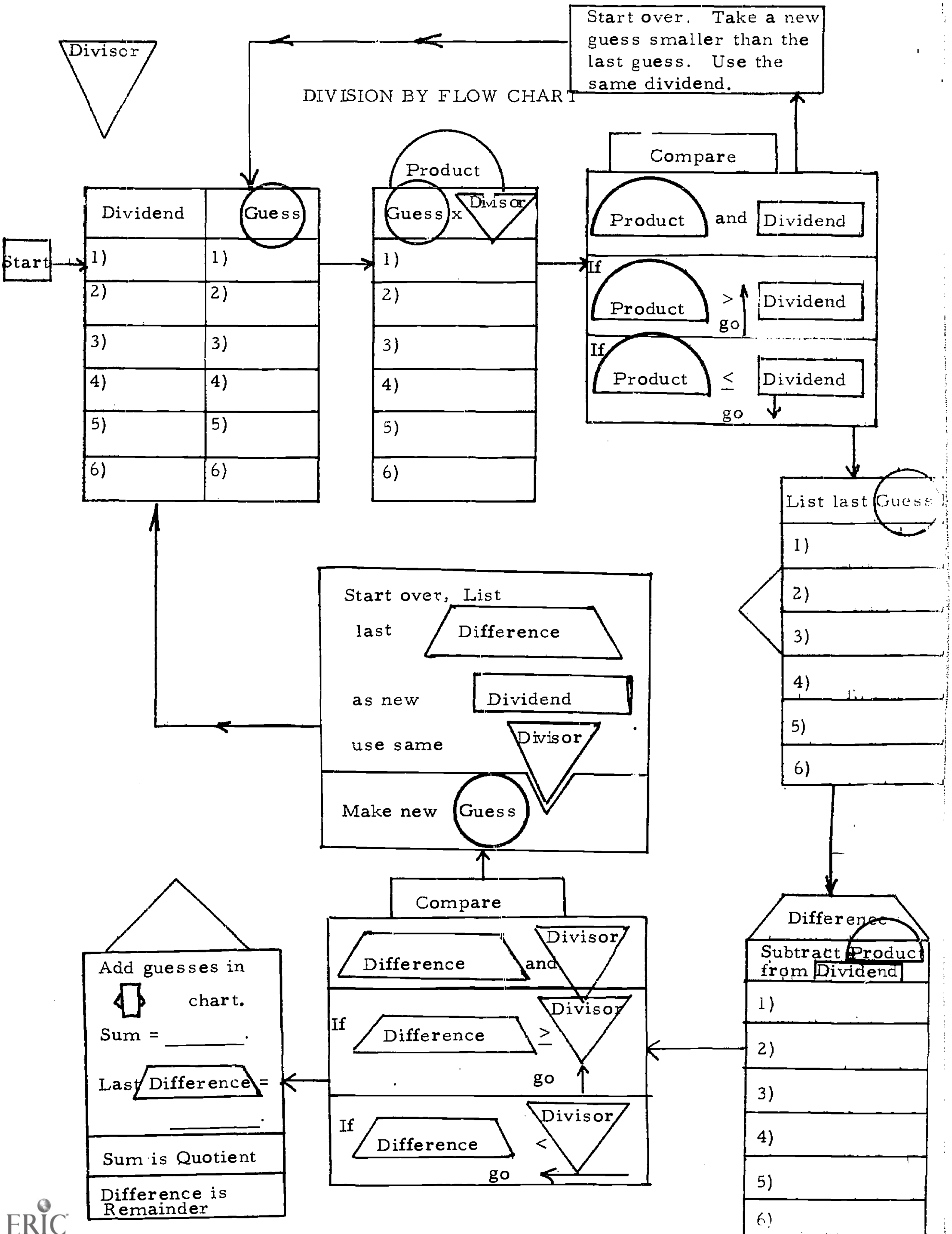
9. If the difference found in step 6 is greater than or equal to the Divisor have students list their last Difference as their next Dividend. They then take a new Guess, use the same Divisor and start over. The process ends when step 8 can be completed.

D. Have students follow several such divisions through this flow chart, some with remainders and some without.

E. When sufficient competence has been gained in using this flow chart, have students perform several divisions on their own using their own guesses.

F. To assess this activity, prepare enough slips of paper for each student in the class. On this paper, list a division problem with a two digit divisor. Have them solve their problem using the flow chart.

DIVISION BY FLOW CHART



NOMOGRAPH
ADD-SUBTRACT FRACTIONS
Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

A. Demonstrate how to construct the sum and difference of two whole numbers using the nomograph

B. Demonstrate how to construct the sum and difference of two fractions and/or mixed forms using the nomograph

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribute the materials to each student.

B. Discuss the three scales A, B and C.

1. Scales A and B begin with zero and end with ten. Each unit is divided into eighths.

2. Scale C begins with zero and ends with twenty. Each unit is divided into fourths.

3. Locate some points on the scales and have the students identify them. On scale C, locate points which are between the one-fourth increment marks.

4. Have the student locate points on the scales. On scale C, make sure that students are able to locate numbers like $\frac{7}{8}$ and $7\frac{3}{8}$.

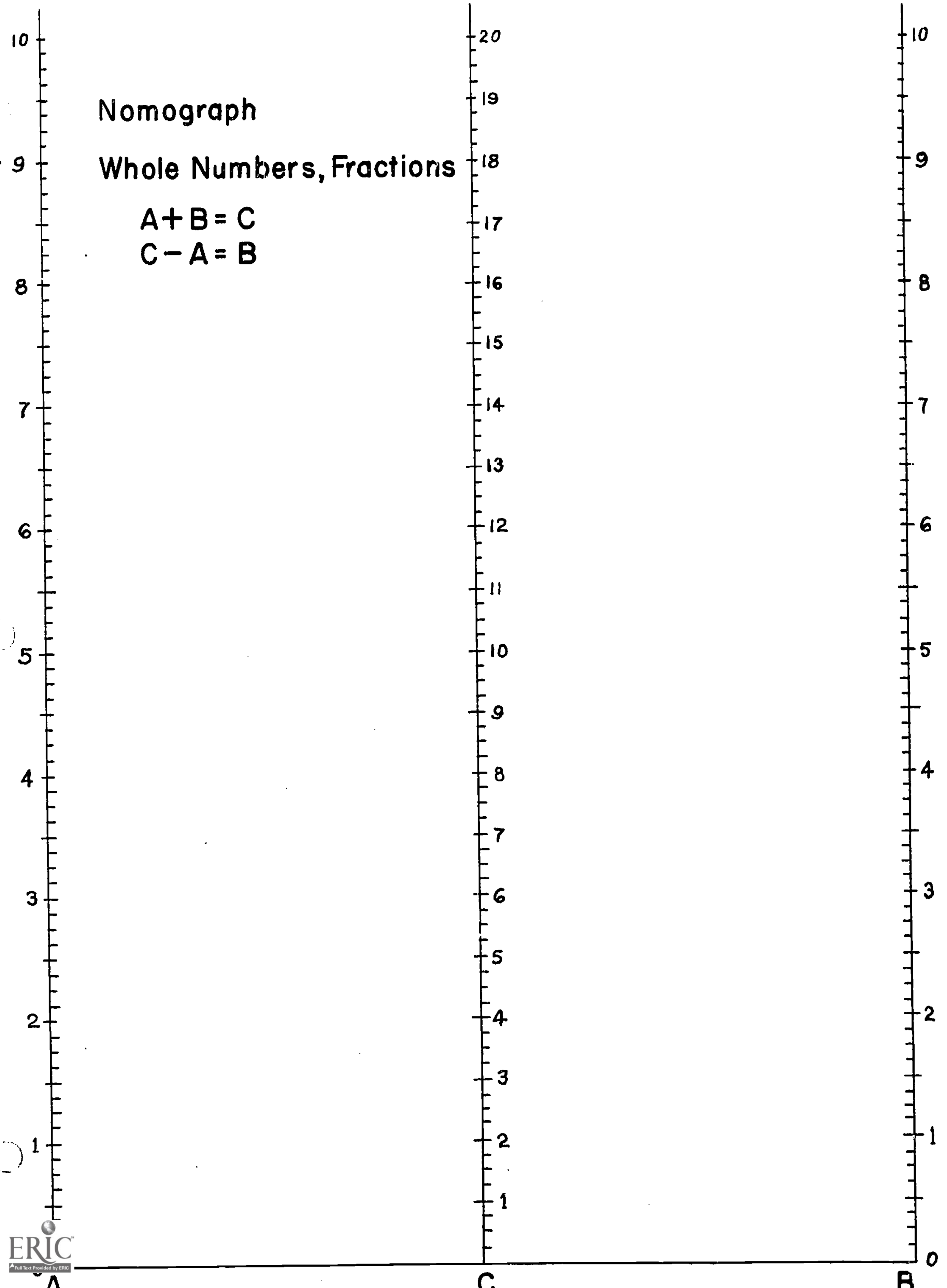
C. In order to add any two numbers $(2\frac{1}{8} + 6\frac{3}{4})$, locate $2\frac{1}{8}$ on scale A and $6\frac{3}{4}$ on scale B. The line joining these two points will cross scale C at a point that represents the sum $(8\frac{7}{8})$. The students will have to estimate their answer since the line will be between $8\frac{3}{4}$ and 9.

D. In order to subtract any two numbers $(3\frac{1}{2} - \frac{3}{4})$, locate the minuend $(3\frac{1}{2})$ on scale C and the subtrahend $(\frac{3}{4})$ on scale A. The line joining these two points will cross scale B at a point the represents the difference $(2\frac{3}{4})$. Be sure to choose only those problems whose answer may be found on the nomograph.

E. This nomograph may be used in the same way to find the sum and difference of any two whole numbers. Only halves, quarters, and eighths should be considered when adding or subtracting fractions and/or mixed forms.

F. The nomograph may also be used to illustrate the commutative property of addition of fractions and whole numbers.

G. After the students have completed some written exercises, they could use the nomograph to check their results.


Nomograph
Whole Numbers, Fractions
A + B = C
C − A = B
10
9
8
7
6
5
4
3
2
1
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
10
9
8
7
6
5
4
3
2
1
0
A
C
B

DUONOMOGRAPH
ASSOCIATIVE PROPERTY
Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

Demonstrate how to construct the sum of three fractions

III. Materials:

A. Student work sheet "Duonomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribute the materials to each student.

B. Discuss the five scales A, B, C, D, and E.

1. Scale A begins with zero and ends with ten. Each unit is divided in half.

2. Scales C and E begin with zero and end with twenty. Each unit is divided in half.

3. Scale B begins with zero and ends with ten. Each unit is divided in half.

4. Scale D begins with zero and ends with forty. Each unit is divided in half.

5. Locate some points on the scales and have the students identify them.

6. Have students locate points on the scales.

C. In order to demonstrate how to use the associative property, a problem will have to be done in two steps. Look at the following example: $(2\frac{1}{2} + 6) + 10\frac{1}{2}$.

1. In order to solve this problem add $2\frac{1}{2}$ and 6. To this sum add $10\frac{1}{2}$.

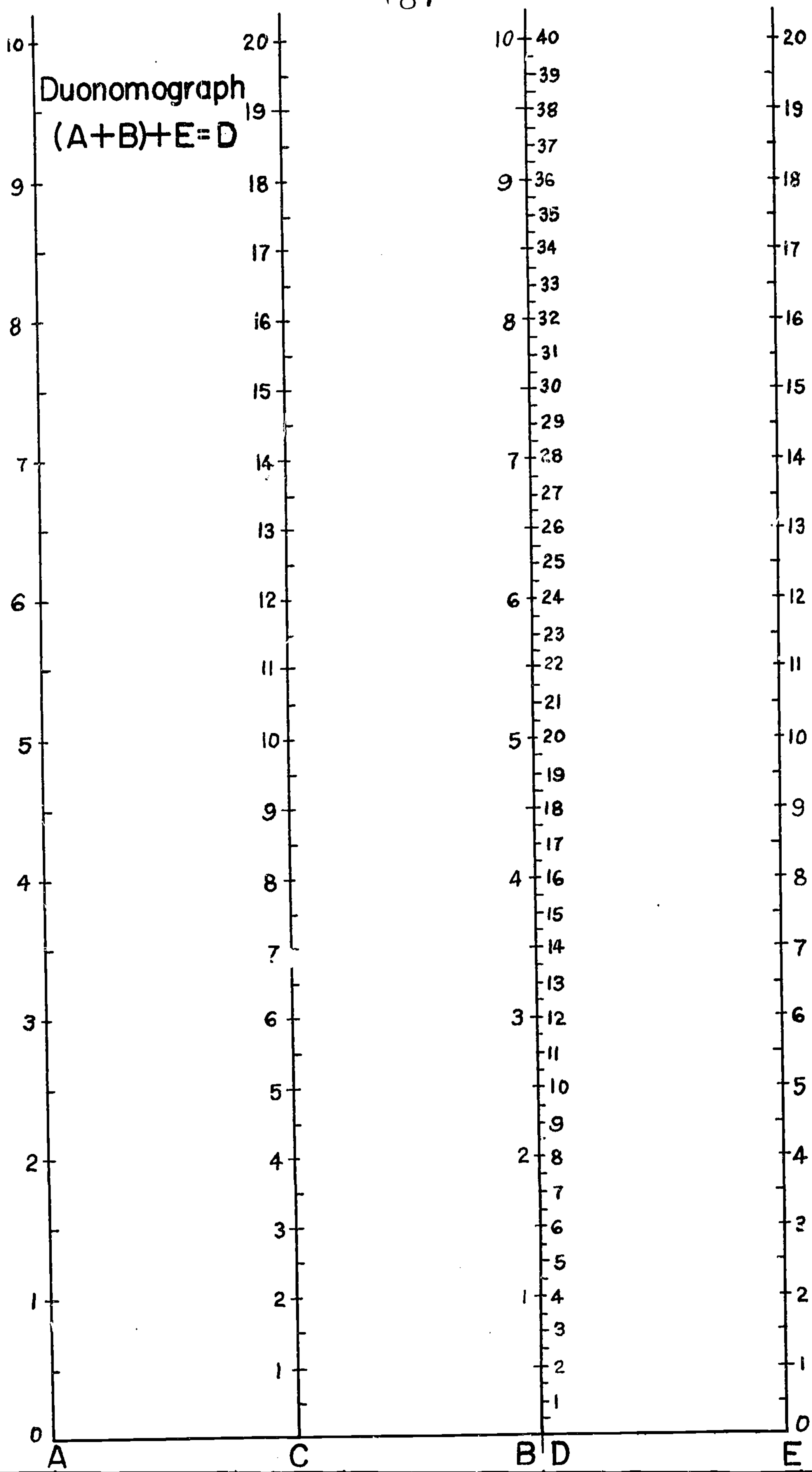
2. On the duonomograph place your ruler on $2\frac{1}{2}$ on scale A and 6 on scale B. The line joining these two points will cross scale C at a point that represents the sum $(8\frac{1}{2})$. Using $8\frac{1}{2}$ on scale C place the ruler on $10\frac{1}{2}$ on scale E. The line joining these two points will cross scale D at a point that represents the sum (19).

3. In working this problem involving the associative law, students would follow the same procedure; namely, add $10\frac{1}{2}$ to the sum of $2\frac{1}{2}$ and 6.

D. The duonomograph may be used to demonstrate the associative property of addition of whole numbers.

E. The duonomograph may be used to find the perimeter of triangles.

F. After the students have completed some written exercises, they could use the duonomograph to check their results.


Duonomograph
(A+B)+E=D
A
C
B D
E

NOMOGRAPH
MULTIPLICATION-DIVISION DECIMALS

Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

Construct the product and quotient of two decimal fractions using the nomograph

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribute the materials to each student.

B. Discuss the three scales A, B and C.

1. Scales A and B begin with one and end with ten. Each unit is divided into tenths.

2. Scale C begins with one and ends with one hundred. The first two units are divided into tenths. Units three to twenty are divided in half.

3. Locate some points on the scales and have the students identify them.

4. Have the students locate points on the scales.

5. Students have to be able to locate and identify given points before learning how to compute.

C. In order to multiply 3.6 and 4, place the ruler on the nomograph that passes through point 3.6 on scale A and point 4 on scale B. The ruler will pace through the product on scale C (14.4).

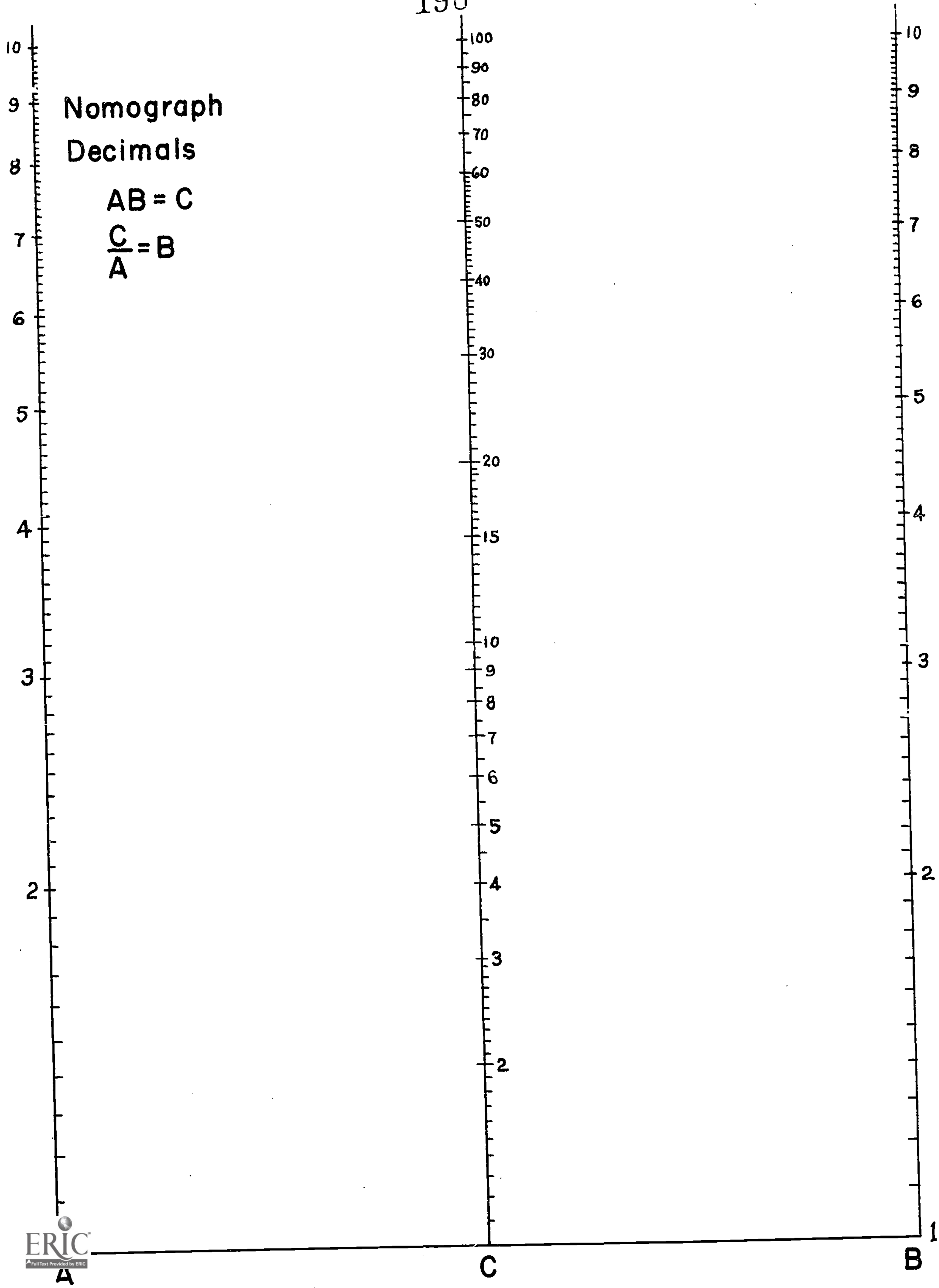
D. In order to divide 4.6 by 2, place the ruler on the nomograph that passes through point 4.6 on scale C (dividend) and point 2 on scale A (divisor). The ruler will pass through the quotient on scale C (2.3). Be sure to select problems whose answers may be found on the nomograph. Another nomograph is provided for division. No restrictions have to be placed on its use.

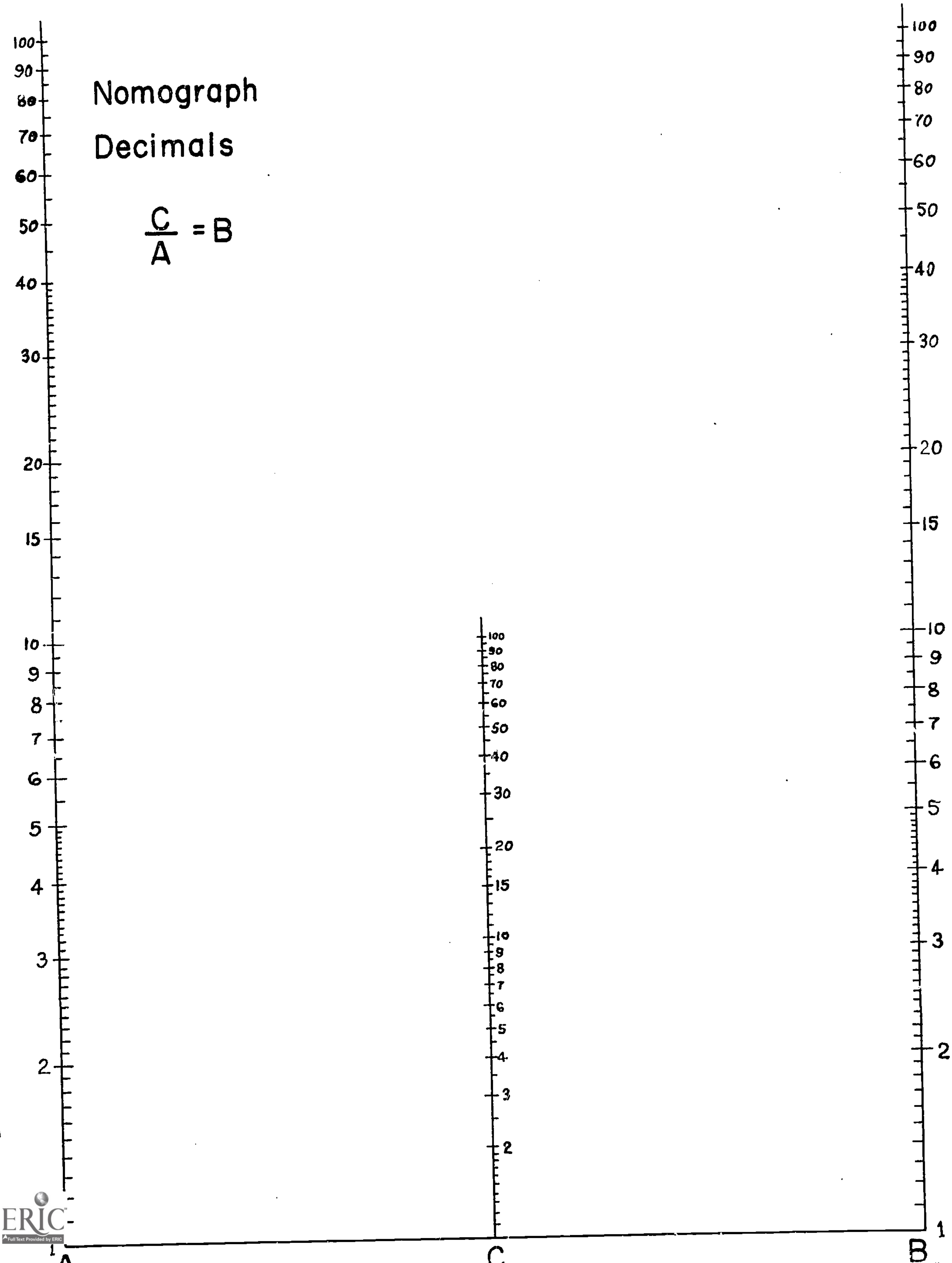
E. In multiplying or dividing, all problems will have to be greater than one. Some instruction in decimal placement will be necessary when multiplying or dividing two mixed decimals.

F. This nomograph may also be used to find the area of a square, where the side is located on scales A and C, a parallelogram and a rectangle.

G. The nomograph may also be used to illustrate the commutative property of multiplication of decimal fractions.

H. After the students have completed some written exercises, they could use the nomograph to check their results.


Nomograph
Decimals
AB = C
C/A = B
A
C
B


Nomograph
Decimals
$\frac{C}{A} = B$
A
C
B
100
90
80
70
60
50
40
30
20
15
10
9
8
7
6
5
4
3
2
1

NOMOGRAPH
POWERS AND SQUARE ROOT
Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

Demonstrate how to construct the power of a number by using the nomograph

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribute the materials to each student.

B. Discuss the three scales A, B and C.

1. Scale A begins with one and ends with ten. Each unit is divided into tenths.

2. Scale B, which is the diagonal of the square, is divided into units from one to six.

3. Scale C begins with one at the top of the page and ends with one hundred at the bottom.

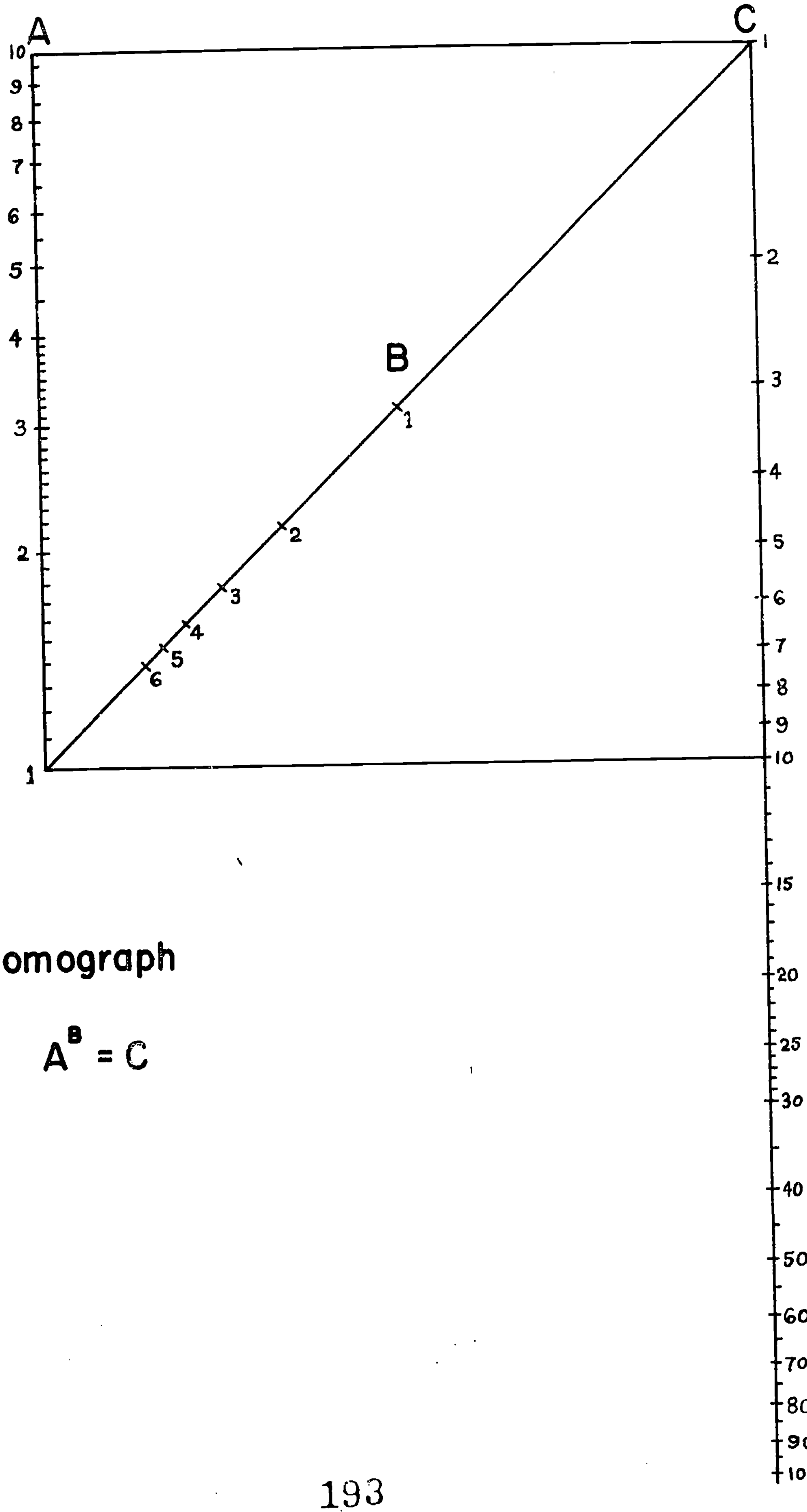
4. Locate points on the scales and have students identify them.

5. Have students locate points on the scales.

C. In order to find 2^3, locate the base 2 on scale A and the exponent 3 on scale B. The line joining these two points will cross scale C at a point that represents the power (8).

D. This nomograph can be used to find the area of a square, where the length of the side is found on scale A, 2 is located on scale B and the area will be found on scale C.

E. After the students have completed some written exercises, they could use the nomograph to check their results.



Nomograph

$A^B = C$

VERBAL PROBLEMS - 2

Teacher Commentary

A Tape Recording of Verbal Problems for Use in Grade 8

I. Materials:

A. Tape recorder

B. Eight-station listening post (optional)

C. Tape entitled, "Verbal Problems - 2"

D. Paper

E. Pencil and eraser

II. Procedure:

A. Preview the tape to determine if it will be used with a small group of students, or if it will be used as a class activity.

B. The tape has 10 problems and is 11 minutes long.

C. Following the presentation of each problem, the answer is given after a suitable interval.

D. Two additional tapes of similar design are also available - Part 1 presented in grade 7 and Part 3 presented in grade 9.

GEOMETRY

GEOMETRY

I. Master Chart - Grades Six through Eleven

II. Grade Eight Chart

III. Behavioral Objectives

IV. Activities

UNIT GEOMETRY GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Point	6	6		6						
Line	6	6		6	6					
Plane	6	6		7	6					
Closed Path	6	6		6	6					6
Segment	6,7	6,7	7	6,7	6					6
Congruent Segments	9	9	9		9					
Ray	6,7	6,7		6	6					6
Angles	6,7	6,7	7	6	6					
Vertex	7	7			7					
Right Angles	6	6	9	6	6					9
Acute Angles	9	9		9	9					9
Obtuse Angles	9	9		9	9					9
Straight Angles	9	9		9	9					9
Vertical Angles	9	9		9	9	9	9			
Supplementary Angles	9	9		9	9					9
Complementary Angles	9	9		9	9					9
Congruent Angles	9	9	9	9	9					
Triangles	6,7	6,7	10	6	6					

UNIT GEOMETRY GRADE(S) Seven through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Equilateral Triangle	8	8	8	8	8					8
Isosceles Triangle	8	8	8	8	8					8
Scalene Triangle	8	8	8	8	8					8
Right Triangle	9	9	9	9	9					9
Acute Triangle	9	9		9	9					9
Obtuse Triangle	9	9		9	9					9
Perpendicular Lines	9	9	9	9	9					
Parallel Lines	7	7	9	7	7					7
Transversal	10	10		10	10					
Corresponding Angles	10	10		10		10	10, 11			
Midpoint	7	7	7		7					
Partitioning a Segment			11							
Quadrilaterals	7	7		7	7					
Trapezoid	7	7		7	7					7
Parallelogram	7	7	10	7	7	10	10, 11			7
Rectangles	7	7	10	7	7	10	10, 11			7
Square	7	7	10	7	7	10	10, 11			7
Rhombus	7	7	10	7	7	10	10, 11			7
Polygon	8	8		8	8					

UNIT GEOMETRY GRADE(S) Eight through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Pentagon	8	8	9	8	8				8	8
Hexagon	8	8	8	8	8				8	8
Octagon	8	8	8, 9	8	8				8	8
Congruent Triangles	10	10			10	10	10			
Similar Triangles	10	10			10	10	10			
Corresponding Sides of Similar Triangles						10	10, 11	10, 11		
Circle	6	6	7	6	6					
Radius	6	6	7	6	6	7				
Diameter	6	6	7	6	6	7				
Chord	7	7	7	7	7					
Tangent	8	8	10	8	8	10				8
Secant	8	8		8	8					8
Central Angle	10	10	10	10	10					10
Inscribed Angle	10	10	10	10	10	10				10
Ellipse	10	10	10	10	10	10				
Angle Bisector	9	9	9	9	9	9				
Sum of Interior Angles of Triangles						9	9			
45°			9	9						
60°			9	9						

UNIT GEOMETRY GRADE(S) Seven through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
30°			9	9						
Median Triangle	7	7	7	7	7	7				
Altitude of Triangle	9	9	9	9	9					
Cube	7	7		7	7					7
Rectangular Solid	7	7		7	7					
Pyramid	8	8		8	8					
Cone	8	8		8	8					
Cylinder	8	8		8	8					
Sphere	8	8		8	8					8
Line of Symmetry	8	8		8	8					
Sum of Interior Angles of Quadrilaterals						9	9			
Sin	11	11				11	11	11		11
Cos	11	11				11	11	11		11
Tan	11	11				11	11	11		11
Trig Tables	11	11		11				11		
Other Polyhedrons	11	11		9, 10						
Pythagorean Theorem						10	10			
Region	7	7		7	7					
Sum of Interior Angles of a Polygon						8	8			

UNIT GEOMETRY GRADE(S) Eight

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Equilateral Triangle	8	8	8	8	8					8
Isosceles Triangle	8	8	8	8	8					8
Scalene Triangle	8	8	8	8	8					8
Polygon	8	8		8	8					
Pentagon	8	8		8	8				8	8
Hexagon	8	8	8	8	8				8	8
Octagon	8	8	8	8	8				8	8
Tangent	8	8		8	8					8
Secant	8	8		8	8					8
Pyramid	8	8		8	8					
Cone	8	8		8	8					
Cylinder	8	8		8	8					
Sphere	8	8		8	8					8
Symmetry	8	8		8	8					

GEOMETRY - Grade 8

Equilateral Triangle, Isosceles Triangle, Scalene Triangle

Page

GE-9
GE-12
GE-14

The student should be able to:

1. Name and identify the three types of triangles
2. Construct a drawing of these triangles using straightedge or freehand sketch
3. Construct a model of these triangles with available materials
4. Demonstrate the construction of triangles by a straightedge and a compass
5. Describe the three types of triangles in terms of their properties
6. Distinguish among the three types of triangles

Polygon

The student should be able to:

1. Name and identify polygons
2. Construct drawings of polygons using straightedge or freehand sketch
3. Describe polygons as closed figures made up of line segments

Pentagons, Hexagon, Octagon

The student should be able to:

1. Name and identify — GE-14
2. Construct drawing of these figures using straightedge or freehand sketch
3. Describe by number of sides
4. Demonstrate the construction of a hexagon by compass and straightedge
5. Demonstrate the construction of a octagon by compass and straightedge
6. Order the figures according to number of sides
7. Distinguish among polygons according to number of sides

Page

Tangent, Secant

The student should be able to:

1. Name and identify a tangent and a secant
2. Construct a drawing of a tangent or secant to a circle using straightedge or freehand sketch
3. Describe a tangent or secant by definition
4. Distinguish between a tangent and a secant

Pyramid, Cone, Cylinder

The student should be able to:

1. Name and identify a pyramid, cone, and cylinder
2. Construct a drawing of these figures using a straightedge or freehand sketch GE-16
3. Construct a model of these figures using available materials
4. Describe these figures in terms of his surroundings

Sphere

The student should be able to:

1. Name and identify a sphere
2. Construct a drawing of a sphere using compass or freehand sketch
3. Describe a sphere in terms of his surroundings
4. Distinguish between spheres and circles

Symmetry

The student should be able to:

1. Name and identify figures which are symmetrical
2. Construct a drawing of a line of symmetry given a symmetrical figure
3. Describe a line of symmetry in terms of his surroundings

THREE TYPES OF TRIANGLES

Teacher Commentary

I. Unit: Geometry

II. Objectives: The student should be able to:

A. Name and identify the three types of triangles

B. Distinguish among the three types of triangles

III. Materials:

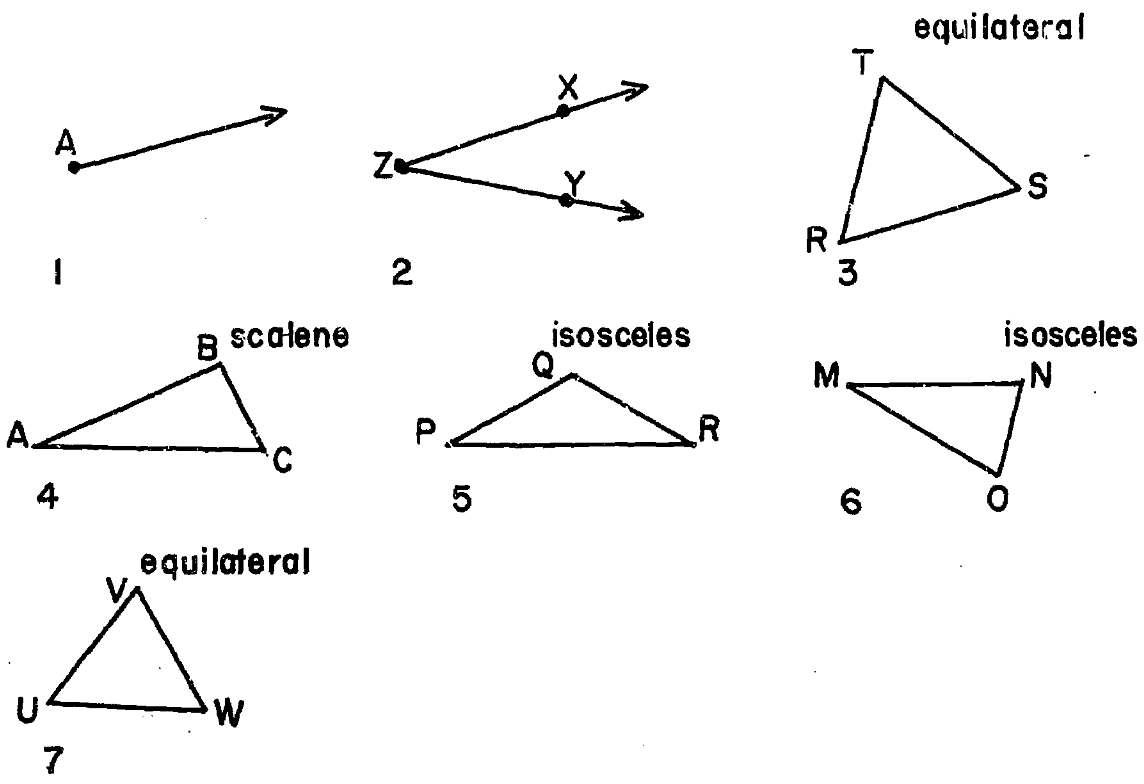
A. Board diagrams

B. Student work sheets, "Three Types of Triangles," and "CATegorize the Triangles."

C. A ruler for each student

IV. Procedure:

A. Have the diagrams below on the board to review the terms: ray; vertex; vertices; angle; line segment; triangle. The triangles should be of the types specified with easily measured dimensions.



B. Discuss these questions with the students:

1. What does figure 1 represent? What is a ray?

2. Figure 2 shows 2 rays with a common end point. What do we call figure 2?

3. How is an angle formed? By 2 rays with a common end point.

4. This angle can be named in two ways. Write $\angle XZY$ on the board. What is another way to name the angle?

5. In both cases the point labeled Z is in the middle. Why? Point Z is the vertex, or the common end point of the rays. When we name an angle the vertex is always the middle letter.

6. How many sides do you see in figure 3? How many angles? What is the figure called?

7. Do any of the line segments in the triangle have common end points? Have a student show them at the board.

8. Illustrate on the board with colored chalk that $\overline{TR}$ and $\overline{RS}$ in figure 3 have a common end point or vertex. What is it? One way the angle formed by $\overline{TR}$ and $\overline{RS}$ can be named is $\angle TRS$. What is the other way?

9. $\overline{TS}$ and $\overline{RS}$ form an angle whose vertex is point ________. In what two ways can this angle be named?

10. What is the vertex of the angle formed by $\overline{RT}$ and $\overline{ST}$? Name the angle in two ways.

11. What are the three angles in $\triangle ABC$, figure 4?

12. In which triangle do you see the angles $\angle MNO$, $\angle NOM$, $\angle OMN$?

13. In $\triangle UVW$ what angle is formed by $\overline{UV}$ and $\overline{VW}$?

C. In preparation for the work sheet, have individuals or small groups come to the board and measure the sides of the triangles. They should record their information beside the triangle as:

$\overline{RT}$	1 foot	or	12 inches
$\overline{TS}$	1 foot	or	12 inches
$\overline{SR}$	1 foot	or	12 inches.

Do not identify the types of triangles at this point. Tell the students they will refer back to them later in the hour.

D. Work sheet, "Three Types of Triangles." This should be an independent exercise; however, the teacher may want to group, and work with, any students who need guidance.

E. Assessment Items

1. As a class, refer to the triangles on the board. Using the dimensions listed, classify them as isosceles, equilateral, or scalene triangles.

2. Student work sheet

   a. The teacher may want to reserve this work sheet, "CATegorize the Triangles," for a review on the next day.

   b. Correct the work sheet together and discuss.

THREE TYPES OF TRIANGLES

1. Below are some examples of polygons. Circle only the triangles.

2. How are the triangles different from the other polygons?

____

How are they alike?

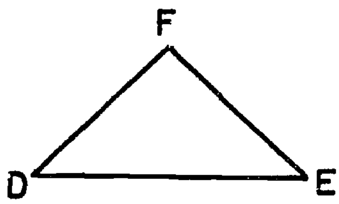
____

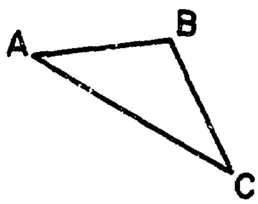
3. Use a symbol to name this triangle

____

4. Name the angles in $\triangle$ ABC: $\angle$BAC , __________ , __________.
Name the vertices in $\triangle$ ABC: A , ______ , ______.
Name the sides of $\triangle$ ABC: $\overline{AB}$, ________ , ________.

5.





a. Measure the sides of $\triangle$DEF and $\triangle$ABC

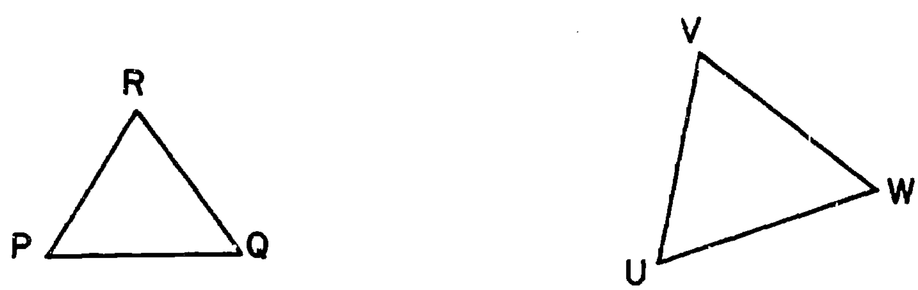
$\overline{DE}$ __________ $\overline{AB}$ __________
$\overline{EF}$ __________ $\overline{BC}$ __________
$\overline{FD}$ __________ $\overline{CA}$ __________

b. What do you notice about $\overline{DF}$ and $\overline{FE}$? $\overline{AB}$ and $\overline{BC}$?

____

c. $\triangle$DEF is called an isosceles triangle because __________ of its sides are equal.

6.



a. Measure the sides of $\triangle$PQR and $\triangle$UVW

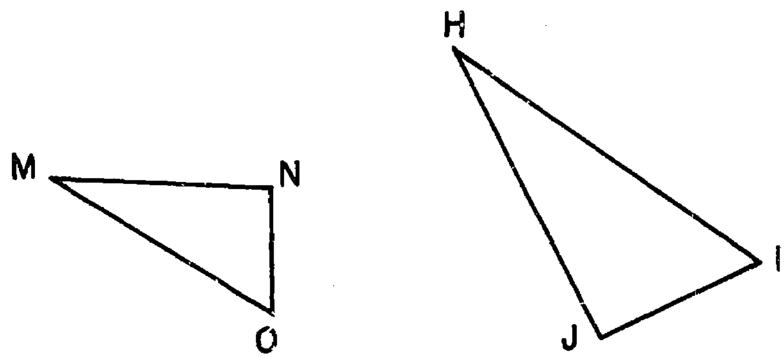
$\overline{PQ}$	__________	$\overline{UV}$	__________
$\overline{QR}$	__________	$\overline{VW}$	__________
$\overline{RP}$	__________	$\overline{WU}$	__________

b. What do you notice about the sides of $\triangle$PQR? $\triangle$UVW?

____

c. $\triangle$PQR is called an equilateral triangle because __________ of its sides are equal.

7.



a. Measure the sides of $\triangle$ MNO and $\triangle$ HIJ

$\overline{MN}$	__________	$\overline{HI}$	__________
$\overline{NO}$	__________	$\overline{IJ}$	__________
$\overline{OM}$	__________	$\overline{JH}$	__________

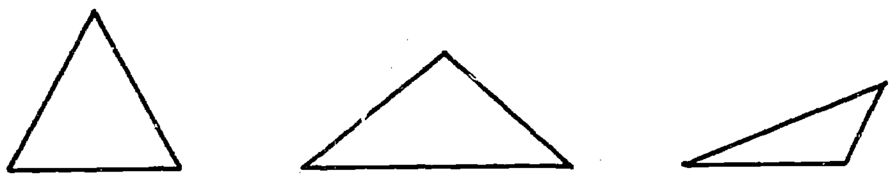
b. Are any of the sides in $\triangle$ MNO or $\triangle$ HIJ equal? ________________

c. $\triangle$ MNO is called a ________________ triangle because ________________ of its sides are equal.

d. What kind of a triangle is $\triangle$ HIJ? ________________

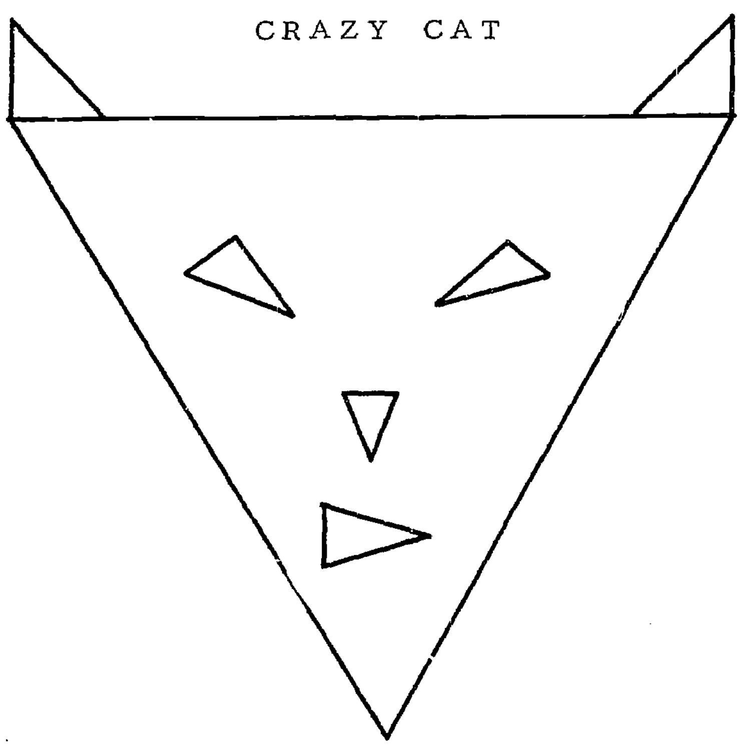
CATegorize the Triangles

Underline the correct name of each triangle below:



isosceles	isosceles	isosceles
equilateral	equilateral	equilateral
scalene	scalene	scalene

1. How many triangles do you see in the drawing and title of "Crazy Cat?" ______________________
2. Shade in the isosceles triangles found in the cat.
3. Stripe the equilateral triangles found in the cat.
4. Outline the scalene triangles found in the cat with a pencil or pen.



PEGBOARD GEOMETRY

Teacher Commentary

I. Unit: Geometry

II. Objectives:

A. Part I "Pegboard Geometry": The student should be able to:

1. Name and identify an equilateral triangle
2. Construct equilateral triangles by using pegboard, pegs, and rubber bands

B. Part II "Pegboard Geometry": The student should be able to:

1. Name and identify a pentagon and a hexagon
2. Construct drawings of the pentagon and hexagon
3. Describe the number of sides
4. Distinguish between hexagon and pentagon

III. Materials:

A. Pegboard Geometry Kit including:

1. Pegboard
2. Pegboard Geometry Booklets, Part I and Part II
3. Rubber bands
4. Pegs

B. Pencil

C. Crayons: red, blue, green, yellow

D. Ruler

E. Patterns I, II, and III for pegboard

IV. Procedure:

A. This kit is designed for small group instruction and may be used either to teach the material initially or to reinforce the learnings of students who are having difficulty. Six to eight students should work in pairs, each pair with a booklet, pegboard, pegs, rubber bands, and pattern for booklet.

B. Permit the group to work through the booklet, each pair working at its own rate.

C. The last section of each booklet is an assessment activity.

Note: The patterns for placing pegs in holes on the pegboard are based on:

Primary Peg Board, #474x, Milton Bradley Company. Springfield, Massachusetts

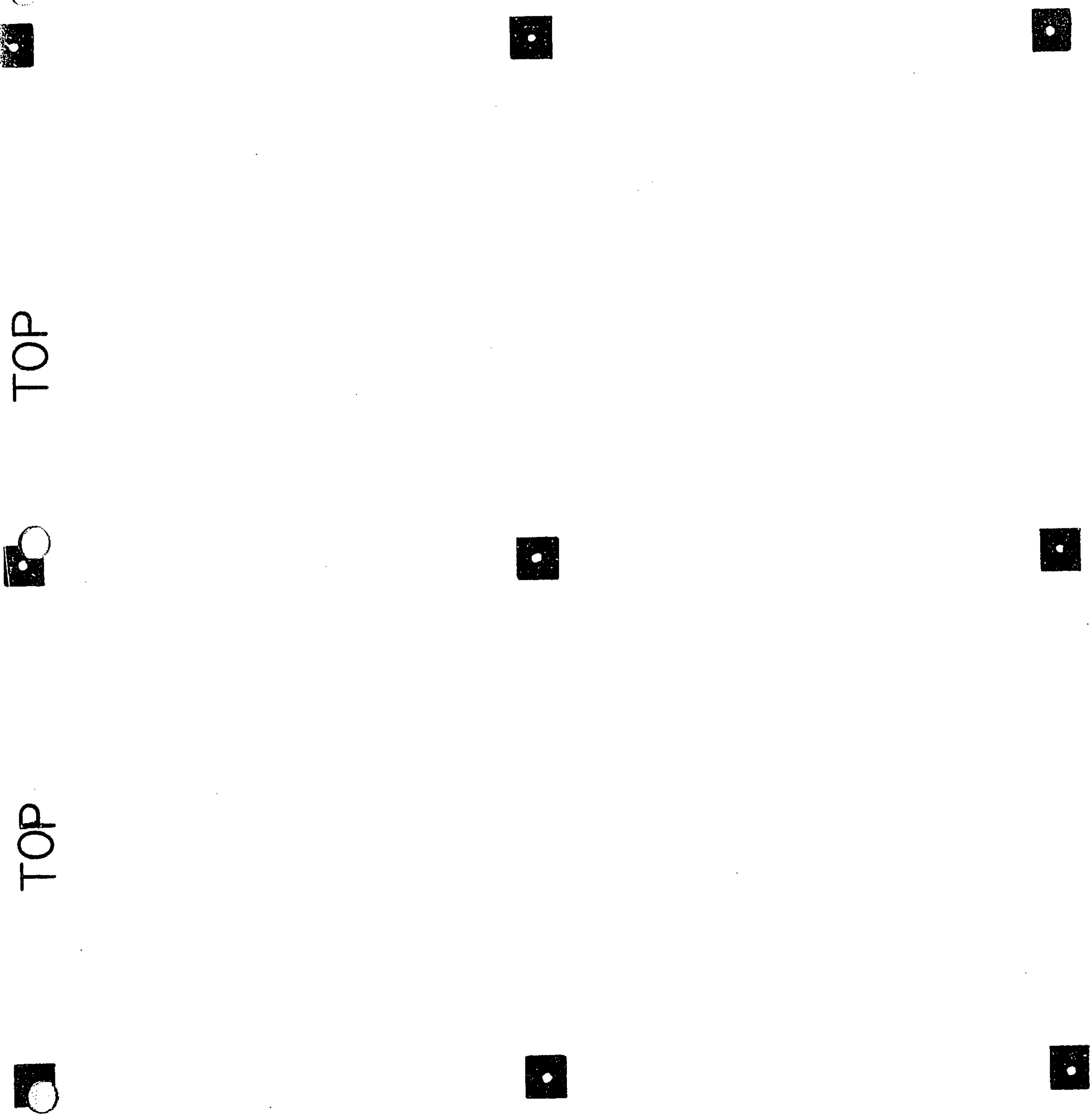
Pegs, #472x or #475x, Milton Bradley Company

Pegboards and pegs may be purchased from Lycetts, North Charles Street, at the following prices:

Pegboards - 90¢ each

Pegs - 65¢ box of 100

PATTERN I

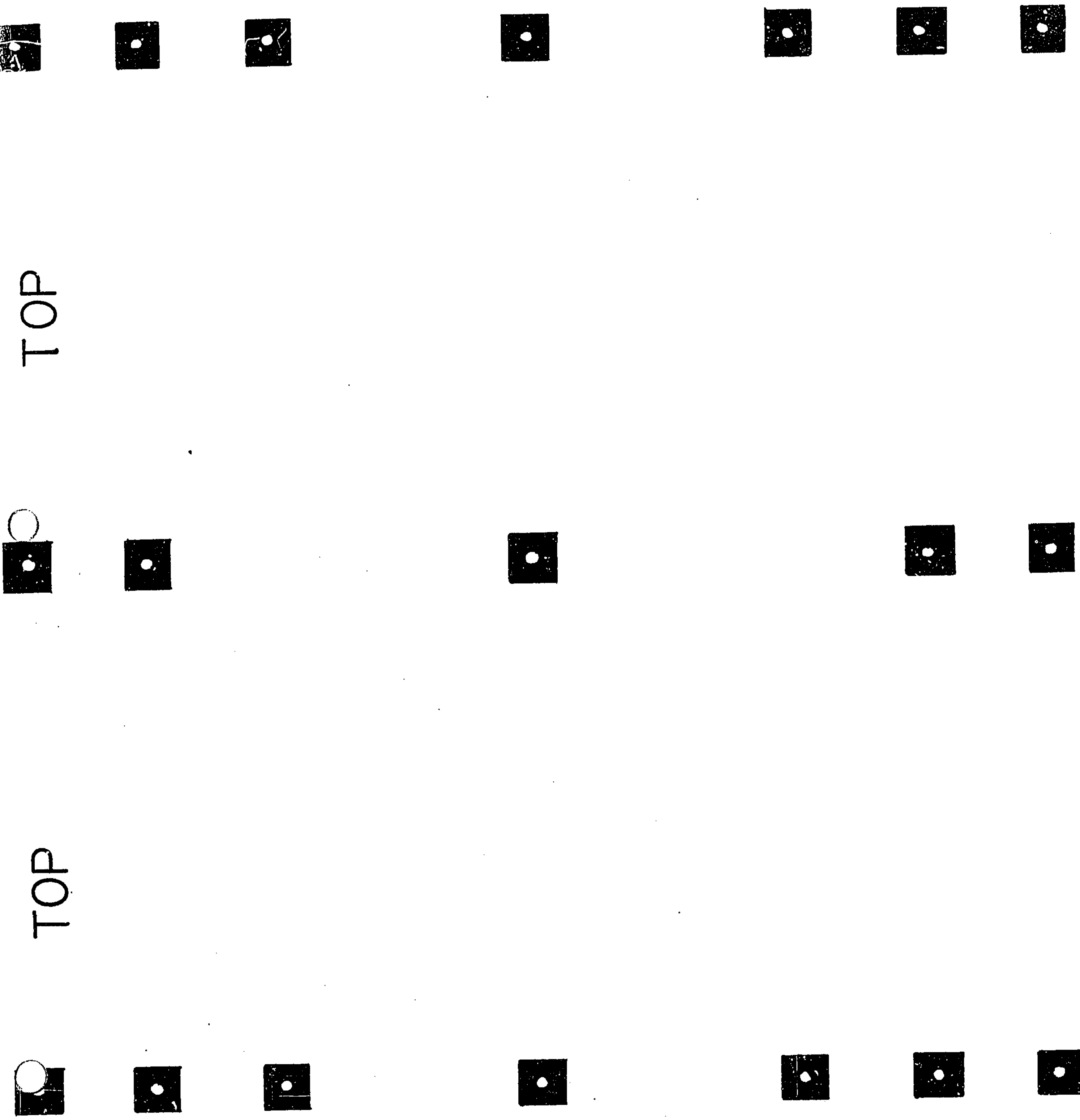


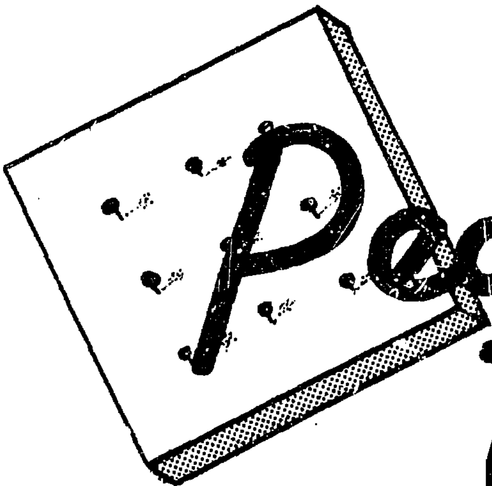
TOP

TOP

PATTERN II

PATTERN III



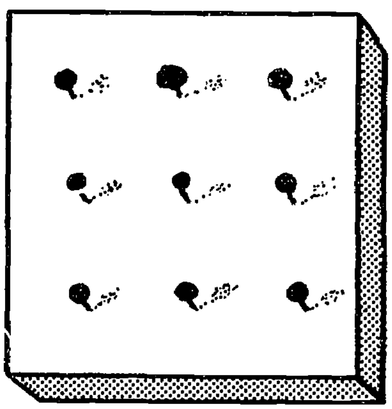


Pegboard Geometry

Part I Triangles

Set up your pegboard in the following way:

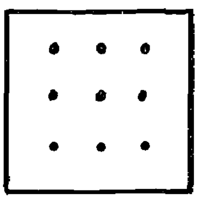
1. Place Pattern I on top of your pegboard.
2. Put pegs through the holes as shown below.



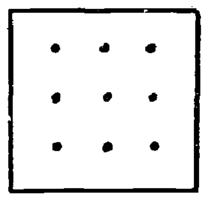
Choose rubber bands to put over the pegs to make triangles.

Remember that a triangle is a polygon having three line segments for sides.

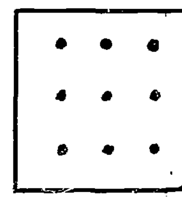
Make pictures of 3 of your triangles here.



1.



2.

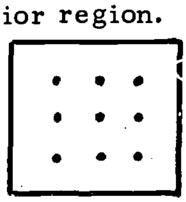


3.

In the word triangle, "tri" means three.

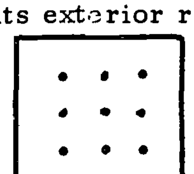
Now use your pegboards to make the following triangles.

1. A triangle that has 3 pegs in its exterior region and 0 peg. in its interior region.



Make a drawing of this triangle.

2. A triangle that has 1 peg showing in its interior region and 4 pegs in its exterior region.

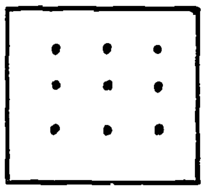


Make a drawing of this triangle.

Look at your drawings of triangles. Count the number of corners each has. Every triangle has exactly ________ corners.

We call these corners vertices.
One corner is called a vertex.

Now use your pegboard to make a triangle with a square corner.

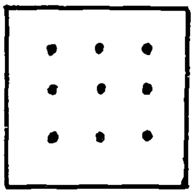


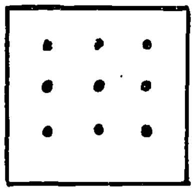
Make a drawing of this triangle.

We call this kind of triangle a right triangle.

Make some right triangles on your pegboard.

Draw pictures of 2 of your right triangles here.





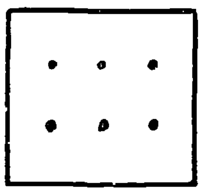
Label each vertex which is a square corner with a capital letter.

Now take the pegs out of your pegboard.

Set up your pegboard in the following way:

1. Place Pattern II on top of your pegboard.
2. Put pegs through the holes as shown below.

Now make a triangle on your pegboard that has sides which are equal.

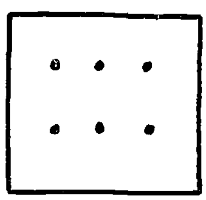


Draw this triangle.

We call a triangle whose sides are equal an equilateral triangle.

Make another equilateral triangle on your pegboard.

Be certain the sides of the triangle are equal.

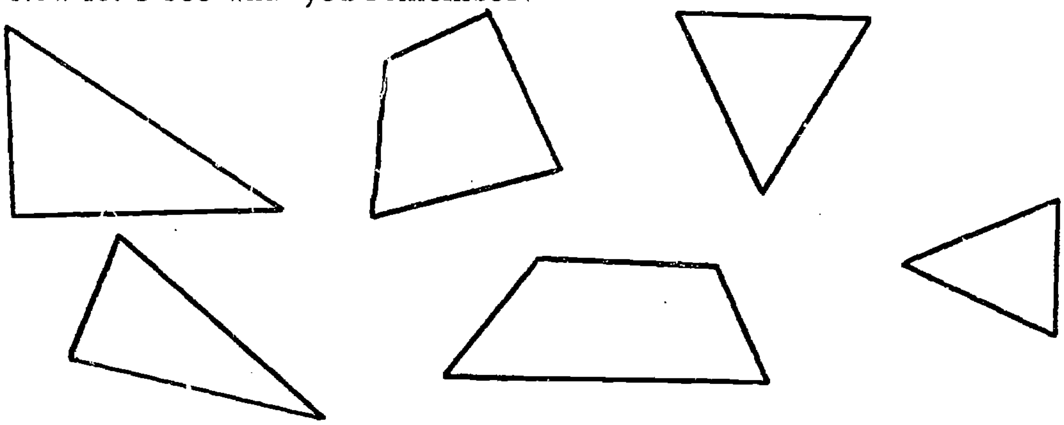


Draw this triangle.

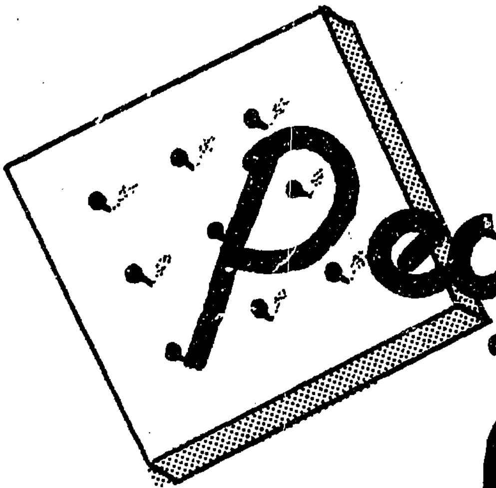
Label the vertices of this triangle with capital letters.

Do these vertices form square corners? Yes _______ No _______

Now let's see what you remember.



1. Color the interior region of the triangles red.
2. Draw a red circle around each right triangle.
3. Put an A on the vertex that is the square corner in each right triangle.
4. Draw a blue circle around the equilateral triangle.



Pegboard Geometry

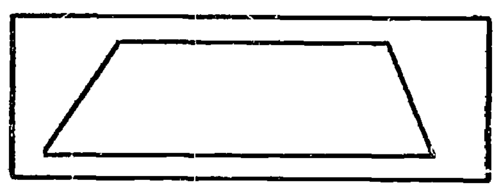
Part II Five-sided and Six sided Polygon

FIVE-SIDED AND SIX-SIDED POLYGONS

LET'S REVIEW!

We have learned that a polygon is a simple closed path made of three or more line segments.

A polygon has an interior region and an exterior region.
Color the interior region of this polygon red.
Color the exterior region of this polygon green.



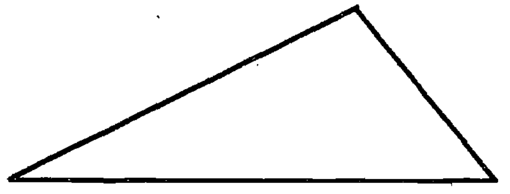
A polygon is named by the number of sides it has.

Draw a picture of a polygon called a triangle.

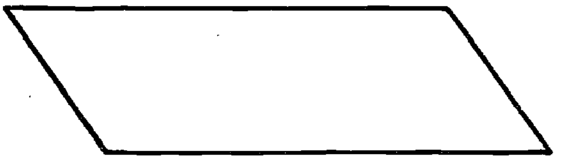
Draw a picture of a polygon called a quadrilateral.

The corners of a polygon are called vertices.

Name the vertices of this triangle C, G, and F.

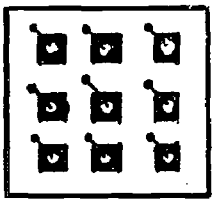


Name the vertices of this quadrilateral N, X, Y, and Z.



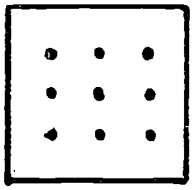
Set up your pegboard in the following way.

1. Place Pattern I on top of your pegboard.
2. Put pegs through the holes as shown below.

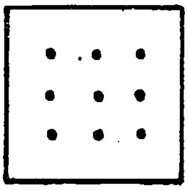


Use rubber bands to make some polygons that have 5 sides.

Draw pictures of 2 of your five-sided polygons here.



1.



2.

We call these five-sided polygons pentagons. (PEN-ta-gons)

Name the vertices of pentagon #1 - A, D, J, K, and L.

Name the vertices of pentagon #2 - R, G, S, T, and V.

How many vertices does each pentagon have?

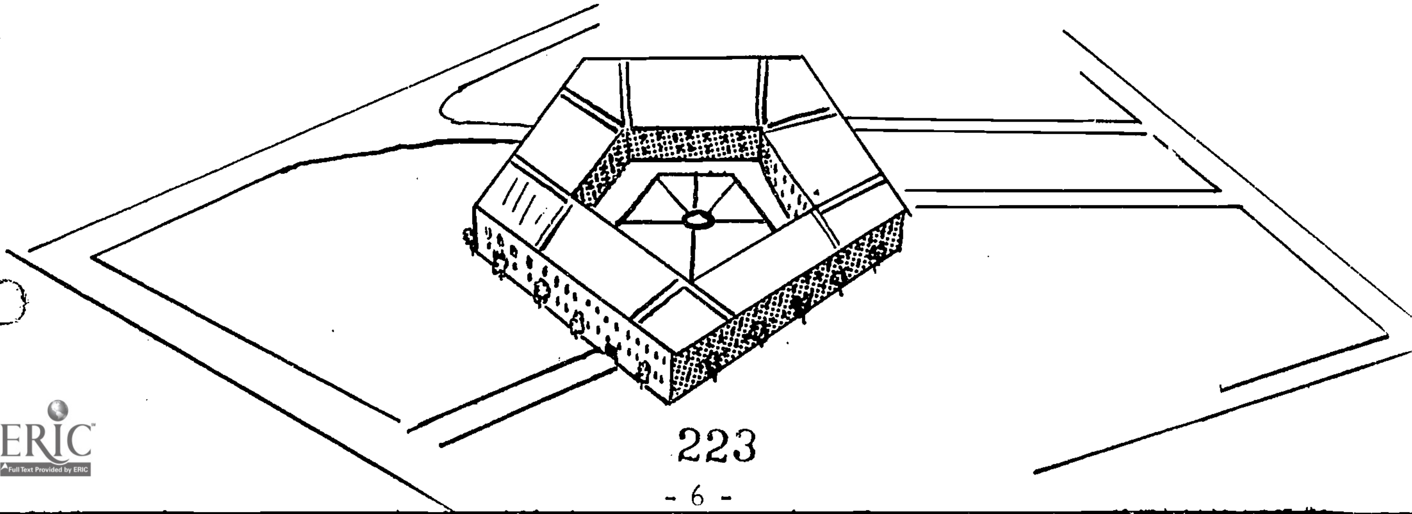
A building in Washington, D. C. is called the Pentagon.

Do you know why?

Look at the picture of the Pentagon building drawn below.

Count the sides.

This building is called the Pentagon because it has exactly ______ sides.



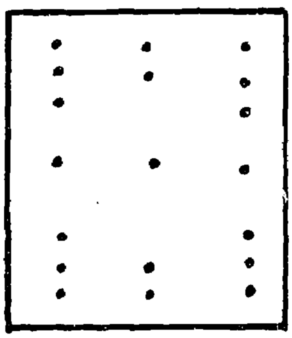
We have learned that:

1. a polygon with 3 sides is called a triangle.

2. a polygon with 4 sides is called a quadrilateral.

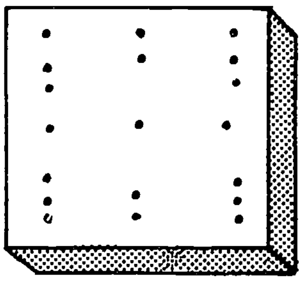
3. a polygon with 5 sides is called a pentagon.

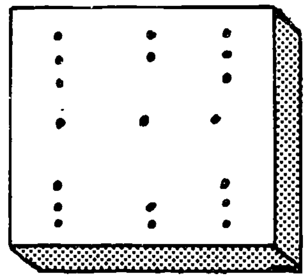
Now set up your pegboard with 19 pegs.
Place Pattern III on top of your pegboard. Put pegs through the holes as shown below:



Use rubber bands to make some polygons that have 6 sides.

Make drawings of your six-sided polygons here.



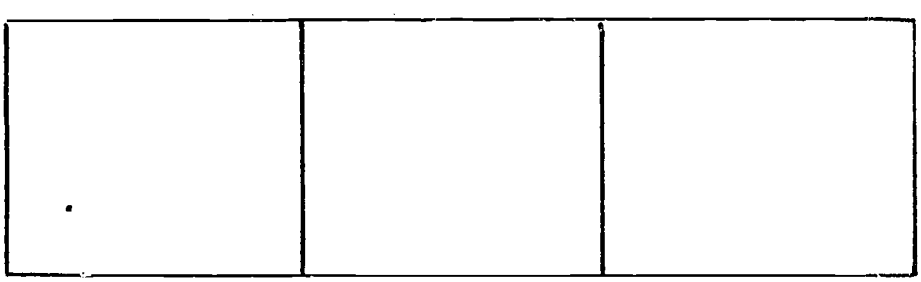


We call these six-sided polygons hexagons. (HEX-a-gons)

Count the number of vertices each of your hexagons has.

Each hexagon has ______ vertices.

Draw pictures of 3 different hexagons in the boxes below.



1. 2. 3.

Name the vertices of hexagon #1 - C, G, F, H, S, and T.

Color the interior region of hexagon #2 blue.

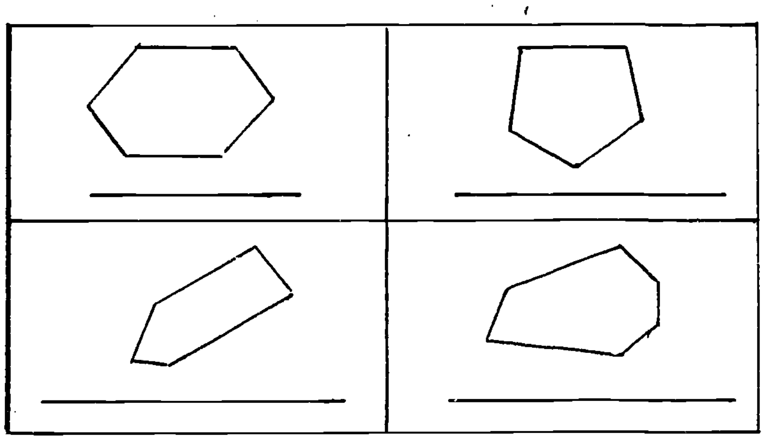
Color the exterior region of hexagon #3 yellow.

Now let's see how much you have learned about polygons.

1. Write the name of each kind of polygon shown in the boxes below.

pentagon

hexagon



2. Name the vertices of each polygon.

3. Make each of the sentences below true by writing a numeral in each blank.

   1. A pentagon is a polygon that has ______ sides.
   2. A hexagon is a polygon that has ______ sides.
   3. A hexagon has ______ more sides than a triangle.
   4. A pentagon has ______ more sides than a quadrilateral.

4. Draw a picture of each of the following:

Pentagon	Hexagon

GEO-KIT

Teacher Commentary

I. Unit: Geometry

II. Objectives: The student should be able to:

A. Construct 4 basic geometric figures (hexagon, pentagon, equilateral triangle, isosceles triangle) using triangles

B. Describe properties associated with each figure

III. Materials:

A. One box for storing all materials

B. Each box should contain 5 sets of each of the following items:

1. Envelope 1 - 6 green equilateral triangles, 6 orange equilateral triangles

2. Envelope 2 - 5 yellow right triangles, 5 pink right triangles

3. Envelope 3 - 1 orange hexagon

4. Envelope 4 - 5 red right triangles, 5 blue right triangles

5. Envelope 5 - 5 green isosceles triangles

6. Envelope 6 - 10 metal rings

7. Set of 11 experiments (divided into 3 sets)

8. Pencil

9. Straightedge (tag board)

IV. Procedure:

A. The materials in this kit are designed as a culminating activity. Therefore, work with the kit should run concurrently with the unit on geometry.

B. The class should be divided into groups containing 1-5 students. Each student is to work at his own rate of speed. While a certain group is working with the geo-kit, other groups could work with related topics in geometry, practice fundamental operations, etc.

C. From the box each student should select:

1. A complete set of envelopes
2. Three sets of experiments
    a. Set 3 (Experiments 12-15)
    b. Set 4 (Experiments 16-18)
    c. Set 5 (Experiments 19-22)

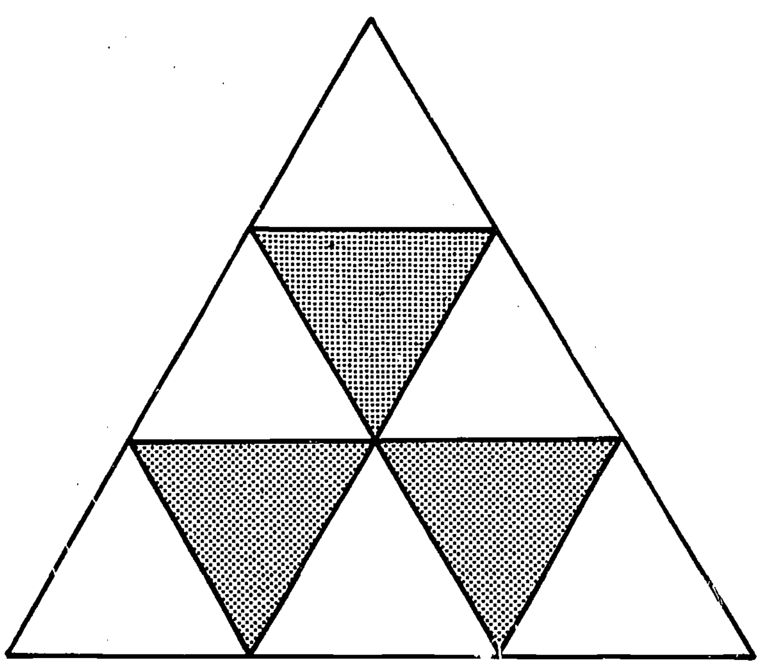
D. All answers should be placed on the experiment sheets.

E. Answer sheets should be collected at the end of each period.

F. The testing devices are built into the experiments.

SET III

grade 8



SHAPES WITH CARDS

EXPERIMENT 12, PART 1

Choose 4 triangles from envelope 1.

These triangles have 3 equal sides. They are called equilateral triangles.

Make as many shapes as you can using these 4 equilateral triangles. Draw these shapes below.

Choose the shape which has only 3 sides.

Are these sides equal? ____________

How do you know? ____

What do you call a three-sided figure? ___________________________

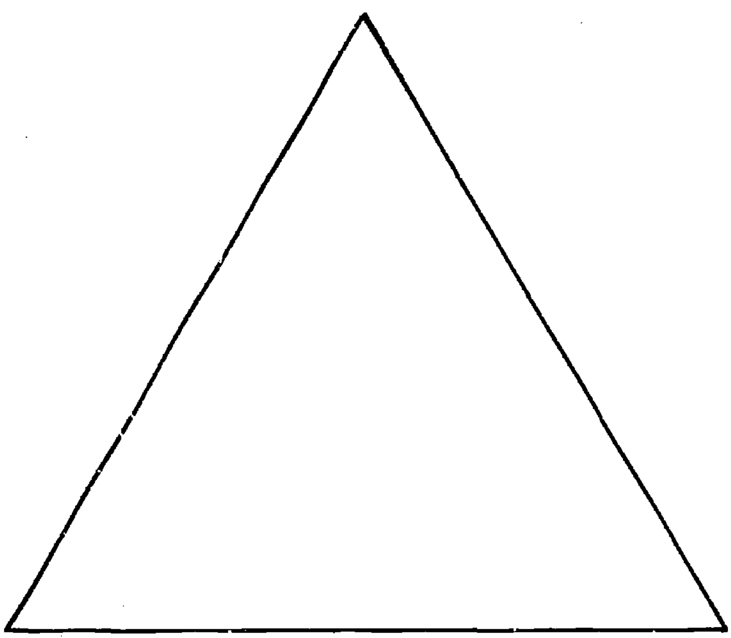
If the three-sided figure has all sides equal, what kind of triangle is this? _________________________

EXPERIMENT 12, PART 2

What is this figure called? ______________

Are the sides equal? ______________

How many sides does it have? ______________



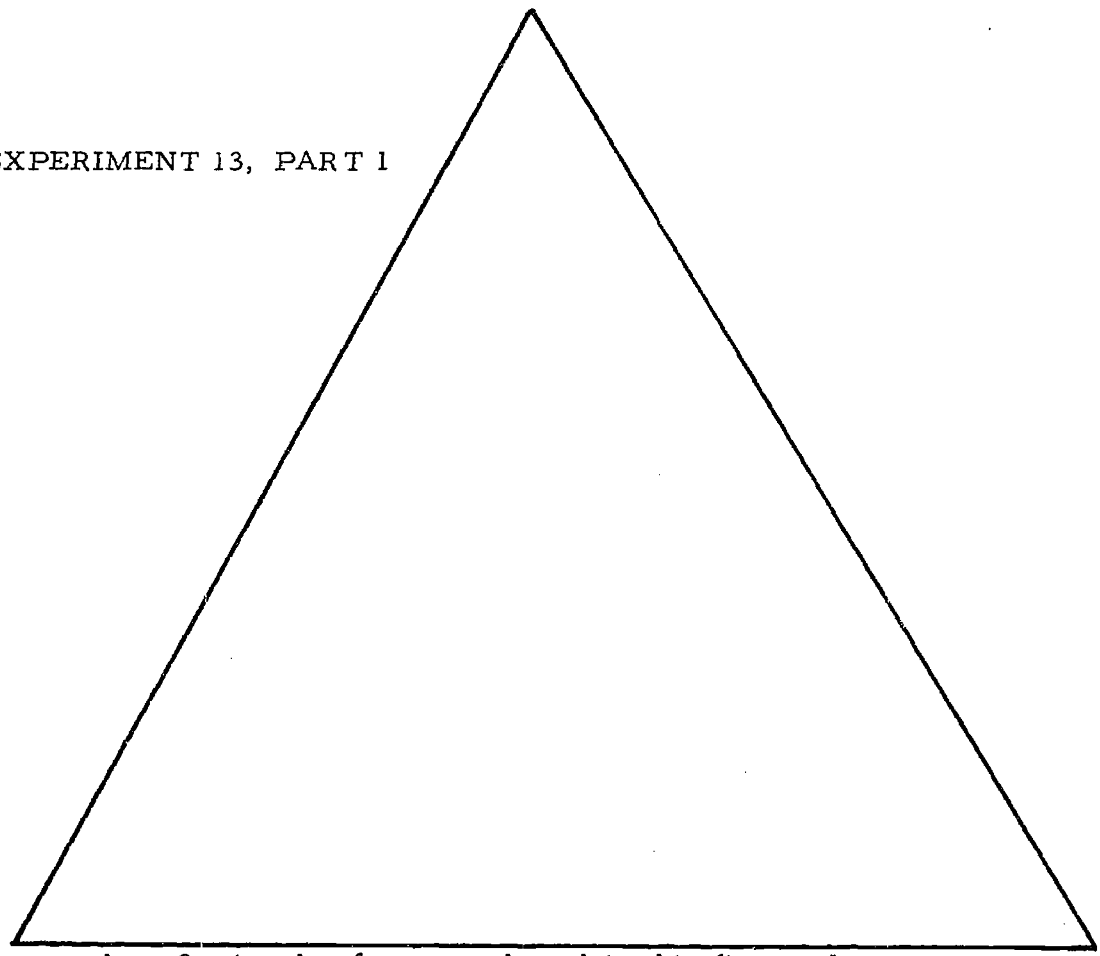
Try to place 4 triangles from envelope 1 in the equilateral triangle.

Show how you placed the triangles in the equilateral triangle.

REMEMBER! You can show this by tracing around each triangle.

PUT THE TRIANGLES BACK IN ENVELOPE 1

EXPERIMENT 13, PART 1



Try to place 9 triangles from envelope 1 in this figure above.

What is this figure called? ________________________

Show how you placed the triangles from envelope 1 in the equilateral triangle.

You can show this by tracing around the triangles.

How many pairs of equal sides does a parallelogram have? ____________

How many pairs of parallel sides does a parallelogram have? ____________

See if you can find some parallelograms in the large equilateral triangle.

How many did you find? ______________

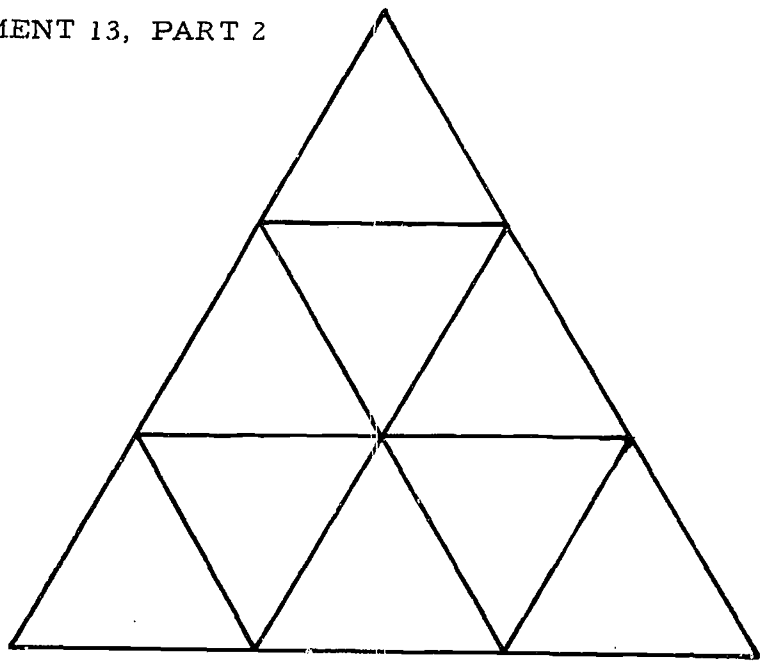
How many pairs of equal sides does a trapezoid have? ______________

How many pairs of parallel sides does a trapezoid have? ____________

Can you find some trapezoids in the large equilateral triangle? __________

How many did you find? ____________

EXPERIMENT 13, PART 2



This drawing is a copy of what you did in the last experiment.

Use it to answer the following questions.

How many pairs of equal sides does a rhombus have? ______

How many pairs of parallel sides does a rhombus have? ______

Are all 4 sides equal? ______

Can you find a rhombus in the equilateral triangle? ______

Are there others? ______

If so, how many? ______

Summarize your results by filling in the chart below.

Geometric Figure	A Picture of it	Number of Sides	Pairs of Equal Sides	Pairs of Parallel Sides
Parallelogram				
Rhombus				
Trapezoid				

PUT THE TRIANGLES BACK IN ENVELOPE 1

EXPERIMENT 14, PART 1

Choose the shape from envelope 3.

Place the side of the shape on the line.

Then trace the shape on your paper.

How many sides does this shape have? ____________

Go on to next page.

EXPERIMENT 14, PART 2

Try to place 6 triangles from envelope 1 in the drawing you made.

Show how you placed the equilateral triangles in the drawn shape.

You can do this by tracing around each triangle.

Are the sides of this shape equal to each other? _______________

How do you know? ____

How many equal sides does this shape have? ____________________

This shape is called a hexagon (Hecks - a - gon).

A hexagon is a six sided figure.

Add 6 triangles on to the hexagon to make a six-pointed star.

Show how you did this by tracing around each triangle.

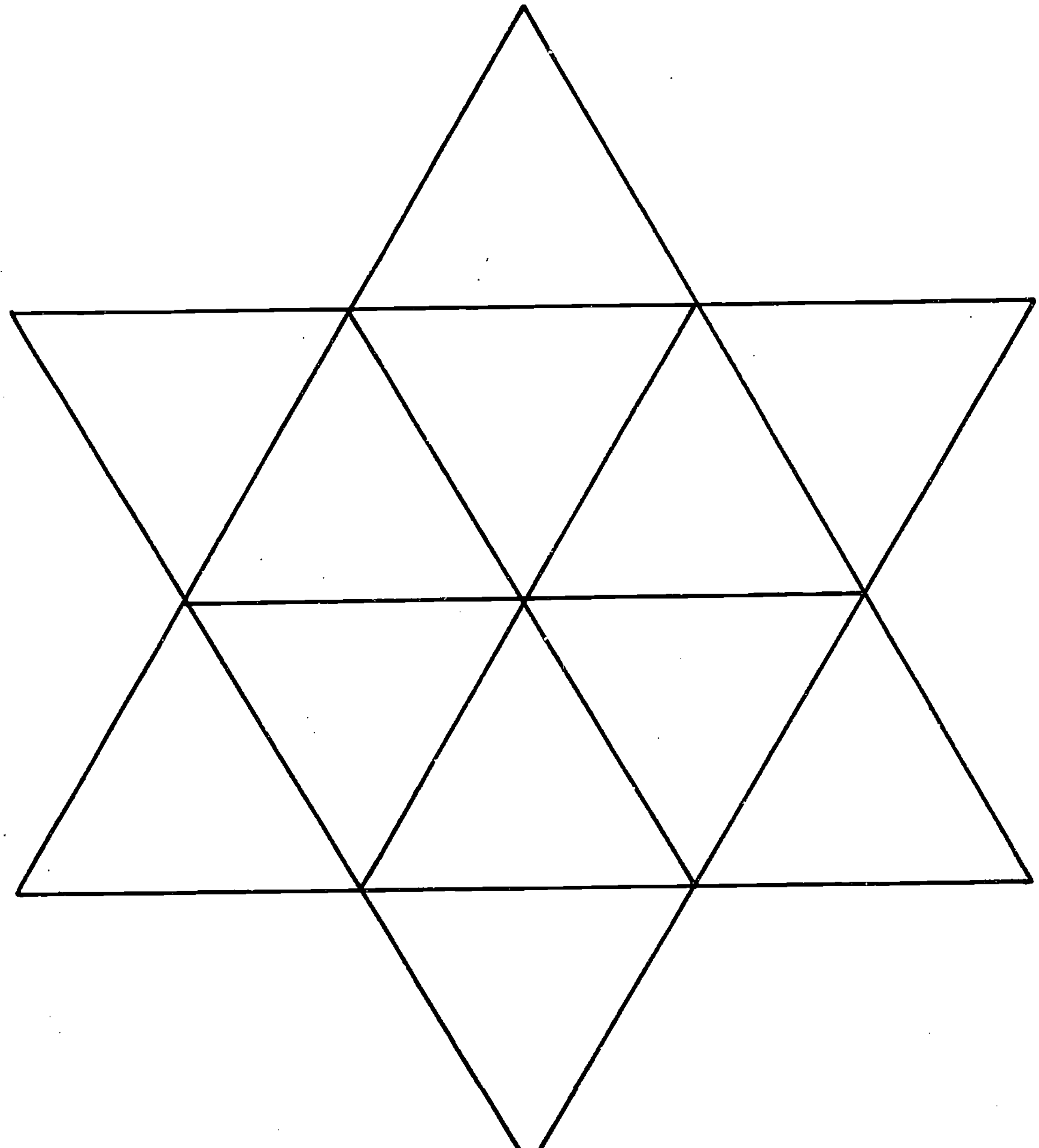
PUT THE HEXAGON BACK IN ENVELOPE 3

EXPERIMENT 14, PART 2

This drawing is a copy of what you did in experiment 14, part 1.

Tear this sheet out of this set.

Use this sheet to answer the questions in experiment 15.

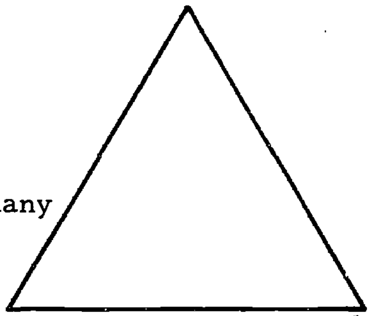


PUT THE TRIANGLES BACK IN ENVELOPE 1

EXPERIMENT 15

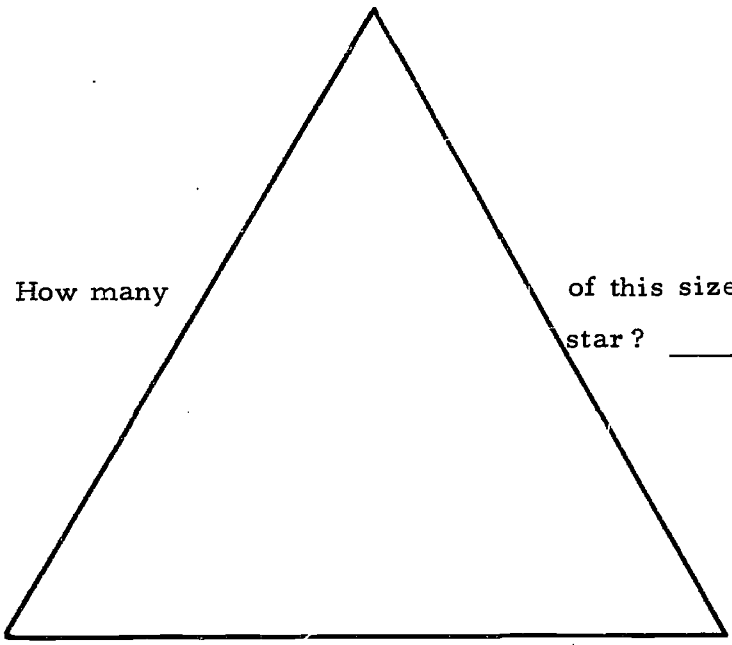
Use the torn out sheet to answer the following questions.

1. How many



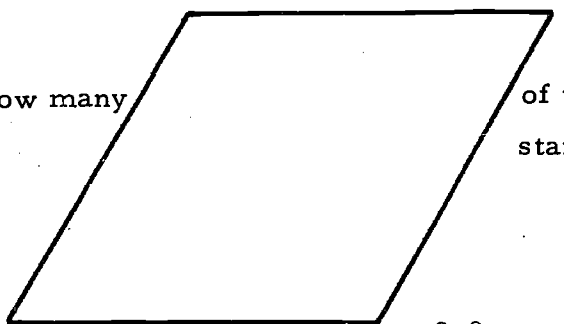
of this size are in the six-pointed star? ____________________

2. How many



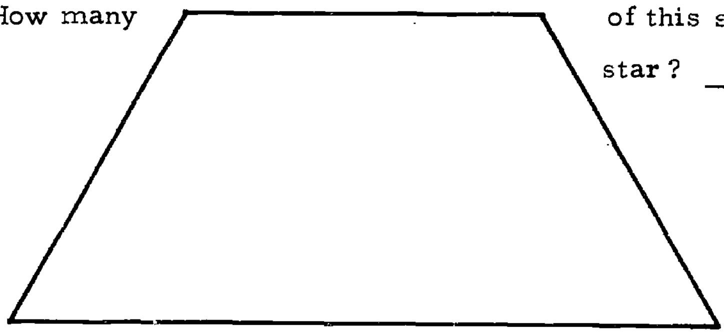
of this size are in the six-pointed star? ____________________

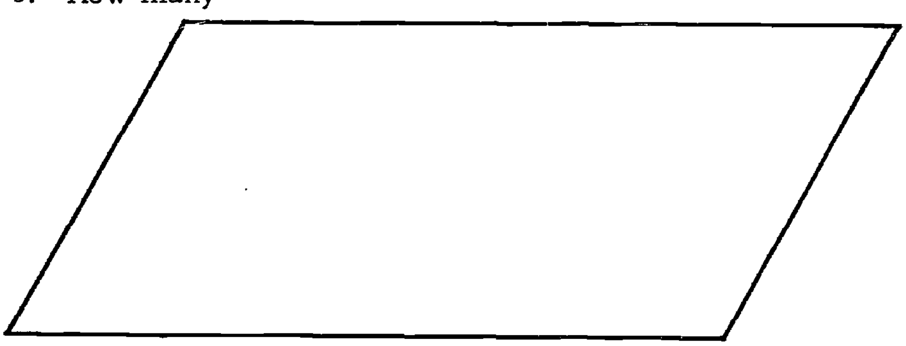
3. How many

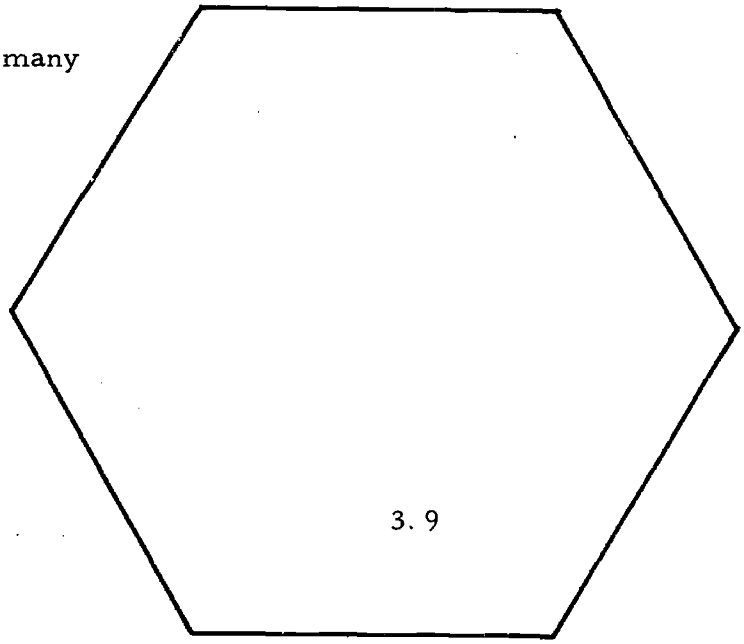


of this size are in the six-pointed star? ____________________

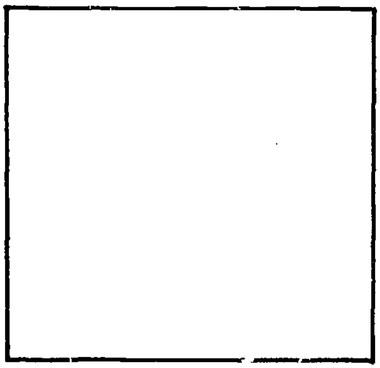
EXPERIMENT 15

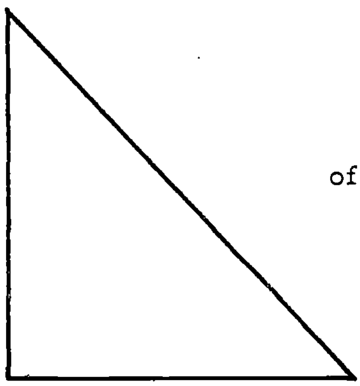
4. How many  of this size are in the star? ____________

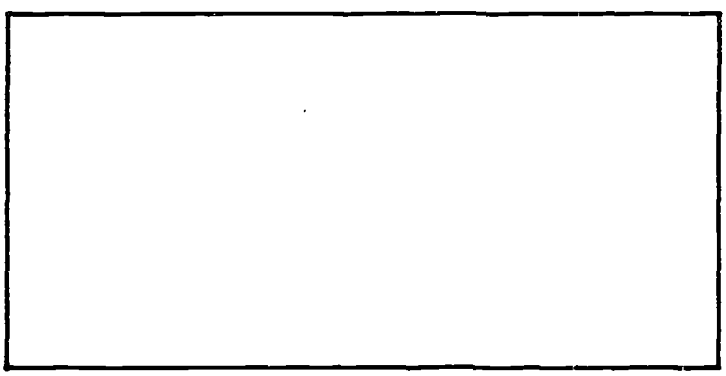
5. How many  of this size are in the star? ____________

6. How many  of this size are in the star? ____________

EXPERIMENT 15

7. How many  of this size are in the star? __________

8. How many  of this size are in the star? __________

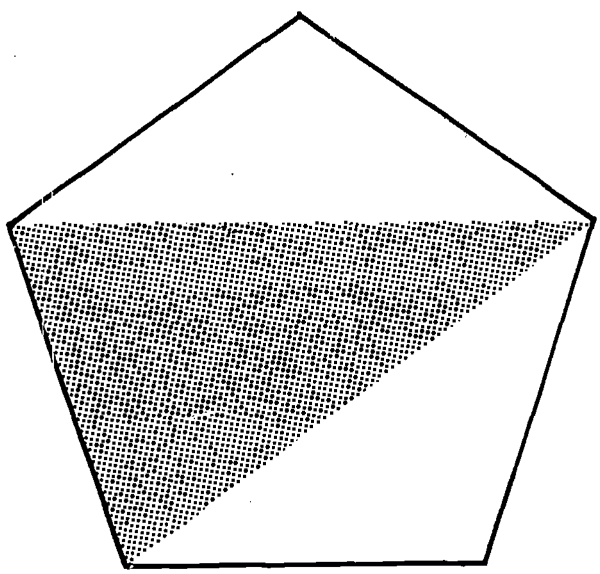
9. How many  of this size are in the star? __________

10. Using the list below, identify figures 1 through 9.
Place your answer inside the figure.

EQUILATERAL TRIANGLE	RIGHT TRIANGLE
PARALLELOGRAM	SQUARE
RECTANGLE	TRAPEZOID
RHOMBUS	HEXAGON

SET IV

grade 8



SHAPES WITH CARDS

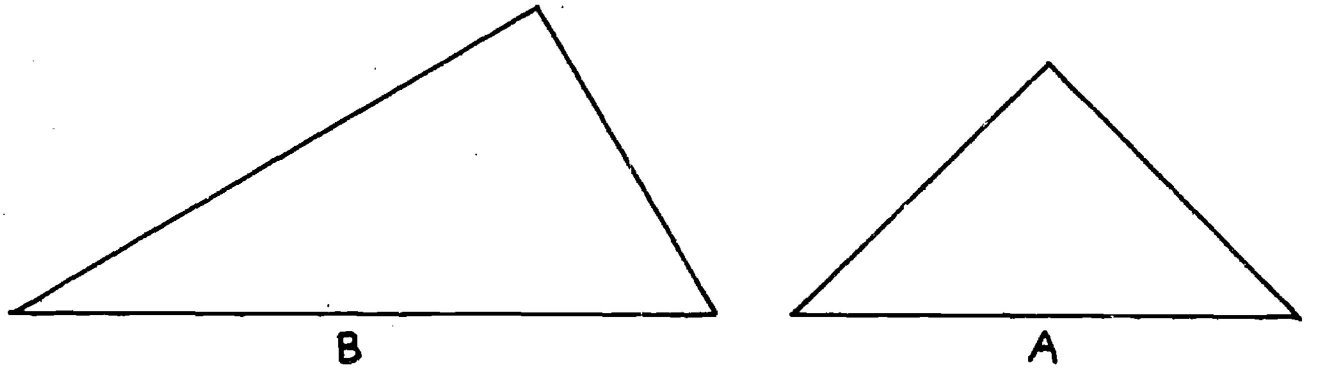
EXPERIMENT 16, PART 1

Choose one triangle from envelope 4.

This triangle has one right angle. Such triangles are called right triangles.

Place this triangle in triangle B, below.

Are they the same? ______________



Locate the right angle in triangles A and B.

Place an X in the right angles.

REMEMBER! The right angle is opposite the longest side.

PUT THIS TRIANGLE BACK IN ENVELOPE 4

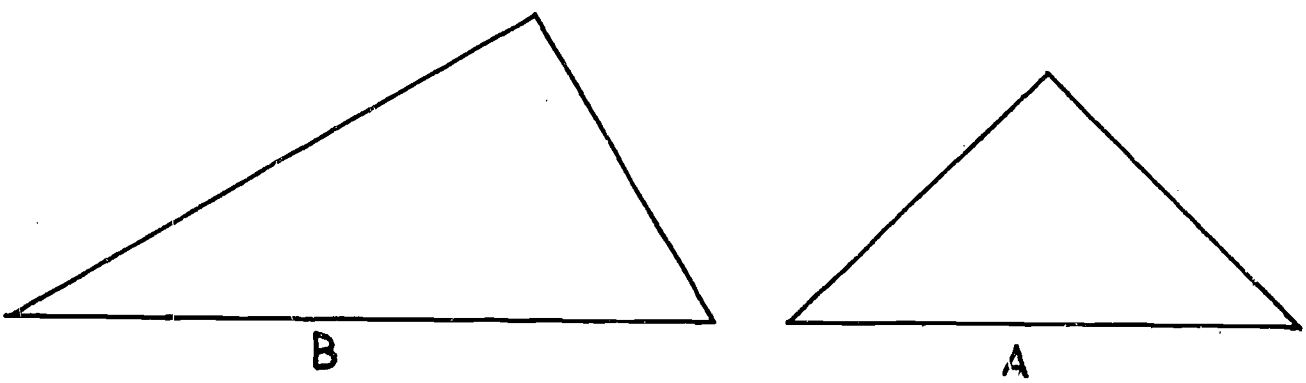
EXPERIMENT 16, PART 1

Choose 1 triangle from envelope 4.

This is a right triangle.

Place this triangle in triangle B.

Are they the same? ___________



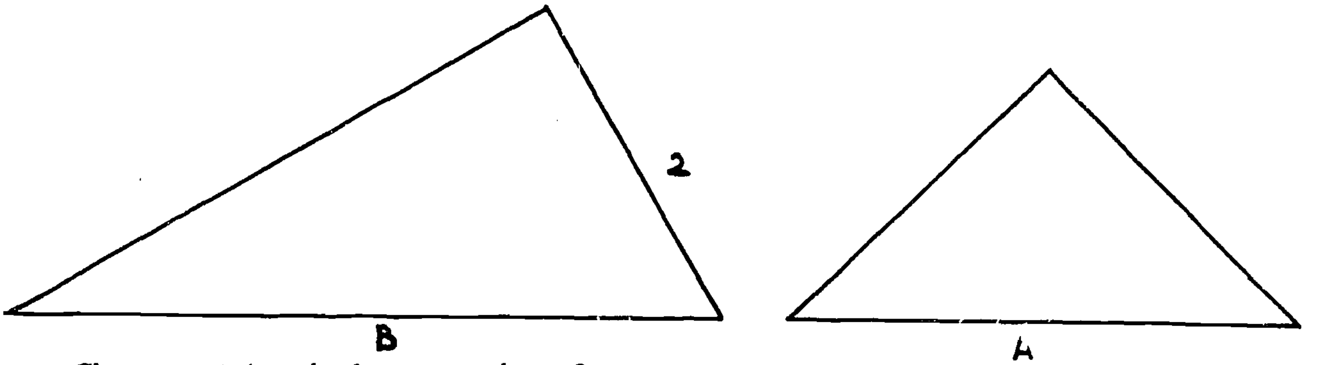
Locate the right angle in triangles A and B.

Place an X in the right angles.

REMEMBER! The right angle is opposite the longest side.

PUT THIS TRIANGLE BACK IN ENVELOPE 4

EXPERIMENT 16, PART 2



Choose a triangle from envelope 2.

Place the triangle in triangle A.

Are they the same? __________

Now place the cut out triangle in triangle B.

Side 1 of the cut out triangle should be next to side 2 of triangle B.

Is the right angle of the cut out triangle on top of the right angle of triangle B? __________ If not, place them so they are.

Show how the cut out triangle was placed on triangle B.

<u>REMEMBER</u>! You can do this by tracing around the triangle.

Is triangle B larger than triangle A? __________

Does triangle A have two equal sides? __________

How do you know? ______________________________

What does each triangle have in common? __________, __________

How is triangle A different from triangle B? __________

PUT THE TRIANGLE BACK IN ENVELOPE 2

EXPERIMENT 17, PART 1

Choose 2 triangles from envelope 4.

These are called ______________________ triangles.

Make as many shapes as you can using these 2 right triangles.

Draw these shapes below.

Choose the drawings which have only 3 sides.

You should have chosen 2 triangles.

If not, try to form two different triangles from the cut out triangles.

Did you form a triangle which has 3 equal sides? ________________

How do you know? ____

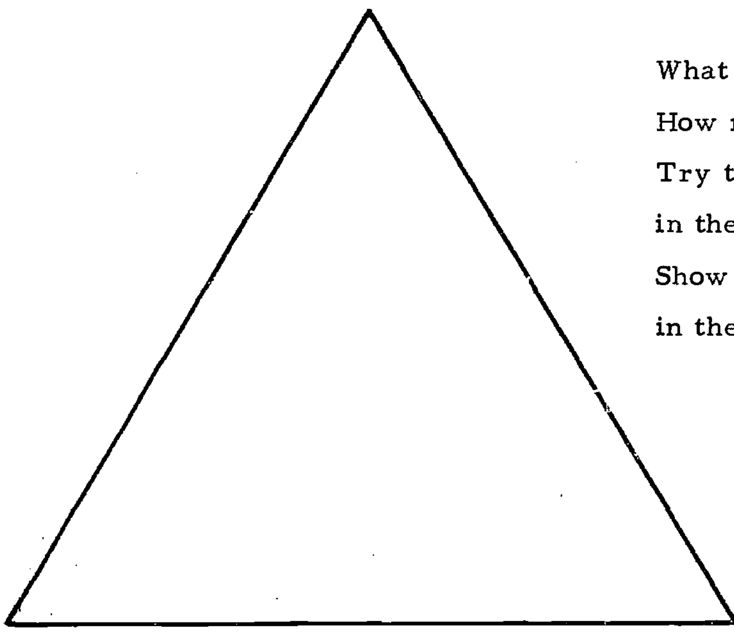
A triangle which has 3 equal sides is called an equilateral triangle.

Does the other triangle have only 2 equal sides? ________________

How do you know? ____

A triangle which has only 2 equal sides is called an isosceles triangle.

EXPERIMENT 17, PART 2



What is this figure called? ______________

How many equal sides does it have? ______

Try to place 2 triangles from envelope 4 in the equilateral triangle.

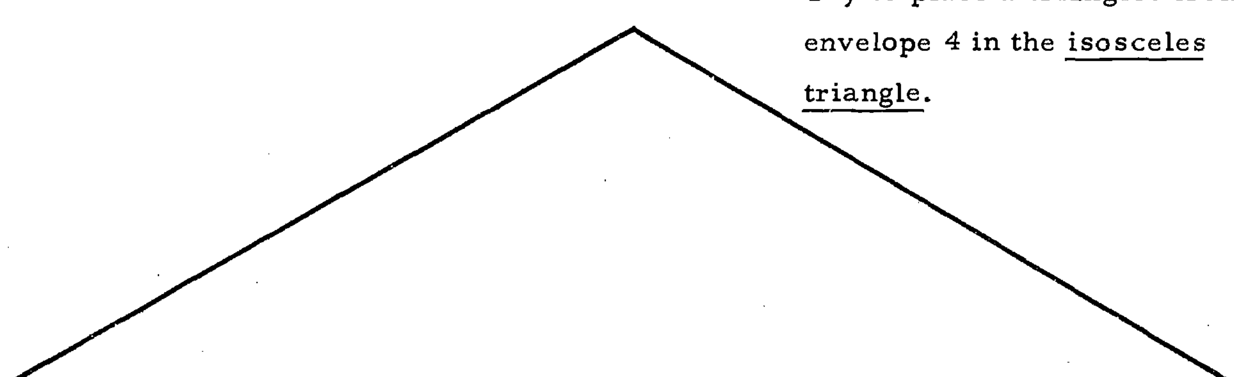
Show how you place the right triangles in the equilateral triangle.

What is this figure called?

____

How many equal sides does it have? ______________

Try to place 2 triangles from envelope 4 in the isosceles triangle.



Show how you place the right triangles in the isosceles triangle.

PUT THE TRIANGLES BACK IN ENVELOPE 4

EXPERIMENT 18

Choose 5 triangles from envelope 5.

Look at one of these triangles.

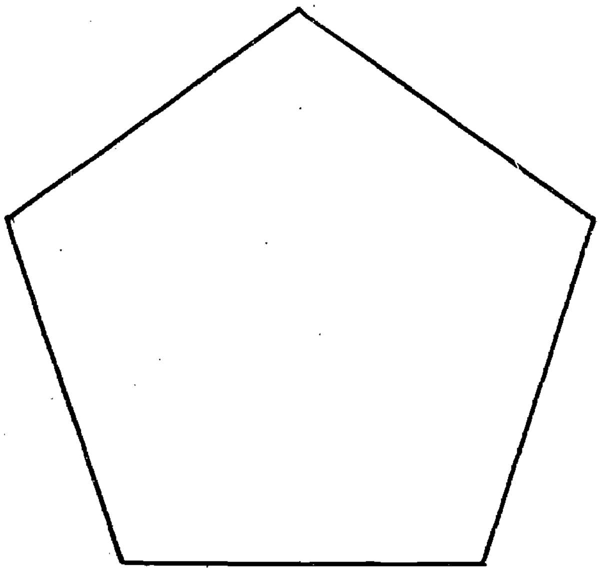
Does it have 3 equal sides? ______________

How do you know? ______________________________________

Does it have only 2 equal sides? ______________

How do you know? ______________________________________

An <u>isosceles triangle</u> is a triangle which has only 2 equal sides.



Place the <u>isosceles triangles</u> in the figure at the left.

Show how you placed the triangles in the figure.

You can do this by tracing around each <u>triangle</u>.

How many sides does the figure at the left have? ______________

Are all the sides equal to each other?

____

How do you know? ______________

This figure is called a <u>pentagon</u> (Pent - a - gon).

A <u>pentagon</u> is a figure which has 5 sides.

PUT THE TRIANGLES BACK IN ENVELOPE 5

SET V

grade 8

SHAPES WITH CARDS

EXPERIMENT 19

Choose 3 circles from envelope 6

Place 2 of the circles side by side on the line.

The edge of each circle should touch the line at the arrow.

Place the third circle above the first two circles, so that it touches both circles.

Do the circles suggest a three sided shape? ________

Draw an outline of this shape.

What is this shape called? ______________________

What kind of triangle is it? ______________________

How many equal sides does it have? ____________

An equilateral triangle is a triangle with __________ equal sides.

PUT THE CIRCLES BACK IN ENVELOPE 6

EXPERIMENT 20

Choose 10 circles from envelope 6.

Place four of the circles side by side on the line.

The edge of each circle should touch the line at the arrow.

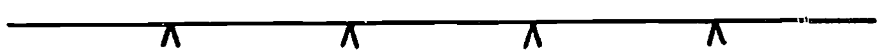
Place 3 circles above the first 4 circles.

Each circle should touch 2 circles on the first row.

Place two circles above the second row of circles.

Each circle should touch 2 circles on the second row.

Place the last circle above the third row of circles.



Do the circles suggest a shape? ____________________

Draw an outline of the shape.

What is this shape called? __________________________

What kind of triangle is this? ______________________

PUT THE CIRCLES BACK IN ENVELOPE 6

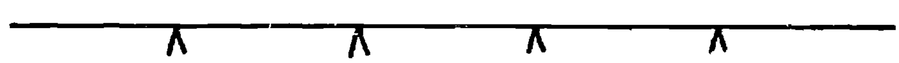
EXPERIMENT 21

Choose 10 circles from envelope 6.

Form the same shape as you did in experiment 20.

Do this on the line.

Now remove the three corner circles.



Do the circles suggest a shape? ________________

Draw an outline of the shape.

How many sides does the figure have? ____________________

This figure is called a ______________________

A hexagon is a figure which has ____________ sides.

PUT THE CIRCLES BACK IN ENVELOPE 6

EXPERIMENT 22

READ ALL OF THE FOLLOWING DIRECTIONS BEFORE YOU START WORKING.

Choose 10 circles from envelope 6.

Make the shapes named on the following pages using circles.

Use as many circles as you can to make each shape.

Make drawings of your shapes below the name of the shape.

Show how circles suggest the shapes you have drawn.

REMEMBER! You can do this by tracing around each circle.

EXPERIMENT 22

Equilateral Triangle

Square

EXPERIMENT 22

Rhombus

Hexagon

EXPERIMENT 22

Parallelogram

Rectangle

PUT THE CIRCLES BACK IN ENVELOPE 6

SPINNING SHAPES

Teacher Commentary

I. Unit: Geometry

II. Objectives: The student should be able to:

A. Name and identify a cylinder, a cone, and a sphere

B. Distinguish between spheres and circles

C. Describe those figures in terms of surroundings

III. Materials: Spinning Shapes Kit including:

A. Student booklets, "Spinning Shapes"

B. Models of triangle, rectangle, and circle

C. Rubber bands

IV. Procedure:

A. The teacher should place the models of a triangle, rectangle, and circle along with a copy of the student booklet, "Spinning Shapes" in a box to form a kit. (See note at end of Teacher Commentary.)

B. Let a small group of students work through the booklet, "Spinning Shapes." Each student should have a booklet. It is assumed that the students already have some knowledge of shapes in two dimensions.

C. After the pupils have completed the materials in the booklet, a group discussion should be held to reinforce the concepts presented.

D. The last page of the booklet provides a quick check for the teacher to evaluate the student's understanding.

E. Assessment - during a group discussion, the following questions may help to evaluate the student's understanding of shapes in three dimensions.

1. Show the student models of a cone, a cylinder, and a sphere. Ask the pupils to name each object.

2. Ask the students to bring to class or to list objects that are in the shape of a cone, a cylinder, and a sphere.

3. Discuss the difference between a circle and a sphere.

Note: The models of the triangle, rectangle, and circle can be purchased from:

Lycett's
North Charles Street
Baltimore, Maryland

Order this set:

Geometric Wire Forms and Patterns
#794
Ideal Supply Company
$3.00

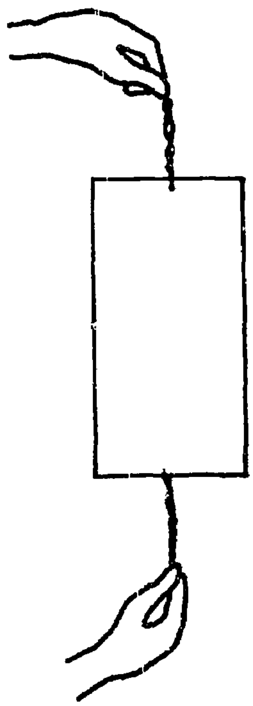
SPINNING SHAPES

SPINNING RECTANGLES

Pick up the rectangle

1. Be sure each rubber band is in the center of the side to which it is attached.
2. Hold one rubber band in each hand.
3. Pull each rubber band tightly.

   The drawing below should help you see what to do.



4. Let a classmate turn the rectangle so that the rubber bands are twisted tightly.
5. Watch the rectangle spin.

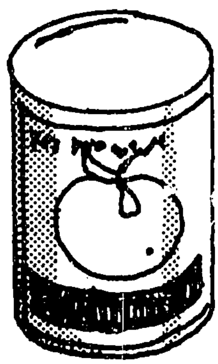
As it spins fast do you still see the rectangle? Circle your answer. Yes no

Did you also see another shape? Circle your answer. Yes No

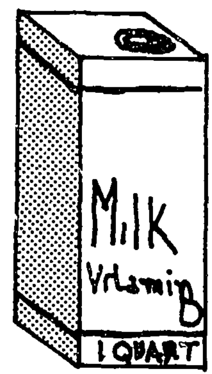
If you did not see another shape, try spinning the rectangle faster.

Look at the drawings below.

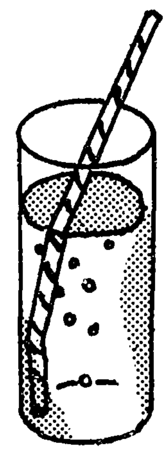
Circle the letters of the drawings that remind you of the shape you saw.



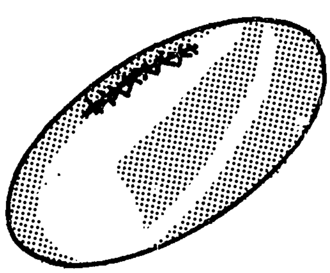
A



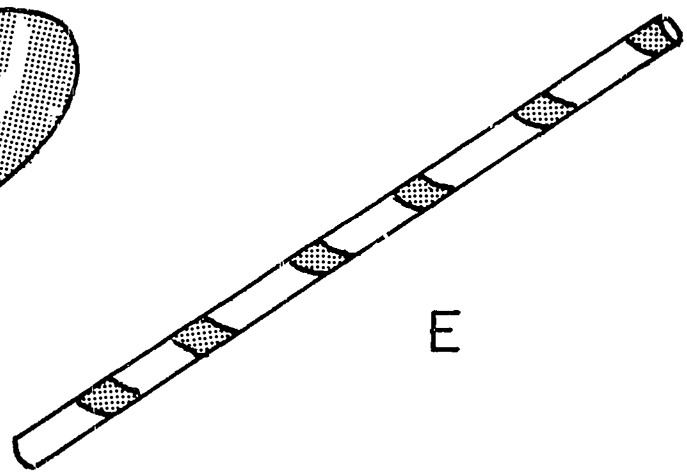
B



C



D



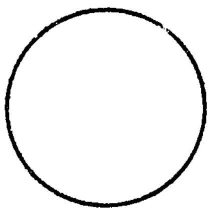
E

You should have circled letters A, C, E.

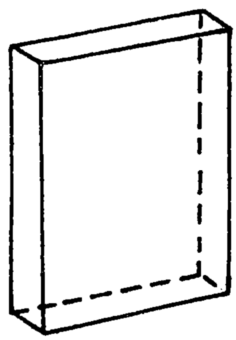
Spin the rectangle again.

Did this shape look like any of these?

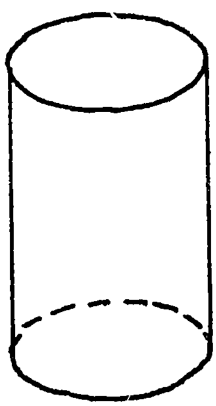
Circle the letter of the drawing that it looked like.



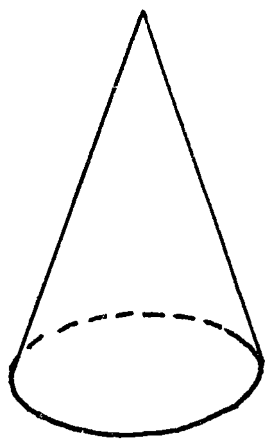
A



B



C



D

You should have circled letter C for page 3.

Shapes of this kind are called cylinders. (SIL-in-ders)

Read the questions below. Circle yes or no for each question.

1.	Is a book shaped like a cylinder?	Yes	No
2.	Is a desk top shaped like a cylinder?	Yes	No
3.	Is a drinking glass shaped like a cylinder?	Yes	No
4.	Is a dinner plate shaped like a cylinder?	Yes	No
5.	Is a tin can shaped like a cylinder?	Yes	No
6.	Is a bicycle wheel shaped like a cylinder?	Yes	No

Your answers for page 4 should be: 1 - no, 2 - no, 3 - yes, 4 - no, 5 - yes, 6 - no.

Look at the rectangle.

Spin it again.

Is the shape of the cylinder you see different from the shape of the rectangle? Circle your answer. Yes No

(Turn this page upside down to see if you are right.)

Yes, the shape of the cylinder is different from the shape of the rectangle.

Shapes like cylinders are called shapes in 3 dimensions.

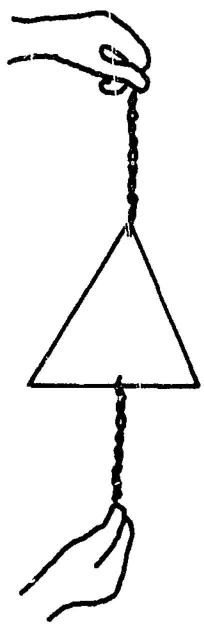
Shapes like rectangles are called shapes in 2 dimensions.

SPINNING TRIANGLES

Pick up the triangle.

1. Be sure one rubber band is in the center of a side.
2. Be sure the other rubber band is on the corner directly opposite.
3. Hold one rubber band in each hand.
4. Pull each rubber band tightly.

   The drawing below should help you.



5. Let a classmate turn the triangle so that the rubber bands are twisted tightly.
6. Watch the triangle spin.

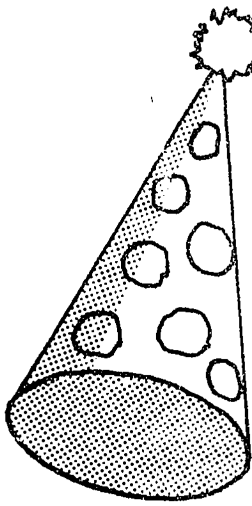
As it spins fast do you still see the triangle? Circle your answer. Yes No

Did you also see another shape? Circle your answer. Yes No

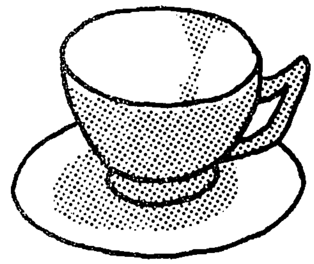
If you did not see another shape, try spinning the triangle faster.

Look at the drawings below.

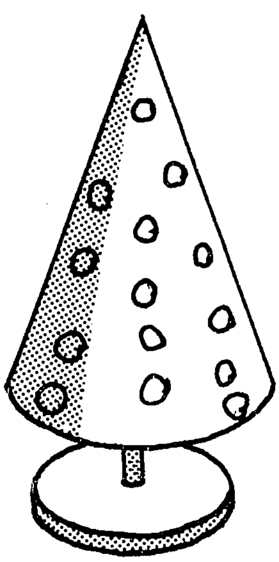
Circle the letters of the drawings that remind you of the shape you saw.



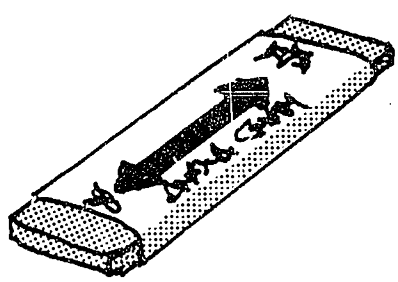
A



B



C



D

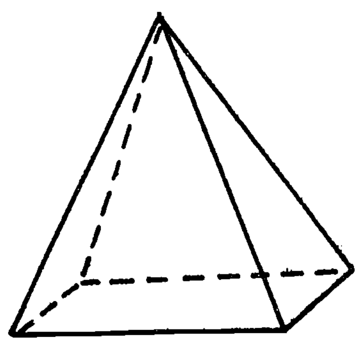
E

Your answers for page 7 should be A, C, E.

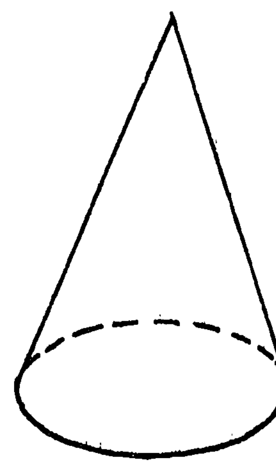
Spin the triangle again.

Did this shape look like any of these?

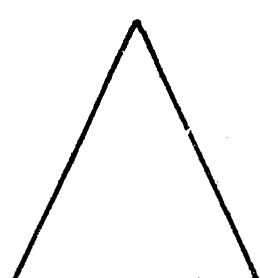
Circle the letter of the drawing that it looked like.



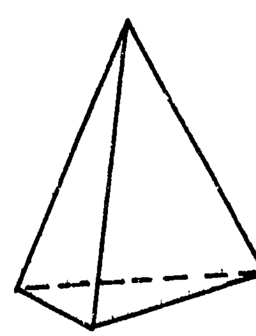
A



B



C



D

Your answer for page 8 should be B.

Shapes of this kind are called <u>cones</u>.

We can remember this by thinking of the shape of an ice cream cone.

Look at the shape of the triangle.

Spin it again.

Is the shape of the triangle different from the shape of the cone?

Circle your answer. Yes No

(Turn this page upside down to see if you are right.)

Yes, the shape of the triangle is different from the shape of the cone.

Shapes like triangles are called shapes in 2 dimensions.

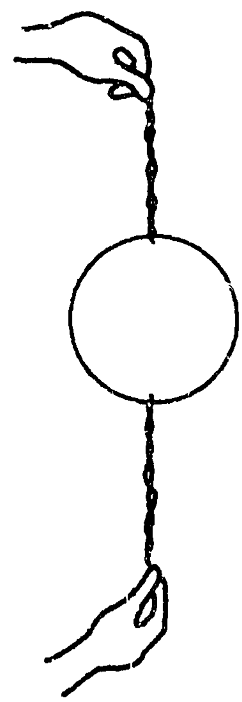
Shapes like cones are called shapes in 3 dimensions.

SPINNING CIRCLES

Pick up the circle.

1. Be sure the rubber bands are attached to opposite sides of the circle.
2. Hold one rubber band in each hand.
3. Pull each rubber band tightly.

   The drawing below should help you.



4. Let a classmate turn the circle so that the rubber bands are tightly twisted.
5. Watch the circle spin.

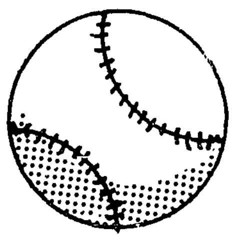
As it spins fast, do you still see the circle? Circle your answer. Yes N

Did you also see another shape? Circle your answer. Yes No

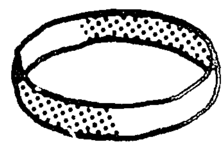
If you did not see another shape, try spinning the circle faster.

Look at the drawings below.

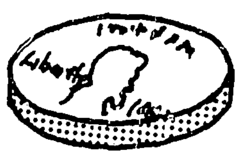
Circle the letters of the drawings that remind you of the shape you saw.



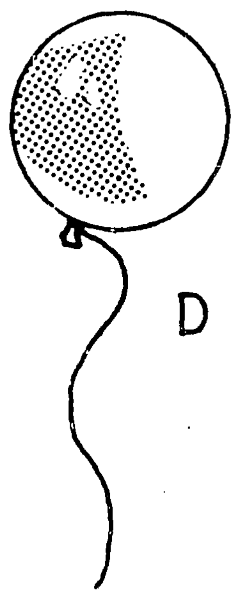
A



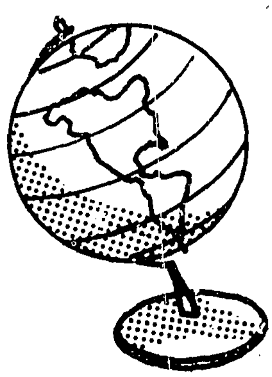
B



C



D



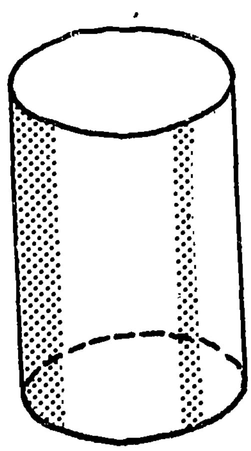
E

Your answers for page 12 are A, D, E.

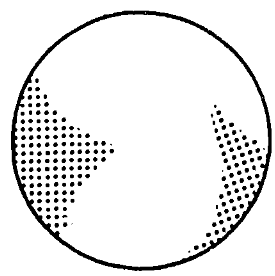
Spin the circle again.

Did this shape look like any of these?

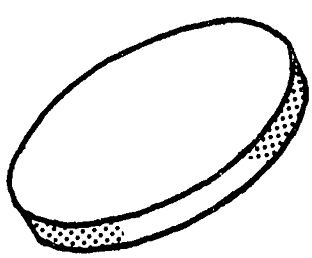
Circle the letter of the drawing that it looked like.



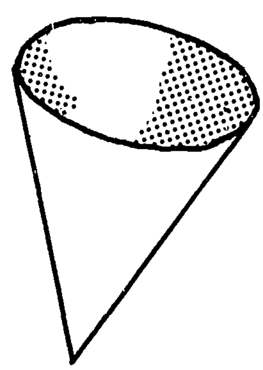
A



B



C



D

Your answer for page 13 should be B.

Shapes of this kind are called <u>spheres</u>.

Read the questions below. Circle yes or no for each question.

1. Is a football in the shape of a sphere? Yes No
2. Is a beach ball in the shape of a sphere? Yes No
3. Is a lemon in the shape of a sphere? Yes No
4. Is a dime in the shape of a sphere? Yes No
5. Is an orange in the shape of a sphere? Yes No

Your answers for page 14 should be 1 - no, 2 - yes, 3 - no, 4 - no, 5 - yes.

Look at the shape of the circle.

Spin it again.

Is the shape of the circle different from the shape of the sphere?

Circle your answer. Yes No

(Turn this page upside down to see if you are right.)

Yes, the shape of the circle is different from the shape of the sphere.

Shapes like circles are called shapes in 2 dimensions.

Shapes like spheres are called shapes in 3 dimensions.

LET'S REVIEW

Look at the drawings below.

What kind of shape does each show?

Write the correct name of each shape under the drawing. You may use the words on the left to help you.

Beside the name of each shape:

1. Write 2 if it is a shape in 2 dimensions.
2. Write 3 if it is a shape in 3 dimensions.

cone

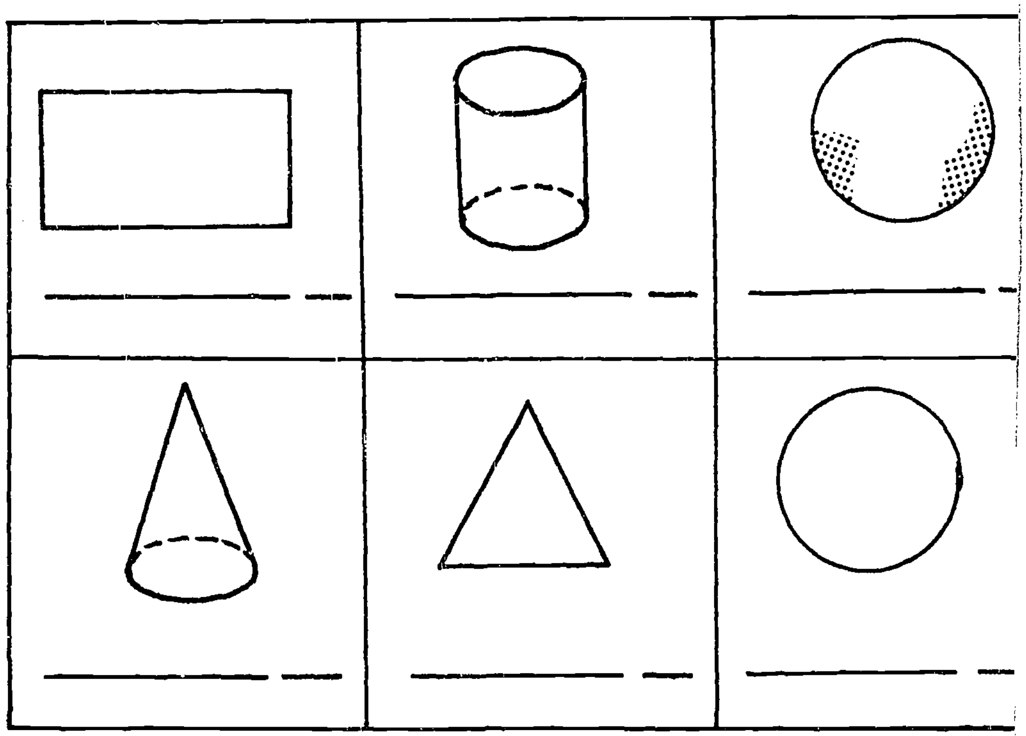
rectangle

circle

sphere

cylinder

triangle



MEASUREMENT

MEASUREMENT

UNIT MEASUREMENT GRADE(S) Seven through Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Area	7	7	7			7				
Square Inch	7	7		7	7	7				
Square Foot	7	7		7	7	7	7			
Square Yard	7	7		7	7	7	7			
Area of Square	7	7	7	7		7	7	7		7
Area of Rectangle	7	7	7	7		7	7	7		7
Area of Triangle	8	8	8		8	8	8			8
Area of Parallelogram	8	8	8		8	8	8			8
Area of Circle	8	8	8		8	8	8			8
Area of Other Polygons			9		9	9	9	9		
Acre	9	9	9		9	9				
Surface Area	9	9	9			9				
Total Surface Area of Cube	9	9			9	9	9			
Lateral Surface Area of Cube	9	9			9	9	9			
Total Surface Area of Box	9	9	9			9	9			
Lateral Surface Area of Box	9	9	9			9	9			
Total Surface Area of Cylinder	9	9	9			9	9			
Lateral Surface Area of Cylinder	9	9	9			9	9			

UNIT MEASUREMENT GRADE(S) Six through Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Inch	6	6	6	6	6	6	6		6	
1/2 Inch	6	6	6	6	6		6		6	
1/4 Inch	7	7	7	7					7	
1/8 Inch	8	8	8							8
1/16 Inch	9	9	9							9
Foot	6	6	6	6	6	6	6		6	
Yard	6	6	6	6	6	6	6		6	8
Meter	8	8	8	8	8				8	8
Centimeter	8	8	8			8	8		8	9
Millimeter	9	9	9			9	9		9	9
Mile	7	7				7	7			
Perimeter	7	7	7			7	7			
Perimeter Square	7	7	7	7		7	7	7		
Perimeter Triangle	7	7	7	7		7	7	7		
Perimeter Rectangle	7	7	7	7		7	7	7		
Perimeter Polygons			8	8		8	8	8		
Circumference	8	8	8			8				
Pi	8	8			8	8	8			

UNIT MEASUREMENT GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Micrometer	10	10	10							
Caliper	10	10	10							
Volume	10	10	10			10				
Cubic Inch	10	10		10	10					
Cubic Foot	10	10		10	10	10	10			
Cubic Yard	10	10		10	10	10	10			
Volume of Cube	10	10	10			10	10	10		
Volume of Rectangular Solid	10	10	10			10	10	10		
Volume of Cylinder	10	10	10			10	10	10		
Angular Measurement	9	9				9				
Degree	9	9			9	9	9	9		
Protractor	9	9	9	9	9	9	9	9		
Central Angle	10	10	10	10		10	10			10
Inscribed Angle	10	10	10	10		10	10			10
Corresponding Angles	10	10		10		10	10	10		
Ounce	6	6	6			6	6	6	6	
Pound	6	6	6			6	6	6	6	
Pint	6	6	6			6	6	6	6	

UNIT MEASUREMENT GRADE(S) Six through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Quart	6	6	6			6	6	6	6	
Fluid Ounce	10	10	10			10	10	10	10	
Gallon	6	6	6			6	6	6	6	
Cup	10	10	10			10	10	10	10	
Degree Fahrinheit	6	6	6			6	6	6	6	
Degree Centigrade	7	7	7			7	7	7	7	
Similar Triangles (Ratio, Proportion)	10	10	10		10	10	10	10		
Scale Drawing			10	10	10	10		10		
Trig Tables	11	11		11	11					
Pythagorean	10	10	10			10	10	10		

UNIT MEASUREMENT GRADE(S) Eight

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
1/8 Inch	8	8	8							8
Yard										8
Meter	8	8	8	8	8				8	8
Centimeter	8	8	8			8	8		8	
Perimeter Polygons			8	8		8	8	8		
Circumference	8	8	8			8				
Pi	8	8			8	8	8			
Area of Triangle	8	8	8		8	8	8			8
Area of Parallelogram	8	8	8		8	8	8			8
Area of Circle	8	8	8		8	8	8			8

$\frac{1}{8}$ Inch

Page

The student should be able to:

1. Name and identify the $\frac{1}{8}$ inch
2. Demonstrate how to measure an object to the nearest eighth inch
3. Distinguish the $\frac{1}{8}$ inch markings on a ruler from $\frac{1}{4}$, $\frac{1}{2}$, and inch markings

Meter

The student should be able to:

1. Name and identify the unit of length--the meter
2. Demonstrate how to measure an object to the nearest meter
3. Construct a drawing or freehand sketch of the unit of length known as the meter using the meter stick
4. Describe the length of a meter by hand motion or familiar objects
5. Order line segments by arranging them into categories of greater than, less than or equal to a meter
6. Distinguish between a meter and a yard

Centimeter

The student should be able to:

1. Name and identify the unit of length--the centimeter
2. Demonstrate how to measure an object to the nearest centimeter
3. State the principle: 100 centimeters = 1 meter
4. Apply the principle for converting from meters to centimeters and centimeters to meters
5. Order line segments by arranging them in categories of greater than, less than, or equal to a centimeter

Perimeter-Irregular Polygons | Page

The student should be able to:

1. Demonstrate how to find the perimeter of polygons by measuring
2. Construct a drawing of a polygon given the perimeter
3. State the principle that the perimeter of a polygon is the sum of the lengths of its sides
4. Apply the principle by finding perimeters of irregular polygons
5. Interpret the principle to find the measure of the missing side or sides when the perimeter and the measures of certain sides are given

Circumference

The student should be able to:

1. Name and identify the circumference of a circle
2. Demonstrate how to find the circumference of a circular object by using a string, tape or other similar procedure
3. State the principle that the circumference is the measure of the distance around the circle

Pi

The student should be able to:

1. Name and identify pi
2. Describe pi as a number
3. State the principles:
   a. Pi is the ratio obtained when comparing circumference and diameter
   b. $c = \pi d$ or $c = 2 \pi r$
4. Apply the principle by finding the circumference of a circle given its diameter or radius

Area of Triangle, Parallelogram, Circle	Page
The student should be able to:	
1. Name and identify the area of a triangle, a parallelogram, and a circle	M-10
2. Demonstrate how to find the area of each figure by using standard units	M-11
3. Describe the area of a triangle, parallelogram, and circle as a number	
4. Describe a procedure for finding area	
5. State the principles: a. Area of a triangle $= \frac{1}{2}bh$; b. Area of a parallelogram $= b \times h$; c. Area of a circle $= \pi r^2$	
6. Apply the principle by finding the areas of these figures when the dimensions are given	
7. Distinguish between the formulas for finding the area of a triangle and the area of a parallelogram, between the area and circumference of a circle, and between the area and perimeter of a triangle and a parallelogram	

NOMOGRAPH
AREA OF A TRIANGLE

Teacher Commentary

I. Unit: Measurement

II. Objectives: The student should be able to:

Demonstrate how to construct the area of a triangle using the nomograph

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribute the materials to each student.

B. Discuss the three scales A, B and C.

1. Scales A and B begin with one and end with ten. Each unit is divided in half.

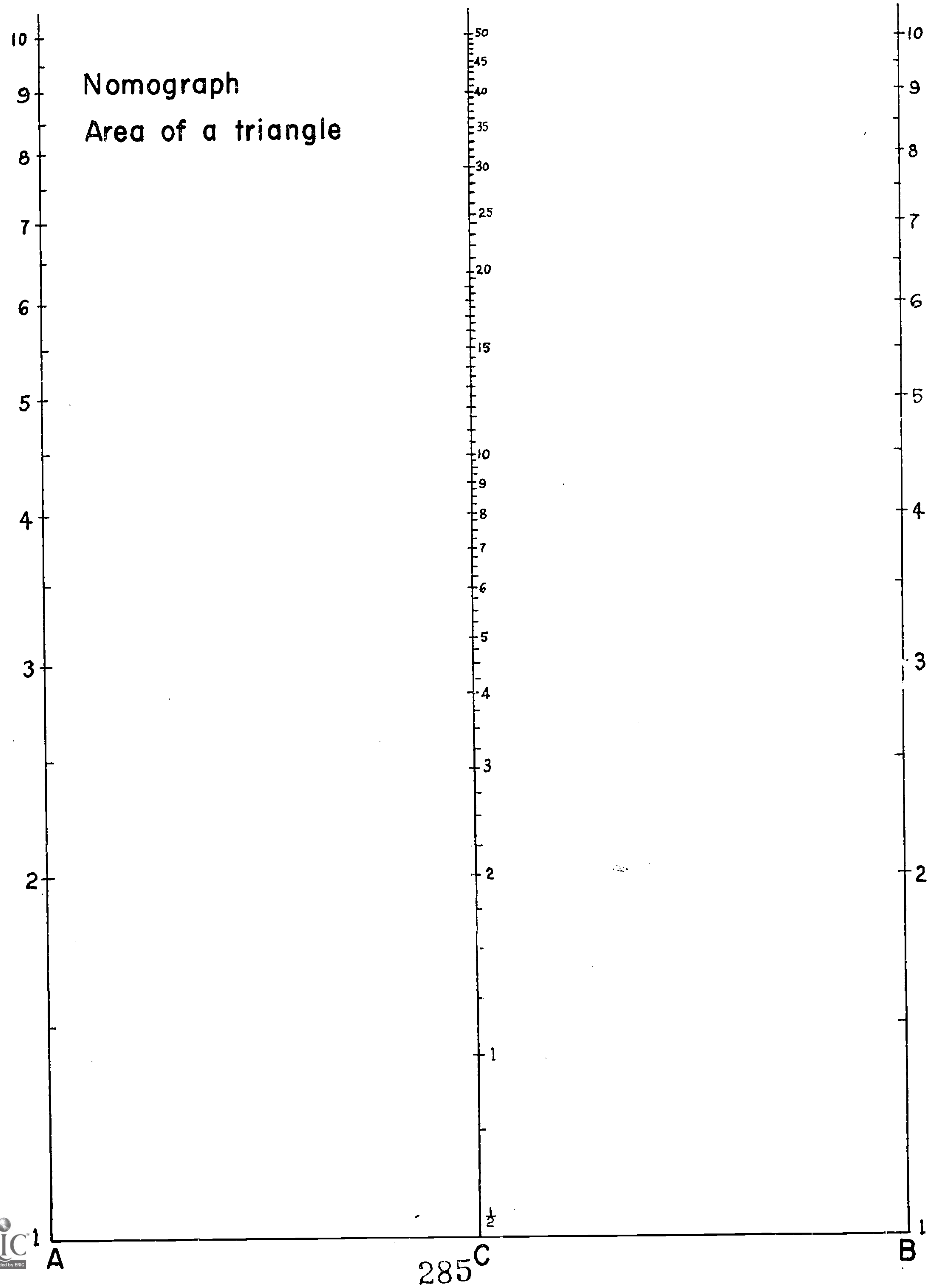
2. The C scale begins with one-half and ends with fifty. The first ten units are divided into fourths. Units ten thru twenty are divided in half.

3. Locate some points on the scales and have the students identify them. On scale C students will have to identify points not designated on the scale.

4. Have the students locate points on the scales. On scale C, make sure that they are able to locate numbers like $10\frac{3}{4}$, $20\frac{1}{4}$, and $40\frac{3}{4}$.

C. In order to find the area of a triangle ($a = 2\frac{1}{2}$, $b = 3$) locate the altitude ($2\frac{1}{2}$) on scale A and the base (3) on scale B. The line joining these two points will cross scale C at a point that represents the area ($3\frac{3}{4}$ sq. units). The student may have to estimate their answer in some problems. ($a = 7\frac{1}{2}$, $b = 8\frac{1}{2}$) The area in this case is $31\frac{7}{8}$ sq. units. The students will probably estimate the answer as $31\frac{1}{2}$ since the space between the unit mark is small.

D. After the students have completed some written exercises, they could use the nomograph to check their results.


Nomograph
Area of a triangle
A
C
B
10
9
8
7
6
5
4
3
2
1
50
45
40
35
30
25
20
15
10
9
8
7
6
5
4
3
2
1
1/2

AREAS OF TRIANGLES AND PARALLELOGRAMS BY SWEEPS

Teacher Commentary

I. Unit: Measurement

II. Objectives: The student should be able to:

A. Name and identify the area of a triangle and a parallelogram

B. Describe a procedure for finding the area

C. State the principles:

1. Area of a triangle $= \frac{1}{2}bh$

2. Area of a parallelogram $= b \times d$

D. Apply the principle by finding the areas of these figures when the dimensions are given

III. Materials:

A. Model of sweep for the triangle

B. Model of the sweep for the parallelogram

C. Colored chalk

IV. Procedure:

A. This material may be used with students who have difficulty understanding area.

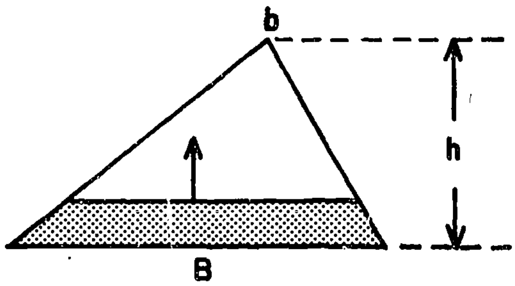
B. Explain to the students that a sweep is a segment which moves across the interior region of a plane figure covering the entire region.

1. Demonstrate this idea by using the window shade as the sweep covering the window.

2. Notice that there is a perpendicular relationship between the sweep and its path across the region. The string on the windowshade may be used to illustrate this idea.

C. If you have a sweep model, show the students the sweep for the triangle and have them observe it as it moves across the region.

D. Discuss the following using a diagram drawn on the board.



1. In the illustrated triangle, b and B represent the measures of their respective sides, b having a measure of zero
2. The distance through which a sweep must move to cover the interior region of the triangle is represented by h.
3. Since the length of the sweep must change uniformly to remain on the triangle, an average length could be computed. Width of sweep is $\frac{1}{2}$ (B + b) = $\frac{1}{2}$ (B + O) = $\frac{1}{2}$ B.

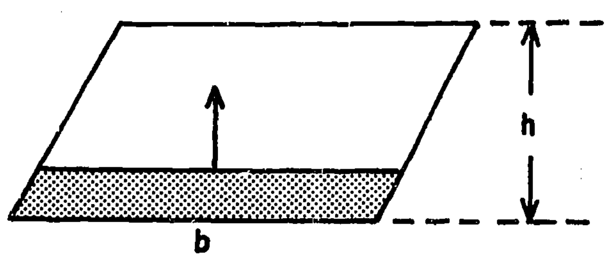
E. Help students to discover the formula for finding the area of a triangle $\frac{1}{2}$ bh--the width of the average sweep and the distance the sweep must move to cover the interior region.

F. Using other triangles drawn on the board, have the students find the area of these and explain how they found it in terms of sweeps. Colored chalk may be helpful in this explanation.

G. Review what the students have learned about sweeps and finding the area of a triangle.

H. If you have a sweep model, show the students the sweep for the parallelogram and have them observe it as it moves across the region.

I. Discuss the following using a diagram drawn on the board.



1. In the illustrated parallelogram the base has a measure represented by b units, and h represents the distance through which a sweep must move to cover the interior region of the parallelogram.

2. The width of the sweep remains constant.

3. The area of the parallelogram is the product of the width of the sweep and the distance the sweep must move to cover the interior--bh.

J. Using other parallelograms drawn on the board, have the students find the areas of these and explain how they found them in terms of sweeps. Colored chalk may be helpful.

K. Summarize the meaning of area by using the models of sweeps or by drawing diagrams on the board. State the formulas using sweeps to explain how the formula was derived.

L. Find the area of parallelograms and triangles, having students demonstrate their procedures on the board.

GRAPHING

GRAPHING

UNIT GRAPHING GRADE(S) Seven through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Pictograph	7	7		7	7		7	7		
Bar Graph	7	7		7	7		7	7		
Line Graph	7	7		7	7		7	7		
Circle Graph	7	7			7		7	7		
Number Line	8	8		8	8	8	8			
Graphing Equalities	8	8		8		8				
Cartesian Products	8	8		8						
Graphing Ordered Pairs in First Quadrant	8	8	8	8	8					8
Coordinate Axes	9	9	9	9	9					9
Graphing Ordered Pairs	9	9	9	9	9	9	9			
Quadrants	9	9			9	9	9			
Table of Values	9	9	9	9	9			10		
Graphing Formulas	9	9		9	9			10		
Graphing Linear Equations and Inequations	10	10		10	10					10
Slope	10	10			10	10	10			
Graphing Simultaneous Equations	11	11		11	11	11	11			
Graphing Quadratic Equations and Inequations	11	11		11	11					11

UNIT GRAPHING GRADE(S) Eight

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Number Line	8	8		8	8	8	8			
Graphing Equalities	8	8		8		8				
Coordinate Axes (Positive Numbers and Zero)	8	8	8	8	8					8
Graphing Ordered Pairs in Quadrant I	8	8	8	8	8	8	8			8
Table of Values	8	8	8	8	8					

Page

Number Line

The student should be able to:

1. Name and identify a number line
2. Construct a number line
3. Describe a number line as a line such that each point on the line is assigned a number
4. State the principle that given two numbers, the point representing the larger number is to the right of the point representing the smaller number on the number line
5. Apply the principle by plotting points

Graphing Equalities and Inequalities

The student should be able to:

1. Name and identify the graph of an equality and an inequality
2. Construct the graphs of equalities and inequalities on the number line
3. State the principle that the graph of an equality or an inequality is a pictorial representation of a mathematical sentence

Coordinate Axes (using only positive numbers and zero)

The student should be able to:

1. Name and identify a pair of axes
2. Demonstrate how to draw and label coordinate axes
3. Construct a pair of coordinate axes using a straight-edge or freehand sketch
4. Describe coordinate axes as a vertical line and a horizontal line which intersects at a point called the origin or by specific example
5. Distinguish coordinate axes from a number line

Graphing Ordered Pairs (use only positive numbers and zero)

Page

The student should be able to:

1. Demonstrate how to plot points in the first quadrant
2. Distinguish between ordered pairs such as (a, b) and (b, a)
3. Name and identify the coordinates given a point in the first quadrant
4. Construct an ordered pair given a point in the first quadrant
5. Describe the procedure for plotting points in the first quadrant
6. State the principle that the first number in the ordered pair indicates the number of units to go to the right and the second number indicates the number of units to go up
7. Apply the principle by plotting points to make a graph
8. Construct a graph given a set of ordered pairs
9. Name and identify graphs of ordered pairs

Table of Values

The student should be able to:

1. Name and identify a table of values
2. Construct a table of values from a given table or mathematical sentence or rule
3. Construct a set of coordinates from a table of values
4. Describe a table of values as an organized way to represent ordered pairs in a chart
5. Demonstrate how to construct a table of values

PROBABILITY AND STATISTICS

PROBABILITY AND STATISTICS

UNIT PROBABILITY GRADE(S) Eight through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Event	8	8	8							
Certainty - Uncertainty	8	8		8	8					
Sample Spaces - Ordered Arrangements	8	8		8	8					8
Equally and Unequally Likely Outcomes	8	8	8			8	8			8
Probability	8	8				10				
Ordered Arrangements (Permutations)	9	9		9	9	9				
Factorial	9	9		9	9		9			
Permutations of N things taken N at a time	9	9		9		9	9			
Permutations of N things taken R at a time	9	9		9		9	9			9
Tree Diagram	9	9		9	9		9			
Box Diagram	9	9		9	9		9			
Decreasing and Increasing Probability	10	10		10		10	10			
Probability of 0 and 1	10	10		10		10	10			
Independent and Dependent Events	10	10		10	10	10	10	10		10
Complementary Events	10	10		10		10	10			
Experimental Probability	10	10	10		10					
Theoretical Probability	10	10			10					10
Sample	11	11	11		11		11	11		11

UNIT PROBABILITY GRADE(S) Eight through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Circular Permutation	11	11		11	11	11	11			
Combination	11	11		11	11					11
Combinations of N things taken N at a time	11	11		11		11				
Combinations of N things taken R at a time	11	11		11	11	11	11			
Preduction	11	11		11	11	11	11			
Chances (Odds)	11	11		11		11	11			11

UNIT STATISTICS GRADE(S) Nine through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Data	9				9				9	
Range	9	9	9		9	9	9			9
Rank	9	9	9		9		9	9		9
Interval	9	9	9			9	9			9
Tally	9	9	9		9		9			
Frequency	9	9	9	9	9					9
Frequency Table	9	9		9	9	9		9		9
Mean	9	9				9	9	9		9
Median	9	9				9	9	9		9
Mode	9	9				9	9	9		9
Pictograph	9	9						9		
Circle Graph	9	9		9				9		
Bar Graph	9	9		9				9		
Line Graph	9	9		9				9		
Normal Curve	10	10		10	10			10		
Pascal's Triangle	10	10	10	10			10	10		
Percentile	10	10		10	10	10	10	10		10
Correlation	11	11			11		11	11		
Scattergram	11	11		11	11	11	11			11

UNIT PROBABILITY GRADE(S) Eight

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Event	8	8	8							
Certainty - Uncertainty	8	8		8	8					
Sample Spaces - Ordered Arrangements	8	8		8	8					8
Equally and Unequally Likely Outcomes	8	8	8			8	8			8
Probability	8	8								

Page

Event

The student should be able to:

1. Name and identify an event
2. Demonstrate that the event is the actual happening

Certainty and Uncertainty

The student should be able to:

1. Name and identify events which are certain and uncertain
2. Construct examples of certain and uncertain events
3. Describe a certain event as one that will happen in all cases
4. Describe an uncertain event as one that may or may not happen

Sample Spaces

The student should be able to:

1. Name and identify sample spaces
2. Construct sample spaces
3. Describe sample spaces as ordered arr gements, listing all possible outcomes
4. Distinguish sample spaces from events

Equally Likely and Unequally Likely Outcomes

The student should be able to:

1. Name and identify equally likely and unequally likely outcomes
2. Demonstrate equally likely and unequally likely outcomes by using familiar objects
3. State the principle that some outcomes are more likely to occur than others

Page

4. Apply the principle by stating the likelihood of outcomes when given events

5. Distinguish between outcome and events

Probability

The student should be able to:

1. Name and identify probability

ALGEBRA

ALGEBRA

I. Master Chart - Grades Six through Eleven

II. Grade Eight Chart

III. Behavioral Objectives

IV. Activities

UNIT ALGEBRA GRADE(S) Six through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Set	6, 8	6, 8		6	6					6
Member	6, 7	6, 7			6					6
Types of Sets	6, 7	6, 7		6	6, 7					6, 7
Relationship Between Sets	6, 8, 9	6, 8, 9		6, 8	6, 8					6, 8
Methods of Describing Sets	6	6			6					6
Operations With Sets	6	6			6					6
Language of Algebra	6, 7, 8 9, 10	6, 7, 8, 9 10		9	7, 8, 9	7			9	6, 7, 8, 9
Symbols for Operations	6, 7, 8	6, 7, 8, 9	6	8, 9	9					6, 7, 8
Symbols for Grouping	6, 10	6, 10	6, 7	10		6, 7, 9, 10	6, 7, 9, 10		10	6, 10
Evaluating Algebraic Expressions			9, 10, 11	10, 11	9, 10, 11	9, 10, 11	9, 10, 11			
Number Line	8, 9	8, 9	9	8, 9		9	9		8, 9	9
Operations With Rationals	9, 10	9, 10	10	9	9	9, 10	9, 10			9
Similar Terms	10, 11	10, 11	10, 11		10, 11	10, 11	10, 11			10, 11
Open Sentences With One Operation			9	9		9	9			
Open Sentences With Combined Operations	9, 11	9, 11	9, 11		9, 11					
Inequalities With One Operation	9	9	9		9					9
Inequalities With Combined Operations	11	11			11	11	11			11
Rationals	9	9		9	9				9	9

UNIT ALGEBRA GRADE(S) Eight

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Sets	8	8								
Equal Sets	8	8		8	8					8
Equivalent Sets	8	8		8	8					8
Overlapping Sets	8	8		8	8					8
Disjoint Sets	8	8		8	8					8
Variable	8	8			8					
Solution Set	8	8			8					
Open Sentence	8	8			8					8
Closed Sentence	8	8			8					8
Symbol of Multiplication	8	8		8						8
Corresp. of Points on No. Line to Positive Rationals and Zero	8	8		8					8	
Exponent Greater Than Two	8	8			8					8

Page

Set

The student should be able to:

1. Name and identify a set using capital letters

Equal Sets

The student should be able to:

1. Name and identify equal sets
2. Construct equal sets
3. Describe equal sets as sets having exactly the same elements
4. Distinguish between equal and equivalent sets

Equivalent Sets

The student should be able to:

1. Name and identify equivalent sets
2. Construct equivalent sets
3. Describe equivalent sets as sets having the same number of elements
4. Distinguish between equivalent and equal sets

Overlapping Sets

The student should be able to:

1. Name and identify overlapping sets
2. Construct overlapping sets using Venn diagrams
3. Describe overlapping sets as sets having at least one element in common
4. Distinguish between overlapping and disjoint sets

Disjoint Sets

The student should be able to:

1. Name and identify disjoint sets
2. Construct disjoint sets using Venn diagrams
3. Describe disjoint sets as sets having no elements in common
4. Distinguish between disjoint and overlapping sets

Variable | Page

The student should be able to:

1. Name and identify a variable
2. Describe a variable as a symbol for which a number may be substituted

Solution Set

The student should be able to:

1. Name and identify a solution set
2. Describe a solution set as the set of answers that make a number sentence true

Open Sentence

The student should be able to:

1. Name and identify an open sentence
2. Describe an open sentence as one in which the truth value cannot be determined until a replacement from the universal set is made for the variable in the sentence
3. Distinguish open sentences from closed sentences

Closed Sentence

The student should be able to:

1. Name and identify closed sentence
2. Describe a closed sentence as one in which the truth value can be determined
3. Distinguish between open and closed sentences

Symbol for Multiplication

The student should be able to:

1. Name and identify parentheses as a symbol which may be used for multiplication
2. Construct examples using parentheses as multiplication symbols
3. Distinguish between parentheses as symbol for grouping and symbol for multiplication

Correspondence of Points on Number Line to Positive Rationals and Zero | Page

The student should be able to:

1. Name and identify points on a number line corresponding to positive rationals and zero
2. Construct a number line whose points represent the positive rationals and zero
3. Order positive rationals and zero on the number line

Exponents Greater Than 2

The student should be able to:

1. Name and identify exponents greater than two
2. Describe the exponent as the symbol, placed above and to the right of the number, which tells how many times the base is used as a factor
3. Distinguish exponent from factor

LOGIC

LOGIC

I. Master Chart - Grades Six through Eleven

UNIT LOGIC GRADE(S) Ten and Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Equivalent Phrases and Sentences	10	10		10						8, 10
Assumptions	10	10		11	10					10
"If - Then" Statements	10, 11	10, 11		10				10		10, 11
Converse	11	11		11	11	11		11		11
Inverse	11	11		11	11	11		11		11
Circular Reasoning (Doubletalk)										11
Valid Arguments					10, 11					10, 11
Non-Valid Arguments					10, 11					10, 11
Syllogism (Chain of Logic)										11
Indirect Proof						11	11			
Venn Diagrams	10	10						10		
Open Sentences	8, 10	8, 10		8, 10	8, 10					8, 10
Closed Sentences	8, 10	8, 10		8, 10	8, 10					8, 10
Deductive Reasoning					11					11
Inductive Reasoning					11					11
Counterexample				10	10	10	10			

RECREATIONAL ACTIVITIES

RECREATIONAL ACTIVITIES

The units contained in this section are supplementary activities for recreation. These may be used to provide variety throughout the year as well as in the daily lesson.

FOUR OPERATIONS WITH WHOLE NUMBERS
CROSS NUMBER PUZZLE

Teacher Commentary

A Recreational Activity on the Four Operations With Whole Numbers

I. Materials: Student work sheet

II. Procedure:

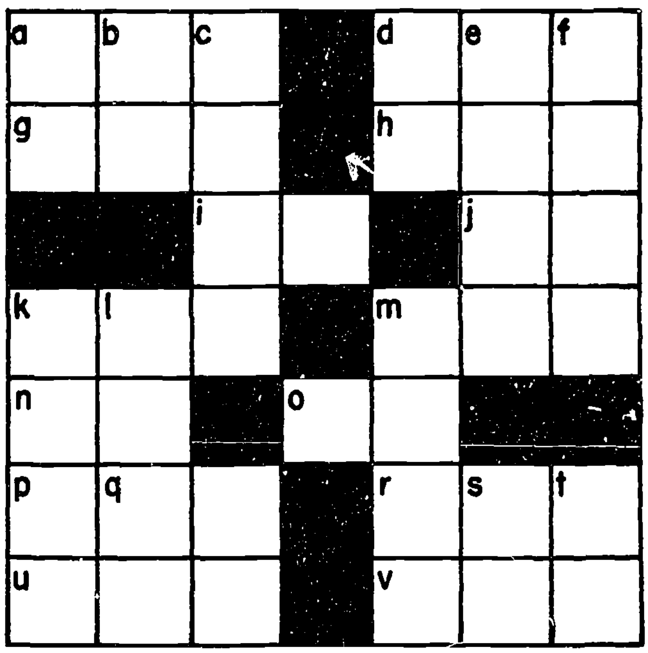
A. Distribute copies of the attached work sheet.

B. Solution:

a 5	b 7	c 9		d 9	e 4	f 2
g 4	6	2		h 1	4	3
		i 9	0		j 1	7
k 9	l 6	6		m 2	7	8
n 5	7		o 1	8		
p 8	q 1	9		r 9	s 4	t 9
u 2	0	6		v 2	2	8

WHOLE NUMBERS CROSS NUMBER PUZZLE

Fill in the blanks.



ACROSS

a. 340 + 90 + 37 + 80 + 32

d. 140 + 436 + 220 + 146

g. 77 x 6

h. 572 ÷ 4

i. 439 - 349

j. 204 ÷ 12

k. 42 x 33

m. 732 - 454

n. 8680 - 8623

o. 198 ÷ 11

p. 13 x 63

r. 39456 - 38507

u. 2472 ÷ 12

v. 6 x 38

DOWN

a. 432 ÷ 8

b. 1520 ÷ 20

c. 8 x 1162

d. 2000 - 1909

e. 39753 ÷ 9

f. 400 + 325 + 843 + 800

k. 6 x 1597

l. 55 x 122

m. 725 + 420 + 927 + 820

q. 752 - 656

s. 7 + 12 + 15 + 8

t. 784 ÷ 8

THE CRAZY CRICKET

Teacher Commentary

A Recreational Activity on Multiplication and Division Facts

I. Materials :

A. Number line calibrated by ones from 0-100

B. Student work sheets

C. Scissors

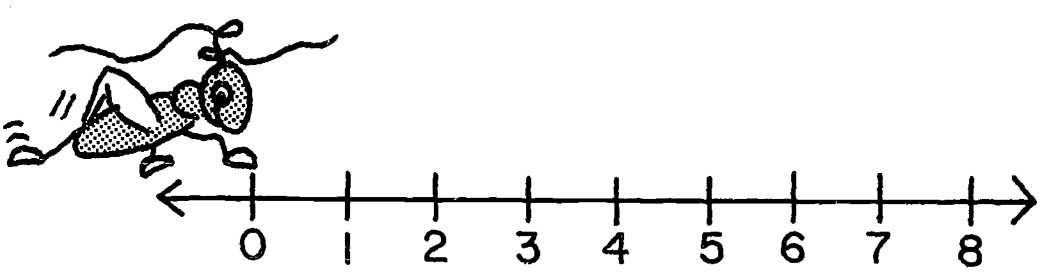
D. Tape

II. Procedure:

A. The student work sheets are designed as drill activities to be completed in five consecutive class periods. The initial activity, Number 6 Crazy Cricket, will probably take a longer period of time than the others because of the additional time needed to teach the student how to use the work sheet and the number line.

B. Distribute the student number lines and have each student cut them out and tape them together.

C. Discuss Crazy Cricket--how he jumps forward and backward, and why he is called crazy. A number line on the board will be helpful in your discussion.



D. Working with the class or a group, do the following exercises orally. Check each exercise by using the number line. This will help the students in thinking in terms of the number line. It will serve to establish the procedure to be used in doing the work sheets independently.

1. Crazy C ̣ ̣ket starts at 0. Makes one jump of three spaces to the right. Where will he land?
2. Crazy Cricket starts at 0. Jumps to the right three spaces at a time. Lands on 24. How many jumps has he made?
3. Crazy Cricket starts at 2 and continues to jump to the right three spaces at a time.
   a. Does he land on 30?
   b. Does he land on 31?
   c. Does he land on 32?
   d. On what number does he land before 32?
4. As long as Crazy Cricket jumps three spaces at a time to the right, let's say he is a positive 3 Crazy Cricket.
5. Let's figure out what Crazy Cricket is doing now?
   a. Crazy Cricket starts at 0.
   b. Makes two equal jumps to 10.
   c. What was the size of each jump?
   d. In what direction did he jump? Now he is a positive 5 Crazy Cricket.
6. Let's see how Crazy Cricket travels this time.
   a. He begins at 26.
   b. Makes five equal jumps.
   c. Lands on 1.
   d. What is the size of each jump?
   e. Why did he land at 1?
   f. In what direction did he travel?
   g. If he travels to the left five spaces at a time, let's call him a negative 5 Crazy Cricket.

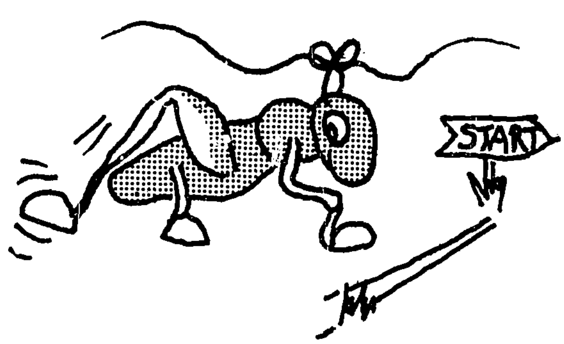
E. When students understand the procedure to be used in Crazy Cricket's travels back and forth on the number line, the first work sheet may be done. Each succeeding work sheet may be completed in successive days as drill activities. The work sheets are as follows:

Work Sheet 1 - "Number 6 Crazy Cricket"
Work Sheet 2 - "Number 7 Crazy Cricket"
Work Sheet 3 - "Number 8 Crazy Cricket"
Work Sheet 4 - "Number 9 Crazy Cricket"
Work Sheet 5 - "Really Crazy Cricket"

Teacher copies of each work sheet are included.

F. Work sheet 5 may be considered an assessment of the students' knowledge of the more difficult facts.

G. Exercises of this type may be used when working with fractional numbers or decimals.



Work Sheet 1

Teacher Copy

NUMBER 6 CRAZY CRICKET

	Direction	Start	Number of Jumps	Land
1.	+6	0	4	24
2.	+6	0	6	36
3.	+6	0	7	42
4.	+6	0	7	42
5.	+6	0	8	48
6.	-6	54	9	0
7.	-6	36	5	6
8.	-6	36	6	0
9.	+6	6	7	48
10.	+6	6	8	54
11.	+6	6	4	30
12.	-6	45	7	3
13.	-6	60	9	6
14.	-6	54	8	6
15.	-6	48	6	12

Work Sheet 2

Teacher Copy

NUMBER 7 CRAZY CRICKET

	Direction	Start	Number of Jumps	Land
1.	+7	0	5	35
2.	+7	0	6	42
3.	+7	0	8	56
4.	+7	0	7	49
5.	-7	63	9	0
6.	-7	63	8	7
7.	-7	49	6	7
8.	-7	56	8	0
9.	+7	7	6	49
10.	+7	7	9	70
11.	+7	14	2	0
12.	-7	65	9	2
13.	+7	21	7	70
14.	-7	35	3	14
15.	-7	77	9	14

Work Sheet 3

Teacher Copy

NUMBER 8 CRAZY CRICKET

	Direction	Start	Number of Jumps	Land
1.	+8	0	4	32
2.	+8	0	6	48
3.	+8	0	5	40
4.	+8	0	8	64
5.	-8	72	9	0
6.	-8	32	4	0
7.	-8	48	6	0
8.	-8	80	9	8
9.	+8	0	7	56
10.	+8	0	8	64
11.	+8	8	4	40
12.	+8	24	4	56
13.	-8	56	6	8
14.	-8	40	4	8
15.	-8	72	6	24

Work Sheet 4

Teacher Copy

NUMBER 9 CRAZY CRICKET

	Direction	Start	Number of Jumps	Land
1.	+9	0	4	36
2.	+9	0	6	54
3.	+9	0	7	63
4.	+9	0	3	27
5.	-9	63	5	18
6.	-9	72	8	0
7.	-9	72	6	18
8.	-9	45	5	0
9.	+9	0	7	63
10.	+9	9	6	63
11.	+9	9	4	45
12.	+9	18	2	36
13.	+9	0	8	72
14.	+9	0	9	81
15.	-9	90	9	9

Work Sheet 5

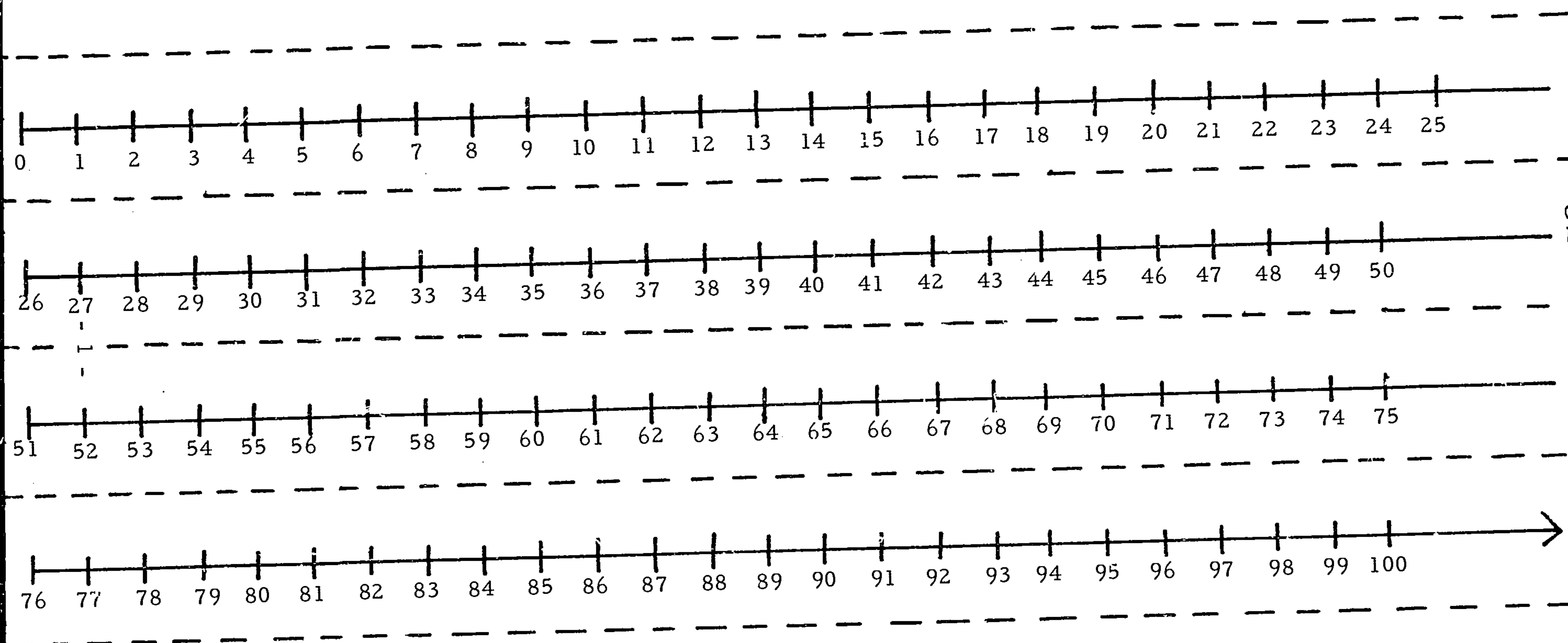
Teacher Copy

REALLY CRAZY CRICKET

	Direction	Start	Number of Jumps	Land
1.	+6	0	7	42
2.	+7	0	6	42
3.	+6	0	9	54
4.	-8	64	8	0
5.	-9	63	7	0
6.	-7	49	7	0
7.	+7	0	8	56
8.	+8	0	9	72
9.	+9	0	6	54
10.	-8	48	5	8
11.	-9	81	9	0
12.	+7	0	8	56
13.	+9	0	8	72
14.	+8	8	7	64
15.	-9	99	9	18

CRAZY CRICKET NUMBER LINE

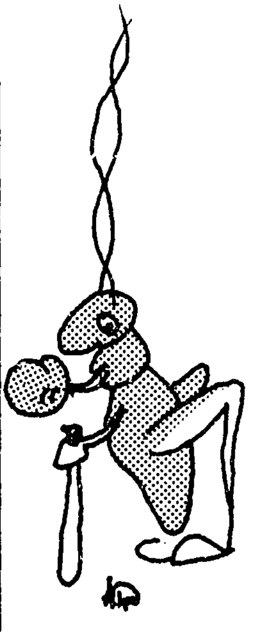
Cut along the dotted line and place the four number lines together end-to-end to form a number line from 0 to 100.



Work Sheet 1

NUMBER 6 CRAZY CRICKET

	Direction	Start	Number of Jumps	Land
1.	+6	0	4	
2.	+6	0	6	
3.	+6	0	7	
4.	+6	0		42
5.	+6		8	48
6.	-6	54		0
7.	-6	36	5	
8.	-6	36		0
9.	+6		7	48
10.	+6	6		54
11.	+6	6	4	
12.	-6	45		3
13.	-6	60		6
14.	-6	54		6
15.	-6	48		12



Directions: Use the number line to fill in the blanks.

+6 means Crazy Cricket jumps to the right 6 spaces at a time

-6 means Crazy Cricket jumps to the left 6 spaces at a time

NUMBER 7 CRAZY CRICKET

	Direction	Start	Number of Jumps	Land
1.	+7	0	5	
2.	+7	0	6	
3.	+7	0	8	
4.	+7		7	49
5.	-7	63		0
6.	-7	63		7
7.	-7	49	6	
8.	-7	56		0
9.	+7	7		49
10.	+7	7		70
11.	+7	14	2	
12.	-7	65		2
13.	+7	21		70
14.	-7	35		14
15.	-7	77		14

Directions: Use the number line to fill in the blanks.

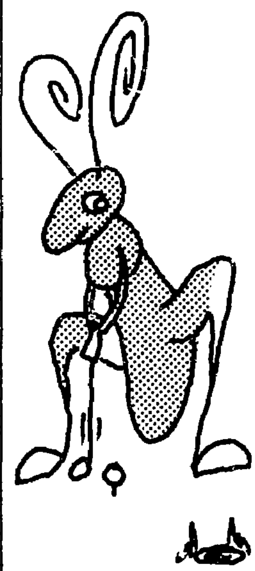
+7 means Crazy Cricket jumps to the right 7 spaces at a time

-7 means Crazy Cricket jumps to the left 7 spaces at a time

Work Sheet 3

NUMBER 8 CRAZY CRICKET

	Direction	Start	Number of Jumps	Land
1.	+8	0	4	
2.	+8	0	6	
3.	+8	0	5	
4.	+8	0	8	
5.	-8	72		0
6.	-8	32		0
7.	-8	48		0
8.	-8	80		8
9.	+8		7	56
10.	+8	0		64
11.	+8	8	4	
12.	+8	24		56
13.	-8	56		8
14.	-8	40	4	
15.	-8	72	6	

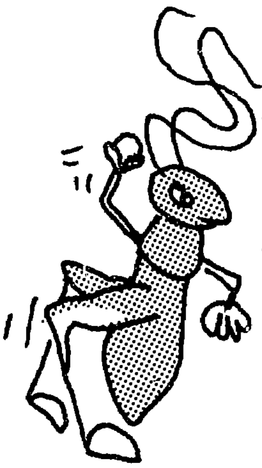


Directions: Use the number line to fill in the blanks.

+8 means Crazy Cricket jumps to the right 8 spaces at a time

-8 means Crazy Cricket jumps to the left 8 spaces at a time

NUMBER 9 CRAZY CRICKET



	Direction	Start	Number of Jumps	Land
1.	+9	0	4	
2.	+9	0	6	
3.	+9	0	7	
4.	+9	0		27
5.	-9	63		18
6.	-9	72	8	
7.	-9	72		18
8.	-9	45		0
9.	+9	0		63
10.	+9	9		63
11.	+9	9		45
12.	+9	18		36
13.	+9	0		72
14.	+9	0	9	
15.	-9	90		9

Directions: Use the number line to fill in the blanks.

+9 means Crazy Cricket jumps to the right 9 spaces at a time

-9 means Crazy Cricket jumps to the left 9 spaces at a time

Work Sheet 5

REALLY CRAZY CRICKET

	Direction	Start	Number of Jumps	Land
1.	+6	0		42
2.	+7	0		42
3.	+6	0		54
4.	-8	64		0
5.	-9	63	7	
6.	-7	49		0
7.	+7	0	8	
8.	+8	0	9	
9.	+9		6	54
10.	-8	48		8
11.	-9	81	9	
12.	+7	0		56
13.	+9	0		72
14.	+8	8		64
15.	-9	99		18

Directions: Use the number line to fill in the blanks.

+ means jumps to the right the given number of spaces at a time

- means jumps to the left the given number of spaces at a time

RUSSIAN PEASANT METHOD OF MULTIPLICATION

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

A. Construct the product of two whole numbers

B. Demonstrate how to find products of whole numbers by using the Russian peasant method of multiplication

III. Materials: Work sheet entitled, "Russian Peasant Method of Multiplication"

IV. Procedure:

The process of doubling and halving is a method used by Russian peasants to multiply any pair of numbers. A person can learn to multiply if he knows how to add, how to multiply by 2, and how to divide by 2.

Example 1 $\boxed{23 \times 18 = 414}$

A Russian sheep herder sold 23 sheep at $18 each. How did he figure out how much money he should get for his sheep? This is what he did:

1. Draw 2 columns and label them "Double" and "Half."

Double	Half

2. Place one factor in each column. It does not matter which factor is placed in which column. However, placing the smaller factor in the half column will reduce the number of steps.

Double	Half
23	18

3. Double the number in the "Double" column and halve the number in the "Half" column.

Double	Half
23	18
46	9

4. Continue the process until the number in the "Half" column is 1. *See explanatory note.

Double	Half
23	18
46	9
92	4
184	2
368	1

5. If results in the "Half" column are even, cross through them and their doubles. Otherwise keep the results.

Double	Half
23	18
46	9
92	4
184	2
368	1

6. The product is the sum of those numbers which are left in the "Double" column.

Double	Half
23	18
46	9
92	4
184	2
+368	1
414	

7. Thus 23 x 18 = 414

*Explanatory Note:

When halving, always drop the remainder. For example: $\frac{1}{2} \times 9 = 4\frac{1}{2}$

Dropping the $\frac{1}{2}$ we get 4.

Example 2

$\boxed{56 \times 14 = 784}$

Double	Half
56	14
112	7
224	3
+ 448	1
784	

Example 3

$\boxed{237 \times 444 = 105,228}$

Have students discover that this method is not so difficult with 2 digit numbers, but becomes extremely involved when multiplying larger numbers.

Double	Half
444	237
888	118
1776	59
3552	29
7104	14
14208	7
28416	3
+ 56832	1
105, 228	

Suggested Assessment Procedures:

Distribute work sheet entitled, "The Russian Peasant Method of Multiplication" or have students demonstrate how to multiply using this method at the chalkboard.

Solutions to the work sheet:

1. 72
2. 294
3. 486
4. 726
5. 12, 361
6. 875, 776

THE RUSSIAN PEASANT METHOD OF MULTIPLICATION

Multiply the following numbers by the Russian Method. You can check your work by the regular method of multiplication. Place your answer in the box.

1. 9 x 8 = ☐

Double	Half

2. 42 x 7 = ☐

Double	Half

3. 27 x 18 = ☐

Double	Half

4. 33 x 22 = ☐

Double	Half

5. 263 x 47 = ☐

Double	Half

6. 256 x 3421 = ☐

Double	Half

MAGIC TRIANGLES

Teacher Commentary

A Recreational Unit on Addition of Whole Numbers

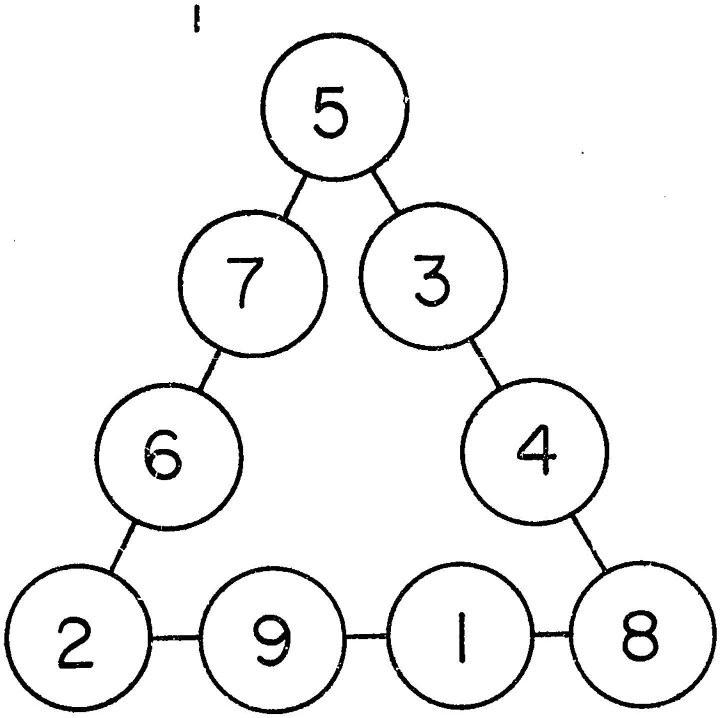
I. Materials: Dittoed sheet with puzzles

II. Procedure:

Puzzles may be used as a recreational activity at the end of the period.

<u>Solution of Magic Triangles</u>

1



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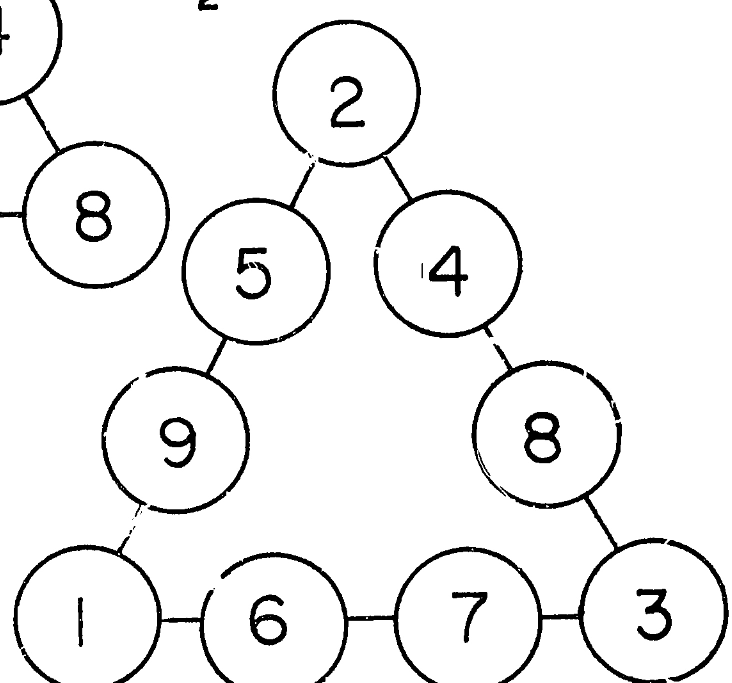
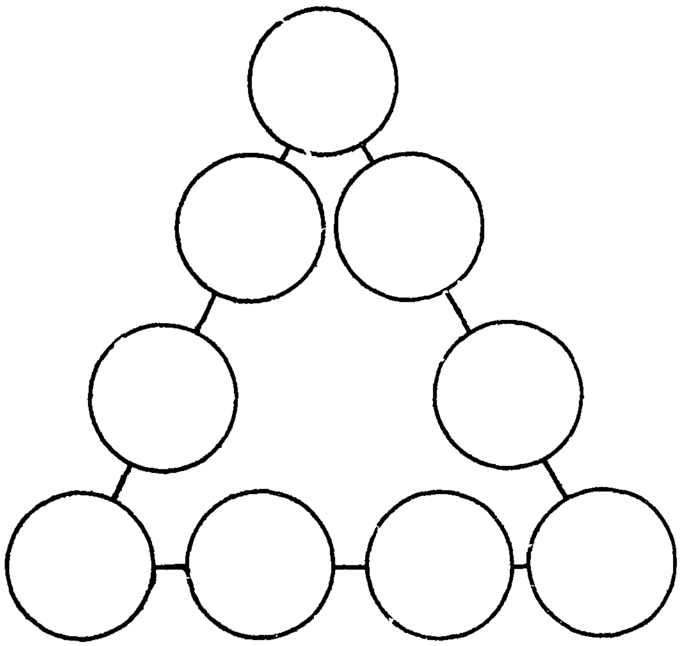
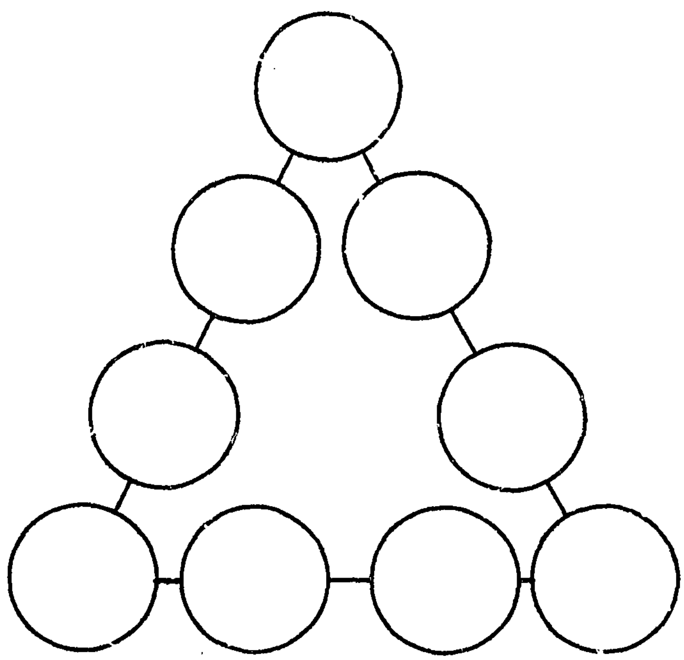


MAGIC TRIANGLES

Write each of the nine digits in the circles below so that you have a total of 20 on each side.



Write the nine digits in the circles below so that you have a total of 17 on each side.



SAM'S DILEMMA

Teacher Commentary

A Recreational Activity on Addition and Multiplication

I. Materials: Story sheet "Sam's Dilemma"

II. Procedure:

A. Distribute the story "Sam's Dilemma".

B. Discuss the first problem to make certain the students understand what Sam is to buy and what his problem is. Note that there are 3 separate problems. It is suggested that only 1 be done each day for three days during the first or third bands.

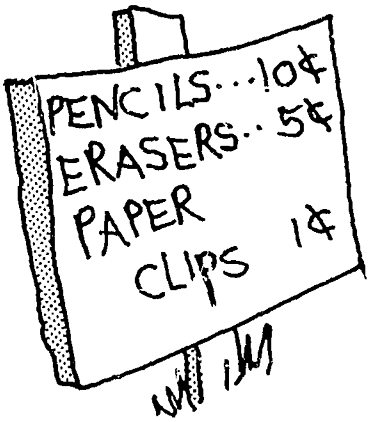
C. Let the students try to discover, by trial and error, the answer to each question on separate days.

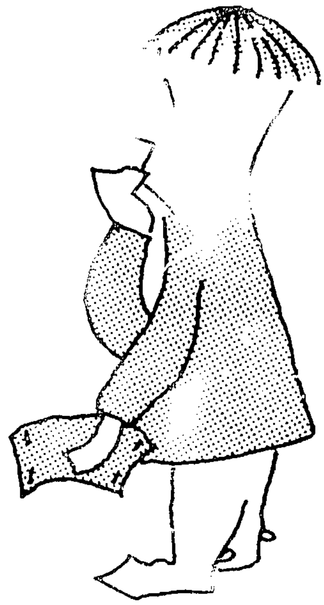
D. Students may wish to set up their own problems using different amounts of money and different items to be purchased.

E. Solution:

	Pencils	Erasers	Paper Things	Total
Fewest Number	9	1	5	15
Largest Number	1	1	85	87
Exactly 50 Items	2	8	40	50

Sam's Dilemma





Sam had $1.00 to spend at the school store. He needed some pencils, erasers, and some paper clips. Since these sold at different prices, Sam found that he could buy a variety of quantities and still spend exactly $1.00. <u>If Sam must buy at least 1 of each item</u>:

1. What would be the fewest number of pencils, erasers, and paper clips that Sam could buy and still spend exactly $1.00?
2. What would be the largest number of pencils, erasers, and paper clips he could buy for exactly $1.00?
3. How many pencils, erasers, and paper clips, could he buy to total exactly 50 items for exactly $1.00?

A CARD GAME

Teacher Commentary

A Recreational Unit on Addition of Whole Numbers

I. Materials:

A set of twenty-four cards, each showing a number from one to six. Four cards are numbered one, four are numbered two, and so on.

II. Procedure:

Two players can play the game. Each player is dealt twelve cards.

Each player picks up his twelve cards and holds them so he can see each one. Alternately the players select a card and place it face up on the desk. After each card has been placed on the pile, the value of the cards (which have been placed on the desk) should be found by adding the numbers on the cards.

A player may win the game in one of two ways:

1. He may place a card on the pile which makes the value of the pile of cards exactly 50, or

2. He may force his opponent to place a card on the pile which makes the value of the pile of cards greater than 50.

A SIMPLE BINARY COMPUTER

Teacher Commentary

A Recreational Activity on the Binary System of Numeration

I. Materials:

A. Five 8" x 8" tag board cards

B. Ruler

C. Scissors

II. Procedure:

A. Construction of cards:

A model for class demonstrations can be constructed from five 8" x 8" tag board cards. The basic plans for this model are included. The small cards on the page of plans can be cut out to make a small model of the computer. By using dittos of these plans, the students could make their own small models.

B. Instructions for using the cards:

1. Ask a student to pick a number from 0 to 31. He may show the number to other members of the class (by writing on the board or a card), but not to you.

2. Ask him if the number he picked appears on the first card. Place the first card face up on the desk. If his number is on the card, the word "yes" should be at the top of the card. If his number is not on the card, turn the card around so that the word "no" is at the top.

3. Go through the remaining four cards in the same way. Place each card in order face up on the top of the preceding card(s). Notice that the location of the word "no" appears on the side of card 5.

4. Pick up the pile of 5 cards, and the number he picked will show through a window in the back of the cards.

C. Why the computer works:

This "Simple Binary Computer" is based on the numbers 0 through 31 as they are written in the binary system of numeration. Each of these numbers uses at most 5 digits and each card represents one of the digits.

The numbers (0-31) must be written in the binary system of numeration to see how the numbers were selected for each card. The chart below shows the numbers from 0 to 31 as they are written in the binary system.

SIXTEEN'S (2^4)
EIGHT'S (2^3)
FOUR'S (2^2)
TWO'S (2^1)
UNIT'S (2^0)

		10	01010	22	10110
		11	01011	23	10111
		12	01100	24	11000
0	00000	13	01101	25	11001
1	00001	14	01110	26	11010
2	00010	15	01111	27	11011
3	00011	16	10000	28	11100
4	00100	17	10001	29	11101
5	00101	18	10010	30	11110
6	00110	19	10011	31	11111
7	00111	20	10100		
8	01000	21	10101		
9	01001				

Card 1 - displays all numbers having the digit 1 in the unit's place. (00001, 00011, 00101, etc.)

Card 2 - displays all numbers having the digit 1 in the two's place. (00010, 00011, 00110, etc.)

Card 3 - displays all numbers having the digit 1 in the four's place. (00100, 00101, 00110, etc.)

Card 4 - displays all numbers having the digit 1 in the eight's place. (01000, 01001, 01010, etc.)

Card 5 - (front) - displays all numbers having the digit 1 in the sixteen's place. (10000, 10001, 10010, etc.)

Card 5 - (back) - is the master card and displays all numbers from 0 to 31.

Notice that the number 0 does not have the digit 1 in any of the 5 places. Therefore, it does not appear on any of the 5 cards. Also, the number 31 has the digit 1 in each of the 5 places. Therefore, it appears on all 5 cards.

There are 32 numbers under consideration. Sixteen of these numbers are displayed on the face of each of the 5 cards. The back of each card narrows the range of numbers from the possible 32 to the one that was chosen. This is accomplished by means of a master card on the back of card 5 and the windows in the back of the other cards.

D. This lesson is recreational in nature and is meant to motivate and interest the student. However, students may discover that they can use the "yes" and "no" answers given for the cards to determine the chosen number. For example, if the cards are answered as follows:

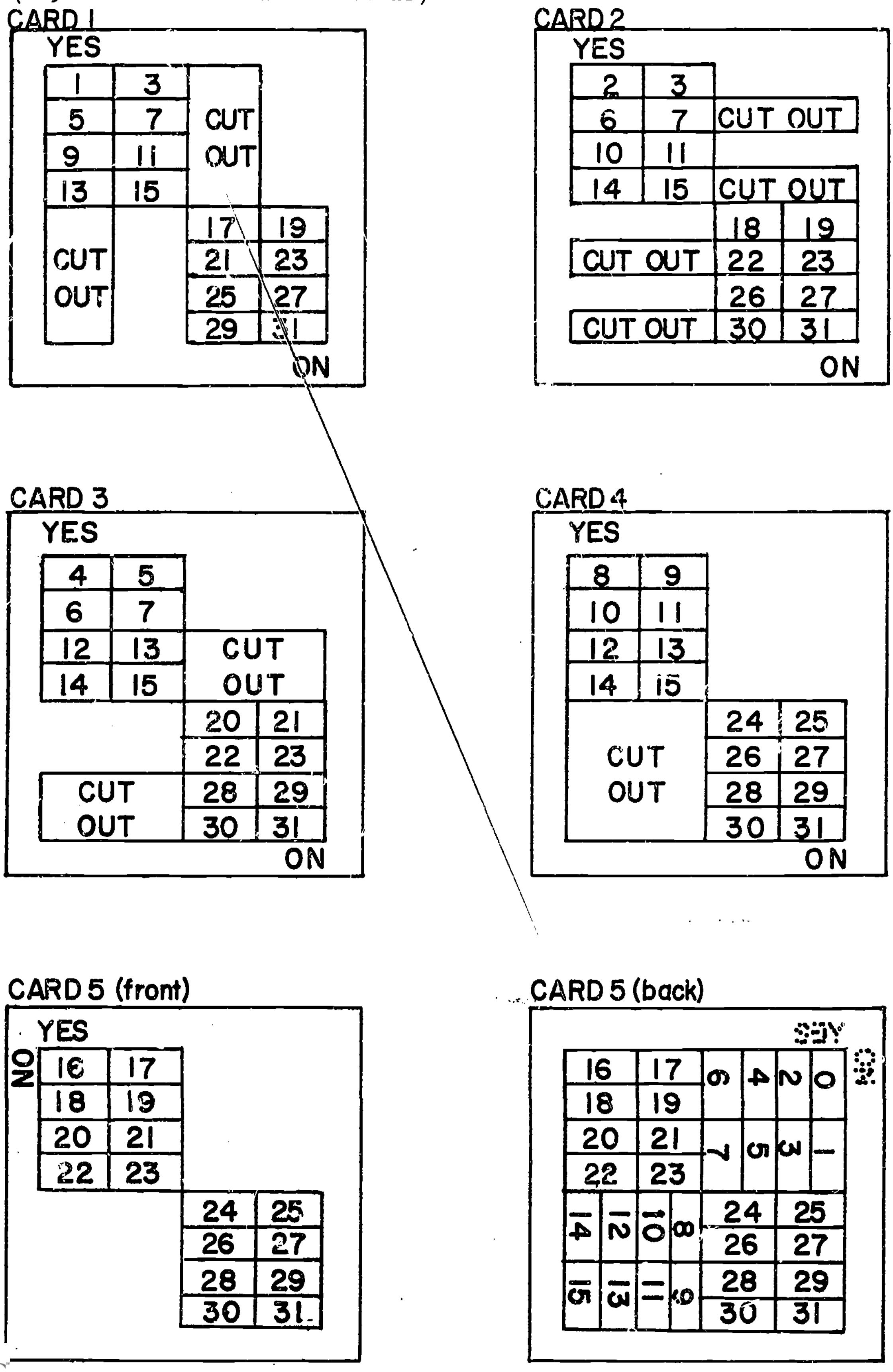
Card 5 - yes
Card 4 - no
Card 3 - no
Card 2 - yes
Card 1 - no

the number is 18.

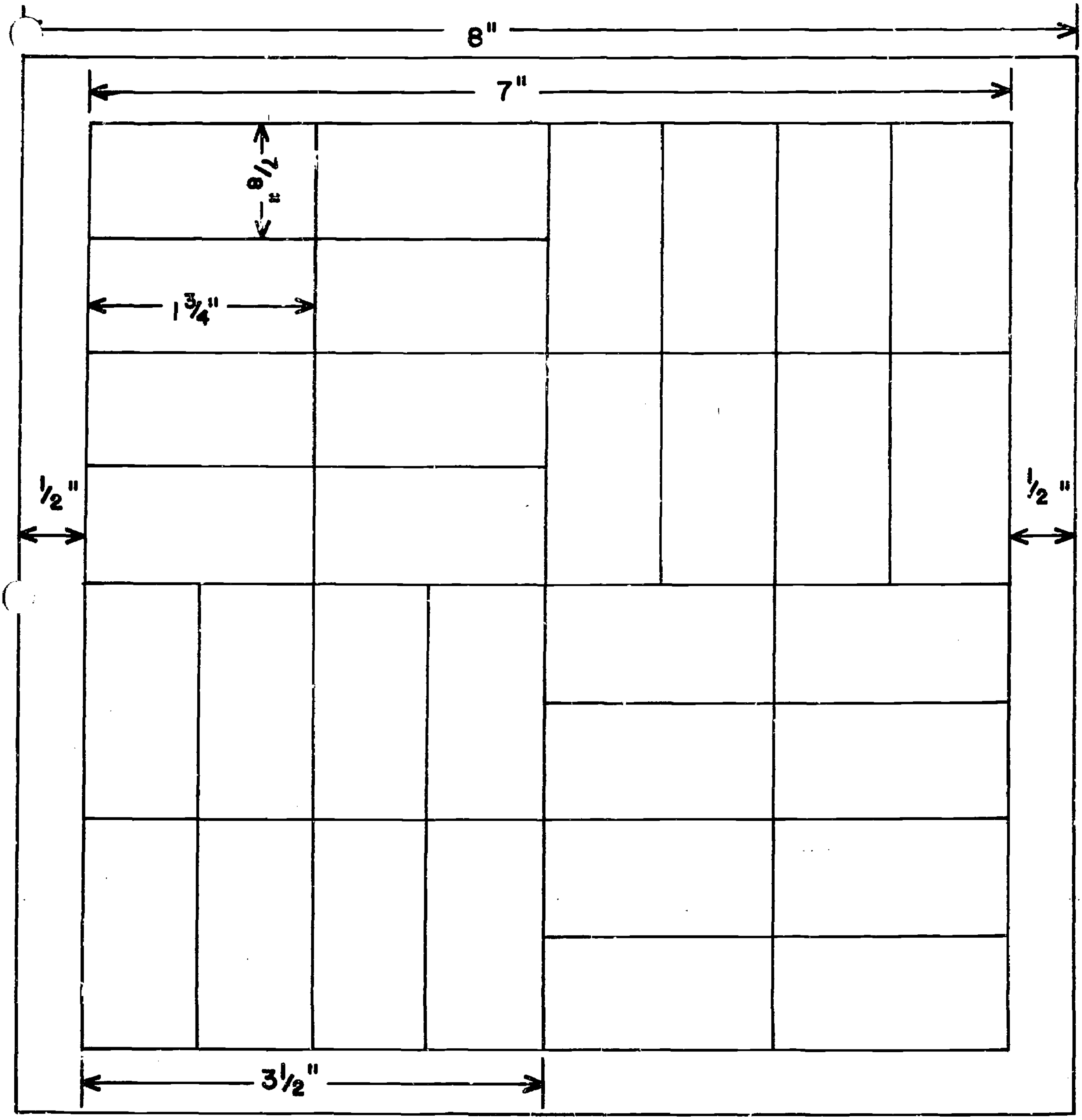
A "yes" answer represents the digit 1, and a "no" answer represents the digit 0. Therefore, yes, no, no, yes, no is the numeral 10010 in the binary system. The numeral 18 in the decimal system represents the same number.

A good exercise is to have one student pick a number. Then use the computer to determine his number while the other students try to determine the chosen number from his "yes" and "no" answers.

BASIC PLANS FOR "SIMPLE BINARY COMPUTER"
(May be used for student models)



BASIC PLANS FOR CLASSROOM DEMONSTRATION MODEL
SIMPLY BINARY COMPUTER



.This is the basic plan for the master card (back of card 5).
All other cards can be made from this basic plan.

ADDITION AND SUBTRACTION OF FRACTIONS
CROSS NUMBER PUZZLE

Teacher Commentary

A Recreational Activity on Addition and Subtraction of Fractions

I. Materials: Student work sheet

II. Procedure:

A. Distribute copies of the attached work sheet.

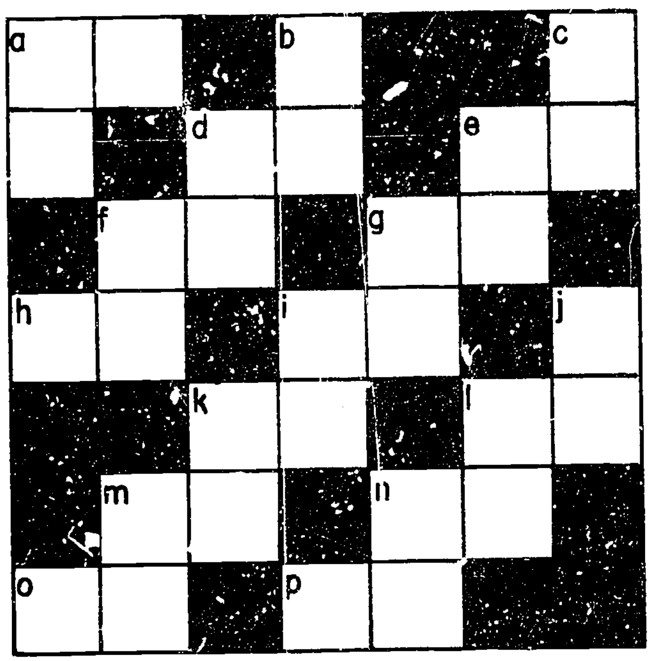
B. Review finding the Least Common Denominator.

C. Solution:

a 1	3		b 1			c 1
3/8		d 2	3/8		e 6	1/6
	f 5	3/5		g 2	2/3	
h 1	5/6		i 3	5/8		j 1
		k 5	1/3		l 2	3/4
	m 1	11/12		n 7	1/12	
o 1	5/16		p 1	5/8		

ADDITION AND SUBTRACTION OF FRACTIONS
CROSS NUMBER PUZZLE

Fill in the blanks. Express your answers in simplest form.



ACROSS

a. $5\frac{1}{2} + 4\frac{1}{2} + 3$

d. $4\frac{5}{8} - 2\frac{1}{4}$

e. $2\frac{7}{12} + \frac{1}{3} + 3\frac{1}{4}$

f. $9 - 3\frac{2}{5}$

g. $8\frac{5}{12} - 5\frac{3}{4}$

h. $\frac{1}{3} + \frac{1}{2} + \frac{1}{6} + \frac{5}{6}$

i. $9\frac{3}{8} - 5\frac{3}{4}$

k. $7\frac{5}{6} - 2\frac{3}{6}$

l. $5 - 2\frac{1}{4}$

m. $4\frac{1}{3} - 2\frac{5}{12}$

n. $2\frac{1}{3} + 4\frac{3}{4}$

o. $\frac{7}{16} + \frac{5}{8} + \frac{1}{4}$

p. $18\frac{7}{8} - 17\frac{1}{4}$

DOWN

a. $4\frac{1}{2} + 3\frac{1}{5} + 2\frac{3}{10}$

b. $\frac{1}{2} + \frac{1}{4} + \frac{5}{8}$

c. $2\frac{3}{6} - 1\frac{1}{3}$

d. $1\frac{1}{10} + 1\frac{1}{2}$

e. $2\frac{2}{9} + 3\frac{1}{3} + 1\frac{1}{9}$

f. $3\frac{2}{3} + 2\frac{1}{6}$

g. $3\frac{3}{8} - \frac{3}{4}$

i. $5 - 1\frac{2}{3}$

j. $\frac{11}{12} + \frac{5}{6}$

k. $2\frac{1}{6} + 3\frac{9}{12}$

l. $7\frac{1}{3} - 5\frac{3}{12}$

m. $2 - \frac{11}{16}$

n. $2\frac{1}{4} + 4\frac{1}{4} + 1\frac{1}{8}$

SOMETHING FOR SQUARES

Teacher Commentary

A Recreational Unit on Quadrilaterals

I. Materials:

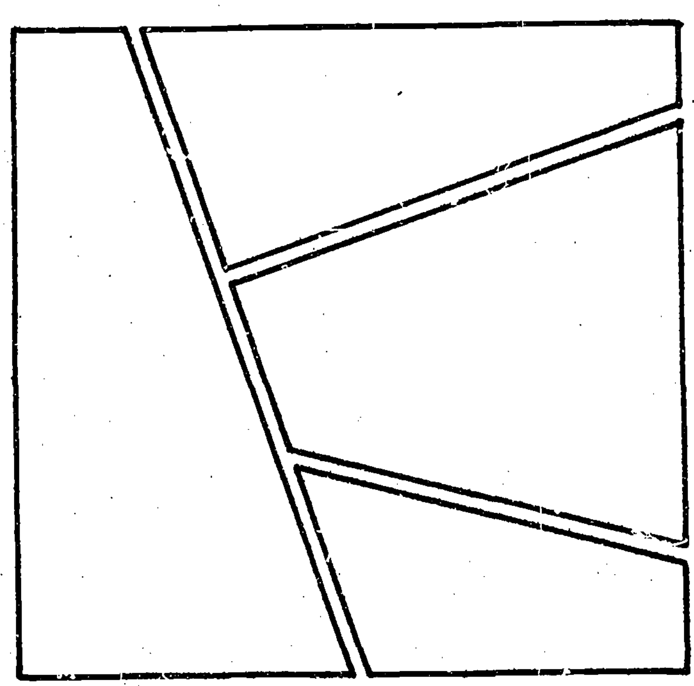
A. Ditto with 4 quadrilaterals (see the next page)

B. Scissors

II. Procedure:

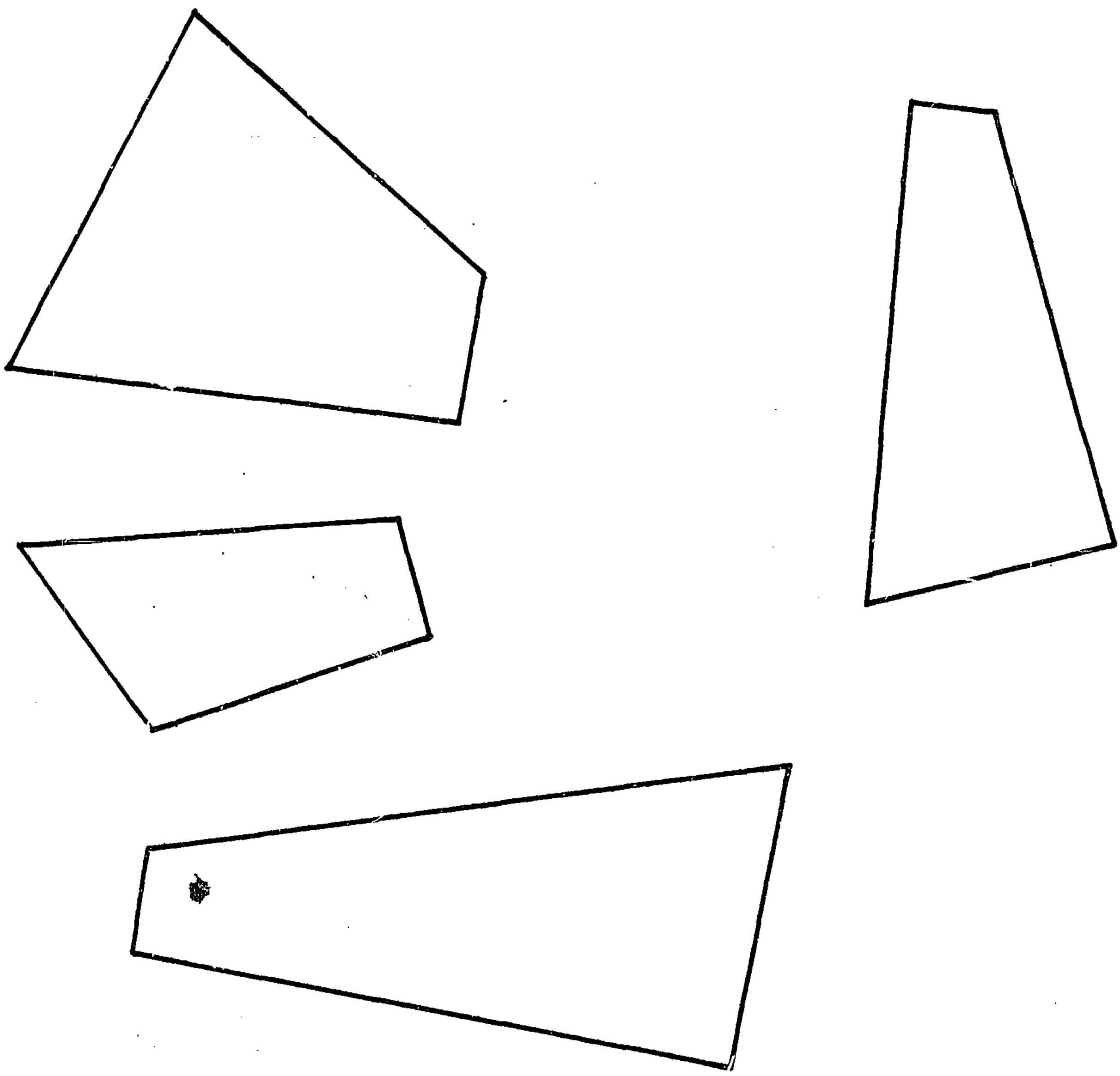
Have students cut out four quadrilaterals on the ditto paper and arrange them to form a square.

Solution



SOMETHING FOR SQUARES

Cut out the pieces and put them together so the outer edge forms a square.



BUILDING A PYRAMID

Teacher Commentary

A Recreational Activity on Constructing a Pyramid

I. Materials:

A. Pencil

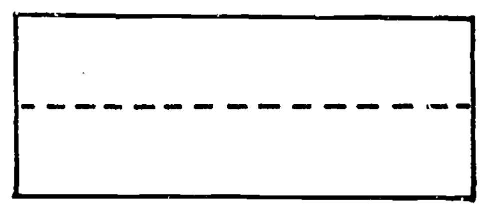
B. Scissors

C. Business size envelope

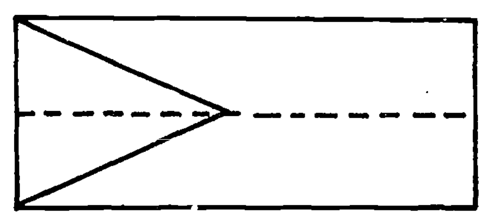
D. Cellophane or masking tape

II. Procedure:

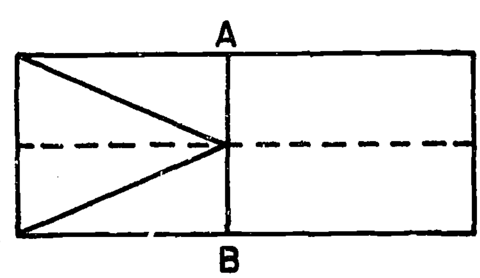
A. Using a sealed envelope, mark the mid-point of the edges at the two narrow ends. Connect dots with a dashed line as shown.



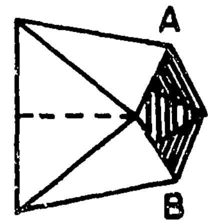
B. Measure the narrow edge. Using this measure, draw line segments from two corners to a point on guide line as shown.

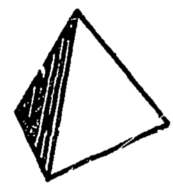


C. Draw a line segment through the top of the triangle parallel to the end of the envelope. Label the end points A and B. Cut along this line segment.



D. Fold back and forth along the line segments you drew in step B. Then lay paper flat again. Open along the cut end. Bring the two open edges together so that A and B meet. Tape these edges together.





GEOMETRICAL-STAINED-GLASS-WINDOWS

Teacher Commentary

A Recreational Activity on Geometry in Art

I. Materials:

A. Colored tissue paper or colored cellophane

B. Black construction paper

C. Scissors

D. Paste

II. Procedure:

A. Take a piece of black construction paper and draw a design as shown on last page of Teacher Commentary.

B. Cut out the parts with a pair of scissors.

C. Paste colored tissue paper in the back of the cut out pattern.

D. Mount the pictures on windows of the classroom.

E. Discuss the figures used in each design. If there is still interest in these "windows," explain that designs may also be made of circles and arcs (see student work sheets). Have students make a stained-glass-window using work sheets. They could then design an original window based on circles and arcs.

F. Display the windows for Christmas, dance decorations, lobby display on the doors and windows.

III. Related Activities:

A. Show some pictures or slides of stained-glass-windows and discuss the geometry found in them.

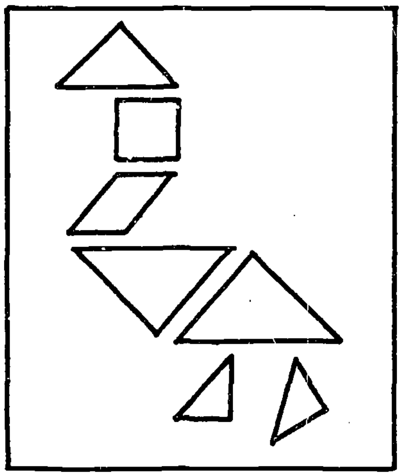
B. Take a trip to a church in the neighborhood to see some examples of stained-glass-windows.

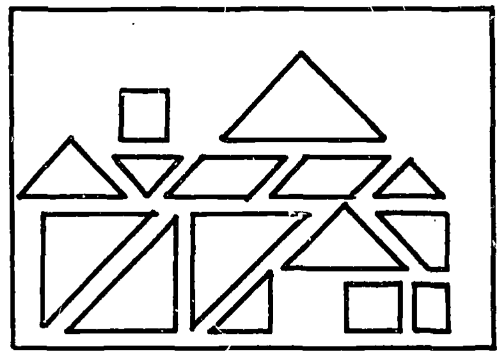
C. Read some simple stories about the history and legend of these windows.[1]

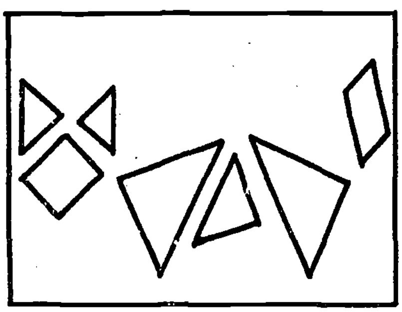
D. Make some 3-dimensional models, (see work sheet on polyhedrons) by cutting out the inside of the faces of the polyhedrons and pasting cellophane in them.

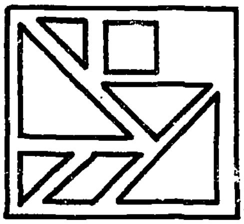
[1] Young, Mary. Singing Windows. New York: Abingdon Press. 1962

GEOMETRICAL-STAINED-GLASS-WINDOWS





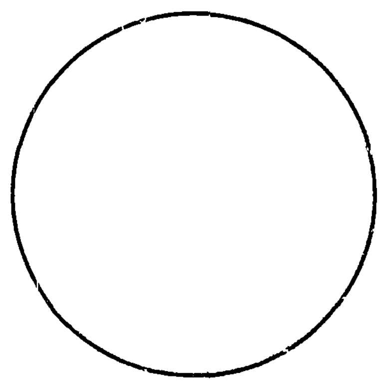




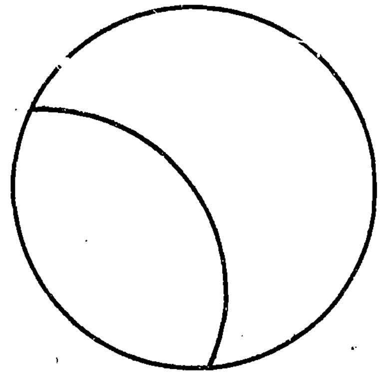
R-28

GEOMETRICAL-STAINED-GLASS-WINDOWS

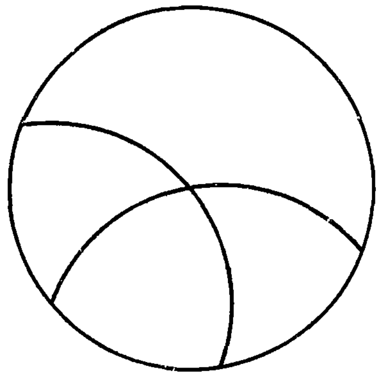
1. Draw a circle with a radius of 4" on a piece of black construction paper.



2. Place the compass at any point A on the circle, and using the same radius draw an inside arc so that the arc intersects the original circle at two points B and C.

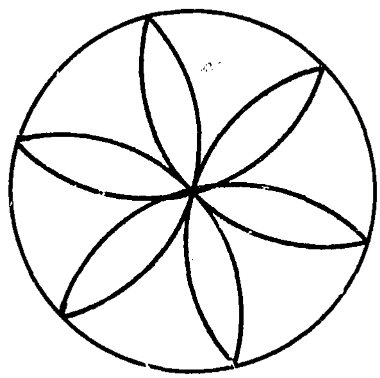


3. Place the compass at point C and using the same radius draw an inside arc from points A to D.



4. Continue in the same manner until you return to point B.

5. Cut out each leaf and using as many colors as you like paste colored tissue paper in back of the cut out pattern.



MEASUREMENT PUZZLE

Teacher Commentary

A Recreational Activity on Measurement

I. Materials: Measurement Puzzle

II. Procedure:

A. Distribute puzzles

B. Go over the directions with the students

C. Permit student to work puzzle

D. Solution:

R	E	C	T	A	N	G	L	E			U	N	I	T	
		T	R	I	A	N	G	L	E						
O									P	I	N	T			
U		F		W	I	D	T	H							M
N		E						S	Q	U	A	R	E		E
C		E			R										A
E		T			U		P	R	E	C	I	S	E		S
					L										U
P					E	I	N	C	H	E	S		Q		R
E		G			R				P		G		U		E
R		R		Y	A	R	D	S	O		A		A		M
I		E							U		L		R		E
M		A	H	E	I	G	H	T	N		L		T		N
E		T			D				D		O				T
T		E			E						N				
E		S			G	L	E	N	G	T	H				
R		T			R						A	R	E	A	
					E										
					E										

MEASUREMENT PUZZLE

How many "measurement words" can you find in this puzzle? Some words go across. Some words go down.

Words you may find are:

rectangle	inches
triangle	pound
square	ounce
ruler	quart
measurement	gallon
unit	pint
precise	yards
perimeter	degree
greatest	length
feet	width
height	area

When you find a word in the puzzle, circle it. This will help you to see the letters you have used. Check each word on the list above as you find it. The first one has been done for you.

MEASUREMENT PUZZLE

R	E	C	T	A	N	G	L	E	C	D	U	N	I	T	R
B	E	T	R	I	A	N	G	L	E	G	I	L	P	Q	S
O	H	J	O	T	W	X	A	F	P	I	N	T	Z	F	V
U	K	F	N	W	I	D	T	H	V	B	E	S	U	Y	M
N	O	E	M	H	W	I	N	S	Q	U	A	R	E	A	E
C	C	E	D	Q	R	G	V	M	T	P	X	E	B	Z	A
E	J	T	K	Y	U	L	P	R	E	C	I	S	E	F	S
D	C	J	Y	V	L	N	U	H	I	T	E	W	A	R	U
P	K	O	C	G	E	I	N	C	H	E	S	P	Q	B	R
E	L	G	M	X	R	E	P	S	P	Z	G	I	U	N	E
R	D	R	P	Y	A	R	D	S	O	G	A	H	A	O	M
I	K	E	J	Q	L	F	R	V	U	M	L	W	R	Z	E
M	S	A	H	E	I	G	H	T	N	X	L	Y	T	A	N
E	T	T	U	B	D	H	I	C	D	J	O	L	D	E	T
T	F	E	Q	G	E	P	K	V	M	A	N	O	U	B	N
E	W	S	R	X	G	L	E	N	G	T	H	S	T	C	H
R	U	T	Y	D	R	A	E	D	Z	F	A	R	E	A	G
V	O	G	Y	N	E	R	Z	P	V	C	M	F	J	O	I
P	H	Q	X	R	E	V	M	D	W	B	L	A	K	Z	T
I	S	W	J	T	B	Q	K	R	C	L	E	N	Y	S	X

MEASUREMENT CROSS NUMBER PUZZLE

Teacher Commentary

A Recreational Activity on Measuring to the Nearest Eighth of an Inch

I. Materials:

A. Work sheet with cross number puzzle

B. Ruler and pencil

II. Procedure:

A. Distribute a cross number puzzle to each student.

B. Measure one segment with the class to make sure students understand what to do.

Example: Across - a segment measures $2\frac{4}{8}'' = 2\frac{1}{2}''$.

Place 2 in the first square.
Place $\frac{1}{2}$ in the second square.

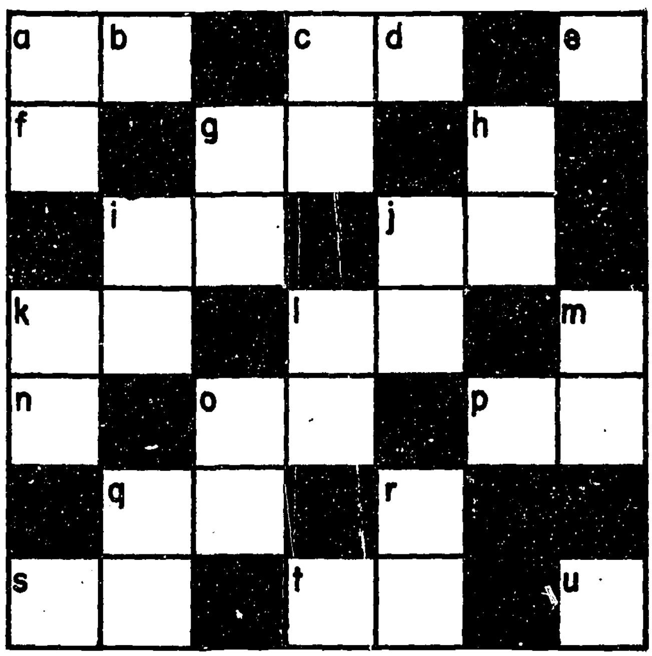
Simplify all fractions. Write each fraction in a single square.

C. Solution:

a 2	b 1/2		c 1	d 1/8		e 1/4
f 3/8		g 3	1/8		h 3	
	i 3	3/8		j 1	1/2	
k 1	5/8		l 1	1/2		m 2
n 7/8		o 3	1/4		p 2	1/4
	q 1	5/8		r 3		
s 3	1/4		t 2	1/4		u 2

MEASUREMENT CROSS NUMBER PUZZLE

Directions: Measure each of the following segments to the nearest eighth of an inch. Place your measures in the correct squares.



Across

a.

c.

e.

f.

g.

h.

i.

j.

k.

l.

m.

n.

o.

p.

q.

r.

s.

t.

u.

Down

a.

c.

d.

g.

h.

i.

j.

k.

l.

m.

o.

p.

q.

r.

s.

t.

TOOTHPICKS AND PI

Teacher Commentary

A Recreational Unit Involving Pi (π)

I. Materials:

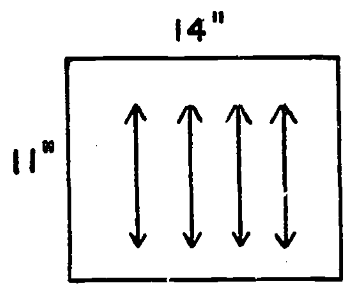
A. One 11" x 14" piece of tag board per student

B. Ten toothpicks per student

II. Procedure:

A. This is an experimental way to obtain an approximation for pi. It should be done as a recreational follow-up activity after pi has been used in circumference problems.

B. Explain - an experiment is going to be performed which should result in a familiar number. Distribute the tag board to each student. Have each student draw several parallel lines on his tag board with a ruler as shown below.



The distance between these lines must be equal to or slightly greater than the length of the toothpicks.

C. Distribute toothpicks to the students. Have students drop all ten toothpicks simultaneously onto their tag board. Have them count the number of toothpicks touching a line and the number not touching a line. Those touching lines are successes, and those not touching lines are failures.

D. This process of dropping and recording should be done a total of 20 times.

E. Have students total all their successes and all their failures.

F. Have students divide the number of successes by the number of failures. An approximation of π will result.

G. Record various student results on the chalkboard.

Ask - Do these numbers look familiar?
Are they close to any number which we have already studied?

You may have to provide some clues.

Explain - Pi is a non-terminating, non-repeating decimal. The values obtained in the experiment are only approximations.

AN INTRODUCTION TO LOGICAL INQUIRY

Teacher Commentary

A Recreational Activity on Asking Logical Questions

I. Materials:

A. Board diagrams

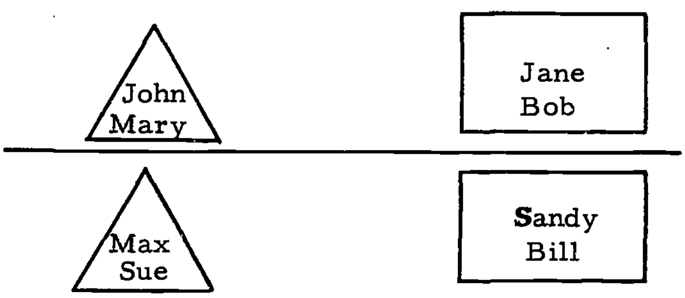
B. Student work sheet entitled, "Think Before You Ask"

II. Procedure:

A. Game of "Twenty Questions"

1. Have one student think of an object in the room. Students can ask a maximum of 20 questions, which can be answered "yes" or "no," to determine what the object is.
2. While the game is played the teacher should note questions which narrow the possibilities best.
3. After the game, discuss which questions were best and why.
4. Emphasize the idea of a "lucky guess" and why asking good questions will bring consistently better results.
5. Play another game, stressing good questions.

B. 1. Have the diagram and questions on the board.



1. Is it Jane?
2. Is it Bill?
3. Is it above the line?
4. Is it in a triangle?
5. Is it a girl?

2. Tell the students you are thinking of one name in the diagram. By asking only 3 questions you want them to narrow the number of choices to one, and tell you the name.
3. Discuss questions 1-5 and compare the merit of each in narrowing the choice. Decide on the three best questions and point out why 1 and 2 were poor.
4. Ask: How many choices there were to begin with? (8) How many choices after the first question? (4) After the second question? (2) After the third question? (1) Each time the choice of names was reduced by $(\frac{1}{2})$.

C. Work sheet, "Think Before You Ask"

1. If possible, have the diagrams on the work sheet duplicated on the board.

2. The work sheet should be presented as a group activity. Discuss each section, and be sure the questions are clear.

   a. Number one - As students select the best questions, have them check to see that each question narrows the choice by one half. Point out why 1 and 3 are not the good questions.

   b. Number two - Discuss what is known from the first two questions. After they have written their third question, take a sampling, and examine each one as a class. It may be wise to point out why some did not work.

   c. Number five - The students should work in pairs - first one guessing the number, then the other.

   d. Number six - The students will make up three of their own questions and test them on someone. Those having trouble may refer to number 1 on the work sheet for help.

D. Follow-up

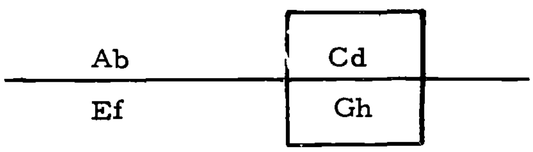
If the students display an interest in this activity, you may want to:

a. Have the children make up their own diagrams and questions and challenge others in the class.

b. Play the game in teams, giving points for good questions.

c. Expand the number of choices, and the number of questions necessary to narrow the possibilities.

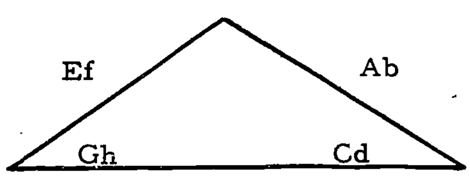
THINK BEFORE YOU ASK



1. Is it E?
2. Is it inside the rectangle?
3. Is it b?
4. Is it a capital letter?
5. Is it above the line segment?

1. Suppose your friend is thinking of one letter in figure A. What are the numbers of the questions above you would ask to tell him what it is? ______, ______, ______

   (Remember that each <u>good</u> question should narrow the number of choices by one half.)



1. Is it inside the triangle?
2. Is it a capital letter?
3.

2. Suppose you have already asked the first two questions above, and the answer to each one was "yes." Do both questions narrow the number of choices by one half? __________

   Write a good third question. How many choices are left? __________

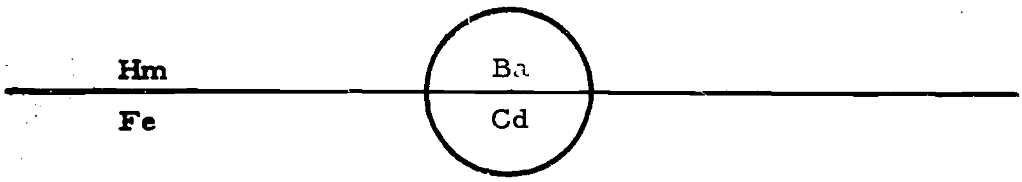
   $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

3. If you were asked to find a number in Set A, how many choices would you have? __________

4. After you ask the first question, how many choices should there be? __________ After the second question? __________ After the third question? __________

5. **Have a classmate** think of a number in Set A. Ask him the first **two questions** below. Then think of a third question to guess his **number.** Write it in space 3. Did it work? ____________

   1. **Is the** number larger than 4?
   2. **Is** it an odd number?
   3.



6. **Write three** <u>good</u> questions to narrow down the choices above.

   1.
   2.
   3.

7. **Ask someone** to think of a letter in this figure. Ask your three **questions** and see if you can guess his letter.

HIDDEN WORD PUZZLE

Teacher Commentary

A Recreational Activity on Vocabulary Words From Measurement

I. Materials: Student work sheet with square array of letters

II. Procedure:

A. Give each student a duplicated sheet.

B. Have him find as many hidden math words as he can.

C. Solution:

S					A	R	E	A		Y		
Q				F						A		
U			P	O	U	N	D	E	G	R	E	E
A				O	U	N	C	E		D		
R	P	I	N	T						G		
E			M			I	N	C	H	A	Q	
I			I							L	U	
N			L							L	A	
C			E							O	R	
H	M	E	A	S	U	R	E	M	E	N	T	

Check List:

1. Foot
2. Pound
3. Ounce
4. Inch
5. Measurement
6. Degree
7. Quart
8. Mile
9. Yard
10. Gallon
11. Pint
12. Area
13. Square Inch

HIDDEN WORD PUZZLE

See if you can find words from measurement in the puzzle. Some words run across and some run down. Altogether there are 13 words.

S	T	I	D	I	A	R	E	A	N	Y	T	L
Q	R	P	I	F	L	G	K	M	O	A	P	A
U	N	W	P	O	U	N	D	E	G	R	E	E
A	T	F	G	O	U	N	C	E	H	D	I	B
R	P	I	N	T	U	B	F	X	G	G	U	C
E	W	N	M	I	P	I	N	C	H	A	Q	U
I	D	H	I	M	R	T	V	A	C	L	U	K
N	P	U	L	T	V	W	B	D	G	L	A	G
C	F	I	E	L	N	O	S	T	V	O	R	R
H	M	E	A	S	U	R	E	M	E	N	T	H

HIDDEN MESSAGES

Teacher Commentary

A Recreational Activity on Multiplication, Division, and Rounding Decimal Fractions

I. Materials: Student work sheets

II. Procedure:

A. Distribute a work sheet to each student

B. Students are to decode the hidden message

C. Solution:

You Do Good Work

A HIDDEN MESSAGE

Secret messages are sometimes in code. Often the code will have numbers or letters in it. Today you will figure out a message by solving some examples.

I. Round all solutions to the nearest whole number. Each solution stands for a letter.

1. 4.9 x 5 ____________
2. 2.6 times 5.6 ____________
3. 62.7 ÷ 3 ____________
4. 8 x .5 ____________
5. 6 ÷ .4 ____________
6. 4.8 x 1.45 ____________
7. 3 x 5.00 ____________
8. 63.14 ÷ 4.1 ____________
9. 3.8 x 1.06 ____________
10. 69.27 divided by 3 ____________
11. 6.16 ÷ .4 ____________
12. 10.854 ÷ .6 ____________
13. 22 x .5 ____________

II. Below is the decoder. Right under each number you have for an answer is a letter. Put this letter in the corresponding numbered space. Then you will see the message.

DECODER

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

HIDDEN MESSAGE

___ ___ ___   ___ ___   ___ ___ ___ ___
1 2 3   4 5   6 7 8 9

___ ___ ___ ___
10 11 12 13

HIDDEN MESSAGE

Teacher Commentary

A Recreational Activity on Multiplying, Dividing, and Rounding Fractions

I. Materials: Student work sheets

II. Procedure:

A. Distribute a work sheet to each student

B. Students are to decode the hidden message

C. Solution:

A Correct Paper

A HIDDEN MESSAGE

Secret messages are sometimes in code. Often the code will have numbers or letters in it. Today you will figure out a message by solving some examples.

I. Round solutions to nearest whole number. Each solution stands for a letter.

1. $\frac{1}{4}$ of 4 ______________

2. $\frac{3}{8} \times 8$ ______________

3. $5 \div \frac{1}{3}$ ______________

4. $18 \div \frac{4}{4}$ ______________

5. $\frac{1}{2} \times 36\frac{3}{4}$ ______________

6. $11\frac{1}{4} \div 2\frac{1}{4}$ ______________

7. $8 \times \frac{3}{8}$ ______________

8. $85 \div 4\frac{1}{6}$ ______________

9. $\frac{1}{3}$ of 49 ______________

10. $\frac{4}{5} \times \frac{5}{6}$ ______________

11. How many 4ths is 4? ______________

12. $\frac{2}{3}$ of what number is $3\frac{1}{3}$? ______________

13. $4\frac{1}{2} \times 4$ ______________

II. Below is the decoder. Right under each number you have for an answer is a let   Put this letter in the corresponding numbered space. Then you will see the message.

DECODER

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

HIDDEN MESSAGE

____ 1   ____ 2   ____ 3   ____ 4   ____ 5   ____ 6   ____ 7   ____ 8

____ 9   ____ 10   ____ 11   ____ 12   ____ 13

WORD MAZE

Teacher Commentary

A Recreational Activity on Vocabulary Words From Math

I. Materials: Work sheet with square array of letters

II. Procedure:

A. Give each student a work sheet

B. Have each student try to find the 10 math words hidden in the maze.

C. You may give the words to be found or you may allow students to find them without hints.

Hidden Words:

1. arc
2. chord
3. circle
4. cube
5. line
6. plane
7. pi
8. ray
9. ten
10. yard

WORD MAZE

Ten math words are hidden in the square. Can you find them? Here's what to do.

You may start in any square and move in any direction to any square next to it. You may continue to move one square in any direction until a word is made. You may not enter the same square twice while spelling a word.

D	O	H	P	L
R	C	U	L	I
A	Y	B	N	A
I	C	P	E	S
P	R	C	L	T

SCRAMBLED WORDS

Teacher Commentary

A Recreational Activity on Vocabulary for Geometry

I. Materials: Work sheet of scrambled words

II. Procedure:

A. Give each student a work sheet.

B. Have each student unscramble the letters to form vocabulary words.

C. Solution:

1. cone
2. hexagon
3. pentagon
4. parallel
5. pyramid
6. triangle

SCRAMBLED WORDS

Put the letters in the right order to find math words:

1. noce
2. gnoxahe
3. ongatpen
4. lelapalr
5. ymraidp
6. gritalen

SCRAMBLED WORDS

Teacher Commentary

A Recreational Activity on Vocabulary Words for Algebra

I. Materials: Work sheet of scrambled words

II. Procedure:

A. Hand out work sheets to students.

B. Have students unscramble words.

C. Solution:

1. member
2. equal
3. exponent
4. variable
5. parentheses

SCRAMBLED WORDS

Unscramble the letters to form algebra words:

1. bermem
2. quale
3. ponexent
4. ablevari
5. esenparthes