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ABSTRACT

Development of a second-grade mathematics instructional unit is discussed in terms of activities, evaluation procedures, and results of evaluation. The report is organized by topic. Behavioral objectives are followed by descriptions of activities and materials used. Topics include: review of symbols, pictorial representation of numbers, writing mathematics sentences, expanding notation, addition and subtraction, geometric shapes, compact notation, grouping ones and tens, number patterns, telling time. Some of the activities and material utilized are worksheets, unit blocks, "Numbars," Unifix Cubes," balance scales, "Lots-a-links," and cuisenaire rods. (JG)

Working Paper No. 46

# Developing Mathematical Processes: Pilot Tryout of Materials for Second Grade Children, Huegel School, 1968-69, Madison, Wisconsin

Report from the Project on Individually Guided Elementary Mathematics, Phase 2: Analysis of Mathematics Instruction


Wisconsin Research and Development
CENTER FOR COGNITIVE LEARNING

Working Paper No. 46

# DEVELOPING MATHEMATICAL PROCESSES: PILOT TRYOUT OF MATERIALS FOR SECOND GRADE CHILDREN, HUEGEL SCHOOL 1968-69, MADISON, WISCONSIN

by

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Diane Planert

Report from the Project on

Individually Guided Elementary Mathematics

Phase Two, Analysis of Mathematics Instruction

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## STATEMENT OF FOCUS

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Working Paper is from Phase 2 of the Project on Individually Guided Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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ABSTRACT

This paper summarized the developmental activities in mathematics conducted in Second Grade at Ray W. Huegel School, Madison, Wisconsin, during the 1968-69 school year. One instructional unit of Developing Mathematical Processes was taught. The instructional unit is characterized through a description of the activities that were tried, an explanation of evaluation procedures, and a report of the results of each evaluation.

I

# INTRODUCTION

The purpose of this working paper is to describe the development of instructional materials and procedures for teaching mathematics at the second grade level. Those activities and procedures found successful will be included in a program entitled Developing Mathematical Processes (DMP) (Romberg & Harvey, 1969).

## Overview of Developmental Procedures

Romberg and Harvey (1969) have written a detailed description of the process involved in developing instructional materials and procedures. Briefly, broad mathematical objectives are identified, each task involved is analyzed and prerequisite behaviors necessary for attainment of objectives are described. The tasks are then organized into sequences, resulting in a topic outline. The next step is making an instructional analysis in which teachers and staff organize and develop activities to accompany the lessons.

During the school year 1968-69, these materials and procedures were tried out at Ray W. Huegel Elementary School located in Madison, Wisconsin. The school, organized on the Multiunit plan (Klausmeier, Morrow, & Walter, 1968), is headed by Principal Jerry Johnson. The pilot study involved only those children in the second year of Unit I, Initial Skills.

## Students

There were 22 children in the Second Grade pilot group.

The children received mathematics instruction 30 minutes a day. All students had participated in the pilot math program the previous year (See Romberg and Roweton, 1969).

Teacher

The teacher was Joan Moscovitch, an employe of the Research and Development Center. The math class was the only class she taught during the 1968-69 year. However, she did have a Master's degree in Education and had taught full time at Huegel school the prior year. She instructed the same group of students in math the previous year (First Grade). She was assisted by Marilyn Roweton.

Materials

The teacher was provided with physical aids and a rough topic outline-task analysis. She planned the daily activities and prepared all the necessary worksheets with the aid of the R & D staff.

Formative Evaluation

Two main sources were used in gathering information: the teacher's daily record book and tests used to evaluate the children's progress. The daily record book included the objectives for the day, the materials used, a description of the activities, and a critical review of the instructional methods and activities used. The tests were given and scored by the teacher and a staff member of the DMP project.

Introduction

Before describing the daily lessons it should be noted that since this was a pilot study, many topics that were introduced had to be dropped

and were re-introduced at a later date because they were found to be too difficult for the children at the time of the first introduction. Therefore, an attempt was made to follow a topic outline but, for the reason given above, some of the lessons within a topic may be found to overlap with lessons included under another topic.

This report is organized by first listing the objectives for each topic. This is then followed by a description of each activity and the materials used, the lesson presentation, and the response of the children. Note that many of the activities were used several days in succession. The results of the evaluations will be presented in the same order as they are given.

## II

## INSTRUCTION

TOPIC 1: REVIEW + AND - SYMBOLS. A total of 6 weeks and 2 days was spent on this topic.

Objective 1: Be able to determine whether + or - is the proper operation.

For the first activity, the children were given worksheets containing problems with only the operation symbol missing, for example 2 = 1 ◯ 1 (the child must fill in the circle). Before beginning work on the problems, the entire class reviewed the meaning of the + and - operations. The teacher reported that there was some confusion when the problems involved adding numbers to the left-hand side of the equation. For instance, when given 7 and 9, they realized that 2 had to be added to 7, but they wrote 7 = 9 + 2.

In the second activity, the children were given cubes and number cards, as well as the activity sheets to work with. Each student was required to make up equations with cubes and to demonstrate the truth of the sentences by adding on or taking away cubes. It was reported that the children were improving.

For the third activity, the class was given Numbars[1], sign cards,

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[1] Numbars - Modified from Unit Blocks, Stern, Stern, and Gould Structural Arithmetic materials (Boston: Houghton Mifflin).

and worksheets. However, this time the worksheets contained equations in which numbers were missing in some, and symbols missing in others. After reviewing the use of Numbars with the entire class, the children were asked to work the problems and to demonstrate the truthfulness of their solutions using the above materials. There were no apparent difficulties. The children were quite excited about working with numbers over 10, and put forth a great attempt at problem solving.

Various materials including Numbars, discs, Unifix Cubes[2], and worksheets were used in the fourth activity. The children were allowed to choose the materials they wished to work with, and then were required to work problems and validate their solutions using the materials. The children enjoyed working with the materials and were meeting with success.

Objective 2: After setting up an equation using objects, be able to write the equation.

In the first activity, the children were given Numbars, discs, and Unifix Cubes to work with. They worked in pairs; one child was to set up an equation using the materials, and the other demonstrated its truth. Then they both were required to write the whole equation. All of the children seemed able to write equations, and enjoyed working with larger numbers.

For the second activity, the children worked individually, setting up and writing equations. The children checked and corrected each other's equations. The same materials were used but included a blank piece of paper for each child. This activity seemed to work quite well with the children. They liked the method of checking each other's papers. Two of the children were reported to work with equations involving patterns (e.g., 10 = 9 + 1, 9 = 8 + 1, 8 = 7 + 1; 8 = 6 + 2, 6 = 8 - 2, 2 = 8 - 6). It

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[2] Unifix Cubes # TN 42-10 (London: Philograph Publications, Ltd.).

was reported that the children were quite able to set up and write equations.

Objective 3: Be able to construct and use the addition table.

In the first activity, the children had to construct their own addition table. They were given graph paper to construct it on and Numbars to aid them in the addition facts. The teacher discussed with them how to use the Numbars in adding. She illustrated the process by putting the 5 and 3 Numbars together and finding another Numbar that was equal in length to the first two. Then she directed the group in setting up the table. First she filled in boxes at random to demonstrate the process.

Addition Table:

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | |
| 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | |
| 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | | |
| 5 | 5 | 6 | 7 | 8 | 9 | 10 | | | | | |
| 6 | 6 | 7 | 8 | 9 | 10 | | | | | | |
| 7 | 7 | 8 | 9 | 10 | | | | | | | |
| 8 | 8 | 9 | 10 | | | | | | | | |
| 9 | 9 | 10 | | | | | | | | | |
| 10 | 10 | | | | | | | | | | |

The children filled in the table with no apparent trouble. However, it was felt they should have been instructed to work most of the table at random since they used the patterns to fill it in rather than performing all of the addition problems. The teacher commented that they probably could not reconstruct it if asked to.

The children were given worksheets in the second activity. They were allowed to use the addition table they had constructed and Numbars or cubes if they wished. All the students worked individually. All but one of the children worked accurately and quickly. One of the students worked every problem as a subtraction problem.

In the third activity, the students were given decks of cards and the addition tables, and were taught the game of 11. In this game, each student works independently. First, he places 12 numbered cards on a table in three rows of four cards each. After finding two cards when combined that equaled 11, he put two more cards down to replace the two that add up to 11. Face cards were set aside. If the child were able to use all the cards, he had won the game. The children enjoyed the game very much.

EVALUATION I

The first evaluation consisted solely of the California Achievement Test in Mathematics, the Addition section. The test included 20 problems on the basic addition facts and 5 on two-place addition, 2 of which were problems on carrying.

As indicated in Table 1 following the children did somewhat better on the CAT math test given in October, 1968 (Grade 2) than they did on the same test in June, 1968 (Grade 1). Two of the children worked all the problems correctly on the October test, whereas no children had perfect papers on the June test. It should be mentioned that at the time of the testings the children had not been taught to carry but that the test included two problems involving the carrying process. Therefore, it was interesting to find that a few of the children worked these two problems correctly (three children answered both problems correctly and seven children got one of the two correct).

EVALUATION II

All six tasks of this evaluation were individually administered.

In the first three tasks, the child was to complete three addition

Table 1

Mean Number of Correct Responses on the

CAT Math Test (Addition)

| Date Given: | n | Problems 1-25 | Problems 1-20 Number Facts | Problems 21-25 2-Place Addition | Problems 6, 8, 11, 18 Adding Zeros | Problems 22, 25 Carrying |
|---|---|---|---|---|---|---|
| June, 1968 Grade 1 | 21 | 20.43 | 18.95 | 1.48 | 3.86 | .52 |
| October, 1968 Grade 2 | 21 | 20.67* | 19.05 | 1.62 | 3.86 | .62 |

* 82.7% of the questions were answered correctly.

equations and use Numbars, paper clips, and Unifix Cubes to demonstrate the accuracy of the solutions (e.g., 10 = 3 ◯ 7, 8 = 5 + □). In the next 3 tasks, the child was given 3 more addition problems to solve using the addition table (e.g., 7 + 8 = □, 4 + 4 = □).

As the results indicate (See Table 2 on page 10), the children did very well on the evaluations. Only two children made mistakes and both of these were on demonstrating the accuracy of a solution using objects. In other words, 20 of the 22 children (90.9%) answered all questions correctly.

Objective 4: Be able to demonstrate equality for the operation of "take away."

A balance scale and Numbars were used in the first activity. The lesson began by demonstrating to the group how the scale showed equality. The Numbars were used in a large group drill in which the teacher would tell them to show 9 - 5 using the Numbars. The children would then hold up the 9 Numbar and place the 5 Numbar in front of it to indicate subtraction. The teacher reported that the scale did not work too well but the children seemed to grasp the concept in spite of it. It was suggested that if it had functioned properly, an excellent activity would have been to have the children experiment with it, and possibly use it to solve problems.

In the second activity, children worked independently on worksheets containing problems of the form 8 - 5 = __. Numbars, cubes, or discs were available for them to use. It was noted that the children still did not know the addition and subtraction facts. However, they worked very well on the worksheets, and all seemed to understand what to do.

The third and fourth activities were quite similar to the second, except that the worksheets contained problems of the form 5 = 8 - __, and

Table 2

Evaluation of Topic 1, Objectives 2 & 3

| Task | Number of Children Answering Correctly n = 22 | Percent |
|---|---|---|
| 1. Using the + (plus) symbol: 10 = 3 ◯ 7 | 22 | 100 |
| Demonstrating with Numbars | 21 | 95.5 |
| 2. Completing the Addition Equation: 8 = 5 + □ | 22 | 100 |
| Demonstrating with paper clips | 21 | 95.5 |
| 3. Completing the Addition Equation: 13 = 9 + □ | 22 | 100 |
| Demonstrating with Unifix Cubes | 22 | 100 |
| 4. Using the Addition Table: 5 + 0 = □ | 22 | 100 |
| 7 + 8 = □ | 22 | 100 |
| 4 + 4 = □ | 22 | 100 |
| Total | 20 | 90.9 |

pairs of children were given flash cards to quiz each other. They continued to enjoy the activities and worked diligently on the problems.

In the next activity, the addition tables were used together with worksheets containing equations of the form 3 + __ = 8. It was felt that this form of the equation would lead into subtraction. However, the students had difficulty with the problems mainly because they did not read the signs carefully enough. It should be noted that the regular teacher was absent on this day and a substitute from the R & D Center taught the class. The teacher felt, on the second attempt at this activity, that it should have been demonstrated how answers could be checked using the table.

In the sixth activity, the addition table and worksheets were used again. The teacher led the class in a discussion of doing and undoing operations as an introduction to equations of the form 3 + __ = 8, and 8 - 5 = __ (the type of equations found on the activity sheet). The children had difficulty using the addition table for subtraction. It was suggested that the second equation did not make clear the process of undoing and perhaps would have been better if it read 8 - __ = 3. After much discussion with the staff, it was decided to use the table for summarizing and validating addition facts only. Some children may discover how the table could be used with subtraction, but it was decided not to spend anymore time on it as it was too frustrating for the children who did not understand.

In the next activity, the children were to translate subtraction problems into addition, i.e., check their answers using addition. They were supplied with various materials to work with, as well as the addition table to check their addition. The teacher reviewed the doing and undoing

process, and then gave the children worksheets to complete. Many of the children were still confused with the addition table and its use. They also seemed to lack motivation. It was decided the task was too difficult for the children at this time and so it was dropped.

Several days were spent on review of subtraction facts before advancing to the next topic. The children were given worksheets to complete using a variety of objects including their fingers.

EVALUATION III

For the third evaluation, the Subtraction section of the California Achievement Test was given to the group. The test included 15 problems on basic subtraction facts and 5 problems involving two-place subtraction, one of which was on borrowing.

Comparing the results of the subtraction evaluation (see Table 3) with those of the addition evaluation (see Table 1), it is apparent that the children did much worse on the subtraction. On the addition, 82.7 the questions were answered correctly, whereas on subtraction only 64.5% were answered correctly. However, five children had been identified by the teacher as not having comprehended the instructions. If this group is not included in the total, the percentage of problems worked correctly increases to 76.2%. The five children in this slow group were given additional lessons in subtraction and then four of the five were retested. The fifth child had done somewhat better than the other slow children; therefore, since he was not included in the retest, the average number of correct answers was lowered on the retest. Again, the children at the time of testing had not been given lessons on borrowing and had not worked with the subtraction facts as much as they had with addition.

Table 3

Mean Number of Correct Responses on the

CAT Math Test (Subtraction)

| Group | n | Problems 1-20 | Percent | Problems 1-15 Subtraction Facts | Problems 16-20 2-Place Subtraction | Problem 20 Borrowing |
|---|---|---|---|---|---|---|
| Regular | 17 | 15.24 | 76.2 | 13.88 | 1.35 | .059 |
| Slow | 5 | 5.0 | | 4.6 | 0 | 0 |
| Slow-Retest | 4 | 4.25 | | 4.25 | .4 | 0 |
| Total: Regular & Slow | 22 | 12.91 | 64.5 | 11.77 | 1.04 | .0405 |

TOPIC 2: REPRESENTING NUMBERS PICTORIALLY.

Objective 1: Be able to represent sets pictorially.

In the first activity, the teacher innovated a large group discussion by asking if there were more girls or boys in the class and how could they find out. The children suggested counting, matching up the boys with the girls, comparing two stacks of Unifix Cubes representing each sex, or putting a dot in each square of chart paper to represent the girls and the boys. The children were given chart paper and crayons to use in solving the problem. Next they were asked if there were more cats or dogs owned by the class. Again there were no problems in remembering how to compare or how to represent numbers.

The children were directed to draw pictures of cars and themselves in the second activity. Next they pasted their pictures on large pieces of graph paper. The children were reported to express an interest in making charts, but the activity did not keep them busy enough, i.e., they had to wait while the pictures were pasted onto the charts. They also made charts representing the number of people in the class, their favorite animals, foods, games, etc. Each child wrote these things on a slip of paper and then the slips of paper were collected. The children worked in pairs; one pair for each category listed. First they counted the data and then made the bar graph accordingly. All of the children were able to construct graphs but some did not start all the bars at the same point. Some also made the mistake of writing the same item two or three times (due to poor spelling and difficulty in reading).

The next activity involved interpreting their findings. After the graphs were finished and mistakes were corrected, the graphs were pro-

jected on an opaque projector and discussed. The teacher had to prod the children with questions but they were able to interpret the results.

In the fourth activity, the children were instructed to measure their wrist, ankle, thumb, head, and one length of their own choosing. They were given lined, margined paper, and one string per student (all the same length). After performing the measurements, they had to represent the lengths on the paper. The children were extremely enthusiastic and had no problems with the representations. It was reported that the children compared their graphs without being asked to.

For the fifth activity, the children assigned a number to the represented lengths. They were given graph paper on which to mark off units of length; however, some students used Numbars to mark off units on large paper.

Objective 2: Be able to represent numbers on a single line.

In the first activity, the class was directed to measure their waists, heights, feet, etc. Each was given a piece of string to use as a measuring tool, and each recorded his measurements on a number line. The children were quite interested in the outcomes and enjoyed reading the graph. They seemed to understand that a mark further out on the line represented a longer length.

In the second activity, the children were directed to measure four items of their own choosing and three objects chosen by the teacher. They again were given string, crayons, and recorded their measurements on the number line used for that object. It was reported that the children were intrigued by the fact that different students had different measures for the same object. They seemed to think that marks at either

extreme were due to the child not measuring the correct part of the object. All the children seemed more interested in performing the measurements than in the accuracy of the measurements. After some discussion, they did see a need to assign numbers to marks on the graph.

Objective 3: Be able to contruct a Number line, and use it to measure objects in terms of units.

Adding machine rolls, Numbars, and pencils were used in the first activity. The children were instructed to draw a straight line down the center of the paper, and to use the Numbar to mark off units. They worked diligently and were challenged by the task. They had some difficulty, however, in drawing a straight line and in completing the long task of writing 100 numerals. It was also noted that the necessity of making the marks equidistant should have been emphasized.

In the second activity, the children were given worksheets with a list of objects next to which they were to write its length in units. Their Number lines were to be used as the measuring devices. The worksheets also included such questions as which object is longest, shortest, and how much longer than your pencil is your paper. The questions were read to the whole class, and then they worked individually. Interest was high, and measurements were roughly accurate.

TOPIC 3: WRITING SENTENCES USING THE SYMBOLS: +, -, =, >, <.

Objective 1: Be able to compare and equalize measurements.

In the first activity, the children were given worksheets listing three sets of two objects. The children were instructed to measure the two objects on the property of length using their Number lines and then to compare them and write three sentences to represent their relationship (e.g., $3 < 5$, $3 = 5 - 2$, $3 + 2 = 5$). After completing the sentences for each set, they

had to order all six objects from smallest to largest. The teacher reported that most of the children could verbally compare the objects and state the relationships but all had difficulty writing the equalization sentences.

For the second activity, the group was again given a worksheet. First, they had to pick two numbers, compare them (e.g., $2 < 8$, $8 > 2$), and then equalize them (e.g., $2 = 8 - 6$, $8 = 2 + 6$). The teacher discussed with the group the properties of the Number line, and introduced the word "origin." It was explained that the further the number was from the origin, the greater the number was. The children still had difficulty writing equations.

Objective 2: Be able to classify and group by tens.

In the first activity, the children were given stacks of tiles and were instructed to detach tiles from their backings and to pack them into boxes according to color. They worked in groups of two and three. They were very interested in the activity.

In the next activity, the students were instructed to count out the number of tiles in each box and then group them by tens. Next, they were to label each box and indicate the number in each using expanded notation (e.g., 18 stacks of 10, or 18(10)s). Later, they made Number lines representing the number of stacks of ten for each color. Another activity involved comparing the number of tiles ordered with the number delivered. After these activities were completed, the children used the tiles to make designs and gave them away as Christmas presents. Only the mathematical aspects of counting and grouping etc., were emphasized since it was felt that discussing geometrical patterns would have been too confusing. Children did enjoy working with the tiles.

After Christmas vacation, the first activity involved reviewing the completion of equations. The children were given worksheets to complete and could use a variety of material (Numbars, discs, beans, cubes, balance beams, etc.) The children worked individually and had no apparent problems.

In the fourth activity, they were given worksheets, commercial Number lines, and instructions on how to use the Number line to translate + and - operations. The children had a great deal of difficulty with this activity. It was believed to be too abstract and, consequently, it was decided they should work more with pictorial representations.

Objective 3: Be able to compare and equalize visual representations of length.

For the first activity, the children were supplied with prepared graph paper (on each sheet two straight lines of different lengths were drawn). The students were directed to compare and equalize the two pictured line segments and then to write the corresponding sentences. The results were very discouraging. The children again had a great deal of trouble writing sentences. Consequently, for the second activity, the teacher dictated sentences to the children (e.g., $4 < 5$, $10 = 6 + 4$, etc.). The children were to write the equations as she stated them. It was reported that the children enjoyed the exercise, but some still had trouble with the symbols $>$ and $<$.

For the next activity, the teacher again dictated a few statements but let the children make some of their own. They were allowed to use objects and Number lines if they wished. The reaction was fantastic. Many found patterns to work on (e.g., $1 < 2$, $2 < 3$, $3 < 4$, etc.; $9 = 0 + 9$, $9 = 1 + 8$, . . . $9 = 9 + 0$; $30 > 0$, $29 > 1$, . . . $15 > 14$, $14 < 15$ . . . etc.). The children were quite interested in the activity.

For the fourth activity the children were again supplied with papers that already had two lines drawn on them. They were instructed to measure the lines using the Numbars, to compare and equalize the lengths, and to write the corresponding equations. It was reported that there was a definite improvement in their writing ability, especially using the symbols > and <. In spite of the fact that most of the children could verbalize the equalization process correctly, some were still having difficulty writing the equation , e.g., they wrote 5 = 11 + 6, 8 = 3 - 5.

In the next activity, the children were instructed to draw their own lines, to compare and equalize them, and to write the appropriate equations. They continued to improve.

Two of the next activities again involved constructing a Number line. In the first one, Numbars were used to make the Number line, while in the second activity, different units were used to construct it. It was stressed to the group that the Number line could be made with any unit, so long as that unit is used consistently. For these activities they were given adding machine paper and a variety of objects. All but one of the children could construct the Number line with no difficulties.

Objective 4: Be able to use pictorial representation to solve word problems.

In the activity for this objective, word problems were read to the entire class. The children were instructed to draw anything that would help them solve the problem. After three problems had been read, the teacher had some of the children show their solutions. Afterwards, more of the children were able to solve the problems but it was noted that they still needed to work on this area.

EVALUATION IV

Since this evaluation was administered individually, the other children not being tested were given worksheets containing addition and subtraction problems of the type:

$$\begin{array}{r} 24 \\ -\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 10 \\ 7 \\ +\ 6 \\ \hline \end{array} \qquad \text{and} \quad 7 + 3 = 2 + \underline{\quad}$$ (more difficult for the children).

The first two tasks of the evaluation were administered individually and the third task was administered as a group test. Testing time per student was approximately 20 minutes.

In task one, the child was given a piece of graph paper, crayons, and a stack of felt cut-outs. The child was to represent the number of cut-outs of each color on the graph paper. Then using only his graph representations, he had to tell which color had the largest number of objects and how he could make the number of one color equal to the number of another color. He then wrote the equation to represent the process.

In the second task, the examiner drew a long, straight line on the back of the graph paper and gave the child 2 strips of construction paper and a piece of string. The child was required to measure the 2 strips with the string and represent each length on the line. Then using the representations only, the child was to determine which strip of paper was longer. The child was given a box and was asked to use it as a unit of length to determine how much longer the one strip was than the other and to write the corresponding mathematical sentence.

The children were given a piece of metal, a strip of adding machine paper and a straight edge for the third task. They were then asked

to construct a Number line using the material.

Results of the evaluation (see Table 4) indicate that the children did very well on most of the items. However, they did have some difficulty writing equalization and comparison sentences and drawing a straight line. Eight children answered all of the questions correctly.

TOPIC 4: GROUPING AND EXPANDED NOTATION.

Objective 1: Be able to group objects by tens and represent them pictorially.

In the first activity, a number of objects were used including Lots-A-Links,[3] discs, porcelain objects, beans, etc. The children were instructed to count out a number of objects at their seats by grouping them by tens, and then to write the number in expanded notation (e.g., 4(10) + 3(1)). It was reported that they enjoyed working with a variety of materials. There were no great problems.

In the second activity they used the tablets in which they had written the numbers for the previous activity. They were instructed to represent these numbers pictorially using graph paper. The teacher reported that almost everyone needed help but that there were some interesting and original responses. One of the children asked if 3(10) + 5(1) was an equation. This was followed by a group discussion on the subject.

In the third exercise, the children were given graph paper, physical objects, and index cards with numbers written in expanded notation on each card. They were instructed to work in pairs. One child represented the number pictorially (with graph paper), and the other used physical objects to represent it; then they switched activities. It was reported that all

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[3] Lots-A-Links - Amsco Industries, Inc., Harboro, Pennsylvania.

Table 4

Evaluation of Topics 2 & 3

| Task | Number of Children Answering Correctly n = 20 | Percent | Number of Children Helped (Answer Counted Correct) | Percent |
|---|---|---|---|---|
| 1. a) Visually representing numerousness of graphs | 18 | 90 | 4 | 20 |
| b) Comparing and equalizing visual representations of sets | 19 | 95 | 1 | 5 |
| c) Written expression of equalization (5 = 3 + 2, 5 - 3 = 2) | 17 | 85 | 1 | 5 |
| 2. a) Visually representing lengths on a line | 20 | 100 | 4 | 20 |
| b) Comparing visual representations of length | 20 | 100 | 0 | 0 |
| c) Comparing lengths with unit of measurement | 18 | 90 | 2 | 10 |
| d) Written expression of comparison (3 > 2, 2 < 3) | 14 | 70 | 2 | 10 |
| 3. Constructing a Number line: | | | | |
| a) Straight line | 16 | 80 | 0 | 0 |
| b) Unit | 18 | 90 | 0 | 0 |
| c) Zero | 19 | 95 | 0 | 0 |
| Total | 8 | 40 | 11 | 55 |

the children worked well under this situation.

In the next activity, the children took turns in "acting out" a number using their fingers while the rest of the class wrote the number in expanded notation. All of the children were able to write the number correctly. It was noted that they enjoyed working with large numbers.

Objective 2: Be able to exchange 10 ones for 1 group of ten, and combine 2 sets.

A variety of materials was used for the first activity, cardboard boxes, cuisenaire rods, Numbars, discs, chains, tablets, etc. The class was divided into groups of 3. The banker gave the other two children two piles of objects. They in turn grouped the objects by tens, and exchanged 10 groups of one object for 1 group of ten (another object) with the banker. Then they wrote the number in expanded notation. The three children took turns being the banker. All of the children liked the "banking" idea and responded very well to the activity. No problems were noted.

The next activity was very much like the one just mentioned. However, rather than each child writing a different equation, they combined their sets and wrote the resulting number.

The worksheets for the next activity contained groups of two sets that were to be combined and the resulting number was to be written. Many required that the child regroup the ones into groups of tens and leftover ones (e.g., 4(10) + 6(1) combined with 3(10) + 5(1) results in 7(10) + 11(1), which can be regrouped into 8(10) + 1(1)). Discs and cuisenaire rods (tens and ones) were available for use.

The children worked in pairs on the fourth activity. One child was given index cards with numerals written in expanded notation on the cards. The other child was given a number of objects from which he took the numbers

represented by the written numerals. These he gave to the other child who combined the two sets of objects and then both children wrote the resulting number in expanded notation. They were again given a variety of objects. The children remained quite enthused about the activities; however, some of them had to be reminded to regroup the ones when there were more than 9.

For the next activity, the children were divided into a fast and slow group. They sat in a circle around the teacher who made a series of sounds using a drum, triangle, and her hands. The students were instructed to put a tally mark on a piece of paper for each sound the teacher made. When the sound stopped, the children were to write the number in expanded notation. The fast group was instructed to combine two sets of sounds and express the resulting number in expanded notation. The teacher reported that the slow group needed a great deal of help; they had to be constantly reminded to circle ten tally marks and express the number in expanded notation. The slow group was not asked to combine two sets. It was also mentioned that interest was low in the slow group but quite high in the fast group.

The children worked individually on the following activity. Each student was given two index cards, each containing a numeral written in expanded notation. They were instructed to represent each numeral using tally marks, and then to combine the two sets of tally marks and write the resulting number in expanded notation. If they did this correctly, they were given a new pair of cards. This task was easily accomplished by all children. It was noted that the children expressed a desire to work with larger numbers (greater than 50).

Worksheets with two sets of objects drawn on each page (the total being less than 100) were given to the children. Each child was to write the number of objects in each set in expanded notation and next to combine the two sets and write the total in expanded notation. It was suggested that many of the careless errors made by the children may have been due to the arrangement of the objects on the page. The objects may have been too close together making it difficult to group them by tens.

Objective 3: Be able to exchange 1(100) for 10(10)s and write 3 digit numbers in expanded notation.

The first activity was a repeat of one used earlier. They worked in pairs; one child was the banker. The other was given a stack of objects that he was to group into hundreds, tens, and ones. Then he was to exchange these for one group of one hundreds, etc. Then both he and the banker were to write the number in expanded notation. There was difficulty with the discs that were used; they continued to break, which kept the attention of the children away from the lesson. However, the children seemed to know without being told how to write expanded notation for hundreds.

The children worked in pairs for the second activity also. They were given play money and cards with numerals written on them in expanded notation. One child was designated as banker and received a sum of money (he was to determine the amount and write it in expanded notation). The other child presented his two cards one at a time to the banker and received the amount specified on the card. He would determine the total amount he was given while the banker would count his remaining money and then both were to write the number in expanded notation. It was noted that most of the children followed only the first direction. They seemed to lack interest since they saw no need to perform the tasks. However, they

were excited about the play money but the money was somewhat confusing because it was not labeled.

For the next activity, the children made objects to be sold at the market and priced each object (price was written in expanded notation). Each child received a sum of play money ($3.83) with the amount written on each piece. They used six cash boxes and wallets made from envelopes. The class was divided into three groups: 6 clerks, 6 buyers, and 6 to write down their transactions. The buyer was required to write the amount of money he had, the amount he spent, and how much he had left after the purchase. If he did not have the exact change, he had to give the clerk a dime for ten pennies, which would give him the correct change. The children were very excited about the activity and saw a real need to be able to exchange tens and ones.

The class was divided into groups of three students for the fourth activity. The children received a large pile of pegs, cubes, or Numbars which they were to group into hundreds and then write the number in expanded notation. Numbar boxes were used to hold ten Numbars to represent one hundred and also were used to hold Unifix cubes. Peg boards were used with the pegs; each board held one hundred pegs. The task was easily accomplished by all of the children.

The children were given computer output sheets each containing a random arrangement of X's (the number varied between 200 and 940, in even tens), and two pencils--red and blue. They were instructed to use one pencil to circle a group of ten and the other pencil to circle a group of a hundred (ten groups of ten). It was noted that the activity became more of a geometric problem than a math problem. The children had difficulty because they did not group X's that were in a roughly circular region;

they missed some X's and consequently could not make another group of ten. However, they did seem to understand how to group by tens.

TOPIC 5: ADDITION AND SUBTRACTION.

Objective 1: Be able to add and subtract numbers written in expanded notation (<100) with the aid of concrete objects.

In the first activity, the children were given a variety of material to choose from (discs, cubes, rods, etc.). All of the materials came in units of ones and tens. The children were instructed to get 9 tens and 18 ones of any material. The teacher handed each child a card with either a subtraction or addition problem on the front and the names of the children in the class on the back. The problems were of graded difficulty. After a child had completed a problem, he was to cross off his name and advance to another problem. He was also told to copy each problem and its solution on paper. However, many of the children failed to copy the problem and consequently, it was often impossible to determine the accuracy of a solution. Many of the children had to be reminded to regroup ones when the group contained a number greater than ten.

Worksheets containing problems of graded difficulty were next given to the children. There were four different sheets containing either: 1) addition without carrying, 2) addition with carrying, 3) addition and subtraction without carrying or borrowing, or 4) addition and subtraction with carrying and borrowing. The children again received a number of objects to work with in solving the problems. They were to solve the problems using expanded notation. It was reported by the teacher that many of the children had difficulty subtracting in expanded notation. For example, many subtracted the tens but added the ones; they failed to see that when subtracting, the numeral in expanded notation was one number

(e.g., $\begin{array}{r} 4(10) + 1(1) \\ -3(10) + 2(1) \\ \hline 1(10) + 3(1) \end{array}$). Other children also had difficulty with borrowing; they would "take away" all the ones that were available and then would put 0(1)s as the answer.

For the third activity, the children were again given worksheets. However, this time there were only two different sheets, and each contained four problems (all involving borrowing). The teacher began the lesson by working one of the problems on the blackboard and drawing an illustration to show the borrowing process (she exchanged a ten Numbar for 10 ones). The children were allowed to use a variety of materials. It was noted that much individual attention was necessary. One common mistake the children made was not using a complete ten, but only the number of ones they had to subtract. The material that worked best was the red discs since they could be broken apart rather than trading it in for 10 ones.

The fourth activity was introduced as a game. Each child was given 9 ten bars and 9 one bar (objects). They were told to write that number on their paper and then to throw a dice (given to them also) and subtract the number shown. After physically taking away the specified number of objects, they again wrote the remainder on their paper. They repeated the above process until there were no more objects remaining on their desks. The teacher reported that learning the process was difficult for the children. She felt they should not have been required to write the numbers on paper; taking away the objects themselves without recording the activity would have simplified the game.

On several occasions, the children were given worksheets to work on independently so that the teacher could help those children having problems. At the end of these lessons, the teacher noted that only half the class could borrow successfully. Therefore, in order to give the children a rest and

reconsider instruction on subtraction, she introduced some geometry.

TOPIC 6: GEOMETRIC SHAPES.

Objective 1: Be able to have experiences with and to use geometric shapes.

Plastic squares, circles, triangles, and rectangles were used in the first activity. Each student received one shape and was directed to trace his shape on a piece of paper as many times as possible. Then he was to cut the shapes out and write the number in expanded notation on the board. The children were fascinated with the activity. When they were finished with one shape they wanted to do another. They were very interested in the numbers on the board.

The children were given duplicated sheets listing the number of objects they had cut out in the previous exercise. Each shape was listed on different sheets. The children received graph paper and physical objects to work with. The class was divided into groups. Two children were assigned to each shape and were to make a graph for that shape. The rest of the group added the total number of that shape made by the class. The children had no trouble making the graphs or adding the numbers (a few had to be reminded to regroup the tens and hundreds). Most of those making the graphs started a new line for each ten rather than using a complete line for each number. Many of the children added successfully without using physical objects.

For the third activity, the children were given trapezoidal shapes (plastic) and paper. They were instructed to trace the shape as many times as possible onto the paper and then to write the number in expanded notation on the back of the paper. As before, the children were very interested in the activity. Next, they were given graph paper and crayons and were told to fill in one square for each shape on their paper. They had no

difficulty with this exercise.

EVALUATION V

All six tasks of this evaluation were administered individually. Testing time per student was approximately 15 minutes.

In task one, the child was to determine the number of discs, Numbars, or Unifix Cubes that was placed on the table and then was asked to express each number in compact notation.

In the second task, the child was presented with the number 26, was asked to represent it with discs, and then to tell how many ones, hundreds, and tens there were in the number. Task three was the same as task two, but the number was changed to 372.

Comparing 2 numbers was the objective of tasks four and five. The child was presented with 2 numbers (53, 59 and 78, 28), was asked to tell which was larger, and by how many tens or ones it was larger.

In task six, the child was presented with 3 numbers, one was in compact notation (54) and the other two were in expanded notation (3(10) + 1(1), 4(10) + 13(1)). The child was asked to write each number in some other form of notation.

Results of the evaluation indicate that the children did very well except for comparing numerals on one's and ten's places (see Table 5). The difficulty with comparing numerals may have been due to the fact that subtraction of 2 digit numerals was involved.

TOPIC 7: COMPACT NOTATION

Objective 1: Be able to represent the spoken numbers with numeral cards and physical objects.

Each student received one set of numeral cards and some paper cups for the first activity. The children met in a large group. The teacher

Table 5

Evaluation of Topics 4, 5, & 6

| Task | Number of Children Answering Correctly n = 19 | Percent |
|---|---|---|
| 1. Representing objects with compact notation: | | |
| 15 | 19 | 100 |
| 63 | 18 | 94.7 |
| 95 | 18 | 94.7 |
| Sub-total: | 18 | 94.7 |
| 2. & 3. Interpreting numerals: with concrete objects: | | |
| 26 | 18 | 94.7 |
| 372 | 18 | 94.7 |
| with ones: | | |
| 26 | 18 | 94.7 |
| 372 | 18 | 94.7 |
| with tens: | | |
| 26 | 18 | 94.7 |
| 372 | 18 | 94.7 |
| with hundreds: | | |
| 26 | 17 | 89.5 |
| 372 | 18 | 94.7 |
| Sub total: | 17 | 89.5 |
| 4. & 5. Comparing numerals: which is larger: | | |
| 53-59 | 19 | 100 |
| 78-28 | 19 | 100 |
| by how many ones: | | |
| 53-59 | 5 | 26.3 |
| 78-28 | 11 | 57.9 |
| by how many tens: | | |
| 53-59 | 9 | 47.4 |
| 78-28 | 5 | 26.3 |
| Sub total: | 4 | 21 |
| 6. Writing numerals in other forms of notation: | | |
| 54 | 18 | 94.7 |
| 3(10) + 1(1) | 18 | 94.7 |
| 4(10) + 13(1) | 10 | 52.6 |
| Sub total: | 10 | 52.6 |
| Total: | 4 | 21 |

said a number and the students were to represent it with numeral cards and/or with cups. It was reported that there was no difficulty using the paper cups; however, there was some confusion with the nine numeral card--upside-down it read six. Otherwise there were no problems.

For the second activity, the children were divided into small groups. One student represented a number with cubes and the rest of the group represented the same number with cards. They were given numeral cards and Unifix Cubes. The teacher reported that not all the children were interested in the activity.

Objective 2: Be able to express numbers in compact notation.

In the first activity, the children were given the numeral cards and worksheets containing visual representations of numbers to work with. First, the teacher met with them in a large group and dictated numbers to them. The children were told to use the numeral cards and to form one compact numeral with them. Next, each child used the worksheets to obtain the numbers and represented them with the cards also. It was reported that the children enjoyed working with the cards and had no problems forming compact numerals.

Stations were set up for the second activity. The students rotated from station to station and worked the problems. Some of the materials used were souffle cups, poker chips (stackable objects), and numeral cards. In the first type of station, the children were required to group objects, represent the number with numeral cards, and then write the number in compact notation. At the second type of station, children found a card with a numeral written in compact notation. They were to represent the numeral with tally marks. The teacher commented that the station set-up

worked very well.

In the next activity, the students were given cards with expanded numerals written on them, pencil and paper but no concrete objects. They were told to write the expanded numeral in compact notation and to represent the numbers physically (tally marks). It was noted that the children were having no problems at all; they seemed quite confident and interested.

TOPIC 8: REVIEW (GROUPING ONES AND TENS).

Objective 1: Be able to express numbers in several forms--to break 1(10) into 10(1)s and vice versa.

The first activity after spring vacation began with a large group discussion about the equivalence of different written expressions of the same number 2(10) + 2(1) = 1(10) + 12(1)). Each child was given a number of verti-pegs, paper and pencil. They were instructed to group the pegs by tens and to write the resulting number in expanded notation. Next they were to break down one group of ten into ones, write the number in expanded notation, and continue the process until all groups of ten had been used. It was commented that all the children showed understanding. It was also noted that the pegs worked quite well.

In the second activity, the teacher worked through one problem, writing every step, and validating each step with pictorial representation (i.e., 23 = 2(10) + 3(1) = 1(10) + 13(1) = 1(10) + 10(1) + 3(1) = 1(10) + 13(1)). Then the children were given problems similar to those given in the first acitivity. The teacher felt the method of presentation was too time consuming and that the "station" method would probably have been better. With the stations, the children could have worked at their own speeds. It was also noted that several of the children performed the task with-

out using objects.

The children were not allowed to use physical objects in the third activity. The teacher wrote compact numbers on the blackboard and had the students write two equivalent expressions in expanded notation. The teacher reported that many had difficulty working without objects and that the entire class seemed poorly motivated--they saw no reason to do the tasks.

For the next activity, stations were set-up with index cards at each. The card had one numeral written on it in one of three forms (compact or a form of expanded notation). Objects were available for those who wanted them. The students were required to copy the numeral on the card and write an equivalent one next to it. All but one child understood the method; the one child interchanged the tens and ones on almost every problem (i.e., $3(10) + 6(1) = 6(10) + 3(1)$).

Stations were again used with the fifth activity. At each station the students were to group the objects into groups of hundreds, tens, and ones and then to record the grouping in expanded notation. Only two of the children had difficulty working the problems; the others did very well.

Objective 2: Be able to write numbers in compact notation.

In the first activity, the children were given numeral cards (hundreds, tens, and ones). The teacher dictated numbers in expanded notation (i.e., 3 hundreds, 4 tens, and 7 ones); the children were told to put the cards in front of them on the table. Next, they were instructed to put the cards together to form a compact numeral. The children were able to follow the directions; however, there was some confusion caused by a number of visitors.

For the second activity, the children were given worksheets and numeral cards. The worksheets contained lists of numbers written in expanded and

compact notation. The students were to write the number in the form not given. It was reported that the students worked well on the assignment and were very enthusiastic with the worksheets put in book form.

EVALUATION VI

All three tasks were individually administered. Testing time per child was approximately 15 minutes.

Task one involved having the child determine the number of objects on a table and writing the number in expanded notation by asking him how many hundreds (when applicable), tens, and ones there were in the number. Numbars, Unifix Cubes, and paper cups were used to illustrate the numbers 12, 31, 84, and 129.

In the second task, the child was presented with the numeral 4(10) + 6(1), was asked to represent it with paper cups and tell how many hundreds, tens, and ones in the number.

For task three, the child was presented with two addition problems written in expanded notation. He had to write the answer in expanded notation showing how many tens and ones were in the number. There were objects available for the student to use. Regrouping was required for one of the problems.

The data from the evaluation suggest that the children had difficulty with the hundred's place, particularly writing in expanded notation. (See Table 6.) Some also had problems with addition facts, regrouping, and carrying. Overall, the children did very well on the evaluation.

TOPIC 9: PATTERNS IN NUMBERS

Objective 1: Be able to find patterns in the number grid, and complete the pattern.

Table 6

Evaluation of Topics 7 & 8

| Task | Number of Children Answering Correctly n = 19 | Percent |
|---|---|---|
| 1. Representing numbers with expanded notation: | | |
| 1(10) + 2(1) | 19 | 100 |
| 3(10) + 1(1) | 19 | 100 |
| 8(10) + 4(1) | 19 | 100 |
| 1(100) + 2(10) + 9(1) | 9 | 47.4 |
| Sub total: | 9 | 47.4 |
| 2. Interpreting written numbers [4(10) + 6(1)]: | | |
| concrete objects | 19 | 100 |
| how many ones | 19 | 100 |
| how many tens | 19 | 100 |
| how many hundreds | 17 | 89.5 |
| Sub total: | 17 | 89.5 |
| 3. Addition Problems: | | |
| a) 3(10) + 7(1) | | |
| 1(10) + 1(1) | 17 | 89.5 |
| b) 6(10) + 8(1) | | |
| 2(10) + 6(1) | 17 | 89.5 |
| c) carrying | 17 | 89.5 |
| Sub total: | 14 | 73.7 |
| Total: | 7 | 36.8 |

For the first activity, the children were required to make a number grid using graph paper. They were very enthused and like seeing patterns emerge. Number Grid:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |

The students were given dittoed sheets with a number grid on it. They were instructed to pick a number between 1 and 10 and color that number and each multiple of it (i.e., they were to count out that number again and again to make a pattern). The teacher reported that there was a wonderful response and it was a great open-ended activity. Everyone was vitally interested in the patterns. With some encouragement and leading questions, some discovered 6 and 12 were similar in that 12 had every other colored square of 6. The teacher reported fair accuracy.

Objective 2: Be able to find a series of numbers.

For the first activity, children were instructed to choose a number, make a bar using that many Unifix Cubes, and place the bar on the Unifix number track to find its pattern (e.g., 8, 16, 24, 32, . . .). The children became quite involved in discovering patterns. However, some were bothered when they did not reach 100 exactly.

The next activity involved counting up or down by 6's, 7's, etc. Unifix Cubes and the number track were again used. The teacher reported that the children did poorly but felt that it was due to the fact that they had been tested the day before. The directions had been written on cards and varied

in difficulty. It was felt they should have been separated by level of difficulty.

Overhead projector transparencies (2 per child), grease pencils, and 100 squares were the materials used in the third activity. The children were to place the transparencies over the 100 squares, and were to color in appropriate squares as directed. At the end of the period, the teacher projected the transparencies and they discussed their similarities and differences. The teacher reported that there was probably more excitement over using the materials than interest in the mathematics.

TOPIC 10: BE ABLE TO TELL TIME (HOUR AND HALF HOUR). The children made a pendulum clock for their first activity. It was noted that they were fascinated by the clock; the boys were interested in the mechanics of it.

Next, they were given toy clocks to work with. The teacher dictated a time, and they represented it on their clocks.

The children worked independently on SRA worksheets on clocks for the third activity. It was observed that they worked through the papers quickly and easily. Almost all of the children finished them within 10 or 15 minutes. Then they played clock solitaire.

TOPIC 11: BE ABLE TO ADAPT TO A TRADITIONAL PROGRAM. Since the R & D Center had decided to change the location of this pilot program to a different school, it was decided that to ease the transisiton the teacher should convert to a more traditional program during the last two weeks of school. Therefore, the children were given SRA worksheets involving computations in addition and subtraction. Generally, they seemed to adapt well to the program. Most of

them used their fingers and tally marks to work the computations; only two or three had their facts memorized. They also did some work on number patterns--counting by 2's, 5's, 10's, and counting backwards. Many had difficulty counting backwards, but otherwise they seemed to know what they were doing.

## III

## SUMMARY

Many problems were encountered throughout the year; some were solved, but many were not. For example, the children were confused when equalizing sentences about which side to adjust, how to use the Number Line to translate + and - operations, and specific problems in borrowing. Several difficulties centered around writing mathematical statements and understanding the greater than (>), less than (<) symbols. These problems along with their suggested solutions will be considered in rewriting Books 3 and 4 prior to the second pilot tryout at Randall Elementary School. Also before the second tryout, Miss Moscovitch is to visit England to collect ideas on their instruction program (see Hello From Merrie England, in preparation). Findings related to the above problems and to math instruction in general will be incorporated into later books.

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