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ABSTRACT

Viewing the number system as a complex of interrelated words, three studies are described: (1) a study of the difficulty of simple addition and subtraction in different decimal positions; (2) a study of variations in the number of digits in the addend or minuend; (3) a study of the effect of irregular morphemes occurring in some number words. Findings proposed were (1) arithmetic tasks involve sequences of operations of reading and arithmetic called shifts; (2) different arithmetic performances require different shifts and thus some performances require more time to complete. (JG)

Effects of morphemic idiosyncracies in number words on performing arithmetic operations

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The number system may be viewed as a complex of interrelated words. What is distinctive about this complex is that the relationships may be described in logical terms. One focus of the research to be discussed is the psychological use made of the logic built into the system. We deal with the question, in Charles Morris' term, of the pragmatics of numbe. use. The finding of non-equivalent difficulty in logically comparable tasks should reflect distinctive characteristics of the cognitive operations of the number user.

Let me insert a methodological comment here. It is the logical equivalence of the relationships between different sets of number words that permits one to instruct a subject to perform the "same" operation on many sets of numbers. Measures such as the total time to perform 90 or 100 simple operations (and it is this sort of measure that has been used in the research to be discussed) then take on the character of averages, summarizing the subject's performance over many individual occasions. The final experiment to be reported shows, however, that this assumption that the operation is the "same" entails some risk, as some logically equivalent or identical relationships are not always processed equally readily by our subjects.

The studies reported here represent a detour undertaken in the course

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of investigating the relationships between free counting, defined as the generation of sequences of count numbers apart from any objects counted, and the parallel arithmetic operations. I was led by way of concern with the carrying operation in addition to the first experiment, which studies the difficulty of simple addition and subtraction in different decimal positions when carrying is precluded by the selection of stimulus numbers. The finding of a decimal position effect led to an analysis of the problem in terms of different sequences of cognitive operations. The second experiment studied variations in the number of digits in the stimulus numbers. This was expected to affect the sequences of cognitive operations without changing the operations themselves. The persistence of the decimal position effect despite variations in context led me to look more closely at the structure of the subjects' utterances. The third experiment tested a specific hypothesis concerning the effect of irregular morphemes occurring in some number words. This level of analysis seems, at the moment, to be on the right track but the problem of the decimal position effect has not been completely resolved.

The first experiment studied the difficulty of adding the digit "1" to or subtracting it from numbers at different decimal positions. I refer to this class of operations as <u>unit arithmetic transformations</u>. This study was reported at APA in 1969 and is summarized in the <u>Proceedings</u> (Chevalier, 1969). Subjects were asked to read a list of 100 specially constructed 3-digit numbers or to add or subtract the numbers 1, 10, or 100 from other sets of numbers derived from the original response list. They were to give their responses as numbers rather than as sequences of digits; e.g., "three hundred twenty-one". Adding 10 or 100 is equivalent to adding 1 in the tens position or in the hundreds position. The digits making up the response numbers included those from 1 to 8 and these were almost completely balanced in the three decimal positions. Stimulus lists for a particular arithmetic instruction, such as that to "Add 10", were derived from the response list by performing the complementary operation upon the numbers in the response list. In this case, 10 was subtracted. The order of the stimulus items was then randomized. The subject regenerated the original response number in performing the indicated operation. This step is later referred to in emphasizing that Ss make the same component responses to two or more lists. The need for carrying to another decimal place was avoided in both addition and subtraction tasks by limiting the digits in the response numbers to the range from 1 to 8. The principal dependent variable to be discussed is the total time to perform 100 operations. Intra-subject designs were used in which each S performed all tasks.

The study of 14 college males and 14 female students at the University of Maine in Portland showed that the speed of adding or subtracting was not independent of the decimal position. Instead, an inverted-V relation appeared between response time and decimal position, with the subjects adding 100 most rapidly, adding 1 almost as rapidly, and adding 10 most slowly. The same function described the results for both males and females and those for addition and subtraction. There were overall differences between the sexes and between the arithmetic operations but there was no evidence of interaction among these variables. The graph for the addition tasks of Experiment 1 is shown in Figure 1 as 3 open dots. The time scores presented here are difference scores obtained by subtracting the time required to read the response list from the times to perform the various arithmetic tasks. The differential time score for adding 1 in the hundreds position was 16.0 sec., in the tens position, 32.1 sec., and in the units position, 21.0 sec.

The likelihood that the perception of the numbers was biased by systematic differences among stimulus lists was diminished by balancing the frequency of occurrence of digits in the different decimal positions. The distributions of digits in the stimulus lists for the arithmetic tasks were comparable even though they may have differed from the stimulus list for the reading task. Because of this control and because the motor aspects of the component responses were identical for all tasks, the observed differences in response times at the different decimal positions is held to reflect differences in central processing up to and including the point at which the response is selected.

After the fact I was led to look at the different arithmetic tasks as sequences of operations of reading and arithmetic performed in different orders. For arithmetic performed in the hundreds position of a 3-digit number, the order of operations would be Arithmetic-Read-Read. For the tens position the order would be Read-Arithmetic-Read. For the units position the order would be Read-Read-Arithmetic. The first and last of these appeared to require only one change in the type of operation, while arithmetic in the tens position appeared to require two such shifts. If such shifts in operation required finite amounts of time, then a condition requiring two such shifts might be expected to take longer than one which required only one. Since the number of shifts depends not only upon the decimal position at which the arithmetic operation is performed but upon the total number of digits in the number, a test of the "number of shifts" hypothesis was readily available. For 3-digit numbers, as I have indicated above, adding 10 requires two shifts, while for 2-digit numbers adding 10 requires only one. Since adding 1 also requires only one shift, addition in the two decimal positions should be carried out equally rapidly.

Following the same logic, again as shown earlier, adding 100 to a 3-digit number requires one shift and is carried out relatively rapidly. Adding 100 to a 4-digit number, however, requires the operations to be performed in the order Read-Arithmetic-Read-Read, making two shifts. Under these conditions the addition should take relatively longer to perform; as long, in fact, as it takes to add 10.

The data of this experiment were collected by Peter Patall at the New York State University College at Geneseo. Ten male and 10 female students were given 12 tasks requiring them to read lists of numbers and to perform unit additions in all decimal positions for sets of 100 2-digit, 3-digit, and 4-digit numbers. All of the 2-digit tasks were given before the 3-digit tasks which, in turn, preceded the 4-digit tasks. Within the set of tasks for a particular number of digits the order of tasks was counterbalanced across subjects. The mean difference scores are represented in the three curves in Figure 2 which are labelled "Experiment 2". The upper curve containing two points represents the data for the 2-digit task. Adding 10 took 55.3 sec. longer than Reading, while adding 1 took 33.4 sec. longer: a difference of 21.9 sec. The curve containing 3 points represents the data for the 3-digit task. Here adding 10 took more than 16 sec. longer than did the other two additions. This curve approximates the curve obtained in Experiment 1. Finally, the four-point curve describes the results for the 4-digit conditions. The difference score for adding 10 was 33.0 sec. while that for adding 1 was 17.8 sec., a difference of 15.2 sec. The difference score for adding 100 was 26.8, which was 6.2 sec. less than that for adding 10.

Several analyses were performed, since the design was not symmetrical. Analyses within each of the number-of-digit conditions showed a significant main effect associated with the decimal position in which addition was performed but no main or interaction effects associated with the sex of subjects. A partial analysis, which might be termed the vertical analysis, compared the Add 10 and Add 1 tasks under the 2-, 3-, and 4-digit conditions. The variables of the decimal position of the addition and the number of digits in the number demonstrated significant main effects, with no evidence of interaction. The fact that adding 10 was performed slowly in the 2-digit condition did not support the hypothesis that requiring only one shift from an Arithmetic to a Reading operation would speed up the performance of this addition.

Addition was slowed most, relative to reading time, for the 2-digit task, less for the 3-digit task, and least for the 4-digit task. This trend may best be explained by supposing that there is some overlap in the performance of the reading and arithmetic operations. While reading and saying as numbers the digits which precede the target decimal position, it is presumed that the subject makes some progress on the processes of the arithmetic operation. When he gets to the point of the actual addition he has only a portion of the process to complete. In the 2-digit task, where no digits precede addition in the tens position, the longest time must be spent in performing the addition. In the 3-digit task, with one digit preceding addition in the tens position, the time attributable to the addition is less. The change in time associated with the 4-digit condition is in the same direction but smaller in magnitude and, as we shall see, not significantly different.

The other step in evaluating the shift hypothesis requires that we look at the addition of 100 in the 3-digit and 4-digit conditions. Adding 100 in the 3-digit condition requires one shift and was expected to be performed faster than was the same addition in the 4-digit condition. These additions took, respectively, 23.0 and 26.8 sec. Within the 4-digit tasks, adding 100 was not significantly different either from adding 10 or adding 1000. What might be called a horizontal analysis was performed for the 3- and 4-digit tasks involving the addition of 1, 10, and 100 but excluding the data from the addition of 1000. Only the decimal position of the addition was significant. There was no evidence of a main effect of the number of digits nor was there an interaction as the experimental hypothesis would demand. While the 4-digit case is not quite as clearcut as the results in the 2-digit case, one can conclude that the number of shifts in Reading and Arithmetic operations do not appear to explain the difficulty of arithmetic in the tens decimal position.

The results of the second experiment rule out contextual effects associated with neighboring digits and point toward some factor specifically connected with addition in the tens position. Number words in the units, hundreds, and thousands positions are straightforward. Adding one thousand to four thousand totals five thousand. Adding one hundred to four hundred totals five hundred. Adding one to four totals five. In the tens position, however, things are more irregular. Adding ten to forty totals _fif_-ty rather than _five_-ty. Other morphomes which are idiosyncratic in our language only in the tens position are _twen_ for _two_ and _thir_ for _three_. _Forty_, _sixty_, _seventy_, _eighty_, and _ninety_ may be considered to contain regular morphemes. It was the principal hypothesis of the third experiment that unit additions in the tens position which generated responses containing idiosyncratic morphemes of the type described would have long latencies. Performance of a series of such additions would take longer than would additions leading to responses containing regular morphemes. Most stringently, it was hypothesized that additions involving regular morphemes in the tens position would not take longer than additions in the units position.

A subsidiary hypothesis concerned the peculiarities associated with eleven and twelve and the reversal of the tens and units morphemes in the names of the numbers from thirteen to nineteen. Transformation into the latter forms particularly was thought to require a step equivalent to a syntactical transformation. This may be illustrated by the example of subtracting 10 from, say, twenty-seven, giving seven-teen; the seven initially follows the number in the tens position but subsequently precedes the morpheme designating the value of ten. Such additional steps were thought to require more time for their completion.

The structure of this experiment simplified the steps used in the earlier ones. The focus was restricted to addition in the tens and units positions. The reading control was omitted because 8 different response lists were used for each S Response lists of 3-digit numbers were used to camouflage the manipulation of the distribution of digits in the tens position. One set of 4 response lists contained numbers whose names had idiosyncratic morphemes: twenty, thirty, and fifty. The other set of 4 response lists contained the regular forms: forty, sixty, and eighty. Orthogonal to this variable in the 2 x 4 factorial design was the inclusion of 4 different percentages of numbers between 11 and 19. Among both the regular and idiosyncratic morpheme lists were lists incorporating 10%, 20%, 30%, and 40% of response numbers whose last two digits were between 11 and 19.

I must confess that the inclusion of this variable was not fruitful except in providing replications for the principal experimental variable. The difficulty stemmed, I think, from overlooking what is now obvious. A stimulus written "3-0-6" which might generate an implicit verbal response, "three hundred six" can be transformed to "three hundred six-teen" without reversing the order of morphemes. The data do not indicate that Ss had any difficulties at this point; in fact, lists with greater percentages of teen numbers were completed more rapidly because they tended to contain fewer syllables: "sixteen" has two syllables, while "twenty-six" has three. This analysis does not deal with the distinctive character of eleven and twelve; any difficulty with these has been effectively diluted by the inclusion of the numbers from 13 to 19 in this design. The original rationale for including this variable would be expected to apply in full force where the task involved subtraction, as in the example given earlier of subtracting 10 from 27.

The eight conditions of the experiment (two levels of morphemic regularity and four levels of admixture of teen numbers) were embodied in 8 response lists. Two stimulus lists were derived from each response list in the manner described for the earlier experiments, one to be used with the instruction to "Add 10" and one for the "Add 1" condition. The table in the handout shows the structure of the elements entering a response list. Each list consisted of 90 3-digit numbers instead of the 100 used earlier. The table consists of 10 rows of 9 numbers. The second, or tens, digits were assigned to rows in the proportions 10%, 20%, 30%, and 40%, totalling 100%. While the variable of interest here was the percentage of teen numbers, each of the other three forms used occurred in all four proportions. Thus, among the four lists containing idiosyncratic morphemes, twenties, thirties, and fifties, as well as the teen numbers, each occurred in some one list at 10%, in some other at 20%, etc. The case was the same with the four lists containing regular morphemes. One purpose of this design was to control response uncertainty in adding 10 between the various experimental conditions. Unavoidably, constrained to have identical sets of component responses for the Add 10 and Add 1 versions of the same response list, the response uncertainties in the two decimal positions were different. The uncertainty in selecting one of the four "tens" responses occurring with probabilities ranging from 10% to 40% was calculated to be 1.85; the uncertainty in selecting one of 9 "units" responses having equal weights of approximately 11% was calculated to be 3.32. The smaller uncertainty associated with addition in the tens position would be expected to lead to more rapid processing. Therefore, the resulting error should be conservative with respect to the hypothesis under test, since the variables under study were expected to slow the performance of adding 10. An experimental effect would have to override the effects of decreased response uncertainty.

Referring again to the table, the first digits of the 3-digit numbers were determined by randomly ordering the digits 1-9 within each row. Independent randomization of the digits 1-9 determined the third digits of the response numbers. Thus, each of the digits 1-9 occurred equally often in the first and third decimal positions over the whole set of 90 numbers. All 8 lists had the same property, although they were constructed independently. They differed in the assignment of digits in the second position.

Sixteen male and 16 female college students were run. The data were again collected by Peter Patall. Each subject was tested under all 16 conditions. The order of the tasks was counterbalanced within sex in a latin square design. The overall analysis showed a significant effect associated with trials, with the decimal position of the addition, and with the percentage of teen numbers. The latter finding, as I have mentioned before, is attributable to the preponderance of two-syllable terms in the teens leading to decreased times for adding both 10 and 1. There was no evidence of interaction of this variable with any other.

The degree of morphemic irregularity did not produce a significant main effect but it did enter into a significant interaction with the decimal position of the addition. This relation is shown in Figure 2. Under conditions of morphemic idiosyncracy, adding 10 took 7.9 sec. longer than did adding 1. Under the conditions of morphemic regularity, adding 10 took only 3.4 sec. longer than did adding 1. That these differences are in fact different is shown in a separate analysis of the distribution of differences scores obtained by subtracting the Add 1 times from the matched Add 10 times for each subject and each experimental condition ($F = 6.16$, 1/30 df, p less than .05). Difference scores derived in the same way are plotted in Figure 3 for the four levels of the percentage-of-teen numbers variable. Each point represents the difference in adding at the two decimal positions for a particular response list. The irregular morpheme condition consistently requires more time.

Another look at the preceding figure, Figure 2, shows another aspect of the data which muddies the waters somewhat. The task of adding 10 takes only 1.1 sec. longer in real time under the idiosyncratic morpheme conditions than it does under the conditions in which the regular morphemes are used. On the other hand, adding 1 to the irregular forms of the numbers takes 3.4

sec. less than does adding 1 to the regular forms. Adding 1 in the units position does not require any step in the tens position other than reading the digit and incorporating it into the appropriate syntactical frame in order to give it the appropriate phonological representation. As far as the actual arithmetic goes, Ss were performing identical operations in the units position on balanced sets of digits from 0 to 8 under both morpheme conditions. It appears, therefore, that the series containing twenties, thirties, and fifties may have been intrinsically easier to read than series containing forties, sixties, and eighties.

One other finding needs to be discussed. Contrary to hypothesis, the 3.4 sec. difference in the time to add 10 and add 1 in the regular morpheme conditions was not negligible but was significantly different from zero (t = 2.58, 30 df, p less than .05). This means that when the factor of morphemic idiosyncracy of the response is controlled, unit addition in the tens position remains more difficult.

In the first two experiments there was almost complete balancing of the stimulus lists in their distribution of digits in the target decimal positions. Under these circumstances the difficulty in adding 10 was clear. In view of the nature of the primary hypothesis in the third experiment, it was not possible to achieve this sort of balance between the Add 10 and the Add 1 tasks. However, it does not appear fruitful to invoke possible stimulus differences to explain the difficulty in adding 10 in this case only. Since responding to the paired stimulus lists by adding in the appropriate decimal positions generates the same sets of component responses, we may eliminate differences associated with the motor aspects of responding. Somewhere in the stage of actually performing the arithmetic operation or in translating the output of this operation to a phonological representation (assuming that these are separable steps) there is a distinctive difficulty associated with the tens position. The results of the third experiment show that this difficulty may be increased when the morphemic form of the output is idiosyncratic or unusual or irregular. One might consider that such morphemes have a lower association value due to their low frequency of occurrence and that Ss require more time to produce them. Such increase in latency might arise through interference from higher frequency or regular morphemes.

After the fact speculation suggests that digits in the tens position are the only ones in the American language whose names occur in bound morphemes; I refer here particularly to the -ty ending for all such numbers from twenty up. If this is a relevant observation, the process of addition may be influenced in two different ways. Subjects seeing the stimulus number may first encode it, digit by digit, in a syntactical frame, "_____ hundred _____-ty _____". The arithmetic operation, if it is in fact a distinct operation, may require the detachment of the bound morpheme, "-ty". This might take additional time. Alternatively, the arithmetic operation may be applied as a figural transformation, with decoding to a verbal output somehow slowed by the necessity to attach a bound morpheme to the name.

I have traced with you the beginning stages of a problem which is not yet solved. The indications are strong that a problem in the pragmatics of number use will be clarified by an understanding of Ss' use of morphological and phonological rules. It is a cliché that something as simple as 2 plus 2 is pretty simple indeed; yet I think it fair to assert that there are some things still to be learned about 1 plus 1.

REFERENCES

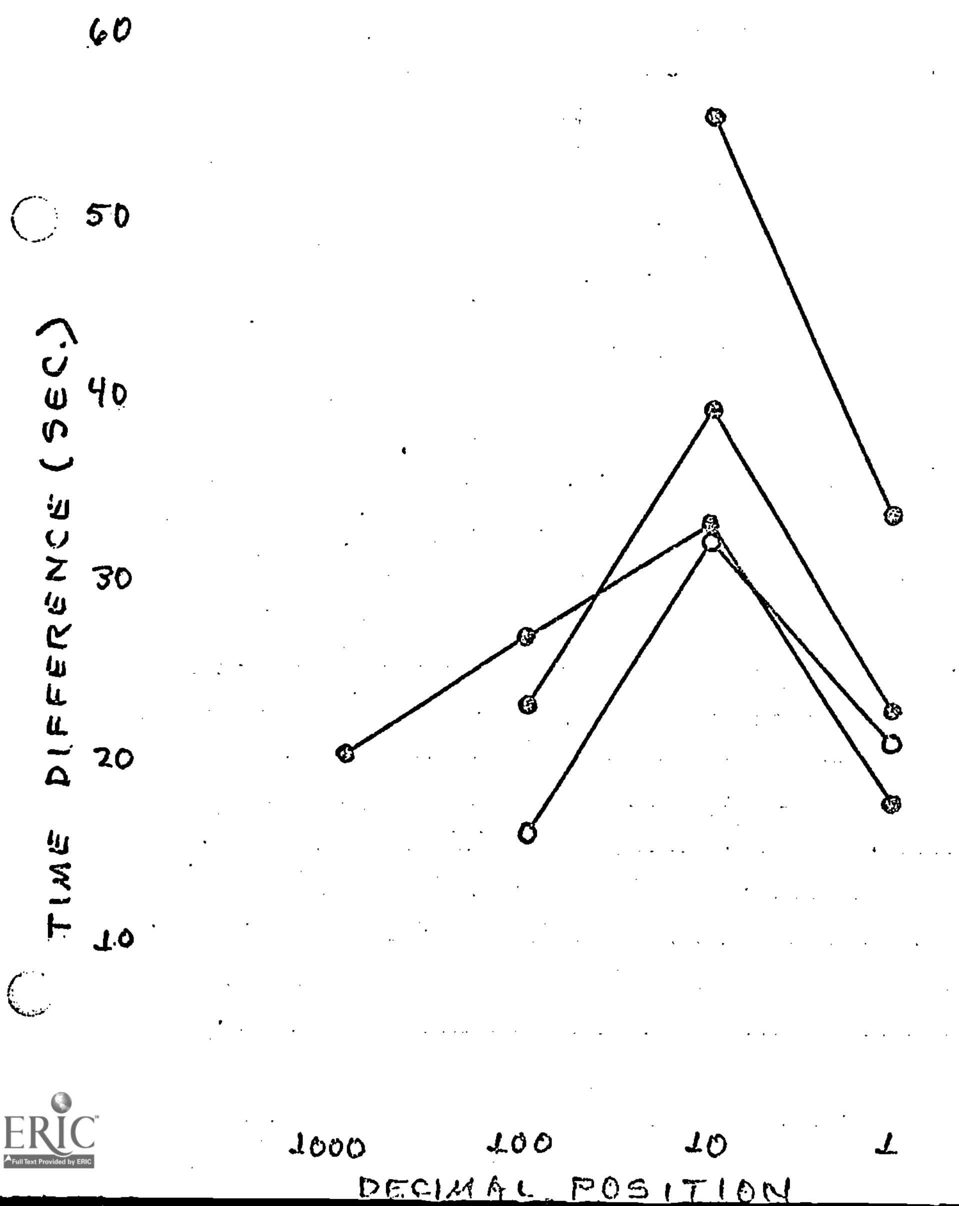
Chevalier, J. A. The difficulty of unit arithmetic transformations in different decimal positions. Proceedings of the 77th Annual Convention of the American Psychological Association, 1969, 73-74.

FIG. 1 DIFFERENTIAL TIME SCORES (ADDITION – READING) FOR ADDITION IN DIFFERENT DECIMAL POSITIONS.

LEGEND:

O – EXPERIMENT 1 (n = 28)

⊛ – EXPERIMENT 2 (n = 20)



Construction of Response List (90 3-Digit Nos.)

% 2nd Digit										
10	_1_	_1_	_1_	_1_	_1_	_1_	_1_	_1_	_1_	Assign 2nd Digit
20	_2_	_2_	_2_	_2_	_2_	_2_	_2_	_2_	_2_	
	62_	92_	42_	32_	22_	52_	72_	82_	12_	First Digit: Randomize 1-9
30	_3_	_3_	_3_	_3_	_3_	_3_	_3_	_3_	_3_	
	339	731	434	235	136	932	537	633	838	1st & 3rd Digits: Randomize 1-9 Separately
	_3_	_3_	_3_	_3_	_3_	_3_	_3_	_3_	_3_	
40	_5_	_5_	_5_	_5_	_5_	_5_	_5_	_5_	_5_	
	_5_	_5_	_5_	_5_	_5_	_5_	_5_	_5_	_5_	
	_5_	_5_	_5_	_5_	_5_	_5_	_5_	_5_	_5_	
	_5_	_5_	_5_	_5_	_5_	_5_	_5_	_5_	_5_	

FIG. 2 EFFECTS OF MORPHEMIC IRREGULARITY OF NUMBER WORDS ON DECIMAL POSITION EFFECT.

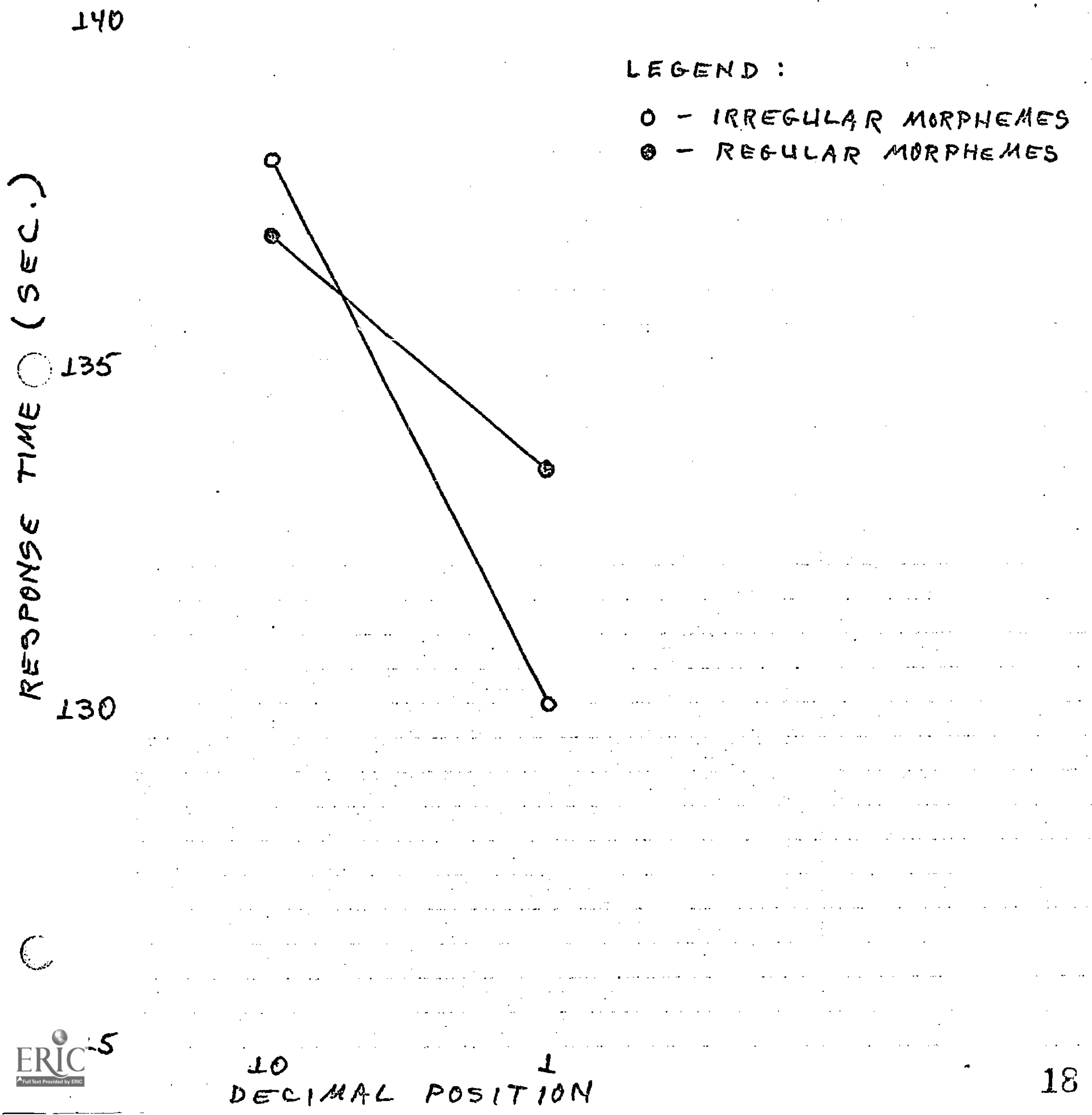


FIG. 3 TIME DIFFERENCE SCORES (ADD 10 MINUS ADD 1) FOR CONDITIONS DIFFERING IN THE PERCENTAGE OF "TEEN" NUMBERS IN THE RESPONSE LISTS.