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ABSTRACT

The effects of three variables on the difficulty of verbal arithmetic problems were examined. Variables included problem form, sequence of information, and problem verb. A total of 32 problems was generated, four in each of four problem forms and two sequences of information. Vocabulary words were not above second-grade level, and numbers used ranged from 2 through 9 with no borrowing or carrying required. Two groups of elementary-grade subjects (63 in all) solved all of the problems. Analysis of variance performed on the data indicated that problem form, sequence of information, and their interaction were significant ($p<.001$) but that the problem verb was not. Reverse sequence problems were most difficult to solve and became more difficult as the problem form became more difficult. It was concluded that subjects need to distinguish sequence of information from sequence of events where these do not coincide and that reverse sequence causes the greatest difficulty in problem solving. Tables and references are included. (MS)

THE SEQUENCE OF INFORMATION IN ARITHMETIC WORD PROBLEMS[1]

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This experiment was designed to examine the effects of 3 variables on the difficulty of verbal problems in arithmetic. These variables included (a) the form of the problem, (b) the sequence of problem information, and (c) the problem verb. The meaning of each of these variables will be discussed below.

Four different problem forms were defined. These word problem forms can be conceptualized as semantic elaborations of forms for number problems defined by Suppes, Hyman, and Jerman (1966) for the numbers m, n, and p. The problem forms appear in Table 1. The 4 number problem forms and the 4 word problem forms can be differentiated on the basis of which set is unknown, and whether the indicated operation is addition or subtraction. In the word problem forms, the grammatical subject always starts out with the m set, gains or loses the n set (depending on the indicated operation in the related number problem form), and ends up with the p set. The underlined elements in the problem forms in column 2 were variable elements, and were replaced by appropriate words from predefined lists. In this way, a set of word problems were generated from each problem form. A sample of the word problems that were generated appears

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in the last column of the table.

The second experimental variable was related to the sequence in which information about events is presented. The semantic information about a sequence of events can be presented in the order in which the events themselves occur, or in some other order. The word problems in Table 1 contain a forward sequence of information. For example, in the first sample item in column 3, Paul started out with 5 boats, and that information is given first. In Table 2, the general forms are transformed into a backward or reverse sequence. The problems that were generated from these forms have the information about events stated in the reverse order from which the events themselves occur. For example, in the first sample item in column 3, note that Paul started out with 5 boats as he did in the corresponding item in Table 1, but that that information is given last.

The third variable was the problem verb, which signified an additive situation in problem forms 1 and 3, and a subtractive situation in forms 2 and 4. Four addition verbs and 4 subtraction verbs were used. Each verb was used once in each relevant problem type. Meaningful comparisons can be made between the effects of the 4 additive verbs and also between the effects of the 4 subtractive verbs.

METHOD

In order to carry out the experiment, 32 word problems were generated, 4 in each of the 4 problem forms and 2 sequences of information. Two groups of subjects (63 in all) solved all 32 problems. Vocabulary words were at the second grade level at most. The numbers used ranged from 2 through 9 with no borrowing or carrying

needed. The problems were presented in written form. The subjects read each problem aloud and the experimenter gave an assist if a subject had difficulty reading a word.

RESULTS

The results of an overall analysis of variance on the collected data appear in Table 3, and indicate that the form of the problem, the sequence of problem information, and their interaction were highly significant. Secondary analyses indicate that the problem verb was not significant. Some of these statistics are reported in greater detail in Tables 4, 5, and 6, and in Figure 1. The form totals on the bottom line of Table 4 show that subjects made many more errors in forms 3 and 4 where the unknown set was the starting set than in forms 1 and 2 where the unknown set was the final or ending set. These totals also show differences in the level of difficulty of problems in each form. The sequence total column shows that problems in the reverse sequence were more difficult to solve. In addition to these main effects there was a significant interaction: the effect of the reverse sequence of information was heightened as the problem form became more difficult (see Figure 1). Problems in form 1 were quite easy for most subjects and the differential effects of information sequence were minimal here. Tables 5 and 6 show mean errors for the additive verbs and the subtractive verbs respectively. There are no significant differences to report.

DISCUSSION

The problem form results support the notion that subjects begin to solve problems by using the information in the starting set. This strategy is adequate in problem forms 1 and 2 (based on $m \pm n = ?$) where information about the starting or m set is given, but not in forms 3 and 4 (based on $? \pm n = p$) where information about the starting set is not given, and where subjects made many more errors. The information sequence results support the hypothesis that subjects need to distinguish the sequence of information from the sequence of events when the 2 sequences do not coincide. In the problems with a forward sequence of information events are mentioned in their proper temporal order, whereas in the problems with a backward sequence of information events are not mentioned in their proper temporal order, and it is in the latter type that subjects made significantly more errors. These results are in keeping with the finding of Clark and Clark (1968) that complex sentences are easier to recall when the order of mention of events directly indicates the temporal order.

REFERENCES

Clark, H. H., & Clark, E. V. Semantic distinctions and memory for complex sentences. The Quarterly Journal of Experimental Psychology, 1968, 20, 129-138.

Suppes, P., Hyman, L., & Jerman, M. Linear structural models for response and latency performance in arithmetic. Technical Report No. 100, Psychology Series, Institute for Mathematical Studies in the Social Sciences, Stanford, California, 1966.

Table 1. Word Problems with a Forward Sequence of Information

Number Problem Forms (Suppes)	Word Problem General Forms	Sample Item
(1) m+n=?	If subject started out with m objects and he verbed n objects, how many objects did he end up with?	If Paul started out with 5 boats and he bought 3 boats, how many boats did he end up with?
(2) m-n=?	If subject started out with m objects and he verbed n objects, how many objects did he end up with?	If John started out with 7 cars and he lost 2 cars, how many cars did he end up with?
(3) ?+n=p	How many objects did subject start out with if he verbed n objects and he ended up with p objects?	How many boats did John start out with if he bought 2 boats and he ended up with 6 boats?
(4) ?-n=p	How many objects did subject start out with if he verbed n objects and he ended up with p objects?	How many balls did Bill start out with if he lost 2 balls and he ended up with 5 balls?

Table 2. Word Problems with a Reverse Sequence of Information

Number Problem Forms (Suppes)	Word Problem General Forms	Sample Item
(1) m+n=?	How many objects did subject end up with if he verbed n objects and he started out with m objects?	How many boats did Paul end up with if he bought 3 boats and he started out with 5 boats?
(2) m-n=?	How many objects did subject end up with if he verbed n objects and he started out with m objects?	How many cars did John end up with if he lost 2 cars and he started out with 7 cars?
(3) ?+n=p	If subject ended up with p objects and he verbed n objects, how many objects did he start out with?	If John ended up with 6 boats and he bought 2 boats, how many boats did he start out with?
(4) ?-n=p	If subject ended up with p objects and he verbed n objects, how many objects did he start out with?	If Bill ended up with 5 balls and he lost 2 balls, how many balls did he start out with?

Table 3. Overall analysis of variance: 2 groups x 4 problem forms x 2 information sequences

	df	MS	F
Main effects			
Groups (G)			
Hypothesis	1	1.9012	2.3776
Error	61	.7996	
Problem Form (A)			
Hypothesis	3	35.3353	8.5028*
Error	183	4.1557	
Information Sequence (B)			
Hypothesis	1	7.6270	46.5447*
Error	61	.1639	
Interactions			
AxB			
Hypothesis	3	1.1693	6.2933*
Error	183	.1858	
GxA			
Hypothesis	3	.3565	.0857
Error	183	4.1557	
GxB			
Hypothesis	1	.0023	.0142
Error	61	.1639	
GxAxB			
Hypothesis	3	.3713	1.9983
Error	183	.1858	

*$p < .001$

Table 4. Mean errors according to problem form and information sequence. Ss solved 4 problems in each cell. N=63.

	Form 1		Form 2		Form 3		Form 4		Sequence Total	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Forward	.41	.66	.46	.88	2.22	1.53	1.49	1.49	4.59	2.92
Reverse	.46	.76	.79	1.02	3.16	1.27	2.14	1.47	6.55	2.66
Form Total	.87	1.17	1.25	1.54	5.38	2.39	3.63	2.65		

Figure 1. Mean errors according to problem form and information sequence. Ss solved 4 problems at each data point. N=63. Problem forms are arranged in sequential order of difficulty.

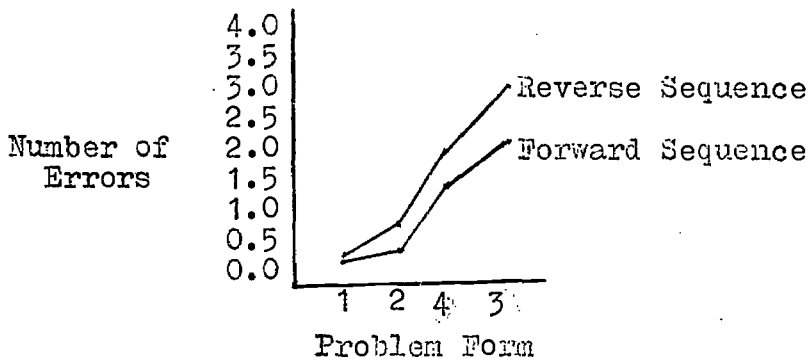

Table 5. Mean errors in addition problem forms (1 & 3). Ss solved 4 problems in each cell. N=63.

	Verb			
	Bought	Found	Took	Got
Mean	1.65	1.48	1.68	1.44
S.D.	.79	.88	.93	.88

Table 6. Mean errors in subtraction problem forms (2 & 4). Ss solved 4 problems in each cell. N=63.

	Verb			
	Sold	Lost	Gave	Sent
Mean	1.25	1.17	1.17	1.29
S.D.	.97	.87	1.00	.97