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ABSTRACT

Three procedures, alpha, image, and uniqueness rescaling, were applied to a joint occurrence probability matrix. That matrix was the basis of a well-known latent class structure. The values of the recurring subscript elements were varied as follows: Case 1 - The known elements were input; Case 2 - The upper bounds to the recurring subscript elements were input; Case 3 - No input parameters, thus incorporating analogs of the strong and weak lower bounds. The uniqueness rescaling and image methods took out the correct number of dimensions in Case 3. Orthogonal rotation failed to reproduce the known latent structure probability parameters. Least squares transformation of the image pattern, however, produced close approximations of the original matrix and the occurrence probability matrix. The results are reported in a comprehensive set of tables. (Author/AE)

Some Factor Analytic Approximations to Latent Class Structure

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It was the purpose of this study to investigate some empirical relationships between factor analysis and latent class structure. A joint occurrence probability matrix P_o was factored using various procedures to determine the feasability of reproducing a known joint item probability matrix L_o.

Green (1951) demonstrated some matrix equations for the point distribution of latent class structure. The assumption basic to the procedure is that all item relationships may be entirely explained by some underlying distribution--items independent for each latent class and item intercorrelations due to varying latent item probabilities. A solution may be realized by factoring two joint occurrence proportion matrices P_{ij} and P_{ijk}. Those matrices contain elements with recurring subscripts (P_{ii} and P_{iii}) which are analogs to communality estimates in the factor analytic sense. The limiting values of those elements have been shown to be $P_i \geq P_{ii} \geq P_i^2$ for the matrix P_{ij}.

Data Source and Methods

The matrix P_o which was used by Green to illustrate a three class, eight item example was used as the data source for this study. The matrix is found in the June, 1951 issue of PSYCHOMETRIKA. It was defined as $P_o = (r + 1) \times (r + 1)$ a symmetric matrix, with elements P_{ij}: $P_{oi} = P_i$, and $P_{oo} = 1$, where r equals the number of items, and i equals the item subscripts from o to r.

The factoring methods used are summarized as follows:

(i) Alpha - Factors a matrix reduced with uniqueness and rescaled with

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Bert F. Green, Jr., "A General Solution for the Latent Class Model of Latent Structure Analysis," PSYCHOMETRIKA, 16 (June, 1951), 151-165.

communality estimates. The method is iterative and produces factors with maximum generalizability in the Cronbach's alpha sense. Unless specified, the number of factors taken was equal to the number of eigenvalues of $H^{-1}(R-U^2)H^{-1}$ greater than one--the weak lower bound. Communality was initially estimated with squared multiple correlations unless other parameters were input, i.e., recurring subscript elements.

(2) Uniqueness Rescaling - Factors the correlation matrix after it has been rescaled with uniqueness estimates. Squared multiple correlations subtracted from one are used to estimate uniqueness unless communality parameters are input. If not specified, the number of factors taken was equal to the number of eigenvalues of $U^{-1}RU^{-1}$ greater than one--the strong lower bound.

(3) Image - Factors the image covariance matrix and, unless specified otherwise, the number of image components taken was equal to the number of eigenvalues of $S^{-1}RS^{-1}$ greater than one--the strong lower bound.

Three types of recurring subscript elements were used with the above solutions:

Case One - The known elements (p_{ii}) indicated by Green were input. In practical situations those elements are unknown.

Case Two - The p_i's, the upper bound to the p_{ii}'s, were input.

Case Three - No parameters were specified so that the reciprocals of the diagonal elements of the inverse of P_o with 1's in the diagonal were incorporated. In a correlation matrix those elements subtracted from one yield squared multiple correlations. In this case, however, those elements were not squared multiple multiple correlations but analogs to them because the joint occurrence probability matrix P_o was not a correlation matrix.

The raw pattern matrices were orthogonally rotated according to the normal varimax criterion.

Results

<u>Case One</u>: The alpha procedure extracted two factors, while the uniqueness rescaling and image procedures took out six. Neither the raw nor rotated pattern matrices yielded close approximations to the know latent class structure. Those procedures took out an incorrect number of factors.

<u>Case Two</u>: As previously, the alpha procedure extracted two factors. The uniqueness rescaling method, however, went to *nine factors, and image went* to nine components. Neither the raw nor rotated solutions proved to be reasonable approximations. These results seem to suggest that the p_j's tend to be over estimates to the recurring subscript elements for methods which feature so [illegible] of uniqueness rescaling.

<u>Case Three</u>: The alpha procedure took out two factors, while the image and uniqueness rescaling both took out three. Once again, the rotated solutions proved to be poor approximations as did the raw pattern matrices. The final two methods did, however, extract the correct number of factors--it was known <u>a priori</u> that the solution involved three latent classes. The fact that the rotated solutions did not prove satisfactory indicated that the requirement $TT' = T'T = I$ was too severe and that a transformation other than orthogonal rotation was needed. Clearly a transformation was called for since the raw solutions indicated the existence of negative proportions. The following procedure was used:

(1) Want A transformed to fit L_o.

(2) Define $T = (A'A)^{-1}A'L_o$

$$\therefore AT = A(A'A)^{-1}A'L_o$$

(3) Known that AT is a least squares approximation of L_o.

That transformation produced a reasonably good approximation.

Image Solution

Raw Image Pattern (Case Three)

-.840	.007	-.003
-.623	-.046	.015
-.769	.004	-.020
-.436	-.095	-.015
-.333	.099	.004
-.614	-.005	.020
-.263	-.085	.013
-.283	-.034	.01[illegible]
-.561	.883	.007

Matrix L_o

1.00	.1	.6	.5	.2	.0	.0	.1	.3
1.00	.5	.7	.0	.7	.7	.0	.2	.9
1.00	.9	.8	.7	.2	.8	.5	.4	.5

$(A'A)^{-1}$

.375	-.178	2.334
-.178	27.634	-2.253
2.334	-2.253	642.338

$(A'A)^{-1}A'$

-.323	-.190	-.313	-.181	-.133	-.182	-.053	-.076	-.206
.349	-1.194	.281	-2.513	2.780	-.073	-2.331	-.5[illegible]	2.377
-.390	8.280	-14.510	-10.438	1.569	11.424	7.928	5.63	2.993

$(A'A)^{-1}A'L_o = T$

.717	-1.061	-1.206
.322	3.806	-2.102
-15.202	3.040	1.890

$AT = L_o$

.651	.204	.814	.510	.210	.135	-.036	.040	.32[illegible]
.910	.531	.707	.056	.743	.694	.004	.202	.93[illegible]
.993	.387	.809	.698	.201	.789	.521	.432	.91[illegible]

S^2 Uniqueness Estimates

Variable		$1-S^2$	Green's P_{ii}
1	.251	.749	1.000
2	.595	.405	.482
3	.465	.535	.539
4	.781	.219	.295
5	.867	.123	.175
6	.611	.389	.467
7	.921	.089	.125
8	.917	.083	.094
9	.655	.455	.386

Determinant .067549

Eigenvalues 8.053000
1.240000
1.053000

Appendix A

Table 1

Green's Original Joint Occurrence Probability Matrix (P_{ij})

1.000	.620	.730	.450	.350	.610	.250	.280	.580
.620	.482	.477	.325	.199	.465	.225	.212	.366
.730	.477	.539	.370	.251	.467	.200	.214	.425
.450	.325	.340	.295	.090	.280	.175	.150	.205
.350	.199	.251	.090	.175	.227	.050	.036	.251
.610	.465	.467	.280	.227	.467	.200	.202	.389
.250	.225	.200	.175	.050	.200	.125	.100	.125
.280	.212	.214	.150	.086	.202	.100	.094	.160
.580	.366	.425	.205	.251	.389	.125	.160	.386

Table 2

Factor Matrix Alpha Procedure Communalities (Case One)

I	II
-.958	-.132
-.680	.092
-.724	-.056
-.464	.219
-.327	-.268
-.660	-.031
-.298	.178
-.303	.050
-.569	-.250

Table 3

Rotated (Varimax) Factor Matrix Alpha Procedure Communalities (Case One)

I	II
.738	.624
.386	.568
.526	.501
.148	.491
.418	.064
.464	.470
.067	.341
.165	.259
.567	.256

Table 4

Raw Factor Matrix Uniqueness Rescaling Procedure Communalities (Case One)

I	II	III	IV	V	VI
.995	.008	-.004	-.000	.000	-.000
.626	-.290	.067	-.000	.002	.017
.734	-.057	.006	-.002	.025	.001
.453	-.198	-.234	-.005	-.039	-.008
.352	.120	.197	-.054	-.023	.009
.616	-.231	.192	-.007	.003	-.023
.253	-.238	-.053	.000	.000	.011
.282	-.118	.005	-.000	.002	.006
.583	.048	.213	.040	-.025	..004

Table 5

Rotated Factor (Varimax) Uniqueness Rescaling Procedure Communalities (Case One)

I	II	III	IV	V	VI
.727	.528	.423	.060	.028	.004
.385	.569	.091	.020	.020	.004
.516	.441	.278	.064	.017	.009
.125	.439	.304	-.015	-.002	-.001
.417	.063	.035	-.007	-.030	-.004
.470	.496	.012	.013	.021	.047
.064	.341	.054	.004	.004	-.001
.164	.249	.067	.011	.008	.001
.563	.244	.093	.009	.071	.005

Table 6

Raw Component Matrix Image Procedure Communalities (Case One)

I	II	III	IV	V	VI
.990	.005	-.002	-.000	.000	-.000
.623	-.159	.032	-.000	.000	.001
.731	-.031	.003	-.000	.002	.000
.451	-.109	-.110	-.000	-.003	-.000
.350	.066	.093	-.004	-.002	.000
.613	-.127	.090	-.001	.000	-.001
.252	-.131	-.025	.000	.000	.000
.281	-.065	.002	-.000	.000	.000
.581	.026	.100	.003	-.002	.000

Table 7

Rotated (Varimax) Component Matrix Image Procedure
Communalities (Case One)

I	II	III	IV	V	VI
.751	.622	.173	.003	.004	-.000
.408	.498	-.000	.002	.001	.000
.539	.483	.106	.002	.001	.000
.240	.395	.116	-.000	-.001	-.000
.337	.146	.014	-.002	-.000	-.000
.442	.451	-.036	.002	.001	.002
.116	.261	-.001	.000	.000	-.000
.182	.223	.014	.001	.001	.000
.496	.318	.027	.006	.000	.000

Table 8

Raw Matrix Alpha Procedure Communalities (Case Two)

I	II
-.957	-.129
-.680	.091
-.724	-.055
-.462	.206
-.330	-.290
-.659	-.030
-.300	.180
-.300	.049
-.566	-.240

Table 9

Rotated (Varimax) Factor Matrix Alpha Procedure
Communalities (Case Two)

I	II
.639	.723
.574	.376
.512	.515
.484	.147
.060	.435
.478	.454
.345	.060
.262	.161
.271	.552

Table 10

Raw Factor Matrix Uniqueness Rescaling Procedure
Communalities (Case Two)

I	II	III	IV	V	VI	VII	VIII	IX
.995	.017	.007	-.008	-.003	.001	.003	-.600	.000
.629	-.374	-.048	-.116	.100	.236	.007	-.035	.04[illegible]
.739	-.160	.069	.391	.005	-.000	-.005	-.001	.004
.454	-.179	.279	-.118	.168	-.199	-.215	-.041	.006
.352	.110	-.220	.057	-.140	.143	-.345	.036	-.04[illegible]
.619	-.312	-.222	-.062	-.214	-.169	.016	-.033	.026
.254	-.218	.071	-.068	.047	-.043	-.003	.025	-.392
.283	-.124	.009	-.040	.019	-.025	.009	.424	.042
.586	.049	-.388	.064	.266	-.094	-.002	-.003	-.006

Table 11

Rotated (Varimax) Factor Matrix Uniqueness Rescaling Procedure
Communalities (Case Two)

I	II	III	IV	V	VI	VII	VIII	IX
.453	.401	.360	.289	.323	.277	.229	.198	.372
.208	.197	.218	.224	.140	.585	.173	.218	.036
.680	.237	.226	.191	.196	.179	.164	.161	.02[illegible]
.136	.085	.602	.098	.039	.114	.113	.156	.022
.095	.130	.033	.083	.556	.062	.055	.019	.016
.199	.222	.163	.603	.178	.198	.161	.180	.034
.069	.048	.106	.073	.018	.077	.073	.463	.013
.074	.069	.082	.070	.051	.067	.495	.076	.015
.176	.658	.101	.158	.210	.128	.114	.080	.025

Table 12

Raw Component Matrix Image Procedure Communalities (Case Two)

I	II	III	IV	V	VI	VII	VIII	IX
.990	.0[illegible]2	.004	-.005	-.002	-.001	.002	-.000	-.000
.626	-.258	-.032	-.073	.053	.120	.003	-.016	.016
.736	-.111	.045	.247	.003	-.000	-.002	-.001	.00[illegible]
.452	-.124	.182	-.074	.089	-.101	-.099	-.019	.026
.351	.076	-.143	.036	-.075	.073	-.159	.016	-.016
.616	-.215	-.145	-.039	-.114	-.086	.007	-.015	.0[illegible]
.253	-.15[illegible]	.046	-.043	.025	-.022	-.001	.011	-.[illegible]
.282	-.086	.006	-.026	.010	-.013	.004	.193	.017
.583	.034	-.253	.040	.142	-.048	-.001	-.001	-.002

Table 13

Rotated (Varimax) Component Matrix Image Procedure Communalities (Case Two)

I	II	III	IV	V	VI	VII	VIII	IX
.528	.437	.330	.328	.318	.314	.178	.131	.215
.259	.251	.346	.182	.237	.154	.167	.302	.017
.354	.292	.268	.208	.234	.461	.128	.087	.009
.117	.417	.225	.089	.157	.098	.068	.050	.003
.396	.068	.066	.079	.087	.062	.049	.033	.004
.307	.194	.298	.197	.224	.146	.357	.098	.016
.060	.112	.279	.052	.096	.054	.042	.025	.007
.101	.009	.116	.065	.285	.057	.046	.031	.007
.368	.148	.156	.431	.170	.131	.105	.006	.009

Table 14

Raw Factor Matrix Alpha Procedure Communalities (Case Three)

I	II
-.958	-.133
-.681	.094
-.725	-.057
-.463	.217
-.32[illegible]	-.257
-.6[illegible]	-.031
-.297	.176
-.303	.051
-.571	-.258

Table 15

Rotated (Varimax) Factor Matrix Alpha Procedure Communalities (Case Three)

I	II
.742	.621
.387	.568
.528	.499
.149	.489
.409	.068
.466	.468
.069	.338
.166	.259
.575	.250

Table 16

Raw Factor Matrix Uniqueness Rescaling Procedure
Communalities (Case Three)

I	II	III
-.897	.017	-.014
-.660	-.105	.065
-.758	.010	-.091
-.466	-.216	-.006
-.356	.226	-.018
-.656	-.011	.090
-.281	-.193	.058
-.303	.077	.047
-.599	.189	.032

Table 17

Rotated (Varimax) Factor Matrix Uniqueness Rescaling Procedure
Communalities (Case Three)

I	II	III
.661	.598	.110
.412	.537	.014
.522	.498	.172
.186	.466	.129
.414	.081	-.007
.470	.466	-.017
.072	.338	-.015
.168	.267	.009
.565	.276	.022

Table 18

Raw Component Matrix Image Procedure
Communalities (Case Three)

I	II	III
-.840	.007	-.003
-.623	-.046	.015
-.709	.004	-.020
-.436	-.095	-.015
-.333	.099	.004
-.614	-.005	.020
-.263	-.085	.013
-.263	-.034	.010
-.561	.083	.007

Table 19

Rotated (Varimax) Component Matrix Image Procedure Communalities (Case Three)

I	II	III
.607	.580	.019
.414	.468	-.001
.511	.491	.034
.246	.372	.025
.308	.161	.000
.436	.432	-.008
.129	.245	-.006
.179	.222	-.004
.460	.332	.002

Appendix B

Least Squares Transformation:
Raw Pattern (Image) and Joint Occurrence Matrix P_o

$A =$

-.840	.007	-.003
-.623	-.046	.015
-.709	.004	-.020
-.436	-.095	-.015
-.333	.099	.004
-.614	-.005	.020
-.263	-.085	.013
-.283	-.034	.010
-.561	.083	.007

$(A'A)^{-1} =$

.375	-.178	2.334
-.178	27.634	-2.253
2.334	-2.253	642.338

$(A'A)^{-1}A' =$

-.323	-.190	-.313	-.181	.133	-.182	-.053	-.077	-.209
.350	-1.194	.282	-2.514	2.786	-.074	-2.331	-.912	2.378
-3.903	8.285	-14.511	-10.439	1.569	11.425	7.928	5.839	3.000

$(A'A)^{-1}A'P_o = T =$

-1.065	-.717	-.792	-.495	-.372	-.706	-.302	-.319	[illegible]
.155	-.369	-.003	-.704	.602	-.041	-.525	-.186	.522
-1.178	1.000	-.428	-.859	.220	1.396	.472	.257	[illegible]

$AT = P_o =$

.899	.567	.666	.413	.316	.589	.249	.266	[illegible]
.639	.479	.487	.327	.207	.463	.219	.211	.375
.779	.487	.570	.365	.262	.472	.203	.220	[illegible]
.467	.332	.352	.295	.102	.291	.175	.153	.220
.365	.206	.262	.091	.184	.237	.050	.089	[illegible]
.630	.462	.478	.290	..230	.462	.198	.202	[illegible]
.252	.233	.203	.179	.050	.207	.130	.103	[illegible]
.284	.225	.220	.155	.087	.215	.108	.099	[illegible]
.602	.379	.441	.212	.260	.402	.129	.165	.400

Squared Residuals Image Procedure

$(AT - P_o)^2$

.010	.000	.004	.001	.001	.000	.000	.000	[illegible]
.000	.000	.000	.000	.000	.000	.000	.000	.000
.002	.000	.001	.000	.000	.000	.000	.000	.000
.000	.000	.001	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000