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ABSTRACT

Many programs for individualization of instruction are based on a curriculum structure involving a careful sequencing of instructional objectives and larger units of learning content. A methodology that may be useful in the formative evaluation of the sequential and structural properties among predefined curriculum units and specific instructional objectives was investigated. The three distinct research populations consisted of the student enrollment in three schools employing the Individually Prescribed Instruction (IPI) system for elementary school mathematics. To examine these structural relationships, as they are found in different content areas and levels, student performance on unit placement tests and pretests in thirty IPI mathematics units consisting of 173 specific objectives was investigated through scalogram and simplex analyses. The usefulness of these procedures in providing suggestions for the revision and refinement of curriculum structures was demonstrated. Alternative structures and sequences suggested by this evaluation program may be helpful in providing guidance for curriculum and testing specialists in the design of future curriculums in IPI mathematics. (Author/PR)

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Among the programs for the individualization of instruction in current use, many are based on a curriculum structure involving a careful sequencing of specific instruction objectives as well as larger units of learning content. The formative evaluation of such systems of individualization should include the study of the extent to which actual pupil performance supports the validity and meaningfulness of the hypothesized learning sequences and structures. Gagné (1967), Resnick (1968), and others have proposed methods for generating and assessing such structures. Among procedures which have been proposed, but investigated only to a limited extent, are Guttman's procedures of scalogram analysis (Guttman, 1950) and simplex analysis (Guttman 1954). The purpose of the present study was to investigate the usefulness of these two procedures for various specific steps in the formative evaluation of an individualized mathematics program, the Individually Prescribed Instruction system.

Scalogram Analysis

Scalogram analysis provides a technique for testing the existence of a single quantitative variable, referred to as the scale variable, in a predefined universe of content. From a person's score on a measure of this scale variable one knows precisely the person's responses to each of the items or objectives constituting the scale. Thus, if a pass-fail response pattern of a student sample for a group of objectives forms a scale, then mastery of a given objective will imply mastery of all subordinate objectives and non-mastery of a given objective will imply non-mastery of all supraordinate objectives.

The definition of a perfect scale, as used in this study, is one in which a person's pass-fail performance with respect to mastery of each group of objectives may be reproduced from his scale score. The scale score is found by counting the total number of objectives which have been mastered. Thus, scales derived from scalogram analysis have the property that responses to individual objectives are reproducible from the scale scores. A two-step procedure may be used for determining the existence of a scale: (1) rank the objectives from the highest to the lowest in terms of the percentage of students indicating mastery, and (2) rank the people from the highest to lowest in terms of total score; i.e., total number of objectives mastered. The resulting pattern of a perfect scale will be triangular in shape when only mastery responses are recorded. An example of a perfect scale is shown in Figure 1.

Perfect scales, however, are rarely found in practice. The measure for the degree of approximation to a perfect scale is defined as the coefficient of reproducibility. This coefficient provides an index of how well a person's response pattern can be predicted from his scale score. The coefficient is computed by counting the total number of errors, or discrepancies from a perfect scale pattern, finding the ratio of the errors to the total number of possible responses and subtracting the ratio from unity.

Lingoes (1963), in a procedure known as multiple scalogram analysis, has developed a method for analyzing the response pattern of a set of dichotomous variables for the purpose of searching out optimal scalable subsets within larger sets of data. Rather than reject the scalogram hypothesis for the total set of variables in cases where a single scale is not obtained, the method seeks out smaller sets of variables which may, indeed, form scales with high coefficients of reproducibility. The multiple scalogram procedure has been programmed and was used in this study. A description and explanation of the program has been reported by Weisberg (1966).

Simplex Analysis

A second method of order analysis which appears to be capable of providing answers to questions regarding order and structure among behavioral objectives is "simplex analysis." Guttman describes this theoretical position as "A New Approach to Factor Analysis: The Radex," (Guttman, 1954) where the term "radex" indicates a radial expansion of complexity. The simplex is one element of the more general radex, and indicates a simple order of complexity among variables of the same kind. The circumplex is the remaining element of the radex and indicates a circular order among different kinds of variables when they are all of the same magnitude of complexity. The analytical approach of simplex analysis, therefore, was developed in order to investigate the hypothesis of complexity hierarchy among observed quantitative variables by accounting for the positive correlations among the variables.

It appears that simplex analysis is ideally designed for investigating order among instructional units and would be a preferred method for studying hypothesized linear sequences when continuous quantitative achievement data were available for each content unit. The simplex hypothesis is that the content units may be arranged in a hierarchical order of complexity such that the intercorrelation matrix of the Pearson product-moment coefficients are accounted for by a single complexity factor. If this hypothesis is verified by student response data, then one has a justifiable basis for considering the existence of a hierarchical or prerequisite relationship among the variables. This analysis may constitute a curriculum design validation procedure when the hierarchical relationship among instructional units has been previously hypothesized. It is suggested here that simplex analysis is better suited for investigating large segments of a curriculum such as units of an arithmetic curriculum in such topics as numeration, addition, etc., whereas scalogram analysis is better suited for analyzing the dichotomized pass-fail data derived from performance on individual instructional objectives.

Kaiser's (1962) method for scaling the variables of a simplex was used in this study. The procedure yields a least squares solution for ordering the variables and provides a measure of the goodness of fit for the empirical data.

The Setting for the Study

To investigate structure and sequence within a curriculum in terms of the learner's performance, it is necessary to work within a setting where the units of the curriculum have been defined in measurable human behavior. These units may vary from a single instructional objective involving a specific skill to an entire content area consisting of many objectives. Since the mathematics curriculum of the Individually Prescribed Instruction (IPI) program developed at the Learning Research and Development Center, University of Pittsburgh, is based on units defined in this way, it has been used to demonstrate the formative evaluation methodology proposed in this study.

In the organizational plan of the present mathematics curriculum of the IPI Project, an effort was made to design objectives with an order of complexity relating to two dimensions, (1) content area and (2) level of achievement. In general, the expected route by which a student would progress through the curriculum is: (1) within a given level the progression would be from the least complex area to the most complex area, and (2) all objectives in a given level would be mastered before advancing to the next higher level, progressing from the least complex level to the most complex level. A curriculum structure such as this implies an order of complexity both across predefined areas (numeration, place value, addition, subtraction, multiplication, division, etc.) and across predefined levels (B, C, D, E, F, G, etc.). The numbers of objectives in each area and level are shown in Table 1. Thus, a student will typically work through the series of content areas at one level before moving on to the next. The work in a given content area at a given level, such as Numeration, B-level, is identified as a unit and is defined on the basis of a limited number of specific instructional objectives. In this study an abbreviated notation will be used to refer to a particular objective within the curriculum. Thus, NB1 will refer to the Numeration area, B-level, and to the first objective listed for that unit. It is assumed that the objectives are arranged in an order of complexity within each unit. Thus, in addition to the study of the hierarchical relationship among the broadly defined units as above, this curriculum organization also provides for the study of the structure and sequence among specific instructional objectives which comprise these units. Empirical evidence concerning both of these types of hierarchies is important in evaluating the validity of this curriculum structure as a guide to pupil placement and the prescription of pupil learning activities.

For the diagnosis of individual needs, several criterion-referenced instruments have been developed for the placement and monitoring of students within the IPI curriculum. Two of these instruments, placement tests and pretests, have been used to obtain data for this study and are described as follows:

Placement Test

Placement tests are administered at the beginning of each school year, or, for a new pupil, when he enters school. These tests are broad in scope as they are intended to provide a general profile of individual pupil achievement over many units of work. The placement tests are content referenced in that each item on every test is coded to one particular objective in the curriculum. Since placement tests must be of minimum length while providing a maximum of information, not every objective in each unit is tested. Generally the most important, or most characteristic objectives in a unit are tested. The basic information provided by the placement tests is a measure of the highest level of mastery that the pupil has attained in each topic area (Cox and Boston, 1967).

Unit Pretest

A pretest is administered when a pupil is about to begin a unit. Since the items are designed to measure the achievement of the skills specified in the curriculum for that unit, each objective is represented and the pupil receives a score on each skill or objective. If the pupil achieves mastery (85 percent) in all skills on the pretest, he does not work in that unit, but is given the pretest for the next unit in his instructional sequence. If mastery is not achieved on a particular skill, the pupil is prescribed work within that skill (Cox and Boston, 1967).

Purposes of the Study

Specifically the study examined the following applications of scalogram and simplex analysis procedures.

1. The study of the extent to which the hypothesized ordered sequences of specific objectives are supported by pupil test performance.
2. The study of the extent to which the hypothesized order of various mathematics topics (numeration, addition, subtraction, etc.) at a given level in the curriculum is supported by pupil test performance.
3. The study of the extent to which the hypothesized order of unit levels within a given topic area is supported by pupil test performance.
4. A comparison of results obtained by scalogram analysis and by simplex analysis when used for the foregoing analyses.

Methods

Guttman's scalogram analysis and simplex analysis were applied to test results obtained from the administration of IPI Test Batteries to students in three schools using Individually Prescribed Instruction. The application involved the computation of reproducibility coefficients and of permutation indices for comparing differences between hypothesized and empirically derived scales.

Data Sources

Data analyzed included IPI Placement Test and Unit Pretest results for thirty units of mathematics consisting of a total of 173 specific objectives. Three samples of students were used, consisting of the enrollment in three schools employing the IPI mathematics program.

Results

The results of this study can be summarized in terms of the four major purposes.

The Study of Hypothesized Ordered Sequences of Specific Objectives

Nineteen homogeneous groupings of objectives were hypothesized within three samples tested by a placement test program at one of the three demonstration schools.

Numeration and Place Value. Eight hypothesized scales within the content areas of Numeration and Place Value are shown in Tables 2, 3, and 4. The results of a multiple scalogram analysis including the suggested optimum empirical scale, the reproducibility coefficient, and the percentage of students indicating mastery of each objective is shown. A minimum reproducibility coefficient of .80 has been used throughout this study as one of the criteria for the existence of a scale. A measure of the difference between the hypothesized scale and the empirical scale has been operationally defined as the permutation index and is computed by counting the number of permutation inversions required to make the hypothesized and empirical scales identical. The total number of inversions in a permutation is, by definition, found by counting the number of smaller integers following each integer of the permutation. Thus 614325 has eight inversions since 6 is followed by 1, 4, 3, 2, and 5; 4 is followed by 3 and 2; 3 is followed by 2. Thus, a permutation index of zero indicates the two scales are identical, and the larger the index, the greater the scale difference.

The homogeneous grouping of objectives in the area of "Expanded Notation - Integer Numbers" is shown in Table 2. The hypothesized scale for the nine objectives in this scale is not verified by the scalogram results. The permutation index of 13 indicates a high degree of difference between the hypothesized and empirical scales. Even though a single scale was derived from the multiple scalogram analysis, the difference in the order of the objectives between this empirically derived scale and the hypothesized scale suggests that either the hypothesized scale is in error or the test items keyed to the objectives are not measuring what they were designed to measure. Thus, depending upon the assumption which the investigator is willing to make, the results of a scalogram analysis which does not verify the original hypothesis may be interpreted as showing that either (1) the hypothesized scale is in error or (2) the test items measuring mastery of the objectives are invalid in the sense that they are not measuring what they were intended to measure. There is no reason why one of these assumptions is basically superior to the other. In some cases, the indicated position of a particular objective may be the result of improperly written or invalid test items. If, however, it is agreed that the test items can be assumed to be valid, then this information provides guidance for re-thinking the logical relationship of the subject matter as defined by the instructional objectives. This way of interpreting scalogram results empha . the importance of jointly seeking solutions to the logical significa e of scales by people interested in both curriculum structure design and test design.

The homogeneous grouping of objectives in the area of "counting" is shown in Table 3. The hypothesized scale for the seven objectives in this group is verified by the multiple scalogram results. A single scale with the same hypothesized order resulted from the analysis. A high degree of consistency in the student responses to the test items is indicated by the single scale, and since the hypothesized order is verified within the homogeneous group, we may interpret these results as a validation of the test items for measuring what they were designed to measure. The reproducibility coefficient of .946 may be interpreted as a validation of the hypothesized order among the objectives and of the test items which measured mastery of these objectives.

The remaining six hypothesized groupings of homogeneous objectives in numeration and place value are shown in Table 4. Only one of these groups, "Expanded Notation--Decimal Numbers," requires a reexamination. The hypothesized scales in all remaining groups were verified by the multiple scalogram analysis.

Addition and Subtraction. Five hypothesized scales within the content area combination of Addition and Subtraction are shown in Tables 5, 6, and 7. The homogeneous grouping of objectives in the area of "Integer Addition--Facts and Algorithm" is shown in Table 5. A single empirical scale is established by the analysis. The permutation index of 2, however, reflects a reversal of the scale positions of AF1 and AE6, and AD8 and AD6, respectively. This analysis suggests that the test items and sequence of these four objectives should be reexamined.

The homogeneous grouping of objectives in the area of "Integer Subtraction--Facts and Algorithm" is shown in Table 6. The hypothesized scale for these five objectives is verified by the analysis except for the reversal of the scale positions of SC1 and SC3. The test items and sequence of these two objectives should therefore be reexamined.

The remaining three hypothesized groupings of homogeneous objectives in Addition and Subtraction are shown in Table 7. The empirical scales resulting from these analyses suggest that the test items and sequence of the objectives in each group should be reexamined with the exception of SF1 in the "Addition-Subtraction Decimal Number" group.

Multiplication and Division. Six hypothesized scales within the content area combination of Multiplication and Division are shown in Tables 8, 9, and 10. The homogeneous grouping of objectives in the area of "Multiplication Algorithm--Integer Numbers" is shown in Table 8. A single empirical scale is established by the analysis for these five objectives. A reversal of objectives MF5 and MF10, however, indicates that the sequence of these objectives and test items should be reexamined.

The homogeneous grouping of objectives in the area of "Division--Concepts and Facts" is shown in Table 9. The order of three of the four objectives in this scale is not verified by the analysis. The sequence of objectives and test items corresponding to DD2, DD5, and DD7 should be reexamined, based on these results.

The remaining four hypothesized groupings of homogeneous objectives in Multiplication and Division are shown in Table 10. Only one empirical scale, "Multiplication--Concept and Facts," suggests an order different from the hypothesized scale. In this case, the sequence of the objectives and test items for MD3 and MD4 should be reexamined.

Due to the extremely low mastery percentages for the remaining three scales, any judgement on the test items corresponding to these objectives should be withheld.

The Study of Hypothesized Order Among Content Areas

As stated earlier, the organizational plan of the IPI Mathematics Curriculum implied an order of complexity among the units relating to two dimensions, (1) content area and (2) content level. The results of the study of these complexity orders will now be presented.

The placement test results from a demonstration school were analyzed for the purpose of investigating the complexity order among the content areas within a given level. Subsets of all students within the school who received unit scores for each of the six content areas were determined for levels D, E, F, and G. Analyses were conducted only for levels D and E, however, because of inadequate sample sizes for levels F and G.

Level D Complexity Study. Each student in the sample group of level D received a percentage score on the unit subtests corresponding to the areas of numeration, place value, addition, subtraction, multiplication, and division. The intercorrelations among the unit scores for these six areas are shown in Table 11 where the topic or content areas are listed in the order used in the IPI curriculum.

Table 12 shows the re-ordering of topic areas which resulted from application of simplex analysis. For this optimal ordering a "Q squared" value is computed to indicate the extent to which this ordering can be explained by a single complexity factor. For the optimal ordering of the content areas as shown in Table 12, the Q squared value is 0.971.

This indicates a high degree of success in explaining these correlations in terms of a single complexity factor among the content areas, when the areas are defined in terms of the objectives included on the unit placement tests. This analysis suggests that a single complexity continuum, from least to most, exists among the content areas of level D in the following order: addition, multiplication, division, subtraction, numeration, and place value. Proper interpretation of these results depend upon the assumptions that (1) the objectives included on the placement tests adequately represent the units and that (2) the test items representing the objectives are valid. Under these assumptions the results of level D simplex analysis would cast doubt upon the hypothesized complexity continuum, from least to most, of numeration, place value, addition, subtraction, multiplication, and division. The results might be interpreted as suggesting the preferred or optimum progression among units within level D of this curriculum. At least these results would tend to discourage a rigid, preconceived notion of how a student should progress through level D of this curriculum.

Level E Complexity Study. Each student in the sample group of level E received a percentage score on the unit subtests corresponding to the areas of numeration, place value, addition, subtraction, multiplication, and division. The intercorrelations among the unit scores for these six areas are shown in Table 13.

The output of the simplex analysis on this matrix, using Kaiser's algorithm for scaling the variables, is shown in Table 14.

For the optimal ordering of the content areas as shown in Table 14, the Q squared value is 0.965. This represents a high degree of "goodness of fit" between the predicted matrix based on the single complexity factor loadings, and the original matrix. Therefore, these correlations can be explained quite successfully in terms of the existence of a single complexity factor among the content areas, when the areas are defined in terms of the objectives included on the unit placement tests. This analysis suggests that a single complexity continuum, from least to most, exists among the content areas of level E in the following order: subtraction, addition, division, multiplication, place value, and numeration.

Under the assumptions of valid test items and the validity of the unit representativeness of the placement tests, the results of level E simplex analysis would cast doubt upon the hypothesized complexity continuum, from least to most, of numeration, place value, addition, subtraction, multiplication, and division. These results represent a rejection of this curriculum. The suggested order might be interpreted as an alternate hypothesis regarding the preferred route for students to progress through this level.

The Study of Hypothesized Order Among Content Levels

The placement test results from a third demonstration school were reviewed for the purpose of investigating the complexity order among the content levels within a given area. One sample group received unit scores for each of the six levels, B, C, D, E, F, and G in the content areas of numeration and place value. A second sample group received unit scores for each of the five levels, C, D, E, F, and G in the content areas of addition and subtraction. A third sample group received unit scores for the four levels D, E, F, and G in the content areas of multiplication and division. The results of the complexity study among levels will now be discussed separately for the content areas of numeration and place value.

Numeration Complexity Study. Each student in the sample group of numeration received a percentage score on the unit subtests corresponding to levels B, C, D, E, F, and G. The intercorrelations among the unit scores for these six levels are shown in Table 15.

An analysis was performed on this intercorrelation matrix by using the computer program employing Kaiser's algorithm for scaling the variables of a Guttman simplex. The output of the simplex program is shown in Table 16.

For the optimal ordering of the levels as shown in Table 16, the Q squared value is 0.992. Therefore, these correlations can be explained quite successfully in terms of the existence of a single complexity factor among the content levels as defined by the unit placement tests. The complexity order, from least to most, of the levels within the content area of numeration as suggested by this analysis is C B D E F G.

Place Value Complexity Study. The same sample group used in the numeration complexity study also received subtest scores on the unit placement tests in six levels of place value for the purpose of investigating their complexity order. The intercorrelations among the unit scores for these levels are shown in Table 17 where the order of levels is the alphabetical order used in the IPI curriculum. The optimal ordering of the levels as determined by simplex analysis is shown in Table 18. The rather complete lack of agreement between these two orderings suggests that in the topic area of place value the units of study, as currently organized, are not arranged in the proper order of increasing complexity. The Q-squared value of 0.957, computed for this ordering, indicates the existence of a single complexity factor for the correlations as reported in Table 18.

A Comparison of Results Obtained by Scalogram Analysis and by Simplex Analysis

Tables 19, 20, 21, 22, 23, and 24 provide a comparison of the results of simplex analysis and scalogram analysis for six different content areas. These tables also permit the comparison of the two empirically derived scales with the original hypothesized scale since the latter is represented by the alphabetical order (starting from the bottom of the table) of the letters assigned to the curriculum levels.

Table 19 shows that the scalogram analysis supported the hypothesized order exactly while the simplex analysis resulted in the interchange of levels B and C. This result, together with the small difference between unit subtest means for these levels and the closeness of percent mastery figures, indicates that there is not a clear-cut prerequisite relationship between levels B and C.

Results such as those summarized in Table 20 reveal a rather disappointing situation for the person attempting to verify his hypothesized sequence or seeking empirical guidance for generating a better sequence. Actually, as far as the IPI math curriculum is concerned, these results may only serve to help to identify some of the causes of the difficulties that pupils were having with the place value sequence. This has been a content area that has been of continuing difficulty with respect to questions of pupil placement and speed of pupil progression. The hierarchical ordering of objectives and units has posed a problem in terms of agreement among curriculum writers in their logical analysis of the abilities involved.

The results summarized in Table 20 verify the real existence of this problem and suggest the need for a rather complete re-analysis of this area combined with a reexamination of all tests involved.

Tables 21, 22, 23, and 24 all reveal situations where there is complete agreement among hypothesized orders, simplex analysis, and scalogram analysis. Obviously this is a most satisfying result to the curriculum developer attempting to evaluate his curriculum hierarchies. A criticism of scalogram analysis as a tool for substantiating the existence of learning hierarchies is that results are ultimately a function of item difficulty. Under many conditions the relative difficulty of an item or of a test may be independent of its position in a hierarchical learning sequence. Since results from a simplex analysis are dependent upon intercorrelations and are independent of difficulty, the fact that the two analyses agree suggests that the hierarchy that is identified is not merely a function of test difficulty. This points to the value of using both the scalogram and the simplex analysis in studying curriculum hierarchies.

Conclusions

This study has attempted to demonstrate the use of scalogram analysis and simplex analysis in the investigation of hierarchical relationships among objectives and among units and levels of study in an individualized mathematics curriculum. Its goal has been to contribute to formative evaluation methodology for use in curriculum development. Although conclusive answers to questions concerning the most effective teaching sequences may require experimental studies, the procedures used here have been found to be helpful in offering suggestions for the revision and refinement of such sequences. This study has demonstrated the usefulness of both simplex and scalogram analysis for the purpose of assessing hypothesized hierarchical relationships among specific behavioral objectives as well as curriculum units. Areas both in curriculum structure and in test structure have been identified which require reexamination and modification. The implications of the results of this study are influencing the current work on the revision of the IPI mathematics curriculum. Some curriculum units have been redefined and certain areas such as Numeration and Place Value have been combined in an effort to create a more logical ordering of the instructional objectives. The alternative structures suggested by these procedures may also be helpful in providing guidance for curriculum and testing specialists in the design of future curriculums in IPI mathematics.

The experience of the investigators in this study has served to confirm the point that empirical procedures such as scalogram analysis and simplex analysis are only useful supplements to the careful logical analysis that must be involved in the original structuring of sequences and hierarchies. Certainly these empirical procedures cannot be used to generate such sequences. Also, the study of curriculum hierarchies should involve a number of case studies in which individual students are closely followed to determine the extent to which specified prerequisite learnings do indeed provide the necessary basis for efficient progression. However, in relatively large curriculum projects, such as IPI, where data on pupil mastery and progression are regularly collected, both scalogram and simplex analysis provide useful procedures for obtaining insights concerning the existence of hypothesized hierarchies. Where both procedures can be applied, the writers have found considerable agreement between results obtained but have felt that being able to study agreements and disagreements makes the joint employment of both analyses a worthwhile step.

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Objectives

		1	2	3	4	5
Individuals	1	X	X	X	X	X
	2	X	X	X	X	
	3	X	X	X		
	4	X	X			
	5	X				
	6					

Figure 1

A Perfect Guttman Scale

TABLE 1

NUMBER OF OBJECTIVES PER UNIT WHICH ARE REPRESENTED ON THE PLACEMENT TESTS

		Number of Objectives Per Area						
		Num.	Pl.V.	Add.	Sub.	Mult.	Div.	Total
Number of Objectives Per Level	G	4	1	2	3	2	3	15
	F	3	2	2	1	3	3	14
	E	2	2	2	2	3	2	13
	D	3	3	2	2	3	3	16
	C	4	2	3	2	0	0	11
	B	3	2	0	0	0	0	5
Total		19	12	11	10	11	11	74

TABLE 2

COMPARISON OF HYPOTHESIZED AND EMPIRICALLY DERIVED SCALES
FOR NUMERATION AND PLACE VALUE OBJECTIVES
EXPANDED NOTATION, INTEGER NUMBERS
N=74

Homogeneous Group	Hypothesized Scale	Empirical Scale	Empirical Mastery Percentage	REP	Permutation Index of Scale Difference
Expanded Notation Integer Numbers	PG1	PG1	.0	.936	13
	PF1	NF2	17.6		
	NF2	PE2	27.0		
	PE2	PB1	71.6		
	PD4	PC1	85.1		
	PC5	PF1	85.1		
	PC1	PB3	86.5		
	PB3	PC5	93.2		
	PB1	PD4	95.9		

TABLE 3

COMPARISON OF HYPOTHESIZED AND EMPIRICALLY DERIVED SCALES
FOR NUMERATION AND PLACE VALUE OBJECTIVES
COUNTING
N=74

Homogeneous Group	Hypothesized Scale	Empirical Scale	Empirical Mastery Percentage	REP	Permutation Index of Scale Difference
Counting	ND3	ND3	60.8	.946	0
	ND2	ND2	64.9		
	NC7	NC7	85.1		
	NC6	NC6	89.2		
	NC5	NC5	93.2		
	NC4	NC4	97.3		
	NB7	NB7	97.3		

TABLE 4

COMPARISON OF HYPOTHESIZED AND EMPIRICALLY DERIVED SCALES
FOR NUMERATION AND PLACE VALUE OBJECTIVES
N=74

Homogeneous Group	Hypothesized Scale	Empirical Scale	Empirical Mastery Percentage	REP	Permutation Index of Scale Difference
Inequality	PE3	PE3	82.4	.936	0
	NB9	NB9	87.8		
	NB8	NB8	93.2		
Rounding Estimates	NF1	NF1	8.1	.972	0
	NE3	NE3	25.7		
Base Conversion	NG5	NG5	2.7	.982	0
	NG4	NG4	4.1		
	NG3	NG3	5.4		
Exponential Form	NG8	NG8	1.4	1.000	0
	PF4	PF4	31.1		
Expanded Notation Decimal Numbers	PD9	PD7	67.6	.946	1
	PD7	PD9	79.7		
Decimal Conversion	NE5	NE5	24.3	.986	0
	ND5	ND5	44.6		

TABLE 5

COMPARISON OF HYPOTHESIZED AND EMPIRICALLY DERIVED SCALES
FOR ADDITION AND PLACE VALUE OBJECTIVES
INTEGER ADDITION - FACTS AND ALGORITHM
N=76

Homogeneous Group	Hypothesized Scale	Empirical Scale	Empirical Mastery Percentage	REP	Permutation Index of Scale Difference
Integer Addition Facts and Algorithm	AF1	AE6	34.2	.828	2
	AE6	AF1	42.1		
	AE4	AE4	48.7		
	AD8	AD6	64.5		
	AD6	AD8	76.3		
	AC3	AC3	80.3		
	AC2	AC2	90.8		
	AC1	AC1	90.8		

TABLE 6

COMPARISON OF HYPOTHESIZED AND EMPIRICALLY DERIVED SCALES
FOR ADDITION AND SUBTRACTION OBJECTIVES
INTEGER SUBTRACTION - FACTS AND ALGORITHM
N=76

Homogeneous Group	Hypothesized Scale	Empirical Scale	Empirical Mastery Percentage	REP	Permutation Index of Scale Difference
Integer Subtraction Facts and Algorithm	SE3	SE3	19.7	.936	1
	SD5	SD5	39.5		
	SD4	SD4	52.6		
	SC3	SC1	81.6		
	SC1	SC3	93.4		

TABLE 7

COMPARISON OF HYPOTHESIZED AND EMPIRICALLY DERIVED SCALES
FOR ADDITION AND SUBTRACTION OBJECTIVES
N=76

Homogeneous Group	Hypothesized Scale	Empirical Scale	Empirical Mastery Percentage	REP	Permutation Index of Scale Difference
Addition Subtraction	SF1	SF1	18.4	.912	1
Decimal Numbers	AF2	SE2	31.6		
	SE2	AF2	35.5		
Addition Subtraction	SG2	SG1	0	.974	2
Negative Numbers	SG1	AG1	7.9		
	AG1	SG2	19.7		
Addition Subtraction	SG3	AG3	14.5	.960	1
Exponential Numbers	AG3	SG3	23.7		

TABLE 8

COMPARISON OF HYPOTHESIZED AND EMPIRICALLY DERIVED SCALES
FOR MULTIPLICATION AND DIVISION OBJECTIVES
MULTIPLICATION ALGORITHM - INTEGER NUMBERS
N=74

Homogeneous Group	Hypothesized Scale	Empirical Scale	Empirical Mastery Percentage	REP	Permutation Index of Scale Difference
Multiplication Algorithm	MF10	MF5	10.8	.946	1
Integer Numbers	MF5	MF10	12.2		
	ME11	ME11	29.7		
	ME10	ME10	29.7		
	ME7	ME7	35.1		

TABLE 9

COMPARISON OF HYPOTHESIZED AND EMPIRICALLY DERIVED SCALES
FOR MULTIPLICATION AND DIVISION OBJECTIVES
DIVISION - CONCEPT AND FACTS
N=74

Homogeneous Group	Hypothesized Scale	Empirical Scale	Empirical Mastery Percentage	REP	Permutation Index of Scale Difference
Division	DE7	DE7	31.1	.878	2
Concept and Facts	DD7	DD5	56.8		
	DD5	DD2	64.9		
	DD2	DD7	77.0		

TABLE 10

COMPARISON OF HYPOTHESIZED AND EMPIRICALLY DERIVED SCALES FOR MULTIPLICATION AND DIVISION OBJECTIVES
N=74

Homogeneous Group	Hypothesized Scale	Empirical Scale	Empirical Mastery Percentage	REP	Permutation Index of Scale Difference
Multiplication	MD8	MD8	77.0	.820	1
Concept and Facts	MD4	MD3	81.1		
	MD3	MD4	87.8		
Multiplication Algorithm	MG6	MG6	2.7	1.000	0
	MG5	MG5	2.7		
Decimal Numbers	MF9	MF9	2.7		
Division Algorithm	DF6	DF6	2.7	.982	0
	DF4	DF4	4.1		
Integer Numbers	DE5	DE5	13.5		
Division Algorithm	DG5	DG5	.0	1.000	0
	DG4	DG4	.0		
Decimal Numbers	DF7	DF7	1.4		

TABLE 11

CORRELATIONS AMONG IPI MATHEMATICS AREAS ON LEVEL D PLACEMENT TESTS (INPUT MATRIX)
N=235

Content Area	Numeration	Place Value	Addition	Subtraction	Multiplication	Division
Num.	1.00	0.36	0.38	0.48	0.54	0.50
P.V.	0.36	1.00	0.12	0.32	0.27	0.28
Add.	0.38	0.12	1.00	0.59	0.69	0.61
Sub.	0.48	0.32	0.59	1.00	0.69	0.68
Mult.	0.54	0.27	0.69	0.69	1.00	0.77
Div.	0.50	0.28	0.61	0.68	0.77	1.00

TABLE 12

CORRELATIONS AMONG IPI MATHEMATICS AREAS ON LEVEL D PLACEMENT TESTS (OPTIMAL ORDERING FROM SIMPLEX ANALYSIS)
N=235

Content Area	Addition	Multiplication	Division	Subtraction	Numeration	Place Value
Add.	1.00	0.69	0.61	0.59	0.38	0.12
Mult.	0.69	1.00	0.77	0.69	0.54	0.27
Div.	0.61	0.77	1.00	0.68	0.50	0.28
Sub.	0.59	0.69	0.68	1.00	0.48	0.32
Num.	0.38	0.54	0.50	0.48	1.00	0.36
P.V.	0.12	0.27	0.28	0.32	0.36	1.00

TABLE 13

CORRELATIONS AMONG IPI MATHEMATICS AREAS ON LEVEL E PLACEMENT TESTS (INPUT MATRIX) N=89

Content Area	Numeration	Place Value	Addition	Subtraction	Multiplication	Division
Num.	1.00	0.34	0.05	0.06	0.21	0.14
P.V.	0.34	1.00	0.13	0.07	0.30	0.15
Add.	0.05	0.13	1.00	0.38	0.27	0.29
Sub.	0.06	0.07	0.38	1.00	0.38	0.21
Mult.	0.21	0.30	0.27	0.38	1.00	0.62
Div.	0.14	0.15	0.29	0.21	0.62	1.00

TABLE 14

CORRELATIONS AMONG IPI MATHEMATICS AREAS ON LEVEL E PLACEMENT TESTS (OPTIMAL ORDERING FROM SIMPLEX ANALYSIS) N=89

Content Area	Subtraction	Addition	Division	Multiplication	Place Value	Numeration
Sub.	1.00	0.38	0.21	0.38	0.07	0.06
Add.	0.38	1.00	0.29	0.27	0.13	0.05
Div.	0.21	0.29	1.00	0.62	0.15	0.14
Mult.	0.38	0.27	0.62	1.00	0.30	0.21
P.V.	0.07	0.13	0.15	0.30	1.00	0.34
Num.	0.06	0.05	0.14	0.21	0.34	1.00

TABLE 15

CORRELATIONS AMONG IPI MATHEMATICS LEVELS ON NUMERATION PLACEMENT TESTS (INPUT MATRIX)
N=74

Content Level	B	C	D	E	F	G
B	1.00	0.63	0.50	0.36	0.24	0.09
C	0.63	1.00	0.45	0.36	0.24	0.09
D	0.50	0.45	1.00	0.67	0.54	0.22
E	0.36	0.36	0.67	1.00	0.64	0.26
F	0.24	0.24	0.54	0.64	1.00	0.50
G	0.09	0.09	0.22	0.26	0.50	1.00

TABLE 16

CORRELATIONS AMONG IPI MATHEMATICS LEVELS ON NUMERATION PLACEMENT TESTS (OPTIMAL ORDERING FROM SIMPLEX ANALYSIS)
N=74

Content Level	C	B	D	E	F	G
C	1.00	0.63	0.43	0.36	0.24	0.09
B	0.63	1.00	0.50	0.36	0.24	0.09
D	0.45	0.50	1.00	0.67	0.54	0.22
E	0.36	0.36	0.67	1.00	0.64	0.26
F	0.24	0.24	0.54	0.64	1.00	0.50
G	0.09	0.09	0.22	0.26	0.50	1.00

TABLE 17

CORRELATIONS AMONG IPI MATHEMATICS LEVELS ON PLACE VALUE PLACEMENT TESTS (INPUT MATRIX)
N=74

Content Level	B	C	D	E	F	G
B	1.00	0.29	0.22	0.45	0.37	0.54
C	0.29	1.00	0.43	0.35	0.41	0.28
D	0.22	0.43	1.00	0.27	0.34	0.17
E	0.45	0.35	0.27	1.00	0.65	0.62
F	0.37	0.41	0.34	0.65	1.00	0.57
G	0.54	0.28	0.17	0.62	0.57	1.00

TABLE 18

CORRELATIONS AMONG IPI MATHEMATICS LEVELS ON PLACE VALUE PLACEMENT TESTS (OPTIMAL ORDERING FROM SIMPLEX ANALYSIS)
N=74

Content Level	D	C	F	E	G	B
D	1.00	0.43	0.34	0.27	0.17	0.22
C	0.43	1.00	0.41	0.35	0.28	0.29
F	0.34	0.41	1.00	0.65	0.57	0.37
E	0.27	0.35	0.65	1.00	0.62	0.45
G	0.17	0.28	0.57	0.62	1.00	0.54
B	0.22	0.29	0.37	0.45	0.54	1.00

TABLE 19

A COMPARISON OF THE COMPLEXITY ORDER OF IPI MATHEMATICS LEVELS IN THE CONTENT AREA OF NUMERATION OBTAINED FROM SIMPLEX AND SCALOGRAM ANALYSES
N=74

Simplex Analysis		Scalogram Analysis	
Unit Subtest Means	Complexity Order	Complexity Order	Percent Mastery
3.2	G	G	1.4
15.7	F	F	5.4
38.4	E	E	25.7
58.1	D	D	47.3
94.6	B	C	89.2
91.2	C	B	91.9

TABLE 20

A COMPARISON OF THE COMPLEXITY ORDER OF IPI MATHEMATICS LEVELS IN THE CONTENT AREA OF PLACE VALUE OBTAINED FROM SIMPLEX AND SCALOGRAM ANALYSES
N=74

Simplex Analysis		Scalogram Analysis	
Unit Subtest Means	Complexity Order	Complexity Order	Percent Mastery
86.8	B	G	25.7
38.2	G	E	27.0
62.5	E	F	43.2
70.5	F	D	79.7
93.5	C	B	81.1
[illegible]3	D	C	93.2

TABLE 21

A COMPARISON OF THE COMPLEXITY ORDER OF IPI MATHEMATICS LEVELS IN THE CONTENT AREA OF ADDITION OBTAINED FROM SIMPLEX AND SCALOGRAM ANALYSES
N-76

Simplex Analysis		Scalogram Analysis	
Unit Subtest Means	Complexity Order	Complexity Order	Percent Mastery
15.5	G	G	9.2
49.0	F	F	46.1
57.4	E	E	52.6
77.1	D	D	69.7
94.3	C	C	96.1

TABLE 22

A COMPARISON OF THE COMPLEXITY ORDER OF IPI MATHEMATICS LEVELS IN THE CONTENT AREA OF SUBTRACTION OBTAINED FROM SIMPLEX AND SCALOGRAM ANALYSES
N=76

Simplex Analysis		Scalogram Analysis	
Unit Subtest Means	Complexity Order	Complexity Order	Percent Mastery
15.3	G	G	1.3
39.2	F	F	28.9
39.2	E	E	30.3
54.0	D	D	47.4
95.5	C	C	93.4

TABLE 23

A COMPARISON OF THE COMPLEXITY ORDER OF IPI MATHEMATICS LEVELS IN THE CONTENT AREA OF MULTIPLICATION OBTAINED FROM SIMPLEX AND SCALOGRAM ANALYSES
N=74

Simplex Analysis		Scalogram Analysis	
Unit Subtest Means	Complexity Order	Complexity Order	Percent Mastery
3.0	G	G	2.7
10.0	F	F	2.7
31.4	E	E	21.6
92.8	D	D	94.6

TABLE 24

A COMPARISON OF THE COMPLEXITY ORDER OF IPI MATHEMATICS LEVELS IN THE CONTENT AREA OF DIVISION OBTAINED FROM SIMPLEX AND SCALOGRAM ANALYSES
N=74

Simplex Analysis		Scalogram Analysis	
Unit Subtest Means	Complexity Order	Complexity Order	Percent Mastery
0.8	G	G	0.0
4.6	F	F	0.0
29.0	E	E	21.6
80.7	D	D	71.6