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ABSTRACT

A review of research reported in 1970, which might be used by the elementary teacher in facilitating mathematics learning is presented. There are 16 basic categories of research included: teaching mathematical operations; children's perception of geometric representation; curriculum sequencing; teaching of non-decimal numeration; variables which affect verbal problem solving; teaching mental computation skills; applications of Piagetian research in education; role of manipulative materials; test and text selection; computer role in math instruction; individualized instruction; discovery learning; effects of reinforcement techniques on achievement; pupil-tutor utilization; relations of pupil attitude and achievement; relationships of teacher characteristics on student attitude and achievement. Research results and teaching conclusions are given for each of these areas. (JG)

THE RESEARCH FROM 1970: WHAT DID IT ADD?

In eleven previous bulletins (Suydam and Weaver), which summarized research on specific topics, it was evident that there are many gaps in our research-based knowledge. On many topics, no research at all could be cited. Has recent research filled any of those gaps? In a few cases, the answer is "Yes".

This bulletin presents some of the questions and answers from research published during 1970. The focus is on research that the teacher might use; there are other studies, of interest principally to researchers, which are not included.* Generally, studies which contribute substantially no new information to that presented in the previous bulletins are also omitted. An attempt has been made to take into consideration the variability in the quality of research as this bulletin was prepared.

1. *How can work with operations be taught more effectively?*

For several years, the number of studies that have focused on one aspect or another of arithmetic content, has been noticeably less than the number which have focused on other instructional variables. During 1970, there were only two studies which seemed to contribute consequential new information.

Dilley compared the teaching of division in grade 4 by using a distributive algorithm and a successive subtractions algorithm. Significant differences were found on an applications test favoring use of the successive subtractions algorithm, and on a retention test favoring the distributive algorithm.

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Green reported that, in teaching multiplication of fractional numbers in grade 5, an approach based on the area of a rectangular region was more effective than one based on finding a fractional part of a region or set. Each approach was studied in relation to two different modes of representation; the "area" approach taught with diagrams was most successful, the "fractional part" approach taught with cardboard strips was second, and the "fractional part" approach taught with diagrams was poorest.

2. *How well are children able to perceive geometric representations?*

Davis found that sixth graders scored significantly below those in grades 8 and 10 in their ability to perceive plane sections of selected solid figures. Palow confirmed that children appeared to be able to acquire this ability to visualize sections of solid figures at about age 12.

According to Cheatham, gains in geometric concepts made by seventh graders were not significantly different for those who constructed models with compass and straightedge or with paper-folding techniques.

3. *Where should content relatively new to the elementary school be placed?*

Burdick used materials on addition of integers in grades 5 through 8. On the basis of a variety of criteria, he concluded that grade 6 is the optimal level for teaching this content.

On the basis of task analysis, Shepler developed a unit on probability and statistics. He found that sixth grade students satisfactorily achieved 11 of the 14 objectives.

King found that although sixth graders could learn methods of deductive proof, teachers had difficulty teaching the material.

4. *Is it effective to teach non-decimal numeration?*

One study lends additional support to recognition that instruction with non-decimal bases does not have the transfer effect that originally was anticipated. Diedrich and Glennon found that for fourth graders, study of the decimal system alone is as effective as study of several non-decimal systems in promoting understanding of the decimal system of numeration. Understanding of place-value systems in general, however, is increased by study of non-decimal systems.

5. *What factors associated with the presentation of verbal problems contribute to their ease or difficulty?*

Bolduc reported that, for first graders to whom addition problems were read, those presented without a visual aid were significantly more difficult than those with a visual aid. However, placing the question first or last did not significantly affect scores.

For fourth graders, Linville found that both vocabulary level and sentence structure were determiners of difficulty level of problems. In another study with fourth graders, Swart found that pupils who were directed to draw a picture "telling the entire story" did as well as those who were taught to write equations to solve the problems. For problems in area measure, those who drew pictures scored significantly higher.

Harmon, from a study with sixth graders, reported that an "expository" approach was more effective than an "inquiry" approach for problems involving ratio concepts, especially for children of average and high intellectual ability.

6. *Is it effective to teach mental computation skills?*

Schall exposed fifth graders to short, frequent periods of oral practice administered in various modes. He found that the exercises resulted in increased ability to compute mentally and in a gain in attitude, although no significant differences were found between groups who used televised lessons, lessons on audio-tape, or programmed materials.

7. *What effect has Piagetian-oriented research on instruction?*

Increasing attention has been directed toward the work of Piaget. This year, as for the past few years, more studies were classified in a "Piagetian related" category than in any other. However, the majority of these studies were attempts to replicate Piaget's work, or to train children in conservation or other developmental behaviors. One of the few which may have greater implications than most for mathematics teaching was done by Biot. He found that the relationship between stage-to-stage matching of the conservation of area ability of six- and seven-year-olds and their ability to subdivide an area into a specified number of equivalent parts was weak. Their level of performance with common fractions was not as high as their level of conservation.

8. *What is the role of manipulative materials?*

For second graders, Fennema found no significant difference in overall learning of a principle when learning was facilitated by either a meaningful concrete or a meaningful symbolic model, but use of the symbolic model resulted in significantly better transfer. Use of concrete materials may not always be as essential to the development of meanings as has been hypothesized.

Weber reported no significant differences between groups of first graders who used manipulative materials for follow-up activities and those who used paper-and-pencil follow-up activities.

Fogelman, in a study with six- and seven-year-olds, reported that boys who manipulated materials demonstrated conservation more frequently than boys who watched the experimenter manipulate the materials. For girls, the opposite was true.

A wide variety of materials were used as teaching aids in a study of a mathematics laboratory by Howard. She reported that use of this approach, planned to facilitate learning a hierarchy of concepts needed by a small group of low achievers, resulted in both achievement and attitude gains.

Games often are included in the mathematics laboratory. Bowen, in a study with fourth graders, reported that pupils who used logic games had significantly higher gain scores than those who used a textbook to study logic. Wynroth reported that kindergarteners and first graders taught new concepts verbally through a series of competitive games, followed by self-paced written work later, had significantly higher scores on achievement tests than those who had a "normal" program. It should be noted that when games are included in the program, they should be carefully selected and supervised, to assure appropriateness and use to meet objectives.

9. *How can we select texts and tests more appropriately?*

McLaughlin compared two seventh grade textbooks and measured the achievement of students on knowledge and understanding of those elements of mathematics which have been included in the curriculum as a result of experimental programs. Groups using the textbook which (1) included more explanation and discussion of subject matter, (2) made greater use of symbolic notation, and (3) provided more examples with the explanations, scored significantly higher.

Text materials and test items often are analyzed in relation to a taxonomy of instructional objectives. Passi analyzed two recent textbook series and one older series. In all cases, low-level cognitive activities were more frequent than were high-level cognitive activities. The Manipulating (of symbols) level dominated the activities; the frequency of Translating, Analyzing, Synthesizing, and Evaluating levels was low.

Dahle used a grid of 120 objectives which ranged across five taxonomic levels. She found that a selected textbook series corresponded more closely to the distribution of objectives than did two standardized tests. She recommended that a study should be made of the effect of teacher involvement in the stating of objectives. Coincidentally, this was done by Piatt. He found that seventh graders whose teachers were trained to write behavioral objectives achieved significantly higher scores on subtests of computation and concepts than those whose teachers had no such training.

10. *How is the computer being used in mathematics instruction?*

Various studies have involved the computer to present instructional materials to pupils, to store resource units for teachers, and to score tests diagnostically. Little strong research evidence on the effectiveness of computer uses is yet available.

11. *How effective are procedures for individualizing instruction?*

Bierden found that for seventh graders an intra-class grouping plan using group instruction followed by independent work for individualized objectives resulted in significant gains in computational skills, concept knowledge, and attitude, with a reduction in anxiety.

Two programs make special provisions for instructional organization. In neither case is there evidence to show that the special provision makes for improved achievement. IPI and PLAN both (1) involve the use of behavioral objectives, (2) identify activities and materials to meet those objectives, and (3) provide sets of test items for those objectives; in short, a systems approach. Earlier research indicated that achievement results with IPI were not, in general, different from those with conventional programs. Recently, Ferney reported that fifth graders not using PLAN achieved significantly higher on arithmetic reasoning than the group using PLAN. Girls using PLAN achieved higher scores than did boys, and thus PLAN may be more appropriate to the learning styles of girls.

Scott matched 25 pairs of low-achieving seventh-graders on computation, concepts, and applications. One-half then used programmed materials appropriate to meet diagnosed needs; they made significantly greater gain scores in computation than did students in the regular classroom, although differences on concepts and applications were not significant.

12. *What is the effect of discovery learning at lower age levels?*

For kindergartners, Anastasiow and others reported that the rule-example method was most efficient for mastery of simple classification tasks, while a guided discovery method appeared to be more efficient for mastery of more complex classification tasks. Since those with low scores on a picture vocabulary test learned best with the rule-example method, while others did well under either treatment, it might be possible to group children and teach with the method which would seem to promise greater success.

13. *What is the effect of reinforcement techniques on achievement?*

The use of token reinforcements -- plastic tokens which may be traded for candy, toys, or other desired items -- has been reported to result in achievement gains in other curricular areas. Hillman reported that fifth graders given per-item knowledge of results, either with or without candy reinforcement, scored significantly higher in achievement with decimals than pupils given knowledge of results 24 hours later. He suggested that low achievers may profit more than high achievers. Heitzman studied pupils aged 6 to 9 in a summer arithmetic program. Those who were rewarded by tokens achieved significantly higher scores on a skills test than those who did not receive tokens (and also who may not have received knowledge of results). Immediate knowledge of results, rather than token reinforcement, may be the determining factor.

Masek reported significant increases in arithmetic performance and level of task orientation of underachieving first and second graders during periods when teachers emphasized reinforcement such as verbal praise, physical contact, and facial expression.

14. *How effective are pupil-tutors?*

Evidence from two studies seems to indicate that use of pupil-tutors may be a valuable as well as inexpensive way of providing some remedial help for other pupils. Ackerman found that pairing low-achieving third graders with either high- or low-achieving sixth-grade tutors resulted in computation scores which were significantly higher than the scores of those third-graders who only talked with sixth graders on non-arithmetic activities or who had no tutor-contact.

Burrow had high-achieving pupils in grades 6, 7, and 8 devise lesson plans organized according to diagnostic test results of low-achieving pupils in grades 3, 4, and 5. Tutored pupils achieved higher gain scores on computational skills than did untutored pupils, regardless of the achievement level of the tutors. Tutors themselves did not achieve significantly more than others who did not tutor.

15. *What is the relationship of pupil attitude and achievement?*

In a study with seventh graders, Burbank found that students' attitudes toward mathematics correlated significantly with achievement in mathematical reasoning, concepts, and computation. And in another study with seventh graders, Bachman also found that their general self-concept and, to a greater extent, their self-concept in mathematics were significantly related to mathematics achievement. Two studies thus lend support to the belief of most educators that a positive relationship exists between attitude and achievement, although it has been shown in few previous studies.

16. *What teacher characteristics are related to pupil achievement and attitudes?*

Caezza, in a survey with over 2700 pupils and 104 teachers in grades 2-6, reported low correlations between pupil achievement and pupil or teacher attitude toward mathematics. Teachers' attitudes did not appear to be affected by their educational background, nor were pupils' attitudes affected by the level of teacher achievement. At the same grade levels, Wess also found no significant relationship between teachers' attitudes toward mathematics and pupils' attitudes or achievement. Phillips found that the attitude toward mathematics of seventh-grade pupils was significantly related to the attitude of their sixth grade teacher. In fact, the type of teacher attitude encountered by a pupil for two or for three of his past three years was significantly related to his present attitude and achievement.

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* A complete annotated listing of studies published during 1970 will be available both from ERIC Information Analysis Center for Science and Mathematics Education and in the November 1971 issue of the Journal for Research in Mathematics Education.

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