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ABSTRACT

Presented is the task analysis for "Developing Mathematical Processes, Arithmetic, Book 1: Comparing and Equalizing Objects and Sets." This task analysis consists of identifying the behavioral objectives necessary for comparing and equalizing objects and sets and arranging them in a logical sequence for instruction. The 52 items in this task analysis may be grouped into five major topic areas. The two initial areas are "Counting and Writing Numerals" and "Description and Classification of Sets and Objects." These two areas are prerequisite of the next two areas, "Comparison of Discrete Sets on the Property of Numerousness" and "Comparison of Objects on Continuous Properties." These two areas are prerequisites for the final area "Ordering and Equalizing Sets and Objects." The booklet contains specific descriptions of each of the 52 items comprising the task analysis. [Not available in hardcopy due to marginal legibility of original document.] (Author/CT)

Working Paper No. 48

The Task Analysis for Developing Mathematical Processes, Arithmetic Book 1: Comparing and Equalizing Objects And Sets

Report from the Project on Individually Guided Elementary Mathematics, Phase 2: Analysis Of Mathematics Instruction

Wisconsin Research and Development
CENTER FOR COGNITIVE LEARNING

THE UNIVERSITY OF WISCONSIN
Madison, Wisconsin

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The Task Analysis for Developing Mathematical Processes, Arithmetic, Book 1: Comparing and Equalizing Objects and Sets

by

Thomas A. Romberg, John G. Harvey, and
Douglas B. McLeod

Report from the Project on

Individually Guided Elementary Mathematics

Phase Two, Analysis of Mathematics Instruction

Thomas A. Romberg and John G. Harvey
Principal Investigators
James M. Moser
Associate Scientist

Wisconsin Research and Development
Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin

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The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Working Paper is from the Project on Individually Guided Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project has developed and tested a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Analysis of Mathematics Instruction is currently the only active phase of the mathematics project and has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focused on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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ABSTRACT

This paper presents the task analysis for Developing Mathematical Processes, Arithmetic, Book 1: Comparing and Equalizing Objects and Sets.

INTRODUCTION

The purpose of this working paper is to present the task analysis for Arithmetic, Book 1: Comparing and Equalizing Objects and Sets, the first book in the Developing Mathematical Processes (DMP) series being prepared by the Analysis of Mathematics Instruction Project of the University of Wisconsin Research and Development Center for Cognitive Learning. The identification of content, the task analysis, and the organization of behaviors identified through task analysis into topics constitute the first steps in the development sequence (Harvey, Romberg, and Fletcher, 1969).

The initial objective of the DMP mathematics program is for students to learn to accurately complete equations of the general form $A = B \pm X$. Conceptually, these equations or mathematical statements simply require the students to compare two objects with respect to a metrizable property and make them equivalent with respect to that property by adding some amount to or taking some amount from one of the objects. This compare-and-equalize process is considered to be fundamental to basic mathematics and is well within the intellectual capabilities of young children (Romberg and Rowston, 1969; Romberg and Cornewicz, 1970; and Romberg and Planert, 1970).

From a mathematical point of view, comparing involves two objects and an identified property which has defined for it a [illegible] metric m. If A and B are two objects having the identified property, then by the trichotomy condition, either $m(A) = m(B)$, $m(A) < m(B)$, or $m(A) > m(B)$. In addition, there is an object X such that $m(A) = m(B) \pm m(X)$. Learning to compare and equalize objects and sets is a behavior prerequisite to using numbers in stating correct mathematical sentences. In the following section the task analysis of this process is presented.

II

THE TASK ANALYSIS

Following identification of the compare-and-equalize process a series of steps followed which identified the behaviors needed to reach this goal. The specification of the behavioral objectives and their arrangement into prerequisite skeins is accomplished by a process known as "task analysis." Here each unit or concept is analyzed in terms of its subconcepts, properties, or attributes, together with the rules necessary for their combination, as well as prerequisite behaviors the student must possess for any unit. These prerequisite behaviors are then used to develop a chart relating the units.

The task analysis provides direction for the staff of the Analysis of Mathematics Instruction Project in sequencing the concepts, but this is only one of its contributions to the development effort. It helps the team to describe general problem-solving processes for mathematics. For example, the task analysis being presented in this Working Paper assisted in the identification of the compare-and-equalize process which is a recurring theme in elementary mathematics. Another contribution is that, since the task analysis is described in terms of student behaviors, it is a complete guide for the generation of valid test items and reliable tests which are used in the evaluation of the curriculum being developed. Finally, the task analysis helps to identify connections between the various subject matter areas.

The major components of the task analysis of Arithmetic Book 1 can be described in terms of five areas. (See Figure 1.) The initial component is "Description and Classification of Sets and Objects." In order to compare sets, one must identify properties of sets upon which they can be compared. The initial objectives have students identify properties and decide whether or not various objects or sets have a particular property common.

Major Components of the Task Analysis for
Developing Mathematical Processes, Arithmetic, Book 1:
Comparing and Equalizing Objects and Sets

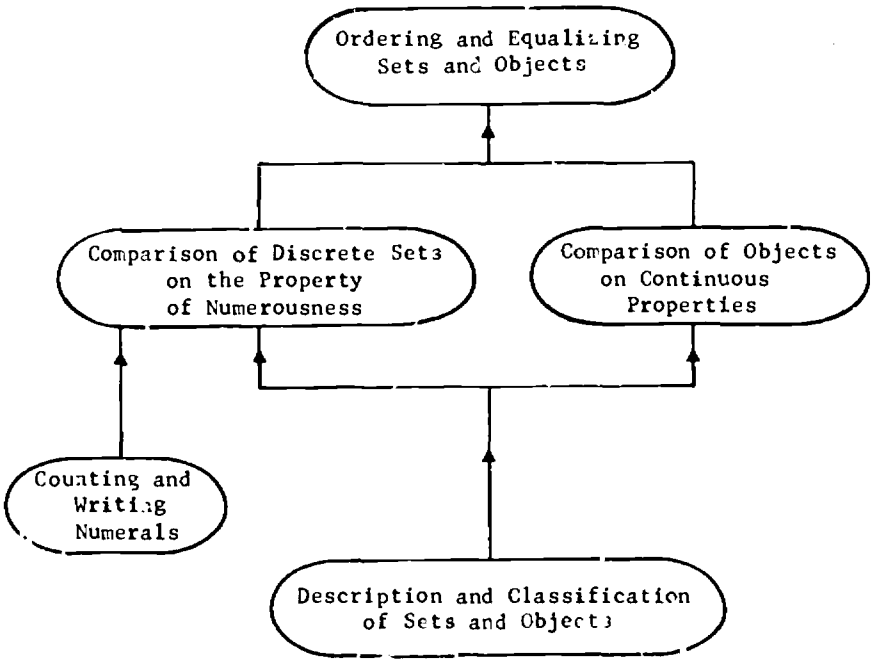


Figure 1

The two components "Comparison of Objects on Continuous Properties" and "Comparison of Discrete Sets on the Property of Numerousness" specify the behaviors needed to make comparisons for the two kinds of physical sets, continuous and discrete. Within the continuous set category, comparisons of objects are specified on a variety of physical properties such as length and weight. Within the discrete set category, the property of numerousness is identified. In each of these categories behaviors proceed from comparing objects directly to comparing them by using physical representations, then pictorial representations, and finally abstract representations. Thus, instruction goes from concrete experience through to abstract experience.

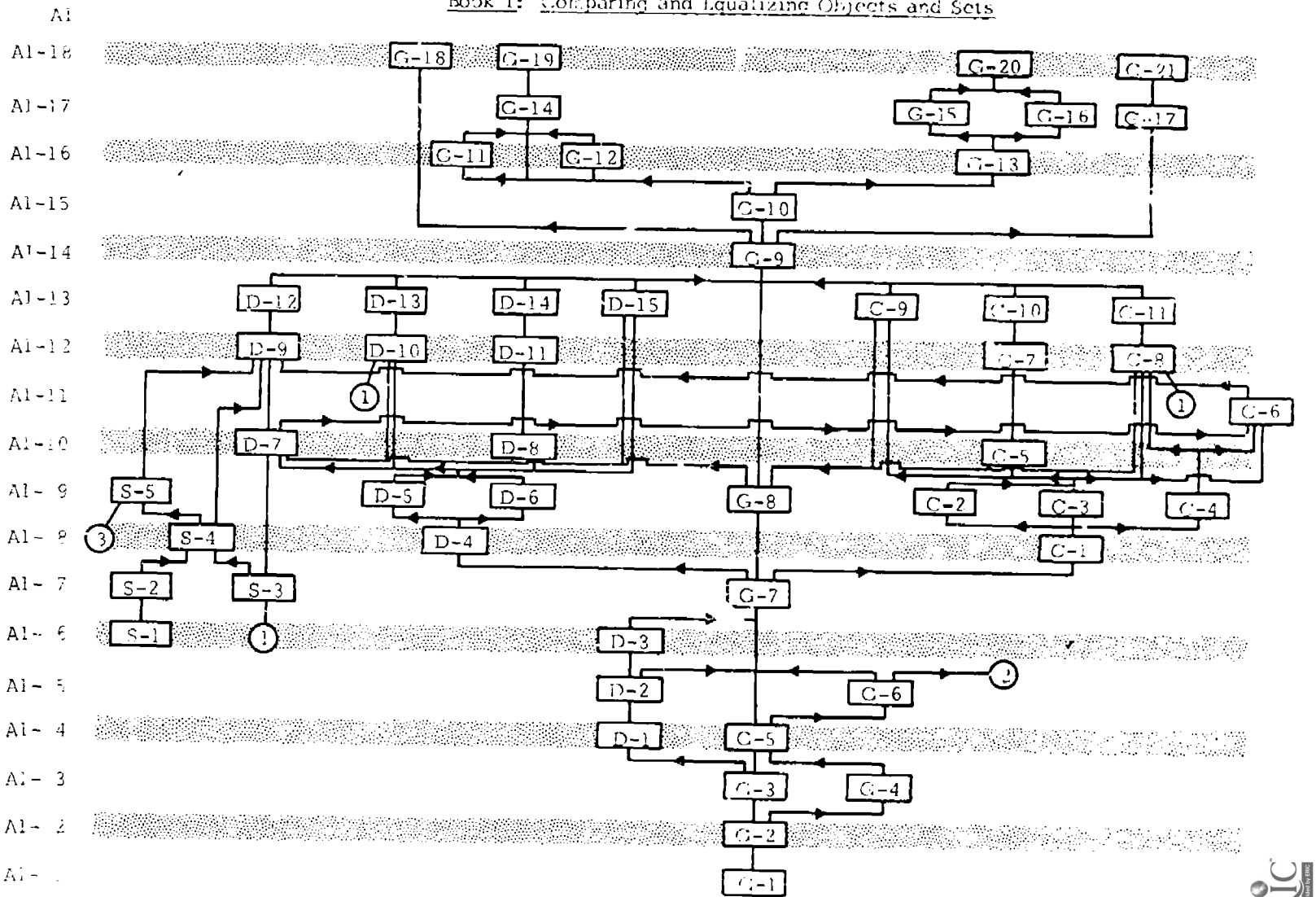
The fourth component is "Ordering and Equalizing Sets and Objects." Within this component linear ordering is considered and its properties analyzed. The analysis of equalization describes "putting with" and "taking from" objects or sets to make them equal on the identified property.

The fifth component is titled "Counting and Writing Numerals." Numerals are arbitrary symbols used to represent properties of objects by measurement or numerousness of sets by counting. Since these symbols are arbitrary, a function of the culture, these objectives can be included at any time instructionally. However, for this program counting and the writing of numerals are introduced only after a need has arisen for the representation of sets or length.

Figure 2 represents the actual task analysis of Arithmetic Book 1. The figure includes behaviors at 18 different levels. Behavioral objectives are indicated in terms of boxes (▭); circles (○) designate

Figure 2

Task Analysis for Developing Mathematical Processes, Arithmetic.
Book 1: Comparing and Equalizing Objects and Sets



related behaviors from other task analyses. Moving upward on the chart indicates a progression from subordinate to superordinate behaviors. The lines show the relationship between behaviors, and the arrowheads (→) indicate the direction of dependence.

Table 1 contains the specific behavioral objectives of the task analysis presented in Figure 2. Each behavioral objective is labeled with a two-part code. The first objective, for example, is A1-1, G-1. The first part of the code, A1-1, tells that this objective is found in the task analysis of Arithmetic Book 1 on the first level; the second part of the code, G-1, corresponds to the label on the box that shows the location of this objective in Figure 2. In the label G-1, the letter refers to a category of objectives, and the number indexes the objectives within the category. The letters and the categories to which they refer are G for general behaviors that are appropriate to both continuous and discrete sets, C for behaviors related to continuous sets, D for behaviors dealing with discrete sets, and S for objectives related to using symbols in counting and writing numerals.

Table 2 gives the titles of the related task analyses that are represented by the circles in Figure 2. The titles are numbered so that they correspond to the numbered circles that represent these related task analyses in the diagram.

A task analysis, however, does not indicate how instruction will take place. Since instruction must proceed sequentially, decisions have been made as to which objectives are to be taught in what order. The sequence of instruction chosen for Arithmetic Book 1 is indicated by a

[Text continued on Page 14]

Table 1

Behavioral Objectives for the Task Analysis of
Developing Mathematical Processes, Arithmetic, Book 1:
Comparing and Equalizing Objects and Sets

Level	Label	Objective
A1-1	G-1	Given at least two objects which are different on some identifiable property, discriminates between the objects.
A1-2	G-2	Given an object(s), differentiates between properties of the object(s).
A1-3	G-3	Given an object(s), identifies an intrinsic property of the object(s).
A1-3	G-4	Given an object(s), identifies an extrinsic property of the object(s).
A1-4	D-1	Given a collection of sets, identifies sets of objects.
A1-4	G-5	Given a set of objects and an identified property, classifies the elements of the set with respect to the given property.
A1-5	D-2	Given a collection of sets and an identified property, classifies the sets with respect to the given property.
A1-5	G-6	Given a set of objects, describes the objects by identifying the properties which they possess.
A1-6	S-1	Given the numerals 0, 1, 2, ..., 10, identifies a specified numeral.
A1-6	D-3	Given a collection of sets, describes the sets by identifying the properties which they possess.

Table 1 (continued)

Level	Label	Objectives
A1-7	S-2	When shown one of the numerals 0, 1, 2, ..., 10, attaches the correct verbal label to it.
A1-7	S-3	When shown a tally (of a set), attaches the correct verbal (numeral) label to the tally.
A1-7	C-7	Given a set of objects, identifies a property on which the given objects can be compared.
A1-8	S-4	When shown a tally (of a set), selects (verbally selects) the correct numeral corresponding to the tally.
A1-8	D-4	Given a collection of sets, identifies numerousness as a property on which the sets can be compared.
A1-8	C-1	Given a set of objects, identifies continuous properties on which the objects of that set can be compared, including length, weight, time, area, volume, angular measure, etc., where appropriate.
A1-9	S-5	Given a tally (of a set), writes the correct numeral which corresponds to the tally.
A1-9	D-5	Given two sets, chooses a means of comparing the two sets on the property of numerousness: directly, physical representation of one or both, pictorial representation of one or both, or numerical representation of both.
A1-9	D-6	Given two sets, matches the elements of one with (some of) the elements of the other.

Table 1 (continued)

Level	Label	Objective
A1-9	C-8	Given two objects and an identified property, compares the two objects on the identified property.
A1-9	C-2	Given two objects and an identified property, correctly chooses a place (internal or external to the objects) at which to begin matching one object with the other object.
A1-9	C-3	Given two objects and an identified property, chooses a means of comparison: directly, physical representation of one or both the objects, pictorial representation of one or both the objects, or numerical representation of one or both the objects.
A1-9	C-4	Given an object and an identified property on that object, identifies an arbitrary unit of measure for the identified property.
A1-10	D-7	Given two sets of objects, counts the number of elements in each of the two sets.
A1-10	D-8	Given two sets of objects, physically represents the numerousness of one or both of the sets.
A1-10	C-5	Given two objects and an identified property, physically represents the identified property of one or both of the objects.
A1-11	C-6	Given two objects, an identified property, and a unit measure appropriate to that property, measures the objects on the identified property and counts the number of unit measures required for each of the objects.

Table 1 (continued)

Level	Label	Objective
A1-12	D-9	Given two sets and a count of the number of elements in each of them, records the number of elements in each by selecting a correct numerical representation.
A1-12	D-10	Given two sets, constructs a pictorial representation of the numerousness of each of the sets.
A1-12	D-11	Given two sets and a physical representation of the numerousness of one or both of the sets, records the representation.
A1-12	C-7	Given two objects and an identified property and given that the property of one or both of the objects has been physically represented, records the representation.
A1-12	C-8	Given two objects and an identified property, constructs a pictorial representation of each of the objects on the identified property.
A1-13	D-12	Given two sets and a numerical representation for the numerousness of each, compares the two numerical representations.
A1-13	D-13	Given two sets and given that the numerousness of each of the sets has been pictorially represented, compares the pictorial representations of the numerousness of the two sets.
A1-13	D-14	Given two sets and given that the numerousness of each of the sets has been physically represented, compares the physical representations of the numerousness of the two sets.

Table 1 (continued)

Level	Label	Objective
A1-13	D-12	Given two sets, directly compares the two sets on the property of numerousness.
A1-13	C-9	Given two objects and an identified property, directly compares the two objects on that property.
A1-13	C-10	Given two objects, an identified property, and a physical representation of the objects on the property, compares the physical representations of the two objects.
A1-13	C-11	Given two objects, an identified property, and a pictorial representation of each of the objects on the property, compares the pictorial representations of the two objects.
A1-14	G-9	Given two objects, an identified property, and a comparison of the two objects on the property, describes if the two objects are or are not the same on the property.
A1-15	G-10	Given two objects, an identified property, a comparison of the two objects on the property, and a description that they are not the same on the property, describes which object has the larger (smaller) measure on the property.
A1-16	G-11	Given three objects A, B, and C such that A is smaller than B on an identified property, describes whether A is smaller than C, A is the same as C, or C is smaller than A on the property.

Table 1 (continued)

Level	Label	Objective
A1-16	G-12	Given three objects A, B, and C such that A is smaller than B on an identified property, describes whether B is larger than C, B is the same as C, or C is larger than B on the property.
A1-16	G-13	Given two objects that are not the same on an identified property, chooses to put with the (representation of the) smaller object or take away from the (representation of the) larger object until the (representation of the) smaller or larger object is modified so that it is the same as the (representation of the) larger or smaller object, respectively.
A1-17	G-14	Given three objects A, B, and C, and an identified property, places the three objects in order from smallest to largest on the property.
A1-17	G-15	Given that it has been decided to take away from the (representation of the) larger object, chooses a means of doing so: directly, using a physical representation, using a pictorial representation, using a numerical representation.
A1-17	G-16	Given that it has been decided to put with the (representation of the) smaller object, chooses a means of doing so: directly, using a physical representation, using a pictorial representation, using a numerical representation.

Table 1 (continued)

Level	Label	Objective
A1-17	G-17	Given two objects, a specific combination of these objects, and an appropriate range, compares the combinations with elements of the range on the given property.
A1-18	G-18	Given objects A, B, and C such that A is the same as B and B is the same as C on the given property, states that A is the same as C on the given property.
A1-18	G-19	Given objects A, B, and C such that A is smaller than B and B is smaller than C on the given property, states that A is smaller than C on the given property.
A1-18	G-20	Given two objects, an identified property, and a (possibly identical) representation of each of the objects, equalizes the representations of the two objects.
A1-18	G-21	Given objects A and B, a specific combination of these objects, and an appropriate range, identifies an element of the range with the same measure on the given property as the combination of A and B.

Table 2

Related Task Analyses

1 PICTORIAL REPRESENTATION

2 SHAPE

3 ORIENTATION

topic outline (Table 3).[1] Instruction begins with description and classification of sets and objects. It then proceeds to the comparison of objects on the continuous property of length followed by ordering and equalizing objects on length. Next comparison of discrete sets on numerousness and ordering and equalizing sets on this property is taught. This is followed by counting. Applying counting to problems involving comparing sets and objects, writing numerals, and writing mathematical sentences based on the compare-and-equalize process are the objectives of Arithmetic, Book 2, of this series.

[1] The rationale for this sequence can be found in Romberg, T. A., Fletcher, H. J. and Scott, J. A. Working Paper from the Wisconsin Research and Development Center for Cognitive Learning, The University of Wisconsin, 1968, No. 12.

Table 3

Topic Outline for Developing Mathematical Processes, Arithmetic, Book 1: Comparing and Equalizing Objects and Sets

Topic	
1	Identifying Properties of Objects
2	Classifying and Describing Objects
3	Identifying Length as a Property of Objects and Comparing Objects on Length
4	Equalizing Objects on Length
5	Ordering Objects on Length
6	Representing Length Physically
7	Representing Length Pictorially
8	Classifying and Describing Sets
9	Identifying Numerousness as a Property of Sets and Comparing Sets on Numerousness
10	Equalizing Sets on Numerousness
11	Ordering Sets on Numerousness
12	Representing Numerousness Physically
13	Representing Numerousness Pictorially
14	Tallying Units of Length
15	Counting from 0 to 10
16	Recognizing the Numerals 0-10

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