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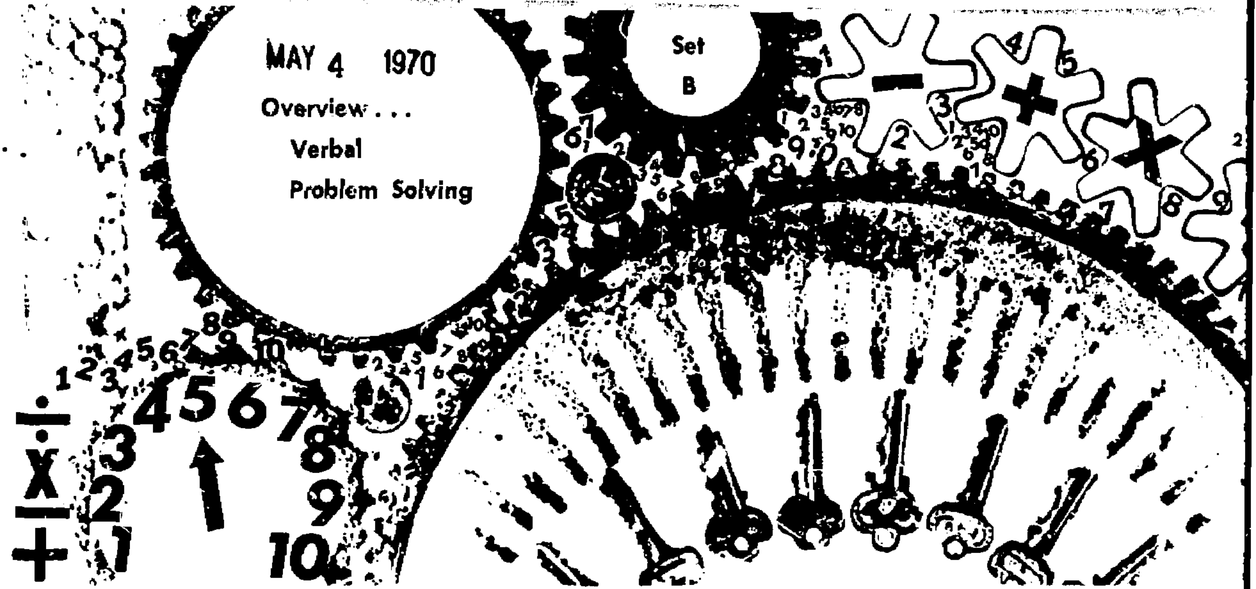
ABSTRACT

This bulletin gives an overview of research studies which pertain to the verbal problem-solving ability of elementary school children. Studies included relate directly to the following questions: (1) What factors are related to problem solving ability? (2) What are the characteristics of good problem solvers? (3) How important is reading to problem-solving ability? (4) What is the role of "understanding?" (5) Is the study of vocabulary helpful? (6) What problem settings are most effective? (7) Does the order of processes affect problem difficulty? (8) Does the order of data affect problem difficulty? (9) Should we place the question first or last? (10) What is the role of formal analysis? (11) What techniques help in improving pupils' ability to solve problems? (12) Is it helpful for pupils to work in groups? (FL)



Using Pesearch: A Key to Elementary School Mathematics

VERBAL PROBLEM SOLVING

What factors are related to problem solving ability?	Intelligence is related to problem solving ability; however, neither sex nor socio-economic status has been found to be related to it.
What are the characteristics of good problem solvers?	Among the factors which characterize high achievers are: ability to note likenesses, differences, and analogies; understanding of mathematical terms and concepts; ability to visualize and interpret quantitative facts and relationships; skill in computation; ability to select correct procedures and data; and comprehension in reading. Reasons for difficulty with problem solving generally focus on computation, reading, and knowledge of fundamental mathematical concepts.
How important is reading to problem solving ability?	Reading is obviously important, since if the child cannot read the problem, he will have difficulty in doing more than guessing how to solve it. It is suggested that reading and other interpretive skills specifically related to problem solving be developed in the problem solving program.
What is the role of "understanding?"	Systematic instruction not only in how to solve a problem but in why that process is appropriate has been found to be effective in increasing problem solving achievement and understanding.
Is the study of vocabulary helpful?	Since knowledge of vocabulary has been found to be important to success in problem solving, it follows that instruction in the vocabulary to be used will increase scores.

What problem settings are most effective?	Evidence on whether settings should be familiar to the child is conflicting. It is apparently not as important as has sometimes been supposed: the child will be interested in a variety in settings.
Does the order of processes affect problem difficulty?	There is some evidence to show that the order in which the processes are presented in multi-step problems may affect their difficulty.
Does the order of data affect problem difficulty?	Significantly higher scores resulted when numerical data were presented in the order in which they would be needed to solve the problem.
Should we place the question first or last?	For some children, it appears that a problem is easier when the question is placed first. This shortens the time needed to solve the problem.
What is the role of formal analysis?	Giving children many opportunities to solve problems and letting children solve problems in a variety of ways appears to be more helpful than formal analysis procedures.
What techniques help in improving pupils' ability to solve problems?	While research evidence supporting each is somewhat limited, researchers have suggested that these techniques should be included in the problem solving program: (1) Provide problems at varying levels of difficulty. (2) Have pupils write mathematical sentences. (3) Have pupils dramatize problem situations. (4) Have pupils make drawings and diagrams. (5) Have pupils formulate problems. (6) Present problems orally. (7) Use problems without numbers. (8) Have pupils designate the process to be used. (9) Have pupils note missing or extra data. (10) Have pupils test the reasonableness of their answers. (11) Use a tape recorder to aid poor readers.
Is it helpful for pupils to work in groups?	The evidence suggests that pupils achieve at least as much by working independently when solving problems as by working in groups of two, three, or four.

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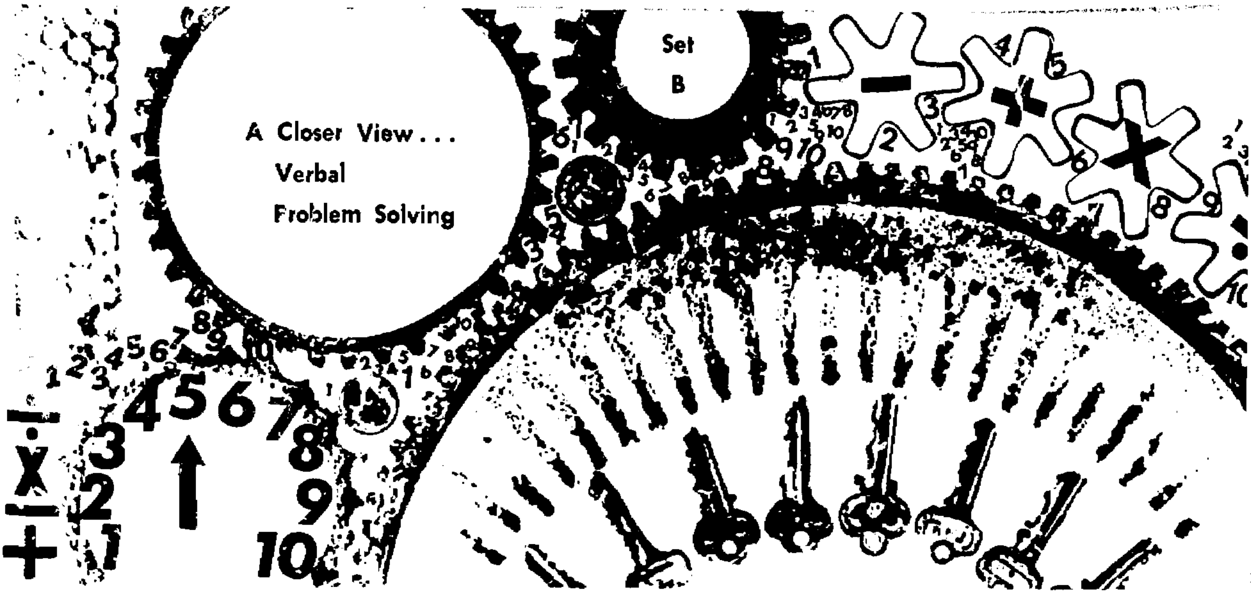
If you would like more information about the research whose findings are cited above, contact MARILYN N. SUYDAM, Project Director, at The Pennsylvania State University, University Park, Pennsylvania, 1680?.



Using Research: A Key to Elementary School Mathematics

VERBAL PROBLEM SOLVING

Verbal problem solving has attracted more attention from researchers than any other topic in the mathematics curriculum. It is considered a plausible way to help children learn how to apply mathematical ideas and skills to the solving of real-life problems--and is a challenge to both pupils and teachers.

It should be noted that virtually all of the research on problem solving has been associated with whole numbers. We lack evidence about the extent to which the research can be generalized to other kinds of numbers. This is a topic for future research.

What factors are related to problem solving ability?

It is generally concluded that:
(1) IQ is significantly related to problem solving ability;
(2) sex differences do not appear to exist in the ability to solve verbal problems; and
(3) socio-economic status alone does not appear to be a significant factor.

What are the characteristics of good problem solvers?

Many researchers have proceeded on the assumption that if we can ascertain what problem solvers who are successful have in common, we may be able to help those who do not do as well. Alexander (1960) and Hansen (1944) compared pupils on selected factors

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The bulletin was prepared by MARILYN N. SUYDAM, The Pennsylvania State University, Project Director, and J. FRED WEAVER, The University of Wisconsin-Madison, Project Consultant. Art by Ed Saffell.

ld be noted that research is variable with respect to its quality; hence, the gree of confidence cannot be placed in all findings. An attempt has been take this fact into consideration in preparing this bulletin.

thought to be related to problem solving ability. Among the factors which characterized high achievers were: (1) ability to note likenesses, differences, and analogies; (2) understanding of mathematical terms and concepts; (3) ability to visualize and interpret quantitative facts and relationships; (4) skill in computation; (5) ability to select correct procedures and data; and (6) comprehension of reading materials.

Related to these findings are the specific errors which John (1930) found that children in grades 4, 5, and 6 made in solving problems: errors in reasoning, in use of fundamentals, and in reading were found to be most frequent. Johnson (1944) noted that other researchers reported similar reasons why children do not succeed in solving problems: (1) ignorance of mathematical principles, rules or processes; (2) insufficient mastery of computational skills; and (3) inadequate understanding of vocabulary. In a more recent study, Chase (1960) reported test data collected from sixth graders showing that the three primary factors related to success in problem solving are computation, reading to note details, and knowledge of fundamental mathematical concepts.

How important is reading to problem solving ability?

Treacy (1944) and Alexander (1960) found that good and poor achievers in problem solving differed on many aspects of reading. Treacy concluded that reading should be regarded as a composite of specific skills rather than as a generalized ability. We may infer that reading and other interpretive skills should be specifically developed in the problem solving program.

Balow (1964) studied 468 sixth graders who had been classified by reading and computational levels. He reported that higher levels of problem solving ability were associated with higher levels of reading and computational ability, but that much of this relationship apparently was the result of the high correlation of these abilities with IQ.

What is the role of "understanding" in problem solving?

We know that many children have difficulty in deciding what process to use to solve a given problem. It therefore has seemed evident to researchers that to make this decision without guessing or using trial and error procedures, pupils must understand both the meanings and the effects of the fundamental processes. Pace (1961) presented one group of fourth graders with systematic instruction in which children not only decided how to solve a problem, but why that process was appropriate, while another group merely solved the problems with no discussion. The first group made statistically significant gains on tests of problem solving. Interviews and other tests used to measure understanding showed that both groups improved, with greater gains for those who received specific instruction.

Is the study of vocabulary helpful in improving problem solving?

Among those who experimented with the teaching of vocabulary was VanderLinde (1964), who reported that such specific instruction on quantitative vocabulary was effective in increasing problem solving scores (for problems in which that vocabulary was used).

What problem settings are most effective?

Whether children's success in solving problems is affected by the familiarity in the settings was studied by many. Brownell and Stretch (1931) reported on the reactions of 256 fifth graders to carefully matched problems at four degrees of familiarity. They concluded that there is "no ground for reasonable belief that problems are made unduly difficult for children by being given unfamiliar settings."

While some other researchers confirmed this finding, there is conflicting evidence on this question. Washburne and Osborne (1926) concluded that unfamiliarity of setting has some influence on success in problem solving, although it is "not as large an element as might be supposed." On the other hand, Sutherland (1942) was among those who found that pupils were decidedly more successful on problems with familiar settings.

It has been concluded by many researchers that children like a variety of problem settings. It seems important that children be interested in problems and in ways of solving them.

In studying a different aspect related to this question, Scott and Lighthall (1967) reported that no statistically significant relationship was found between "need content" in problems and degree of "disadvantage." ("Need content" was defined low if problems concerned food and shelter, and high if they concerned such factors as belongingness, education, travel, etc. "Disadvantage" was determined by whether or not pupils were assured of food and shelter.)

Does the order in which fundamental processes appear affect problem difficulty?

Citing data from 4,444 pupils in grades 4, 6, and 7, Berglund-Gray and Young (1940) said "yes." They reported that the easier order for each pair of operations with whole numbers in two-step problems was: addition before subtraction or division; subtraction before division; and multiplication before any of the three others. However, we should note that this study was conducted at a time when there was considered to be only one way of solving a problem.

Does the order of data affect problem difficulty?

Burns and Yonally (1964) reported that, when the data in each of ten multi-step problems were in the order required to solve them, significantly higher scores resulted than when data were not in the order in which it would be used. For the 95 fifth graders they studied, reasoning ability was positively related to pupil success with problems which presented numerical data in mixed order.

Should we place the question at the beginning or the end of a problem?

Williams and McCreight (1965) concluded that for fifth and sixth graders, there was "some advantage to the child when the question was placed first," though no significant difference between mean scores was found. Time to solve was less when the question was placed at the beginning.

What is the role of formal analysis in problem solving?

Research evidence does not show that formal analysis (that is, requiring pupils to answer a specific set of questions in order) is an effective procedure (e.g., Burch, 1953). Washburne and Osborne (1926) noted that "merely giving many problems...appears to be most effective." Pace (1961) also suggested that giving many opportunities to solve problems and letting children solve problems in a variety of ways were especially helpful.

What other techniques help in improving pupils' ability to solve problems?

Many specific techniques have been reported to be helpful, though how helpful has been impossible to determine from the structuring of the research studies. Among the techniques which researchers suggest are:

(1) Provide a differentiated program, with problems at appropriate levels of difficulty.
(2) Have pupils write the number question or mathematical sentence for a problem.
(3) Have pupils dramatize problem situations and their solutions.
(4) Have pupils make drawings and diagrams using them to solve problems or to verify solutions to problems.
(5) Have pupils formulate problems for given conditions.
(6) Present problems orally.
(7) Use problems without numbers.
(8) Have pupils designate the process to be used.
(9) Have pupils note the absence of essential data, or the presence of unnecessary data.
(10) Have pupils test the reasonableness of their answers.
(11) Use a tape recorder to aid poor readers.

Some evidence exists to support each of these. Keil (1965) found that pupils who wrote and solved problems of their own were superior in problem solving ability to pupils who had the "usual textbook experiences." Riedesel (1964) reported that sixth grade classes using specific procedures plus 30 sets of verbal problems at two levels of difficulty achieved higher mean gains on problem solving tests than did control groups who followed the regular textbook program. For instance, Arnold (1969) reported evidence from sixth graders favoring the expression of problem relationships in number sentences. It should be noted that emphasis upon isolated word cues ("left," "in all," etc.) can be grossly misleading as a problem solving procedure. They may lead pupils away from recognition of the relationships inherent in the problem, which are crucial to its solution.

How should equations for problems be stated?

In a well-controlled study, Wilson (1967) studied two problem solving procedures, one using equations which express the real or imagined actions in the problem (an "action-sequence" structure) and the other using equations which emphasize operations by which the problem may be solved directly (a "wanted-given" structure), and a third practice-only control treatment. He reported that differences for ability to choose the correct operation, accuracy, and speed favored those taught the "wanted-given" structure over those taught the "action-sequence" structure on tests given during instruction and after a nine-week retention period. The "wanted-given" structure was also significantly better than the practice-only treatment on the immediate posttest and the retention test. On the other hand, Lindstedt (1963) reported many differences favoring a group who used a text program in which equations are structured in terms of the action, over a group using a "traditional type of problem solving program."

Could it be that one of these procedures is better than the other for certain children?

Is it helpful for pupils to work together in solving problems?

Evidence by investigators in other areas has indicated that children can learn more by working with partners or small groups than by working alone. In relation to verbal problem solving, however, this evidence has not been so clear.

Hudgins (1960) reported that fifth graders who worked on sets of verbal problems in groups of four solved significantly more problems than those who worked alone. When they then worked individually, no significant differences were found among their scores. In an extension of this study, Hudgins and Smith (1966) found that for pupils in groups of three, group solutions to problems were no better than the independent solutions of the most able member of the group, if he is perceived to be most able. (If he is not so perceived, the group will do better than he--or change their perception of him.)

Klugman (1944) found that two children working together at grades 4, 5, and 6 solved more problems correctly, but took a longer time than pupils working alone. In another study with fourth, fifth, and sixth graders, Dembo (1969) reported that there were no significant differences in the improvement of peer relations, attitude toward mathematics, or mathematical achievement between pupils working in small groups or independently.

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