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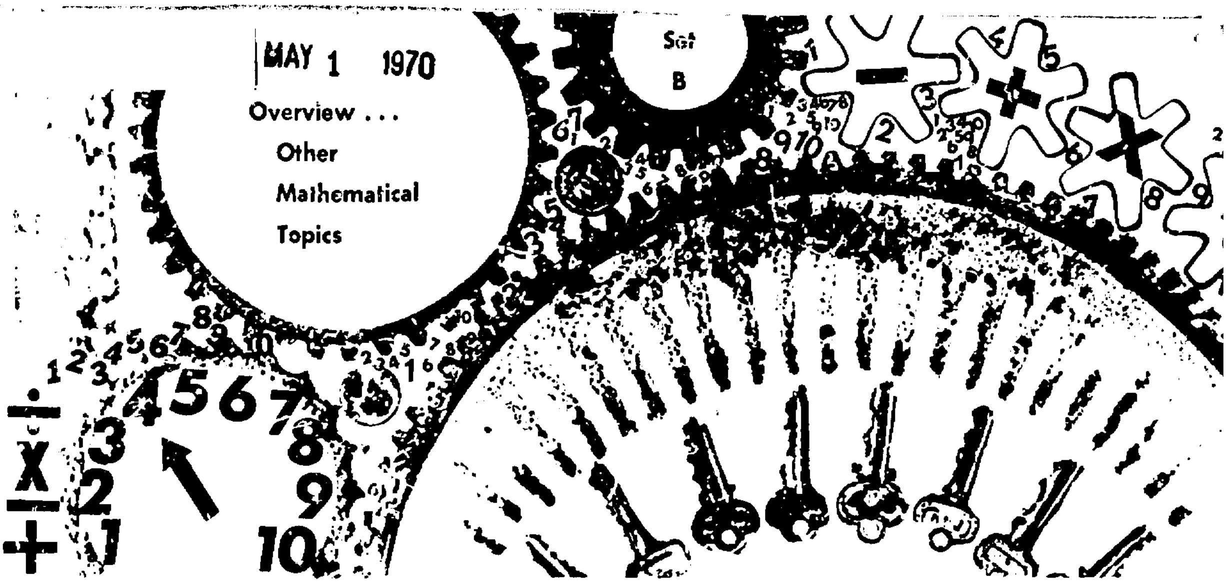
ABSTRACT

This bulletin provides an overview of research studies which relate to the teaching of certain mathematical topics in the elementary school. The studies cited pertain directly to the following questions: (1) What measurement and geometry is included in textbooks and programs? (2) What do children know about geometry and measurement? (3) What can they learn about geometry and measurement? (4) What aspects of graphing can be learned? (5) How can we help pupils understand our numeration system? (6) What effect does the teaching of properties and relations have? (7) What can pupils learn about integers? (8) What set concepts facilitate achievement? (9) What can children learn about probability and statistics? (10) What can children learn about logic? (FL)



Using Research: A Key to Elementary School Mathematics

OTHER MATHEMATICAL TOPICS

What measurement and geometry is included in textbooks and programs?

Beginning in most third grade textbooks, <u>measurement</u> content is organized by units, with emphasis on relationships among standard units developed by grade 6. Few experiences in creating measures, applying measuring ideas, and actually measuring were noted. The amount of <u>geometry</u> in the program has increased threefold since 1900, with separation of two- and three-dimensional ideas common.

What do children know about geometry and measurement?

There is evidence that children can learn many geometric ideas associated with plane figures. They can learn to make simple constructions, though lack of precision in using the compass results in many errors.

Wide differences in familiarity with measurement ideas are evident. It has been suggested that (1) some ideas now taught in first grade are probably already part of the child's knowledge when he enters school, and (2) teachers need to take into account the age, socioeconomic level, and mental ability when planning measurement activities.

How can we help pupils understand our numeration system?

There is some evidence that learning about other bases increases understanding of the decimal numeration system. However, emphasizing the structure and properties of the decimal system seems just as effective.

What effect does the teaching of properties and relations have?

Teaching the commutative, associative, and distributive properties and various relations <u>may</u> facilitate other mathematical learning, but research on this is limited.

What can pupils learn about . . .

integers?

The little research evidence on this topic indicates only that concrete and abstract approaches may each be effective.

. . . set concepts?

Ideas about sets appear to be useful in introducing both numerical and geometric concepts. A teaching sequence using (1) physical action, (2) manipulation of concrete materials, and (3) observation of semi-concrete illustrations seemed effective in teaching about sets. Several studies have suggested that pictures of objects and groupings should be kept relatively simple.

. . . probability and statistics?

Intermediate grade children apparently have acquired considerable familiarity with probability from everyday experiences, and can apply knowledge about finite sample spaces and the probability of certain events occurring. The mode, the mean, and possibly the median can be introduced as early as grade 4.

. . . logic?

Children aged 6 through 8 may be able to recognize valid conclusions derived from sets of given premises, though they may have difficulty testing the logical necessity of a conclusion.

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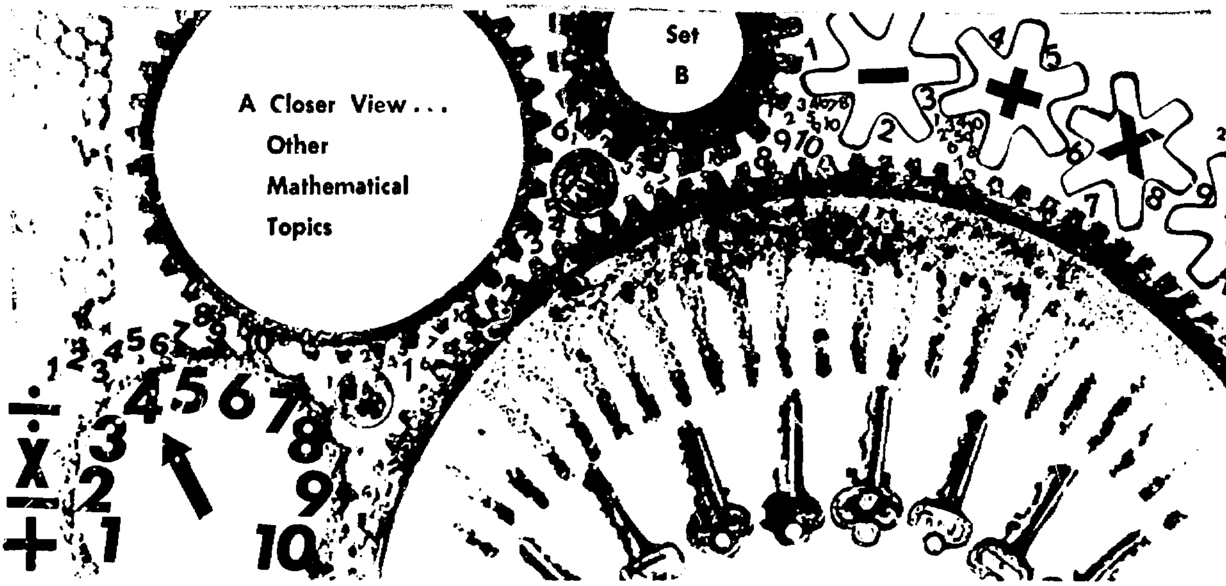
If you would like more information about the research whose findings are cited above, contact MARILYN N. SUYDAM, Project Director, at The Pennsylvania State University, University Park, Pennsylvania, 16802.



Using Research: A Key to Elementary School Mathematics

OTHER MATHEMATICAL TOPICS

What measurement is included in elementary school textbooks?

Is there common agreement on what geometry will be presented?

Paige and Jennings (1967) surveyed 39 textbook series, summarizing the measurement content. Starting in third grade, about half of the books put measurement concepts in a separate chapter. In most fourth grade books, problems generally involved regrouping with measures and conversions. By grade 5 most series had developed the ideas of standard units and errors in measuring. Other relationships between measures were introduced in many series in grade 6. Paige and Jennings noted that there were few experiences where students created their own units of measure, too little emphasis on practical application, and too few problems requiring actual measuring.

Neatrour (1969) analyzed 16 textbook series and surveyed 156 middle schools to determine the status of geometric content in their curricula. He found that while the amount of geometric content varied greatly, three times as much was included as in 1900, with emphasis on informal geometry. Compartmentalization of geometric content into two- and three-dimensional ideas was common.

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The bulletin was prepared by MARILYN N. SUYDAM, The Pennsylvania State University, Project Director, and J. FRED WEAVER, The University of Wisconsin-Madison, Project Consultant. Art by Ed Saffell.

It should be noted that research is variable with respect to its quality; hence, the ... gree of confidence cannot be placed in all findings. An attempt has been ... take this fact into consideration in preparing this bulletin.

What geometric ideas can children learn?

From a set of tests administered after two weeks of teaching, Shah (1969) reported that children aged 7 to 11 learned concepts associated with plane figures, nets of figures, symmetry, reflection, rotation, translation, bending and stretching, and networks. In a pilot study, Denmark and Kalin (1964) found that fifth graders could satisfactorily (1) bisect an angle, (2) construct the perpendicular bisector of a line segment, (3) copy a triangle, (4) construct a perpendicular to a line through a point on the line, and (5) copy a quadrilateral. Lack of precision in the use of the compass accounted for many errors.

D'Augustine (1966) used programmed texts on topics such as paths and their properties, simple closed curves, and polygons with pupils in grades 5, 6, and 7. He reported that reading and mathematics achievement significantly affected success, but age, length of class period, grade, or sex did not.

What do children know about measurement?

What can they learn about measurement?

Four- and five-year-olds exhibit wide differences in familiarity with ideas of time, linear and liquid measures, and money, with little mastery evident (Davis, Carper, and Crigler, 1959). In another survey with first graders, Mascho (1961) reported that as age, socioeconomic level, or mental ability increased, the children's familiarity with measurement increased. Familiarity was greater when the terms were used in context. It was suggested that (1) some ideas now considered appropriate for first grade should be considered part of the child's knowledge when he enters school, and (2) teachers need to study the composition of their groups in terms of age, socioeconomic level, and mental ability when planning curricular activities with measurement. This may be especially important in view of Piaget's findings, which suggest that general concepts of linear measurement are not attainable for children until approximately age 8, when the child appreciates that a linear segment may be conserved even when subdivided.

Friebel (1967) found seventh graders using SMSG materials were significantly superior to those using "traditional" materials in understanding of and skill in using measurement concepts. However, "in process estimation of the measures of common quantities," both groups were equally adept except when dealing with area and volume, where the SMSG students were better.

Corle (1960) substantiated the need for experiences with measurement. He found that sixth graders could estimate weight, size, temperature, and time more accurately than fifth graders, but error was 45% for sixth grade and 61% for fifth grade. Sixth grade pupils measured with acceptable accuracy only about half the time; fifth graders, one-third of the time.

What aspects of graphing can be learned?

Dutton and Riggs (1969) used a programmed text to present pictographs and circle, bar, and line graphs to 393 fourth and fifth graders. The text was effective in improving skills on both a graph test and on graph interpretation items from a standardized test. There is some evidence from other research that, for third graders, pictographs and bar graphs are easier to interpret than line graphs.

How can we help pupils understand our numeration system?

Flournoy, Brandt, and McGregor (1963) found that the items missed very frequently by pupils in grades 4-7 on tests measuring understanding of our numeration system related to: (a) the additive principle; (b) making "relative" interpretations; (c) meaning of 1000 as 100 tens or 10 hundreds, etc.; (d) expressing powers of ten, as 1000 = 10 x 10 x 10; and (3) the 10-1 place value relationship. Thus greater emphasis on these is necessary as we teach.

The study of non-decimal numeration systems was included in many modern mathematics programs because it was presumed that such work would strengthen understanding of the decimal numeration system. There is some evidence that kindergarteners, first graders, and fourth graders showed an increase in their understanding of the decimal system after a study of another base. Jackson (1965) concluded that fifth graders taught non-decimal systems did significantly better than pupils taught only the decimal system, on tests measuring understanding of the decimal system, properties, and problem solving. Those receiving instruction only in the decimal system did significantly better on computation in that system.

On the other hand, Scrivens (1968) concluded that study of non-decimal numeration systems is "inappropriate" for third graders and Schlingsog (1968) reported no significant differences on tests of understanding and computation in base ten between groups who were taught about other systems and those who studied only the decimal system. Kavett (1969) reported similar results for the reasoning scores of fourth and sixth graders, though retention scores were significantly higher for the groups taught non-decimal numeration. Smith (1968) found that study of non-decimal numeration systems by fourth graders produced a greater understanding of non-decimal systems but not of the decimal system.

What effect does the teaching of properties and relations have?

We believe that learning about properties will facilitate understanding, but research on this is very limited. Schmidt (1966) reported that teaching the commutative, associative, and distributive properties significantly increased fourth graders' ability to apply the fundamental processes to examples and problems. Sixth graders learned a significant amount about topics such as the reflexive, symmetric and transitive properties of some relations, equivalence relations, and graphing relations, but no significant difference was found in their ability to perform on traditional problems (Gravel, 1968).

Other researchers have reported that the properties may be too difficult for second and fourth graders to understand, and that seventh graders apply properties better than fifth graders.

What can pupils learn about integers?

There has been almost no research which provides an answer to this question. An exploratory study with six primary grade children showed that they could be taught some concepts about integers when the number line is used. Coltharp (1969) reported no significant difference in achievement between sixth graders taught addition and subtraction of integers from an abstract, algebraic approach and those taught by means of a concrete, visual approach. According to Tremel (1964), success in learning to add and multiply integers was not related to numerical and spatial abilities, but was related to verbal and problem solving abilities.

What set concepts facilitate achievement?

This is another example of a topic which has influenced modern programs tremendously, yet evidence is woefully lacking. It is generally accepted that many of the elementary terms and operations of set theory are useful and desirable in the elementary mathematics program. In fact, the ideas of "sets" are unavoidable in the introduction of number concepts and intuitive geometry, though the formal terms may not be used.

There has been some concern with how to picture groups of object In two older studies, Carper (1942) and Dawson (1953) concluded that the greater the complexity of the objects and the group cor figuration, the greater the difficulty children have in determir ing how many are in the group. Thus in the primary grades it seems important to picture relatively simple objects and groupir

Suppes and McKnight (1961) found that concepts and operations with sets could be taught in grade 1, noting that "operations or sets are more meaningful to the student than operations on numbers," since sets are concrete objects. As long as the notatio introduced is explicit and precise and corresponds to simple co cepts, no difficulties of comprehension seemed to arise. Holme (1963), however, reported that first graders scored below the 5 level for tests on equality concepts, ordinal number, subsets, and number property of sets.

Harper, Steffe, and Van Engen (1969) reported success in teachi conservation of numerousness, including one-to-one corresponden and equivalent and non-equivalent sets, to children at the firs grade level. They noted that "the teaching sequence used in these lessons, i.e., a progression from physical action of the children, to their manipulation of concrete materials, to their observation of semi-concrete illustrations, seems to be an effe tive approach to use in teaching early number concepts." [*Underlining added.*]

What can children learn about probability and statistics?

Intermediate grade children apparently have acquired considerab familiarity with probability from everyday experiences, and can apply knowledge about (1) a finite sample space, (2) the probability of a simple event in a sample space, (3) the probability of the union of non-overlapping events, (4) the difference between mutually independent and mutually exclusive events, and (quantification of probabilities (Doherty, 1966; Leffin, 1969).

Smith (1966) concluded that the following topics of probability and statistics seem to be appropriate for most seventh grade st dents: (1) possible outcomes of an experiment, (2) probability of events that are equally likely and events that are not equal likely, (3) mutually exclusive events, (4) Pascal's triangle, (histograms, (6) continuous and discrete data, (7) central tendency, and (8) measures of variation. There is some evidence f another study that the mode, the mean, and possibly the median can be introduced as early as grade 4.

What can children learn about logic?

If the child is to learn to think critically, it is important that he make logically correct inferences, recognize fallacies, and identify inconsistencies among statements. Hill (1961) concluded that children aged 6 through 8 are able to recognize valid conclusions derived from sets of given premises. There seems to be a "gradual, steady growth which is nearly uniform for all types of formal logic." Differences in difficulty were associated with type of inference, but these difficulties were specific to age. Difficulties associated with sex were not significant. Children can learn to recognize identical logical form in differing content. The addition of negation very significantly increased difficulty in recognizing validity. Roberge (1969) reported that negation in the major premise also had a marked influence on the development of logical ability in children in grades 4, 6, 8, and 10.

O'Brien and Shapiro (1968) confirmed Hill's findings, except that "little growth was detected between ages 7 and 8." Using a modification of Hill's test, they found that children experienced great difficulty in testing the logical necessity of a conclusion, and showed slow growth in this ability, which supports Piaget's theory that children reach the stage of ability to think logically later than age 8. They caution that Hill's research should be interpreted and applied with caution: hypothetical-deductive ability cannot be taken for granted in children of this age.

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