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ABSTRACT

A word problem is more difficult to solve when the minimum number of different operations to reach the correct solution is large, when it is of a different type than a problem preceding it, when the indexed complexity of its most complex sentence is great, when there are a large number of words in the problem, and when a conversion of units (as from days to weeks) is required. These variables of problem difficulty were determined to be significant in an experiment using 16 disadvantaged sixth-grade students, who were given access to a computer-based teletype. Variables that did not make a significant contribution to the regression analysis were: the "verbal-clue" variable, the "order" variable, and the "steps" variable. (MF)

AN ANALYSIS OF THE STRUCTURAL VARIABLES THAT DETERMINE PROBLEM-SOLVING DIFFICULTY ON A COMPUTER-BASED TELETYPE

BY

ELIZABETH JANE FISHMAN LOFTUS

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50 R. C. Atkinson and R. C. Calfee. Mathematical learning theory. January 2, 1963. (In B. B. Wolman (Ed.), Scientific Psychology. New York: Basic Books, Inc., 1965. Pp. 254-275)

51 P. Suppes, E. Crothers, and R. Weir. Application of mathematical learning theory and linguistic analysis to vowel phoneme matching in Russian words. December 28, 1962.

52 R. C. Atkinson, R. Calfee, G. Sommer, W. Jeffrey and R. Shoemaker. A test of three models for stimulus compounding with children. January 29, 1963. (J. exp. Psychol., 1964, 67, 52-58)

53 E. Crothers. General Markov models for learning with inter-trial forgetting. April 8, 1963.

54 J. L. Myers and R. C. Atkinson. Choice behavior and reward structure. May 24, 1963. (Journal math. Psychol., 1964, 1, 170-203)

55 R. E. Robinson. A set-theoretical approach to empirical meaningfulness of measurement statements. June 10, 1963.

56 E. Crothers, R. Weir and P. Palmer. The role of transcription in the learning of the orthographic representations of Russian sounds. June 17, 1963.

57 P. Suppes. Problems of optimization in learning a list of simple items. July 22, 1963. (In Maynard W. Shelly, II and Glenn L. Bryan (Eds.), Human Judgments and Optimality. New York: Wiley. 1964. Pp. 116-126)

58 R. C. Atkinson and E. J. Crothers. Theoretical note: all-or-none learning and intertrial forgetting. July 24, 1963.

59 R. C. Calfee. Long-term behavior of rats under probabilistic reinforcement schedules. October 1, 1963.

60 R. C. Atkinson and E. J. Crothers. Tests of acquisition and retention, axioms for paired-associate learning. October 25, 1963. (A comparison of paired-associate learning models having different acquisition and retention axioms, J. math. Psychol., 1964, 1, 285-315)

61 W. J. McGill and J. Gibbon. The general-gamma distribution and reaction times. November 20, 1963. (J. math. Psychol., 1965, 2, 1-18)

62 M. F. Norman. Incremental learning on random trials. December 9, 1963. (J. math. Psychol., 1964, 1, 336-351)

63 P. Suppes. The development of mathematical concepts in children. February 25, 1964. (On the behavioral foundations of mathematical concepts, Monographs of the Society for Research in Child Development, 1965, 30, 60-96)

64 P. Suppes. Mathematical concept formation in children. April 10, 1964. (Amer. Psychologist, 1966, 21, 139-150)

65 R. C. Calfee, R. C. Atkinson, and T. Shelton, Jr. Mathematical models for verbal learning. August 21, 1964. (In N. Wiener and J. P. Schoda (Eds.), Cybernetics of the Nervous System: Progress in Brain Research. Amsterdam, The Netherlands: Elsevier Publishing Co., 1965. Pp. 333-349)

66 L. Keller, M. Cole, C. J. Burke, and W. K. Estes. Paired associate learning with differential rewards. August 20, 1964. (Reward and information values of trial outcomes in paired associate learning. (Psychol. Monogr., 1965, 79, 1-21)

67 M. F. Norman. A probabilistic model for free-responding. December 14, 1964.

68 W. K. Estes and H. A. Taylor. Visual detection in relation to display size and redundancy of critical elements. January 25, 1965, Revised 7-1-65. (Perception and Psychophysics, 1966, 1, 9-16)

69 P. Suppes and J. Donio. Foundations of stimulus-sampling theory for continuous-time processes. February 9, 1965. (J. math. Psychol., 1967, 4, 202-225)

70 R. C. Atkinson and R. A. Kinchla. A learning model for forced-choice detection experiments. February 10, 1965. (Br. J. math stat. Psychol., 1965, 18, 184-206)

71 E. J. Crothers. Presentation orders for items from different categories. March 10, 1965.

72 P. Suppes, G. Groen, and M. Schlag-Rey. Some models for response latency in paired-associates learning. May 5, 1965. (J. math. Psychol., 1966, 3, 99-128)

73 M. V. Levine. The generalization function in the probability learning experiment. June 3, 1965.

74 D. Hansen and T. S. Rodgers. An exploration of psycholinguistic units in initial reading. July 6, 1965.

75 B. C. Arnold. A correlated urn-scheme for a continuum of responses. July 20, 1965.

76 C. Izawa and W. K. Estes. Reinforcement-test sequences in paired-associate learning. August 1, 1965. (Psychol. Reports, 1966, 18, 879-919)

77 S. L. Blehart. Pattern discrimination learning with Rhesus monkeys. September 1, 1965. (Psychol. Reports, 1966, 19, 311-324)

78 J. L. Phillips and R. C. Atkinson. The effects of display size on short-term memory. August 31, 1965.

79 R. C. Atkinson and R. M. Shiffrin. Mathematical models for memory and learning September 20, 1965.

80 P. Suppes. The psychological foundations of mathematics. October 25, 1965. (Colloques Internationaux du Centre National de la Recherche Scientifique. Editions du Centre National de la Recherche Scientifique. Paris: 1967. Pp. 213-242)

81 P. Suppes. Computer-assisted instruction in the schools: potentialities, problems, prospects. October 29, 1965.

82 R. A. Kinchla, J. Townsend, J. Yellott, Jr., and R. C. Atkinson. Influence of correlated visual cues on auditory signal detection. November 2, 1965. (Perception and Psychophysics, 1966, 1, 67-73)

83 P. Suppes, M. Jerman, and G. Groen. Arithmetic drills and review on a computer-based teletype. November 5, 1965. (Arithmetic Teacher, April 1966, 303-309.

84 P. Suppes and L. Hyman. Concept learning with non-verbal geometrical stimuli. November 15, 1968.

85 P. Holland. A variation on the minimum chi-square test. (J. math. Psychol., 1967, 3, 377-413).

86 P. Suppes. Accelerated program in elementary-school mathematics -- the second year. November 22, 1965. (Psychology in the Schools, 1966, 3, 294-307)

87 P. Lorenzen and F. Binford. Logic as a dialogical game. November 29, 1965.

88 L. Keller, W. J. Thomson, J. R. Tweedy, and R. C. Atkinson. The effects of reinforcement interval on the acquisition of paired-associate responses. December 10, 1965. (J. exp. Psychol., 1967, 73, 268-277)

89 J. I. Yellott, Jr. Some effects on noncontingent success in human probability learning. December 15, 1965.

90 P. Suppes and G. Groen. Some counting models for first-grade performance data on simple addition facts. January 14, 1966. (In J. M. Scandura (Ed.), Research in Mathematics Education. Washington, D. C.: NCTM, 1967. Pp. 35-43.

91 P. Suppes. Information processing and choice behavior. January 31, 1966.

92 G. Groen and R. C. Atkinson. Models for optimizing the learning process. February 11, 1966. (Psychol. Bulletin, 1966, 66, 309-320)

93 R. C. Atkinson and D. Hansen. Computer-assisted instruction in initial reading: Stanford project. March 17, 1966. (Reading Research Quarterly, 1966, 2, 5-25)

94 P. Suppes. Probabilistic inference and the concept of total evidence. March 23, 1966. (In J. Hintikka and P. Suppes (Eds.), Aspects of Inductive Logic. Amsterdam: North-Holland Publishing Co., 1966. Pp. 49-65.

95 P. Suppes. The axiomatic method in high-school mathematics. April 12, 1966. (The Role of Axiomatics and Problem Solving in Mathematics. The Conference Board of the Mathematical Sciences, Washington, D. C. Ginn and Co., 1966. Pp. 69-76.

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Chapter I.

INTRODUCTION

There exists a great diversity of approaches to the investigation of human problem solving. A wide range of materials, techniques, and "problems" has been used for such study. Subjects have been required to solve an anagram, a matchstick problem, a water-jar problem, a pendulum problem, a concept-identification problem, an analogy problem, a number-series problem, or an arithmetical word problem, to name a few. Some of these problems require the student to restructure a patterned situation to achieve a novel result (Duncker, 1945; Wertheimer, 1945), while others require him to find the commonality in a group of disparate situations (Heidbreder, 1947; Bruner, Goodnow, and Austin, 1956). Some problems are solved in a sequence of well-defined steps (Hayes, 1965), while others are solved suddenly in a single step (Maier, 1931). Several theoretical formulations have been offered and many facts have been discovered, but there is still no single adequate theory into which they can be integrated. In addition, there is very little analysis of why arithmetic word problems, specifically, are difficult for students. We know that students have notorious difficulty in solving word problems. The present study is an attempt to find out why. It is an attempt to explore the notion that in solving a set of word problems, certain items are more difficult to solve than others. It is an attempt to understand what variables cause some word problems to be hard to solve while others are easy. Once we have a grip on these variables, we can attempt to organize a set of word problems in terms of them. It is assumed that understanding the variables relevant to problem solving is a worthwhile and scientifically important goal, for only then can we hope to be able to formulate a coherent theory of problem solving.

The purpose of the present study is to examine the relative influence of structural factors in word problems to be solved. The use of the term "structural" indicates that the focus of attention is on the variables which characterize the specific problems themselves (for example, the number of words in the problem) and on the variables which characterize the relationship between individual problems (for example, the structural similarity of two adjacent problems). The emphasis, then, is on the problems themselves and the relationship between problems, rather than on variables which characterize the students (student variables) or variables which characterize the experimental methods used to present the problems (presentation variables). Presentation variables which have been examined in previous research include such ones as massed vs. distributed practice, immediate vs. delayed reinforcement, etc. They are generally thought to be independent of the student's past learning history. Student variables which have been studied previously include such ones as the chronological age, sex, training, reading ability, and capacity for mathematics of the student. Problem-solving ability has been shown to depend on all of these (Lazerte, 1933). Possible inter-relationships between the structural, presentation, and student variables are not denied here. However, a number of experiments suggest that the effects of most presentation variables (massed vs. distributed practice, for example) hold in diverse student populations and over a wide range of experimental materials (Underwood, 1961; Mednick, 1964, pp. 84-87). In this study, therefore, we feel justified in having restricted ourselves to an analysis of structural variables alone.

One aspect of this research is unique to investigations of problem solving. It was conducted in the context of a computer-assisted instructional system. At Stanford, the Institute for Mathematical Studies in the Social Sciences (IMSSS) has been developing over the last six years a working computer-assisted instruction (CAI) system for classroom use. This research is a small part of an investigation of the potential use and value of such systems. This study, in conjunction with Suppes, Loftus, and Jerman (1969), demonstrates a new use for such systems.

A computer program was used to teach sixth-grade students the mechanics of how to solve arithmetic word problems on a computer-based teletype. The assumption was made that the students had a basic understanding of the four arithmetical operations: addition, subtraction, multiplication, and division. The students were required to know which operation(s) should be performed for problem solution, and to tell the computer which one(s); however, the actual computations were done by the computer. Following the initial instruction set, a series of 100 word problems was presented to the students. For each problem, the students were required to find a quantitative answer. The arithmetical operations were not explicitly indicated. An example of a problem in arithmetic providing the pupil with an opportunity to use his knowledge of multiplication is the following:

> A bushel of corn weighs 56 pounds.
> How much do 44 bushels weigh?

The solutions of these problems were analyzed to determine the structural variables related to problem difficulty. The length of a problem and the number of steps required to reach the correct solution are examples of what we mean by factors related to problem difficulty. It is quite probable that a problem containing a large number of words or one requiring a large number of steps is harder to solve than one containing fewer words or requiring fewer steps.

Chapter II.

TRADITIONS IN PROBLEM-SOLVING RESEARCH

There has been a great deal of writing, speculating, and research on problem solving. Although much of this work is not directly relevant to the investigation of word problem solving, it did provide a number of important hints and suggestions as to which variables might be influencing problem-solving performance. It would be neither practical nor useful to present a comprehensive survey of all remotely relevant research. We refer interested readers to the following excellent reviews: Johnson, 1955; Gagne, 1959; Duncan, 1959; Davis, 1966; Kleinmuntz, 1966. In this section, we instead describe the types of work that have been done and attempt to give the flavor of these previous approaches to the investigation of problem solving.

Research on problem solving has evolved within two main traditions. One tradition has its roots in Gestalt psychology. The Gestalt psychologists used many different kinds of problems, ranging from mechanical puzzles to abstract mathematical problems. Many of the problems they chose for study were selected from a "true life" situation such as troubleshooting electronic equipment. Others tried to capture the flavor, if only partially, of problems we meet in everyday life. The Gestalt approach, in remaining devoted to the analysis of internal processes, emphasized the tendency of the mind to organize and integrate and to perceive situations, including problems, as <u>total</u> structures. They emphasized the structure of the problem, then, and the process of reorganization of the perceptual field which leads to insight. This insight leads to a solution of the problem. This emphasis on perceptual phenomena dominated their research on problem solving.

Working within the Gestalt tradition, Maier (1931) demonstrated how the perception of the solution of his famous pendulum problem was like perceiving a hidden figure in a puzzle picture--the solution appeared suddenly and as a complete idea. In this problem, two strings are hanging from the ceiling and the subject is required to tie the ends together. However, the strings are too far apart for the subject to grasp one, walk to the other, and tie them. The solution is to tie a weight on one string and set it in motion as a pendulum, then hold the other string and wait until the swinging string comes within reach. The only object in the room available to serve as a weight is a pair of pliers. Because the pliers normally function as a tool rather than as a weight, they are not easily seen as a pendulum bob. This has been called functional fixedness. In order to solve the problem, the pliers must be seen as a weight. Maier pictured the solution of this problem as the sudden combination and organization of elements. This characterization of problem solving, in terms of changes in organization and meaning, is typical of the Gestalt view. However, these results can easily be interpreted in terms of an all-or-none conditioning model. If the phrase "sudden combination and organization of elements" is replaced by "all-or-none conditioning," the underlying formal model is unchanged.

Whereas Maier emphasized the perceptual aspects of constructional problems, Duncker (1945) emphasized the structural aspects of practical and mathematical ones. He used clearly defined problems requiring the discovery of a novel relationship. For example, how can rays which destroy organic tissues at sufficient intensity be used to treat an inoperable stomach tumor? Or, why are all six-place numbers of the form abc,abc, such as 276,276, divisible by 13? His interesting experiments with such problems were designed to evoke general facts about reasoning and problem solving. They were meant to open up the field and supply data for further investigation rather than to answer questions or confirm theories. From a number of diverse experiments he drew general conclusions, suggesting that in solving a problem, a subject usually applies previous experience to the present situation by means of cognitive-perceptual responses. These cognitive-perceptual responses are set off through reactions to some signals from the immediate environment in which the problem is set. The subject perceives in the present, and he reacts to present stimuli. Whether he solves the problem correctly depends more or less on chance reformulations and changes in those presented stimuli. The solution itself emerges from a particular reorganization of the entire "psychological field."

Max Wertheimer (1945), a founder of Gestalt psychology and one-time teacher of Duncker, completed a whole series of problem-solving experiments. From this series he arrived at several notions which could comprise his theory of problem solving. It is a good statement of the Gestalt position. For Wertheimer, problem solving depended on a grasping of the structural and functional relationships of the problem situation. The key to solving a problem was discovering the "inner relations" of the situation and reorganizing the situation in light of that discovery. For example, suppose a child who is capable of finding the area of a rectangle is asked to get the area of a parallelogram. Wertheimer claimed that if the child thinks about it he will notice that a parallelogram differs from a rectangle in that the former has a "protuberance" on one side and a "gap" on the other. (See Figure 1.) Then, he realizes that

---

Insert Figure 1 about here

---

the "protuberance" and the "gap" are equivalent (discovering "inner relations" of the situation). If he moves the "protuberance" to fill in the gap, the parallelogram is converted into a rectangle of the same base and altitude. He has essentially reorganized the situation in light of his discovery. Now he knows that the formula for the area of a parallelogram is the same as it is for a rectangle.

Problem solving, then, involved striving and struggling with a configuration the structure of which changed with the effort. Problem solving was not the automatic application of established habits or behavior patterns to stereotyped situations. It was a dynamic process growing out of, and shaped by, each specific situation.

Hoffman's (1961) theory of problem solving was essentially a statement of the conditions which stimulate creative problem solving. The following conditions appear to be necessary:

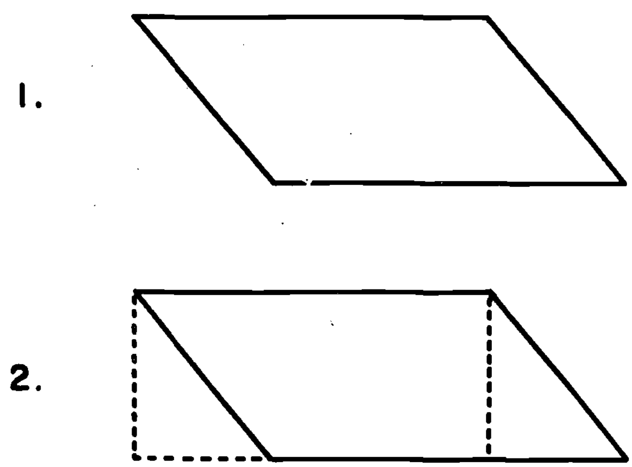

Fig. 1. Wertheimer's parallelogram. The parallelogram (1) is equal in area to a rectangle of the same base and altitude because the "protuberance" at one end is equal to the "gap" at the other end (2) (adapted from Scheerer, 1963).

1. Differing, but comparable, cognitions must coexist. Examples of such cognitions are solutions, or approaches to the problem.

2. At least 2 differing cognitions must acquire approximately equal positive valence, so that none of the alternatives can be accepted and an impasse is reached.

3. Problem solving must occur in a situation in which the individual or group which is solving the problem is required to arrive at the best possible decision. Thus, the possibility of leaving the problem or accepting a quick solution just "to get it over with" is omitted.

4. The points of conflict between the alternatives should be recognized.

Hoffman cites some examples from research literature which illustrate the usefulness of viewing problem solving in terms of these conditions.

The work of Sheerer (1963) centered on the phenomenon of fixation. Sheerer observed that insight into many different problems and puzzles is often delayed or thwarted by "fixation" on an inappropriate solution. The problems he discussed illustrated several causes of fixation. A person may start with an incorrect premise or fail to perceive a required novel use of a familiar object, or be unwilling to accept a detour that delays the achievement of his goal. Too much motivation can amplify any type of fixation and is detrimental to a solution. Fixation can often be overcome and insight attained through a sudden "recentering" or shift in the ways the problem or objects are perceived.

Cognitive considerations have entered into the studies reported by Bruner, Goodnow, and Austin (1956). Following in the tradition of Wertheimer and Duncker, they report nine new experiments on the phenomenon of categorizing or conceptualizing, a type of problem solving. In a typical task, S has to select stimuli, one by one, from a whole set of stimuli, being told each time whether the instance selected was a member of the concept which E had in mind. The authors claim, rightly, that if we are to analyze the sequences of behavior involved in such a problem-solving activity, we must externalize the components of those sequences. As their unit of analysis, they have used "cognitive strategy," a temporarily extended segment of behavior. The sequence of choices enables E to make inferences about the particular strategy which S has adopted. But these cognitive strategies are little more than descriptive terms; analysis of such strategies may be useful for understanding concept formation, but it does not provide any sort of framework for estimating parameters for the prediction of differential difficulty of concept formation items or any other stimulus items (Suppes, Jerman, and Brian, 1968). Nor can any predictions be derived within the framework of Wertheimer's "inner relations," Hoffman's "differing cognitions," or Sheerer's "fixation" on an inappropriate solution. It is our conviction that until such predictions can be derived from well-developed theoretical ideas, we are not even close to having a deep understanding about cognitive processes.

A more recent development within this tradition, the information-processing approach, has been due to the impact of computer technology. Computer simulation is a method for investigating human behavior that uses a computer program as a precise, well-defined theoretical model of the behavior being simulated. Some of the computer models directly concerned with problem solving are reported by Neisser (1963a), Feigenbaum and Feldman (1963), Newell, Shaw, and Simon (1958a), Newell and Simon (1963a), and Paige and Simon (1966). The computer program serves as a useful notation and the computer itself serves as a helpful mechanism for testing the implications of the models and for dealing with the complexities involved.

The basic notion behind the computer simulation approach is that complex thinking processes are built of elementary symbol manipulation processes; human subjects are thought to solve problems in a manner very similar to the symbol manipulations carried out by a computer. The general strategy of these information-processing psychologists, then, is to develop a computer program which produces a sequence of rules for manipulating symbols (i.e., solving problems) in a manner which closely matches the behavior of human subjects. Often information gained in comparing computer program sequences and human protocols is used to modify the program and make it a more accurate simulation of human performance. When the program can produce a protocol which closely resembles human behavior in terms of the sequence of steps used in solving a problem then that program constitutes a theory of how subjects solve problems (Newell and Simon, 1963b).

Computer simulation is a difficult process. Among other things, prior information, possible solutions, and the sequence of steps in problem solving must all be specified (Hovland, 1960). Yet it has been done with reasonable success. Newell, Shaw, and Simon (1958a; 1958b; 1959; 1963a; 1963b) have developed programs that solve logic problems, prove theorems, and play chess in a manner much like that of humans: traces of the moves considered by the computer compare line by line with human protocols. Newell, Shaw, and Simon have elaborated their research into a theory of problem solving which emphasizes the discovery and understanding of systems of heuristics. Their work has significantly influenced recent research on higher mental processes, but their programs can be considered only tentative theories; additional evidence is still needed to determine the extent to which the problem-solving processes used in the theory resemble, or differ from, processes used by humans on the same tasks. One difficulty with this approach is that, from the examination of a computer program alone, is often very hard to understand just what the theory is. Because discussions of the internal structure of a program typically involve pages and pages of very technical information, a high level of sophistication is usually necessary to understand the program. Another major problem associated with computer simulation models lies in evaluating the goodness of fit of the model to the data. Hilgard and Bower (1966, p. 421) point out two drawbacks to the typical method for testing the validity of such models (i.e., direct comparison of computer output and human performance). One drawback is that the content of the subject's protocol may be influenced by incidental, selective reinforcements by the experimenter. The effects of certain facial expressions, tones of voice and other casual reinforcement upon behavior, whether intentional or not, are well-established (Krasner, 1958; Verplanck, 1962). A second drawback is that in many cases the standard statistical data analysis techniques cannot be applied. Because of these technical complexities and other practical problems, some researchers feel that computer simulation may be as much a hinderance as a help in theory construction (Scandura, 1968).

The other main tradition in problem-solving research has its roots in Behaviorism. Behaviorism is an approach which considers the relations between observable stimuli and responses. Proponents of behaviorism are concerned not with what people think and feel, but with what people do. They maintain that the phenomena termed "problem solving" can be accounted for most efficiently by means of elementary relationships among stimuli and responses. The laboratory study of learning has been successful in establishing these elementary principles; these can then be used to explain problem solving and other complex processes. Since problem solving itself is concerned with mental processes and thus is often not observable, psychologists working in this tradition have been forced to assume implicit or internal stimuli and responses which mediate the overt response indicating the solution of the problem. This attempt to broaden the theoretical system typically results in one's predictions being applicable to more situations, but not closely to any.

A typical experiment on problem solving within this tradition is one by Judson, Cofer, and Gelfand (1956). Their subjects learned lists of words prior to working on Maier's two-string pendulum problem. The list learned by one group included names of objects relevant to the solution of the problem (e.g., rope, swing, pendulum, weight). This group subsequently did better in solving the problem than control groups exposed to lists containing no "key words." These key words referred to different uses of the problem material, and learning them was assumed to mediate the overt responses which constituted solution of the problem.

Along these same lines, Saugstad (1952) showed that Ss tended to solve problems more readily when verbal response sequences concerning the function of problem-solving objects were present. Gagné and Smith (1962) found that Ss who are required to verbalize while practicing in a problem-solving situation perform significantly better than Ss who are not required to verbalize.

A series of experiments by Kendler and his associates (Kendler and Vineberg, 1954; Kendler and D'Amato, 1955; Kendler and Mayzner, 1956; Kendler and Karasik, 1958) have demonstrated that problem solving seems to depend upon the prior establishment of verbal discriminations. Two specific findings were (1) the proportion of children who respond most effectively in problem solving increased with age, and (2) younger children who were required to tact the stimuli to which they were responding solved problems more quickly. These suggest that problem solution is dependent upon the availability of verbal responses (tacts) to the objects involved in the problem. A stronger position suggested by these studies is that some verbal mediation must be assumed to occur between the external stimulus and the overt responses if we are to account adequately for human concept learning.

Other behaviorists, arguing that complex behavior can be explained by the same elementary principles that explain simple discrimination learning, have attempted to analyze problem solving in the language of operants, habit family hierarchies, and chains of association. Osgood's (1953) model of problem solving, for example, emphasizes the role of meaning responses. Briefly, an object involved in a problem elicits in the subject a hierarchy of meaning responses (ways of perceiving the object or its significance). Each meaning response itself elicits a hierarchy of problem-solving behaviors. Chains of these meaning responses constitute the mechanism controlling problem-solving behavior. The problem-solving models of Maltzman (1955), Staats (1961), and Cofer (1954) provide additional examples of well-developed models with which the present discussion is congruent.

We will not review the pros and cons of the Gestalt or Behaviorist viewpoints; for the interested reader, such reviews exist in several places (see, for example, Hebb, 1949; Estes et al., 1954; or Hilgard, 1956).

It is difficult to find crucial differences between the predictions which follow from the theoretical notions of these two traditions, and it is not our purpose here to try to do this. It is, however, in the context of the latter tradition that we have chosen to experiment. We believe that problem solving is compounded of elementary behavioral processes, and thus we have devised some simple problems in which the relationships of fundamental variables to problem solving are highlighted. We believe that in order to fully understand problem-solving behavior, we must discover new variables or new relationships between old variables. Like Duncker, however, we hope to evoke some general facts about problem solving and to supply data for further investigation, rather than to put forward a neat theory which, once submitted to scrutiny, turns out to be unsound. For clearly an all-inclusive theory which seeks to explain a wide range of behavior in terms of a few simple principles is open to some question. Although problem solving is not a total mystery, the facts available at present, as the result of experimental studies, are scanty. What is needed are much more empirical data, not more abstract, empty theory. This can only be undertaken through a large number of separate investigations, each concerned with a highly specific situation in which one type of problem is studied. Each type of problem situation has to be investigated separately and related to the others only if connections are revealed. Until this has been done, we believe it is unwise to attempt to put forth an all-encompassing theory of problem solving.

Let us turn briefly to the specific problem situation with which we are dealing, that is, the solving of arithmetic word problems. According to Davis (1966) there are no systematic investigations of variables which may determine some sources of difficulty in solving various types of arithmetic problems.

There are studies on problem solving which are concerned with the effects of stress (Kurz, 1964; Woodhead, 1964), the effects of hypoxia (Phillips, Griswold, and Pace, 1963), and the effects of sleep deprivation (Orr, 1964). These experiments seem to suggest that minor stresses do not particularly disturb problem-solving performance.

Numerous experiments are concerned with problem-solving ability. Most of these examine the relationship bet· ·en an individual's success in problem solving and some other characteristics of his personality. Klausmeier and Loughlin (1961) analyzed the problem-solving behaviors of 11-year-olds as a function of IQ. They found that the high-IQ Ss made significantly fewer errors, were more persistent and more efficient than low-IQ Ss. Martin (1963) found that problem solving as measured by the Arithmetic Problem-solving Test of the Iowa Tests of Basic Skills given to fourth and eighth graders was correlated with the following factors: reading comprehension, computation, abstract verbal reasoning, and arithmetic concepts. In addition, problem-solving ability has been reported to be related to arithmetic reading ability (Stevens, 1932), ability to note details in reading (Chase, 1960), and a General Reasoning factor (Werdelin, 1966). Although it is evident that there is some relationship between a number of abilities and success in problem solving, the nature of the relationship and the relative contribution of each of these factors is still unclear.

From the standpoint of a deeper scientific view of either what determines problem difficulty or how problems should be organized in order to optimize student learning, the existing research has been disappointing. One reason is that few of the studies purporting to show evidence of variables affecting problem-solving performance can be accepted without considerable reservation. Many of them are faulty, either in design or interpretation. Very few of these experiments have dealt with the specific structural variables in word problems or in the sequence of word problems.

A search of the literature reveals a few studies on the effects of content of word problems (Washburne and Morphett, 1928; Travers, 1967), a few on the effect of language used in the problem (Hydle and Clapp, 1927; Steffe, 1967), and a few on the effects of readability (Thompson, 1967). A handful of others that have been particularly relevant to our choice of variables are discussed in Chapter III. However, these are pitifully few. Many more detailed studies dealing with specific structural variables are needed as an important step to the development of a general theory of word-problem solving. The present study is meant to be a modest contribution toward the development of such a theory; to attempt to put forth such a theory now is far beyong the theoretical reach of this study.

Chapter III.

THE THEORY

For the word problems analyzed in this paper, the main task was to identify the factors that contributed to the difficulty of the item. Examples of factors that we examined are the number of words in the problem and the minimum number of steps required to solve that problem. Exactly how each factor was defined is a matter that we take up in detail below. We would like to attach weights to the various factors, and then to use estimates of the weights to predict the relative difficulty of each of a large number of items.

Before we can formulate any linear structural models from which parametric predictions of relative difficulty can be made, some notation is needed. Let the jth factor of problem i in the set of problems be denoted by X_{ij}. The statistical parameters estimated from the data are the weights attached to the factors. We denote the weight assigned to the jth factor by a_j. We emphasize that the factors identified and used in the model presented in this paper are always objective factors identifiable by the experimenter in the problems themselves. The response data themselves never influence the decision as to what is the numerical value of a factor for a given word problem. The definitions of all the factors used in the analyses presented here are quite straightforward; each factor has an intuitive and direct relevance to commonsense ideas of difficulty.

Consider the analysis of the response data. For a given problem i, let p_i be the observed proportion of correct responses for a group of students. The main task of a model is to predict the observed proportion p_i. The natural linear-regression model in terms of the factors X_{ij} and the weights a_j is

$$p_i = \sum_j a_j X_{ij} + a_0 .$$

In order to guarantee preservation of probability, that is, to insure that predicted p_i's will always lie between 0 and 1, we make the following transformation and define a new variable z_i:[1]

$$z_i = \log \frac{(1 - p_i)}{p_i} . \qquad (1)$$

We then use as the regression model

$$z_i = \sum_j a_j X_{ij} + a_0 . \qquad (2)$$

The rest of this section is devoted to the discussion of how each variable used in the regression analysis is defined.

We consider two types of variables. Variables of the first type are 0,1-variables. A 0,1-variable would be appropriate if, for example, we were dealing with a problem that required a conversion of units, such as from days to weeks. If a problem requires such a conversion, the conversion variable for that problem would receive a value of 1. If no conversion is required, the conversion variable is given a value of 0. Variables of the second type assume a finite set of values, with the set being greater than 2. Such a variable would be appropriate, for example, if we were concerned with the length of a problem; the length variable is given a value which is equal to the number of words in the problem.

Three other variables of the second type are the operations variable, the steps variable, and the depth variable. The operations variable refers to the minimum number of different operations required

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[1]To take care of the case when the observed p_i is either 0 or 1, we use the following transformation

$$z = \begin{cases} \log (2n_i - 1) & \text{for } p_i = 0 \\ \log \dfrac{1}{2n_i - 1} & \text{for } p_i = 1 \end{cases}$$

where n_i = the total number of subjects responding to item i. It should be noted that the reason for putting $1 - p_i$ rather than p_i in the numerator of equation (1) is that it is desirable to make the variables z_i increase monotonically in difficulty. For example, if the length of a problem or the number of steps needed to solve a problem increases with the difficulty of the problem, it is desirable that the model reflect this increase directly rather than inversely.

to solve a problem. For any given problem, this variable could take on a value of 1, 2, 3, or 4. The steps variable refers to the minimum number of steps required to reach the correct solution.[2] These two variables may be distinguished more clearly if we consider a problem that asks the student to find the average of 8 numbers. Such a problem would give a value of 8 to the steps variable and a value of 2 to the operations variable. Seven steps of addition and one step of division are required to solve this problem.

Before discussing the depth variable, a few words must be said about the length variable. Sentence length is frequently proposed as the most obvious and plausible factor contributing to sentence difficulty. This factor is generally determined by the total count of the number of words in the sentence. Studies in language acquisition (Miller and Ervin, 1963; Ervin, 1964) give evidence of a gradual progression of children's language development from one-word sentences, holophrases, to two-word pivot sentences, to sentences consisting of greater numbers of words. When children first begin to combine words, and when they begin to imitate adult sentences, they tend to use a "telegraphic code," a grammatically incomplete sentence which is a shortening of adult sentences that retains only content words (Brown and Fraser, 1963). Many other developmental studies have shown similar increases in chronological age (McCarthy, 1930; Davis, 1937; Loban, 1963; Menyuk, 1963). For Menyuk, mean sentence length is taken to be a valid and quantitative measure of increased verban maturity. Deutsch and Cherry-Peisach (1966) found sentence length to be a significant variable in distinguishing the speech of first-grade children of different socio-economic groups. Braun-Lamesch (1962) found that younger children cannot recall whole sentences easily. Because this evidence indicates that younger children in early language development lack the ability to process long sentences, it seems safe to say that long sentences are more difficult for children to comprehend than shorter sentences. For the present, we shall generalize these results and assume they imply that longer word problems will be more difficult than shorter ones.

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[2]To avoid any ambiguity, we always first minimize the number of steps and then the number of operations.

Modern linguists agree that total comprehension of a sentence includes recognizing and understanding the structural relationships in the sentence. Factors which focus on element counts (e.g., number of words, number of pronouns, number of syllables per one hundred words) have been successful in accounting for only 26 to 51 percent of the variance in comprehension scores (Ruddell, 1964). This low percentage makes obvious the need for more attention to be given to the organization of language structure. The measure of structural complexity that we will use is based on the depth hypothesis of Yngve (1960). Yngve describes a procedure which assigns a number to each word of a sentence. The number reflects how embedded the word is in the sentence; the more embedded in the sentence the word is, the higher the number assigned to the word. Yngve's procedure for determining the characterizing set of numbers for any sentence consists of drawing a phrase structure tree diagram of the sentence in question and then counting the number of left branches leading to each word. The number of left branches which terminates the longest string of left branches represents the maximum depth, d_{max}, of the sentence. Figure 2 illustrates the constituent structure tree represented by the sentence THE MAN SAW THE BOY. The sentence can be characterized by the following set of numbers: 2, 1, 1, 1, 0; these are the respective number of left branches leading to each word in the sentence.

Insert Figure 2 about here

The first occurrence of THE terminates the longest string of left branches. Since THE terminates two left branches, the maximum depth for this sentence is two. Yngve (1964) claims that the depth hypothesis explains many of the complexities of language in terms of their function in allowing a maximum depth of about seven, but no more.

Martin and Roberts (1966) have modified Yngve's depth measure by using the average number of left branches per word in a sentence as their measure of structural complexity. The depth of the sentence THE MAN SAW THE BOY is equal to the mean of its Yngve numbers, or $(2 + 1 + 1 + 1 + 0)/5 = 1.33$. Martin and Roberts presented to subjects sentences which differed

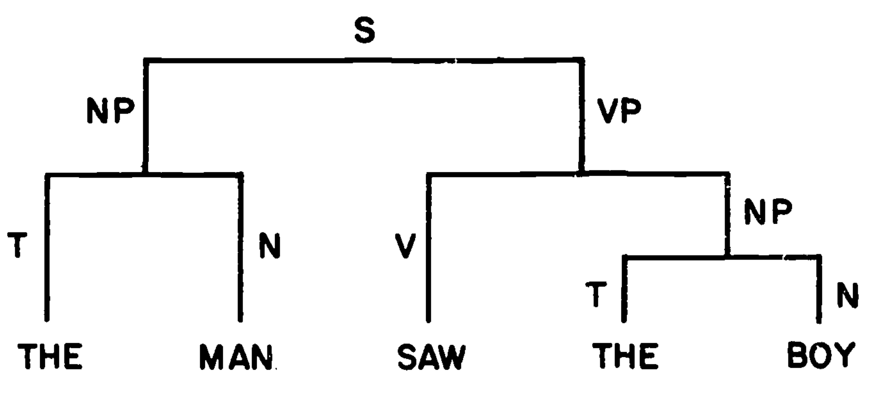

Fig. 2. The constituent-structure tree represented by THE MAN SAW THE BOY. This constituent-structure tree is based on the following grammar and vocabulary:

1. Grammar

    a. The whole sentence is symbolized by S.

b.	S → NP + VP	S can be rewritten as noun phrase, NP, plus verb phrase, VP.
c.	NP → T + N	The noun phrase is rewritten as an article, T, and a noun, N.
d.	VP → V + NP	The verb phrase is rewritten as a verb, V, and a noun phrase, NP.

2. Vocabulary

    a. N = man
    b. N = boy
    c. T = the
    d. V = saw

in depth. Out of 6 "low-depth" sentences, subjects correctly recalled an average of 3.9 sentences; recall for "high-depth" sentences was 3.1 sentences. Martin, Roberts, and Collins (1968) demonstrated additional support for the depth hypothesis in a task of recall of single sentences. Other investigators (Rohrman, 1968; Perfetti, 1969) have found no support for the depth hypothesis in recall tasks.

The conflicting reports cast some doubt on the general value of the Yngve hypothesis in recall tasks. However, the hypothesis may have some value for our understanding of word problem difficulty. The notion of quantifying the structural complexity of a word problem and relating that complexity to problem difficulty is appealing. For a given problem, then, let its structural complexity, or <u>depth</u>, be formally defined by the following procedure:

1. Compute the mean of the Yngve numbers for each sentence in the problem.

2. The highest value of this set of what might be called Yngve means is taken as a measure of the structural complexity of the problem as a whole. In other words, we assume that a problem is as complex as its most complex sentence.

The procedure is illustrated by the following simple example. Suppose the problem is:

> Jim has 40 bottles. Ken has 30 bottles.
> They have how many bottles together?

Sentence 1 can be characterized by the following numbers: 1, 1, 1, 0, with a mean of .75. Sentence 2 can be characterized by the numbers 1, 1, 3, 2, 1, 0, with a mean of 1.33. 1.33 is the structural complexity, or depth, of the problem.

At this point, it is important to mention that coding the depth of a sentence objectively is not an easy matter. Any discussion of the Yngve metric that does not consider this difficulty is quite naive. The coding problem has been hinted at recently by Rohrman (1968) in his attack on Martin and Roberts (1966). Martin and Roberts characterized the sentence "children are not allowed out after dark" by the numbers: 1, 4, 3, 2, 1, 1, 0; "are" was assigned a 4. Rohrman claims that it is very difficult to see what kind of tree could possibly give more than two branches leading to the auxiliary verb, "are." It is certainly possible for a given sentence to have more than one derivation tree, in which case there would be a different mean depth for each of the trees. This is often the case with ambiguous sentences; typically they have more than one tree and a different mean depth for each. However, in the context of a complete word problem, none of the sentences used in the study is ambiguous. The problem of coding still exists, however, because Yngve has failed to provide an explicit set of rules for assigning numbers to words in a sentence. To assess the degree of reliability between two people coding these problems independently, a graduate student in psycholinguistics was given the job of coding a sample of 20 of the problems. His results correlate extremely well with those obtained by the author on that 20-item sample. Further discussion of this procedure is presented in Chapter V.

The first 0,1-variable is the sequential variable. This is the only variable in this study which concerns the relationship between individual problems rather than emphasizing the structure of the individual problems themselves. If a problem cannot be solved by the same operation(s), in the same order, as the problem that preceded it, the sequential variable for that problem is assigned the value of 1. If a problem is of the same type as the preceding problem, the value for this variable is 0. Successful use of a sequential variable has been made in the analysis of fractions (Suppes, Jerman, and Brian, 1968, Chapter 7) and in the analysis of arithmetic word problems (Suppes, Loftus, and Jerman, 1969).

The emphasis on such a sequential variable is very much in the spirit of recent work on verbal learning. In free recall, for example, the importance of the relationship between items in a list is well documented. Underwood and Schulz (1960) and Postman (1964) have stated quite explicitly that recall may be facilitated by associations among items in the list. In other words, recall of a particular item depends not only on the item qua item but also on the relationship between the item and other items in the list. Other psychologists have postulated the relationship between list-items and the general experimental context to account for the response-learning stage in paired-associate learning

(Keppel, 1964; McGovern, 1964; Underwood, 1964). Using a reaction-time technique, Carey, Mehler, and Bever (1970) presented _Ss_ with a picture, then with a sentence, and asked them to judge the sentence true or false with respect to the picture. Results showed that the response latency for an ambiguous sentence clearly depended upon the particular syntactic structure of _prior_ sentences that the _Ss_ had heard. This abundance in the literature of evidence for the effects of inter-item relationships indicates that this matter is of psychologically great importance.

The _verbal-clue_ variable is the second 0,1-variable. Brownell and Stretch (1931) felt that a problem could be analyzed into several elements or factors, one of which was a verbal clue to the operations. This factor was not varied systematically, and so no conclusions could be drawn about it.

Kendler and Kendler (1962), who discuss problem solving in S-R terms, claim that verbal behavior is necessary for problem solving. Furthermore, they say, problem-solving ability depends on the development of verbal behavior which mediates between the problem stimulus and the problem-solving behavior. At one point they suggest that investigation of the cue function of words might prove fruitful (p. 10). The work of Kendler and associates, discussed in Chapter II, has demonstrated the critical role of verbal discriminative responses in problem solving. These findings suggest that the provision of a verbal clue to the operation(s) required to solve a word problem may facilitate solution.

In the following problem,

> A wooden box contains 23 red beads and 83 blue beads. How many beads does it contain in all?

the word "and" should help the person to discriminate between the four operations he _could_ use, and to choose the one (addition) that he _should_ use.

In a sense, the word "and" is a cue or a label for the operation of addition. The importance of the verbal responses of labeling in a multitude of situations is very well known (Miller, 1948).

We have defined the verbal-clue variable as follows:

1. The verbal clue for problems requiring a single addition is the word "and"; if the problem does not contain this word, the verbal-clue variable for that problem is to be assigned a value of 1, and 0 otherwise.

2. The corresponding verbal clues for the other operations are:
(a) "left" or a comparative for subtraction;
(b) "each" for multiplication;
(c) "average" or "each" appearing in the question sentence of the problem for division.

3. Problems requiring multiple operations must contain all of the verbal clues pertaining to the required operations in order that the verbal-clue variable be assigned a value of 0.

The order variable is the third 0,1-variable. Burns and Yonally (1964) asked the question, "does the order of presentation of numerical data in multi-step problems affect their difficulty?" In other words, if problems are stated with numerical data not given in the order needed to solve the problem, will students solve as many of them successfully as problems stated with numerical data given in an order in which they could be used to solve the problems? Their results indicated that students were less successful in getting the correct answer to word problems when the numerical data were presented in some order other than an order in which they could be used to solve the problem. These results suggest a new factor, the order variable, assigned a value of 0 if the problem can be solved by using the numerical data in the order given in the verbal statement of the problem. Note that the numerical data need not necessarily be so used, but if it can be used in the order presented, the value for the order variable is 0. If the order of the numerical data must be reversed, then the value of the order variable is 1.

The conversion variable is the last 0,1-variable. If a problem requires a conversion of units (such as from months to weeks), the conversion variable for that problem is assigned a value of 1, and 0 otherwise. The importance of this variable was suggested by the results of Suppes, Loftus, and Jerman (1969).

In summary, the variables we investigated are:

X_1 = the <u>operations</u> variable, that is, the minimum number of different operations required to reach the correct solution;

X_2 = the <u>steps</u> variable, that is, the minimum number of steps required to reach the correct solution;

X_3 = the <u>length</u> variable, that is, the number of words in the problem;

X_4 = the <u>depth</u> variable, that is, the Yngve mean for the most complex sentence in the problem;

X_5 = the <u>sequential</u> variable, assigned a value of 1 if the problem is not of the same type (i.e., cannot be solved by the same operation(s)) as the problem that preceded it, and 0 otherwise;

X_6 = the <u>verbal-clue</u> variable, assigned a value of 1 if the problem does not contain a verbal clue to the operation(s) required to solve the problem, and 0 otherwise;

X_7 = the <u>order</u> variable, assigned a value of 1 if the numerical data are presented in some order other than an order in which they could be used to solve the problem, and 0 otherwise;

X_8 = the <u>conversion</u> variable, assigned a value of 1 if a conversion of units is required to solve the problem, and 0 otherwise.

It should be noted that the higher the value assigned to a variable, the more difficult the problem is assumed to be.

To illustrate how a word problem is coded on each of the 8 variables, consider the following example:

> A truck and its load of coal weighed 14,875 pounds.
> The empty truck weighed 5,996 pounds.
> Find the weight of the coal.

The <u>operations</u> variable receives a value of 1 because the minimum number of operations required to reach the correct solution is one. The problem can be solved by using only subtraction.

The <u>steps</u> variable receives a value of 1 because the minimum number of steps required to reach the correct solution is one. One step of subtracting will do it.

The _length_ variable receives a value of 22 because there are 22 words in the problem. Each number was counted as one word.

The _depth_ variable receives a value of 1.60. The Yngve means for the three sentences in the problem are 1.60, 1.17, and .83, respectively. The first sentence yields the highest Yngve mean and the depth variable receives that number as its value.

The _sequential_ variable receives a value of 1. The problems were arranged sequentially such that this problem never followed a problem of the same type. A value of 1 is thus appropriate here.[3]

The _verbal-clue_ variable receives a value of 1. The problem does not contain a verbal clue to the operation required for solution; it does not contain either the word "left" or a comparative for subtraction.

The _order_ variable receives a value of 0. The numerical data are presented in an order in which they can be used to solve the problem. No rearranging is required.

The _conversion_ variable receives a value of 0 because a conversion of units is not required to solve the problem.

[3]The randomization and sequencing of problems will be explained further on in the next chapter.

Chapter IV.

DESIGN AND EXPERIMENTAL PROCEDURE

Subjects

The 16 subjects who completed the problem-solving program were seven sixth-grade students in one elementary school (School A) and nine sixth-grade students at a second elementary school (School B). Both schools are in the Ravenswood City School District in California. The district, in which 35,000 people live in a 17-square-mile area, comprises 5 percent of the total county school population. Thirty-three percent of the county welfare families live within this school district. Both schools are essentially "depressed area" schools. In School A, 82 percent of the children are black. The average sixth-grade IQ is 93. In School B, 59 percent of the children are black. The average sixth-grade IQ is 99.

Because these children are slow learners, the generality of the statements that can be made from the results of this investigation is limited. However, this investigation potentially contributes information about the disadvantaged student who has been neglected in previous research.

Equipment

The student terminals used in this project were commercially available teletype machines, connected by private telephone lines to a computer at the Institute for Mathematical Studies in the Social Sciences at Stanford. There were five teletypes at School A and four at School B. All teletypes at a particular school operated in a single classroom at that school.

The control functions for the entire system were handled by the PDP-1, a medium-sized computer with a 32,000-word core and a 4,000-word core interchangeable with any of 32 bands of a magnetic drum, together with two large IBM-1301 disc files. All input-output devices were processed through a time-sharing system. Two high-speed data channels permitted simultaneous computation and servicing of peripheral devices.

Instructional Program

Initial instruction on the teletype consisted of explaining to each student the general procedure of taking turns on the machine and the general program logic. Each student was given assistance in finding the letters to type his name for the first two lessons. No student had any trouble learning how to type his name or answer the questions on the teletype.

The program began each day by asking the student to type his assigned number and his name. If the student made an error or gave a fictitious name, such as Napoleon, he was asked to try again. If he correctly typed his number and name, the computer consulted his file and began with the item following the last one completed. The items were divided into two parts, with the set of instructions presented before the set of problems.

The set of instructions. The students were taught how to command the computer to perform operations on given numbers by a set of instructions presented via computer. The complete set of instructions is given in Appendix A.. We will briefly list and give an example of each of the abbreviated operation names that the student learned in the instruction set. Student entries are underlined.

1. X is the answer key.

   Suppose the student sees on the printout sheet before him:

   G 1) 21

   He would indicate that 21 was his answer by typing <u>1X</u>, which says to the computer "my answer is on line 1." The line number followed by X indicates what line the final answer is on.[4]

---

[4] "G" stands for "given number." Whenever a student is given a word problem to solve, all the numbers in the problem are typed out as given numbers just after the word problem itself has been typed out. The reason for designing the program in this way was to reduce the time required for students to input large numbers themselves. Requiring students to input very large numbers slows down the learning and can be relatively demanding, especially during early stages of learning. Instead we ask the child to input on the keyboard what rule he wishes to apply to what given numbers. All he has to input, then, is the rule and the lines to which that rule is to be applied.

2. A is the abbreviation for ADD.

An example of how a student might use the A rule is:

G	1)	36
G	2)	41
1.2A	3)	77

By typing "1.2A," the student tells the computer to add the number on line 1 to the number on line 2. The computer then prints the result of doing the addition operation.

3. S is the abbreviation for SUBTRACT.

An example is:

G	1)	500
G	2)	48
1.2S	3)	452

4. M is the abbreviation for MULTIPLY.

An example is:

G	1)	59
G	2)	4
1.2M	3)	236

5. Q is the abbreviation for DIVIDE. Q rather than D was used for divide because D was used for something else in the system.

An example is:

G	1)	77
G	2)	7
1.2Q	3)	11

6. E means ENTER, and is used to enter a number that is not entered by the computer program. For example, in a problem that asks the student to find the number of days in 8 weeks, the student would be required to enter the number 7, the number of days in one week. The number 8 would be entered by the computer as a "given number."

An example is:

G	1)	8
E	2)	7

The following sequence of interactions between the student and the computer illustrates how a word problem is solved in this context. Again, student entries are underlined. The computer first types out the problem, and then types out the numbers in that problem. The student sees on the printout sheet before him:

```
At the tree nursery, Tom counted 28 rows of pine trees.
The forester said that there were 575 trees in each row.
How many trees were there at the nursery...

                G        1)  28
                G        2)  575
```

At this point, the student tells the computer the operation he wants the computer to perform, and the line numbers to which the operation should apply. For this problem, the student typically types out "1.2M," meaning "multiply the number on line 1 by the number on line 2." The computer responds by typing the result of applying the operation, or by typing an error message if the operation could not be applied validly.

The student is still not finished with the problem. He must also indicate where his final answer is by typing the line number on which the answer appears followed by an X. The complete protocol for a correct response in the above example, then, might be:

```
At the tree nursery, Tom counted 28 rows of pine trees.
The forester said that there were 575 trees in each row.
How many trees were there at the nursery...

                G        1)     28
                G        2)    575
                1.2M     3)  16100
                3X
                Correct
```

If the answer is incorrect, "answer is wrong" appears in pleace of "correct." If the student has not yet indicated his final answer by using "X," and if he asks the computer to perform an operation that cannot be applied validly, he receives an error message. In the above example, if instead of typing "1.2M" the student had typed "1.2MT," the computer would respond by typing "There is no rule name 'MT.'" If the student had erroneously typed "1.2," the computer would respond by typing "No rule name given." A flow chart of the program logic is shown in Figure 3.

Insert Figure 3 about here

A given word problem can often be solved in many ways. The students own experience and ingenuity determines which rule he uses and what strategy he takes. The computer allows any valid step, regardless of whether it helps reach the solution. Any combination of steps reaching a solution, valid within the rules, is entirely acceptable. A problem such as the following could be solved in several ways:

> For an experiment, Susan mixed 7 ounces of glycerin and 14 ounces of alcohol with some water. The resulting mixture contained 45 ounces. How many ounces of water were used?

It could be solved:

$45 - (7 + 14)$ or $(45 - 7) - 14$.

A more idiosyncratic solution, such as $45 - (7 \times 3)$, is equally acceptable.

In the instruction set, students solved easier problems before being presented with more difficult ones. On several of the problems, the student was invited to ask for a hint after a certain time lapse by the message, "Type H and a space if you want a hint." If the student asked for a hint on the problem "What is (486 + 390) + 707?" he was told "First find 486 + 390. Then add that sum to 707." No hints were available on multiple-choice problems; the student had to guess until he got the problem correct.

While the student was trying to reach a solution, the computer did four things.

1. Every student command was examined to see if it was a valid step and if the syntax was correct. If incorrect, the computer printed out an error message.

2. If valid, the computer performed whatever step the student commanded.

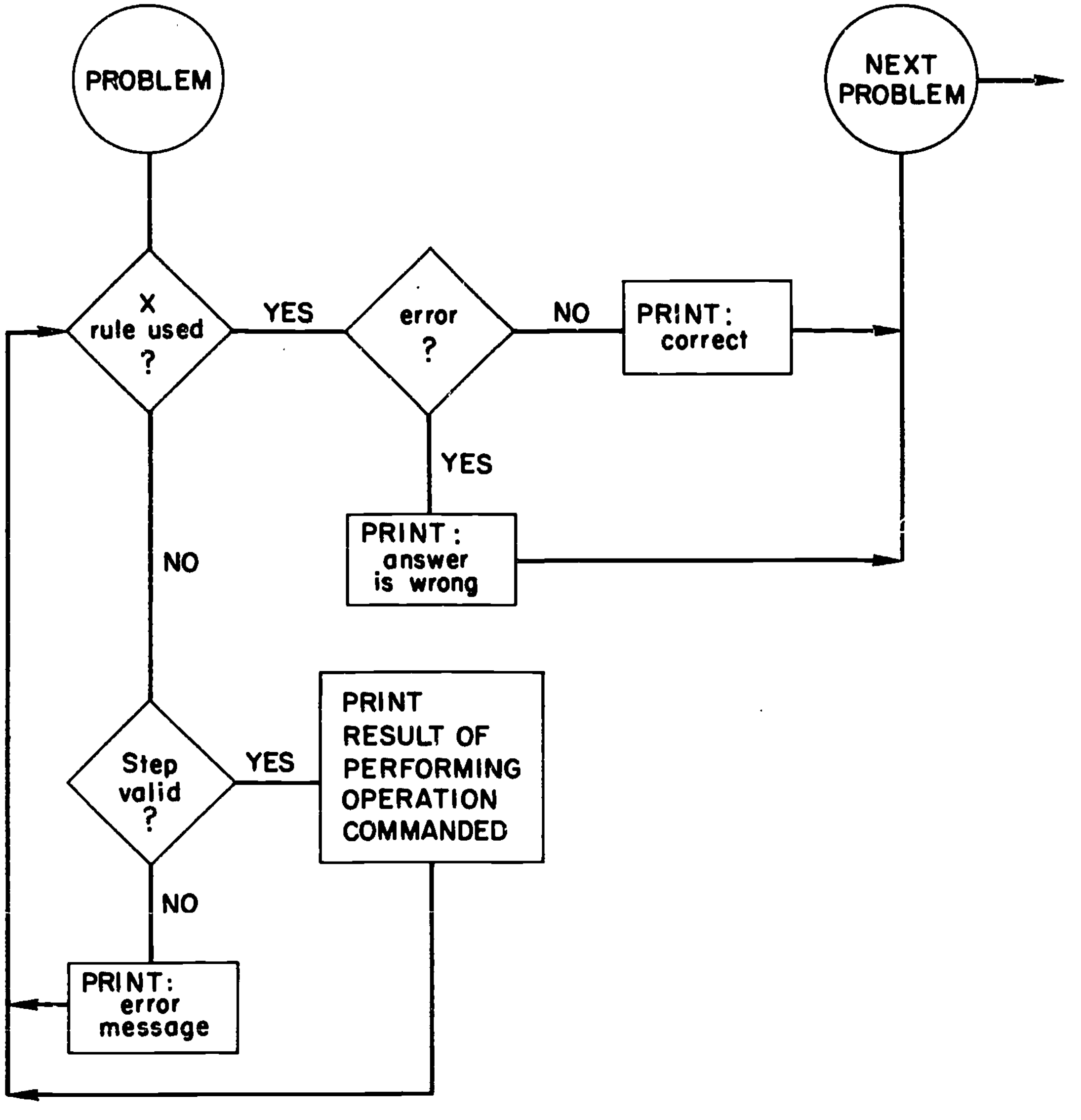

Fig. 3. Flow chart of the program logic for presentation of problems and classification of responses.

3. The desired solution was compared to the final answer indicated by "X." If the two were identical, the computer terminated the problem after typing "correct." If they were not identical, the computer typed "answer is wrong."

4. After a fixed time lapse, the computer offered a hint on certain problems. Hints were available only for certain problems in the instruction set, not for those in the problem set.

The word-problem set. The 100 word problems used in this study were designed to be of appropriate difficulty for sixth-grade students. The word problems are listed in Appendix B. These 100 problems were divided into 50 pairs; a pair consisted of two problems both of which could be solved by the same operation or sequence of operations. The 50 pairs were then randomly permuted with the following restriction: no two pairs whose problems required the identical operation(s) for solution could be presented adjacent to each other. Five randomizations were obtained, and each S was assigned to one of the five random sequences. The purpose of creating sequences of problems in this way is that for a given pair of problems, the first problem never followed a problem of the same type and the sequential variable for that problem always received a value of 1. The second problem in the pair always followed a problem of the same type; the sequential variable for that problem always received a value of 0. Since the study was designed to permit investigation of factors that might contribute to problem difficulty, the problem set was designed so that there would be as many different combinations of variable values as possible.

To solve the set of problems, students used the rules they learned in the instruction set. As before, the computer first typed out the problem, and then typed out the numbers in that problem. Then, using any of the rules mentioned above, the student told the computer what to do with these numbers. Figure 4 illustrates how a student could have gone about solving a word problem in this way.

Insert Figure 4 about here

```
IN ITS FIRST 3 GAMES A FOOTBALL TEAM SCORED
14 POINTS, 35 POINTS, AND 20 POINTS.
LAST YEAR THE TEAM AVERAGED 20 POINTS PER GAME.
THIS YEAR'S AVERAGE SCORE IS HOW MUCH ABOVE LAST
YEAR'S AVERAGE.

G              (1) 3
G              (2) 14
G              (3) 35
G              (4) 20
2.3A           (5) 49
4.5A           (6) 69
6.1Q           (7) 23
7.4S           (8) 3
8X
CORRECT
```

Fig. 4. Sample solution of a word problem.

After the computer typed out all the numbers in the problem as "given numbers," the type wheel of the teletype was positioned at the left-hand side of the paper. The student made his response, and then the computer positioned the type wheel at the center of the page, typed the line number, and finally typed the result of the operation the student had commanded the computer to perform. If the final answer was correct, the computer typed the message "correct" and went to the next problem. If the final answer was incorrect, the computer typed "answer is wrong" and went to the next problem.

When working on the teletype, the students were not allowed to use pencil or paper. Every problem was worked on the machine, so that all responses could be recorded.

Following the "goodbye" message the student was told "please tear off on dotted line." A dotted line was printed, and the student tore off his printout and gave it to the experimenter.

Typically, it takes about 8 weeks to complete both the instruction set and the word-problem set. Each portion takes 4 weeks. However, the students at School A had such initial difficulty with the program that they were allowed to repeat portions of the instruction set before beginning the problem set. The notion was that we wanted them to learn the rules as well as possible before beginning to solve the test problems. The School A group took a mean of 12 weeks to finish the program: 8 weeks for the instruction set and 4 weeks for the word-problem set. The School B group took a total of 8 weeks to complete both portions of the project.

Chapter V.

RESULTS

Before reporting the results, a few words are in order about the measure of depth based on Yngve. The problem with the measure of depth discussed in Chapter III arises when we attempt to find the Yngve numbers for a particular sentence. It is not an easy matter to ascertain the precise tree structure to be associated with a given sentence. Since there exists no agreed-upon, explicit set of grammatical rules to which you can refer, it follows that there is no absolute, agreed-upon way to assign Yngve numbers to words in a sentence. What is needed is a system of generative rules of such a kind that they will explicitly assign the correct constituent-structure to sentences. In the absence of such a system, and because Yngve has not completely formalized the rules of structural assignment, we have had to make some arbitrary decisions on the assignment of structure. When Yngve offers no basis for a particular decision, we have based the decision on Hockett (1958), or Wells' (1947) methods for determining immediate constituents, or on our own best judgment of linguistic notions.

To get an estimate of reliability, a random sample of 20 word problems was given to J. Dexter Fletcher, a graduate student concentrating in psycholinguistics. For each problem, he computed the mean of the Yngve numbers for each sentence. The complexity of a problem was taken to be the highest value of the set of Yngve means for that problem. The Pearson correlation was .84 ($r^2 = .71$) between the values we obtained and those obtained by Mr. Fletcher for the sample of 20 problems. The correlation is sufficiently high to give us reasonable confidence that our methods of structural assignment were well-founded.

In this chapter, the main task is to report the predictive worth of the eight variables described earlier. The objective is successfully to predict the probability of a correct response for each item. The first step in analysis was to obtain regression coefficients for each of the factors. A stepwise, multiple linear regression analysis program, BMD 02R, adapted for Stanford University's IBM 360 computer, was used to obtain regression coefficients, multiple correlation R and R^2.

For purposes of analysis, we have initially combined the data from both School A and School B into one group composed of 16 Ss. We will present the results of the combined data first, and then present the results of the two groups taken separately.

Combined Group

The mean percentage of correct solutions for 16 Ss was 47.09 percent.

The regression equation was

$$z_i = -3.24 + .48X_{i1}^{***} + .04X_{i2} + .02X_{i3}^{**} + .88X_{i4}^{***} + .61X_{i5}^{***}$$
$$+ .20X_{i6} + .13X_{i7} + .49X_{i8}^{*}$$

(* indicates $p < .05$; ** indicates $p < .01$; *** indicates $p < .001$)

with a multiple R of .83, a standard error of estimate of .52, and an R^2 of .70. The reason that X_{i3} is significant in spite of the fact that its regression coefficient is so small is because the standard error of the regression coefficient is .006. The T-value is computed by dividing the regression coefficient by its standard error. Table 1 presents the regression coefficients, standard errors of the regression coefficients, computer T-values, and partial correlation coefficients for each of the eight independent variables. Table 2 presents the independent variables in order, as introduced in the stepwise regression, with corresponding multiple correlations. Table 3 presents the analysis of variance for the multiple regression.

Insert Tables 1, 2, and 3 about here

TABLE 1

Regression Coefficients, Standard Errors of Regression Coefficients, Computed T-values, and Partial Correlation Coefficients (8 Variables, 16 Subjects)

Variable	Regression coefficient	Standard error	Computed T-value	Partial correlation coefficient
X_1 operations	.48329	.10251	4.71479	.44308
X_2 steps	.04135	.05431	.76137	.07956
X_3 length	.01740	.00611	2.84970	.28623
X_4 depth	.87853	.22884	3.83898	.37334
X_5 sequential	.61081	.10617	5.75318	.51644
X_6 verbal-clue	.19603	.11872	1.65125	.17056
X_7 order	.13303	.12467	1.06703	.11116
X_8 conversion	.49446	.21960	2.25165	.22972

TABLE 2

Order of Introduction of the Variables in the Regression with Corresponding Correlations (Variables, 16 Subjects)

Variable	Multiple Regression
1. X_1 = <u>operations</u>	.67
2. X_2 = <u>sequential</u>	.77
3. X_3 = <u>length</u>	.79
4. X_4 = <u>depth</u>	.81
5. X_5 = <u>conversion</u>	.83
6. X_6 = <u>verbal-clue</u>	.83
7. X_7 = <u>order</u>	.83
8. X_8 = <u>steps</u>	.83

TABLE 3

Analysis of Variance for Multiple Linear Regression
(8 Variables, 16 Subjects)

Source of variation	d.f.	Sum of squares	Mean squares	F value
Due to regression	8	56.92740	7.11592	26.0892*
Deviation about regression	91	24.82055	0.27275	
Total	99	81.74795		

* p < .001

Consideration of the partial correlation coefficients indicates that X_5, the sequential variable is the most important of the eight variables. The operations variable, X_1, the depth variable, X_4, and the length variable, X_3, are also valuable predictors of the probability of a correct response for each item. The conversion variable, X_8, is moderately valuable. A rough indication of the goodness of fit of the regression line is given by the multiple correlation coefficient, R, and its square, R^2, which is an estimate of the amount of variance accounted for by the regression model. In this case, 70 percent of the variance in probability of a correct response is accounted for by the model.

Figure 5 presents a graph of the predicted and observed proportions of correct responses for each of the 100 items. The probabilities are plotted as a function of the rank of observed proportion of correct responses. Consequently, the curve of the observed probabilities is

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Insert Figure 5 about here

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monotonically decreasing and smoother than the predicted curve. An inspection of the two curves shows a reasonable fit for the regression model, especially in view of the heterogeneity of problem types. The model does not fit for very difficult or for very easy items as well as it does for items in the middle range of difficulty. For an analysis of goodness of fit of the probability of a correct response predicted from the regression model and the observed probability of a correct response, the predicted probability, p_i, of a correct response for problem i, was first calculated for each item. As a measure of fit, X^2 was then calculated, where

$$X^2 = \sum_i (f_i - p_i N)^2/[p_i(1 - p_i)N]$$

and f_i = observed frequency of correct response, N = number of students. For the above model, $X^2 = 206.74$.

This rather high value for X^2 is an indication that the correspondence between the observed and expected frequencies is not very close. A

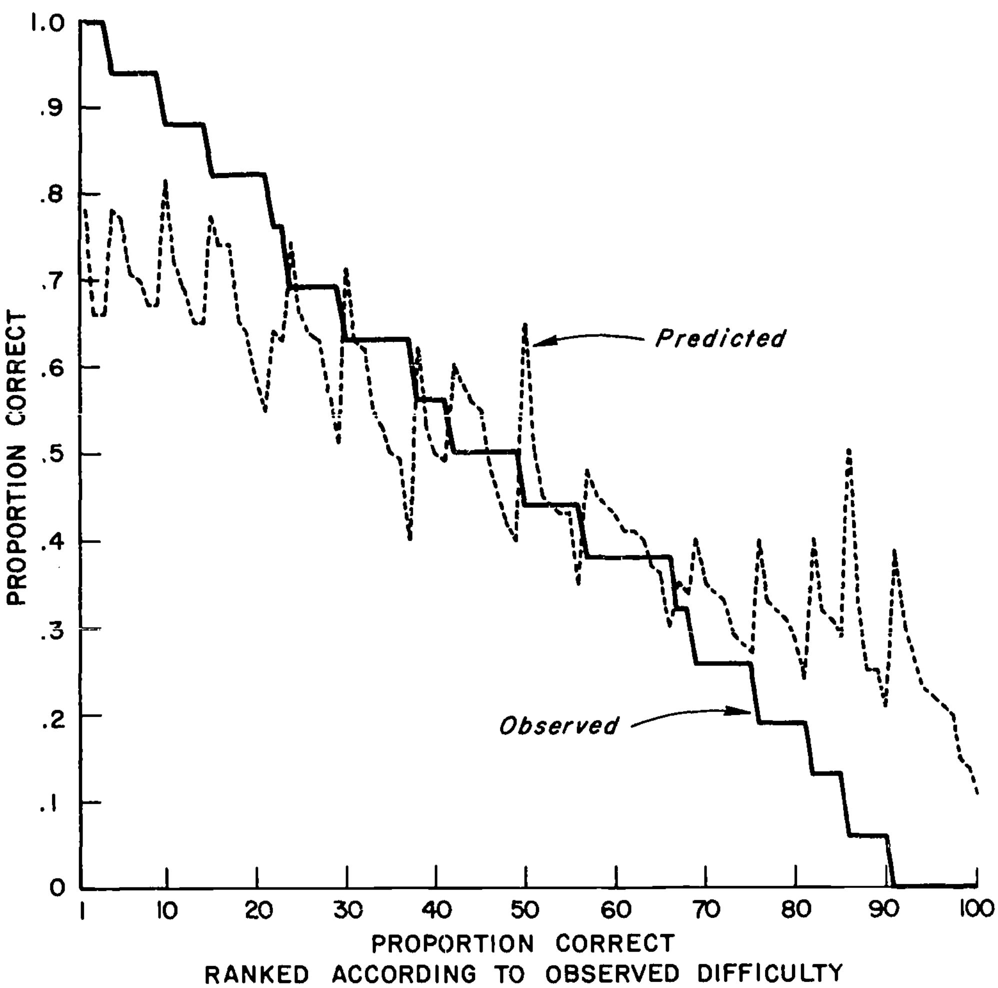

Fig. 5. Problem Rank Order according to proportion correct.

closer look at the components of X^2, however, shows that a few problems made extremely large contributions to the total X^2. The following problem, for example, contributed 6.3 percent to the total X^2 obtained:

> "A school playground is rectangular, 273 feet long and 21 feet wide. What is the total length of the fence around the playground..."

The observed proportion of correct responses for this item was .06, while the predicted proportion was .50; clearly, this is a very poor fit. As a second example, the following problem contributed 5.3 percent to the total X^2 obtained:

> "Mary is twice as old as Betty was 2 years ago. Mary is 40 years old. How old is Betty..."

None of the 16 Ss solved this problem correctly, although .39 was the predicted proportion of correct responses. The large deviations between the observed and predicted results for certain problems, such as the two just mentioned, emphasize the need for a more elaborate theory.

Often, most of the prediction achieved can be attributed to a small number of variables, and the inclusion of additional variables contributes only small amounts to prediction. In this case, most of the variance can be accounted for by variables X_1, X_3, X_4, and X_5. If we reduce the number of variables in the regression equation to include only these, the reduction in multiple R and R^2 is very slight. Considering only these four variables, the regression equation becomes

$$z_i = -2.89 + .64X_{i1} + .02X_{i3} + .64X_{i4} + .63X_{i5}$$

with a multiple R of .81, a standard error of estimate of .54, and R^2 of .66. The standard errors of the regression coefficients are X_1, .081; X_3, .006; X_4, .225; and X_5, .109. All four variables are significant. Table 4 presents the analysis of variance for the multiple linear regression using these four variables.

Insert Table 4 about here

TABLE 4

Analysis of Variance for Multiple Linear Regression
(4 Variables, 16 Subjects)

Source of variation	d.f.	Sum of squares	Mean squares	F value
Due to regression	4	53.900	13.475	45.969*
Deviation about regression	95	27.848	0.293	
Total	99	81.748		

* $p < .001$

School A Group

The mean percentage of correct solutions for 7 Ss was 50.30 percent. The regression equation was

$$z_i = -3.74 + .52X_{i1}^{***} + 0.0X_{i2} + .02X_{i3}^{**} + 1.03X_{i4}^{***} + .72X_{i5}^{***}$$

$$+ .21X_{i6} + .28X_{i7} + .27X_{8}$$

(** indicates $p < .01$; *** indicates $p < .001$)

with a multiple R of .80, a standard error of estimate of .66, and an R^2 of .64. Table 5 presents the regression coefficients, standard errors of regression coefficients, computed T-values, and partial correlation coefficients for each of the eight independent variables. Table 6 presents the independent variables in order, as introduced in the stepwise regression, with corresponding multiple correlations. Table 7 presents the analysis of variance for the multiple linear regression.

Insert Tables 5, 6, and 7 about here

School B Group

The mean percentage of correct solutions for these 9 Ss was 43.58 percent. The regression equation was

$$z_i = -2.96 + .51X_{i1}^{***} + .03_{i2} + .02X_{i3}^{**} + .76X_{i4}^{***} + .61X_{i5}^{***}$$

$$+ .20X_{i6} + .07X_{i7} + .60X_{i8}^{**}$$

(** indicates $p < .01$; *** indicates $p < .001$)

with a multiple R of .83, a standard error of estimate of .51, and an R^2 of .70. Table 8 presents the regression coefficients, standard errors of the regression coefficients, computed T-values, and partial correlation coefficients for each of the eight independent variables. Table 9 presents

TABLE 5

Regression Coefficients, Standard Errors of Regression Coefficients, Computed T-values, and Partial Correlation Coefficients (8 Variables, School A)

Variable	Regression coefficient	Standard error	Computed T-value	Partial correlation coefficient
X_1 operations	.51821	.11039	4.69438	.44154
X_2 steps	.00014	.00009	1.63630	.16906
X_3 length	.02064	.00740	2.79040	.28075
X_4 depth	1.03102	.28891	3.56865	.35038
X_5 sequential	.72709	.13450	5.40597	.49303
X_6 verbal-clue	.21291	.14790	1.43956	.14922
X_7 order	.28089	.15877	1.76918	.18235
X_8 conversion	.26669	.29792	.89518	.09343

TABLE 6

Order of Introduction of the Variables in the Regression with Corresponding Correlations (8 Variables, School A)

Variable		Multiple r
1.	operations	.57
2.	sequential	.69
3.	depth	.75
4.	length	.77
5.	order	.78
6.	steps	.79
7.	verbal-clue	.80
8.	conversion	.80

TABLE 7

Analysis of Variance for Multiple Linear Regression
(8 Variables, School A)

Source of variation	d.f.	Sum of squares	Mean squares	F value
Due to regression	8	69.80213	8.72527	20.0795*
Deviation about regression	91	39.54274	.43454	
Total	99	109.34487		

* $p < .001$

the independent variables in order, as introduced in the stepwise regression, with corresponding multiple correlations. Table 10 presents the analysis of variance for the multiple linear regression.

Insert Tables 8, 9, and 10 about here

TABLE 8

Regression Coefficients, Standard Errors of Regression Coefficients, Computed T-values, and Partial Correlation Coefficients (8 Variables, School B)

Variable	Regression coefficient	Standard error	Computed T-value	Partial correlation coefficient
X_1 operations	.50698	.10098	5.02084	.46575
X_2 steps	.03171	.05350	.59272	.06201
X_3 length	.01756	.00602	2.91922	.29262
X_4 depth	.76093	.22543	3.37547	.33358
X_5 sequential	.61000	.10458	5.83262	.52164
X_6 verbal-clue	.19509	.11695	1.66822	.17226
X_7 order	.06686	.12281	.54443	.05698
X_8 conversion	.59629	.21632	2.75652	.27760

TABLE 9

Order of Introduction of the Variables in the Regression
with Corresponding Correlations
(8 Variables, School B)

Variable		Multiple r
1.	operations	.67
2.	sequential	.77
3.	length	.79
4.	depth	.81
5.	conversion	.83
6.	verbal-clue	.83
7.	steps	.83
8.	order	.83

TABLE 10

Analysis of Variance for Multiple Linear Regression
(8 Variables, School B)

Source of variation	d.f.	Sum of squares	Mean squares	F value
Due to regression	8	55.09598	6.88700	26.0210*
Deviation about regression	91	24.08506	.26467	
Total	99	79.18104		

* $p < .001$

Chapter VI.

DISCUSSION

The results reported in Chapter V indicate the following variables to be significant determinants of word problem difficulty: sequential, operations, depth, length, and conversion. These findings imply that a word problem will be difficult to solve if it is of a different type than the problem that preceded it, if its solution requires a large number of different operations, if its surface structure is complex, if it has a large number of words, or if it requires a conversion of units. The multiple correlations and thus the predictive results of this analysis of the data are rather impressive. There is considerable difficulty in intuitively rank-ordering the expected proportions of correct responses obtained in word problems. We believe that our results give a sense of the real possibility of analyzing and predicting in terms of meaningful variables, the response performance of children who are solving arithmetical word problems. At first glance, the problem set appears to be quite complex. Yet, with a few variables we have brought a considerable amount of order to it. In view of the intrinsic complexity of this type of problem solving, the fit obtained is excellent.

It is interesting and potentially instructive to compare the results of performance of this "disadvantaged" group with the results of a similar study using bright Ss (Suppes, Loftus, and Jerman, 1969). The important variables reported by Suppes et al., were the sequential, operations, and conversion variables. Depth and order were not investigated in that study. The most important variables in the present study were operations, sequential, depth, and length; conversion was of secondary, although substantial, importance. The findings regarding the conversion variable are extremely interesting. For School A (average IQ = 93), the conversion variable was the last variable to be introduced into the stepwise regression. It had the lowest partial correlation coefficient. For School B (average IQ = 99), the conversion variable was the fourth most important variable. For the bright group (average IQ > 120), the conversion variable was the second most important variable in terms of predictive worth. It appears that there may be some relationship between mental ability and the importance of the conversion variable. No attempt has been made to estimate parameters that would reflect mental ability, or to explore the relationship between mental ability and the conversion variable, because accounting for differential mental ability is beyond the scope of this research.

The most suggestive finding in all the analyses is the importance of the sequential and operations variables. These two variables are highly significant determinants of difficulty for bright as well as disadvantaged students. Whether one is a bright or a dull student, one is more likely to solve a problem correctly if it is similar to the problem that preceded it or if its solution requires a small number of different operations. The implication is that many aspects of the processing done internally by the students when they solve problems do not differ for children of differing mental ability. The next step is to acquire a better understanding of these variables and to use this understanding to develop better predictive models.

Recall that in Chapter V two problems were mentioned as having contributed most heavily to the total X^2 obtained. The two problems are:

1. A school playground is rectangular, 273 feet long and 21 feet wide. What is the total length of the fence around the playground?

2. Mary is twice as old as Betty was 2 years ago. Mary is 40 years old. How old is Betty?

The discrepancies between the observed and predicted proportions of correct responses for these two problems were quite large. In the first problem, Ss typically multiplied the two numbers together, or added the two numbers together only once. The difficulty here seems to be due to the confusion about what a perimeter is, as distinguished from an area, and how to find a perimeter. In problem 2, the word "ago" seemed to worry all of the Ss. They tended to subtract two years at the beginning of the problem. Suppes et al., report that the very same two problems contributed most heavily to the total X^2 obtained in their study with bright Ss. It appears that the regression models investigated cannot account for performance on these two types of problems, for either above or below average Ss. A more elaborate theory is needed to handle them. That theory should be able to handle problems which require additional knowledge (such as the formula for finding the perimeter) as well as problems which involve a change in tense.

An interesting side result is revealed by comparison of the percentage of correct responses for School B vs. School A group. Recall that the B group, drawn from a class with an average IQ of 99, and in which 59 percent of the children were black, solved 43.58 percent of the problems correctly. The A group (82 percent black, average IQ = 93) solved 50.9 percent of the problems correctly. We assume that the reason for the superior performance of the A Ss was they they were allowed to repeat portions of the instruction set before beginning the problem set. They may have learned the rules better than the B group. In addition, their motivation seemed to be elevated by the mere successful solving of a problem, regardless of whether that problem had been missed the first time. It appears that if a student has failed to solve a problem correctly, to allow him a second chance at the problem is a better procedure than to force him to go on to a new problem.

A disadvantage of the data reported in this study is that even though the students did participate in the program for a number of weeks, the number of students completing the program was small. A main objective for the future is to increase considerably the number of students involved in order to provide the quantity of data required for meaningful inferences about problem-solving processes.

The results obtained in this study give a clear indication of the difficulty in constructing an explanatory theory that is adequate to account for all the difficulties students encounter in solving word problems. We have just scratched the surface of the complete syntactic and semantic analysis that will be required to predict all the details that must be accounted for in the behavior of students. The present study represents only a tentative, preliminary effort at the construction of a more mature theory. Further development of such a theory, as well as any discussion of the implications of the various predictive analyses for the teaching of problem solving, must await much needed additional research. In particular, more refined analysis with data from larger numbers of students is needed. We are convinced that deeper investigations in this direction are essential to a better understanding of problem solving.

Chapter VII

SUMMARY

The research reported here examines the problem-solving performance of 16 sixth-grade students. These students were taken from two "depressed area" schools. The students were first taught the mechanics of how to use a computer-based teletype to solve arithmetic word problems. The assumption was made that all students had a basic understanding of the four arithmetical operations: addition, subtraction, multiplication, and division. The students were required to know which operation(s) should be performed for problem solution, and to tell the computer which one(s). The actual computations were done by the computer. Following the initial instruction set, a series of 100 word problems was presented to the students. For each problem the students were required to find a quantitative answer. The arithmetical operation(s) required were not explicitly indicated. An example of a problem in arithmetic providing the pupil with an opportunity to use his knowledge of multiplication is the following:

> A bushel of corn weighs 56 pounds. How much does 44 bushels weigh?

The solution of these problems were analyzed to determine the variables related to problem difficulty. A linear regression analysis revealed the vollowing variables to be significant:

1. The operations variable, that is, the minimum number of different operations required to reach the correct solution. The larger the required minimum, the harder the problem was to solve.

2. The sequential variable. A problem was easier to solve if it was of the same type (i.e., could be solved by the same operation(s)) as the problem that preceded it.

3. The depth variable, that is, the mean number of left branches per word in the constituent-structure tree for the most complex sentence in the word problem. This mean, sometimes called the Yngve mean, is a

measure of the structural complexity of a sentence. Problems of lesser indexed complexity were easier to solve than problems of greater complexity.

4. The <u>length</u> variable, that is, the number of words in the problem. The larger the number of words in a problem, the harder the problem was to solve.

5. The <u>conversion</u> variable. Problems which required a conversion of units (such as from days to weeks) were harder to solve than problems which did not.

Three additional variables did not make a significant contribution to the regression analysis:

1. The <u>verbal-clue</u> variable. A problem that contained a verbal clue to the operation(s) required for solution was not easier to solve than a problem which did not contain such a clue.

2. The <u>order</u> variable. A problem which contained the numerical data in an order in which they could be used to solve the problem was not easier to solve than a problem which did not.

3. The <u>steps</u> variable, that is, the minimum number of steps required to reach the correct solution. No relationship was found between the minimum number of steps and problem difficulty.

REFERENCES

Braun-Lamesche, M. Le role du contexte dans la comprehension du langage chez L'enfant. Psychologie Francaise, 1962, 7, 180-189.

Brown, R., and Fraser, C. The acquisition of syntax. In Cofer, C. N., and Musgrave, B. (Eds.), Verbal behavior and learning: problems and processes, New York: McGraw-Hill, 1963.

Brownell, W., and Stretch, L. B. The effect of unfamiliar settings on problem-solving. Duke University Research Studies in Education, No. 1, Durham, North Carolina. Duke University Press, 1931.

Bruner, J. S., Goodnow, J. J., and Austin, G. A. A study of thinking. New York: Wiley, 1956.

Burns, P. C., and Yonally, J. L. Does the order of presentation of numerical data in multi-steps affect their difficulty? School Science and Mathematics, 1964, 64, 267-270.

Carey, P. W., Mehler, J., and Bever, T. G. Judging the veracity of ambiguous sentences, Journal of verbal learning and verbal behavior, 1970, 9, 243-254.

Chase, C. I. The position of certain variables in the prediction of problem solving in arithmetic. Journal of Educational Research, 1960, 54, 9-14.

Cofer, C. N. The role of language in human problem solving. Paper presented at the Conference on Human Problem Solving, sponsored by the National Science Foundation, New York University, April, 1954.

Davis, E. A. Mean sentence length compared with long and short sentences as a reliable measure of language development. <u>Child Development</u>, 1937, <u>8</u>, 69-79.

Davis, G. A. Current status of research and theory in human problem solving. <u>Psychological Bulletin</u>, 1966, <u>66</u>, 36-54.

Deutsch, M., and Cherry-Peisach, E. A study of language patterns. <u>The Instructor</u>, 1966, <u>75</u> (No. 7), 95.

Duncan, C. P. Recent research on human problem solving. <u>Psychological Bulletin</u>, 1959, <u>56</u>, 397-429.

Duncker, K. On problem-solving. <u>Psychological Monographs</u>, 1945, <u>58</u>, No. 5 (Whole No. 270).

Ervin, S. Imitation and structural change in children's language. In E. Lenneberg (Ed.) <u>New Directions in the study of language</u>. Cambridge, Massachusetts: M.I.T. Press, 1964. Pp. 163-189.

Feigenbaum, E. A., and Feldman, J. (Eds.) <u>Computers and thought</u>. New York: McGraw-Hill, 1963.

Gagne, R. M. Problem solving and thinking. <u>Annual Review of Psychology</u>, 1959, <u>10</u>, 147-172.

Gagne, R. M. Human problem solving: internal and external events. In Kleinmuntz, B. (Ed.) <u>Problem solving: research, method, and theory</u>. New York: Wiley, 1966.

Gagne, R. M., and Smith, E. C., Jr. A study of the effects of verbalization on problem solving. <u>Journal of Experimental Psychology</u>, 1962, <u>63</u>, 12-18.

Hayes, J. R. Problem topology and the solution process. <u>Journal of Verbal Learning and Verbal Behavior</u>, 1965, <u>4</u>, 371-379.

Heidbreder, E. The attainment of concepts: III. The process. Journal of Psychology, 1947, 24, 93-138.

Hilgard, E. R., and Bower, G. H. Theories of learning. New York: Appleton-century-crofts, 1966.

Hockett, C. F. A course in modern linguistics. New York: The Macmillan Co., 1958.

Hoffman, L. R. Conditions for creative problem solving. Journal of Psychology, 1961, 52, 429-444.

Hovland, C. I. Computer simulation of thinking. American Psychologist, 1960, 15, 682-693.

Hydle, L. L., and Clapp, F. L. Elements of difficulty in the interpretation of concrete problems in arithmetic. Bureau of Educational Research Bulletin, No. 9, Madison, Wisconsin: University of Wisconsin, 1927.

Johnson, D. M. The psychology of thought and judgment. New York: Harper and Row, 1955.

Judson, A. J., Cofer, C. N., and Gelfand, S. Reasoning as an associative process: II. "Direction" in problem solving as a function of prior reinforcement of relevant responses. Psychological Reports, 1956, 2, 501-507.

Kendler, H. H., and D'Amato, M. F. A comparison of reversal shifts and non-reversal shifts in human concept formation behavior. Journal of Experimental Psychology, 1955, 49, 165-174.

Kendler, H. H., and Karasik, A. D. Concept formation as a function of competition between response produced cues. Journal of Experimental Psychology, 1958, 55, 278-283.

Kendler, H. H., and Kendler, T. S. Vertical and horizontal processes in problem solving. Psychological Review, 1962, 69, 1-16.

Kendler, H. H., and Mayzner, M. S. Reversal and non-reversal shifts in cardsorting tests with two or four sorting categories. Journal of Experimental Psychology, 1956, 51, 244-248.

Kendler, H. H., and Vineberg, R. The acquisition of compound concepts as a function of previous training. Journal of Experimental Psychology, 1954, 48, 252-258.

Kendler, T. S. Development of mediating responses in children. Monograph of Society for Research in Child Development, 1963, 28 (Whole No. 86), 33-48.

Keppel, G. Facilitation in short- and long-term retention of paired associates following distributed practice in learning. Journal of Verbal Learning and Verbal Behavior, 1964, 3, 91-111.

Klausmeier, H. J., and Loughlin, L. J. Behaviors during problem solving among children of low, average, and high intelligence. Journal of Educational Psychology, 1961, 52, 148-152.

Kleinmuntz, B. (Ed.). Problem solving: research, method, and theory. New York: Wiley, 1966.

Krasner, L. Studies of the conditioning of verbal behavior. Psychological Bulletin, 1958, 55, 148-170.

Kurz, R. B. Effects of three kinds of stressors on human learning and performance. Psychological Reports, 1964, 14, 161-162.

Lazerte, M. E. The development of problem solving ability in arithmetic. Toronto: Clarke, Irwin and Company, 1933.

Loban, W. The language of elementary school children. National Council of Teachers of English Research Bulletin No. 1, 1963.

Luce, R. D., Bush, R. R., and Galanter, E. (Eds.) Handbook of mathematical psychology, Vol. 1, New York: Wiley, 1963.

Maier, N. R. F. Reasoning in humans: II. The solution of a problem and its appearance in consciousness. Journal of Comparative Psychology, 1931, 12, 181-194.

Maltzman, I. Thinking: from a behavioristic point of view. Psychological Review, 1955, 62, 275-286.

Martin, E., and Roberts, K. H. Grammatical factors in sentence retention. Journal of Verbal Learning and Verbal Behavior, 1966, 5, 375-380.

Martin, E., Roberts, K. H., and Collins, A. M. Short-term memory for sentences. Journal of Verbal Learning and Verbal Behavior, 1968, 7, 560-566.

Martin, M. D. Reading comprehension, abstract verbal reasoning, and computation as factors in arithmetic problem solving. Unpublished doctoral thesis. Iowa City: State University of Iowa, 1963.

McCarthy, D. A. The language development of the pre-school child. Minneapolis: The University of Minnesota Press, 1930.

McGovern, J. B. Extinction of associations in four transfer paradigms. Psychological Monographs, Vol. 78, No. 16, 1964.

Mednick, S. A. Learning. Englewood Cliffs, New Jersey: Prentice Hall, 1964.

Menyuk, P. Syntactic structures in the language of children. Child Development, 1963, 34, 407-422.

Miller, N. E. Theory and experiment relating psychoanalytic displacement to stimulus response generalization. Journal of Abnormal and Social Psychology, 1948, 43, 155-178.

Miller, W., and Ervin, S. Language development. Child Psychology, 1963, 62nd Yearbook for the National Society for the Study of Education. Pp. 108-143.

Moore, E. F. (Ed.) Sequential machines. Addison-Wesley, 1964.

Neisser, U. The multiplicity of thought. British Journal of Psychology, 1963, 54, 1-14. (a)

Neisser, U. The imitation of man by machine. Science, 1963, 139, 193-197. (b)

Newell, A. Discussion of papers by Dr. Gagne and Dr. Hayes. In Kleinmuntz, B. (Ed.) Problem solving: research, method, and theory. New York: Wiley, 1966.

Newell, A., Shaw, J. C., and Simon, H. A. Elements of a theory of human problem solving. Psychological Review, 1958, 65, 151-166. (a)

Newell, A., Shaw, J. C., and Simon, H. A. The processes of creative thinking. Paper P-1320. Santa Monica: Rand Corporation, 1958. (b)

Newell, A., Shaw, J. C., and Simon, H. A. Report on a general problem-solving program. In Proceedings of the International Conference on Information Processing. Paris: UNESCO House, 1959. Pp. 256-264.

Newell, A., Shaw, J. C., and Simon, H. A. Chess-playing programs and the problem of complexity. In E. A. Feigenbaum and J. Feldman (Eds.) Computers and thought. New York: McGraw-Hill, 1963. Pp. 39-70. (a)

Newell, A., Shaw, J. C., and Simon, H. A. Empirical explorations with the Logic Theory Machine: a case study in heuristics. In E. A. Feigenbaum and J. Feldman (Eds.), Computers and thought. New York: McGraw-Hill, 1963. Pp. 109-133. (b)

Newell, A., and Simon, H. A. GPS, a program that simulates human thought. In E. A. Feigenbaum and J. Feldman (Eds.), Computers and thought. New York: McGraw-Hill, 1963. (a)

Newell, A., and Simon, H. A. Computers in psychology. In Luce, R. D., Bush, R. R., and Galanter, E. (Eds.), Handbook of mathematical psychology, Vol. I, New York: Wiley, 1963. Pp. 361-428. (b)

Orr, D. B. Research on behavior impairment due to stress: an experiment in long-term performance. Journal of Experimental Psychology, 1964, 68, 94-102.

Osgood, C. E. Method and theory in experimental psychology. New York: Oxford University Press, 1953.

Paige, J. M., and Simon, H. A. Cognitive processes in solving algebra word problems. In B. Kleinmuntz (Ed.), Problem-solving: research, methods, and theory. New York: Wiley and Sons, 1966.

Perfetti, C. A. Lexical density and phrase structure depth as variables in sentence retention. Journal of Verbal Learning and Verbal Behavior, 1969, 8, 719-724.

Post, R. A study of certain factors affecting the understanding of verbal problems in arithmetic. Unpublished doctoral dissertation, Teacher's College, Columbia University, New York, 1958.

Postman, L. Short-term memory and incidental learning. In A. W. Melton (Ed.), Categories of Human Learning. New York: Academic Press, 1964. Pp. 145-201.

Reitman, W. R. Cognition and thought. New York: Wiley and Sons, 1965.

Rohrman, N. L. The role of syntactic structure in the recall of English nominalizations. Journal of Verbal Learning and Verbal Behavior, 1968, 7, 904-912.

Ruddell, R. B. The effect of oral and written patterns of language structure on reading comprehension. The Reading Teacher, 1964, 18, 270-275.

Russell, D. H. Children's thinking. Boston: Ginn and Co., 1956.

Russell, D. H. Higher mental processes. In C. W. Harris (Ed.), Encyclopedia of Educational Research. New York: Macmillan, 1960.

Saugstad, P. Problem solving as dependent upon availability of functions. Unpublished doctoral thesis, University of Chicago, 1952.

Scandura, J. M. New directions for theory and research on rule learning: I. A set-function language. Acta Psychologica, 1968, 28, 301-321.

Schultz, R. W. Problem solving bahavior and transfer. Harvard Educational Review, 1960, 30, 61-77.

Sheerer, M. Problem-solving. Scientific American, 1963, 208 (4), 118-128.

Staats, A. W. Verbal habit-families, concepts, and the operant conditioning of word classes. Psychological Review, 1961, 68, 190-204.

Steffe, L. P. The effects of two variables on the problem-solving abilities of first-grade children. Technical Report No. 21. Madison: Wisconsin Research and Development Center for Cognitive Learning, University of Wisconsin, March 1967.

Stevens, B. A. Problem solving in arithmetic. Journal of Educational Research, 1932, 25, 253-260.

Suppes, P., Jerman, M., and Brian, D. Computer-assisted instruction: the 1965-66 Stanford arithmetic program. New York: Academic Press, 1968.

Suppes, P., Loftus, E. F., and Jerman, M. Problem-solving on a computer-based teletype. Educational Studies in Mathematics, 1969, 2, 1-15.

Travers, K. J. A test of pupil preference for problem-solving situations in junior high school mathematics. Journal of Experimental Education, 1967, 35, 9-18.

Thompson, E. N. Readability and accessory remarks: factors in problem solving in arithmetic. Unpublished doctoral thesis. Stanford: Stanford University, 1967.

Underwood, B. J. Ten years of massed practice on distributed practice. Psychological Review, 1961, 68, 229-247.

Underwood, B. J. The representativeness of rote verbal learning. In A. W. Melton (Ed.), Categories of human learning. New York: Academic Press, 1964. Pp. 47-87.

Underwood, B. J., and Schultz, R. W. Meaningfulness and verbal learning. Chicago: J. B. Lippincott Co., 1960.

Verplanck, W. S. Unaware of where's awareness: some verbal operants - notates, monents, and notents. In C. W. Eriksen (Ed.), Behavior and awareness. Durham: Duke University Press, 1962.

Washburne, C. W., and Morphett, M. V. Unfamiliar situations as a difficulty in solving arithmetic problems. Journal of Educational Research, 1928, 18, 220-224.

Wendelin, I. A synthesis of two factor analyses of problem solving in mathematics. Didakometary, No. 8. Malmo, Sweden: Malmo School of Education, 1966.

Wells, R. G. Immediate constituents. Language, 1947, 23, 81-117.

Wertheimer, M. Productive thinking. New York: Harper and Row, 1945.

Woodhead, M. M. The effect of bursts of noise on an arithmetic task. American Journal of Psychology, 1964, 77, 627-633.

Yngve, V. H. A model and an hypothesis for language structure. Proceedings of the American Philosophical Society., 1960, 104, 444-466.

Yngve, V. H. Implications of mechanical translation research. Proceedings of the American Philosophical Society, 1964, 108, 275-281.

APPENDIX A

Problem-Solving Program
Instruction Set

570.1

HELLO. WELCOME TO PROBLEM SOLVING. THESE LESSONS WILL SHOW YOU A NEW, FUN WAY TO SOLVE PROBLEMS.

THIS PROBLEM ENDS WITH CHOICE A), B), OR C). TYPE THE LETTER OF THE CORRECT CHOICE, THEN PRESS THE SPACE BAR. THE LETTER ALONE IS NOT ENOUGH, YOU MUST ALSO...
A) WINK AN EYE
B) PRESS THE SPACE BAR
C) CROSS YOUR FINGERS

B

570.2

WHAT DOES 2 + 3 EQUAL...
A) 2
B) 3
C) 5

C

570.3

HOW MUCH IS 14 MINUS 8...
A) 14
B) 6
C) 8

B

570.4

NOW YOU WILL LEARN A NEW WAY TO TELL THE COMPUTER WHAT YOU THINK THE ANSWER IS. WHEN THE COMPUTER STOPS TYPING DO THESE THINGS:

1. FIND THE ANSWER TO THE PROBLEM.
2. FIND THE LINE NUMBER THAT THE ANSWER IS ON.
3. TYPE THE LINE NUMBER, THEN TYPE 'X', THEN PRESS THE SPACE BAR.

NOW, LET'S TRY IT...
THE PROBLEM IS: WHAT IS 2 + 3...

G	(1)	4
G	(2)	5
2X		

570.5

'X' IS THE ANSWER KEY. '2X' TELLS THE COMPUTER, 'MY FINAL ANSWER IS ON LINE 2'.

HOW DO YOU TELL THE COMPUTER YOUR ANSWER IS ON LINE φ...
A) 3
B) X
C) 3X

C

571.1

TO DO THIS PROBLEM TYPE '3X' AND THEN A SPACE.
2+3...

G	(1)	2
G	(2)	3
G	(3)	5
3X		

571.2

'3X' MEANT THAT ON LINE 3 IS THE ANSWER TO...
A) 2+3
B) 3+5

A

571.3

USE A LINE NUMBER AND X TO TELL THE COMPUTER WHERE THE ANSWER IS.
4+7=...

```
G            (1)  10
G            (2)  11
G            (3)  12
2X
```

571.4

IN PROBLEM SOLVING, WE CAN USE THE COMPUTER TO ADD NUMBERS. TO ADD THE NUMBERS ON LINES 1 AND 2, TYPE '1.2A'. THEN SPACE.
ADD 48 AND 37.

```
G            (1)  48
G            (2)  37
                       TYPE H AND SPACE IF YOU WANT A HINT.
H                 DID YOU TYPE '1.2A' AS THE COMPUTER SAID.
1.2A         (3)  85
                       TYPE H AND SPACE IF YOU WANT A HINT.
H                 IF YOU TYPED '1.2A', NOW USE 'X' TO TELL
                  THE COMPUTER WHERE YOUR ANSWER IS.
3X
```

571.5

TO ADD THE NUMBERS ON LINES 2 AND 3, YOU TYPE...
A) A2.3
B) 2.3A

B

571.6

THE COMMAND FOR ADDITION IS...
A) A
B) ADD

A

571.7

TO TELL THE COMPUTER THAT YOUR ANSWER IS ON LINE 4, YOU TYPE...
A) 4
B) 4X
C) X4

B

571.10

```
ADD 491 AND 510.
G              (1)  491
G              (2)  510
                         TYPE H AND SPACE IF YOU WANT A HINT.
H              TYPE '1.2A', THEN SPACE.  REMEMBER X.
1.2A           (3)  1001
3X
```

571.11

'G' STANDS FOR GIVEN NUMBER.
IN PROBLEM 571.10 THE FIRST G SHOWS THAT
491 IS A...
A) GRAND TOTAL
B) GREEDY NUMBER
C) GIVEN NUMBER

C

571.12

IN THIS PROBLEM TRY 1.2A AND THEN TRY 2.1A TO SEE IF YOU GET THE SAME ANSWER.

```
4587+3089=...
G              (1)  4587
G              (2)  3089
1.2A           (3)  7676
2.1A           (4)  7676
3X
```

571.13

YOU GET THE SAME RESULT IF YOU ADD TWO NUMBERS IN THE OTHER ORDER BECAUSE...
A) ADDITION IS COMMUTATIVE
B) ADDITION IS ASSOCIATIVE

A

571.14

THE NUMBERS BEFORE THE COMMAND ARE...
A) LINE NUMBERS
B) THE NUMBERS TO ADD

A

571.15

TO ADD THE NUMBER ON LINE 2 TO THE NUMBER ON LINE 3 YOU WOULD TYPE...
A) 1.2A
B) A3.2
C) 3.2A

C

571.16

SOMETIMES THERE ARE EXTRA NUMBERS GIVEN.
FIND 486+390.

```
G          (1) 707
G          (2) 486
G          (3) 390
2.3A       (4) 876
4X
```

571.17

WITH 'A' YOU CAN ONLY ADD TWO NUMBERS AT A TIME.
TO FIND THE SUM (5+7)+4, FIRST ADD 5 AND 7 TO GET 12, THEN...
A) ADD 4 TO 7.
B) ADD 4 TO THE 12.
C) ADD 4 TO 5.

```
571.20

FIND (486+390)+707.
G               (1)  486
G               (2)  390
G               (3)  707
                         TYPE H AND SPACE IF YOU WANT A HINT.
H                    FIRST FIND 486+390.
                     THEN ADD THAT SUM TO 707.
1.2A            (4)  876
3.4A            (5)  1583
5X
```

```
571.21

FIND 523+(341+56).
G               (1)  523
G               (2)  341
G               (3)  56
2.3A            (4)  397
1.4A            (5)  920
5X
```

```
571.22

ADD 947, 382, 410, AND 523.
G               (1)  947
G               (2)  382
G               (3)  410
G               (4)  523
1.2A            (5)  1329
3.4A            (6)  933
5.6A            (7)  2262
7X
```

```
571.23

WHAT IS THE SUM OF 450, 301, 271, AND 638.
G               (1)  450
G               (2)  301
G               (3)  271
G               (4)  638
1.2A            (5)  751
3.4A            (6)  909
5.6A            (7)  1660
7X
```

571.24

ADD 43 AND 43.

```
G            (1)  43
                      TYPE H AND SPACE IF YOU WANT A HINT.
H                 TYPE '1.1A'.
1.1A         (2)  86
2X
```

571.25

ADDING A NUMBER TO ITSELF IS...

A) TRIPLING THE NUMBER.
B) DOUBLING THE NUMBER.
C) SQUARING THE NUMBER.

B

571.26

DOUBLE 577.

```
G            (1)  577
1.1A         (2)  1154
2X
```

571.27

JOHN HAD 55 APPLES. TOM GAVE HIM ANOTHER 39 APPLES. HOW MANY APPLES DOES JOHN HAVE NOW...

```
G            (1)  55
G            (2)  39
1.2A         (3)  94
3X
```

571.30

A TANK HAD 957 GALLONS OF WATER IN IT. 1188 GALLONS ARE ADDED. HOW MANY GALLONS DOES THE TANK HAVE NOW...

```
G            (1)  957
G            (2)  1188
1.2A         (3)  2145
3X
```

571.31

DURING ONE WEEK, MISS BROWN'S CLASS USED 120 CARTONS OF MILK AT LUNCH TIME. MRS. SMITH'S CLASS USED 132 CARTONS AND MRS. GUGGENHEIMER'S USED 143 CARTONS. HOW MANY CARTONS OF MILK DID THE 3 CLASSES USE DURING THE WEEK...

G	(1)	120
G	(2)	132
G	(3)	143
G	(4)	3
1.2A	(5)	252
3.5A	(6)	395
6X		

571.32

A WOODEN BOX CONTAINS 23 RED BEADS, 5 GREEN BEADS, 30 YELLOW BEADS, AND 83 BLUE BEADS. HOW MANY BEADS DOES IT CONTAIN IN ALL...

G	(1)	23
G	(2)	5
G	(3)	30
G	(4)	83
1.2A	(5)	28
3.4A	(6)	113
5.6A	(7)	141
7X		

572.1

WE CAN SUBTRACT TWO NUMBERS USING THE 'S' COMMAND.
IN THIS PROBLEM, TYPE '1.2S', SPACE. REMEMBER X.
FIND 4258-256.

G	(1)	4258
G	(2)	256
1.2S	(3)	4002
3X		

572.2

THE '1' IN '1.2S' IS...
A) THE LINE NUMBER OF THE LARGER NUMBER.
B) THE NUMBER YOU WANT TO SUBTRACT.

A

572.3

THE '2' IN '1.2S' IS...
A) THE NUMBER WHICH IS LARGER.
B) THE LINE NUMBER OF THE SMALLER NUMBER.

B

572.4

```
FIND THE DIFFERENCE BETWEEN 9613 AND 912.
G             (1)  9613
G             (2)  912
1.2S          (3)  8701
3X
```

572.5

```
CAREFUL HERE.
SUBTRACT 912 FROM 9613.
G             (1)  912
G             (2)  9613
                       TYPE H AND SPACE IF YOU WANT A HINT.
H                  TYPE '2.1S', SPACE.
2.1S          (3)  8701
3X
```

572.6

```
BE CAREFUL OF EXTRA GIVEN NUMBERS.
SUBTRACT 1396 FROM 6054.
G             (1)  1396
G             (2)  4890
G             (3)  6054
3.1S          (4)  4658
4X
```

```
572.7

USE S TWICE HERE.  DO WHAT IS IN THE PARENTHESES FIRST.
FIND (8376-649)-700.
G            (1)  8376
G            (2)  649
G            (3)  700
                      TYPE H AND SPACE IF YOU WANT A HINT.
H                 TYPE '1.2S' FIRST.  THEN SUBTRACT 700 FROM
                  WHAT IS INSIDE THE PARENTHESES.
1.2S         (4)  7727
4.3S         (5)  7027
5X
```

```
572.10

USE A AND S HERE.  DO WHAT IS IN THE PARENTHESES FIRST.
WHAT IS (320+168)-167...
G            (1)  320
G            (2)  168
G            (3)  167
1.2A         (4)  488
4.3S         (5)  321
5X
```

```
572.11

WHAT IS (9131-4275)+25...
G            (1)  9131
G            (2)  4275
G            (3)  25
1.2S         (4)  4856
3.4A         (5)  4881
5X
```

```
572.12

WORD PROBLEMS ARE EASY NOW.

READ EACH PROBLEM.
THE COMPUTER WILL PRINT THE GIVEN NUMBERS.
DECIDE WHICH OPERATIONS TO USE.
USE THEM.  REMEMBER X.

AFTER THE COMPUTER PRINTS THE GIVEN NUMBERS, YOU SHOULD...
A)  STAND UP AND SMILE.
B)  DECIDE WHICH OPERATIONS TO USE.
C)  DESTROY THE TELETYPE MACHINE

B
```

572.13

TOM COLLECTED 500 SEASHELLS AND PLACED 48 OF THEM IN A SHOWCASE. HOW MANY SHELLS WERE NOT PLACED IN THE SHOWCASE...

```
G             (1)  500
G             (2)  48
1.2S          (3)  452
3X
```

572.14

USE TWO OPERATIONS HERE.

MARY HAD 128 POSTCARDS. HER MOTHER GAVE HER 17 MORE. HER BROTHER TOOK AWAY 10 POSTCARDS. HOW MANY POSTCARDS DID SHE HAVE LEFT...

```
G             (1)  128
G             (2)  17
G             (3)  10
1.2A          (4)  145
4.3S          (5)  135
5X
```

573.1

USE M TO MULTIPLY TWO NUMBERS.

MULTIPLY 19 BY 46.

```
G             (1)  19
G             (2)  46
                        TYPE H AND SPACE IF YOU WANT A HINT.
H                  TYPE '1.2M', SPACE.  REMEMBER X.
1.2M          (3)  874
3X
```

573.2

FIND 72X21.

```
G             (1)  72
G             (2)  21
1.2M          (3)  1512
3X
```

573.3

'1.2M' MEANS...
A) MULTIPLY THE NUMBER '1' BY THE NUMBER '2'.
B) TAKE THE NUMBERS ON LINE 1 AND LINE 2 AND MULTIPLY THEM.

B

573.4

2.1M GIVES THE SAME RESULT AS 1.2M BECAUSE...
A) MULTIPLICATION IS COMMUTATIVE.
B) MULTIPLICATION IS ASSOCIATIVE.
C) ADDITION IS COMMUTATIVE.

A

573.5

MULTIPLY 34 BY 32.
TRY MULTIPLYING BOTH WAYS BEFORE USING X.

```
G          (1)  34
G          (2)  32
1.2M       (3)  1088
2.1M       (4)  1088
3X
```

573.6

TO FIND THE PRODUCT OF THREE NUMBERS, USE M TWICE.
FIND (6X9)X10.

```
G          (1)  6
G          (2)  9
G          (3)  10
1.2M       (4)  54
3.4M       (5)  540
5X
```

573.7

MULTIPLY 15X(2X1003).

```
G          (1)  15
G          (2)  2
G          (3)  1003
2.3M       (4)  2006
1.4M       (5)  30090
5X
```

573.10

MULTIPLY 43 BY 43.

G	(1)	43
		TYPE H AND SPACE IF YOU WANT A HINT.
H		TYPE '1.1M', SPACE.
1.1M	(2)	1849
2X		

573.11

WHAT IS (68X68)X7...

G	(1)	68
G	(2)	7
1.1M	(3)	4624
2.3M	(4)	32368
4X		

573.12

THE GIRL SCOUTS SOLD 54 BOXES OF COOKIES.
EACH BOX HAD 12 COOKIES IN IT.
HOW MANY COOKIES DID THE GIRL SCOUTS SELL...

G	(1)	54
G	(2)	12
1.2M	(3)	648
3X		

573.13

EACH WEEK HAS 7 DAYS IN IT.
EACH DAY HAS 24 HOURS IN IT.
HOW MANY HOURS ARE THERE IN 6 WEEKS.

G	(1)	7
G	(2)	24
G	(3)	6
1.2M	(4)	168
3.4M	(5)	1008
5X		

573.14

```
FIND (214X122)+36.
G             (1)  214
G             (2)  122
G             (3)  36
1.2M          (4)  26108
3.4A          (5)  26144
5X
```

573.15

```
JOHN HAD 25 PACKS OF BASEBALL CARDS WITH 4 CARDS IN EACH PACK.
ROGER GAVE HIM 29 MORE BASEBALL CARDS.  HOW MANY BASEBALL CARDS
DID JOHN HAVE THEN...
G             (1)  25
G             (2)  4
G             (3)  29
                       TYPE H AND SPACE IF YOU WANT A HINT.
H                  THIS PROBLEM IS WORKED JUST LIKE 573.14.
                   FIRST USE M, THEN USE A.
1.2M          (4)  100
3.4A          (5)  129
5X
```

573.16

TO SEE HOW TO MULTIPLY BY A NUMBER THAT IS NOT GIVEN, STUDY THIS PROBLEM.

```
MULTIPLY 8 BY 12.
G             (1)  8
E             (2)  12
1.2M          (3)  96
3X
```

YOU MAY TELL THE COMPUTER YOU WANT TO ENTER A NUMBER BY TYPING...

A) 'X', SPACE.

B) 'E', SPACE.

B

573.17

AFTER TYPING 'E', SPACE, WAIT UNTIL THE MACHINE STOPS. THEN TYPE...
A) THE NUMBER YOU WANT TO ENTER, SPACE.
B) A LINE NUMBER, SPACE.

A

573.20

HOW MUCH IS 51X32...

G	(1)	51
		TYPE H AND SPACE IF YOU WANT A HINT.
H		TO ENTER A NUMBER THAT IS NOT GIVEN, TYPE 'E', SPACE.
E	(2)	32
1.2M	(3)	1632
3X		

573.21

HOW MANY DAYS ARE THERE IN 3 WEEKS...

G	(1)	3
E	(2)	7
1.2M	(3)	21
3X		

573.22

HOW MANY MONTHS ARE THERE IN 7 YEARS...

G	(1)	7
E	(2)	12
1.2M	(3)	84
3X		

573.23

PAT HAD TO WAIT 14 WEEKS AND 3 DAYS UNTIL HIS BIRTHDAY.
HOW MANY DAYS DID HE HAVE TO WAIT ALTOGETHER...

G	(1)	14
G	(2)	3
		TYPE H AND SPACE IF YOU WANT A HINT.
H		YOU MUST ENTER 7, THE NUMBER OF DAYS IN A WEEK.
E	(3)	7
1.3M	(4)	98
2.4A	(5)	101
3X		

574.1

THE RESULT OF DIVIDING ONE NUMBER BY ANOTHER IS CALLED THEIR...
A) SUM
B) DIFFERENCE
C) QUOTIENT

C

574.2

WHEN WE DIVIDE TWO NUMBERS, WE USE 'Q'
BECAUSE 'D' ALREADY MEANS SOMETHING ELSE.

TO DIVIDE ONE NUMBER BY ANOTHER, USE...
A) Q
B) DIV
C) D

A

574.3

DIVIDE 91 BY 13 BY TYPING '1.2Q', SPACE.

G	(1) 91
G	(2) 13
1.2Q	(3) 7
3X	

574.4

WHAT IS 1750 DIVIDED BY 50...

G	(1) 1750
G	(2) 50
1.2Q	(3) 35
3X	

574.5

DIVIDE 3750 BY 30.

G	(1) 3750
G	(2) 25
G	(3) 30
1.3Q	(4) 125
4X	

574.6

```
WATCH OUT HERE.
DIVIDE 75 INTO 2625.
G             (1)  75
G             (2)  2625
                        TYPE H AND SPACE IF YOU WANT A HINT.
H             WHEN YOU USE Q, PUT THE LINE NUMBER
              OF THE LARGER NUMBER FIRST.
2.1Q          (3)  35
3X
```

574.7

```
DICK'S FATHER DROVE 96 MILES IN 3 HOURS.
WHAT WAS HIS AVERAGE SPEED PER HOUR...
G             (1)  96
G             (2)  3
1.20          (3)  32
3X
```

574.10

```
JIM HAS 78 CENTS.
HOW MANY 6-CENT STAMPS CAN HE BUY...
G             (1)  78
G             (2)  6
1.2Q          (3)  13
3X
```

574.11

```
SUBTRACT 205 FROM 268.
THEN DIVIDE THE RESULT BY 7.
G             (1)  205
G             (2)  268
G             (3)  7
2.1S          (4)  63
4.3Q          (5)  9
6X                      THERE IS NO LINE 6.
5X
```

574.12

```
USE E.
DIVIDE THE NUMBER OF MONTHS IN 13 YEARS BY 6.
G            (1)  13
G            (2)  6
                      TYPE H AND SPACE IF YOU WANT A HINT.
H                 YOU MUST ENTER 12, THE NUMBER OF MONTHS IN A YEAR.
                  MULTIPLY 12 BY 13, DIVIDE THE RESULT BY 6.
E            (3)  12
1.3M         (4)  156
4.2Q         (5)  26
5X
```

574.13

```
FIND THE NUMBER OF WEEKS IN 77 DAYS.
G            (1)  77
E            (2)  7
1.2Q         (3)  11
3X
```

574.14

```
FIND 3844/4.
G            (1)  3844
E            (2)  4
1.2Q         (3)  961
3X
```

574.15

```
IN 13 DAYS, THE JONES FAMILY TRAVELED 4212 MILES ON A VACATION TRIP.
HOW MANY MILES A DAY DID THEY AVERAGE...
G            (1)  13
G            (2)  4212
2.1Q         (3)  324
3X
```

574.16

WATCH HOW WE SOLVE AVERAGE PROBLEMS.

WHAT IS THE AVERAGE OF 5 AND 15...

G	(1)	5
G	(2)	15
1.2A	(3)	20
E	(4)	2
3.4Q	(5)	10
5X		

WHICH OPERATION DID WE DO FIRST...
A) ADD
B) DIVIDE

A

574.17

IN 574.16 WE ENTERED THE NUMBER OF THINGS WE WANT TO AVERAGE.
IT WAS...
A) 10
B) 2

B

574.20

WHAT IS THE AVERAGE OF 23, 14, AND 8...

G	(1)	23
G	(2)	14
G	(3)	8
		TYPE H AND SPACE IF YOU WANT A HINT.
H		ADD THE THREE NUMBERS. THEN DIVIDE BY 3.
1.2A	(4)	37
3.4A	(5)	45
E	(6)	3
5.6Q	(7)	15
7X		

574.21

```
RUTH HAD 37 STAMPS, MARY HAD 58, AND JUDY HAD 31.
WHAT WAS THE AVERAGE NUMBER OF STAMPS AMONG THE GIRLS...
G             (1)  37
G             (2)  58
G             (3)  31
1.2A          (4)  95
3.4A          (5)  126
E             (6)  3
5.6Q          (7)  42
7X
```

574.22

```
FIND (10620/45)+33.
G             (1)  10620
G             (2)  45
G             (3)  33
                       TYPE H AND SPACE IF YOU WANT A HINT.
H                  REMEMBER TO DO WHAT IS IN
                   THE PARENTHESES FIRST.
1.2Q          (4)  236
3.4A          (5)  269
5X
```

574.23

```
WHAT IS (7X12)+(7X3)...
G             (1)  7
G             (2)  12
G             (3)  3
G             (4)  5
1.2M          (5)  84
1.3M          (6)  21
5.6A          (7)  105
7X
```

574.24

DO WHAT IS INSIDE THE INNER PARENTHESES FIRST.
FIND (34-((486/162)+6))X58.

G	(1)	34
G	(2)	486
G	(3)	162
G	(4)	6
G	(5)	58
2.3Q	(6)	3
1.6S	(7)	31
4.7A	(8)	37
4.6A	(9)	9
1.9S	(10)	25
5.10M	(11)	1450
11X		

575.1

TRY THESE WORT PROBLEMS.

HARROLD WENT FISHING. HE CAUGHT 112 FISH IN THE MORNING AND 127 IN THE AFTERNOON. HOW MANY FISH DID HE CATCH THAT DAY...

G	(1)	112
G	(2)	127
1.2A	(3)	239
3X		

575.2

HENRY SAW 291 WILD DUCKS ON THE LAKE. 120 OF THE DUCKS FLEW AWAY. HOW MANY DUCKS WERE LEFT...

G	(1)	291
G	(2)	120
1.2S	(3)	171
3X		

575.3

BILL HAS 16 BANANAS, 34 APPLES, AND 28 ORANGES. HOW MANY PIECES OF FRUIT DOES HE HAVE...

G	(1)	16
G	(2)	34
G	(3)	28
1.2A	(4)	50
3.4A	(5)	78
5X		

575.4

CHICKEN LITTLE LAYS 98 EGGS A DAY.
HOW MANY EGGS DOES SHE LAY IN 2 WEEKS...

G	(1)	98
G	(2)	2
E	(3)	7
2.3M	(4)	14
1.4M	(5)	1372
5X		

575.5

FOR THANKSGIVING THE BAKER MADE 3405 LITTLE PUMPKIN PIES.
HE DIVIDED THEM EQUALLY AMONG EACH OF HIS 5 CHILDREN.
HOW MANY PUMPKIN PIES DID EACH CHILD GET...

G	(1)	3405
G	(2)	5
1.2Q	(3)	681
3X		

575.6

ONE MARSMAN HAS 71 GOLLOPS, ANOTHER HAS 24 GOLLOPS, AND A THIRD HAS 35 GOLLOPS. WHEN THEY ARE ALL HERDED TOGETHER, 20 OF THE GOLLOPS ESCAPE. HOW MANY ARE LEFT...

G	(1)	71
G	(2)	24
G	(3)	35
G	(4)	20
1.2A	(5)	95
3.5A	(6)	130
6.4S	(7)	110
7X		

575.7

IN 1 DAY, FAT ALBERT EATS 4 STRAWBERRY, 3 VANILLA, 2 RASPBERRY, AND 8 CHOCOLATE ICE CREAM CONES. HE WILL GAIN WEIGHT IF HE EATS MORE THAN 20 ICE CREAM CONES A DAY. HOW MANY MORE MAY HE EAT THAT DAY WITHOUT BEING SICK...

G	(1)	1
G	(2)	4
G	(3)	3
G	(4)	2
G	(5)	8
G	(6)	20
2.3A	(7)	7
7.4A	(8)	9
8.5A	(9)	17
6.9S	(10)	3
10X		

575.10

27 CHILDREN GOT 13 PIECES OF CANDY EACH. GENEROUS GEORGIA GAVE AWAY 9 PIECES. HOW MANY PIECES DID SHE HAVE REMAINING...

G	(1)	27
G	(2)	13
G	(3)	9
1.2M	(4)	351
4.3S	(5)	342
5X		

APPENDIX B

Word Problem Set

1. How much larger is 456 than 402.

2. The number 301 is how much smaller than 586.

3. A bushel of corn weighs 56 lb. How much will 44 bushels weigh.

4. Dick's uncle weighed 161 pounds. Dick weighed 87 pounds. How much less did Dick weigh than his uncle.

5. A school superintendent transferred 275 pupils out of a school having an enrollment of 1080 pupils. How many pupils were left.

6. A trailer was loaded with 4 automobiles weighing 3185 pounds each. What was the total weight of the load.

7. Mr. Jackson drove 12,903 miles in 11 months. On an average, how many miles did he drive each month.

8. Miss Allen had 789 books in the library and added 350 books during the summer. How many books were then in the library.

9. Pike's Peak is 14,110 feet high. How high above the peak would an airplane fly, if it flew at an altitude of 30,000 feet.

10. A truck and its load of coal weighed 14,875 pounds. The empty truck weighed 5996 pounds. Find the weight of the coal.

11. The driver of a school bus drove the bus 38 miles each day. How many miles would he drive the bus in 9 days.

12. As an advertising stunt a new car was driven 2880 miles in 24 hours. What was its average speed per hour.

13. A driver estimated that he averaged 16 miles per gallon of gasoline. How many gallons of gasoline would he use in driving 10,000 miles.

14. In one city, there were 1737 pupils enrolled in elementary school and 713 pupils enrolled in high school. How many pupils were enrolled in the schools.

15. Ruth bought 200 Mexican stamps. She traded some of them for United States stamps. Then she had 163 Mexican stamps. How many Mexican stamps had she traded.

16. The Castle School has 6 new bicycle racks. Each rack holds 54 bicycles. There is room for how many bicycles in the 6 new racks.

17. Jerry counted 444 names listed on a page in the telephone book, and there were 55 pages in the book. How many names were listed in his telephone book.

18. A car traveled 391 miles in 1 day and used 23 gallons of gasoline. About how many miles did the car travel on 1 gallon of gasoline.

19. The estimated September school enrollment of a city was 12,404 pupils and there was to be an average of 28 pupils per classroom. How many classrooms were needed.

20. In March the number of bicycles produced in the United States was 4084 the first week and 1370 the second week. How many bicycles were produced during the 2 weeks.

21. At the equator the diameter of Mercury is 3100 miles and that of the Earth is 7927 miles. How much greater is the diameter of the Earth than that of Mercury.

22. At the tree nursery, Tom counted 28 rows of pine trees. The forester said that there were 575 trees in each row. How many trees were in the 28 rows of trees.

23. David's older brother built a ham radio station. He said that he spent about 60 hours working on the station and that he finished it in 5 weeks. On the average, he spent how many hours per week working on the station.

24. Mr. Andrews can drive to work in about 35 minutes less than the time it takes when he rides the bus. He can drive to work in about 45 minutes. How long does it take Mr. Andrews to get to work when he rides on the bus.

25. Mr. Phillips averaged 800 miles of driving a month. About how far did he drive in 3 years.

26. A school playground is rectangular, 273 feet long and 21 feet wide. What is the total length of the fence around the playground.

27. A football team gained 215 yards rushing, lost 12 yards passing, and lost 25 yards on penalties. What was their net gain in yards.

28. One day the girls gathered leaves for a science project. Ann found 23 different leaves, Susan found 31, and Marie found 29. How many leaves did the girls gather.

29. A dramatics club had 109 guests at its first play, 129 guests at its second play, and 135 guests at its third play. How many guests came altogether.

30. Some empty crates were stacked outside a shed. There were 13 rows of stacks, with 15 stacks in a row, and there were 9 crates in each stack. How many crates were there.

31. For an experiment, Susan mixed 7 ounces of glycerin and 12 ounces of alcohol with some water. The resulting mixture contained 45 ounces. How many ounces of water were used.

32. It is 713 miles by airplane from New York to Chicago, 1858 miles from Chicago to San Francisco, and 2407 miles from there to Honolulu. What is the distance between New York and Honolulu by this route.

33. Ranger VII transmitted 4304 pictures of the moon to Earth. Ranger VIII transmitted 7137 pictures of the moon and Ranger IX transmitted 5814 pictures of the moon back to Earth. How many pictures of the moon have these space probes sent back to Earth altogether.

34. Bob had 75 stamps. He gave 18 to Dan and 23 to John and 12 to Mike. How many stamps did Bob have left.

35. John scored 21 points in the first football game, 7 in the second, and 9 in the third. He was 12 points short of the school record. What was the school record for 3 games.

36. A homeowner paid 90 dollars for a specially made front door. The carpenter charged 18 dollars for installing the new door. Hardware cost 13 dollars, and the man who weather-stripped the door charged 24 dollars. What was the total cost of the door.

37. Don bought 15 dozen cookies. He ate 3 cookies. Then how many cookies were left in the box.

38. Janice is 14 years of age. Her brother is 5 years less than twice Janice's age. How old is her brother.

39. Mary is twice as old as Betty was 2 years ago. Mary is 40 years old. How old is Betty.

40. David read 2 books in 11 days. One book had 266 pages, and the other had 119 pages. David read an average of about how many pages per day.

41. Paul delivered 140 papers. Of these he delivered 61 on Poplar Street, 58 on Garfield Avenue, and the rest on York Road. How many did he deliver on York Road.

42. John and his father drove 387 miles in 9 hours. They had hoped to average 45 miles an hour. By how many miles did they miss their guess.

43. Mr. Ellis bought a car for 2768 dollars. He made a payment of 950 dollars and agreed to pay the rest in 18 payments. If there were no additional charges, how much would each payment be.

44. At one place in the warehouse there are 23 stacks of canned peaches in cases, with 16 cases in each stack. At another place there are 27 stacks with 16 cases of peaches in each stack. How many cases of canned peaches are there in all.

45. Steve has 13 toy soldiers, Tom has 18, and Richard has 41. What is the average number of toy soldiers.

46. An airplane flying at 14,000 feet climbs 3000 feet to avoid a storm. Then it drops 6000 feet and finally climbs 2000 feet. What is its final altitude.

47. There were 1500 textbooks to be stored on shelves. 48 of them were sent to the library. 28 shelves have been filled with 34 books on each shelf. How many books remain to be stored.

48. A ship sailed 746 miles. Then it sailed 9 days at 287 miles per day. The total distance planned for the voyage is 3765 miles. How

49. A football team had the ball on its own 15-yard line. On three successive plays the team made a gain of 7 yards, a loss of 12 yards, and then a gain of 21 yards. On what yard line was the football then.

50. John and his brothers weighed 74 pounds, 83 pounds, 69 pounds, and 70 pounds. What was their average weight.

51. June helped her mother with the housework for 50 min. on Monday, 35 min. on Tuesday, 40 min. on Thursday, and 55 min. on Saturday. She helped with the housework for how many hours that week.

52. There are 638 sixth-grade pupils, 395 fifth-grade pupils, and 205 fourth-grade pupils in the Crystal Lake schools. Last year each sixth-grade pupil wrote 4 book reports. Each fifth-grade pupil and each fourth-grade pupil wrote 3 book reports. What is the total number of book reports written by these pupils last year.

53. Mr. Taylor was making a 700-mile trip. Before lunch he drove for 5 hours at an average speed of 42 miles per hour. Before dinner he drove for 3 hours at an average speed of 39 miles per hour. How many more miles did he have to drive after dinner.

54. In 5 months, a dealer sold 165 tons, 206 tons, 210 tons, 274 tons, and 115 tons of coal. What was the average amount of coal sold each month.

55. The sixth graders sold 264 children's tickets at 1 dollar each, 53 student tickets at 2 dollars each, and 72 adult tickets at 3 dollars each. How much money did the sixth graders receive for all of the tickets.

56. Find the average of the following scores on a bicycle safety test: 12, 23, 15, 30, 8, 6, 18.

57. Mark has test grades of 79 and 86 on 2 tests. What score must he make on the next test in order to have an average of 85 on all 3 tests.

58. John and Alice were the only candidates for class president. Alice received 75 votes more than John. There were 519 votes cast. How many votes did Alice receive all together.

59. Bob had 1 gross, or 144, pencils to sell. He sold 5 dozen on Wednesday. On Thursday and Friday he sold all but 15 of the remaining pencils. How many pencils did he sell on Thursday and Friday together.

60. A square-shaped school playground is 87 feet on a side. One third of the playground is used for basketball courts. The rest is used for football fields. How much land is used for football. Give your answer in square feet.

61. Judy bought 3 pounds of steak at 98 cents a pound and 24 oranges at 45 cents a dozen. If 4 girls shared the cost, what did each girl pay.

62. In its first 3 games a football team scored 14 points, 35 points, and 20 points. Last year the team averaged 20 points per game. This year's average score is how much above last year's average.

63. Alice practiced on the piano for 20 min. on Monday, 35 min. on Tuesday, and 15 min. on Wednesday. Her sister Ruth practiced 40 min. a day on these 3 days. Ruth practiced how much longer than Alice.

64. Committee members bought 3 jars of candy with 14 ounces in each jar, and 2 boxes of candy with 27 ounces in each box. They put the candy into bags that contained 4 ounces each. How many bags of candy did they fill.

65. 3 classes of 32 pupils each, 1 class of 34 pupils, 4 teachers, and 7 parents took a trip on 3 buses. Each bus took the same number of riders. How many riders were on each bus.

66. A pump has a capacity for pumping water at the rate of 57 gallons per minute.

67. Mrs. Tulip's flower garden is 507 feet long and 39 feet wide. Every day she walks along the perimeter of the flower garden once. How far does Mrs. Tulip walk each day.

68. Gordon bought 140 guinea pigs, and his sister gave him 28 more. He gave away 19 of the guinea pigs. How many did he have left.

69. If a book has 95 names per column, and 5 columns per page, how many names does it have on 64 pages.

70. Mr. Dumpty bought a new home costing $28,000. He used $6500 which he had in the bank and $2876 from the sale of some land. The rest of the money he borrowed. It cost him $175 to have a lawyer make out the necessary papers to buy the house. What amount did he have to borrow in order to pay for everything.

71. The aviary at the zoo has 3584 birds. 224 birds eat at each outdoor feeding station. There are 7 indoor feeding stations where no birds eat. How many feeding stations does the aviary have altogether.

72. 124 Girl Scouts have 100 boxes of mint cookies and 260 boxes of chocolate chip cookies to sell. If each girl sells the same number of boxes of cookies, how many boxes will each girl have to sell.

73. The Jolly Green Giant weighed 16,000 pounds. He had hoped to weigh 95 times as much as a man weighing 200 pounds. What is the difference between how many times heavier he wanted to be and how many times heavier he actually was.

74. The sixth graders were decorating the gym for a party. Although they bought a 606 foot roll of crepe paper ribbon, they lost 30 feet of it. How many 24 foot lengths could they get from the remainder of the roll.

75. Yesterday Willy the whale drank 16 gallons of water in the morning and 17 gallons of water in the afternoon. Water weighs 8 pounds per gallon. How many pounds of water did Willy drink yesterday.

76. Mr. Daniels drove 295 miles on Monday, 330 miles on Tuesday, and 395 miles on Wednesday. On the average, how many miles did he drive each day.

77. In the beginning of June, Dr. Pill deposited 56 5-dollar bills and 27 10-dollar bills into the bank. At the end of June, he deposited 37 more 5-dollar bills. How much money did Dr. Pill deposit in the bank in June.

78. A boat and its contents weigh 17,000 pounds. Its contents consist of 130 gallons of gasoline and 12 gallons of oil. Gasoline weighs 6 pounds per gallon and oil weighs 7 pounds per gallon. What is the weight of the boat without its contents.

79. During one week Mr. Loaderman reported the number of pounds of freight loaded for each of five days as follows: 1294, 2010, 2413, 1999, and 1079. What was the average number of pounds loaded per day.

80. Mrs. Sewer had a new spool of ribbon. From it she cut 8 pieces each 4 yards long and 6 pieces each 3 yards long. She had 2 yards of ribbon left. How many yards of ribbon did the spool hold.

81. The Swiggle family went south for its vacation. Mr. Swiggle drove 35 miles the first day of the trip, 41 miles the second day, 59 the third day, 53 the fourth day, 39 the fifth day, 44 the sixth day, and 65 the seventh day. Find the average number of miles that Mr. Swiggle drove per day.

82. Sound travels at a speed of about 1100 feet per second. A blast from a whistle travels for 5 seconds. Then it travels another 1000 feet. How much further will it have to go to pass 9990 feet.

83. My uncle weighs 70 pounds more than John (computers also have relatives). John and Jim are identical twins who together weigh 4 pounds less than I. I weigh 130 pounds. How much does my uncle weigh.

84. Tom has 331 toy cars. George has twice as many as Tom. Bill has 28 fewer cars than George. John has 188 fewer cars than Bill. How many toy cars does John have.

85. At the beginning of the year there were 1620 people registered at the employment office. At the end of the year there were 6 times as many people registered, one third of whom were under 25 years old. How many people were over 25 years old.

86. There are 2 computers that teach children how to solve problems. One computer breaks down 4 times a day. The other one breaks down 12 times a week. What is the average number of breakdowns per computer in 21 days.

87. The Podunk Daily Post had an average daily circulation of 9305 papers the first week, 8000 papers the second week, and 10,127 papers the third week. The average Sunday circulation for those 3 weeks was 13,455 papers. By how much did the average Sunday circulation exceed the average daily circulation.

88. If a student's grades on the first 3 tests were 70, 80, and 95, and his average on the 4 tests was 75, what was his grade on the fourth test.

89. A fast snail averaged 35 inches per hour for 6 hours and 40 inches per hour for 4 hours. What was his average speed per hour on his 10-hour trip.

90. Every day the Greasy Spoon Restaurant makes 37 poached eggs, 46 fried eggs, 15 cheese omelettes with 3 eggs in each omelette, and 43 scrambled eggs. If 57 people eat in the Greasy Spoon, and each person eats the same number of eggs, how many eggs does each person eat.

91. Euclid wanted to measure the area of his back yard. Since it was L-shaped he could break it up into 2 rectangles--one measuring 42 feet by 13 feet, the other 13 feet by 23 feet. What was the total area.

92. The Hobby Club decided to have a party one Saturday night. There were 40 members in the club--24 boys and 16 girls. If each boy brought 4 guests and each girl brought 5 guests, how many guests came to the party.

93. Mr. Larsen used 396 pounds of apples to fill baskets with 44 pounds in each. He sold the baskets for 2 dollars each. How much did he receive for his apples.

94. If a man can bind 124 sets of books in 4 days and there are 17 books in a set, how many books does the man bind in a day.

95. Sam had enough books to pack 13 boxes with 28 books in each box. He had 5 books left over. How many books did he have in all.

96. An old pad of paper had 47 sheets of paper left in it. 21 new pads of paper each had 151 sheets of paper. How many sheets of paper were there in all the pads.

97. Someone has yelled at me once every day for the last 69 days, except for 5 days while I was gone on vacation. If my 9 aunts and 7 uncles each yelled at me an equal number of times, how many times did each person yell at me.

98. There were 154 popcorn balls on a table where 8 boys and 4 girls were seated. A dog came along and ate 10 of the popcorn balls. If the children shared the rest equally, how many popcorn balls were left for each person.

99. An empty airplane weighs 2600 pounds. Each pilot and passenger weighs 170 pounds. If the plane has 2 pilots and 10 passengers, what will be the total weight of the airplane, pilots, and passengers.

100. A milk can that weighed 18 pounds when empty was filled with 7 gallons of water in the morning, and 6 gallons were added in the afternoon. Water weighs 8 pounds per gallon. How much did the can full of water weigh.

(Continued from inside front cover)

96 R. C. Atkinson, J. W. Breisford, and R. M. Shiffrin. Multi-process models for memory with applications to a continuous presentation task. April 13, 1966. (J. math. Psychol., 1967, 4, 277-300).
97 P. Suppes and E. Crothers. Some remarks on stimulus-response theories of language learning. June 12, 1966.
98 R. Bjork. All-or-none subprocesses in the learning of complex sequences. (J. math. Psychol., 1968, 1, 182-195).
99 E. Gammon. The statistical determination of linguistic units. July 1, 1966.
100 P. Suppes, L. Hyman, and M. Jerman. Linear structural models for response and latency performance in arithmetic. (In J. P. Hill (ed.), Minnesota Symposia on Child Psychology. Minneapolis, Minn.: 1967. Pp. 160-200).
101 J. L. Young. Effects of intervals between reinforcements and test trials in paired-associate learning. August 1, 1966.
102 H. A. Wilson. An investigation of linguistic unit size in memory processes. August 3, 1966.
103 J. T. Townsend. Choice behavior in a cued-recognition task. August 8, 1966.
104 W. H. Batchelder. A mathematical analysis of multi-level verbal learning. August 9, 1966.
105 H. A. Taylor. The observing response in a cued psychophysical task. August 10, 1966.
106 R. A. Bjork. Learning and short-term retention of paired associates in relation to specific sequences of interpresentation intervals. August 11, 1966.
107 R. C. Atkinson and R. M. Shiffrin. Some Two-process models for memory. September 30, 1966.
108 P. Suppes and C. Ihrke. Accelerated program in elementary-school mathematics--the third year. January 30, 1967.
109 P. Suppes and I. Rosenthal-Hill. Concept formation by kindergarten children in a card-sorting task. February 27, 1967.
110 R. C. Atkinson and R. M. Shiffrin. Human memory: a proposed system and its control processes. March 21, 1967.
111 Theodore S. Rodgers. Linguistic considerations in the design of the Stanford computer-based curriculum in initial reading. June 1, 1967.
112 Jack M. Knutson. Spelling drills using a computer-assisted instructional system. June 30, 1967.
113 R. C. Atkinson. Instruction in initial reading under computer control: the Stanford Project. July 14, 1967.
114 J. W. Brelsford, Jr. and R. C. Atkinson. Recall of paired-associates as a function of overt and covert rehearsal procedures. July 21, 1967.
115 J. H. Stelzer. Some results concerning subjective probability structures with semiorders. August 1, 1967
116 D. E. Rumelhart. The effects of interpresentation intervals on performance in a continuous paired-associate task. August 11, 1967.
117 E. J. Fishman, L. Keller, and R. E. Atkinson. Massed vs. distributed practice in computerized spelling drills. August 18, 1967.
118 G. J. Groen. An investigation of some counting algorithms for simple addition problems. August 21, 1967.
119 H. A. Wilson and R. C. Atkinson. Computer-based instruction in initial reading: a progress report on the Stanford Project. August 25, 1967.
120 F. S. Roberts and P. Suppes. Some problems in the geometry of visual perception. August 31, 1967. (Synthese, 1967, 17, 173-201)
121 D. Jamison. Bayesian decisions under total and partial ignorance. D. Jamison and J. Kozielecki. Subjective probabilities under total uncertainty. September 4, 1967.
122 R. C. Atkinson. Computerized instruction and the learning process. September 15, 1967.
123 W. K. Estes. Outline of a theory of punishment. October 1, 1967.
124 T. S. Rodgers. Measuring vocabulary difficulty: An analysis of item variables in learning Russian-English and Japanese-English vocabulary parts. December 18, 1967.
125 W. K. Estes. Reinforcement in human learning. December 20, 1967.
126 G. L. Wolford, D. L. Wessel, W. K. Estes. Further evidence concerning scanning and sampling assumptions of visual detection models. January 31, 1968.
127 R. C. Atkinson and R. M. Shiffrin. Some speculations on storage and retrieval processes in long-term memory. February 2, 1968.
128 John Holmgren. Visual detection with imperfect recognition. March 29, 1968.
129 Lucille B. Mlodnosky. The Frostig and the Bender Gestalt as predictors of reading achievement. April 12, 1968.
130 P. Suppes. Some theoretical models for mathematics learning. April 15, 1968. (Journal of Research and Development in Education, 1967, 1, 5-22)
131 G. M. Olson. Learning and retention in a continuous recognition task. May 15, 1968.
132 Ruth Norene Hartley. An investigation of list types and cues to facilitate initial reading vocabulary acquisition. May 29, 1968.
133 P. Suppes. Stimulus-response theory of finite automata. June 19, 1968.
134 N. Moler and P. Suppes. Quantifier-free axioms for constructive plane geometry. June 20, 1968. (In J. C. H. Gerretsen and F. Oort (Eds.), Compositio Mathematica. Vol. 20. Groningen, The Netherlands: Wolters-Noordhoff, 1968. Pp. 143-152.)
135 W. K. Estes and D. P. Horst. Latency as a function of number of response alternatives in paired-associate learning. July 1, 1968.
136 M. Schlag-Rey and P. Suppes. High-order dimensions in concept identification. July 2, 1968. (Psychom. Sci., 1968, 11, 141-142)
137 R. M. Shiffrin. Search and retrieval processes in long-term memory. August 15, 1968.
138 R. D. Freund, G. R. Loftus, and R.C. Atkinson. Applications of multiprocess models for memory to continuous recognition tasks. December 18, 1968.
139 R. C. Atkinson. Information delay in human learning. December 18, 1968.
140 R. C. Atkinson, J. E. Holmgren, and J. F. Juola. Processing time as influenced by the number of elements in the visual display. March 14, 1969.
141 P. Suppes, E. F. Loftus, and M. Jerman. Problem-solving on a computer-based teletype. March 25, 1969.
142 P. Suppes and Mona Morningstar. Evaluation of three computer-assisted instruction programs. May 2, 1969.
143 P. Suppes. On the problems of using mathematics in the development of the social sciences. May 12, 1969.
144 Z. Domotor. Probabilistic relational structures and their applications. May 14, 1969.
145 R. C. Atkinson and T. D. Wickens. Human memory and the concept of reinforcement. May 20, 1969.
146 R. J. Titiev. Some model-theoretic results in measurement theory. May 22, 1969.
147 P. Suppes. Measurement: Problems of theory and application. June 12, 1969.
148 P. Suppes and C. Ihrke. Accelerated program in elementary-school mathematics--the fourth year. August 7, 1969.
149 D. Rundus and R.C. Atkinson. Rehearsal in free recall: A procedure for direct observation. August 12, 1969.
150 P. Suppes and S. Feldman. Young children's comprehension of logical connectives. October 15, 1969.

(Continued on back cover)

(Continued from inside back cover)

151 Joaquim H. Laubsch. An adaptive teaching system for optimal item allocation. November 14. 1969.
152 Roberta L. Klatzky and Richard C. Atkinson. Memory scans based on alternative test stimulus representations. November 25, 1969.
153 John E. Holmgren. Response latency as an indicant of information processing in visual search tasks. March 16, 1970.
154 Patrick Suppes. Probabilistic grammars for natural languages. May 15, 1970.
155 E. Gammon. A syntactical analysis of some first-grade readers. June 22, 1970.
156 Kenneth N. Wexler. An automaton analysis of the learning of a miniature system of Japanese. July 24, 1970.
157 R. C. Atkinson and J. A. Paulson. An approach to the psychology of instruction. August 14, 1970.
158 R.C. Atkinson, J.D. Fletcher, H.C. Chetin, and C. M. Stauffer. Instruction in initial reading under computer control: the Stanford project. August 13, 1970.
159 Dewey J. Rundus. An analysis of rehearsal processes in free recall. August 21, 1970.
160 R.L. Klatzky, J.F. Juola, and R.C. Atkinson. Test stimulus representation and experimental context effects in memory scanning.
161 William A. Rottmayer. A formal theory of perception. November 13, 1970.
162 Elizabeth Jane Fishman Loftus. An analysis of the structural variables that determine problem-solving difficulty on a computer-based teletype. December 18, 1970.