DOCUMENT RESUME

ED 047 115 VT 012 435

AUTHOR	McNamara, James F.
TITLE	A Mathematical Programming Model for the Efficient Allocation of Vocational Technical Education Funds.
INSTITUTION	Pennsylvania State Dept. of Education, Harrisburg. Bureau of Educational Research.
PUB DATE	70
NOTE	92p.
EDRS PRICE	EDRS Price MF-$0.65 HC-$3.29
DESCRIPTORS	Administration, *Decision Making, Educational Economics, Information Needs, *Mathematical Models, Program Evaluation, *Resource Allocations, *School Funds, *Vocational Education

ABSTRACT

This mathematical programing model was developed to provide the State Board of Education with complete information for evaluating decisions about the efficient allocation of vocational education funds to local school districts. The model, based on a supply-demand criterion, was tested on a set of occupational training programs within a given Labor Market Area of Pennsylvania. It was thus demonstrated that the model provides a generalizable procedure that can be applied to all labor markets in the State. The study integrates theories and concepts from the fields of vocational education, operations research, educational administration, and mathematical economics. The generalizability of the model is also enhanced by the fact that the administration of vocational education programs in every state must follow the regulations set forth in the guidelines provided by the U.S. Office of Education. This report is based on a doctoral dissertation. (Author)

C1

A Mathematical Programming Model

For the Efficient Allocation of Vocational Technical Education Funds

A Mathematical Programming Model

For the Efficient Allocation of Vocational Technical Education Funds

by James F. McNamara
Research Associate
Bureau of Educational Research
Pennsylvania Department of Education
1970

Commonwealth of Pennsylvania
Raymond P. Shafer, Governor

Department of Education
David H. Kurtzman , Secretary
Neal V. Musmanno, Deputy Secretary

Office of Educational Research and Statistics
Paul B. Campbell, Director

Bureau of Educational Research
Robert B. Hayes, Director

Pennsylvania Department of Education
Box 911
Harrisburg, Pa. 17126

Table of Contents

Table of Contents (Continued)

List of Tables

Preface

The past few years have witnessed a rapid development in the field of management science. The successful application of management science models in the fields of business management, military operations research, traffic engineering and regional planning has encouraged educational planners to test the utility of such models within the educational environment. This study represents one modest attempt to contribute to this emerging literature on the development of mathematical planning models in the educational field. In this case, a linear programming model was developed to provide a State Department of Education with an improved knowledge base to evaluate alternative strategies for the allocation of vocational education funds to local school districts.

This study represents an extension of the recent developments in the Pennsylvania state-wide comprehensive study of vocational education. The Pennsylvania Research Coordinating Unit wishes to acknowledge the assistance provided by Dr. Donald J. Willower, Professor of Education, and Dr. David A. Walker, Associate Professor of Quantitative Business Analysis at the Pennsylvania State University.

This study provides an excellent example of how the resources of a major university might be utilized to approach numerous and complex problems which confront educational decision-makers within State Departments of Education. The approach of this study is interdisciplinary in nature and exhibits the integration of theories and concepts from the fields of vocational education, operations research, educational administration and mathematical economics.

Since the model in this study was developed with the intention of providing a generalizable procedure that could be used by any State Department of Education, the Pennsylvania Research Coordinating Unit is disseminating this monograph to a selected set of educational research and development personnel. I hope that the publication will prove to be of some value for other states faced with similar problems in developing alternative strategies and policies for the allocation of vocational education resources.

Jay Smink, Director
Research Coordinating Unit
for Vocational Education

May 20, 1970

Chapter One
Introduction

The reality that education is the second largest single contributor to the Gross National Product in this country has been emphasized in recent literature. The expenditures of the Vocational Education Act of 1963 represent a portion of this contribution. For example, Benson shows that the 1967 expenditures for education administered by this act exceeded 225 million dollars.[1] In Pennsylvania alone, Arnold calculates that in the same year this act provided the state with more than 13.8 million dollars for the development and maintenance of vocational education programs.[2] Public expenditures of this magnitude place an obligation on State Departments of Education to administer funds according to state plans which are based on sound financial management, a knowledge of the economics of demand and supply, and an understanding of the organizational structure of its educational institutions. These elements taken collectively form a basis for vocational education program planning at the state level which is the concern of this investigation.

1.1 An Overview of the Study

In this investigation an educational planning model was developed to provide a State Department of Education with a set of guidelines for the efficient allocation of vocational funds to public school systems. Following its construction, the model's utility was illustrated by applying it to Pennsylvania data. It should be noted that, although the Pennsylvania Department of Education does have a state plan for administering these funds, this plan did not utilize quantitative economic theory, nor did the plan give sufficient consideration to the relationships between public schools and other educational institutions within a labor market. The development of the model was based on both of the above considerations.

In the investigation, all testing of the model was based on a set of occupational training programs within a given Labor Market Area (LMA) of

[1] Charles S. Benson, *The Economics of Public Education*, Second Edition. (Boston: Houghton Mifflin, 1968), p. 214.

[2] Walter M. Arnold (ed), *Vocational, Technical and Continuing Education in Pennsylvania: A Systems Approach to State-Local Program Planning*. (Harrisburg: Pennsylvania Department of Education, 1969), p. 6.

Pennsylvania.[3] It was demonstrated that the model provides a generalizable procedure that can be applied to all labor markets in the state. In fact, since labor markets are federally defined subdivisions and since schools across the country exhibit similar organizational characteristics, this model should provide a procedure with some level of application in any state. The generalizability of the model is also enhanced by the fact that the administration of vocational education programs in every state must follow the regulations set forth in the guidelines provided by the U. S. Office of Education.[4]

1.2 Educational Planning

Since the prime objective of this investigation is the construction of an educational planning model, it seems appropriate that a section on developments in educational planning should be included in the introductory chapter. The discussion focuses on recent developments in the literature and is divided into three parts. These are the necessity and scope of educational planning, present planning methods, and models and mathematics in educational planning.

THE NECESSITY AND SCOPE OF EDUCATIONAL PLANNING

The necessity for additional knowledge about educational planning and its importance to economists and government personnel as well as educational administrators is evidenced in the articles of the 1967 World Yearbook of Education.[5] This publication is devoted entirely to the topic of educational planning. Other recent educational writers such as Hartley,[6] Davis,[7] and Banghart[8] concur on the necessity for additional knowledge about educational planning. Hartley advances the notion that general systems theory could provide the conceptual framework for studying the re-

[3] For a description and definition of LMA's see U. S. Department of Labor, *Handbook Defining Labor Market Areas*, Employment Security Research Methods, BES No. — R 186, (Washington, D. C.: Bureau of Employment Security, 1960).

[4] For example see the proposed regulations for use by State Boards for Vocational Education in *Vocational Education Act of 1963 as Amended by Title One of the Vocational Education Amendments of 1968—Regulations for State Plan Programs*, (Washington, D. C.: Bureau of Adult, Vocational and Library Programs, U. S. Office of Education; Department of Health, Education, and Welfare; April, 1969).

[5] George Z. Bereday and Joseph A. Lauwerys (eds.), *The World Yearbook of Education 1967: Educational Planning*, (London: Evans Brothers Limited, 1967).

[6] Harry Hartley, *Educational Planning-Programming-Budgeting: A Systems Approach*, (Englewood Cliffs, New Jersey: Prentice-Hall, 1968).

[7] Russell G. Davis, *Planning for Human Resource Development: Educational Models and Schemata*, (Chicago: Rand McNally, 1966).

[8] Frank W. Banghart, *Educational Systems Analysis*, (Toronto: Collier-MacMillan, Canada, 1969).

lationships among educational variables and a program-planning-budgeting system (PPBS). Banghart recommends new and more extensive use of the computer to assist in the decision-making process, while Davis points out the need for using the economically based human resources approach in educational planning.

While the necessity for additional knowledge about educational planning is widely accepted among scholars and administrators, the same level of agreement does not hold for its definition. Coombs states:

> There does not exist as yet any simple and generally agreed definition of educational planning. This is perhaps a good thing, for it is still too early in the career of this young subject to stunt its growth by verbal constraints. Nevertheless, there does seem to be a rapidly emerging consensus of opinion concerning the general character, desirable dimensions, and primary functions of educational planning as applied to the kind of situation in which most nations find themselves today.[9]

Parnes notes:

> Where general economic development plans exist, it is clear that educational planning must be related to the overall production targets established by the economic plan. But even in the absence of economic planning, education is in all countries primarily a public responsibility and decisions with respect to the amount and nature of educational expenditures are continuously being taken by public authorities, presumably in terms of some conception of the social goals that are to be served. In a sense, therefore, there is educational planning under any circumstances, and the only question is how rational or scientific it is to be.[10]

Finally, Anderson and Bowman define educational planning as "the process of preparing a set of decisions for future action pertaining to education."[11]

The need for integration of educational and economic planning to provide patterns for efficient allocation of scarce resources is, however, one idea that does not suffer from a lack of agreement among educational planners. This agreement is noted by Chirikos and Wheeler who state:

> . . . recent years have witnessed an increasing emphasis on the need to design educational policy in relation to an overall set of objectives for economic and social development. This stems

---

[9] Philip M. Coombs. "What Do We Still Need to Know about Educational Planning?," in Bereday and Lauwerys, *op. cit.*, p. 58.

[10] Herbert S. Parnes, *Forecasting Educational Needs for Economic and Social Development*, (Paris: The Organization for Economic Cooperation and Development, 1962), p. 9.

[11] C. Arnold Anderson and Mary Jean Bowman, "Theoretical Considerations in Educational Planning," in Don Adams (ed.), *Educational Planning*, (Syracuse: Syracuse University, School of Education, 1964), p. 9.

principally from research carried out during the past decade on formation of human capital and sources of economic growth.[12]

Hence, a comprehensive plan for the allocation of resources for vocational-technical education programs should not be developed independently of economic demands.

PRESENT EDUCATIONAL PLANNING METHODS

While the need for the utilization of economic theory and method in educational planning is well documented in the literature, agreement on its most efficient and proper use in relation to educational organizations is not so clearly defined.[13]

In terms of application, however, one general group of methods can plan an important role in developing resource allocation models designed specifically for vocational-technical education. This group is generally referred to as the manpower-requirements approach. The widespread use of this approach started to accelerate in the early part of the 1960's.[14] A review of the literature in the manpower-requirements approach and its utility as a tool for educational planning can be found in the following chapter.

MODELS AND MATHEMATICS IN EDUCATIONAL PLANNING

During the past few years there has been a rapid development of the field called *management science* (not to be confused with the older tradition of "scientific management" in the sense of time and motion studies).[15] Management science models, which express the organizational environment

---

[12] Thomas N. Chirikos and A. C. Wheeler, "Concepts and Techniques of Educational Planning," "*Review of Educational Research*, 38: 265; June, 1968. For a discussion of the relationship between educational planning and economic growth see also the extensive bibliography provided by Mark Blaug, *Economics of Education: A Selected Annotated Bibliography*, (Oxford: Pergamon Press, 1965).

[13] A general review of a number of planning methods can be found in William G. Bowen, *Economic Aspects of Education*, Princeton: Princeton University Press, 1964); Mark Blaug, "Approaches to Educational Planning," *Economics Journal*, 77: 262-287 and Chirikos and Wheeler. op. cit., pp. 264-276.

[14] Almost every recent bibliography on the economics education, including the 1967 and 1968 issues of the *Review of Educational Research*, point to two major works as representative of the manpower-requirements approach. These are Parnes op. cit. and The Organization for Economic Cooperation and Development, *Econometric Models of Education: Some Applications*, (Paris: OECD, 1965).

[15] The discussion on management science in this section is based on Karl A. Fox, Francis P. McCamley and Yakir Plessner. *Formulation of Management Science Models for Selected Problems of College Administration*, U. S. Department of Health, Education, and Welfare; Office of Education, Final Report Contract No. OEC-3-6-068058, (Ames, Iowa: Iowa State University, 1967).

and its dynamics in mathematical relationships, have successfully been applied in the fields of business management, military operations research, traffic engineering and regional planning. Although the utility of such models has been proven in the areas listed above, Fox and his colleagues observe that academic administrators are making *little* use of these modern scientific approaches. He mentions that not all aspects of academic life are or should be subject to quantification; however, he claims that many decisions are made by administrators which affect in *quantitative* ways the "input" and "output" of the educational process. In this work, Fox and his associates build and demonstrate the efficiency of three management science models which can be applied by educational planners to the problems of resource allocation in a university.

Correa, reviewing the current status of mathematical models in educational planning, offers one explanation for the lack of adoption by educational planners. He notes:

> . . . models must be and have always been used in educational science and planning, but that mathematical models are an innovation. Probably many educationists and educational planners will find this statement unacceptable. The only reason for this is that most model-builders have been busy constructing models instead of explaining what models are.[16]

Hence, it seems instructive to describe what models are. Hagget states:

> In everyday language the term model has at least three different usages. As a noun, model implies a representation; as an adjective, model implies ideal; as a verb, to model means to demonstrate. We are aware that when we refer to a model railway or a model husband we use the term in different senses. . . . In model building we create an idealized representation of reality in order to demonstrate certain of its properties.[17]

It should be mentioned that models and mathematics are not synonymous terms.[18] Educational models without mathematics are often utilized in scientific analysis of behavior in education.[19]

---

[16] Hector Correa, "Models and Mathematics in Educational Planning." In Bereday and Laufwerys (eds.), *op. cit.*, p. 398.

[17] Peter Hagget, *Location Analysis in Human Geography*, (London: Edward Arnold Publishers, 1965), p. 17.

[18] Russell Ackoff describes three types of models (iconic, analogue and symbolic) in *Scientific Method: Optimizing Applied Research Decisions*, (New York: John Wiley and Sons, 1962). Richard Chorley carries this classification process further and creates a 'model of models' in "Geography and Analogue Theory," *Annuals of the Association of American Geographers*, *54*: 127-137; 1964. He classifies models as *mathematical*, *experimental* or *natural*. *Mathematical* models are further classified as stochastic or deterministic.

[19] See the treatment of models of organizations in Willard R. Lane, et al., *Foundations of Educational Administration: A Behavioral Analysis*, (New York: Macmillan Co., 1966), pp. 39-46 and the literature cited therein.

Up to this point, the treatment of models and mathematics in educational planning focuses on three points. These are: (1) management science models (mathematical models) are innovative and are not widely utilized in academic administration, (2) an explanation of the concept of a model, and (3) the distinctions between models in general and a specific subset of models; namely, mathematical models. In the concluding paragraphs of this section, a selected set of recent developments in mathematical planning models is presented. This should demonstrate some of the capabilities and also the range of applicability of mathematical models to educational planning. A more comprehensive treatment of planning models that are related directly to this study is deferred until Chapter Two.

Correa provides educational planners and administrators with a mathematical model to help the decision-maker formulate a set of parameters for the distribution of resources between general and vocational education.[20] Cohn uses a set of mathematical models to develop economies of scale for an Iowa sample of 377 high school districts.[21] Bruno uses a linear programming model to determine a school district salary schedule which can meet specific teacher union and school board demands.[22] Stankard and Sisson developed a model to simulate the operation of a large urban school district.[23] In this case, one result of the simulation is to portray various conditions such as staff salaries, space per student, and students per staff member and, then, to estimate the financial requirements and operating statistics for each condition. Projections could then be made of the cost of operating the schools under various sets of policies.

For macroeconomic educational planning, several mathematical models have been published by the Organization for Economic Cooperation and Development (OECD).[24] These models are almost exclusively of the manpower requirement variety and have been designed to operate at a national level. Since education in this country is a state function, similar models are not commonly developed on a national level.

---

[20] Hector Correa, "Optimum Choice Between General and Vocational Education," *Kyklos*, 18: 107-113; 1965.

[21] Elchanan Cohn, "Economics of Scale in Iowa High School Operations," *The Journal of Human Resources*, 3: 423-429; Fall, 1968.

[22] James E. Bruno, "Using Linear Programming Salary Evaluation Models in Collective Bargaining Negotiations with Teacher Unions," *Socio-Economic Planning Science*, 3: 103-117; 1969.

[23] Martin Stankard and Roger L. Sisson, *Operations Research and Improved Planning for an Urban School District: Definitions and a Simulation Model*, (Philadelphia: Management Science Center, University of Pennsylvania, Technical Memo #11667, 1967).

[24] These OECD publications are all outlined in Chirikos and Wheeler, *op. cit.*, pp. 264-276.

The types of mathematical models currently used in this country for educational planning are briefly discussed in Kraft.[25] A survey of econometric models applicable to the educational system can also be found in Fox.[26] The mathematical planning models described above offer the reader some insight into the wide range of applications of these models in educational planning.

1.3 The Need for the Study

In this section the discussion on the need for the study will be extended. It will be demonstrated that the research is timely and also that the results of the study can make a contribution to the existing literature on vocational-technical education program planning at the state level. The section is divided into four parts which focus on different aspects of the problem. These topics are: (1) the emerging role of State Departments of Education, (2) the future role of Divisions of Vocational-Technical Education, (3) long-range state plans, and (4) the survey of the states.

THE EMERGING ROLE OF STATE DEPARTMENTS OF EDUCATION

The structure of the United States educational system can be briefly summarized as a federal-state-local partnership. In an effort to gain a new perspective on how the State Department of Education (SDE) might strengthen its contribution to this relationship, investigators at the University of Chicago have conducted research on a number of SDE's and have presented their findings in a seminar held during the summer of 1966. These results are summarized by Campbell who outlines a model for the emerging role of the SDE.[27] In general, the model suggests that SDE's should have less preoccupation with regulations and more concern with leadership. Specifically, these leadership functions should include services which are relevant to the needs of local school systems and should be based on systems management rather than merely reflecting experiences in school administration.

Roe, depicting an SDE operating in 1980, claims that their services will include application of up-to-date and tested business management techniques and suggests that SDE personnel will program computers so that contempo-

---

[25] Richard H. Kraft (ed.), *Strategies of Educational Planning: Proceedings of the Second Annual Conference on the Economics of Education*, (Tallahassee: Educational Systems Development Center, The Florida State University, 1969).

[26] Karl A. Fox and J. K. Sengupta, "The Specification of Econometric Models for Planning Educational Systems: An Appraisal of Alternative Approaches," *Kyklos*, 21: 665-693; 1968.

[27] Roald F. Campbell, "The Emerging Role," in Roald F. Campbell, Gerald E. Sroufe and Donald M. Layton (eds.), *Strengthening State Departments of Education*, (Chicago: The Midwest Administration Center, The University of Chicago, 1967), pp. 76-92.

rary information dealing with education requirements will be readily available.[28] Rackley and Carroll assert that:

> . . . while the educational planning is growing more complex and increasingly involves such other governmental agencies as health, welfare, the United States Office of Education, etc., it is the responsibility of the *state education agency* to coordinate the educational effort within a state.[29]

Culbertson sees the SDE of the future:

> . . . serving as an interpreter of quantitative data on education and of important state and national studies which bear upon and have implications for educational planning.[30]

Clearly, if the SDE's of the future are to adapt to the changing role they must play in the federal-state-local partnership of the future, the development of management science models built to reflect the properties of the educational system must become an integral part of their planning procedures.

THE FUTURE ROLE OF DIVISIONS OF VOCATIONAL-TECHNICAL EDUCATION

A more immediate concern of this study is the future activities of a specific subdivision of SDE, the Division (or Bureau) of Vocational-Technical Education (DVTE). The future leadership role that DVTE can play in the federal-state-local partnership of the education system is clearly outlined in a recent publication by The Ohio State University Center for Vocational and Technical Education.[31] This report contains nine background papers on major forces and factors relevant to SDE operations and three papers prepared by individuals responsible for synthesizing the various viewpoints and drawing major implications for the emerging role of the DVTE.

Rice translates the inputs of these nine papers into specific functions that DVTE's can perform.[32] He concludes: (1) DVTE's should be concerned with developing data systems for educational planning purposes and these data

[28] William H. Roe, "The State Education Agency of the Future," In Edgar L. Morphet and Charles O. Ryan (eds.), *Designing Education for the Future: Planning and Effecting Needed Changes in Education*, (New York: Citation Press, 1969), pp. 246-260.

[29] J. Ralph Rackley and Donald M. Carroll, "Supplementary Statements," in Morphet and Ryan, *op. cit.*, p. 290.

[30] Jack Culbertson, "State Planning for Education," in Morphet and Ryan, *op. cit.*, p. 281.

[31] Dick C. Rice and Powell E. Toth (eds.), *The Emerging Role of State Education Departments with Specific Implications for Divisions of Vocational-Technical Education*, (Columbus: The Center for Vocational-Technical Education, The Ohio State University, 1971).

[32] Dick C. Rice, "State Education Departments and Vocational-Technical Education," In Rice and Toth, *op. cit.*, pp. 353-374.

should include information such as manpower needs, cost of training programs for different occupations, and follow-up statistics on graduates, (2) the DVTE's should forge stronger cooperative ties with other local, state, federal and private agencies concerned with meeting the increasing need for vocational education, and (3) DVTE's should sponsor in-depth studies to determine the existing real situation and to develop effective strategies for influencing the state legislative process.

The Stanford Research Institute, commissioned in 1967 by the USOE, examines the existing planning and allocation process for vocational-technical education in six states and eleven communities. The position papers and the results of this project are published and edited by Kotz.[83] The recommendations that follow include those summarized by Rice as well as others relating specifically to the economic aspects of vocational-technical education planning. Kotz notes:

> (1) Programs funded by the Vocational Education Act of 1963 should be required to meet tests of economic efficiency. Analytical studies should be conducted of alternative ways of achieving objectives and goals, using benefit/cost and systems analysis techniques.
>
> (2) The basic concept of and new approaches to decision theory and PPB (planning, programming and budgeting) on a systematic basis should be adopted and installed by all governmental jurisdictions having major responsibilities for the allocation of resources for occupational education.
>
> (3) Goals should be stated in quantitative terms and represent the final purposes of the occupational education process. Illustrative of goals stated in such terms could be the number and percent of college-bound students who should graduate from secondary schools with academic diplomas Output goals could also be stated in such quantitative terms as the number and percent of fully qualified students to be graduated from specific occupational course sequences and placed in jobs. Progress toward attaining these goals should be measured.
>
> (4) Major objectives should be clearly identified and priorities for them should be established as a guide to program development and allocation of resources at the state and community levels.[84]

It should be noted that Kotz aims these recommendations at vocational-

[83] Arnold Kotz (ed.), *Occupational Education: Planning and Programming*, Volumes One and Two, (Menlo Park, California: Stanford Research Institute, SRI Project No. MU-6157, 1967).

[84] *Ibid.*, pp. 7-15. Additional discussions about the allocation of resources in vocational-technical education planning can be found in J. Robert Warbrod, *Review and Synthesis of Research on the Economics of Vocational Education*, (Columbus: The Center for Vocational and Technical Education, The Ohio State University, 1967).

technical education planners in federal and local levels as well as to DVTE personnel.

Lee has recently completed a study of the role, organization and administration of vocational education at the state level in 41 states.[35] Based on the results of this study and a review of recent research in vocational education, Lee and Hamlin identify certain shortcomings of vocational education planning research. They conclude that "the policy and policy making arrangements responsible for introducing change into the system" need far more attention in future research efforts.[36]

At this point the implications of the research for the future role of the DVTE are clear. DVTE's must: (1) provide vocational-technical education planners at the local level with accurate information on manpower needs, (2) develop and apply decision theory models to the problem of resource allocation at the state level, (3) communicate vocational-technical education needs to the appropriate government decision-making agencies, and (4) cooperate with the private education sector and other governmental agencies to develop a systems approach for labor demand and supply relationships within the state.

LONG-RANGE STATE PLANS

Based on recent developments in vocational-technical education at the state level, it can be shown that this investigation is both timely and extremely useful for assisting SDE's to develop their long-range plans. For example, on the local level, Russo shows that enrollments in vocational programs increased from 349,000 in 1964 to an estimated 8.2 million in 1968 and will continue to rise to an estimated enrollment of 14 million by 1975.[37] If SDE's assemble accurate trends on future enrollments, they can begin now to consider alternative policies for planning their occupational programs of the seventies. In this regard, simulation can be used to determine the relative benefits of various alternative policies on future output of the state system.

At the national level, the USOE now requires State Boards of Education to submit state plans for vocational-technical education programs.[38] States are required to project, by various occupational categories, labor market

---

[35] Allen Lee, "Study of Perceptions of State Level Administration," *A Nationwide Study of the Administration of Vocational-Technical Education at the State Level*, Volume 4, (Washington, D. C.: U. S. Government Printing Office, 1967).

[36] Allen Lee and Herbert M. Hamlin, "Organization and Administration," *Review of Educational Research*, 36: 359-402; October, 1968.

[37] Michael Russo, "14 Million Vocational Students by 1975," *American Education*, 5: 10-11; March, 1969.

[38] See the discussion in footnote 4 in this chapter.

supply and demand for the current year and for a target year five years following. Models, developed for investigating long-term funding policies of SDE's, can now use standardized data inputs. Hence, a model such as the one developed in this investigation could be used by an SDE to determine the relative merits of alternative policy decisions.

A status report on current mathematical planning models used by SDE's is deferred to the next paragraph. At this point, however, it can be mentioned that few SDE's are applying these models to their long-range projections. In 1968, Benson attributed this lack of application to the fact that analytic tools of planning such as linear programming and simulation techniques are considerably better than the educational data to which they can be applied.[39] With the new USOE mandate requiring supply-demand projections for long-term planning, lack of data can no longer be used as an argument for not utilizing mathematical planning models to assist in establishing future resource allocation policies.

THE SURVEY OF THE STATES

A survey of all 50 SDE's, conducted by the author in September, 1969, shows that 46 states do not currently use a model similar to the one utilized in this investigation nor are they presently developing one. Colorado and Utah indicate that they have started to develop a model but due to staff limitations and other work priorities have postponed their projects. Oklahoma intends to do some research on the value of using linear programming techniques to assist with vocational-technical education funding policies. Minnesota is developing a planning form for local school districts that will provide the state in the near future with the necessary data to use simulation techniques for long-range planning in the state vocational education system.

The results of this survey add additional evidence for the potential capability of the proposed model. Since (1) the literature on the future leadership role prescribed for the SDE's shows that their planning agencies should utilize mathematical planning models to assist in determining their resource allocations and (2) the survey shows that these models are not currently used or yet developed, the need for an appropriate model which has been

[39] Benson, op. cit., p. 68. See also a similar discussion by Benson on application of econometric types of educational planning models in "Economics and Education," *Review of Educational Research*, 37: 1, 96-102; February, 1967. Lochene attributed this gap between developed management science models and their educational application not only to educational data limitation but also to the nature of current educational planning. He noted that the mathematical instruments presently available are more elaborate than the concepts being used by educational planners. See Rene Lochene, "The Application of Operational Research to Educational Planning," in Organization for Economic Cooperation and Development, *Efficiency in Resource Utilization in Education*, (Paris: OECD, 1969), p. 33.

empirically tested is apparent. Based on the discussion above, the model developed in this investigation can make a significant contribution to the existing literature on the financing of vocational-technical education program planning at the state level.

1.4 Statement of the Problem

The remaining portion of this chapter extends the discussion in Section 1.1 on the nature and scope of the present study. This study, dealing with the problem of the efficient resource allocation of vocational-technical education funds to local school districts, is based on concepts from economic theory which Hartley notes can:

> . . . provide a broad framework within which the desired educational objectives can be expressed and accomplished in the most reasonable and efficient manner.[40]

Recognition of the important contribution of education to economic growth has heightened the interest of economists and educational planners in the development of an economically rational basis for the allocation of resource in the educational sector. They have attempted to develop methods of resource allocation and enrollments within the educational system. For example, Bowles constructs a model for the efficient allocation of resources which views the educational system as "an aggregation of production activities" where "each of these processes used a variety of inputs (both human and otherwise) to transform raw materials (the uneducated) into a producer's good."[41] This view of an educational system as a set of input-output or production relationships, which can be controlled in a way that will optimize the use of scarce educational resources, can also be found in other recent economic investigations of the educational system.[42] It also provides a basis for this investigation.

Following the example set forth by Bowles and others, the State Board of Education can adapt this input-output relationship to the state educational system. Using this economic conceptualization, the Board could formulate the following question about its allocation of vocational-technical education funds:

---

[40] Hartley, *op. cit.*, p. 15.

[41] Samuel S. Bowles, "The Efficient Allocation of Resources in Education," *Quarterly Journal of Economics*, 81: 191; May, 1967.

[42] For example, see Joseph A. Kershaw, "Productivity in Schools and Colleges," in Seymor E. Harris and Alan Levensohn (eds.), *Education and Public Policy*, (Berkeley, California: McCutchan Publishing Co., 1965), pp. 185-191; Jesse Burkhead, Thomas G. Fox and John W. Holland, *Input and Output in Large-City High Schools*, (Syracuse: Syracuse University Press, 1967); H. Thomas James, J. Alan Thomas and Harold T. Dyck, *Wealth, Expenditures and Decision Making for Education*, Office of Education, Final Report, Cooperative Research Project No. 1241, (Stanford, California, Stanford University, School of Education, 1963) and Richard Stone, "A Model of the Educational System," *Minerva*, 3: 172-186; Winter, 1965.

> How can the SDE most efficiently allocate a fixed level of vocational-technical education funds (input) to the public schools so that the output of graduates from these vocational programs at the local level makes the most significant contribution toward reducing the existing demands of the labor market?

The question above provides the statement of the problem for this study.

The purpose of the mathematical planning model developed in this study is to provide the decision-maker, the State Board of Education, with new knowledge to evaluate decisions about the efficient allocation of resources. The model is based on the supply-demand criterion outlined in the question above.

Simon shows the relationship of new knowledge about the future performance of a system and its relationship to the decision-making process in the following statement:

> The function of knowledge in the decision-making process is to determine which consequencies follow upon which of the alternative strategies. It is the task of knowledge to select from the whole class of possible consequences a more limited subclass, or even (ideally) a single set of consequences correlated with each strategy. The behaving subject cannot, of course, know directly the consequences causality would be operating here—future consequences would be determinants of present behavior. What he does is to form expectations of future consequences, these expectations being based upon known empirical relationships, and upon information about the existing situation.[13]

Hence, following the selection of any specific resource allocation policy, the mathematical planning model can provide the decision-maker with new knowledge about the future output of graduates generated as a result of that policy. Further, for a fixed level of input (funds allocated for programs), the model can be employed to calculate an optimal solution in terms of the output of graduates in the system. In this manner, the decision-maker can clearly view the state level system as a set of input-output relationships.

At this juncture, it should be mentioned that the mathematical planning model does not make decisions nor can it replace judgment on the part of decision-makers. Rather, the model is designed to aid and support decision-makers by providing pertinent data on alternative programs and courses of action for funding vocational-technical education programs.

1.5 Definitions of Terms

Labor Market Demands. Demands are notices of job vacancies that exist in the labor market. These notices are expressed in terms of graduates

[13] Herbert A. Simon, *Administrative Behavior: A Study of the Decision-Making Process in Administrative Organizations*, Second Edition, (New York: Macmillan Company, 1961), p. 68.

needed from specific vocational-technical education programs defined in the Office of Education classification system.[44]

Labor Market Supply. Supply consists of graduates of vocational-technical education programs defined in the Office of Education classification system.[45]

Labor Market Area (LMA). Labor market areas are geographic boundaries defined by the U. S. Department of Labor for manpower planning and statistical reporting.

Input-Output Analysis. Hartley defines input-output analysis as:

> . . . an economic technique designed to examine the effect of changes in certain input variables to the outcome or output of the system under study; a form of systems analysis; inputs are the resources employed to achieve objectives and outputs are the products of a program, often expressed numerically.[46]

Input. Following the general definition by Hartley, an input in this study is defined as an amount of vocational-technical education funds given to a local school district by an SDE to subsidize the existing local funds spent on a vocational-technical education program.

Output. Applying Hartley's definition, an output in this study is a supply of vocational-technical education graduates who have completed their occupational training and are prepared to enter the labor force.

Vocational-Technical Education Programs. Vocational-technical education programs considered in this investigation include only those set forth in the SDE guidelines for administering the Vocational Education Act. The document clearly states that:

> Funds under the 1963 Act will not be available for instruction which is designed to fit individuals for employment in recognized occupations which are generally considered to be professional or as requiring a baccalaureate or higher degree.[47]

Vocational-Technical Education Funds. These funds represent a subsidy given to local school districts to defray the increased cost of vocational-technical education programs. Payments to local districts are made on an enrollment basis as outlined by Arnold.[48]

---

[44] U. S. Department of Health, Education, and Welfare, *Vocational Education and Occupations*, OE-80016, (Washington, D. C.: Government Printing Office, July, 1969).

[45] *Ibid.*

[46] Hartley, *op. cit.*, p. 255.

[47] U. S. Department of Health, Education, and Welfare, *Administration of the Vocational Education Act: Rules and Regulations*, Vocational Education Bulletin No. 1, Revised 1966, (Washington, D. C.: Government Printing Office, 1967).

[48] *Arnold, op. cit.*, p. 296.

A Typology of Educational Organizations in the LMA. This study uses the *typology of educational organizations that Arnold has shown provides* a valid input for the labor market supply in a supply-demand model of the vocational-technical education system.[49] These are graduates from:

1. Public Secondary Schools.
2. Community Colleges.
3. Private Trade and Technical Schools.
4. Private Business Schools.
5. State Trade and Technical Schools.
6. Manpower Development Training Programs.
7. State Retraining Act Programs.
8. Two-Year Programs in Four-Year Schools.
9. Private Junior Colleges.

The following standard definitions pertaining to mathematical models of decision making can be found in most sources dealing with organizational decision making. The definitions used below are taken from Nemhauser.[50]

Optimization. The process of finding a best solution among several feasible alternatives. The term "best solution" is used because there may be more than one optimal solution.

Variables. Variables are those factors that can be manipulated to achieve the desired objective. These variables are commonly referred to as independent or decision variables.

Parameters. Parameters are those factors that effect the objective but are not controllable (cannot be manipulated as are decision variables).

The Measure of Effectiveness. The measure of effectiveness is the value, utility, or return associated with particular values of the decision variable and parameters. The measure of effectiveness, alternatively called the utility measure, criterion function or *objective function*, is a real-valued function of the decision variables and parameters.

Constraints. Constraints are relationships that determine the values that the decision variables can assume.

Feasible Solution. Any solution of the objective function which also satisfies the constraints is known as a feasible solution.

[49] Arnold, *op. cit.* p. 160.

[50] George L. Nemhauser, *Introduction to Dynamic Programming*, (New York: John Wiley and Sons, 1967), pp. 2-3. For a similar discussion of mathematical models of decision making see also Marcus Alexis and Charles Z. Wilson (eds.), *Organizational Decision Making*, (Englewood Cliffs, New Jersey: Prentice-Hall, 1967).

Optimal Solution. An optimal solution is defined as a feasible solution producing the greatest possible return.

Efficiency in Educational Decision Making. Burkhead notes that:

> Economists use the term efficiency in two senses. First, a given pattern of resource allocation is said to be efficient if other things remaining the same, consumer satisfactions are maximized. This is conventionally known as economic efficiency. Second, a given production mechanism is said to be efficient if outputs are maximized in relation to inputs; this is technological or engineering efficiency.[51]

The present study (as was Burkhead's) is concerned only with the second of these concepts which, however, is an appropriate economic inquiry because choices are involved among alternative use of a fixed level of vocational-technical education funds.

1.6 Properties of the Model

The function of the mathematical planning model in this investigation is to provide the decision-maker with new knowledge that can provide guidelines for selecting an optimal policy for allocating vocational-technical education funds to local school districts. McGivney shows that PPBS is a valuable tool for organizing information to improve decisions having to do with the allocation of resources.[52] Hence, the guidelines from PPBS can be used to evaluate the ability of the model to provide the decision-maker with appropriate new knowledge that can improve the decision-making process.

McGivney shows that PPBS attempts:

1. To focus on inputs and outputs rather than inputs alone.
2. To assure the decision maker a choice of valid comparable alternatives.
3. To express the ingredients for decisions in concrete quantifiable terms, and when they cannot be quantified, it attempts to be explicit about the incommensurables.
4. To build in a dimension over time that tries to see today's decisions in terms of their longer term consequences.[53]

Using these guidelines the following comments can be made about the ability of the planning model to provide appropriate new knowledge for the decision-maker:

---

[51] Burkhead, op. cit., p. 4.

[52] Joseph M. McGivney and William C. Nelson, *Program, Planning, Budgeting Systems for Educators, Volume One*, (Columbus: The Center for Vocational and Technical Education, The Ohio State University; August, 1969), p. 20.

[53] Ibid., p. 33.

1. Using a supply-demand criterion, the state system is viewed as a set of input-output relationships. Inputs are funds allocated at state level. Outputs are the future graduates of educational organizations in the LMA.

2. Using linear programming techniques, the model provides a set of feasible solutions for a given set of variables and parameters. In this manner, a choice of valid comparable alternatives is provided.

3. Formulating the resource allocation problem in a linear programming format, the ingredients for decisions are expressed in quantifiable terms.

4. Using an input-output relationship to analyze vocational-technical education allows the decision-maker to view a current resource allocation plan in terms of future program outputs.

The analysis above shows how the planning model utilizes the guidelines of PPBS to structure information in a form that it can be used by the decision-maker to evaluate resource allocation policies. What remains to be shown is the types of planning information necessary to construct and operate the model.

1.7 Data Requirements for the Model

For this model, the decision-maker's objective is to maximize the contribution of vocational-technical education graduates toward reducing the existing demands in the labor market. Given this objective, this model utilizes the following information to determine the decision variables and parameters:

1. Labor market demands expressed in terms of specific vocational-technical programs.

2. The total amount of state funds to be allocated to public secondary schools.

3. The cost to the SDE expressed in terms of dollars per pupil per program.

4. Statistics on the enrollments in all vocational-technical education programs sponsored by public secondary schools.

5. Statistics on the output of graduates in each type of educational institution in the typology (see Section 1.5).

6. Data on the school capacities per program.

7. Decision parameters provided by the SDE on the desired ratios among the expected outputs of vocational-technical education programs.

The data described in items 1 through 5 are published in existing state government publications or federal reports. Data in item 6 can be obtained from an interview or survey of local school administrators. Data in item 7 can be obtained by interviewing the appropriate SDE personnel.

1.8 Application of the Model

Following the construction of the model and the collection of the appropriate data, the model is applied for specific vocational-technical education programs in the Philadelphia, Pennsylvania LMA. The results of this application provide a sample of the type of information the model offers to the decision-maker.

The optimal solution of the model for the Pennsylvania LMA indicates the new enrollment figures by vocational-technical education programs that (1) can be supported with the fixed level of predetermined funds allocated to the LMA and (2) reflect the maximum future output of graduates designed to reduce existing labor market demands. The future output of graduates is based on existing enrollment data on the rate of completion by type of program.

1.9 Summary

Chapter One contains sections on (1) introductory notes on the fiscal dimensions of the Vocational Education Act of 1963, (2) an overview of the study, (3) a general discussion on the theory and application of educational planning which includes the role of management science models, (4) the need for the study which focuses on the emerging role of the SDE and its implications for long-range planning in vocational-technical education, (5) the statement of the problem which deals with the application of a supply-demand criterion to decisions regarding the allocation of funds to local school districts, (6) the definition of terms, (7) a discussion of the properties of the model which indicates how the model generates new knowledge for the decision-maker about future consequences of present decisions, (8) data requirements for the model which show the type of data needed for the input-output analysis of the state system, and (9) a discussion on the application of the model in a specific LMA.

The remaining chapters will now be described briefly. In Chapter Two, the literature on the manpower requirement approach to educational planning is reviewed. Chapter Three is devoted to the construction of the model. In Chapter Four, the application of the model to a Pennsylvania LMA is explained. The final chapter consists of the summary, recommendations and conclusions.

Chapter Two

Review of Related Literature

The literature reviewed in the introductory chapter clearly indicates that (1) the basic concepts and new approaches in decision theory should be adopted and installed by government agencies having a major responsibility for the allocation of resources for vocational-technical education, (2) public occupational educational programs should be required to meet the tests of economic efficiency, and (3) analytical studies should be conducted of alternative ways of achieving the objectives and goals of vocational-technical education. One important criterion for the economic efficiency relating to resource allocation in vocational-technical education is the existing relationship of the supply and demand for occupationally trained graduates found in the labor force. The discussion of this criterion and the extent to which it can be utilized in determining a policy for resource allocation by SDE's is the topic of this chapter.

2.1 The Supply-Demand Criterion in Vocational-Technical Education Planning

Kaufman and Brown note that "One of the basic principles underlying the Vocational Education Act of 1963 was that youth would be trained for occupations (supply) for which society has a need (demand)."[1] His review of the literature in the manpower supply and demand points out that (1) developments in manpower policy are currently made without sufficient support of research, (2) future planning in vocational-technical education can be improved if very detailed and accurate knowledge of the labor market developments are available, and (3) it is more realistic to plan and train for occupational clusters than for specific occupations. This latter recommendation allows greater flexibility and smoother adjustments of supply in response to changes in demand.

Woodhall, reviewing the literature on educational planning, comes to the following conclusion:

> The interdependency of the educational system and the occupational structure of the labor force has been so frequently emphasized that many countries, both advanced and underdeveloped, have drawn up detailed estimates of future manpower requirements

[1] Jacob J. Kaufman and Anne F. Brown, "Manpower Supply and Demand," *Review of Educational Research*, 38: 325; October, 1968.

which are used to determine the rate of expansion of secondary or higher education.[2]

Sanders and Barth, reviewing the literature on the relationship between educational policy and human resource development, note:

> Efforts to develop educational policy along human resource development lines typically assumed that the primary link between education and economic growth lay in the intervening manpower preparation, the process of preparing persons for the more complicated and sophisticated economic roles they would play in a more industrialized society.[3]

Harbison observes that "for analytical purposes it is necessary to have some definition in terms of both occupations and educational levels" if human resource development plans are to integrate educational and economic planning.[4] In this country, a recently developed publication, jointly sponsored by the Office of Education and Manpower Administration, provides educational planners and manpower analysts a method for linking the *Office of Education Classification System* and the *Dictionary of Occupational Titles Classification and Code*. The intent of the document is described in the foreword which states:

> The joint education and manpower responsibilities of the Department of Health, Education and Welfare and the Department of Labor involve many common goals. . . . There has been a need for a common occupational language that would aid the cooperative efforts of both departments in relating education and the world of work. . . . By facilitating more efficient planning this publication should make possible more realistic matching of educational output with occupational requirements.[5]

The Pennsylvania Vocational Education Study generates manpower supply and demand data which can be used as a guide by SDE's to develop future programs and to provide a guide for future resource allocation.[6] In

[2] Maureen Woodhall, "The Economics of Education," *Review of Educational Research*, 37: 391; October, 1967.

[3] Donald P. Sanders and Peter S. Barth, "Education and Economic Development," *Review of Educational Research*, 38: 217; June, 1968.

[4] Frederick Harbison and Charles A. Meyers, *Education, Manpower and Economic Growth: Strategies of Human Resource Development*, (New York: McGraw-Hill Book Company, 1964), pp. 15-16. See also Russell C. Davis, *Planning Human Resource Development: Educational Models and Schemata*, (Chicago: Rand McNally and Company, 1966) and Jacob J. Kaufman (et. al.), *The Development and Utilization of Human Resources: A Guide for Research*, (University Park, Pennsylvania: The Pennsylvania State Unievrsity, The Institute for Research on Human Resources; July, 1967), p. 47.

[5] U. S. Department of Health, Education, and Welfare, *Vocational Education and Occupations*, OE-80061, (Washington, D. C.: Government Printing Office; July, 1969), p. v.

[6] Walter M. Arnold (ed.), *Vocational, Technical and Continuing Education in Pennsylvania: A Systems Approach to State-Local Program Planning*, (Harrisburg: Pennsylvania Department of Education, 1969).

this study a methodology is developed for matching labor market requirements (demand) provided by the Department of Labor, with the existing output of educational institutions having occupational training programs (supply). A similar approach, which attempts to estimate directly the manpower requirements by vocational-technical skill categories and to provide quantitative information needed for educational planning, is outlined by Eckaus.[7]

Warbrod's publication, which represents the most extensive review and syntheses of the research on the economics of vocational-technical education, clearly indicates that the costs and benefits of occupational education cannot be made independent of labor market considerations.[8] The fact that the labor market plays a significant role in research on the costs and benefits of vocational education appears in some more recent studies. For example, Hu compares the benefits of vocational and nonvocational education at the secondary level.[9] The criterion variable of this investigation is the labor market performance of noncollege graduates. Kraft in his conclusion about vocational education expenditures notes:

> The author is a firm advocate of manpower planning and the rational adaption of our system of education and training to the needs of the economic system. It seems absurd to invest, annually, more than 40 billion in human capital without asking whether, from an economic standpoint, this money could not be allocated more efficiently. (Needless to say we do not want to be interpreted as asserting that the only criterion to be used is the investment or productivity criterion. But it is obvious that unless the economic impact of education is to be given no weight at all, some form of manpower planning is both desirable and inevitable.)[10]

Katz, summarizing the results of the famous U. S. Office of Education sponsored Airlie House conference, notes:

> The economic analysis of manpower demand and supply, including projections and their validity, is of great importance to vocational educators. . . . Among other labor market considera-

[7] R. S. Eckaus, "Economic Criteria for Education and Training," *Review of Economics and Statistics*, 46: 183-184; May, 1964.

[8] J. Robert Warbrod, *Review and Synthesis of Research on the Economics of Vocational Education*, (Columbus: The Center for Vocational-Technical Education, The Ohio State University, 1967).

[9] Teh-wei Hu, Jacob Kaufman, Maw Lin Lee and Ernst W. Stromsdorfer, *A Cost-Effectiveness Study of Vocational Education: A Comparison of Vocational and Nonvocational Education in Secondary Schools*, (University Park, Pennsylvania: The Pennsylvania State University, Institute for Research on Human Resources; February, 1969).

[10] Richard H. Kraft, *Cost Effectiveness: Analysis of Vocational-Technical Education Programs*, Florida State Department of Education, Final Report, Project No. 569-124, (Tallahassee, Florida: The Florida State University, Department of Educational Administration, Educational Systems and Planning Center; July, 1969), p. 116.

tions, the educator must be concerned with trends in employment by occupational categories and by job family, skill requirements, the relationship between filled jobs and job vacancies as forecasts for the state or major metropolitan area, and the size of the existing work force to meet the demand.[11]

Based on the statements from the existing literature, it should be clear that the manpower supply and demand relationship in the labor force is a valid criterion which can be used as one measure to test the economic efficiency of the SDE's policy for resource allocation of public funds to local school districts.[12]

2.2 Theoretical Criterion for Optimum Allocation of Public Expenditures for Education

In a recent study where the theory of public expenditures for education is examined, Hu sets forth two assumptions that should govern the comparisons of alternate expenditures within the educational system. The first states:

> A basic assumption in economics is that goods are scarce and that consumers prefer to have more goods than less. Therefore, it is generally desirable to employ resources in those uses where they have the highest productivity.
>
> Given the total amount of resources available for public education, it is relevant to determine the optimum allocation of expenditures on different programs such as vocational-technical education and academic education.[13]

Second, he assumes "that the goal of government programs is to maximize the social welfare."[14] Using these assumptions he develops a theoretical criterion for measuring resource allocation. This criterion, based on the theory of marginal analysis, is briefly outlined below.[15]

---

[11] Arnold Katz, "Major Recommendation and Conclusions," in Arnold Katz (ed.), *Occupational Education: Planning and Programming*, Volume One, (Menlo Park, California: Stanford Research Institute; September, 1967), p. 23.

[12] This investigation is concerned with resource allocation at the state level. The economic efficiency of resource allocation at the federal level also utilizes manpower criteria. This position is clearly stated in Leonard A. Lecht, *Manpower Requirements for National Objectives in the 1970's*, (Washington, D. C.: National Planning Association, Center for Priority Analysis, 1968).

[13] Hu (et. al.), *op. cit.*, p. 11.

[14] *Ibid.*, p. 12.

[15] The discussion that follows can also be found in most elementary economic texts. For example, see the treatment of marginal analysis and theorems on resource allocation found in William J. Baumol, *Economic Theory and Operations Analysis*, (Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1965), pp. 21-40.

The social welfare function, with respect to different government programs, may be written in the form:

$$(1) \qquad W = w(g_1, g_2, \ldots, g_n)$$

where W is the social welfare (or it can be denoted as social benefits) and the g's represent the output of different government programs. The maximization of function (1) is subject to the constraint of the government budget, namely

$$(2) \qquad B = \sum_{i=1}^{n} (a_i + c_i g_i)$$

where a_i is the fixed cost of the ith government program, c_i is the marginal cost of the ith government program, and B is the total government revenue.

The Lagrangian multiplier technique is used to solve the maximization problem, that is:

$$(3) \quad w(g_1, g_2, \ldots, g_n) - \lambda \left[\sum_{i=1}^{n} (a_i + c_i g_i) - B \right] = 0$$

where λ is the Lagrangian multiplier. Differentiating this expression with respect to g_i leaves

$$(4) \qquad w_i - \lambda c_i = 0$$

where $w = \frac{\partial W}{\partial g_i}$ is the marginal benefit of the ith program.

From this it follows that:

$$(5) \qquad \frac{w_i}{w_j} = \frac{c_i}{c_j} \qquad (i, j = 1, 2, \ldots, n)$$

and also that:

$$(6) \qquad \frac{w_i}{c_i} = \lambda.$$

Thus, in equilibrium, as shown in equation (5), the maximization of social benefits is achieved if the ratio of marginal benefit in this example of two government programs is equal to the ratio of the marginal cost of these programs; that is, the marginal benefit is proportional to the marginal cost.

The theoretical criterion which governs models built for optimum allocation of educational funds is clear-cut. In the model to be built for the SDE's, the g_i's represent the output of graduates from various vocational-technical education programs.

2.3 Manpower Models in Educational Planning

Davis notes that various points of departure are possible for building human resource development planning models.[16] These are:

1. Departure from a set of political, cultural or social goals which state that some specific portion of the population has a right to some specified amount of education and training.
2. Departure from estimates of the resources (human and fiscal) available for assignment to education and training so that returns are maximized.
3. Departure from a set of human resource requirements or targets in the work force. The objective is to equal or exceed the targets with allocations minimized.[17]

This investigation builds a model hich is based on the human resource requirement approach outlined in (3) above. In the following section, the manpower-requirement approach is briefly described and is followed by a review of the existing educational planning models based on this approach.

THE MANPOWER-REQUIREMENTS APPROACH

Parnes briefly describes the manpower-requirements approach as follows:

> An attempt is made to foresee the future occupational structure of the economy and to plan the educational system so as to provide the requisite number of personnel with the qualifications which that structure demands.[18]

In a similar manner Kraft outlines a strategy for educational planning based on the manpower-requirements approach consisting of the following steps:

1. A calculation of the future occupational structure of the labor force.

[16] Davis, *op. cit.*, p. 137.

[17] *Ibid.*

[18] Herbert S. Parnes, *Forecasting Educational Needs for Economic Development*, (Paris: The Organization for Economic Cooperation and Development, 1962), p. 15.

2. The translation of the labor requirements by occupational categories into requirements by educational qualification.[19]

More detailed outlines of the methodology of the manpower-requirements approach have been offered by Benson,[20] Goldstein,[21] Parnes,[22] and Harbison.[23]

The important note to be extracted from any outline of the manpower approach is that it provides the educational planner with (1) estimates of the required additions to the labor force during the planning period and (2) for each occupational category of the labor market estimates there exists an educational program. This provides the basis for indicating the required outputs (graduates) during the planning period, which in turn permits the calculation of required enrollments, teacher requirements and needed educational plant and equipment.

A common criticism of the manpower approach is that there currently exists a poor rationale that underlies the fitting of educational preparation to occupational requirements in the work force. In the United States this problem need no longer exist for vocational education planners. A common occupational language for the Department of Labor and the Department of Health, Education, and Welfare is currently available.[24]

A second criticism of the manpower approach to educational planning is that it focuses exclusively on economic criteria. In this regard Parnes notes:

> To be sure the "manpower requirements approach" alone cannot answer the question "how much education is needed," but it provides useful guides to the desirable structure of whatever educational expenditure is decided upon.[25]

[19] Richard H. Kraft, "Inter-Firm Correlations: The Contribution of Educationally-Heavy Inputs to Increasing Profitability," in Richard H. Kraft (ed.), *Education and Economic Growth: Proceedings of the First Annual Conference on the Economics of Education*, (Tallahassee, Florida: The Educational Systems Development Center, The Florida State University, 1968), p. 123.

[20] Charles S. Benson, *The Economics of Public Education, Second Edition*, (Boston: Houghton Mifflin Company, 1968), pp. 67-75.

[21] Harold Goldstein, "Manpower Requirements and Educational Organization" in Organization for Economic Cooperation and Development, *Organizational Problems in Planning Educational Development*, (Paris: OECD, 1963), pp. 37-49.

[22] Herbert S. Parnes, "Assessing the Educational Needs of a Nation," in Don Adams (ed.), *Educational Planning*, (Syracuse: Syracuse University Press, School of Education, 1964), pp. 55-56.

[23] Frederick Harbison, "Human Resource Assessments," in United Nations, Educational, Scientific and Cultural Organization, *Economic and Social Aspects of Educational Planning*, (Paris: UNESCO, 1964), pp. 118-119.

[24] See footnote 5 and the discussion which preceded it.

[25] Parnes (1962), *op. cit.*, p. 15. See also Parnes (1964), *op. cit.*, pp. 56-60, for an extended discussion of the value of the manpower approach and how it interacts with what he calls the "cultural approach."

Kraft suggests that the manpower criterion can be a significant but not exclusive factor in planning human resource development.[26] Burkhead notes "specific goals for a state's vocational education program, for example, can be outlined and these can be related to manpower requirements and needs for identifiable skills for several years ahead."[27] Finally, Culbertson, discussing educational planning based upon manpower requirements, mentions, "the planning techniques developed and used by economists in developing nations have utility in developed nations—especially in relation to vocational education."[28] Hence, the literature shows that the manpower-requirements approach can be effectively utilized as a criterion to determine educational resource allocation policy at the state level.

OPTIMIZATION MODELS IN MANPOWER PLANNING

Given the labor market needs expressed in terms of educational requirements, Davis shows that the problem can be expressed in a linear programming framework.[29] He also demonstrates that the model can be developed to yield problems amenable to linear programming in static and dynamic situations.[30] In either case, the linear programming model will yield optimal solutions for educational enrollments which are based on labor market requirements. Davis suggests that linear programming models can assist education planners in formulating a resource allocation policy; however, the optimization models do not provide a substitute for other necessary kinds of empirical analysis. For example, his models do not consider the existing school capacities and teacher requirements necessary to implement the outcomes suggested by the optimum enrollment solution.

Chirikos and Wheeler, reviewing the use of manpower models in educational planning, note that:

> The view of an educational system as a set of input-output or production relationships which can be controlled in a way that will optimize the use of scarce educational resources is almost entirely a consequence of recent interest in educational planning.[31]

---

[26] See Kraft's statement previously quoted and referenced in footnote 10 of this chapter.

[27] Jesse Burkhead, *Public School Finance: Economics and Politics*, (Syracuse: Syracuse University Press, 1964), p. 364.

[28] Jack Culbertson, "State Planning for Education," in Edgar L. Morphet and Charles O. Ryan (eds.), *Designing Education for the Future: Planning and Effecting Needed Changes in Education*, (New York: Citation Press, 1967), pp. 278-279.

[29] Davis, *op. cit.*, p. 28.

[30] Davis, *op. cit.*, pp. 160-162. See also Russell G. Davis, "On the Development of Educational Planning Models at Harvard, CSED: An Algebraic History of Activity in One Small Place," in Kraft (1968), *op. cit.*, pp. 42-84.

[31] Thomas N. Chirikos and A. C. Wheeler, "Concepts and Techniques of Educational Planning," *Review of Educational Research*, 38: 264-276, June, 1968.

One of the first attempts to deal with manpower models in educational planning is the Tinbergen-Bos-Correa model. This is outlined by Correa and Tinbergen[82] and the Organization for Economic Corporation and Development (OECD).[83] Chirikos and Wheeler provide an abstract of the model which states:

> Built on simple input-output principles, the basic model is a set of six linear difference equations which relate the operation of the educational system to various types of educated manpower required to produce a given level of gross output. The aggregate nature of the model, in addition to the fact that it eliminates the intermediate step of estimating occupational needs by relating fixed proportions of educated manpower directly to output, is an obvious limitation in its usefulness to educational planners.[84]

While this model does not directly reflect the environment and constraints of the problem in this investigation, it serves as a guide to show that the input-output method can be used to assess educational policy options.

Other attempts to use linear programming techniques for assessing educational policy options include the work of Adelman,[85] Bowles,[86] Galladay,[87] and Schiefelbein.[88] The objective functions of these models are used to maximize the gross output (subject to linear constraints) to test alternative educational and economic strategies at the national level for Argentina, Nigeria, Morocco and Chile. National educational requirements and the organizational properties of the various countries represented in these studies are not identical to the similar educational requirements and properties of a state in this country. However, these studies show that linear programming techniques can be effectively used by educational policy makers to determine resource allocation strategies based on manpower criteria.[89]

---

[82] Hector Correa and Jan Tinbergen, "Quantitative Adaptation of Education to Accelerated Growth," Kyklos, 15: 776-786, 1962.

[83] Organization for Economic Cooperation and Development, *Econometric Models of Education: Some Applications*, (Paris: OECD, 1965).

[84] Chirikos and Wheeler, *op. cit.*, p. 270.

[85] Irma Adelman, "A Linear Programming Model of Educational Planning: A Case Study of Argentina," in Irma Adelman and Erik Thorbecke (eds.), *The Theory and Design of E o-nomic Development*, (Baltimore: Johns Hopkins Press, 1964).

[86] Samuel S. Bowles, "The Efficient Allocation of Resources in Education," *Quarterly Journal of Economics*, 81: 189-219, May, 1967.

[87] Frederick L. Galladay, "A Dynamic Linear Programming Model for Educational Planning with Application to Morocco," (Doctoral Dissertation, Northwestern University, August, 1968).

[88] Ernesto Schiefelbein, "A Model for Assessing the Quantitative Results of Alternate Educational Policies," (Doctoral Dissertation, Harvard University, 1969). The model in this dissertation is reviewed in Davis (1968), op. . . pp. 49-60.

[89] The reader may now wish to again refer to Culbertson's comment on the potential application of the manpower planning models for developing countries to vocational education in this country. See footnote 28 in this chapter.

Chance shows conceptually how a general model (linear programming) employing occupational output coefficients on the demand side and a Markov chain on the education or supply side can be manipulated to produce consistent economic and education policies.[40] Correa shows that linear programming techniques can be used by educational planners to deal with policy decisions relating to enrollment policy, financial resources, and the problem of the adaptation of the educational system to socio-economic development.[41]

The results of the survey conducted in this investigation (see Section 1.3) show that no SDE's are currently using this approach. Bruno, however, demonstrates that a linear programming model can be used to determine optimal resource allocation strategies for a foundation type state support program.[42] Commenting on the value of the linear programming techniques, he notes: "The important contribution of this approach is that it realistically places the problem of education finance in a system constrained by political, social and economic considerations."[43]

Judy[44] and Koenig[45] observe that one of the greatest values of optimizing models is not so much that it shows the specific solution to a particular problem with particular numbers, as that within a given conceptual framework, it defines what can and cannot be done. Hence, optimization models (and the one which this study builds is no exception) can give information which permits the decision-maker himself to be much clearer about the implications of different alternatives. Clearly, in the model of this study, the decision-maker is ultimately the State Board of Education.

2.4 Summary

The purpose of this chapter has been to review the literature relating to the application of optimization models of manpower planning to educa-

---

[40] W. A. Chance, "Long-Term Labor Requirements and Output of the Educational System," *Southern Economics Journal*, 32: 417-428; April, 1966.

[41] Hector Correa, "A Survey of Mathematical Models in Educational Planning," in Organization for Economic Cooperation and Development, *Mathematical Models in Educational Planning*, (Paris: OECD, 1967).

[42] James E. Bruno, "A Mathematical Programming Approach to School System Finance," *Socio-Economic Planning Science*, 3: 1-12; 1969.

[43] *Ibid.*, p. 1. See also a similar discussion in James E. Bruno, "An Alternative to the Use of Simplistic Formulas for Determining State Resource Allocation in School Finance Programs," *American Educational Research Journal*, 6: 479-500; November, 1969.

[44] Richard W. Judy, "Simulation and Rational Resource Allocation in Universities," in Organization for Economic Cooperation and Development, *Efficiency in Resource Utilization in Education*, (Paris: OECD, 1969), pp. 255-285.

[45] M. G. Keeney, H. E. Koenig and R. Zemack, "State Space Models of Educational Institutions," in Organization for Economic Cooperation and Development (1969), *op. cit.*, pp. 99-135.

tional resource allocation policies at state level. In Section 2.1 it has been shown that a supply-demand criterion can and should be utilized in vocational-technical education policy plans at the state level. In Section 2.2 a theoretical criterion that should govern the efficient allocation of public expenditures for education has been explained. Section 2.3 contains the important concepts of the manpower-requirements approach and shows various mathematical models that can be used to determine the efficient allocation of resources given the labor market demands expressed in terms of specific educational programs.

The results of this review indicate that linear programming can be of important practical value to educational planners for designing a state-level policy about the efficient resource allocation of vocational-technical education funds to local school systems. In the following chapter the resource allocation model is presented.

Chapter Three

The Educational Planning Model

In this chapter the mathematical planning model is presented. It is designed to provide the decision-maker, the State Board of Education, with new information to evaluate decisions about the efficient allocation of vocational-technical education funds based on a supply-demand criterion. In the model, the state education system is viewed as a set of input-output or production relationships which can be designed so that the use of vocational-technical funds will be optimized. Alternative funding policies are examined and in each case the net output of future graduates is determined. Finally, from a set of feasible solutions, an optimal solution is selected to maximize the output of vocational-technical education graduates. This optimal solution is derived using linear programming techniques.

The resources to be allocated by the SDE in this model are vocational-technical education funds which are given to local school districts to defray the additional costs of vocational programs. The funds allocated to each school are based on a specific amount per student. These amounts, although fixed for students matriculating in a given vocational-technical education program, may vary among different programs.

The model is built to operate on a specific set of vocational-technical education programs within a labor market area (LMA). Hence, future references to inputs and outputs apply to *funds allocated to* and *students enrolled in* programs which are physically located in one LMA. The planning period for the model is three years and the vocational-technical education programs require two academic years to complete.[1]

Ideally, the model should be run during the operation of the base (first) year when the acquisition of the data necessary for estimating the output of graduates for the entire planning period is available. The model can be utilized following the completion of the base year; however, this will decrease the time for formulating policy decisions about future program operations.

Solutions from the model require the completion of two phases. These are briefly outlined below.

---

[1] Although the model is currently applied to programs requiring two academic years to complete, it could easily be applied to programs requiring more or less formal training.

Phase One provides the information necessary to calculate the parameters for the linear programming model. First, supply and demand projections for each year in the planning period must be determined. Using these projections, a supply-demand posture is derived. This posture indicates the LMA demands not satisfied when current vocational-technical program levels (enrollment trends) are unchanged for the second and third year of the planning period.

Phase Two involves the actual operation of the linear programming model. Using the results of phase one, the linear programming model is set forth and solutions are generated. These solutions provide the State Board of Education with new information on the alternate funding policies they could implement during the second and third (final) years of the planning period. Following the linear programming solutions, a formula for allocating vocational-technical education funds to individual school districts is described.

3.1 Phase One: The Supply-Demand Posture

Focusing on the output of the educational system (graduates), a supply-demand posture for a specific vocational-technical education program at the close of the third year of the planning period can be expressed as:

$$A + B + C = D \tag{1}$$

where

A = the output of graduates from the public schools for the three-year planning period.

B = the output of graduates from all other types of training institutions for the three-year planning period.

C = the unsatisfied LMA demand.

D = the LMA demand for trained graduates in the three-year period.

This equation states that the demand for vocational graduates from a specific vocational program in the LMA for a given number of years can be described in terms of satisfied and unsatisfied demands. The total of A and B in (1) represents the satisfied demand. Further, A represents the number of graduates who receive vocational-technical education funds from the SDE while B represents the number of graduates who are not subsidized with these funds. In general, graduates reflected in B cannot be controlled by the SDE.

The entire set of statistics necessary to complete (1) would not be available until the termination of the planning period. For example, public school graduates in the final year of the planning period do not begin their training until the second year of the three-year period. Hence, as in all long-range planning, projections of graduates for future years are calculated to provide the decision-maker with some estimates of the future performance of the system.

Outlined below is a method for generating projections to substitute in (1), the supply-demand posture. The projections are made in the initial year of the planning period and are based on existing enrollments.

SUPPLY ESTIMATES

The output of graduates from all types of educational institutions in the LMA can be expressed in terms of yearly supply estimates. Let

$$A' = \sum_{j=1}^{n} \sum_{t=1}^{3} y_{jt} \tag{2}$$

and

$$B' = \sum_{k=1}^{8} \sum_{t=1}^{3} z_{kt} \tag{3}$$

where

A' = an estimate of A from (1) above derived in the first year of the planning period.

B' = an estimate of B from (1) above derived in the first year of the planning period.

y_{jt} = the public school output from county j in the year t.

z_{kt} = the output of graduates from school type k in the year t.

j = 1, 2, . . ., n (corresponding to the number of counties in the LMA).

t = 1, 2, 3 (corresponding to each year in the planning period).

k = 1, 2, . . ., 8 (corresponding to types of educational organizations other than the public school that provide trained graduates for the LMA).

The values for k in (3) are defined such that

1 = Community Colleges.

2 = Private Business Schools.

3 = Private Trade and Technical Schools.

4 = State Trade and Technical Schools.

5 = Manpower Development Training Act Programs.

6 = State Retraining Act Programs.

7 = Two-Year Programs in Colleges and Universities.

8 = Private Junior Colleges

The values for y_{j1} and y_{j2} can be estimated using the existing data in the base year such that

$$y_{j1} = e_{j1} \, y'_{j1} \qquad (4)$$

and

$$y_{j2} = f_{j2} \, y''_{j1} \qquad (5)$$

where

y'_{j1} = the graduating students in county j enrolled during the base year of data collection.

y''_{j1} = the first year students in county j enrolled during the base year.

e_{j1} = the percentage of students in county j who will graduate in the base year.

f_{j2} = the percentage of students in county j who will graduate in the second year.

Further, it is assumed that graduates in the final year of the planning period do not increase beyond the level reflected by y_{j2}. In this case, $y_{j3} = y_{j2}$ for all j.

The values for z_{kt} can be determined using statistics on graduates from the year prior to the planning period. Let

$$z_{kt} = (1 + p_k)^{t-1} \, z_{k,\, t-1} \qquad (6)$$

where p_k = the annual rate of change in the output from school type k.

The estimates in (6) provide information on the number of LMA demands that have been satisfied by other training agencies. Thus, the supply-demand posture is based on the concept that each educational institution in the LMA can simultaneously make a significant contribution in the area of occupational training.

DEMAND ESTIMATE

In a similar manner the demand for occupationally trained graduates can be expressed in terms of annual demands. Let

$$D = \sum_{t=1}^{3} D_t$$

where D_t = the LMA demand for trained graduates in the year t. The data for D_t is usually published by the State Department of Labor and Industry. The methodology for linking labor market data and vocational-technical education programs has been discussed in Section 2.1 of Chapter Two.

THE SUPPLY-DEMAND POSTURE

Using the format set forth in (1) and the estimates described above, a supply-demand posture for the planning period can be described by (8) below. In this case

$$A' + B' + C' = D \tag{8}$$

where C' = an estimate of C from (1). Note $C' = D - (A' + B')$. This posture indicates the LMA demands that would be satisfied as well as the demand unmet if the current vocational-technical program levels (enrollment and graduate trends) remained unchanged for the second and third years of the planning period.

In (8) it is assumed that all graduates enter directly in the LMA upon completion of their training. This is known to be a tenuous assumption. It has been documented, for example, that high school graduates from vocational education programs often enter the armed forces upon completion of their secondary program or that some graduates go directly into postsecondary training institutions.

A^* and B^* are formed below to reflect the actual number of graduates who enter directly into the labor force upon the completon of their vocational-technical education program. Let

$$A^* = aA' \tag{9}$$

and

$$B^* = bB' \tag{10}$$

where

A^* = the output of graduates from the public school who enter the LMA upon the completion of their vocational training.

B^* = the output of graduates from all other types of training institutions who enter the LMA upon completion of their vocational training.

a = the percentage of A' who enter the LMA immediately following their training.

b = the percentage of B' who enter the LMA immediately following their training.

Using A* and B*, (8) can be adjusted to reflect the actual participation of vocational-technical education in the labor force. Let

$$A^* + B^* + C^* = D \tag{11}$$

where C^* = the unmet labor market demands. Note $C^* = D - (A^* + B^*)$.

In addition to the output represented by A*, the SDE may also use vocational-technical education funds to encourage public schools to produce additional graduates that reduce the unsatisfied demands expressed by C. Following this approach, the public school output for the three-year planning period could include additional graduates who would start their training in the second year and complete their training in the third (final) year of the planning period. This is shown in (12).

$$F = A^* + c \sum_{j=1}^{n} x_j \tag{12}$$

where

F	=	the potential output of graduates from the public schools who enter the LMA upon completion of their vocational training.
x_j	=	the additional number of graduates in county j which the SDE would be willing to support.
c	=	the percentage of additional graduates who enter the LMA immediately following their training.

Since present research on the enrollments of public school vocational-technical education programs indicates that enrollments in existing programs are increasing and the number of new programs are growing, the model developed here focuses on providing the SDE with new knowledge about the potential additional output of graduates that the SDE can support.[3] A* and x_j from (12) above can be viewed as a parameter and a variable respectively in the linear programming model which follows in the next section.

The maximum number of additional graduates that the SDE would be willing to support in the LMA can be controlled so that C, the unmet demand, is not exceeded. Let

---

[3] For example see Michael Russo, "14 Million Vocational Students by 1975," *American Education*, 5: 10-11; March, 1969. He shows that enrollments in vocational programs increased from 349,000 in 1964 to an estimated 8.2 million in 1968 and will continue to rise to an estimated enrollment of 14 million by 1975.

$$0 \leq \sum_{j=1}^{n} x_j \leq d \qquad (x_j \geq 0 \text{ for all } j) \tag{13}$$

and

$$d = gC^* \qquad (g \geq 1.0) \tag{14}$$

where

d = the unmet LMA demands expressed in terms of the number of graduates necessary to generate C graduates who will enter the LMA upon completion of their vocational training.

g = the percentage of graduates necessary to train in order that C graduates will enter the LMA upon completion of their training. (Note: If it is assumed that all additional graduates enter the LMA when their training is completed, then g = 1.0).

In effect, (12) and (13) now restrict the potential additional enrollment in the occupational program so that each output from the public schools satisfies a LMA demand that no other educational organization satisfies.

In a similar fashion the equations developed in (1) through (14) can be derived from different vocational-technical education programs. The following set of equations would then summarize these activities in phase one.

$$A_i + B_i + C_i = D_i \tag{15}$$

$$A'_i = \sum_{j=1}^{n} \sum_{t=1}^{3} y_{ijt} \tag{16}$$

$$B'_i = \sum_{k=1}^{8} \sum_{t=1}^{3} z_{ikt} \tag{17}$$

$$y_{ij1} = e_{ij1} \, y'_{ij1} \tag{18}$$

$$y_{ij2} = f_{ij2} \, y''_{ij2} \tag{19}$$

$$z_{ikt} = (1 + p_{ik})^{t-1} \, z_{k,\,t-1} \tag{20}$$

$$D_i = \sum_{t=1}^{3} D_{it} \tag{21}$$

$$A'_i + B'_i + C'_i = D_i \tag{22}$$

$$A^*_i = a_i A' \tag{23}$$

$$B^*_i = b_i B' \tag{24}$$

$$A^*_i + B^*_i + C^*_i = D_i \tag{25}$$

$$F_i = A^*_i + C_i \sum_{j=1}^{n} x_{ij} \tag{26}$$

$$0 \leq \sum_{j=1}^{n} x_{ij} \leq d_i \qquad (x_{ij} \geq 0 \text{ for all } i \text{ and } j) \tag{27}$$

$$d_i = g_i C^*_i \qquad (g_i \geq 1.0 \text{ for all } i) \tag{28}$$

where

A_i = the public school output in program i for the three-year planning period.

B_i = the output of other educational institutions in program i for the three-year planning period.

C_i = the unsatisfied demand for trained graduates in program i.

D_i = the LMA demand for trained graduates in program i for the three-year planning period.

A'_i = an estimate of A_i derived in the first year of the planning period.

B'_i = an estimate of B_i derived in the first year of the planning period.

y_{ijt} = the public school output in program i from county j in the year t.

z_{ikt} = the output in program i from school type k in the year t.

y'_{ij1} = the enrollment of terminal year students in program i in county j during the base (first) year.

y''_{ij2} = the enrollment of first year students in program i in county j during the base year.

e_{ij1} = the percentage of students in program i in county j who will graduate in the base year.

f_{ij2} = the percentage of students in program i in county j who will graduate in the second year.

p_{ik} = the annual rate of change in the output in program i from school type k.

D_{it} = the LMA demand for trained graduates from program i in the year t.

A^*_i = the public school output who enter the LMA upon completion of their vocational training.

B^*_i = the output from all other k type institutions who enter the LMA upon completion of their vocational training.

a_i = the percentage of A'_i who enter the LMA following their training.

b_i = the percentage of B'_i who enter the LMA following their training.

F_i = the potential output from the public school in program i who will enter the LMA upon completion of their training.

x_{ij} = the additional output in program i which the SDE would be willing to support.

c_i = the percentage of additional output in program i who enter the LMA folowing their training.

d_i = the unmet LMA demands for program i expressed in terms of the number of graduates necessary to generate C graduates who will enter the LMA upon completion of their vocational training.

g_i = the percentage of graduates necessary to train in program i in order that C graduates will enter the LMA upon completion of their training. (Note: If it is assumed that all additional graduates enter the LMA when their training is completed, $g_i = 1.0$.)

i = 1, 2, . . ., m (vocational-technical education programs).

j = 1, 2, . . ., n (corresponding to the number of counties in the LMA).

k = 1, 2, . . ., 8 (corresponding to the types of educational organizations other than the public school that provide trained graduates in the LMA).

t = 1, 2, 3 (corresponding to each year in the planning period).

The information in (15) through (28) provides the input data necessary to construct the linear programming model. The variables in this model will be the x_{ij}'s discussed in (26) through (28). Each d_i in (28) provides the necessary data to formulate one of the i labor market area constraints in the model.

The optimal solution to the linear programming model is defined as Z^* such that

$$Z^* = \sum_{i=1}^{m} \sum_{j=1}^{n} x^*_{ij} \qquad (29)$$

where x^*_{ij} = the optimal level for the variable x_{ij} in the solution set and Z^* is a scalar representing the sum of the additional number of graduates for the optimal level of each activity in the model.

The data necessary to complete (15) through (28) can be found in the existing statistical records kept by the SDE's. The one exception to this generalization might be the LMA demands projected by occupational categories. Since the U. S. Office of Education guidelines for vocational education state plans require SDE's to assemble this type of labor force in the coming years, all states will soon have the data necessary to operate the *linear programming model in phase two.*[3]

3.2 Phase Two: The Linear Programming Model

The purpose of developing a linear programming model in this phase is to generate an optimal solution for an objective function given information on (1) the unmet demands of the LMA, (2) constraints on the existing additional capacity of the public schools, and (3) a set of budget constraints. The solution of the model can then be presented to the State Board of Education to use as new information about alternative actions the Board can use to determine future resource allocation policies. In this case, the model is based on a supply-demand criterion and it is indicative of the models generated by the manpower-requirement approach used by human resource development planners.

The linear programming model can be solved using the IBM 360 MPS mathematical programming algorithm. This package or a similar one is available in most computer centers. The average computer time for obtaining the initial optimal solution as well as other optimal solutions resulting from parameterization of selected variables should be less than two or three minutes.

THE OBJECTIVE FUNCTION

The objective function in this investigation is linear and is based on the question posed by the State Board of Education, which is:

---

[3] *For example,* the Vocational Education Amendments of 1968 authorize the Commissioner of Education to transfer to the Secretary of Labor an amount not to exceed $5 million in any fiscal year to finance national, regional, state and local studies providing projections of occupational manpower needs. This information will be for use in vocational education planning and for the guidance of federal, state and local officials and advisory councils responsible for implementing of this legislation.

> How can the SDE most efficiently allocate a fixed level of vocational-technical education funds (input) to the public schools so that the output of graduates from these vocational programs at the local level make the most significant contribution toward reducing the existing demands of the labor market?

Since the model is based on a supply-demand criterion, it is assumed that the most significant contribution the public schools can make is to produce the maximum number of graduates (output) which will satisfy demands in the LMA not satisfied by any other education institution. Hence, the objective function in (30) is designed to maximize the number of additional graduates that the public school can produce in the m different occupational programs.

$$\text{Maximize} \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \tag{30}$$

where

x_{ij} = the output in program i from county j.

i = 1, 2, . . ., m (vocational-technical education programs).

j = 1, 2, . . ., n (counties in the LMA).

First, x_{ij} represents the output of additional students which the SDE would be willing to support. These are represented by the output vector Z^* found in (29). Second, these additional students would begin their vocational program at the start of the second year and complete their occupational training in the third year of the planning period.

THE CONSTRAINTS

In mathematical programming language, the constraints may be viewed as equations or inequalities that restrict the set of values the decision variables may assume. This restricted set of values forms the region of feasibility. In a nontechnical sense constraints are mathematical formulations of environmental conditions which place limitations on the set of alternate decisions available to the decision-maker. In this model, three general categories of constraints are formulated. There are LMA constraints, public school constraints and SDE constraints.

LABOR MARKET AREA CONSTRAINTS

The LMA constraints are based on the supply-demand information calculated in phase one. Using the d_i's from (28) above, the constraints are formed below.

$$\sum_{j=1}^{n} x_{ij} \leq d_i \qquad i = 1, 2, \ldots, m. \qquad (31)$$

These m inequalities insure that the additional output from the public school does not exceed the unsatisfied demands of the LMA for each of the m occupations in the model.

SCHOOL CAPACITY CONSTRAINTS

The variables in this model are the set of all x_{ij} which represent the additional output of graduates that the SDE would be willing to support in m different occupational programs in each of the j counties. The j's represent a classification by counties rather than by school districts. This provides the SDE greater flexibility to investigate the effects of increasing existing program enrollments and/or initiating new programs in the different occupational areas. For example, when counties are used as a basis for planning, more realistic parameterizations and ranges of sensitivity analyses can be performed to determine optimal allocation policies based on labor market areas. Also, area vocational-technical schools are usually designed to service a large number of school districts within a county or a regional area. With this trend towards larger intermediate units and centralization of resources, the county dimension becomes a more realistic basis for planning.

The school capacity constraints developed below allow the SDE to reflect its judgments about the future performance of the system. Capacity constraints are expressed in terms of the increased number of graduates that the system should produce in various occupational programs. It is assumed that the SDE decisions about the desired future outcomes of the vocational education system are based on a knowledge of the current long-range occupational trends in the labor force as well as updated information on occupational programs where student requests for admission exceed the existing level of program enrollments. For example, in this latter case, area vocational-technical schools have given participating school districts quotas in specific occupational programs. With a financial incentive to defray the cost of increased enrollments, these quotas could be subsequently raised, then more students desiring training could be accommodated.

The SDE decisions about increasing the output of graduates in m different occupational programs can be formed as constraints. Let

$$t_{ij} \leq x_{ij} \leq t'_{ij} \qquad \begin{array}{l} i = 1, 2, \ldots, m \text{ and} \\ j = 1, 2, \ldots, n. \end{array} \qquad (32)$$

$$t_{ij} = m_{ij}\ y_{ij2} \tag{33}$$

$$t'_{ij} = n_{ij}\ y_{ij2} \tag{34}$$

$$\sum_{i=1}^{m} x_{ij} \leq T_j \qquad j = 1, 2, \ldots, n. \tag{35}$$

$$T_j = q_j \sum_{i=1}^{m} y_{ij2} \qquad j = 1, 2, \ldots, n. \tag{36}$$

where

t_{ij} = the minimum increase that the SDE desires in the output of *program* i in county j.

t'_{ij} = the maximum increase that the SDE desires in the output of program i in county j.

m_{ij} = the minimum percentage of increase in *program* i in county j.

n_{ij} = the maximum percentoge of increase in program i in county j.

y_{ij2} = the output of program i in school j during the second yeor of the planning period (see (18) from phase one).

T_j = the maximum increose thot the SDE desires in the total output of all m programs in county j.

q_j = the moximum percentage of increose for all m progroms in county j.

Notice, if

$$\sum_{j=1}^{n} t'_{ij} \leq d_i \tag{37}$$

then the appropriote LMA constraint from (31) is not binding and may be *removed from the constraint set.*

STATE LEVEL BUDGET CONSTRAINT

Since the SDE does not have unlimited resources to finance vocotional-technical education programs that produce odditional groduates, a fixed moximum level of vocational educotion funds that con be allocated for this purpose must be specified.[4] It should be noted that the fixed allocation

[4] *See the original question posed by the State Board of Education. This question asks how the SDE can most efficiently allocate a fixed level of funds.*

also places a limitation on the feasible solutions that the model can produce. This limitation can be put in budget constraints such that

$$\sum_{j=1}^{n} h_i r_{ij} x_{ij} \leq H_i \quad i = 1, 2, \ldots, m. \tag{38}$$

where

h_i = the fixed amount of vocational education funds allocated by the SDE to the public schools in the n counties for each additional student enrolled in program i.

r_{ij} = the percentage of students in program i in county j necessary to produce the desired number of graduates at the completion of the two-year program ($r_{ij} \geq 1.0$ for all i and j).

H_i = the total amount of vocational education funds available for allocation by the SDE to the public schools to support additional students in program i.

It is assumed that the h_i's, the fixed allocation per student, would be sufficient to defray the additional instructional costs of the increasing vocational enrollments.[5] The r_{ij}'s are a weighting factor to compensate for the differences between first-year program enrollees and the graduates that will emerge from these enrollments following the completion of the two-year program.

The model can be optimized for different budget constraints depending on the values the SDE wishes to assign to each h_i and H_i in (38).

SUMMARY OF THE LINEAR PROGRAMMING MODEL

Given the objective function and constraints outlined above, the model can now be solved using standard linear programming techniques. In the model the additional output of the public school is maximized in such a manner that:

1. No output from any of the public schools in the LMA represents a duplication of a labor market demand.

2. No county capacity for training additional graduates is exceeded.

[5] For example, the National Educational Finance Project recently noted that the cost of training a vocational-technical education graduate is approximately 1.2 to 1.3 the per pupil cost in the general education program. See Erick L. Lindman and Edwin L. Kurth, "Dimensions of Need for Vocational Education," in Roe L. Johns, Kern Alexander and Richard Rossmiller (eds.), *Dimensions of Educational Need*, (Gainesville, Florida: National Educational Finance Project, 1969), p. 150.

3. No vocational education funds in excess of the fixed predetermined budget are necessary to completely support the additional graduates in all i programs in the LMA.

The linear programming model with its objective function and all of its constraints is outlined below.

THE OBJECTIVE FUNCTION

$$\text{Maximize} \qquad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \qquad (30)$$

THE CONSTRAINTS

$$\sum_{j=1}^{n} x_{ij} \leq d_i \qquad \text{(for all i)} \qquad (31)$$

$$t_{ij} \leq x_{ij} \leq t'_{ij} \qquad \text{(for all i and j)} \qquad (32)$$

$$\sum_{i=1}^{m} x_{ij} \leq T_j \qquad \text{(for all j)} \qquad (35)$$

$$\sum_{j=1}^{n} h_i r_{ij} x_{ij} \leq H_i \qquad \text{(for all i)} \qquad (38)$$

where

x_{ij} = the output from program i from county j.

i = 1, 2, . . . , m (vocational-technical education programs).

j = 1, 2, . . . , n (counties in the LMA).

d_i = the unsatisfied demand in the LMA for graduates in program i.

t_{ij} = the minimum increase that the SDE desires in the output of program i in county j.

t'_{ij} = the maximum increase that the SDE desires in the output of program i in county j.

T_j = the maximum increase that the SDE desires in the total output of all m programs in county j.

h_i = the fixed amount of vocational education funds allocated by the SDE to the public schools in the n counties for each additional student enrolled in program i.

r_{ij} = the percentage of students in program i in county j necessary to produce the desired number of graduates at the completion of the two-year program.

H_i = the total amount of vocational education funds available for allocation by the SDE to the public schools to support additional students in program i.

The optimal solution to the linear programming model provides the SDE with the maximum number of additional future graduates from the m occupational programs that it can support with a fixed level of vocational funds.

THE ALLOCATION FORMULA FOR LOCAL SCHOOL DISTRICTS

Using the solution from the model to govern the increased enrollments authorized in program i for any school district in the LMA, the allocation of vocational education funds for the second year of the planning period can be calculated using the following equation. Let

$$J_p = \sum_{i=1}^{m} \sum_{j=1}^{n} (h'_i K_{ip} + h_i L_{ip} + h'_i M_p) \qquad (39)$$

where

J_p = the total amount of funds allocated by the SDE to school p for the operation of all i programs.

K_{ip} = the terminal year enrollment in program i in school p.

L_{ip} = the additional initial year enrollment in program i in school p that is authorized to receive the special per pupil allocation.

M_{ip} = the balance of the initial year enrollment in program i in school p.

h'_i = the existing per pupil allocation for program i.

h_i = the special per pupil allocation for program i specified in the budget constraint.

Unless the additional initial year enrollment exceeds the amount authorized by the SDE, M_{ip} should equal the comparable initial year enrollment from the previous year. A similar formula could be derived for the final year of the planning period. This formula could be slightly altered if the model were run again in the third year of the planning period when the year t = 1, 2, 3, 4 would be considered. This is an extension of the model and is discussed below.

3.3 Model Extensions and Limitations

A logical extension of the model would be to use the optimal solution for t years of the planning period and run the model again for the years 2 through t + 1. In this case, it would be assumed that the optimal solution for the first three years is implemented. This process could be repeated an infinite number of times. For example, in the third stage it would be run for the years 3 through t + 2, in the fourth stage it would be run for the years 4 through t + 3, etc. In this manner, the performance of the system over time can be determined during the first year of the initial three-year planning period.

The model could also be operated once each year using a three-year planning period. In this case, the supply-demand posture for the cumulative number of years could be used. All other data would be based on a three-year planning period beginning with the year a and ending with the year t + a – 1. This second extension would approximate the existing decision-making process in the SDE's, i.e., a decision about program operation is made once each year.

LIMITATIONS OF THE MODEL

Using a supply-demand posture involves matching the output of graduates with the occupational demands of the LMA in which the training has been completed. This matching is a one-to-one correspondence and no mobility factor is applied to the graduate output data. This assumes that all graduates who enter the labor force upon completion of their training also enter the LMA in which the training has been completed. While this assumption does not hold for all cases, there are reasons to believe that the absence of a mobility factor does not limit the value of the model as an effective tool for program planning.

First, LMA's are geographic boundaries built on the basis of transportation accessibility and labor force mobility. Major LMA's (where a large portion of vocational-technical education programs are conducted) usually include more than one county. In Pennsylvania, for example, the Philadelphia and Pittsburgh LMA's each include a five county area while the Harrisburg LMA includes three. Hence, demands assembled on a LMA basis can account for a portion of student mobility.

Second, Eninger, reporting on the follow-up of a nationwide survey of public school vocational education graduates, notes that "the percentage of graduates that moved to another city to get their first job is negligible.

Of those who do move to other cities, the mojoriy of cases are for distonces less than 300 miles."[6] In an analysis of the Pennsylvania component of the nationwide survey, he notes that for the yeors in which the survey was conducted "less than six per cent of the Pennsylvania graduates moved to a new community for their first job. . . . There is no evidence of a trend toward increased geographic mobility."[7] Based on his findings he asserts:

> Vocational graduates are not a highly mobile population. Almost 95 per cent will find their first full-time job in the city in which they went to school. This would seem to imply that vocational curriculum planning is better based upon job opportunity forecasts that are local in nature rather than national or even regional.[8]

Based on studies discussed above and similar follow-up studies indicating the large percentage of vocational-technical education graduates employed in occupations for which they were trained, the formulation of the supply-demand posture without a mobility factor should not limit the utility of the analysis and the subsequent program planning policy decisions.[9]

Those who intend to interpret or utilize the model should be aware of the rationale which governs the LMA constraints and therefore ultimately the solution set of the linear programming model. The LMA constraints are based on the assumption that the d_i's in (31) represent LMA demands that no training institution in the LMA satisfies if the existing projected levels of student output are unchanged for the second and third year of the planning period. Recall from (31) $\sum_{j=1}^{n} x_{ij} \leq d_i$ for all i. Hence, no x^*_{ij} in the optimal solution set of the model duplicates the demands already assumed to be satisfied by the appropriate A^*_i and B^*_i from (25) calculated in phase one.

An interesting case arises for any occupational program i where $A^*_i + B^*_i \geq D_i$ from (25) in phase one. In this case, without increasing the public school output in the planning period, the total output of graduates

[6] Max U. Eninger, *The Process and Product of T and I High School Level Vocational Education in the United States: The Product*, (Pittsburgh, Pennsylvania: American Institutes for Research; September, 1965). This publication has no set of page numbers. The quote above can be located in the summary of Chapter 12.

[7] Max U. Eninger, *Report on Pennsylvania Data from a National Follow-Up Study of High School Level T and I Vocational Graduates*, (Pittsburgh, Pennsylvania: Educational Research Institute; May, 1968), p. 26.

[8] *Ibid.*, p. 26.

[9] For example see the discussion on the follow-up of vocational graduates in J. Robert Warbrod, *Review and Synthesis of Research on the Economics of Vocational Education*, (Columbus: The Center for Vocational-Technical Education, The Ohio State University, 1967), pp. 34-38.

(including those from the public school and those from all other training institutions) who will enter the LMA upon completion of their training exceeds the LMA demand.

The decision-maker, the State Board of Education, may wish to adhere to the policy that no increased financial incentive, h_i from (38), should be allocated to local school districts who provide additional graduates in occupational programs where the projected supply already exceeds the demand. In this case, the appropriate set of x_{ij}'s in the objective function and their corresponding constraints can be removed from the model, since in the optimal solution set the comparable x^*_{ij}'s would all be zero. This is not to infer that local school districts will not receive reimbursement for additional students in that occupational program. The standard reimbursement h'_i from (38) will be allocated to the local district for all enrollments not authorized to receive the special financial incentive h_i.

On the other hand, the decision-maker may not wish to be constrained by the outputs of the other training institutions in the LMA. In this case, the model can easily be adapted to allow for increases in the x_{ij}'s for that particular occupation. The appropriate LMA constraint in (31) can be replaced by

$$\sum_{j=1}^{n} x_{ij} = Q_i \qquad (39)$$

where

Q_i = the output of program i in excess of d_i which the SDE will support with the financial incentive h_i.

In either case above, the model can be utilized. It should be clear, however, that decisions about honoring the appropriate constraint in (31) or the modified constraint in (39) rests ultimately with the decision-maker not the model builder. His role is not to determine policy but rather to provide the decision-maker with information on the performance of the system if a particular alternative funding policy would be implemented.

3.4 Summary

The optimal solution for the model discussed in this chapter provides the State Board of Education a solution to its original question as shown below.

> How can the SDE most efficiently allocate a fixed level of vocational-technical education funds (input) to the public schools so that the output of graduates from these vocational programs at the local level make the most significant contribution toward reducing the existing demands of the labor market?

The model, based on a supply-demand criterion, provides new knowledge that the State Board of Education can use to determine its actual allocation policy. Viewed in its proper perspective, the economically efficient pattern of resource allocation yielded by the model is in competition with other allocation plans based on noneconomic criteria. The purpose of the model is not to make decisions for the State Board of Education (no model makes decisions), but rather to provide the Board with new information which can be used to formulate decisions about the allocation of vocational education funds to public schools.

In the next chapter, the model is applied to a selected set of vocational-technical education programs in the Philadelphia, Pennsylvania LMA. A solution such as the one described in (29) above will be generated.

Chapter Four

Application of the Model in a Pennsylvania Labor Market Area

In this chapter, the educational planning model is applied to a Pennsylvania LMA. Using the procedures developed in the previous chapter and the appropriate data from the Philadelphia LMA, optimal solutions are generated for a selected set of vocational-technical programs. The solutions provide the SDE with new information that can be used to determine future funding policies for public school vocational-technical education programs.

Using the Philadelphia LMA in the initial application of the model has a number of advantages. These can best be seen in a few of the characteristics of the region.

1. This five county area has a population of 3.7 million, which is approximately one-third of the entire population of the Commonwealth.

2. Vocational-technical education programs in this LMA are currently expanding to accommodate the manpower needs in the labor force which now exceeds 1.69 million workers.

3. This geographic area contains a significant number of comprehensive high schools and area vocational-technical schools with vocational education programs. For example, all school districts in the LMA operate at least two programs in Business Education.

4. The LMA has the largest concentration of postsecondary vocational-technical education schools and programs in the Commonwealth.

5. The training institutions in the LMA provide better than 29.2 percent of the total vocational-technical education graduates in the state.

6. Local school districts in this LMA vary in size from a small rural district to a large urban district such as Philadelphia with a combined total full-time elementary and secondary enrollment in excess of 293,000.

The Philadelphia LMA contains a labor force who are employed in five Pennsylvania and three New Jersey counties. This analysis, however, deals only with the supply and demand data for the Pennsylvania counties. These are Bucks, Chester, Delaware, Montgomery and Philadelphia. This five-county area coincides with the geographic boundaries of the Southeastern

Region, a subdivision of the Commonwealth, used by the Pennsylvania State Planning Board for economic planning and development.[1]

AN OVERVIEW OF PROGRAMS IN THE MODEL

The vocational-technical education programs included in this application of the model are from the area of Business and Distributive Education.[2] The total output of public school graduates in these two program areas currently account for approximately:

1. 68.3 per cent of the graduates in Business and Distributive Education programs in the LMA.

2. 72 percent of all public school vocational-technical education graduates in the LMA.

3. 36.4 percent of the total output of vocational-technical education graduates from all programs operating in the various training institutions found in the LMA.[3]

Specifically, the model includes three instructional programs in Business Education and one general category for instruction in Distributive Education. In this application, there are 20 variables, one for each instructional program in each of the five counties.

The instructional programs in Business Education include:

1. Accounting Clerk and Bookkeeper (14.0101 to 14.0105).
2. General Clerical Occupations (14.0301 to 15.0303).
3. Stenographic and Secretarial (14.0701 to 14.0703).[4]

Hereafter, these above will be labeled Bookkeeping, Clerical, and Stenographic, respectively.

---

[1] A description of additional characteristics and demographic measures about this region can be found in the Pennsylvania State Planning Board publication entitled, *Pennsylvania's Regions: A Survey of the Commonwealth*, (Harrisburg: PSPB, 1967).

[2] Other subdivisions of vocational-technical education programs in Pennsylvania include Agricultural, Technical, Trade and Industrial, Health and Home Economics Education. These are explained in Walter M. Arnold (ed.), *Vocational, Technical and Continuing Education in Pennsylvania: A Systems Approach to State-Local Program Planning* (Harrisburg: Pennsylvania Department of Education, 1969).

[3] *Ibid.*, pp. 184-188. The statistics above were derived using the data published in Tables 94 and 95.

[4] These instructional programs and their corresponding code numbers from the U. S. Office Classification Scheme can be found in *Vocational Education and Occupations*, OE-80061, (Washington, D. C.: Government Printing Office; July, 1969). This publication describes the characteristics of all occupational programs and allows the researcher to link vocational education programs with the U. S. Department of Labor's *Dictionary of Occupational Titles*.

Distributive Education is comprised of programs of occupational instruction in the field of distribution and marketing. Distributive Education programs are relatively new and not common in most public secondary schools. In this application, the different types of Distributive Education programs are aggregated. The appropriate U. S. Office of Education code numbers are 04.0100 to 04.1200, and Distributive Education will, hereafter, be replaced by the term, Marketing.

DATA PRESENTATION

The data necessary to complete phase one of the model, the supply-demand postures, are presented in Section 4.1. The data for the parameters and the formulation of the linear programming model of phase two are contained in Section 4.2. In each case, these sections will follow the outline of phases one and two from the previous chapter. Model solutions are presented in Section 4.3.

The planning period in this application is three years. It begins with the school year 1968-9 and ends with the school year 1970-71. Since this study is conducted in the early part of 1970, the system is theoretically in the second year of operation. For the purpose of this investigation, it is assumed that the application is carried out at the termination of the base year of the planning period. This assumption does not change the procedures or outcomes of the investigation.

4.1 The Supply-Demand Posture for the Philadelphia LMA

The purpose of this phase is to calculate a supply-demand posture for the four vocational programs in this application of the model. The supply-demand posture indicates the LMA demands not satisfied when current program levels (enrollment trends) are unchanged for the second and third year of the planning period. The results of this phase provide the data to determine the LMA and school capacity constraints for the linear programming model.

Table 1 contains the graduates of all four programs by county for the first year of the planning period. Table 2 contains the projections of graduates in these programs for the three-year planning period. Here it is assumed that graduates in the third year are equal to the comparable number of graduates for the previous year. This assumption is outlined in the discussion following (5) in Chapter Three.[6]

---

[6] All numbers in parentheses; such as the one above, refer to the functions and relationships outlined in Chapter Three.

The column totals in Table 2 correspond to the A'_i's in (16). The values for i in this application are defined such that

1 = Bookkeeping.

2 = Clerical.

3 = Stenographic.

4 = Marketing.

and the values for j, the counties, are defined such that

1 = Bucks.

2 = Chester.

3 = Delaware.

4 = Montgomery.

5 = Philadelphia.

The values for t, the years in the planning period, are defined such that

1 = the fiscal year ending June 30, 1969.

2 = the fiscal year ending June 30, 1970.

3 = the fiscal year ending June 30, 1971.

The adjusted totals in Table 2 represent A^*_i in (23) where $a_i = 0.80$ for all i.

Table 3 contains the projections of postsecondary vocational education graduates in the LMA for the base year of the planning period. Since certain types of training institutions described in Chapter Three do not offer programs in the area of Business and Distributive Education, the values for k in (17) are modified, such that

1 = Community Colleges.

2 = Private Business Schools.

3 = Manpower Development Training Act Programs.

4 = Two-Year Programs in Colleges and Universities.

5 = Private Junior Colleges.

Table 4 shows the three-year projections of postsecondary graduates. The column totals in Table 4 correspond to the B'_i in (17). The adjusted totals in Table 4 correspond to the B^*_i in (24) where $b_i = 0.95$ for all i.

Table 5 shows the decision-maker the end product of phase one, the shortage of trained graduates in the Philadelphia LMA by occupational program. This table corresponds to (25). At this point, the decision-maker

TABLE 1. PROJECTED PUBLIC SCHOOL VOCATIONAL EDUCATION GRADUATES BY COUNTY IN THE PHILADELPHIA LABOR MARKET AREA FOR THE SCHOOL YEARS ENDING JUNE 30, 1969 AND 1970

County	Bookkeeping[a]		Clerical[a]		Stenographic[a]		Marketing[b]	
Name	1969	1970	1969	1970	1969	1970	1969	1970
Bucks	314	237	395	388	443	379	77	123
Chester	159	188	316	239	229	320	61	95
Delaware	282	531	557	356	510	525	255	319
Montgomery	281	351	285	281	420	523	123	77
Philadelphia	1,102	688	1,334	3,053	1,324	791	969	2,008
Total	2,138	1,995	2,887	4,317	2,926	2,538	1,485	2,622

[a] Projections of graduates derived from the 1969 Business Education Annual Reports, Pennsylvania Department of Education Form PIBE-50 (Rev. 3/68) submitted by individual school districts.

[b] Projections of graduates derived from the 1969 Distributive Education Annual Reports, Pennsylvania Department of Education Form PIBE-192 (Rev. 3/68) submitted by individual school districts.

TABLE 2. PROJECTED PUBLIC SCHOOL VOCATIONAL EDUCATION GRADUATES BY COUNTY IN THE PHILADELPHIA LABOR MARKET AREA FOR THE THREE-YEAR PLANNING PERIOD ENDING JUNE 30, 1971

County Name	Bookkeeping	Clerical	Stenographic	Marketing
Bucks	788	1,171	1,201	323
Chester	535	794	869	251
Delaware	1,334	1,269	1,560	893
Montgomery	983	847	1,466	277
Philadelphia	2,478	7,440	2,906	4,985
Total	6,128	11,521	8,002	6,729
Adjusted Total[a]	4,902	9,217	6,402	5,383

[a] Adjusted totals represent the percentage of graduates assumed to enter the labor force upon completion of their occupational training.

TABLE 3. PROJECTED POSTSECONDARY VOCATIONAL EDUCATION GRADUATES IN THE PHILADELPHIA LABOR MARKET AREA BY TYPE OF INSTITUTION FOR THE SCHOOL YEAR ENDING JUNE 30, 1969

Type of Institution	Bookkeeping	Clerical	Stenographic	Marketing
Community College[a]	19	9	45	7
Private Business School[b]	383	431	566	205
Manpower Development Training Act[c]	20	31	106	37
Two-Year Programs in Colleges and Universities[a]	24	—	14	—
Private Junior Colleges[a]	116	—	164	120
Total	*562*	*471*	*895*	*369*

[a] Data for these types of institutions are actual rather than projected graduates. Data were calculated from Roger G. Hummel, *Degrees and Other Formal Awards Conferred by Pennsylvania Institutions of Higher Education 1968-1969*, (Harrisburg: Pennsylvania Department of Education, 1969).

[b] Projections of graduates derived from Annual Instructional Program Summary for Private Schools, Pennsylvania Department of Education Form PIBE-357B (3/69) submitted by individual private schools following the completion of the school year ending June 30, 1968.

[c] Projections of graduates based on Bureau of Vocational, Technical and Continuing Education Manpower Development Training Act Reports for authorized programs in Pennsylvania for the year ending June 30, 1968.

TABLE 4. PROJECTED POSTSECONDARY VOCATIONAL EDUCATION GRADUATES IN THE PHILADELPHIA LABOR MARKET AREA FOR THE THREE-YEAR PLANNING PERIOD ENDING JUNE 30, 1971

Type of Institution	Bookkeeping	Clerical	Stenographic	Marketing
Community College	63	30	150	24
Private Business School	1,207	1,360	1,786	646
Manpower Development Training Act	60	93	318	111
Two-Year Programs in Colleges and Universities	78	—	45	—
Private Junior Colleges	346	1,486	2,754	1,162
Total	1,754	2,969	5,053	1,943
Adjusted Totals[a]	1,666	2,821	4,800	1,846

[a] Adjusted totals represent the percentage of graduates assumed to enter the labor force upon completion of their occupational training.

TABLE 5. THE SUPPLY-DEMAND POSTURE FOR THE PHILADELPHIA LABOR MARKET AREA FOR THE THREE-YEAR PLANNING PERIOD ENDING JUNE 30, 1971

Occupations	Demand[a]	Public School Supply	Postsecondary Supply	Occupational Shortages
Bookkeeping	9,813	4,902	1,666	3,245
Clerical	28,197	9,217	2,821	16,159
Stenographic	15,687	6,402	4,800	4,485
Marketing	17,823	5,383	1,846	10,594
Total	71,520	25,904	11,133	34,483

[a] The projected three-year demand derived from the annual demand for specific occupations found in Walter M. Arnold (ed.), *Vocational, Technical and Continuing Education in Pennsylvania: A Systems Approach to State-Local Program Planning*, (Harrisburg: Pennsylvania Department of Education, 1969), pp. 196-200.

can use these projections of unmet demands in Table 5 to formulate his choices about program changes he wishes to initiate in the public schools. These decisions will then be translated into parameters which are used in phase two. In this application, no supply figure exceeds the corresponding demand. In fact, there exists a large discrepancy in all four cases.

4.2 The Linear Programming Model for the Philadelpha LMA

The purpose of developing a linear programming model in phase two is to generate an optimal solution for the Philadelphia LMA given information on (1) the unmet demands of the LMA, (2) constraints on the desired additional program capacity of the public schools, and (3) a set of budget constraints. This latter set of information is based on the vocational-technical education funds that the State Board may use to encourage public schools to produce additional graduates in the third year of the planning period. Since the vocational-technical education programs in this application take two years to complete, these additional students would begin their program in the second year of the planning period.

The optimal solution to the linear programming model provides the State Board of Education with new information to improve future resource allocations for the public schools in the LMA. Various alternatives that the State Board may wish to investigate are formulated in the constraint set of the model. The objective function is linear. It is designed to maximize the output of additional graduates in each vocational-technical education program.

THE OBJECTIVE FUNCTION

The objective function in this application can be written

$$\text{Maximize} \qquad Z = \sum_{i=1}^{4} \sum_{j=1}^{5} x_{ij} \qquad (40)$$

where

Z = a scalar representing the total output of additional graduates in the third year of the planning period.

x_{ij} = the output in program i from county j.

i = 1, 2, 3, 4 (vocational-technical education programs).

j = 1, 2, 3, 4, 5 (counties in the Philadelphia LMA).

The constraints that follow represent restrictions on the values of the x_{ij}'s. Specifically, the constraints represent various alternatives that the State Board wishes to put on the additional number of graduates they will support.

LABOR MARKET AREA CONSTRAINTS

LMA constraints are formed using the data from Table 5. In this application, $g_i = 1$ for all i. Hence, $C^*_i = d_i$ for all occupational programs. Using the d_i's from (28), the constraints are formed below.

$$\sum_{j=1}^{5} x_{1j} \leq 3245 . \tag{41}$$

$$\sum_{j=1}^{5} x_{2j} \leq 16159. \tag{42}$$

$$\sum_{j=1}^{5} x_{3j} \leq 4485 . \tag{43}$$

$$\sum_{j=1}^{5} x_{4j} \leq 10594. \tag{44}$$

These four inequalities insure that the additional output from the public school does not exceed the unsatisfied demands of the LMA for each occupational in the model.

SCHOOL CAPACITY CONSTRAINTS

The school capacity constraints developed below allow the SDE to reflect its judgments about the future performance of the public schools in the Philadelphia LMA. Capacity constraints are expressed in terms of the increased number of graduates that the SDE desires to produce in each county. Table 6 provides a typical set of program increases that the decision-maker would be expected to provide. Upper and lower bounds on the x_{ij}'s indicate minimum and maximum program growth that the SDE desi...

Notice that for Philadelphia County (j = 5), the upper and lower bounds for each program are less than those for the other counties. This is because the Philadelphia School District is the only district in that county. All other counties have at least nine districts offering one or more programs.

The data in Table 6 can easily be translated into the constraints described in (32). For example, using the table entry in row one, the appropriate constraint is $24 < x_{11} < 48$. The other nineteen constraints can be set up in a similar manner.

TABLE 6. SUMMARY OF THE SCHOOL CAPACITY CONSTRAINTS BY COUNTY BY OCCUPATIONAL PROGRAM

Occupational Program	County	y_{ij2}	m_{ij}	n_{ij}	t_{ij}	t'_{ij}
1	1	227	0.10	0.20	24	48
1	2	188	0.10	0.20	18	36
1	3	531	0.10	0.20	53	106
1	4	351	0.10	0.20	35	70
1	5	688	0.05	0.10	34	68
2	1	388	0.15	0.25	58	98
2	2	239	0.15	0.25	36	60
2	3	356	0.15	0.25	53	89
2	4	281	0.15	0.25	42	70
2	5	3,053	0.10	0.20	305	610
3	1	379	0.10	0.20	38	76
3	2	320	0.10	0.20	32	64
3	3	525	0.10	0.20	53	106
3	4	523	0.10	0.20	52	104
3	5	791	0.05	0.10	40	80
4	1	123	0.20	0.40	25	50
4	2	95	0.20	0.40	19	38
4	3	319	0.20	0.40	64	128
4	4	77	0.30	0.40	23	46
4	5	2,008	0.10	0.20	201	402

TABLE 7. SUMMARY OF THE SCHOOL CAPACITY CONSTRAINTS BY COUNTY

County	Total Number of Graduates in 1970	q_j	T_j
1	1,127	0.20	225
2	842	0.20	168
3	1,667	0.20	332
4	1,232	0.20	246
5	6,540	0.20	1,308

The data in Table 7 indicate the total maximum number of additional graduates that the decision-maker wishes to generate in each county. Using the data in Table 7 and description of the constraints in (36), the total school capacity constraints can be formed for each county.

$$\sum_{i=1}^{4} x_{i1} \leq 225. \tag{45}$$

$$\sum_{i=1}^{4} x_{i2} \leq 168. \tag{46}$$

$$\sum_{i=1}^{4} x_{i3} \leq 332. \tag{47}$$

$$\sum_{i=1}^{4} x_{i4} \leq 246. \tag{48}$$

$$\sum_{i=1}^{4} x_{i5} \leq 1308. \tag{49}$$

Although the maximum percentage of total increase in graduates from all four programs varies from county to county (as can the t_{ij} and t'_{ij} for each variable in the model), all q_j from (36) are set at 0.20 for this particular application.

TABLE 8. SUMMARY OF THE BUDGET CONSTRAINTS BY OCCUPATIONAL PROGRAM

Occupational Program	County	h_i	r_{ij}	$h_i r_{ij}$
1	1	150.00	1.11[a]	167.00
1	2	150.00	1.16	174.00
1	3	150.00	1.09	164.00
1	4	150.00	1.06	159.00
1	5	150.00	1.15	173.00
2	1	100.00	1.19	119.00
2	2	100.00	1.12	112.00
2	3	100.00	1.10	110.00
2	4	100.00	1.17	117.00
2	5	100.00	1.12	112.00
3	1	200.00	1.09	218.00
3	2	200.00	1.17	234.00
3	3	200.00	1.06	212.00
3	4	200.00	1.11	222.00
3	5	200.00	1.18	236.00
4	1	150.00	1.08[b]	162.00
4	2	150.00	1.16	174.00
4	3	150.00	1.09	164.00
4	4	150.00	1.07	161.00
4	5	150.00	1.19	179.00

[a] The r_{ij}'s for Business Education programs derived from the Business Education Annual Reports, Pennsylvania Department of Education Form PIBE-50 (Rev. 3/68) submitted by individual school districts.

[b] The r_{ij}'s for Distributive Education programs derived from the Distributive Education Annual Reports, Pennsylvania Department of Education Form PIBE-192 (Rev. 3/68) submitted by individual school districts.

BUDGET CONSTRAINTS

Since the state does not have unlimited resources to finance vocational-technical education programs, the decision-maker must specify a fixed maximum level of funds per program that can be allocated for this purpose. Table 8 provides the information necessary to form the required budget constraints outlined in (38). The value of each h_i in this table is the fixed amount of vocational education funds allocated to a school district for every additional student enrolled in program i.

Notice that the allocation is based on additional students enrolled rather than additional graduates. This allows the SDE to support an initial additional enrollment sufficient to generate the appropriate number of additional graduates two years later. Hence, $r_{ij}x_{ij}$ indicates the initial additional enrollment necessary to generate x_{ij} graduates two years later. Further, there is no reason to expect that each r_{ij} from (38) must equal the corresponding f_{ij2} from (19). This would hold only when the attrition rate is constant from year to year. With the expansion of program enrollments and the continued improvement of vocational guidance, this is not likely.

Using the data from Table 8, a budget constraint can be formed for each occupational program in the model.

$$\sum_{j=1}^{5} h_1 r_{1j} x_{1j} \leq 50000.00. \tag{50}$$

$$\sum_{j=1}^{5} h_2 r_{2j} x_{2j} \leq 100000.00. \tag{51}$$

$$\sum_{j=1}^{5} h_3 r_{3j} x_{3j} \leq 80000.00. \tag{52}$$

$$\sum_{j=1}^{5} h_4 r_{4j} x_{4j} \leq 100000.00. \tag{53}$$

The entire set of constraints and the objective function have been formulated. These equations and inequalities can now be transformed into the appropriate form to run the IBM 360 mathematical programming algorithm.

4.3 Solutions to the Model

Using the objective function and the constraints outlined in Section 4.2, an optimal solution can now be calculated. The optimal solution and the subsequent information that can be presented to the decision-maker are shown in Tables 9 through 13.

Table 9 contains the summary of the solution set outlined in (29); Z^* is the total additional graduates and each x^*_{ij} in the optimal solution is defined by using the entry in the table next to the appropriate occupational program and county. The optimal solution appears in the column labeled graduates. The other column labeled enrollments includes the number of enrollees necessary to generate the x^*_{ij} graduates at the completion of the two-year program. The enrollments equal $r_{ij}x^*_{ij}$. First year allocations per county are based on enrollments. This can be seen in the budget constraints in (50) through (53).

The x^*_{ij}'s found in the column of Table 9 labeled Graduates are not all integers in the optimal solution. Where the activity level is not expressed as an integer in the optimal solution, that x^*_{ij} is rounded to the nearest integer satisfying the constraints. Hadley notes in such circumstances "this is frequently done in practice and is a perfectly valid approach when the values of the variables are sufficiently large that rounding has a negligible effect."[8] If the budget constraints, (50) through (53), are removed from the constraint set; each activity level in the optimal solution would be expressed as an integer.

Based on the optimal solution, a set of additonal information can be prepared for the decision-maker. Table 10 shows the allocation of funds to each county for each program in the model. Also, no budget limitation expressed in the budget constraints is exceeded.

Table 11 provides the decision-maker with the actual percentage of increase in graduates by county and by program. The lower and upper bounds for each county in the table represent the corresponding t_{ij} and t'_{ij} specified by the decision-maker in Table 6 prior to the calculation of the optimal solution. Notice that no actual increase in program output for any entry exceeds the range for x^*_{ij} set by the upper and lower bounds in Table 6.

In Table 12 the decision-maker is provided with the actual percentage of increase in the total number of graduates per county resulting from the optimal solution. The column labeled Maximum Increase represents the q_i's specified by the decision-maker in Table 7. Notice that the actual percentage of increase in the total output is less than or equal to the corresponding maximum increase specified by the decision-maker prior to calculating the optimal solution.

Table 13 shows the new supply-demand posture that would result from the additional 1971 graduates described in Table 9. In this table, it is assumed that 90 percent of the additional 1971 graduates would enter the labor market upon completion of their training.

The information in Tables 9 through 13 provides the decision-maker with a set of information describing the long-range consequences of implementing an optimal solution. This type of planning data increases the information

[8] George Hadley, *Nonlinear and Dynamic Programming*, (Reading, Massachusetts: Addison-Wesley Publishing Company, 1964), p. 251.

TABLE 9. SUMMARY OF THE OPTIMAL SOLUTION TO THE LINEAR PROGRAMMING MODEL FOR THE PHILADELPHIA LABOR MARKET AREA

Item Description	Graduates	Enrollments
Total Additional Graduates	2,119	2,395
SUMMARY BY PROGRAM		
Accounting		
Bucks	48	53
Chester	36	42
Delaware	78	85
Montgomery	70	74
Philadelphia	68	78
Total	300	332
General Clerical		
Bucks	58	69
Chester	60	67
Delaware	89	98
Montgomery	70	82
Philadelphia	610	683
Total	887	999
Stenographic		
Bucks	68	74
Chester	49	57
Delaware	101	107
Montgomery	60	67
Philadelphia	80	94
Total	358	399
Marketing		
Bucks	50	54
Chester	23	27
Delaware	64	70
Montgomery	46	49
Philadelphia	391	465
Total	574	665
SUMMARY BY COUNTY		
Bucks	224	250
Chester	168	193
Delaware	332	360
Montgomery	246	272
Philadelphia	1,149	1,320
Total	2,119	2,395

TABLE 10. SUMMARY OF THE ALLOCATION OF VOCATIONAL EDUCATION FUNDS RESULTING FROM THE OPTIMAL SOLUTION TO THE LINEAR PROGRAMMING MODEL

Item Description	Allocations
Total Allocation	329,250.00
SUMMARY BY PROGRAM	
Accounting	
Bucks	7,950.00
Chester	6,300.00
Delaware	12,750.00
Montgomery	11,100.00
Philadelphia	11,700.00
Total	49,800.00
General Clerical	
Bucks	6,900.00
Chester	6,700.00
Delaware	9,800.00
Montgomery	8,200.00
Philadelphia	68,300.00
Total	99,900.00
Stenographic	
Bucks	14,800.00
Chester	11,400.00
Delaware	21,400.00
Montgomery	13,400.00
Philadelphia	18,800.00
Total	79,800.00
Marketing	
Bucks	8,100.00
Chester	4,050.00
Delaware	10,500.00
Montgomery	7,350.00
Philadelphia	69,750.00
Total	99,750.00
SUMMARY BY COUNTY	
Bucks	37,750.00
Chester	28,450.00
Delaware	54,450.00
Montgomery	40,050.00
Philadelphia	168,550.00
Total	329,250.00

TABLE 11. SUMMARY OF THE PERCENTAGE OF INCREASE IN 1971 GRADUATES BY OCCUPATIONAL PROGRAM RESULTING FROM THE OPTIMAL SOLUTION TO THE LINEAR PROGRAMMING MODEL

Item Description	Lower Bound	Upper Bound	Actual[a] Increase
Accounting			
Bucks	10.0	20.0	20.00
Chester	10.0	20.0	20.00
Delaware	10.0	20.0	14.69
Montgomery	10.0	20.0	20.00
Philadelphia	5.0	20.0	10.00
Total	—	—	15.04
General Clerical			
Bucks	15.0	25.0	15.00
Chester	15.0	25.0	25.00
Delaware	15.0	25.0	25.00
Montgomery	15.0	25.0	25.00
Philadelphia	10.0	20.0	20.00
Total	—	—	20.55
Stenographic			
Bucks	10.0	20.0	17.94
Chester	10.0	20.0	15.31
Delaware	10.0	20.0	19.24
Montgomery	10.0	20.0	11.47
Philadelphia	5.0	10.0	10.00
Total	—	—	14.11
Marketing			
Bucks	20.0	40.0	40.00
Chester	20.0	40.0	24.21
Delaware	20.0	40.0	20.00
Montgomery	30.0	40.0	40.00
Philadelphia	10.0	20.0	19.47
Total	—	—	21.89

[a] Actual Increase is expressed as a percentage derived from the ratio of additional graduates in 1971 to corresponding number of total graduates in 1970.

TABLE 12. SUMMARY OF THE PERCENTAGE OF INCREASE IN 1971 GRADUATES BY COUNTY RESULTING FROM THE OPTIMAL SOLUTION TO THE LINEAR PROGRAMMING MODEL

County	Maximum Increase	Actual Increase[a]
Bucks	20.0	19.86
Chester	20.0	19.95
Delaware	20.0	20.00
Montgomery	20.0	20.00
Philadelphia	20.0	17.57
Total	—	18.57

[a] Actual Increase is expressed as a percentage derived from the ratio of additional graduates in 1971 to corresponding number of total graduates in 1970.

base upon which policy decisions about future resource allocations are made. The planning data are generated in keeping with the guidelines of PPBS, which allow the decision-maker first to focus on inputs and outputs rather than inputs alone and second to build in a dimension over time that tries to see today's decisions in terms of their longer term consequences.

4.4 Discussion of the Model

In terms of the model described in Chapter Three, phase two requires the decision-maker to provide the following parameters:

1. m_{ij}, the minimum percentage of increase in program i in school j.
2. n_{ij}, the maximum percentage of increase in program i in school j.
3. q_j, the maximum percentage of increase in the total output of county j.
4. h_i, the fixed amount of vocational education funds allocated to the public schools for each additonal student authorized to enroll in program i.
5. H_i, the fixed amount of vocational education funds to be allocated to support additional students in program i.
6. r_{ij}, the percentage of students in program i in county j necessary to produce the desired number of graduates at the completion of the two-year program.

These parameters are used to define the constraint set of the model. This application of the model in the Philadelphia LMA requires only 17 seconds of computer time to generate a solution using the mathematical program-

TABLE 13. THE NEW SUPPLY-DEMAND POSTURE FOR THE PHILADELPHIA LABOR MARKET AREA RESULTING FROM THE OPTIMAL SOLUTION TO THE LINEAR PROGRAMMING MODEL

Occupations	Demand	Public School Supply	Additional Public School Supply	Postsecondary Supply	Occupational Shortages
Bookkeeping	9,813	4,902	270	1,666	2,975
Clerical	28,197	9,217	798	2,821	15,361
Stenographic	15,687	6,402	322	4,800	4,163
Marketing	17,823	5,383	517	1,846	10,077
Total	71,520	25,904	1,907	11,133	32,576

ming algorithm and an IBM 360-67 computer. Since it has been demonstrated that this is a relatively inexpensive model to run, a number of optimal solutions for different alternatives specified by the decision-maker in 1 through 6 above could be generated. For example, the model could be run using higher or lower budget constraints for each program. The decision-maker might also wish to alter the minimum and maximum program growth or reduce the fixed amount of subsidy for each student. The model can easily reflect any of these changes in the parameters specified by the decision-maker.

The purpose of this application has been to exhibit the capability of the model to provide the decision-maker with new information to formulate policy decisions about the future resource allocations to certain vocational-technical education programs in the Philadelphia LMA. No attempt is made in this study to investigate other selected optimal solutions that could result from changing the decision-maker's parameters in 1 through 6. An infinite number of possibilities exist. Other optimal solutions would be generated by formulating the linear programming model in the same manner as outlined in Section 4.2.

VARIABLES IN THE MODEL

In Chapter Three the model has been constructed for i programs in j counties. In this application there were only four programs and five counties which resulted in 20 variables in the model. It should not be concluded that future applications of the model are limited to such a small number of variables. For example, Bruno has shown in a similar educational planning model that 92 variables resulting in a 182 by 92 matrix can be easily and inexpensively solved using the IBM 360 mathematical programming algorithm.[7]

FUTURE ALLOCATIONS OF VOCATIONAL FUNDS

The budget necessary to implement the results of the optimal solution in the local school districts for the second year of the planning is described in Table 9. If the SDE wishes to provide the local school with the same financial incentive for these additional students when they enter the second year of their two-year vocational-technical education program, a similar budget would be prepared the following year.

The model does not specify that the financial incentive should be given for the additional students in their second year of the program. This option

[7] James E. Bruno, "An Alternative to the Use of Simplistic Formulas for Determining State Resource Allocation in School Finance Programs," *American Educational Research Journal*, 6: 495; November, 1969.

rests with the decision-maker. Possibly the financial incentive, h_i, could be viewed as "seed money" given only in the initial year. In this case, "seed money" could be used the following year to generate new enrollments in other occupational programs also having a critical shortage in the LMA but not included in the current application of the model.

IMPLEMENTING THE SOLUTION

The solution to the model in Table 9 is expressed in terms of additional graduates per program per county rather than per program per school district. The value of using counties rather than school districts has been discussed in Chapter Three. To implement the solution, however, the SDE must ultimately specify the number of additional students per school district who will receive the financial incentive h_i.

One approach to this problem would be to allow each school district to claim an appropriate reimbursement for additional students using the formulas below.

$$X_{ijp} \leq w_{ij} Y_{ijp}. \tag{54}$$

$$W_{ijp} = h_i X_{ijp}. \tag{55}$$

where

X_{ijp} = the additional students in program i in school district p in county j who are eligible to receive the financial incentive h_i.

w_{ij} = the actual increase in students in program i in county j reflected in the optimal solution. (The figures are contained in Table 11 as percents and should be converted to decimals.)

Y_{ijp} = the first year students in program i in school p of county j during the year ending June 30, 1969.

W_{ijp} = the allocation of vocational funds to school district p in county j for additional graduates in program i authorized to receive the financial incentive h_i.

p = 1, 2, ..., r (the local school districts within a county).

Possibly all schools would not wish to expand their enrollments to the level of the optimal solution. In this case, the inequality in (54) would hold. Hence, an alternative method for allocating the financial incentive to specific local school districts should be developed. It should be clear, however, that the solution to the linear programming model can be calculated independent of any particular policy developed for allocating the budgets in Table 10 to local school districts within counties.

4.5 Summary

The application of the educational planning model to selected vocational-technical education programs in the Philadelphia LMA indicates that linear programming techniques can be utilized effectively by states to provide valuable information for determining future resource allocation policies in vocational education. Specifically, the model provides a method for examining the long-term consequences and budget requirements for alternative strategies that the decision-maker may wish to implement in the state system.

It is important to note that the *optimal solution* to the model not only provides a measure of the maximum number of graduates that the public schools could generate, but also forms the basis for constructing other decision-making information that should influence resource allocation strategies. This information and the manner in which it should be presented to the decision-maker has been outlined in Tables 9 through 13. This additional planning information includes:

1. The initial enrollments necessary to generate a specific number of graduates in a program taking two academic years to complete. In this case, the decision-maker is forced to review not only the producton of graduates but also to consider the attrition rates that exist within programs.

2. The budget necessary to support each program in each county. This allows the decision-maker to review the state system as a set of input-output or production relationships. The inputs are the vocational education funds allocated by the state while the outputs of the system become the number of trained graduates resulting from the porticular allocation of funds.

3. The net effect of the output of graduates on the supply-demand posture in the LMA. This allows the decision-maker to view the result of his alternative in terms of the number of LMA demands that would be satisfied by the production of additional graduates. He is forced to distinguish between *graduates* of occupational programs and its actual number of graduates who enter the labor market upon completion of their program.

It should be clearly understood that the mathematical planning model does not make decisions nor can it replace judgment on the part of the decision-maker. The model is designed to aid and support the decision-maker by providing information that allows him to be much clearer about the implications of different alternatives.

Finally, it has been shown that the educational planning model developed in this study provides a relatively inexpensive method for investigating various alternatives regarding the *future performance* of the vocational education system. It is inexpensive for two reasons. First, the model uses existing data which have already been gathered for accounting and reporting purposes. Second, the computer time necessary to derive solutions for the model is less than two minutes.

Chapter Five

Summary and Conclusions

The purpose of developing the mathematical programming model in this investigation has been to provide the decision-maker, the State Board of Education, with new information to evaluate decisions about the efficient allocation of vocational education funds to local school districts. The mathematical model has been designed specifically to answer the following question posed by a State Board of Education.

> How can the SDE most efficiently allocate a fixed level of vocational-technical education funds (input) to the public schools so that the output of graduates from these vocational programs at the local level make the most significant contribution toward reducing the existing demands of the labor market?

Given this question, it has been demonstrated that linear programming techniques can be effectively utilized to provide valuable information for determining future resource allocation policies in vocational education. Specifically, the model can provide a method for examining the long-term consequences of alternative strategies that the decision-maker may wish to implement in the state system.

The model has been developed in keeping with the guidelines set forth in the PPBS which force the decision-maker

1. To focus on inputs and outputs rather than inputs alone.
2. To assure the decision-maker a choice of valid comparable alternatives.
3. To build in a dimension overtime that tries to see today's decisions in terms of their longer term consequences.

The application of the model to the Philadelphia LMA shows that these guidelines have been utilized to generate appropriate planning information for the decision-maker. In the application of the model, emphasis has been placed not only on the solution to the model, but also on the type of additional information that can be given to the decision-maker as a result of the solution. This information includes budget requirements by program, enrollments necessary to generate future graduates and the net long-term effect of the decision-maker's strategy on the future supply-demand posture of the labor market.

It has been demonstrated that the model formulates a generalizable procedure that can be applied to any labor market in the state. In fact, since labor markets are federal subdivisions, schools across the country exhibit similar organizational characteristics and State Departments of Education have similar decision-making structures, this model provides a procedure that can be applied in any state. The generalizability of the model is also enhanced by the fact that the administration of vocational education programs in every state must follow the regulations set forth in the guidelines provided by the U. S. Office of Education.

One valuable property of the model developed in this investigation is that it uses data which currently exist in State Departments of Education. These data have been used in the past few years almost exclusively for accounting and reporting purposes. The model has provided a guide for demonstrating how such data can be analyzed and structured to improve the current information base upon which decisions about future resource allocations are made.

Another principal value of a model such as the one constructed in this investigation is that it forces the educational planner and the decision-maker to face explicitly fundamental questions. For example, the planner must establish an adequate measure of the performance of the educational system. Also he must be able to express explicitly in terms of the model such things as differences in the cost of various training programs as well as variations in the attrition rates in different geographic areas. The point to be emphasized is that models and the research associated with them can help to make these kinds of facts explicit, but the models do not by themselves provide the basis for deciding which allocations of resources are appropriate.

5.1 Recommendations for Future Research Using Mathematical Models

The results of the study reported in this dissertation suggest several interesting areas for further study. The objective function in the present model has been designed to maximize the output of graduates who will enter the labor market and thus reduce the critical occupational shortages that currently exist. Future research using a model such as the one set forth in this investigation might consider other objective functions that measure benefits of vocational training programs. For example, an objective function could be constructed to measure long-term gains to the economy that would result from considering the different lifetime earnings of graduates employed in various occupations.

If an index could be constructed to reflect the number of different related occupati ; opportunities that would be available for graduates from the various vocational-technical education programs, an objective function could be formulated to maximize the number of employment opportunities that would accrue to individual students. In this case the objective might be to

$$\text{Maximize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_i x_{ij}$$

where

x_{ij} = the number of graduates in program i in county j.

c_i = a weight reflecting the number of different occupations that the graduate of program i would be able to enter upon completion of his training program.

Z = student benefits resulting from vocational education programs.

This suggestion relates to the concept of "cluster training" which is currently receiving much attention from vocational education curriculum specialists.[1]

There are many possible mathematical refinements that one might consider. Suppose the objective is to

$$\text{Maximize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

where $c_{ij} = f_{ij}(x_{ij})$ and f_{ij} is a linear function of x_{ij}. Then Z is a quadratic function. When the objective function is nonlinear (quadratic or more complex) and the constraints are linear, the model can often be formulated as a convex programming problem. In this case, the separable programming option in the IBM 360 mathematical programming algorithm could be utilized to provide close approximations for the convex programming model.

In some educational planning models the c_{ij}'s and the decision-maker's parameters (such as those listed in Section 4.4) might be random variables. Then the mathematical programming problem involves a stochastic model. There are a number of methods that have been applied to stochastic programming problems.[2] The active and passive approaches to stochastic

[1] For an excellent discussion of the cluster concept see Nevin R. Frantz, "The Cluster Concept as a Program in Vocational Education at the Secondary Level," in Cathleen Quirk and Carol Sheehan (eds.), *Research in Vocational Education*, (Madison: Center for Studies in Vocational and Technical Education, University of Wisconsin, 1967, pp. 83-94.

[2] For a summary see Jati K. Sengupta, "The Stability of Truncated Solutions in Stochastic Linear Programming," *Econometrica*, 34: 77-104; September, 1966.

programming could be applied. Chance-constrained programming is a likely candidate, since this approach includes the possibility that a constraint is violated. In chance-constrained programming (31) and (35) from this study could be written as

$$P\left(\sum_{i=1}^{m} x_{ij} \leq T_j \right) \geq \alpha_j \qquad \text{(for all j)}$$

$$P\left(\sum_{j=1}^{n} x_{ij} \leq d_i \right) \geq \beta_i \qquad \text{(for all i)}$$

where α_j and β_i are probabilities not far from 1.0. In the model developed in Chapter Three the α_j's and the β_i's have been set at 1.0. Hence, the model in this study is the deterministic case.

The method of certainty equivalence is the most common approach that has been applied to mathematical programming problems with random variables. In this case, usually the expected value of the objective function is to be maximized. Theil has applied this technique to models in macro-economics and the theory of the firm.[3]

In the near future, as refinements appear in the area of nonlinear and dynamic programming, one can expect that some of these features might be incorporated into education planning models. This study considered basic constraints and a simple objective function since its purpose was merely to demonstrate the potential usefulness of this approach as a practical alternative to present methods which do not utilize management science or decision theory models.

5.2 Recommendations for Vocational Education Planners

Vocational education planners should attempt to improve their data on the employment patterns of graduates. This type of information becomes extremely important to the decision-makers in vocational education. If long-term planning is to be more effective, the decision-maker must have accurate information on such factors as mobility patterns, occupational longevity of graduates and the relevance of vocational training to job performance. The utility of future models to determine the benefits of specific vocational education programs would be greatly increased if student follow-up data such as those described above could be collected and analyzed.

[3] Henri Theil, *Optimal Decision Rules for Government and Industry*, (Amsterdam, Netherlands: North-Holland Publishing Company, 1964).

State Departments of Education have played a key role in the administration and supervision of federally-financed vocational education since 1917. With the expansion of federal programs in 1963 and 1968, they have been called upon to play a more important role in planning and financing vocational education programs. To maintain a sound financial program, additional research will be necessary to develop more precise measures of the excess costs of specific vocational programs in comparison with general education. In dealing with large vocational education enrollments, statements such as "for general planning purposes, it may be assumed that vocational education classes cost 1.2 or 1.3 times the cost per student for general classes" will not suffice. More accurate measures of the costs of specific vocational education programs will be necessary to conduct thorough economic investigations of expenditures.

Labor market area information, such as the supply-demand postures developed in phase one of this model, should be disseminated to the Private Business Schools and the Private Trade Schools. Research on the effect of forwarding this type of information to the private schools should be conducted. Possibly, this flow of information assembled periodically by the State Department of Education will increase the productivity of these institutions. If this is true, then State Departments of Education will have played a key role in reducing the occupational shortages in the labor market without directly allocating funds for program expansion.

5.3 Conclusions

During the past few years there has been a rapid development of the field called management science (not to be confused with the older tradition of "scientific management" in the sense of time and motion studies). Management science models, which express the organizational environment and its dynamics in mathematical relationships, have successfully been applied in the fields of business management, military operations research, traffic engineering and regional planning. Although the utility of such models has been demonstrated in areas listed above, few educational planners are making use of thse modern management approaches.

The results of this study have demonstrated that management science models can be of practical value to education planners at the state level. Specifically, this investigation has shown that linear programming techniques, a management science tool, can be effectively utilized by State Departments of Education to provide valuable new information for determining future resource allocation policies in vocational education. The flexibility of linear programming techniques demonstrated in this study seems to offer some evidence that this type of model can be effectively applied to similar educational planning operations at the federal, state, regional and local levels.

BIBLIOGRAPHY

Ackoff, Russell A. *Scientfic Method: Optimizing Applied Research Decisions.* New York: John Wiley and Sons, 1962.

Adams, Don (ed.). *Educational Planning.* Syracuse: Syracuse University, School of Education, 1964.

Adelman, Irma. "A Linear Programming Model of Educational Planning: A Case Study of Argentina," in Irma Adelman and Erick Thorbecke (eds.), *The Theory and Design of Economic Development.* Baltimore: Johns Hopkins Press, 1964.

Alexis, Marcus and Wilson, Charles Z. (eds.). *Organizational Decision Making.* Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967.

Anderson, Arnold C. and Bowman, Mary Jean. "Theoretical Considerations in Educational Planning," in Don Adams (ed.), *Educational Planning.* Syracuse: Syracuse University, School of Education, 1964.

Arnold, Walter M. (ed.). *Vocational, Technical and Continuing Education in Pennsylvania: A Systems Approach to State-Local Program Planning.* Harrisburg: Pennsylvania Department of Education, 1969.

Banghart, Frank W. *Educational Systems Analysis.* Toronto: Collier-Macmillan, Canada, 1969.

Baumol, William J. *Economic Theory and Operations Analysis.* Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1965.

Benson, Charles S. "Economics and Education," *Review of Educational Research,* 37: 1, 96-102; February, 1967.

Benson, Charles S. *The Economics of Public Education.* Boston: Houghton Mifflin Company, 1968.

Bereday, George Z. and Laufwerys, Joseph A. (eds.). *The World Yearbook of Education 1967: Educational Planning.* London: Evans Brothers Limited, 1967.

Blaug, Mark. "Approaches to Educational Planning," *Economics Journal,* 77: 262-287; June, 1967.

Blaug, Mark. *Economics of Education: A Selected Annotated Bibliography.* Oxford: Pergamon Press, 1965.

Bowen, William G. *Economic Aspects of Education*. Princeton: Princeton University Press, 1964.

Bowles, Samuel S. "The Efficient Allocation of Resources in Education," *Quarterly Journal of Economics*, 81: 189-219; May, 1967.

Bruno, James E. "Using Linear Programming Salary Evaluation Models in Collective Bargaining Negotiations with Teacher Unions," *Socio-Economic Planning Science*, 3: 103-117; 1969.

Bruno, James E. "A Mathematical Programming Approach to School System Finance," *Socio-Economic Planning Science*, 3: 1-12; 1969.

Bruno, James E. "An Alternative to the Use of Simplistic Formulas for Determining State Resource Allocation in School Finance Programs," *American Educational Research Journal*, 6: 479-514; November, 1969.

Burkhead, Jesse. *Public School Finance: Economics and Politics*. Syracuse: Syracuse University Press, 1964.

Burkhead, Jesse, et al. *Input and Output in Large-City High Schools*. Syracuse: Syracuse University Press, 1967.

Campbell, Roald F. "The Emerging Role," in Roald F. Campbell, Gerald E. Sroufe and Donald H. Layton (eds.), *Strengthening State Departments of Education*. Chicago: The Midwest Administration Center, The University of Chicago, 1967.

Chance, W. A. "Long-Term Labor Requirements and Output of the Educational System," *Southern Economics Journal*, 32: 417-428; April, 1966.

Chirikos, Thomas N. and Wheeler, A. C. "Concepts and Techniques of Educational Planning," *Review of Educational Research*, 38: 264-276; June, 1968.

Chorley, Richard. "Geography and Analogue Theory," *Annuals of the Association of American Geographers*, 54: 127-137; 1964.

Cohn, Elchanan. "Economics of Scale in Iowa High School Operations," *The Journal of Human Resources*, 3: 423-429; Fall, 1968.

Coombs, Philip M. "What Do We Still Need to Know about Educational Planning?," in George Z. Bereday and Joseph A. Laufwerys (eds.), *The World Yearbook of Education 1967: Educational Planning*. London: Evans Brothers Limited, 1967.

Correa, Hector. "A Survey of Mathematical Models in Educational Planning," in Organization for Economic Cooperation and Development, *Mathematical Models in Educational Planning*. Paris: OECD, 1967.

Correa, Hector. "Optimum Choice between General and Vocational Education," *Kyklos*, 18: 107-113, 1965.

Correa, Hector. "Models and Mathematics in Educational Planning," in George Z. Bereday and Joseph A. Laufwerys (eds.), *The World Yearbook of Education 1967: Educational Planning*. London: Evans Brothers Limited, 1967.

Correa, Hector and Tinbergen, Jan. "Quantitative Adaptation of Education to Accelerated Growth," *Kylos*, 15: 776-786, 1962.

Culbertson, Jack. "State Planning for Education," in Edgard L. Morphet and Charles O. Ryan (eds.), *Designing Education for the Future: Planning and Effecting Needed Changes in Education*. New York: Citation Press, 1967.

Davis, Russell G. "On the Development of Educational Models at Harvard, CSED: An Algebraic History of Activity in One Small Place," in Richard H. Kraft, *Education and Economic Growth: Proceedings of the First Conference on the Economics of Education*. Tallahassee: The Florida State University, The Educational Systems Development Center, 1968.

Davis, Russell G. *Planning for Human Resource Development: Educational Models and Schemata*. Chicago: Rand McNally, 1966.

Eckaus, R. S. "Economic Criteria for Education and Training," *Review of Economics and Statistics*, 46: 183-184; May, 1964.

Eninger, Max U. *The Process and Product of T and I High School Level Vocational Education in the United States*. Pittsburgh, Pennsylvania: American Institutes for Research; September, 1965.

Eninger, Max U. *Report on Pennsylvania Data for a National Follow-Up Study of High School Level T and I Vocational Graduates*. Pittsburgh, Pennsylvania: Educational Systems Research Institute; May, 1968.

Fox, Karl A., et al. *Formulation of Management Science Models for Selected Problems of College Administration*, U.S. Department of Health, Education, and Welfare, Office of Education, Final Report Contract No. OEC-3-068058. Ames, Iowa: Iowa State University, 1967.

Fox, Karl A. and Sengupta, J. K. "The Specification of Econometric Models for Planning Educational Systems: An Appraisal of Alternative Approaches," *Kyklos*, 21: 665-693; 1968.

Frantz, Nevin R. "The Cluster Concept as a Program in Vocational Education at the Secondary Level," in Cathleen Quirk and Carol Sheenan (eds.), *Research in Vocational Education*. Madison: Center for Studies in Vocational Technical Education, University of Wisconsin, 1967.

Golladay, Frederick L. "A Dynamic Linear Programming Model for Educational Planning with Application to Morocco." Doctoral Dissertation, Northwestern University; August, 1968.

Goldstein, Harold. "Manpower Requirements and Educational Organization," in Organization for Economic Cooperation and Development, *Organizational Problems in Planning Educational Development*. Paris: OECD, 1963.

Hadley, George. *Nonlinear and Dynamic Programming*. Reading, Massachusetts: Addison-Wesley Publishing Company, 1964.

Hagget, Peter. *Location Analysis in Human Geography*. London: Edward Arnold Publishers, 1965.

Harbison, Frederick and Meyers, Charles A. *Education, Manpower and Economic Growth: Strategies of Human Resource Development*. New York: McGraw-Hill Book Company, 1964.

Harbison, Frederick. "Human Resource Assessments," in United Nations, Educational, Scientific and Cultural Organization, *Economic and Social Aspects of Educational Planning*. Paris: UNESCO, 1964.

Hartley, Harry. *Educational Planning-Programming-Budgeting: A System Approach*. Englewood Cliffs, New Jersey: Prentice Hall, 1968.

Hu, Teh-wei, et al. *A Cost-Effectiveness Study of Vocational Education: A Comparison of Vocational and Nonvocational Education in Secondary Schools*. University Park, Pennsylvania: The Pennsylvania State University, Institute for Research on Human Resources; February, 1969.

Hummel, Rodger G. *Degrees and Other Formal Awards Conferred by Pennsylvania Institutions of Higher Education 1968-1969*. Harrisburg: Pennsylvania Department of Education, 1969.

James, H. Thomas, et al. *Wealth, Expenditures and Decision Making for Education*, Office of Education, Final Report, Cooperative Research Project No. 1241. Stanford, California: Stanford University, School of Education, 1963.

Judy, Richard W. "Simulation and Rational Resource Allocation in Universities," in Organization for Economic Cooperation and Development, *Efficiency in Resource Utilization in Education*. Paris: OECD, 1969.

Kaufman, Jacob J., et al. *The Development and Utilization of Human Resources: A Guide for Research*. University Park, Pennsylvania: The Pennsylvania State University, The Institute for Research on Human Resources; July, 1967.

Kaufman, Jacob J. and Brown, Anne F. "Manpower Supply and Demand," *Review of Educational Research*, 32: 326-345; October, 1968.

Keeney, M. G., et al. "State Space Models of Educational Institutions," in Organization for Economic Cooperation and Development, *Efficiency in Resource Utilization in Education*. Paris: OECD, 1969.

Kershaw, Joseph A. "Productivity in Schools and Colleges," in Seymor E. Harris and Alan Levensohn (eds.), *Education and Public Policy*. Berkeley, California: McCutchen Publishing Company, 1965.

Kotz, Arnold (ed.). *Occupational Education: Planning and Programming*, Volumes One and Two. Menlo Park, California: Stanford Research Institute, SRI Project No. MU-6157, 1967.

Kraft, Richard H. (ed.). *Education and Economic Growth: Proceedings of the First Annual Conference on the Economics of Education*. Tallahassee: The Florida State University, The Educational Systems Development Center, 1968.

Kraft, Richard H. (ed.). *Strategies of Educational Planning: Proceedings of the Second Annual Conference on the Economics of Education*. Tallahassee: The Florida State University, The Educational Systems Development Center, 1969.

Kraft, Richard H. *Cost Effectiveness: Analysis of Vocational Technical Education Programs*, Florida State Department of Education, Final Report, Project No. 569-124. Tallahassee: The Florida State University, Educational Systems and Planning Center; July, 1969.

Lachene, Rene. "The Application of Operational Research to Educational Planning," in Organization for Economic Cooperation and Development, *Efficiency in Resource Utilization in Education*. Paris: OECD, 1969.

Lane, Willard R., et al. *Foundations of Educational Administration: A Behavioral Analysis*. New York: Macmillan Company, 1966.

Lecht, Leonard A. *Manpower Requirements for National Objectives in the 1970's*. Washington, D.C.: National Planning Association, Center for Priority Analysis, 1968.

Lee, Allen. "Study of Perceptions of State-Level Administration," *A Nationwide Study of the Administration of Vocational-Technical Education at the State Level*, Volume 4. Washington, D.C.: U.S. Government Printing Office, 1967.

Lee, Allen and Hamlin, Herbert M. "Organization and Administration," *Review of Educational Research*, 36: 359-402; October, 1968.

Lindman, Erick L. and Kurth, Edwin L. "Dimensions of Need for Vocational Education," in Roe L. Johns, Kern Alexander and Richard Rossmiller (eds.), *Dimensions of Educational Need.* Gainesville, Florida: National Educational Finance Project, 1969.

McGivney, Joseph M. and Nelson, William C. *Program, Planning, Budgeting System for Educators, Volume One.* Columbus: The Center for Vocational and Technical Education, The Ohio State University; August, 1969.

Nemhauser, George L. *Introduction to Dynamic Programming.* New York: John Wiley and Sons, 1967.

Organization for Economic Cooperation and Development. *Econometric Models in Education: Some Applications.* Paris: OECD, 1965.

Organization for Ecnomic Cooperation and Development. *Mathematical Models in Educational Planning.* Paris: OECD, 1967.

Organization for Economic Cooperation and Development. *Efficiency in Resource Utilization in Education.* Paris: OECD, 1969.

Parnes, Herbert S. *Forecasting Educational Needs for Economic Development.* Paris: The Organization for Economic Cooperation and Development, 1962.

Parnes, Herbert S. "Assessing the Educational Needs of a Nation," in Don Adams (ed.), Educational Planning. Syracuse: Syracuse University Press, School of Education, 1964.

Pennsylvania State Planning Board. *Pennsylvania's Regions: A Survey of the Commonwealth.* Harrisburg: PSPB, 1967.

Rice, Dick C. "State Education Departments and Vocational-Technical Education," in Dick C. Rice and Powell E. Toth (eds.), *The Emerging Role of State Education Departments with Specific Implications for Divisions of Vocational and Technical Education.* Columbus: The Center for Vocational and Technical Education, The Ohio State University, 1967.

Roe, William H. "The State Education Agency of the Future," in Edgar Morphet and Charles O. Ryan (eds.), *Designing Education for the Future: Planning and Effecting Needed Changes in Education.* New York: Citation Press, 1969.

Russo, Michael. "14 Million Vocational Students by 1975," *American Education,* 5: 10-11; March, 1969

Sanders, Donald P. and Barth, Peter S. "Education and Economic Development," *Review of Educational Research,* 38: 217; June, 1968.

Schiefelbein, Ernesto. "A Model for Assessing the Quantitative Results of Alternate Educational Policies." Doctoral Dissertation, Harvard University, 1969.

Sengupta, Jati K. "The Stability of Truncated Solutions in Stochastic Linear Programming," *Econometrica*, 34: 77-104; September, 1966.

Simon, Herbert A. *Administrative Behavior: A Study of the Decision Making Process in Administrative Organizations*, Second Edition. New York: Macmillan Company, 1961.

Stankard, Martin and Sisson, Rodger L. *Operations Research and Improved Planning for an Urban School District: Definitions and a Simulation Model.* Philadelphia: Management Science Center, University of Pennsylvania, Technical Memo No. 11067, 1967.

Stone, Richard. "A Model of the Educational System," Minerva, 3: 172-186; Winter, 1965.

Theil, Henri. *Optimal Decision Rules for Government and Industry.* Amsterdam, Netherlands: North-Holland Publishing Company, 1964.

Tinbergen, Jan and Bos, H. C. (eds.). *Econometric Models in Education.* Paris: Organization for Economic Cooperation and Development, 1965.

U.S. Office of Education. *Vocational Education Act of 1963 as Amended by Title One of the Vocational Education Amendments of 1968--Regulations for State Plan Programs.* Washington, D.C.: Bureau of Adult, Vocational and Library Programs; April, 1969.

U.S. Department of Health, Education and Welfare. *Administration of the Vocational Education Act: Rules and Regulations,* Vocational Education Bulletin No. 1. Washington, D.C.: Government Printing Office, 1967.

U.S. Department of Health, Education, and Welfare. *Vocational Education and Occupations,* OE-80061. Washington, D.C.: Government Printing Office; July, 1969.

U.S. Department of Labor. *Handbook Defining Labor Market Areas,* Employment Security Research Methods, BES No. — R 186. Washington, D.C.: Bureau of Employment Security, 1960.

Warbrod, J. Robert. *Review and Synthesis of Research on the Economics of Vocational Education.* Columbus: The Center for Vocational-Technical Education, The Ohio State University, 1967.

Woodhall, Maureen. "The Economics of Education," *Review of Educational Research,* 37: 387-398; October, 1967.