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ABSTRACT

This paper presents a mathematical model for use in determining student tuition charges at public and private institutions. This model treats higher education as an economic commodity, with the price to the consumer--in the form of tuition--as an algebraic function of supply, demand, and quality. The model provides one set of solutions to such problems as the optimum size and the pricing policy of private institutions, the pricing policies associated with various public objectives, and the extent to which the public sector should subsidize higher education. The model, which is simple and abstract, offers no definitive answers, but does provide a framework for the creation of a rational pricing policy for higher education. (RA)

STUDENT TUITION MODELS IN PRIVATE AND PUBLIC HIGHER EDUCATION

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TABLE OF CONTENTS

I. INTRODUCTION

The problem of student tuition in public institutions of higher education, student charges in private institutions of higher education, and state support of both public and private higher education are inevitably intertwined. However, recent discussions of these issues[1] have focused on either the repayment scheme or the derivation of student-borne charges. There has been little attempt to construct a behavioral model on the consumer-firm analogy which can potentially suggest answers to such important questions as: (1) If higher education is highly prized, why can't private institutions meet this demand? (2) What is the economically optimal size and pricing policy for private institutions? (3) What are the pricing policies associated with various public objectives? and (4) To what extent should the public sector subsidize higher education? This paper is a brief attempt to address these questions utilizing a simple, abstract model of student demand and institutional behavior. While this analysis is neither conclusive nor definitive, it does provide a framework for thinking about this problem and yields several counter intuitive results.

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[1] Including the Collier [2], Monagan [4], and Zacharias [6] plans, and the Leven-Osman [3] study.

See Burns and Chiswick [1] for a discussion of the recent literature on the effects of tuition programs at public universities.

II. THE ECONOMIC MODEL OF STUDENT DEMAND

We assume that a total pool of N individuals are eligible for and could potentially attend the institutions of higher education in this state. Based on individual (or family) preference patterns and constrained by family wealth, there is some highest total price that an individual is willing to pay for higher education. This price differs even among members of the same social or economic group and, therefore, we observe a distribution of this highest price. This is shown graphically in Figure 1.

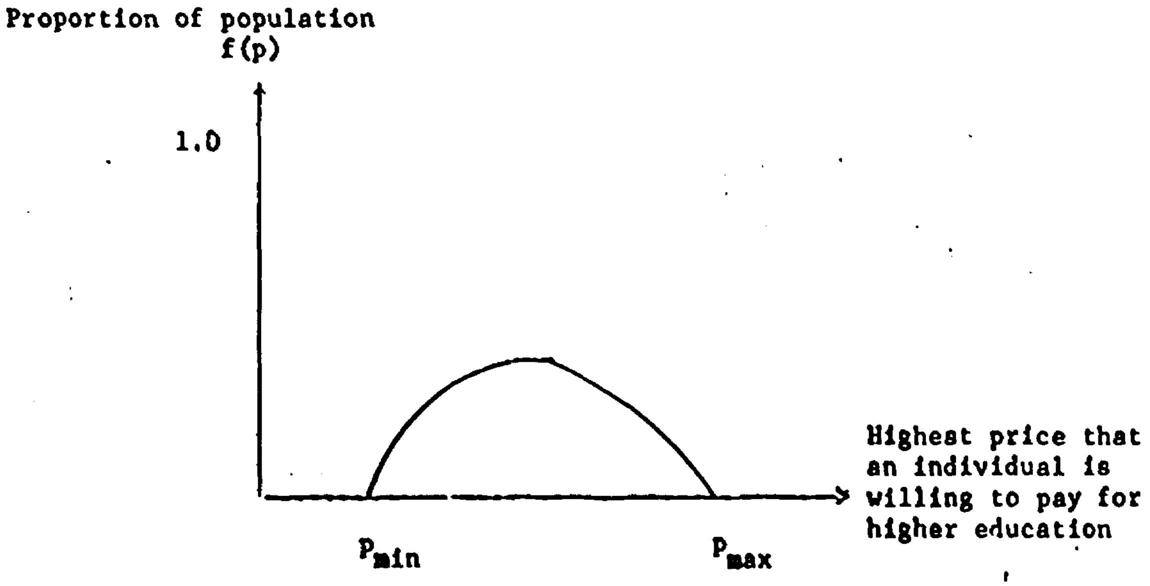


Figure 1

This is a true density function whose integral is one; i.e.,

$$\int_{P_{min}}^{P_{max}} f(p)dp = 1 \quad . \tag{1}$$

Notice that the price involved is a total price to the individual which includes the personal costs of attending college (including foregone income) plus the fees imposed by the college. For some social or economic groups, the college imposed fees might have to be negative (i.e., subsidy) for their p_{max} to be non-negative. An important issue not explored at this time is the difference between perceived personal costs and actual personal costs.

Each population sub-group, classified by income, ethnic background, education of parents, or in whatever manner is relevant to the decision maker, potentially has a different distribution of this highest price. The weighted sum of all the component sub-groups is the population envelope.

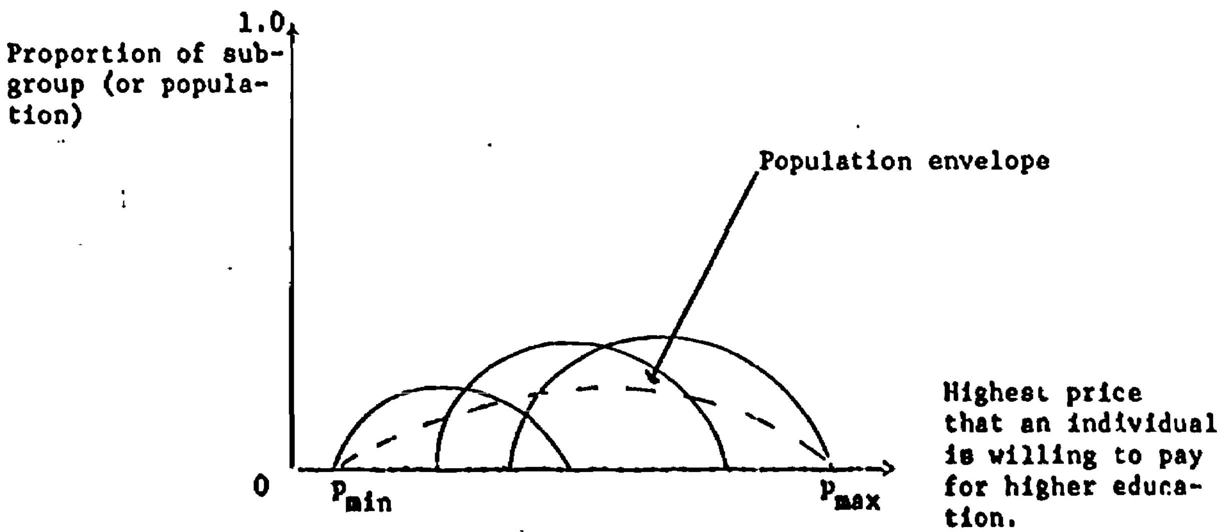


Figure 2

The remainder of this analysis will focus on the population envelope but the same results apply to each sub-group's density function.

It is also important to know what proportion of the population would attend a college if the total cost was p. This is given by the cumulative function $F(p)$ defined by

$$F(p) = \int_{p}^{p_{max}} f(\rho)d\rho \qquad (2)$$

where $f(\rho)$ is the population density function. This cumulative density function is the sum of all individuals willing to pay p or more for their education and has the following graph:

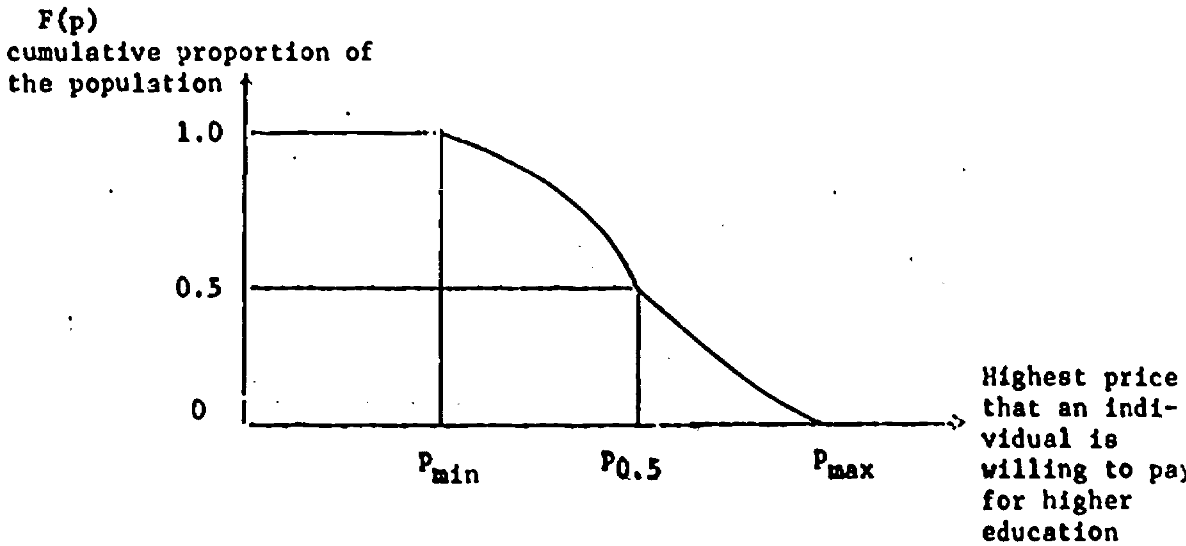


Figure 3

In other words, if the total cost of attending college is less than p_{min}, everyone who is eligible would like to attend. On the other hand, if the total costs of attending college rose to $p_{0.5}$, only one half of those eligible would want to attend. Finally, as the total cost exceeded p_{max}, no one would choose to attend college because their return from alternative investments would exceed their net return from higher education. In other words, $NF(p)$ is the population's demand function for college admission as a function of the total cost of attendance.

The excess of willingness to pay, or consumers' surplus, associated with any price p is simply given by

$$CS\ (p) = \int_{p}^{p_{max}} f(\rho)\rho d\rho \quad . \tag{3}$$

The consumers' surplus is not distributed evenly amongst all population subgroups; this can be seen from Figure 2.

Finally, for this analysis we assume that all students have the same personal costs, pc, of attending college and, therefore, the total cost, p, of attending an institution charging a tuition t is

$$p = pc + t \quad . \tag{4}$$

III. ANALYSIS FOR PRIVATE INSTITUTIONS

If n is the number of students attending private colleges and if all students who desire to attend college at total price p are accomodated in private colleges, then we observe that

$$n(p) = NF(p) \quad . \tag{5}$$

If private colleges have a cost function $C(n)$ that is related to the number of students, then the financial return (or profit) to a private college, Π, is

$$\Pi = n(p)t - C[n(p)] \tag{6}$$

$$= NF(p)t - C[NF(p)] \quad . \tag{7}$$

Certainly private colleges can currently serve a variety of objectives including religious instruction, specialized training, transmission of particular life styles, as well as instruction. However, if all student demand is accomodated in private colleges, the financial burden of their current mode of operation would become acute and the institutions would be forced towards a profit maximizing mode of behavior both for institutional survival and to finance the other missions of the institution.

Maximizing profit is equivalent to maximizing equation (7). The first order condition that $d\Pi = 0$ is

$$\frac{d\Pi}{dt} = N\frac{dF(p)}{dt}t + NF(p) - \frac{dC}{dn}\frac{dn}{dt} = 0 \quad . \tag{8}$$

Since the total cost p is the sum of a constant plus the tuition t, we may write $F(p)$ as

$$F(p) = \int_{pc+t}^{p_{max}} f(\rho)d\rho \tag{9}$$

Then,

$$\frac{dF(p)}{dt} = -f(pc + t)\,\frac{d(pc + t)}{dt}$$

$$= -f(pc + t) \quad . \tag{10}$$

Similarly,

$$\frac{dn(p)}{dt} = \frac{d[NF(p)]}{dt} = -N\,f(pc + t) \quad . \tag{11}$$

Therefore, we may write (8) as

$$\frac{d\Pi}{dt} = -N\,f(p)t + NF(p) + \frac{dc}{dn}\,Nf(p) = 0 \tag{12}$$

or

$$t = \frac{dc}{dn} + \frac{F(p)}{f(p)} \quad . \tag{13}$$

To derive a specific result, we need to know the functional forms of $F(p)$, $f(p)$, and $C(n)$. However, this formulation can be simplified if $C(n)$ is linear

$$C(n) = a_o + b_o n \quad . \tag{14}$$

In this case, $\frac{dc}{dn} = b_o$ and (13) becomes

$$t = b_o + \frac{F(pc + t)}{f(pc + t)} \quad . \tag{15}$$

The optimal tuition is then the t° chosen so that (15) is satisfied. The corresponding total cost, p°, total enrollment, n°, and resulting profit, Π°, are then

$$p^\circ = pc + t^\circ ,$$

$$n^\circ = NF(p^\circ) ,$$

and

$$\Pi^\circ = n^\circ t^\circ - C(n^\circ) \quad .$$

From (13) [or (15)] and in the absence of perfect competition, we observe that the optimal tuition charges should exceed the marginal cost to account for the tuition responsiveness of enrollment. If demand were inelastic, the optimal tuition would be equal to the marginal cost. The term $F(p)/f(p)$ is the price elasticity of demand which can easily be seen by writing it as

$$\frac{F(p)}{f(p)} = \frac{NF(p)}{Nf(p)} = \frac{-NF(p)}{\frac{dNF(p)}{dt}} \quad . \tag{16}$$

One way to interpret the "uniqueness" espoused by many private (and public) colleges is to recognize it as an attempt at product differentiation which would decrease competition and allow the school to charge p° or a higher tuition. Alternatively, maintaining a differentiated product enables institutions to establish a more expensive operating mode than could otherwise be justified.

IV. ANALYSIS FOR THE PUBLIC SECTOR

Suppose that a public decision maker decides that society needs n^* students in school to satisfy future manpower needs of the state or for any other reason. There are two possibilities: (1) either $n^* \leq n^\circ$ in which case no public action is needed because the private colleges are producing enough students to satisfy the state's desires; or (2) $n^* > n^\circ$ in which case public intervention is required. Therefore, this section will consider the case of $n^* > n^\circ$.

From the previous section, we know that n° is the optimal number of students private colleges should accept. Assuming that n° are enrolled in private colleges, the resulting number to be accommodated in public institutions is then

$$n_s = n^* - n^\circ \quad .$$

However, if students have the option of enrolling in either a public or a private institution, the student demand will be a function of both the private tuition, t_p , and the state tuition, t_s . We may write the corresponding joint density as $f(t_p , t_s)$ and the cumulative as $F(t_p , t_s)$ where

$$F(t_p , t_s) = \int_{pc+t_p}^{p_{max}} f(\rho, t_s)d\rho \quad .$$

We assume that the state chooses its tuition rates and then private institutions respond with new rates reflecting their own cost patterns and the state's rates. Therefore, when the option of attending a state institution exists, the optimal tuition for the private college is not equation (13), but

$$t_p = \frac{dCp}{dn} + \frac{F(t_p\ ,\ t_s)}{f(t_p\ ,\ t_s)} \quad . \tag{13'}$$

In other words, t_p° is a function of t_s and, therefore, n° is also a function of t_s . Consequently, at the prevalent state tuition rate, the student demand for public education is

$$NF(pc + t_s) - n^\circ(t_s) \quad .$$

The state could decide to accept a proportion α of this demand, where α is defined by

$$\alpha = \frac{n_s}{NF(pc + t_s) - n^\circ(t_s)} \quad . \tag{17}$$

If $\alpha < 1$, there will be excess demand at the tuition rate t_s . If $\alpha = 1$, the market will be cleared and the needs and enrollment goals of the state will be met.

Analagous to the case of private colleges, the state also has a profit function Π_s , and a cost function $C_s(n)$, which for a fixed private college tuition are related by

$$\Pi_s = n_s t_s - C_s(n_s) \quad . \tag{18}$$

The state has a number of pricing policies available to it exclusive of subsidizing private institutions which will be discussed in section V. These various state policies are examined below.

(1) Choose t_s to clear the market while meeting manpower requirements, i.e., $\alpha = 1$. In this case, (17) becomes*

*This assumes t_p is fixed and we abbreviate

$$F(pc + t_p, pc + t_s) = F(pc + t_s) \quad .$$

$$F(pc + t_s) = \frac{n_s + n^\circ(t_s)}{N} \quad . \tag{19}$$

The optimal tuition is that t_s such that $pc + t_s$ represents the $[n_s + n^\circ(t_s)]/N$ fractile of the cumulative distribution of F .

If $t_s \leq t$ and assuming perfect substitution and no quality distinctions, there would be little incentive for students to enroll in private colleges. To prevent the abandonment of private colleges, the state can and does:

(1) sets admission standards to restrict the eligibility for public education;

(2) subsidizes students thus reducing the total costs faced by the individual at private schools;

(3) sets quotas on public enrollment;

(4) restricts certain degree offerings among public institutions.

Observe that any redistributional effects whereby individuals who are willing to pay more choose a less expensive school and will decrease aggregate demand for private colleges.

(2) If future manpower requirements are uncertain, one could quantify society's reaction to uncertainty by choosing $\alpha < 1$. In this case, the appropriate fractile is

$$F(pc + t_s) = \frac{n_s + n^\circ(t_s)}{\alpha N} \tag{20}$$

and the comments of the previous section are once again relevant.

(3) Another possible public policy would be to set a tuition charge such that all costs are covered but no profit is made. In this case,

$$\Pi_s = n_s t_s - C_s(n_s) = 0$$

or

$$t_s = \frac{C_s(n_s)}{n_s} \qquad . \tag{21}$$

Again, if we assume a linear cost function

$$C_s(n_s) = a_1 + b_1 n_s \quad ,$$

equation (21) becomes

$$t_s = \frac{a_1 + b_1 \alpha NF(pc + t_s)}{\alpha NF(pc + t_s)} \tag{22}$$

$$= \frac{a_1}{\alpha NF(pc + t_s)} + b_1 \qquad . \tag{23}$$

(4) An alternative objective of the state is to choose the lowest tuition which achieves the manpower and other objectives while absorbing a fixed amount of subsidy. In this case, the society recognizes that an educated populace is worth a subsidy of at least S to the state. Thus, we have

$$S = C_s(n_s) - n_s t_s \tag{24}$$

and, therefore,

$$t_s = \frac{C_s(n_s) - S}{n_s}$$

$$= \frac{a_1 - S}{n_s} + b_1$$

$$= \frac{a_1 - S}{\alpha NF(pc + t_s)} + b_1 \qquad . \tag{25}$$

The solution to (25) is the desired tuition charge.

Observe from (25) and (23) that the state tuition charge exceeds the marginal cost, b_i , by an amount proportional to the fixed cost (or effective fixed cost), which has an obvious capital outlay interpretation, and inversely proportional to the cumulative willingness to pay of the constituency. In other words, to subsidize students at or below their marginal cost will lead, in this analysis, to production in excess of the state's manpower objectives. This is in fact the current status of our major public institution at the current time. However, as Burns and Chiswick [1] point out,

> "There are two hypotheses consistent with university administrators' charging a less-than-market clearing price. First, the administrators' behavior could be explained by assuming that they are utility maximizers whose pecuniary incomes are regulated by public (or alumni) censure. . . Second, the administrators' behavior could be explained by long-run profit considerations for their respective universities." (p. 239)

Alternatively, we could presume that the state is trying to maximize the "value" of the educational enterprise subject to the available resources. If S state dollars are available for investment in public higher education, and if the state recognizes NC numbers of clientele served by higher education (ethnic, income, occupational groups, or whatever), then the state could choose a (possible) graduated tuition, t_i , to

$$\max_{t_i} c'n$$

subject to

$$t'n \geq C(n) - S$$

where c_i is the relative value to the state of a student in group i . This says that the state should set a tuition level t_i for each group i to maximize the weighted sum $\Sigma c_i n_i$ subject to the constraint that the income to the institution, $\Sigma t_i n_i + S$ is greater than the cost, $C(n)$. This is a nonlinear programming* problem because $n_i = N_i F_i(pc + t_i)$ is nonlinear.

More generally, we could assume that the state has a utility function over the number and composition of student enrollment, $U(n)$, which it is trying to maximize subject to demand and financial constraints. This problem could be expanded to include dynamic system constraints, uncertainty, and other realistic attributes of public sector resource allocation decision problems.**

* See Zangwill, [7].
** See Weathersby, [5].

V. STATE SUPPORT OF PRIVATE HIGHER EDUCATION

There are several alternatives open to a state desiring to support in some part its private educational institutions.* These include:

(1) direct subsidy to students which would shift up their willingness to pay;

(2) lump sum subsidies to institutions;

(3) subsidies proportional to student enrollment which diminish the costs to the institution;

(4) payment in kind to reduce the capital or operating costs of an institution, such as a gift of land; and

(5) any combination of the above.

Throughout this discussion, we will assume that the state tuition rate is fixed in advance, and, thus, we will refer only to the private tuition charge.

(1) Under direct subsidy to students, the optimal tuition charge for an institution would be, by (13),

$$t = \frac{dC}{dn} + \frac{F(pc + t - s)}{f(pc + t - s)} \quad . \tag{26}$$

Depending on the ratio of the cumulative to the density function, state subsidy could lead to either a higher or lower level of tuition. However, provided any increase in tuition is less than the subsidy, aggregate enrollment in private colleges will increase by the amount

$$N[F(pc + t_{\sigma}^{\circ} - s) - F(pc + t^{\circ})] \tag{27}$$

*See Levin - Osmand [3] for a review of the recent literature and a traditional economic discussion of this topic.

where t°_{σ} is the optimal tuition associated with a per student subsidy s .

(2) Direct lump sum subsidies to institutions would improve their profit but leave their tuition and enrollment patterns unchanged. To see this, the profit function with a subsidy S would be

$$\Pi = nt - C(n) + S \quad . \tag{28}$$

The first order conditions,

$$\frac{d\Pi}{dt} = n + t\frac{dn}{dt} - \frac{dC}{dn}\frac{dn}{dt} = 0 \quad , \tag{29}$$

are identical to (8) with no subsidy. Therefore, the optimal tuition charges are the same as before and, consequently, the student enrollment will be the same. In other words, in this framework lump sum subsidies enrich private college treasuries but induce neither lower costs nor higher enrollments. On the contrary, lump sum subsidies to public colleges directly decrease student tuition charges as shown by equation (25).

(3) Subsidies proportional to student enrollment can be incorporated in this framework by modifying the private college profit function

$$\Pi = n(t + s) - C(n) \quad . \tag{30}$$

In this case, the first order conditions are

$$\frac{d\Pi}{dt} = n + (t + s)\frac{dn}{dt} - \frac{dC}{dn}Nf(p) = 0$$

or

$$t = \frac{dC}{dn} + \frac{F(p)}{f(p)} - s \quad , \tag{32}$$

The optimal tuition is then that t which satisfies (32).

It is of interest to compare the tuition charge associated with direct student subsidies with institutional proportional subsidies, or equations (26) and (32). Assuming that the marginal cost is unchanged by the method of subsidy, we see that the relevant comparison is between

$$\frac{F(p-s)}{f(p-s)} \overset{?}{\gtrless} \frac{F(p)}{f(p)} - s \quad .$$

When s is small, we may write

$$F(p-s) \approx F(p) + s\, f(p)$$

and

$$f(p-s) \approx f(p) \quad .$$

Then we see that, for small s, tuition under direct student subsidies varies as

$$t \propto \frac{F(p)}{f(p)} + s \tag{33}$$

while tuition under proportional institutional subsidies varies as

$$t \propto \frac{F(p)}{f(p)} - s \quad . \tag{34}$$

In other words, tuition under direct student subsidy is *higher* than tuition under proportional institutional subsidy by an additive factor of approximately *twice* the per student subsidy for small subsidies. Correspondingly, the student enrollment will be lower with direct student subsidy than with proportional institutional sub:`'dy.

To expand on this last point, we show that the state would achieve greater enrollments under proportional institutional subsidies rather than direct student subsidies. With direct subsidies to students, the optimal tuition would be

$$t = \frac{dC}{dn} + \frac{F(pc + t - s)}{f(pc + t - s)}$$

$$\approx \frac{dC}{dn} + \frac{F(pc + t) + sf(pc + t)}{f(pc + t)}$$

$$\approx \frac{dC}{dn} + \frac{F(pc + t)}{f(pc + t)} + s \quad . \tag{35}$$

The corresponding enrollment with direct student subsidies, E_D, is

$$E_D = NF(pc + t - s)$$

$$\approx NF[pc + \frac{dC}{dn} + \frac{F(pc + t)}{f(pc + t)} + s - s]$$

$$\approx NF[pc + \frac{dC}{dn} + \frac{F(pc + t)}{f(pc + t)}] \quad . \tag{36}$$

With institutional subsidies proportional to student enrollment, the optimal tuition is given by (32) to be

$$t = \frac{dC}{dn} + \frac{F(p)}{f(p)} - s$$

and the corresponding enrollment, Ep, is

$$Ep = NF(pc + t)$$

$$\approx NF[pc + \frac{dC}{dn} + \frac{F(pc + t)}{f(pc + t)} - s]$$

$$\approx N\{F[pc + \frac{dC}{dn} + \frac{F(pc + t)}{f(pc + t)}] + sf[pc + \frac{dC}{dn} + \frac{F(pc + t)}{f(pc + t)}]\} \quad . \tag{37}$$

Observe that the increase in enrollment achieved through proportional institutional subsidies over direct student subsidies is

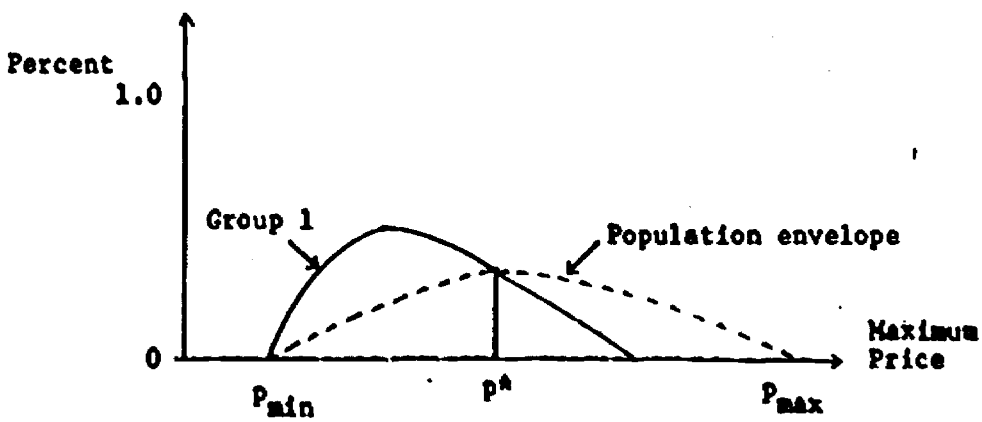
$$E_P - E_D \approx Nsf[pc + \frac{dC}{dn} + \frac{F(pc + t)}{f(pc + t)}] \quad . \tag{38}$$

For no subsidy, $s = 0$, E equals E_D and the enrollment is unchanged. For small subsidies, the enrollment is increased proportional to the subsidy as described by equation (38). From equations (13) and (36), we observe that for small subsidies there is no enrollment increase when students are supported directly. One conclusion of this analysis is that to increase aggregate enrollment with limited resources, $s \ll pc + t$, the state should subsidize institutions proportionally to the number of students, rather than subsidize students directly. This qualitative result is independent of the cost function of the institution, but the quantitative increase depends upon the cost function as seen in (38).

VI. FURTHER CONSIDERATIONS

While this analysis has focused on the financial and manpower objectives of higher education, there are a large number of critically important questions still unanswered. There is no quantitative description of the change in the quality of the student body in response to changing student tuition, although there have been some qualitative discussions.* However, we can use the framework of this analysis to shed a little light on this issue.

We can define the subgroups of the college-age population in terms of "quality," begging the question of definition of quality whether by SAT score, grade point averages, or whatever. Consider the lowest quality group, group 1, and the overall population envelope. Logically, the willingness and ability to pay for higher education is distributed over all ability groups, and, therefore, group 1 will have a distribution in its maximum price as shown below.



*See Burns and Chiswick [1], p. 241.

At any price p the proportion of the enrolled population in group 1 will be

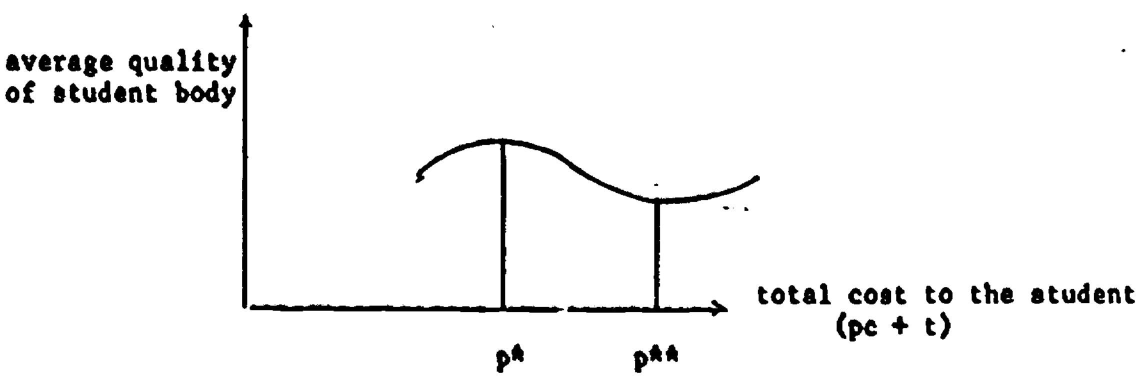
$$\frac{N_1F_1(p)}{NF(p)} = \alpha \ , \ 0 \leq \alpha \leq 1 \quad .$$

If tuition increases a small amount Δt, the new proportion of the total in group 1 will be

$$\frac{N_1F_1(p + \Delta t)}{NF(p + \Delta t)} \approx \frac{N_1[F_1(p) - \Delta t f_1(p)]}{N[F(p) - \Delta t f(p)]} \tag{39}$$

There exists a p^* such that $f_1(p^*) = f(p^*)$ (see graph). If $p < p^*$, $f_1(p) > f(p)$, and $\Delta t f_1(p) > \Delta t f(p)$. Therefore, the numerator of (39) will decrease faster than the denominator and the average quality of the enrolled population will increase with increased tuition. On the other hand, if $p > p^*$, $f_1(p) < f(p)$, $\Delta t f_1(p) < \Delta t f(p)$, and by a similar argument, the average quality of the enrolled population will decrease with increased tuition. The critical point is p^* where the slope of the demand curves are equal.

If group 1 is the only "low quality" group, then the average quality of the enrolled student body as a function of tuition would increase until $p = p^*$ and then decrease, as shown below.



If p_{max} for group 1 is less than p_{max} of any other group, then there is some p^{**}, $p^{**} \geq p_{max}$ of group 1, beyond which the average quality will no longer decrease and may increase. This clarifies the ambiguity in [1].

Another question that remains unsolved is that of increasing student control with increasing tuition. Currently, states provide about 25% of the total resources of higher education (the bulk of the remainder is the foregone income of students' time) and makes most of the major policy decisions affecting a student's education. It seems only a matter of time until students demand and receive a larger share of the decision making commensurate with their increasing share of the total cost of their education.

Finally, there is the effect of uncertainty in resource acquisition on institutional cost patterns. Income from student fees should be both more controllable and more certain than dependence upon state funds. This increased certainty may enable institutions to reduce their marginal costs which now reflect the institutional concern for its uncertain future.

VII. SUMMARY OBSERVATIONS

(1) The highest total price that an individual is willing to pay for higher education is distributed between some upper and lower limits according to both his preferences and his wealth.

(2) The excess willingness to pay associated with any price is the consumers' surplus related to that price.

(3) This analysis assumes all students have the same personal costs of attending college including out-of-pocket costs, foregone earnings, and tuition or fees.

(4) If private colleges set their fees to maximize profits, they will price at their marginal cost unless they are either monopolistic or sufficiently differentiated that they can charge more than their marginal cost.

(5) The public sector should supply higher education only if its perceived manpower and social needs exceed the production in private colleges.

(6) If additional training is desirable, the public sector has several pricing policies available:

a. Choose the tuition level that both clears the market and meets desired manpower requirements.

b. Choose the tuition level that clears the market while meeting future stochastic manpower requirements.

c. Choose the tuition level that covers all instructional costs but shows no profit.

d. Choose the lowest level of tuition which achieves the manpower and social objectives while absorbing a fixed amount of subsidy.

(7) Alternate policies of state support of private higher education include:

a. direct subsidies to students;

b. lump sum subsidies to institutions;

c. institutional subsidies proportional to student enrollment; or

d. payment in kind.

(8) Direct student subsidies may lead to higher tuition but, as long as any increase in tuition is less than the subsidy, the aggregate enrollment in private colleges will increase.

(9) Lump sum subsidies enrich private college treasuries without inducing either lower tuition charges or higher enrollments.

(10) Tuition under direct student subsidy is higher than tuition under proportional institutional subsidy by an additive factor of approximately twice the per student subsidy for small subsidies.

(11) For small subsidies, the enrollment achieved through proportional institutional subsidies exceeds the comparable enrollment achieved through direct subsidies independent of the form of the institutional cost function.

(12) To maximize enrollment gains with fixed resources, the state should proportionally support institutions rather than directly support students. However, other objectives of freedom of choice or equity might justify direct student support.

(13) The average quality of the student body is a function of tuition charged and may first increase, then decrease, and then increase again with an increasing tuition charge.

BIBLIOGRAPHY

[1] Burns, J. M. and B. R. Chiswick, "Analysis of the Effects of a Graduated Tuition Program at State Universities," Journal of Human Resources, Vol. V, No. 2, Spring 1970, pp. 237 - 245.

[2] Collier, John, "The Learn, Earn, and Reimburse Plan," Assembly Bill No. 625, California Legislature, Regular Session, introduced February 9, 1970.

[3] Levin, H. M. and Osman, J. W., "Alternative Methods of State Support for Independent Higher Education in California," Report to the California Coordinating Council for Higher Education, February 1970.

[4] Monagan, Robert, "Graduated Student Charges: an Equitable Plan for Raising Revenue for Improved Support of Higher Education Based on Ability to Pay," Assembly Bill No. 468, California Legislature, Regular Session, introduced February 12, 1969.

[5] Weathersby, George B., "The Allocation of Public Resources: a Decision and Control Theory Analysis," unpublished doctoral dissertation, Harvard University, 1970.

[6] Zacharias, Jerrold R., Chairman, Educational Opportunity Bank: A Report of the Panel on Educational Innovation to U.S. Commissioner of Education Harold Hower II, Director of the National Science Foundation, Leland J. Haworth, and Special Assistant to the President for Science and Technology, Donald F. Hornig, U. S. Government Printing Office, August 1967.

[7] Zangwill, W. I., Nonlinear Programming, Prentice-Hall, Inc., Englewood Cliffs, 1969.