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ABSTRACT

Without a conscious effort to achieve optimum resource allocation, there is a real danger that educational resources may be wasted. This document uses input-output analysis to develop a model for rational decision-making in secondary education. (LLR)

TOWARDS RATIONAL DECISION-MAKING IN SECONDARY EDUCATION

by

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TOWARDS RATIONAL DECISION-MAKING IN SECONDARY EDUCATION

I. Introduction

Education is the largest single industry in the United States. Public elementary and secondary education occupy an important portion of the industry with estimated current expenditures in 1968 over 26 billion dollars.[1] If we take a broader view of costs of schooling, total resources entering education have been estimated by this author at over 60 billion dollars in 1968 (compared to only 31 billion dollars in 1960).[2] A good deal of this is spent annually by the public secondary schools.

Since the products of education are not easily visible or marketable, the educational industry has not been subjected to ordinary market forces. And in the absence of a conscious effort to achieve optimal resource allocation, there is a real danger that valuable resources--most of which are drawn from the middle-class tax payer--are wasted with impunity.

We are not about to suggest that the educational industry be turned into private hands.[3] Perhaps this _is_ the best solution; but considerable experimentation is needed before such a drastic step is taken on a large scale. Still, there might be some less grandiose--yet useful--approaches that could be followed to improve decision-making in education. The analysis here will be both general and specific, the latter intended to provide a simple illustration of how the general principles could be applied by a high school principal when the necessary information is available.

There are a number of factors which mitigate the chances of providing useful decision criteria for education. In the first place, the objectives of secondary education vary considerably from school to school, and further, these are often highly ambiguous. It is not possible to construct aggregate statistical experiments without knowledge of educational objectives, and micro-analysis is useless when such objectives are vague. Secondly, school data on a wide range of variables are often lacking, and where they exist they are of questionable validity and reliability. Finally, even if sufficient data were available and objectives clearly and unambiguously delineated, the tradition of ad-hoc decision-making processes in education must still be replaced by a willingness on the part of school administrators to attempt an optimal allocation of the funds allotted for their schools' operation.

In Section II we shall discuss the notion of input and output in education. So far, it will be observed, this notion is rather elusive. Unable to provide an empirically workable optimization model we turn to suboptimization in Section III. An illustration of the application of such a model will be given in that section.

II. Input and Output in Education

An assessment of the (lack of) optimality of resource allocation in secondary education requires a knowledge of the production process of education.[4] Whereas in some manufacturing industries the production process is quite unambiguous, this is not the case in education. Firstly, it is a most difficult task to enumerate and quantify the educational outputs. Secondly, the relationship between the numerous inputs and the outputs is most difficult to discern--even if all outputs could be specified and quantified. Finally, while in recent years considerable progress has been made in estimating school inputs, we have a far less than satisfactory set of inputs. Evidently, any effort to estimate educational production functions is almost assuredly doomed to failure.

A production function is a mathematical relation, relating physical inputs to (maximum) physical outputs. It is different from the engineer's production process in so far as it presents variables in <u>economic</u> units (such as dollars, weight in pounds, number of workers, pieces of equipment, etc.) as opposed to non-economic units (such as diameter of pipes). Still, the economist would normally be required only to translate the engineer's formula into economic terms, rather than specify the production process himself. In education, the economist's job is to translate the educational process, as specified by the educational experts, into economic terms. Unfortunately, the educational experts have not as yet been able to provide the economist with a precise formula of the educational process. Still, some general characteristics of the educational process have been discussed in the literature, upon which a generalized production function could be formulated.

Suppose there are n different educational outputs, Q_1, Q_2, ..., Q_n. These might include academic achievement, fostering individual study and work habits, good citizenship, vocational skills, and so on. Suppose, further, that the set of educational inputs is divided into two parts: (1) "endogenous" inputs, z_1, z_2, ..., z_k, over which the school system has some control--such as the quality of teaching services, the breadth and depth of course offering, laboratory equipment, library volumes, etc; and (2) "exogenous" variables, s_1, s_2, ..., s_m, over which no direct control may be exercised by the school---such as the socio-economic composition of the community (family income, parental education and occupation, etc.), the location of the community, and the amount of federal and/or state aid. Given the sets of inputs and outputs, an implicit production function of education will have the form

(1) $$F(Q_1,Q_2,\ldots,Q_n;\ z_1,z_2,\ldots,z_k;\ s_1,s_2,\ldots,s_m) = 0.$$

The meaning of (1) is that any of the Q_i are determined, according to the function F, by (1) the set of endogenous inputs; (2) the set of exogenous inputs; and (3) the set of outputs, Q_j, for all $j \neq i$. The function is <u>implicit</u> since we have not specified <u>how</u> each of the three variable sets influences Q_i. An <u>explicit</u> function will make such a specification. For example, we might want to simplify the analysis and assume that there is only one output, Q (or, alternatively, that all the Q_i are independent of one another), and that the effect of changes in any one input on Q is additive to the effect of changes in other inputs. Then the production function might be written as:

(2) $$Q = a + b_1 z_1 + \ldots + b_k z_k + c_1 s_1 + \ldots + b_m s_m$$

where a, $b_1, \ldots, b_k$, and $c_1, \ldots, c_m$ are constants. Note that while (2) is an explicit function it still is not a complete description of the educational process since it does not provide numerical values for the coefficients a, b_1, etc.

There are good reasons to suspect that a production function of type (2) is highly unsatisfactory. First, it implies that there is either only one output or that the various outputs are independent in some sense. Now it is quite obvious that the educational process is too complex to be described by one output alone. Further, there is no single output which is of overwhelming importance in relation to the entire set of outputs. For example, achievement is often construed to be <u>the</u> educational output. But where vocational skills are emphasized, academic achievement is of secondary importance only. Further, even where academic education is predominant, achievement competes with the holding power of the school (the inverse of the drop-out rate). As Burkhead has noted, "if students of less than average performance are encouraged to remain in school, test score averages will decline."[5]

Second, the assumption of additivity is difficult to accept.[6] The implication of this assumption is that the endogenous input for which b_i/p_i (where p_i is the price per unit of input i) is largest should be expanded indefinitely at the expense of all other inputs. For example if the ratio b_i/p_i is largest for library volumes, the implication is that an optimal policy would require expanding the library facilities indefinitely, leaving all other inputs constant. Moreover, the form in (2) assumes that any levels of Q can be reached by using just any <u>one</u> of the inputs (perhaps at an extremely costly level). That is, we assume perfect substitutability among inputs. In most cases we are interested only in marginal changes in inputs, and only in the effect of changes in the inputs on potential <u>changes</u> in Q. Then the assumption of additivity may not be too difficult to accept.

An alternative form to (2) which has been widely applied is to transform the input and output variables into logarithms.[7] Then our production function becomes

(3) $$Q = a\, z_1^{b_1} \ldots z_k^{b_k}\, s_1^{c_1} \ldots s_m^{c_m}.$$

or

(4) $$\log Q = \log a + \sum_{i=1}^{k} b_i \log z_i + \sum_{j=1}^{m} c_j \log s_j$$

where constants (a, b_i, and c_j) are different from those in (2). Finally, production functions in which some of the variables were transformed into the logs, some were left in linear form, and some were specified in a quadratic form (for example, both $b_i z_i$ and $b_i' z_i^2$ were included in the equation) can be found in recent works on the educational production function. The advantage of the logarithmic form is in that it allows for some substitution of inputs but not for perfect substitution. Therefore b_i no longer represents both the infintesimal and finite marginal productivity of the i inputs as in (2).[8] Instead it gives the elasticity of input i with respect to Q, when all other inputs are held constant.[9] It is easy to show that the marginal product of z_i, in this case, is sensitive to changes in z_i such that as more and more of input i is utilized, the lower and lower will its productivity be. This will eliminate the awkward policy implications made above with respect to model (2).

There are no *a priori* reasons to select any of the above-mentioned functions. What is needed is a careful examination of the objectives of secondary schools, the inputs used to achieve them, and the resulting outputs. The process which relates the inputs to outputs could then be examined, and suggestions regarding the best way in which available resources ought to be utilized could then be made. Despite recent efforts at estimating the production function of education, the technique could not as yet be used due to the inherent flaws in the analysis. This is not to say that such efforts are useless; nothing is farther from the truth. But for our purposes here, such a tool cannot as yet be used.

III. Optimization and Suboptimization

Suppose that the main objective of the high school principal is to prepare pupils for post-secondary education.[10] Then, with limited funds, he has the option of choosing among various school inputs to obtain the maximum product, call it Q. We assume that Q is one dimensional and measurable, that all of the relevant inputs are known and quantifiable, and that the process by which the inputs affect Q is also known (at least to an extent). To simplify matters, let us assume that educational funds are limited on a per pupil in average daily attendance (ADA) basis, that ADA is fixed, and that the relevant inputs which can be varied at the discretion of the principal are limited to the following:

A = number of subject matter assignments;

k = number of courses taken each term by the average pupil;

S = number of sections per unit taught;

T = number of teachers in the school;

U = number of units taught;

A/T = number of subject matter assignments per teacher;

ADA/T = the students-teachers ratio;

F/T = average teachers' salary;

kADA/SU = average class size;

SU/T = number of courses per teacher.

Since ADA is fixed, F, the total amount of funds available for compensation of teachers, can be assumed to be approximately proportional to ADA, and hence also fixed.[11] (This is not precisely so since funds could be switched from other categories to teachers' salaries, and vice versa. The possibilities for such transfers are, however, extremely narrow in actual practice.) The model described here is therefore limited to the choice of the "best" resource allocation of (1) the teachers' salary fund;

(2) the available supply of teachers and (3) the number and composition of courses offered.[12]

Each school board has discretionary powers over the salary schedule. With limited funds at its disposal it could offer relatively low average salaries[13] and hire more teachers, presumably of lower average quality, or it could hire fewer, but better qualified teachers, by offering relatively high salaries. In either case the total salary bill, F, will be the same, but T, the number of teachers, will be different. Moreover, the quality of each teacher, measured by the average teacher salary, F/T, will also differ.

Suppose, for now, that the principal (or the school board, or both) decided on the magnitude---and hence the quality---of T. The principal must now choose the type, breadth, and depth of curriculum desired which, it will be seen, could conflict with a desire to achieve maximum quality per course taught. The choices open to the principal are usually wide and varied. Although most states require some minimum number of units in specified subject matters, the minimum rarely serves as a constraint for all but the smallest high schools.[14] (When such requirements become important constraints, the analysis must be modified considerably.) The principal might choose to offer fewer subject matters (sacrifice breadth) and instead offer advanced courses (perhaps equivalent to college freshman courses) in a limited number of subject matters (introduce added depth). For example, instead of offering a course in psychology he might offer a course in calculus.

Additionally, with a fixed number of teachers, any choice to increase (or decrease) the total number of courses offered will affect the average number of courses taught per teacher. There is a further dimension of some interest, the number of different subject matter assignments per teacher. While the number of courses per teacher is indicative of the teaching load, the number of assignments per teacher is indicative of the extent of specialization allowed. Changes in U invariably affect SU/T, unless T (or S) is changed proportionately (inversely). Yet changes in U may have no effect on A/T if the extra assignments are so arranged as to leave the extent of specialization unchanged; still, other things equal, changes in U are likely to affect both SU/T and A/T.

A further evidence on the complexity of the decision-making process required of the principal is clearly seen when we consider the possibilities of changing not the total number of courses (the <u>product</u> of S and U) but rather the <u>magnitude</u> of U and S. That is, instead of offering <u>new</u> units, the principal could conceivably increase the number of sections per (old) unit.

What of the "average class size?" Most studies define this by ADA/T, i.e., the students - teachers ratio. This measure may or may not correspond to the true average class size, which is defined by enrollment divided by the number of courses offered. If each student takes, on the average, k courses per term, enrollment in each term will be kADA. Therefore the average class size is defined by kADA/SU. The latter will equal to ADA/T only if $1/T = k/SU$, or when $U/T = k/S$--i.e., when the number of units per teacher equals the ratio of courses per pupil to sections per course. Offhand there seems to be no reason to expect the two measures to be equal; hence the use of ADA/T instead of kADA/SU cannot be justified <u>a priori</u>.

<u>Course Quality</u>: How do we measure quality per course? It seems that some of the factors mentioned about <u>ought</u> to influence the quality of the "average" course. First, the quality of the teacher is very important. Average salary is probably correlated with teacher's quality, the latter being a function of training, experience, innate ability, and teaching aptitude. Second, even the best teacher could not be expected to perform well when his teaching load is excessive. Therefore we would expect course quality to vary inversely with the teaching load, measured by the number of courses per teacher, SU/T. Another dimension of quality is the extent of specialization allowed. It might be argued that schools that allow sufficient specialization in teaching

could sacrifice some teaching quality in terms of experience and educational attainment since the process of self-study and self-improvement might more than compensate for the lack of formal education and/or teaching experience. Our proposed measure for the extent of specialization is A/T.

Although our earlier comments on SU/T indicated that changes in S vary inversely with changes in quality--because of the implied increase in teaching load--changes in S, coupled with offsetting changes in U (leaving SU constant), appear to vary directly with course quality (sacrificing either curriculum breadth or depth). This is so because increased number of sections per unit taught is likely to result in increased communication and co-operative efforts among teachers.[15]

Finally, it is the opinion of many educational psychologists that a smaller class size is always preferable to a larger one (i.e., a tutorial system is best). The controversy about class size is mostly concerned with empirical varification of the hypothesis. But such a verification is not possible unless and until we can specify and estimate an educational production function. Meanwhile, we will assume that class size is inversely associated with quality per course.

We might conclude, therefore, that course quality will vary directly with S and F/T, and inversely with SU/T, A/T, and class size, kADA/SU.

The Optimization Model: The school administrator is charged with the responsibility of allocating the available resources among competing inputs in such a manner as to provide the best preparation of students for subsequent study in college or other post-secondary education. He must decide on whether to select more, but less qualified, teachers; greater curriculum breadth at the expense of depth; greater curriculum breadth and/or depth at the expense of increased teaching load, and so on. To assist the administrator in this formidable task we need, first, a production function which will describe the relationships between the inputs and the output. The production function is given in (5):

(5) $$Q = f(U, F/T, S, SU/T, A/T, kADA/SU; u)$$

In other words, we presume that the total product of the educational system is given by Q, where Q is influenced by the number of units offered, the elements of the course-quality index, and other noncontrollable variables (including a random disturbance variable) symbolized by u.

Each of the input variables in (5) is subject to some constraints. In most states, U has a lower limit for accredited schools. Although F/T could be varied, there are institutional as well as practical limits on the extent of the variation. Similarly, the school administrator may impose practical limits on S, SU/T, A/T and kADA/SU. Another important constraint is that the total sum spent on inputs must not exceed the budget (we assume here that F is fixed).

If the production function is of the form described in (2) above, the technique of linear programming could be used to find the optimal levels for each of the inputs (whenever an optimal solution exists). When (5) has a non-linear form, mathematical programming might still be used, but then it will be far more difficult to obtain an optimal solution.[16] In any event, a necessary, though not sufficient, condition for specifying the optimal mix of inputs is the specification and estimation of a production function.

The Suboptimization Model: Since we are not ready as yet to specify an educational-production function--even with the limited number of inputs and objectives of this illustration--we must search for other methods to obtain a more effective resource allocation. We will continue to assume that function (5) exists, though we shall make no attempt to estimate it directly.

We note that some of the variables in (5) are inherently related to one another. For example, a change of one unit in U, holding all other inputs constant, will result in a change in SU/T equal to S/T, and a change in kADA/SU equal to $kADA/SU^2$. Similarly, a change in the average salary, given that F if fixed, will increase the number of teacher, T, and hence affect SU/T and A/T--all other inputs being constant. These relationships, which we might call "barter terms of trade," are summarized in Table 1 (the cells to the right of the diagonal of Table 1 could be computed by taking the inverse of the respective cells to the left of the diagonal).

The process of suboptimization will require some knowledge of the effect of inputs on Q--but not to the extent required by the production function method. Given ADA and F, let the principal manipulate, first, only the number of teachers hired, T, leaving all other (non-related) inputs constant. Manipulating T would, of course, affect F/T, SU/T and A/T. Since the presumed effect on Q of F/T is positive, a larger core of teachers is negatively related to Q through its effect on F/T, but positively related to Q through its effect on both SU/T and A/T. At this stage all that we require of the principal is to be able to judge the effect on Q of changes in T when the three variables (F/T, SU/T and A.T) are thereby affected. A suboptimal position will be attained when the algebraic sum of the effects of the three inputs on Q is zero.[17]

It is conceiveable that some constraints might limit our ability to reach the suboptimal point. Then we vary T until such a point in which Q is maximized subject to the constraint.[18]

The next step might be the varying of U. But changes in U affect SU/T--which has already been suboptimized--and kADA/SU, which affect Q in the same direction as U. The process requires that U is varied until the algebraic sum of the effects on Q of U, SU/T, and kADA/SU are zero.[19] This would lead us immediately to a third step in which F/T, SU/T and A/T must again be brought to a suboptimum. This process may be repeated until no changes in any of the variables so far mentioned could improve Q. The next step would involve changing S, which would probably require further adjustments in all affected variables. Finally, we

TABLE 1

"Barter Terms" Among Six Educational Inputs*

	U	F/T	S	SU/T	A/T	kADA/SU
U	1					
F/T	0	1				
S	0	0	1			
SU/T	S/T	SU/F	U/T	1		
A/T	0	A/F	0	A/SU	1	
kADA/SU	$\frac{-kADA}{SU^2}$	0	$\frac{-kADA}{U \cdot S^2}$	$\frac{-(T)kADA}{(SU)^2}$	0	1

*Each "barter term" was calculated by taking the partial derivative of one input with respect to another. For example, the barter term for SU/T and F/T was computed as follows. Let $X = F/T$. Then $T = F/X$. So $\partial(SU/T)/\partial(F/T) = \partial(SU/F/X)/\partial X = SU/F$. Note that these barter terms hold for small (infintesimal) as well as large (finite) changes in each of the variables.

would select the level of A such that the algebraic sum of the effects on Q of A/T, F/T, and SU/T be zero, repeating all suboptimization steps which might be required as a consequence. At the end, the resulting levels of the input variables will be such that no reallocation of resources could produce significantly superior consequences as measured by Q.[20]

Since we have used here a quasi input-output model, it might well be asked, what is the advantage of the suboptimization model? To comprehend the important difference between the optimization and suboptimization models we recall that in the former case sufficient information is needed to describe the entire production process where all inputs enter the process simultaneously. To specify such a process in education is indeed quite difficult. On the other hand, in the suboptimization framework we only require that the administrator weigh the consequences of varying at most three inputs simultaneously; since it might be assumed that most administrators are aware, at least to an extent, of the potential effects of inputs on Q, the process of suboptimization offers a more promising tool for educational decision-making than the production function method. Moreover, if school administrators acquire the habit of using rational decision formulas, the time when sufficient information will be available to embark on a full-scale optimization decision-making process might come much sooner.

FOOTNOTES

1. Projections of Educational Statistics to 1977-78 (Washington: U.S. Office of Education, 1968), p. 78.

2. Elchanan Cohn, "The Costs of Education," chapter in a manuscript under preparation entitled The Economics of Education, Table 15.

3. Such a suggestion was originally proposed by Milton Friedman in 1955. See his "The Role of Government in Education," in Robert A. Solo, Editor, Economics and the Public Interest (Rutgers University Press, 1955); reprinted in Milton Friedman, Capitalism and Freedom (Chicago: University of Chicago Press, 1962). For additional arguments in favor of private production of education see A. Peacock and J. Wiseman, Education for Democrats (London: Institute of Economic Affairs, 1964); R.M. Parish, "The Economics of State Aid to Education," Economic Record, XXXIX (1963), 292-304; and W.E. Laird and D.L. Schilson, "Financing Investment in Education," Journal of General Education, XVII (1965), 55-61.

4. For an excellent review of input and output in high school education see Jesse Burkhead, Thomas G. Fox and John W. Holland, Input and Output in Large-City High Schools (Syracuse: Syracuse University Press, 1967). For a recent survey, including some critical comments on the educational production function, see Samuel Bowles, "Towards an Educational Production Function," paper presented at the National Bureau of Economic Research, Conference on Research in Income and Wealth, November 15-16, 1968. Other studies on production in secondary schooling include Elchanan Cohn, "Economies of Scale in Iowa High School Operations," Journal of Human Resources, III (1968), 422-434; James S. Coleman, et al., Equality of Educational Opportunity,

2 vols. (Washington: Government Printing Office, 1966); Herbert J. Kiesling, "Measuring a Local Government Service: A Study of School Districts in New York State," Review of Economics and Statistics, XLIX (1967), 356-67; and Richard Raymond, "Determinants of the Quality of Primary and Secondary Public Education in West Virginia," Journal of Human Resources, III (1968), 450-70.

5. Burkhead, *et al.*, *op. cit.*, p. 26. In statistical terminology, the utilization of equation (2) with only one output may lead to a "simultaneous equation bias." Recent evidence by Thomas G. Fox indicates that such a bias may be considerable. Reworking his earlier model of production in the Chicago high schools (see Burkhead, *et al.*, Chapter III), using achievement and holding power simultaneously--instead of one or the other as in the earlier work--he showed that (1) the explanatory power of the model (R^2) increases considerably; and (2) more input variables are statistically significant: whereas in the earlier work only some of the variables in the set $s_1,\ldots,s_m$ were found to be statistically significant, in the joint-product model some of the school variables, in the set $z_1,\ldots,z_k$, were also significant. See Thomas G. Fox, "Joint Production and Cost Functions for a Big-City High School System," paper presented to the Joint National Meeting of the American Astronautical Society and the Operation Research Society, June 17-20, 1969.

6. See John E. Brandl, "Comment on 'Towards an Educational Production Function' by S. Bowles," in Conference on Research in Income and Wealth, *op. cit.*

7. In most cases, the transformation improves the results only slightly. Also, an objection to the logarithmic form has been voiced by Bowles. See Bowles, *op. cit.*, and Brandl, *op. cit.*

8. Mathematically, the infintesimal marginal productivity of input i, at any given level of z_i, is $\partial Q_i/\partial z_i$. The finite marginal productivity is given by $\Delta Q_i/\Delta z_i$, where Δ stands for a "finite change in...." When the production function is non-linear the two marginal productivities will differ, in general.

9. The elasticity of z_i with respect to Q is given by the percentage change in Q divided by the percentage change in z_i. Mathematically, this is given by $(\partial Q/Q)/(\partial z_i/z_i)$. For a simple exposition on the concept of elasticity see, e.g., Richard H. Leftwich, The Price System and Resource Allocation, 3rd Edition (New York: Holt, Rinehart and Winston, 1966), especially pp. 33-44.

10. Other objectives might include (1) the preparation of students for the world of work; (2) changing (or, perhaps, maintaining) students' attitudes towards self, school, family, and society; and (3) the maximization of the excellence or reputation of the secondary school.

11. A study of the Iowa high school system indicates that schools with higher ADA hire, as might be expected, proportionately more teachers. What is surprising is that ADA and T are extremely closely correlated (the simple correlation coefficient, r, is 0.975). ADA was also found to be correlated (directly) with U (r=0.8) and (inversely) with A/T (r=-0.5). The correlation between ADA and median teachers' salaries (not F/T) was considerable (r=0.4), while no significant correlation was found between ADA and the students-teachers ratio, ADA/T. For more detail see my "Economies of Scale in Iowa High School Operations," op. cit.

12. This involves the selection of the optimal number of assignments per teacher (A/T), number of units (U), and the number of course per unit (S).

13. This could be done, for example, by lowering the "base salary," by awarding smaller increments for educational attainment and/or experience above and beyond the levels required for the base salary, or by seeking inexperienced or less educated teachers. We assume here that the so-called "single salary schedule" is used by the school administrators. For an authoritative discussion of teacher salary schedules see Charles S. Benson, The Economics of Public Education, 2nd Edition (Boston: Houghton Mifflin Company, 1968), Chapter 10.

14. In Iowa, for example, each high school was required, in 1968, to offer 27 units in specific subject matters. The average number of units offered for a sample of 375 Iowa high schools, in 1961-2, was 33.35 (with a standard deviation of 10.18). It can be safely assumed that the average number of units offered, in 1968, for the same sample of schools was considerably higher than that in 1961-2.

15. This type of an argument has recently been used to support the "educational parks" idea, i.e., huge school centers, where many sections of each unit are likely to be taught. See John Sessions, "A New Approach to Urban Education," Changing Education (May 1966), cited in August C. Bolino, "Education, Manpower, and Economic Growth, Journal of Economic Issues, II (1968), 323-41.

16. A classic on mathematical programming is Robert Dorfman, Paul A. Samuelson, and Robert M. Solow, Linear Programming and Economic Analysis (New York: McGraw-Hill Book Company, 1958).

17. Suppose we have initially the following: T = 100, F/T = $10,000, SU = 200 (SU/T = 2), and A = 50 (A/T = 0.5). All that we require of the principal, at this point, is to weigh the possibilities of increasing Q by changing T alone. We might ask him the following question: If F/T were to be reduced to $9,000, so that we can now hire approximately 11 more teachers, would the reduction in Q due to the supposedly reduced quality of the average teacher be more or less than compensated for by the reduction in the teaching load (the new SU/T is now only 1.8) and the increase in specialization (A/T is reduced to only 0.45)? If he is able to provide answers to such questions, marginal changes in T would then be made until a small change in T would result in no appreciable increase in Q.

18. That is, we stop at a point, short of the suboptimum as described above, at which changes in T could still increase Q (i.e., the sum of the marginal productivities of F/T, SU/T and F/T is still positive) but at which all feasible changes in T have been made to get a maximum Q.

19. Again, if constraints limit our ability to achieve this condition, the sum of the marginal productivities of U, SU/T and kADA/SU will be positive at the constrained suboptimum.

20. This is true only when a full optimization is not possible. The sub-optimization procedure may lead to a solution inferior to that obtainable by the optimization process as described above. However, the latter was not considered practicable at this time.