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ABSTRACT

Presented is the scope and sequence of mathematics for Grades 4 and 5. It is based on the program which is introduced in prekindergarten through Grade 3 and is designed to extend and strengthen the computational skills and mathematical understanding of children. The units described are organized around four main themes--sets, number, and numeration; operations; geometry and measurement; and algebraic concepts, graphs, probability, and statistics. (FL)

# MATHEMATICS

- Grade 4
- Grade 5

## *Scope and Sequence*



BUREAU OF CURRICULUM DEVELOPMENT
BOARD OF EDUCATION OF THE CITY OF NEW YORK

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# MATHEMATICS

## Grade 4
## Grade 5

## *Scope and Sequence*

BUREAU OF CURRICULUM DEVELOPMENT
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## FOREWORD

This publication is designed to extend and strengthen the computational skills and mathematical understandings of children in Grades 4 and 5. It presents an overall scope and detailed sequence for implementation by teachers and supervisors, based on the modern program already introduced in Prekindergarten through Grade 3. Teaching manuals are now being prepared to assist teachers to translate scope and sequence into specific learning activities for children.

The bulletin has been jointly developed by staff members of the Bureau of Curriculum Development and the Bureau of Mathematics. We express appreciation to them for this contribution to relevancy in mathematics instruction.

*Deputy Superintendent of Schools*
SEELIG LESTER

C O N T E N T S

INTRODUCTION

The scope and sequence for Mathematics: Grade 4 and for Mathematics: Grade 5 is presented in the form of units, each of which may require one or more lessons.

The units are organized around four central themes. These themes are coded as shown below:

* Sets; Number; Numeration
** Operations
*** Geometry and Measurement
**** Algebraic Concepts; Graphs; Probability; Statistics

The sequence is structured to provide a spiral or cyclic approach for presentation of the concepts and skills.

The teacher should make provision for review, reinforcement, practice, and evaluation to meet the particular needs of the class.

Units or parts of units marked ⊙ are optional units and may be used at the discretion of the teacher.

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The preparation of this bulletin was under the general direction of Seelig Lester, Deputy Superintendent, Office of Instructional Services; David A. Abramson, Acting Director, Bureau of Curriculum Development; and George Grossman, Director, Bureau of Mathematics.

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Edythe Kahn, Editor, Bureau of Curriculum Development, had overall responsibility for design and production. Simon Saulman designed the cover.

## Mathematics: Grade 4

The scope and sequence for Grade 4 is presented in the form of 68 units.

The development of concepts and operations is presented in a cyclic pattern. For example, the development of the algorithm for recording division in vertical format begins in Unit 14, continues in Units 16, 21, 28, 36, 43, and the understanding is reinforced and extended in Units 48 and 55.

To provide an overview of the mathematical understandings and skills for Grade 4, the following is presented:

## Mathematical Goals for Grade 4

### 1. Sets; Number; Numeration

Finite and infinite sets
Set notation
Place value through hundred-thousands; record numerals in expanded notation; read and write numerals through hundred-thousands
Round numbers to nearest ten-thousand; to nearest dollar
Roman numerals L, C

### 2. Addition and Subtraction of Whole Numbers

Standard algorithm for computing sums, two or more addends, numbers in the ten-thousands and beyond
Observe and use properties of addition
Standard algorithm for computing differences
Addition and subtraction in dollars-cents system

### 3. Multiplication of Whole Numbers

Algorithm: horizontal and vertical format, numbers in the thousands and beyond multiplied by a number less than 10, with and without exchange
Use commutative property of multiplication; the distributive property of multiplication over addition
Multiplication in dollars-cents system

### 4. Division of Whole Numbers

Algorithm: vertical format, dividends-numbers in the thousands and beyond, divisors-numbers less than 10
Division in dollars-cents system
One as a divisor and zero as a dividend

5. Fractional Numbers

Concept: fifths, tenths, ninths, and twelfths of a whole
Algorithms: finding unit and non-unit fractional parts of numbers
Algorithms: computing sums and differences of fractional numbers expressed with like and related denominators, horizontal and vertical format
Algorithms: multiplication of fractional numbers by a whole number
Observe and use properties of operations

6. Geometry

Concepts: line segments in a circle; perimeter; circumference as a perimeter
Properties of space figures: cone, cylinder, cube, sphere

7. Measurement

Linear: quarter-inch; eighth-inch; mile; perimeter of rectangle
Weight: ton
Time: Clock - seconds; number of seconds in a minute; record time as 3:06 P.M.
Calendar - relationship between number of days in a year, in a leap year; number of weeks in a year
Temperature: freezing point; boiling point
Concept of speed and frequency as rate

8. Algebraic Concepts

Solution set for relationship such as $\square > 4$ and $\square < 10$, when replacement set is set of whole numbers
Graph: number on a line
solution set of an open sentence with one place holder (variable)
Tabulate solutions of open sentences
Write open sentence to describe relationship between two variables shown in table

9. Statistics and Probability

Interpret circle graphs when sectors represent the same unit fraction
Interpret and construct bar graphs
Concept of mean (average)
Coin tossing: one coin - tabulate results of several tosses
express probability as a fraction

## MATHEMATICS: GRADE 4

### Scope and Sequence

***

1. Geometry

   Reinforce and/or develop concepts of point; of space as a set of points; of plane.

   Reinforce recognition of geometric figures in a plane, such as: quadrilateral, rectangle, square, triangle, circle.

   Develop concept that plane figures divide a plane into 3 sets of points. Use terms: inside, outside, on.

*

2. Sets; Number

   Recall and/or develop the concept of infinite set, such as the set of counting numbers and the set of whole numbers; record in set notation: set of counting numbers, {1,2,3,...}; set of whole numbers, {0,1,2,3,...}.
   Develop the concept of finite set using clock arithmetic.

   Use the set of odd numbers and the set of even numbers and reinforce addition and subtraction of whole numbers by guiding the children to discover patterns, such as: the sum of 2 odd numbers is an even number; the sum of an odd number and an even number is an odd number; etc.

*

3. Numeration

   Use representations of 1 thousand, such as squared materials, to

   a. reinforce place value to thousands

      recall regrouping tens for ones, etc.

   b. group and count by tens to 10 tens or 1 hundred; by hundreds to 10 hundreds or 1 thousand; by thousands to 10 thousands or 1 ten-thousand

c. introduce place value to ten-thousands

| Ten Thousands | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |

d. prepare period chart for reading numerals in ten-thousands

e. read and write numerals for numbers in ten-thousands

f. record numerals using expanded notation, e.g., 12,534

1 ten-thousand + 2 thousands + 5 hundreds + 3 tens + 4 ones
10,000 + 2000 + 500 + 30 + 4

*

4. Number

Reinforce and/or develop concept of rounding numbers to the nearest ten and nearest hundred.

A suggested procedure is,

231 is between 200 and 300
250 is halfway between 200 and 300
231 < 250 Therefore, 231 is nearer to 200 than to 300
231, rounded to the nearest hundred, is 200

Develop rounding numbers to the nearest thousand.

****

5. Algebraic Concepts

Reinforce concept of mathematical sentences.

Use these concepts and the relation symbols +, -, =, >, <, ≠ to reinforce basic addition and subtraction facts and higher decade facts.

Use addition properties: commutative, associative, identity element.

The use of mathematical sentences and other algebraic concepts should be emphasized wherever possible.

**

## 6. Addition of Whole Numbers

Reinforce and/or develop procedures for computing sums in vertical format. Use two addends, each named by a 3-digit numeral. Estimate, compute, compare sum with estimate

An illustrative teaching sequence follows:

a. No exchange

$$\begin{array}{r} 532 \\ +463 \end{array}$$

b. Exchange ones for tens

$$\begin{array}{r} 529 \\ +432 \end{array}$$

c. Exchange tens for hundreds

$$\begin{array}{r} 592 \\ +326 \end{array}$$

d. Sums in thousands

$$\begin{array}{r} 602 \\ +598 \end{array}$$

e. Two exchanges sums in hundreds

$$\begin{array}{r} 687 \\ +214 \end{array}$$

f. Two exchanges sums in thousands

$$\begin{array}{r} 786 \\ +435 \end{array}$$

**

## 7. Subtraction of Whole Numbers

Reinforce and/or develop procedures for computing differences in vertical format. Use minuends and subtrahends named by 3-digit numerals.
Estimate, compute, compare result with estimate.

An illustrative teaching sequence follows:

a. No exchange

$$\begin{array}{r} 883 \\ -162 \end{array} \qquad \begin{array}{r} 634 \\ -321 \end{array}$$

b. Exchange tens for ones

$$\begin{array}{r} 684 \\ -187 \end{array} \qquad \begin{array}{r} 460 \\ -135 \end{array}$$

c. Exchange hundreds for tens

$$\begin{array}{r} 854 \\ -162 \end{array} \qquad \begin{array}{r} 406 \\ -133 \end{array}$$

d. Exchange tens for ones and hundreds for tens

$$\begin{array}{r} 834 \\ -357 \end{array} \qquad \begin{array}{r} 800 \\ -348 \end{array}$$

*

8. Number: Fractional Numbers

Use objects, diagrams, number lines to reinforce understanding of halves, fourths, eighths, thirds, and sixths as parts of a whole.

Review: symbol for fractional number
meaning of numerator and denominator
concept of equivalence (many names for the same number)
counting; comparing; expressing comparison as $\frac{1}{2} > \frac{1}{4}$
expressing fractional numbers in simplest form
concept of an improper fraction
expressing a fractional number as a mixed numeral and as a whole number: $\frac{12}{4} = 3$; $\frac{13}{4} = 3\frac{1}{4}$
concept of division: how many halves in 1 whole; 2 wholes, etc.
concept of addition and subtraction

In a similar manner, develop concepts of fifths and tenths.

***

9. Measurement: Linear

Reinforce the understanding that: measurement involves comparing the length of an object with a selected unit of length; all measurement is approximate; measurement is expressed as a number and a unit of measure; e.g., 4 feet.

Use experiences to reinforce understanding of foot, inch, half-inch, yard, and equivalent measures; e.g., the length of the ribbon is 13 inches, or 1 foot, 1 inch.

Develop concept of $\frac{1}{4}$ inch, and develop finding the number of $\frac{1}{4}$ inches in 1 inch; 2 inches; $2\frac{1}{2}$ inches; etc.

Develop concept of $\frac{1}{8}$ inch and develop finding the number of $\frac{1}{8}$ inches in 1 inch; 2 inches; $2\frac{1}{2}$ inches; etc.

**

10. Multiplication of Whole Numbers

Develop concept of factor x factor = product relationship and reinforce basic facts.
Construct a multiplication table and observe commutative property, identity element "1", zero as a factor, closure.

Develop concept that a multiple of a whole number is a product of that number and any other whole number, e.g., 6 is a multiple of 3 because $2 \times 3 = 6$.
Use counting by threes, fours, etc. to reinforce concept of multiples.

**

## 11. Multiplication of Whole Numbers

Develop understanding of multiples of 10 and reinforce and/or develop multiplying multiples of 10 by numbers less than 10.

To provide readiness for division, use commutative property to develop multiplying numbers less than 10 by multiples of 10.

$2 \times 10 = 20$ $\quad 10 \times 2 = 20$ $\quad 2 \times 20 = 40$ $\quad 20 \times 2 = 40$

So $\quad 2 \times 10 = 10 \times 2 \quad 2 \times 20 = 20 \times 2$

Reinforce and/or develop algorithm for multiplication of numbers 10 through 99 by 2,3,4, and 5.

Suggested developments of algorithm for computing products:

a. 23 × 2

2 tens + 3 ones
x2
4 tens + 6 ones = 46

23
x 2
46

26 × 2

2 tens + 6 ones
x2
4 tens + 12 ones = 40+12 = 52

26
x 2
12(2x6)
40(2x20)
52(2x26)

26
x 2
52

b. $2 \times 23 = 2 \times (20+3) = (2x20)+(2x3) = 40 + 6 = 46$

23
x 2
40(2x20)
6(2x3)
46(2x23)

23
x 2
46

$2 \times 26 = 2 \times (20+6) = (2x20)+(2x6) = 40 + 12 = 52$

26
x 2
40(2x20)
12(2x6)
52(2x26)

26
x 2
52

**

## 12. Division of Whole Numbers

Reinforce basic facts using concept of missing factor and concept of multiplication and division as related operations.

Recall: recording and reading symbols for division such as

$2\overline{)8}$

$8 \div 2 = n$

$\square \times 2 = 8$

meaning of divisor, dividend, and quotient

role of 1 as a divisor, role of zero as a dividend

**

13. Operations: Fractional Numbers

Use problems involving sharing things to reinforce a procedure for finding $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$ of numbers.

Develop algorithm for finding non-unit fractional parts of numbers. To find $\frac{2}{3}$ of 12 inches, think:

$\frac{1}{3}$ of 12 = 4 because 12 ÷ 3 = 4, then

$\frac{2}{3}$ of 12 = 4 + 4 or 2 x 4

$\frac{2}{3}$ of 12 = 8

**

14. Division of Whole Numbers

Use experience situations to develop the concept that division may be interpreted as repeated subtractions. Children's thinking may be recorded in a variety of ways.

For $5\overline{)95}$ a child may suggest taking out 5 fives first.

| | | |
|---|---|---|
| $5\overline{)95}$ | $5\overline{)95}$ | $5\overline{)95}$ |
| 5 fives = 25 | 5x5 = 25 | -25 \| 5 |
| 70 still to divide | 70 | 70 |
| 8 fives = 40 | 8x5 = 40 | -30 \| 6 |
| 30 still to divide | 30 | 40 |
| 6 fives = 30 | 6x5 = 30 | -40 \| 8 |
| 0 | 0 | 0 \| |
| 19 fives in 95 | 19x5 = 95 | 19 |

Use divisors 2,3,4, and 5 to develop recording division as repeated subtraction.

***

15. Measurement: Time-Clock

Use projects involving experiences to recall number of hours in a day, number of minutes in an hour, and develop concept of seconds, number of seconds in a minute.

Reinforce understanding of A.M. and P.M. and telling time in quarter hours, in five-minute intervals.

Develop recording time as 3:06 P.M.

**

16. Division of Whole Numbers

Reinforce and/or continue to develop procedures for recording division in vertical format.

Divisors: 2,3,4, and 5
Dividends: Numbers less than 100

Reinforce multiplying multiples of ten and numbers less than 10 (limit to 2,3,4,5).

An illustrative teaching procedure is suggested: $2\overline{)48}$; $\square \times 2 = 48$; $48 \div 2 = n$.

a. Estimate:

| | |
|---|---|
| $10 \times 2 = 20$ | $20 < 48$ |
| $20 \times 2 = 40$ | $40 < 48$ |
| $30 \times 2 = 60$ | $60 > 48$, |

Therefore, $n > 20$ and $n < 30$.

b. Possible ways to arrive at solution:

$$\begin{array}{rr} & 24 \\ & 2\overline{)48} \\ 20\times2 = & \underline{40} \\ & 8 \\ 4\times2 = & \underline{8} \\ & 0 \\ \hline 24\times2 = & 48 \end{array} \qquad \begin{array}{rr} & 24 \\ & 2\overline{)48} \\ & \underline{-40} \quad 20 \\ & 8 \\ & \underline{-8} \quad \underline{4} \\ & 0 \quad 24 \end{array}$$

c. Compare quotient with estimate.

d. Check by multiplication.

**

17. Addition of Whole Numbers

Continue to develop procedures for computing sums in vertical format using more than 2 addends (column addition), sums less than 10,000.

A suggested teaching sequence follows:

a. No exchange

b. One exchange: exchange ones for tens; tens for hundreds

c. Two exchanges, etc.

Estimate, compute, compare sum with estimate.

Include dollars and cents and problem solving.

**

18. Multiplication of Whole Numbers

Use a procedure similar to the one described in Unit 11 to develop multiplication of numbers 10 through 99 by 6,7,8, and 9.

*

19. Number: Fractional Numbers

Use objects, diagrams, and number lines to develop concept of ninths and twelfths.

The following experiences should be included:

counting
ordering
comparing ninths and twelfths with halves, fourths, eighths, thirds, sixths
expressing as equivalent fractions $\frac{2}{6} = \frac{1}{12}$; $\frac{1}{4} = \frac{3}{12}$, etc.
expressing in simplest form
expressing a fractional number as a mixed numeral and as a whole number $\frac{24}{12} = 2$; $\frac{13}{12} = 1\frac{1}{12}$; $\frac{18}{9} = 2$; $\frac{19}{9} = 2\frac{1}{9}$
finding the number of ninths and twelfths in a whole; in 2 wholes; 3 wholes, etc. (concept of division)

**
20. Operations: Fractional Numbers

Use a procedure similar to the one indicated in Unit 13 to develop finding sixths, eighths, and ninths of numbers.

**
21. Division of Whole Numbers

Use a procedure similar to the one described in Unit 16 to develop dividing numbers less than 100 by 6,7,8, and 9.

****
22. Graphs: Bar Graph

Use experience situations to construct and interpret bar graphs.

**
23. Addition of Whole Numbers

Continue to develop addition algorithm, vertical format, 2 addends in the thousands.

A suggested teaching sequence follows:

a. Adding whole thousands to whole thousands

b. Adding whole thousands to numbers in the thousands

c. Adding numbers in the thousands to numbers in the thousands

Estimate, compute, compare sum with estimate.

**
24. Subtraction of Whole Numbers

Continue to develop subtraction algorithm, vertical format, numbers in the thousands.

A suggested teaching sequence:

a. Subtracting whole thousands from whole thousands $\begin{array}{r} 8000 \\ -3000 \\ \hline \end{array}$

b. Subtracting whole thousands from numbers in the thousands $\begin{array}{r} 8723 \\ -2000 \\ \hline \end{array}$

c. Subtracting numbers in the thousands, no exchange $\begin{array}{r} 8723 \\ -2413 \\ \hline \end{array}$

d. Subtracting numbers in the thousands, one exchange

| 1 ten for 10 ones | 1 hundred for 10 tens | 1 thousand for 10 hundreds |
|---|---|---|
| $\begin{array}{r} 4831 \\ -2318 \\ \hline \end{array}$ | $\begin{array}{r} 4836 \\ -2351 \\ \hline \end{array}$ | $\begin{array}{r} 4386 \\ -2814 \\ \hline \end{array}$ |
| $\begin{array}{r} 4830 \\ -2318 \\ \hline \end{array}$ | $\begin{array}{r} 4806 \\ -2351 \\ \hline \end{array}$ | $\begin{array}{r} 4086 \\ -2814 \\ \hline \end{array}$ |

Estimate, compute, compare result with estimate.

***

25. Geometry

Experiment with models of space figures to recognize cube, sphere, disc, cone, and cylinder, and to discover some of their characteristics.

****

26. Problem Solving

Reinforce procedure for analyzing and solving problems.

Include:

problems with insufficient and extraneous data
problems that may be solved in more than one way
problems that apply computational skills developed in previous units

Verbal problems should be presented throughout the school year.

**

27. Multiplication of Whole Numbers

Develop understanding of multiples of one hundred and introduce multiplying multiples of one hundred by numbers less than 10.

To provide readiness for division, use commutative property to develop multiplying numbers less than 10 by multiples of 100.

Develop algorithm for multiplication of numbers in the hundreds by 2,3,4, and 5.

Adapt procedure suggested in Unit 11.

**

28. Division of Whole Numbers

Continue to develop procedures for recording division as repeated subtractions.

Divisors: 2,3,4, and 5
Dividends: Numbers in the hundreds, less than 1000

Reinforce: multiplying multiples of ten and numbers less than 10
multiplying multiples of 100 and numbers less than 10

Adapt procedure indicated in Unit 16.

Estimate, compute, check results with estimate.

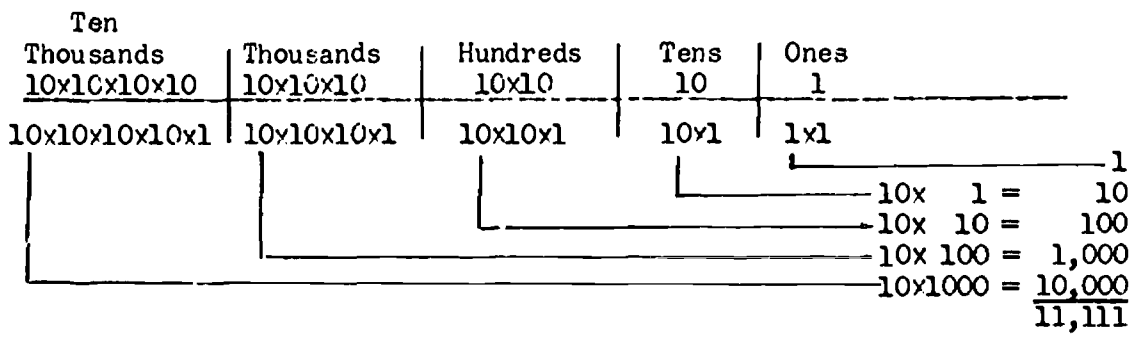
*

29. Numeration

Extend the understanding of place value through 10,000 by developing the idea that each place in a numeral has ten times the value of the place at its right.

Recall: symbols or digits used - 0,1,2,3,4,5,6,7,8,9
grouping by tens

For the numeral 11,111, the value of the digits may be shown as:




Adapt steps b,c,d,e, and f of Unit 3 to develop place value through hundred thousands.

*

30. Number

Use a procedure similar to the one indicated in Unit 4 to develop rounding numbers to the nearest 10,000.

**

31. Addition of Whole Numbers

Continue to develop procedures for computing sums in vertical format using more than 2 addends (column addition), sums less than 100,000.

Follow a procedure similar to the one suggested in Unit 17.

**

32. Subtraction of Whole Numbers

Continue to develop subtraction algorithm, vertical format, numbers in the thousands and introduce two exchanges.

Use a sequence similar to the one suggested in Unit 24.

***

33. Measurement: Weight

Use a procedure similar to the one indicated in Unit 9 to reinforce the understanding of units of weight: pound, ounce, fractional parts of a pound and equivalent measures. The bread weighs 1½ pounds or 1 pound, 8 ounces.

Develop understanding of concept of ton.

**

34. Operations: Addition of Fractional Numbers - Like Denominators

Reinforce: counting forward by fractional numbers; procedures for expressing fractional numbers in simplest form, as mixed numerals and as whole numbers.

Use experiences, projects, number lines to develop algorithms for computing sums of fractional numbers with like denominators.

Use horizontal and vertical format.
In computing the sums, use commutative and associative properties of addition.
Relate circle graph, when the sectors represent the same unit fraction, to addition of fractional numbers with like denominators.

**

35. Operations: Subtraction of Fractional Numbers - Like Denominators

Reinforce: counting backward by fractional numbers; procedures for expressing fractional numbers in simplest form, as mixed numerals and as whole numbers.

Use experiences, projects, number lines to develop algorithms for finding remainders.

Use horizontal and vertical format.

In subtracting fractional numbers, use the principle of inverse operations.

***

36. Geometry and Measurement

Use projects and/or physical models of rectangles to develop the concept of perimeter of a polygon; of a rectangle.

Reinforce units of measure, addition of whole numbers, addition of fractional numbers with like denominators by computing perimeters.

**

37. Multiplication of Whole Numbers

Use a procedure similar to the one described in Unit 27, and continue to develop procedures for multiplying numbers in the hundreds by 6,7,8, and 9.

**
38. Division of Whole Numbers

Use a procedure similar to the one indicated in Unit 28, and continue to develop procedures for recording division as repeated subtractions.

Divisors: 6,7,8, and 9
Dividend: Numbers in the hundreds less than 1000

***
39. Measurement: Temperature

Use projects involving experiences to recall use of thermometer to measure temperature, meaning of scale, meaning of above and below zero, reading thermometer.

Develop concept of freezing point, boiling point; construct scale.

****
40. Graphs: Line Graph

Use experience situations to construct and interpret line graphs.

*
41. Number; Numeration

Reinforce recording money values in dollars and cents system.

Develop the concept of rounding to the nearest dollar, e.g., $3.45 to the nearest dollar is three dollars or $3.

**
42. Multiplication of Whole Numbers

Use experiences to develop multiplication of numbers less than 10, and numbers expressed as dollars and cents (less than $10).
In computing products, express dollars and cents as cents and use algorithm for multiplication of whole numbers.

| | |
|---|---|
| $3.21 | 321 cents |
| x 4 | x 4 |
| | 1284 cents or $12.84 |

Estimate, compute, compare result with estimate.

**
43. Division of Whole Numbers

Use experiences and continue to develop procedures for recording division as repeated subtractions as suggested in Unit 28.

Divisors: Numbers less than 10
Dividends: Numbers expressed as dollars and cents (less than $10)

In computing quotients, express dollars and cents as cents and use algorithm for division of whole numbers, e.g., $2\overline{)234}$ cents.
Use open sentence to state division problem, e.g., $2.34 ÷ 2 = \square$
$\square \times 2 = \$2.34$.

**

44. Operations: Multiplication of Fractional Numbers

Use problems involving experiences, diagrams, number lines to develop an understanding of multiplication of fractional numbers by whole numbers as repeated addition.

$3 \times \frac{1}{2}$ may be thought of as $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$ or $1\frac{1}{2}$

*

45. Numeration: Roman Numerals

Use activities to reinforce the repetitive, additive, and subtractive principles of the Roman system of numeration, and record numbers 1 through 39 as Roman numerals.

Develop understanding of the symbols, L, C, and apply repetitive, additive, and subtractive principles to record numbers 39 through 100.

**

46. Subtraction of Whole Numbers

Continue to develop subtraction algorithm, vertical format, numbers in the thousands and introduce three exchanges.

Use a sequence similar to the one suggested in Unit 24.

**

47. Multiplication of Whole Numbers

Reinforce procedures for multiplying numbers in the hundreds by numbers less than ten leading to recording in concise form, e.g.,

$$\begin{array}{r} 243 \\ \times\ 3 \\ \hline 729 \end{array}$$

Extend procedures to multiplication of numbers in the thousands by 2,3,4, and 5.

**

48. Division of Whole Numbers

Continue to develop procedures for recording division as repeated subtractions.

Divisors: 2,3,4, and 5
Dividends: Numbers in the thousands, less than 10,000

Reinforce: multiplying multiples of 1000 and numbers less than 10
multiplying multiples of 100 and numbers less than 10
multiplying multiples of 10 and numbers less than 10

Adapt procedure indicated in Unit 16.

****

49. Statistics: Mean

Use objects and experience situations to develop understanding of the meaning of average (mean) of a set of numbers.

**

50. Operations: Addition of Fractional Numbers - Related Denominators

Reinforce: concept of equivalence, e.g., $\frac{1}{8} = \frac{2}{4}$; $\frac{2}{4} = \frac{4}{8}$; $\frac{1}{3} = \frac{2}{6}$; etc.

procedures for expressing fractional numbers in simplest form, as mixed numerals, and as whole numbers

Use experiences, projects, number lines to develop algorithms for computing sums of fractional numbers with related denominators.

An illustrative teaching sequence follows:

$$\frac{1}{2} + \frac{5}{6} = n$$

$$3\frac{1}{2} + \frac{5}{6} = n$$

$$3\frac{1}{2} + 7\frac{5}{6} = n$$

Use horizontal and vertical format. In computing sums, use the commutative and associative properties of addition.

**

51. Operations: Subtraction of Fractional Numbers - Related Denominators

Reinforce: concept of equivalence

procedures for expressing fractional numbers in simplest form

Use experiences, projects, number lines to develop algorithms for subtraction of fractional numbers with related denominators.

An illustrative teaching sequence follows:

$$\frac{3}{8} - \frac{1}{4} = n$$

$$3\frac{3}{8} - \frac{1}{4} = n$$

$$3\frac{3}{8} - 2 = n$$

$$3\frac{3}{8} - 2\frac{1}{4} = n$$

Use horizontal and vertical format. Check solutions by using the related addition operation.

***

52. Geometry and Measurement: Circle

Reinforce: meaning of simple closed curve

meaning of plane

meaning of polygon

meaning of circle

Reinforce and/or develop understanding of line segments in a circle: radius, diameter, chord.

Use projects and models of circles to develop the concept of circumference as perimeter.

**

53. Operations: Subtraction of Fractional Numbers - Related Denominators: Regrouping

Reinforce renaming numbers such as $2 = \frac{1}{3}$ $\quad 2 = 1\frac{3}{3}$ $\quad 2\frac{1}{4} = 1\frac{5}{4}$

Continue to develop algorithms for subtraction of fractional numbers with related denominators involving renaming.

An illustrative teaching sequence follows:

$$8 - \frac{3}{4} = n$$

$$8 - 3\frac{5}{6} = n$$

$$5\frac{1}{4} - 2\frac{3}{4} = n$$

$$3\frac{2}{3} - \frac{5}{6} = n$$

$$9\frac{1}{3} - 7\frac{3}{4} = n$$

Use horizontal and vertical format.

**

54. Multiplication of Whole Numbers

Continue to develop algorithms for multiplication of numbers in the thousands and beyond by 6, 7, 8, and 9.

Use concise form as indicated in Unit 47.

**

55. Division of Whole Numbers

Use a procedure similar to the one indicated in Unit 48, and continue to develop recording division as repeated subtractions.

Divisors: 6,7,8, and 9
Dividends: Numbers in the thousands and beyond.

****

56. Graphs: Graphing a Number on a Line

Reinforce the concept of ray and associating numbers with points on a number ray.

Reinforce and/or develop meaning of replacement set and solution set (truth set).

Develop understanding of the graph of the solution set of an open sentence with one place holder (variable), e.g., when the replacement set is the set of whole numbers, the graph of $\square + 3 = 8$ is indicated as:


****

57. Algebraic Concepts: Number Sentences and Graphs

Develop finding the solution set when two open sentences are used to state relationships such as $\square > 4$ and $\square < 10$.

Develop graphing of the solution of such relationships when the replacement set is the set of whole numbers. For example, the graph of $\square > 4$ and $\square < 10$ is


** 58. Addition and Subtraction of Whole Numbers

Applying the principle of inverse operations, continue to develop skill in use of algorithms for addition and subtraction.

Continue to develop skill in computing sums, using more than two addends (column addition).

*** 59. Measurement: Time - Calendar

Use projects involving experiences to develop understanding of number of days in a month, number of weeks in a year, and number of days in a year, in a leap year.

** 60. Multiplication and Division of Whole Numbers

Reinforce and/or develop an understanding of:
"1" as the identity element for multiplication
the role of zero in multiplication
the role of "1" as a divisor
the role of zero as a dividend
Patterns such as the following may be used:

| | | | |
|---|---|---|---|
| $6 \times \square = 6$ | $6 \times \square = 0$ | $\square \overline{)6}$ | $6 \overline{)\square}$ |
| $7 \times \square = 7$ | $7 \times \square = 0$ | $\square \overline{)7}$ | $7 \overline{)\square}$ |
| $8 \times \square = 8$ | $8 \times \square = 0$ | $\square \overline{)8}$ | $8 \overline{)\square}$ |
| $\square \times 1 = \square$ | $\square \times 0 = 0$ | $1 \overline{)\square}$ | $\square \overline{)0}$ |

*** 61. Geometry: Symmetry

Experiment with mirrors, paper folding, etc., to discover and recognize symmetrical patterns involving line symmetry.

⊙* 62. Number; Numeration: Place Value

Adapt suggested procedures for Unit 29 to develop understanding of meaning of numbers through millions.

⊙ ** 63. Multiplication of Whole Numbers

Develop multiplication using multiples of ten as factors.
Use horizontal and vertical forms.

Some drill patterns are shown.

a.
$10 \times 10 = 100$
$10 \times 20 = 200$
$10 \times 30 = 300$
etc.

b.
$10 \times 10 = 100$
$20 \times 10 = 200$
$30 \times 10 = 300$
etc.

c.
$10 \times 10 = 100$
$20 \times 20 = 400$
$30 \times 30 = 900$
etc.

d.

| | | |
|---|---|---|
| 10 | 10 | 10 |
| x10 | x20 | x30 |
| 100 | 200 | 300, etc. |

***

64. Measurement: Linear

Use projects to develop the concept of mile and the concept of speed and frequency as rate.

****

65. Algebraic Concepts

Reinforce operations and properties of operations:
by tabulating solution sets of open sentences such as: $\square + \diamond = 8$
$\square = (2 \times \diamond) + 1$

by writing an open sentence derived from a table such as:

| $\square$ | $\diamond$ |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 3 | 5 |

etc.

The open sentence is: $\diamond = (2 \times \square) - 1$.

**

⊙ 66. Multiplication of Whole Numbers

Use the distributive property of multiplication over addition and continue to develop multiplication with one factor, 10, and the other factor named by a two-digit numeral.

$13 \times 10 = n$
$13 \times 10 = (10 \times 10) + (3 \times 10)$
$= 100 + 30$
$= 130$

| | | | | | |
|---|---|---|---|---|---|
| 10 | 10 | 10 | 10 | | 10 |
| x13 | x10 | x 3 | x13 | | x13 |
| | 100 | 30 | 100(10x10) | | 30(3x10) |
| | | | 30(10x3) | | 100(10x10) |
| | | | 130 | | 130 |

****

67. Probability

Experiment with tossing one coin to reinforce the understanding that the probability of obtaining a head is one chance out of two, and develop the concept of a sample event.

Develop expressing this probability as a fraction.

Continue to experiment with tossing one coin. Tally, tabulate, and graph results after 8 throws; after 10 throws, etc.

**

⊙68. Multiplication of Whole Numbers

Use the distributive property of multiplication over addition, and continue to develop multiplication when each factor is named by a two-digit numeral.

$12 \times 34 = n$

$12 \times 34 = (10 \times 34) + (2 \times 34)$

$= 340 + 68$

$= 408$

| 34 | 34 | 34 | 34 | | 34 | |
|---|---|---|---|---|---|---|
| x12 | x10 | x 2 | x12 | | x12 | |
| | 340 | 68 | 340 | (10x34) | 68 | (2x34) |
| | | | 68 | (2x34) | 340 | (10x34) |
| | | | 408 | | 408 | |

## Mathematics: Grade 5

The scope and sequence for Grade 5 is presented in the form of 66 units.

The development of concepts and operations is presented in a cyclic pattern. For example, the development of multiplication of fractional numbers begins in Unit 14, continues in Units 15 and 18, and is reinforced and extended in Units 28 and 47.

To provide an overview of the mathematical understandings and skills for Grade 5, the following is presented:

### Mathematical Goals for Grade 5

1. Sets; Number; Numeration

   Intersection of sets; symbols ∪ and ∩
   Venn diagrams
   Place value through millions; read and write numerals through millions
   Round numbers to the nearest hundred-thousand
   Decimal system of numeration through thousandths
   Round numbers in decimal form to nearest whole number
   Relationship between decimal system of notation and dollars-cents system
   Roman numerals D, M

2. Addition and Subtraction of Whole Numbers

   Standard algorithm for computing sums, two or more addends, numbers in the hundred-thousands and beyond
   Observe and use properties of addition
   Standard algorithm for computing differences
   Addition and subtraction in dollars-cents system

3. Multiplication of Whole Numbers

   Algorithm: any whole number by numbers 10-99
   Use commutative property of multiplication; the distributive property of multiplication over addition
   Multiplication in dollars-cents system

4. Division of Whole Numbers

   Algorithm: short form with divisors less than 10
   Algorithm: vertical format, dividends numbers in the thousands and beyond, divisors 10-99
   Division in dollars-cents system

5. Fractional Numbers

Concepts: division of fractional numbers; sixteenths and other fractional numbers; density; a fraction as an indicated division
Equivalence using multiplicative identity, "1"
Procedure for expressing a whole number as a fraction; mixed numeral as improper fraction; an improper fraction as a mixed numeral or as a whole number
Algorithm: multiplication of any two fractional numbers
Algorithm: finding least common denominator
Algorithms: computing sums and differences of fractional numbers
Algorithms: computing sums and differences of numbers in decimal form through thousandths

6. Geometry

Relationships: between planes and lines; between planes; between parallel lines
Plane figures: ellipse, pentagon, parallelogram
Angle: concept, naming, symbols, classification
Circle: central angle, arc
Area: concept

7. Measurement

Linear: tenth of an inch; read measurements to nearest inch, half-inch, quarter-inch, eighth-inch; rulers calibrated in sixteenths of an inch
Perimeter: formula for perimeter of equilateral polygons of 3,4,5 sides
Area: rectangular region
Time: decade, century
Capacity: liquid and dry measure
Scale drawings

8. Algebraic Concepts

Concept of ordered number pair
Associate an ordered number pair with a point in a plane
Graph: solutions of an equation with two variables (first quadrant)

9. Statistics and Probability

Interpret circle graphs when sectors represent fractions with related denominators
Read, interpret, and construct bar graphs; line graphs
Compute mean
Median (odd number of cases)
Coin tossing: two coins; possible combinations; probability of a favorable outcome expressed as a fraction; tabulate results of several tosses

## MATHEMATICS: GRADE 5

### Scope and Sequence

***

1. Geometry

Reinforce concepts of: point, space, plane, line segment, ray, line, closed curve, simple closed curve.

Develop understanding of relationships between planes and lines: two intersecting lines in a plane, two parallel lines in a plane; relationship between two planes: two intersecting planes, two parallel planes.

*

2. Numeration

Reinforce and/or develop place value through hundred-thousands by using the idea that each place in a numeral has ten times the value of the place at its right.

Introduce place value to millions:

a. Develop a place value chart and show the meaning of 1,111,111.

| Millions | Hundred Thousands | Ten Thousands | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|---|---|
| 10x100,000 | 10x10,000 | 10x1000 | 10x100 | 10x10 | 10x1 | 1 |

| | | |
|---|---|---|
| 1x | 1 = | 1 |
| 10x | 1 = | 10 |
| 10x | 10 = | 100 |
| 10x | 100 = | 1,000 |
| 10x | 1,000 = | 10,000 |
| 10x | 10,000 = | 100,000 |
| 10x100,000 | = | 1,000,000 |
| | | 1,111,111 |

b. Count by tens to 1 hundred, by hundreds to 1 thousand, etc.
c. Prepare a period chart.
d. Read and write numerals in millions.
e. Use relation symbols >, < to compare numbers.

*

3. Number

Reinforce concept of rounding numbers to the nearest hundred; to nearest thousand; to nearest ten-thousand.

A suggested procedure is:

46,239 is between 40,000 and 50,000
45,000 is halfway between 40,000 and 50,000
46,239 > 45,000
Therefore, 46,239 is nearer to 50,000

46,239, rounded to the nearest ten-thousand, is 50,000.

Develop rounding numbers to the nearest hundred-thousand.

****

4. Algebraic Concepts

Reinforce understanding of:

mathematical sentences: true, false, open
meaning of place holder (variable)
relation symbols >, <, =, ≠
replacement set, solution set
rules for substitution

Use these understandings and the properties of operations to reinforce basic addition, subtraction, multiplication, and division facts.

It is recommended that to maintain skill, such drill activities be ongoing throughout the school year.

**

5. Addition of Whole Numbers

Reinforce algorithm for computing sums in vertical format, two addends in the thousands.

Develop procedure for computing sums in vertical format, two addends in the ten-thousands.

Estimate, compute, compare sum with estimate.

Use commutative property of addition to check.

Include: word problems
computations with dollars and cents

**

6. Subtraction of Whole Numbers

Reinforce algorithm for computing differences in vertical format, numbers in the thousands, with exchange.

Develop procedure for computing differences in vertical format, numbers in the ten-thousands.

Use the principle of inverse operations to check.

Include: word problems
computations with dollars and cents

*

7. Number: Fractional Numbers

Use diagrams, number lines to reinforce understanding of halves, fourths, and eighths as parts of a whole.

Include: symbol for fractional number
meaning of numerator and denominator
concept of equivalence
counting; comparing - expressing comparison as $\frac{1}{2} > \frac{1}{4}$
expressing fractional numbers in simplest form
concept of improper fraction (a fraction whose numerator is $\geq$ the denominator)
expressing a fractional number such as $\frac{12}{4}$ as 3 and a fractional number such as $\frac{13}{4}$ as $3\frac{1}{4}$.
concept of division: how many eighths in 1 whole; 2 wholes, etc.
concept of addition and subtraction

Use a similar procedure to develop concept of sixteenths.

Develop the idea that between any two fractional numbers there is another fractional number.

***

8. Measurement: Linear

Reinforce understanding that measurements are approximate and counting is exact.

Use experiences to reinforce reading of rulers calibrated in inches, half-inches, quarter-inches, and eighth-inches.

Develop concept of $\frac{1}{16}$ inch and develop finding number of $\frac{1}{16}$ inches in 1 inch; in 2 inches; in $\frac{1}{2}$ inch; $\frac{1}{4}$ inch; $2\frac{1}{2}$ inches; etc.

Develop reading measurements to nearest inch; to nearest half-inch; to nearest quarter-inch; to nearest eighth inch.

**

9. Multiplication of Whole Numbers

Reinforce basic multiplication facts:
Use concept of factor x factor = product relationship.
Use concept that a multiple of a whole number is the product of that number and any other whole number, e.g., 6 is a multiple of 3 because 2x3=6.
Observe and use commutative property of multiplication; distributive property of multiplication over addition; identity element "1"; zero as a factor.

Reinforce multiplying numbers in the thousands and beyond by numbers less than ten.

Reinforce and/or develop multiplication using multiples of ten as factors.

Use horizontal and vertical format. Some drill patterns are shown:

a. 10x10=100
10x20=200
10x30=300
etc.

b. 20x10=200
20x20=400
20x30=600
etc.

c. 10x10=100
20x20=400
30x30=900
etc.

d.
$$\begin{array}{r} 10 \\ \times 10 \\ \hline 100 \end{array} \qquad \begin{array}{r} 10 \\ \times 20 \\ \hline 200 \end{array} \qquad \begin{array}{r} 10 \\ \times 30 \\ \hline 300 \end{array} \quad \text{etc.}$$

**

10. Division of Whole Numbers

Reinforce basic facts:
Use concept of missing factor.
Apply concept of multiplication and division as inverse operations.
Recall meaning of dividend, divisor, quotient, and record and read symbols for division.
Observe role of one and zero in division.

Reinforce procedures for recording division as repeated subtraction.

Divisors: Numbers less than 10
Dividends: Numbers in the thousands and beyond

*

11. Number: Fractional Numbers

Develop concept of a fraction as representing an indicated quotient, e.g., $1 \div 3$ is the same as $\frac{1}{3}$.

Develop concept that any whole number can be expressed as a fraction with a denominator of 1. For example, another name for 5 is $\frac{5}{1}$.

Develop a procedure for expressing an improper fraction such as $\frac{8}{2}$ as 4, and an improper fraction such as $\frac{9}{2}$ as $4+\frac{1}{2}$ or $4\frac{1}{2}$ (expressing an improper fraction in simplest form).

***

12. Measurement: Perimeter

Reinforce understanding of meaning of perimeter of a polygon; of a rectangle.

Develop procedures for computing the perimeter of a rectangle such as:

a. combining the sum of the measures of the length of the rectangle with the sum of the measures of the width of the rectangle

b. doubling the sum of the measure of the length of the rectangle and the measure of the width of the rectangle, etc.

Reinforce addition of whole numbers and addition of fractional numbers with like and related denominators by computing perimeters of polygons; of rectangles.

**

13. Division of Whole Numbers

Reinforce expressing numbers such as 64, as 6 tens and 4 ones.

Develop procedures for recording division using the distributive property of division over addition.

Divisors: Numbers less than 10
Dividends: Numbers in tens; numbers in hundreds

An illustrative teaching sequence follows:

a. Present dividends that require no exchange.

For $2\overline{)24}$ 24 expressed as 2 tens + 4 ones

$$\begin{array}{r} \text{1 ten + 2 ones} \\ \hline 2)\overline{\text{2 tens + 4 ones}} \end{array} \qquad 1 \text{ ten} + 2 \text{ ones} = 10 + 2 = 12$$

For $2\overline{)648}$ 648 expressed as 6 hundreds + 4 tens + 8 ones, etc.

b. Present dividends in which the first digit names a number greater than the divisor (one exchange).

For $2\overline{)34}$ 34 expressed as 3 tens + 4 ones, then regrouped as 2 tens + 14 ones

$$\begin{array}{r} \\ \hline 2)\overline{\text{3 tens + 4 ones}} \end{array} \qquad \begin{array}{r} \text{1 ten + 7 ones} \\ \hline 2)\overline{\text{2 tens + 14 ones}} \end{array} \qquad 10 + 7 = 17$$

For $2\overline{)748}$ 748 expressed as 7 hundreds + 4 tens + 8 ones, then regrouped as 6 hundreds + 14 tens + 8 ones, etc.

**

14. Operations: Multiplication of Fractional Numbers

Use a pattern to develop use of x (multiplication sign) to represent the product of a fractional number and a whole number, such as $\frac{1}{2}$ of 4, e.g.

| | | |
|---|---|---|
| 4 sets of 4 = 16 | 4 fours = 16 | 4 x 4 = 16 |
| 2 sets of 4 = 8 | 2 fours = 8 | 2 x 4 = 8 |
| 1 set of 4 = 4 | 1 four = 4 | 1 x 4 = 4 |
| $\frac{1}{2}$ of a set of 4 = 2 | $\frac{1}{2}$ of four = 2 | $\frac{1}{2}$ x 4 = 2 |
| $\frac{1}{4}$ of a set of 4 = 1 | $\frac{1}{4}$ of four = 1 | $\frac{1}{4}$ x 4 = 1 |

Use experiences, diagrams, etc. to develop procedures for multiplication of fractional numbers less than 1, e.g.,

a. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ b. $\frac{1}{2} \times \frac{3}{3} = \frac{2}{6}$

Lead to the generalization: to find the product of two fractional numbers multiply the numerators and multiply the denominators.

**

15. Equivalent Fractional Numbers: Multiplication Property of "1"

Reinforce:
understanding of identity element of multiplication, "1"
naming in various ways, such as $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, etc.

understanding that to find the product of two fractional numbers multiply the numerators and multiply the denominators

Develop procedures for renaming fractional numbers in higher terms using the multiplicative identity.

Develop procedures for renaming fractional numbers in lower terms (simplest form) using the multiplicative identity.

**

16. Division of Whole Numbers

Continue to develop procedures for recording division using the distributive property of division over addition.

Divisors: Numbers less than 10
Dividends: Numbers in tens; numbers in hundreds

An illustrative teaching sequence continues from Unit 13.

a. Present dividends that require exchange in the second digit.

For $2\overline{)456}$ 456 expressed as 4 hundreds + 5 tens + 6 ones, then regrouped as 4 hundreds + 4 tens + 16 ones

$2\overline{)4 \text{ hundreds}+5 \text{ tens}+6 \text{ ones}}$ $\quad$ $\dfrac{2 \text{ hundreds}+2 \text{ tens}+8 \text{ ones}}{2\overline{)4 \text{ hundreds}+4 \text{ tens}+16 \text{ ones}}}$ $\quad$ 200+20+8=228

b. Present dividends that require exchange in the first and second digits, such as

$2\overline{)372}$ and adapt the procedure suggested in a.

***

17. Measurement: Perimeter of Polygons

Reinforce: meaning of polygon; recognition of polygons of 3, 4, and 5 sides.

Develop procedures for computing perimeter of polygons whose sides have the same length.

Reinforce multiplication of whole numbers by computing perimeter of polygon.

**

18. Operations: Multiplication of Fractional Numbers

Reinforce understanding that any whole number can be expressed with a denominator of 1.

Use generalization developed in Unit 14 to develop procedures for computing the product of a whole number and a fractional number.

a. $2 \times \frac{1}{3} = n$

$2 \times \frac{1}{3}$ can be expressed as $\frac{2}{1} \times \frac{1}{3}$, therefore $\frac{2}{1} \times \frac{1}{3} = \frac{2}{3}$

b. $\frac{1}{3} \times 2 = n$

$\frac{1}{3} \times 2$ can be expressed as $\frac{1}{3} \times \frac{2}{1}$, therefore $\frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$

Products may also be computed by using the meaning of multiplication as repeated addition, and using the commutative property of multiplication.

**

19. Division of Whole Numbers

Continue to develop procedures for recording division using the distributive property of division over addition.

Divisors: Numbers less than 10
Dividends: Numbers in the hundreds; numbers in the thousands

An illustrative teaching sequence continues from Unit 16.

a. Present dividends in which the first digit names a number less than the divisor.

For $2\overline{)126}$ 126 expressed as 1 hundred + 2 tens + 6 ones, then regrouped as 12 tens + 6 ones.

$$\begin{array}{r} 6 \text{ tens} + 3 \text{ ones} \\ 2\overline{)12 \text{ tens} + 6 \text{ ones}} \end{array} \qquad 60 + 3 = 63$$

b. Lead to finding quotients without showing exchange.

c. Use dividends in the thousands and beyond and continue to develop procedures for recording division using the distributive property of division over addition.

****

20. Statistics: Computing Mean (Average)

Reinforce understanding of mean (average).

Use experiences to develop procedures for computing mean of a set of numbers.

Vary one element in a set of scores and explore the effect on the mean.

****

21. Graphs: Bar and Line Graphs

Use experience situations to reinforce constructing and interpreting bar and line graphs.

Develop understanding of scale and axis to help children read and interpret bar and line graphs which appear in textbooks, newspapers, magazines, etc.

**

22. Addition of Whole Numbers

Reinforce procedures for computing sums in vertical format using more than 2 addends (column addition).

Continue to develop skill in computing sums using more than two addends, sums less than 1 million.

Estimate, compute, compare sum with estimate; use commutative property of addition to check.

Include: word problems
computations with dollars and cents

**

23. Subtraction of Whole Numbers

Reinforce procedure for computing differences in vertical format, numbers in the ten-thousands.

Develop procedures for computing differences in vertical format numbers in the hundred-thousands.

Use principle of inverse operations to check.

Include: word problems
computations with dollars and cents

*

24. Numeration: Roman Numerals

Use activities to reinforce the repetitive, additive, and subtractive principles of the Roman system of numeration and record numbers 1 through 100 as Roman numerals.

Develop understanding of the symbols D and M and apply repetitive, additive, and subtractive principles of the Roman system of numeration and record numbers through 1000.

**

25. Multiplication of Whole Numbers

Reinforce multiplication using multiples of ten as factors.

Use the distributive property of multiplication over addition and continue to develop multiplication algorithm when one factor is ten and the other factor is named by a two-digit numeral. Use horizontal and vertical format, e.g., 13 x 10 = n.

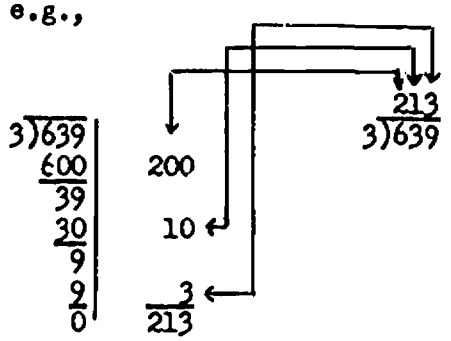
Rename 13 as 10 + 3. Then $13 \times 10 = (10+3) \times 10$
$= (10 \times 10) + (3 \times 10)$
$= 100 + 30$
$= 130$

| | | | | | |
|---|---|---|---|---|---|
| 10 | 10 | 10 | 100 | 10 | 10 |
| x13 | x10 | x 3 | + 30 | x13 | x13 |
| | | 30 | 130 | 100 (10x10) | 30 ( 3x10) |
| | | | | 30 ( 3x10) | 100 (10x10) |
| | | | | 130 (13x10) | 130 (13x10) |

**

26. Division of Whole Numbers

Develop understanding of relationship between finding quotients using the distributive property of division over addition, and finding quotients using repeated subtractions. Through discussion lead pupils to discover relationships among partial quotients, e.g.,




***

27. Geometry

Reinforce recognition of plane geometric figures: polygon; circle. Introduce ellipse.

Use objects, models, geoboard activities, etc. to develop an understanding of the characteristics of the members of the set of polygons through pentagons; of the members of the set of quadrilaterals through parallelograms.

**

28. Operations: Multiplication of Fractional Numbers

Reinforce procedures for naming a mixed numeral as an improper fraction.

Use generalization developed in Unit 14 to develop procedures for computing the product of a whole number and a number named by a mixed numeral.

a. $2 \times 3\frac{1}{3} = n$

2 can be expressed as $\frac{2}{1}$ and $3\frac{1}{3}$ can be expressed as $\frac{10}{3}$. Therefore, $\frac{2}{1} \times \frac{10}{3} = \frac{20}{3}$ or $6\frac{2}{3}$.

b. $3\frac{1}{3} \times 2 = n$

$3\frac{1}{3}$ can be expressed as $\frac{10}{3}$ and 2 can be expressed as $\frac{2}{1}$. Therefore, $\frac{10}{3} \times \frac{2}{1} = \frac{20}{3}$ or $6\frac{2}{3}$.

Products may also be computed by using the distributive property of multiplication over addition.

a. $2 \times 3\frac{1}{3} = n$

$2 \times 3\frac{1}{3} = (2 \times 3) + (2 \times \frac{1}{3})$

$= 6 + \frac{2}{3}$

$= 6\frac{2}{3}$

$$\begin{array}{r} 3\frac{1}{3} \\ \times\ 2 \\ \hline 6\ (2 \times 3) \\ \frac{2}{3}(2 \times \frac{1}{3}) \\ \hline 6\frac{2}{3} \end{array}$$

b. $3\frac{1}{3} \times 2 = n$

$3\frac{1}{3} \times 2 = (3 \times 2) + (\frac{1}{3} \times 2)$

$= 6 + \frac{2}{3}$

$= 6\frac{2}{3}$

$$\begin{array}{r} 2 \\ \times\ 3\frac{1}{3} \\ \hline 6\ (3 \times 2) \\ \frac{2}{3}(\frac{1}{3} \times 2) \\ \hline 6\frac{2}{3}(3\frac{1}{3} \times 2) \end{array}$$

***

29. Measurement: Area of Rectangular Region

Reinforce the concept that plane figures such as a rectangle divide a plane into three sets of points:

the set of points within the rectangle (interior)

the set of points on the rectangle

the set of points outside the rectangle (exterior)

Develop the concept that the set of points inside the rectangle together with the set of points on the rectangle form a region.

Develop: the concept that the measure of the region is called area
the understanding of the need for a standard unit of measure
standard unit of square measure

Use materials such as graph paper and geoboard to develop procedures for computing the area of regions by counting the number of unit squares contained in each.

*

30. Number; Numeration: Decimals

Use objects, diagrams, number lines to reinforce understanding of tenths as parts of a whole.

Reinforce understanding of multiplicative relationships developed in Unit 2 and develop the understanding that each place in a numeral (digit) has $\frac{1}{10}$ the value of the place at its left.

Use physical representations such as squared materials, activities to extend the numeration system to tenths.

a. Prepare a place value chart.
b. Group and count by tenths to ones; to ones and tenths.
c. Record in various ways.

2 and 1 tenth

$2\frac{1}{10}$

2.1

21 tenths

d. Read and write numerals.
e. Compare, e.g., $.6 < 1$.
f. Round to nearest whole number.

**

31. Operations: Addition and Subtraction of Decimals

Reinforce: reading and writing decimal numerals in tenths
counting forward and backward by tenths (decimal form)

Develop procedures for computing sums and differences using tenths, horizontal and vertical format.

An illustrative teaching sequence follows:

a. addends and minuends less than 1, no exchange

$.4 + .3 = n$

4 tenths
+ 3 tenths
7 tenths

$.4$
$+ .3$
$.7$

$.4 + .3 = n$

$\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$

$.4 + .3 = .7$

$.7 - .3 = n$

7 tenths
- 3 tenths
4 tenths

$.7$
$- .3$
$.4$

$.7 - .3 = n$

$\frac{7}{10} \quad \frac{3}{10} = \frac{4}{10}$

$.7 - .3 = .4$

b. addends and minuends greater than 1, no exchange

$2.6 + 5.3 = n$

2 + 6 tenths
+ 5 + 3 tenths
7 + 9 tenths

$2.6$
$+ 5.3$
$7.9$

$2.6 + 5.3 = n$

$2\frac{6}{10}$ $\quad 2.6$
$+ 5\frac{3}{10}$ $\quad + 5.3$
$7\frac{9}{10}$ $\quad 7.9$

$7.9 - 5.3 = n$

7 + 9 tenths
-(5 + 3)tenths
2 + 6 tenths

$7.9$
$- 5.3$
$2.6$

$7.9 - 5.3 = n$

$7\frac{9}{10}$ $\quad 7.9$
$- 5\frac{3}{10}$ $\quad - 5.3$
$2\frac{6}{10}$ $\quad 2.6$

c. addends and minuends greater than 1, with exchange

$3.5$
$+ 2.7$

3 + 5 tenths
2 + 7 tenths
5 + 12 tenths
or 6 + 2 tenths

$3.5$
$2.7$
$6.2$

$6.2$
$- 2.7$

6 + 2 tenths
-( 2 + 7)tenths

5 + 12 tenths
-( 2 + 7)tenths
3 + 5 tenths

$6.2$
$-2.7$
$3.5$

Estimate, compute, compare results with estimate.

Use commutative property to check addition.

Use principle of inverse operations to check subtraction.

Include word problems.

**

32. Multiplication of Whole Numbers

Reinforce multiplication using multiples of ten as factors.

Use the distributive property of multiplication over addition and continue to develop multiplication algorithms when each factor is named by a 2-digit numeral.

Use horizontal and vertical format, e.g., 12 x 34 = n.

Rename 12 as 10 + 2. Then 12 x 34 = (10x34) + (2x34)
= 340 + 68
= 408

| 34 | 34 | 34 | 340 |
|---|---|---|---|
| x 12 | x 10 | x 2 | + 68 |
| | 340 | 68 | 408 |

| 34 | | 34 | |
|---|---|---|---|
| x 12 | | x 12 | |
| 340 | (10x34) | 68 | ( 2x34) |
| 68 | ( 2x34) | 340 | (10x34) |
| 408 | (12x34) | 408 | (12x34) |

**

33. Division of Whole Numbers

Continue to develop procedures for recording division in vertical format.

Divisors: Numbers in the tens less than 50
Dividends: Numbers in the hundreds

Reinforce multiplying multiples of ten and numbers in the tens.

An illustrative teaching procedure follows:

$23\overline{)851}$ $\qquad$ $n \times 23 = 851$ $\qquad$ $851 \div 23 = n$

a. Estimate:

| | |
|---|---|
| 10 x 23 = 230 | 230 < 851 |
| 20 x 23 = 460 | 460 < 851 |
| 30 x 23 = 690 | 690 < 851 |
| 40 x 23 = 920 | 920 > 851, therefore n > 30 and n < 40 |

b. Possible ways to arrive at solution:

$$\begin{array}{r|l} 37 & \\ 23\overline{)851} & \\ -690 & 30 \\ \hline 161 & \\ -115 & 5 \\ \hline 46 & \\ -46 & \frac{2}{37} \\ \hline 0 & \end{array} \qquad \begin{array}{r|l} 37 & \\ 23\overline{)851} & \\ -690 & 30 \\ \hline 161 & \\ -161 & \frac{7}{37} \\ \hline 0 & \end{array}$$

c. Compare quotient with estimate.

d. Check by multiplication.

***

34. Geometry: Angles

Reinforce concept of ray.

Develop the concept of angle as the union of two rays not on the same line which have a common end point.

Discuss: vertex, sides, symbol $\angle$; naming angles.

Use models to observe right angles, angles less than right angles (acute), angles greater than right angles (obtuse).

**

35. Addition and Subtraction of Whole Numbers

Continue to develop skill in computing sums and differences, in vertical format, addends and minuends in the hundred thousands and beyond.
Estimate, compute, compare results with estimate.

Use commutative property of addition to check addition.

Use principle of inverse operations to check subtraction.

Include: word problems; computations with dollars and cents.

Note: It is recommended that to maintain skill, practice in computing sums and differences be provided throughout the school year.

*

36. Sets: Intersection of Sets

Reinforce: meaning of set; set notation; element of a set; number of a set; equivalent sets; finite set; infinite set; empty set; union of sets; subset.

Develop understanding of set intersection.

Introduce symbols: $\cup$ for union; $\cap$ for intersection.

Use Venn diagrams.

*

37. Number: Common Denominator; Least Common Denominator

Reinforce expressing fractions as equivalent fractions (renaming).

Use concept of equivalent fractions and set intersection to develop procedures for finding common denominator.

Develop understanding of least common denominator.

Use procedures for finding common denominator to compare fractions with unlike denominators.

****

38. Graphs: Circle Graph

Reinforce understanding of mathematical relationships on a circle graph when the sectors represent fractional numbers with like denominators.

Develop ability to interpret circle graphs when the sectors represent fractional numbers with unlike denominators.

**

39. Operations: Addition of Fractional Numbers - Unlike Denominators

Reinforce: addition of fractional numbers with like denominators
expressing a fraction in simplest form
finding common denominator; least common denominator

Develop procedures for adding fractional numbers with unlike denominators. Use common denominator method.

An illustrative teaching sequence follows:

$$\frac{1}{6} + \frac{1}{8} = n$$

$$2\frac{1}{6} + \frac{1}{8} = n$$

$$2\frac{1}{6} + 3\frac{1}{8} = n$$

$$5\frac{5}{8} + 2\frac{3}{8} = n, \text{ etc.}$$

Use vertical and horizontal format.

**

40. Operations: Subtraction of Fractional Numbers - Unlike Denominators

Reinforce: subtraction of fractional numbers with like denominators
finding common denominator and least common denominator
expressing fractional numbers in simplest form

Develop procedures for subtracting fractional numbers with unlike denominators.

Use common denominator method.

An illustrative teaching sequence follows:

$\frac{1}{6} - \frac{1}{8} = n$

$\frac{3}{8} - \frac{1}{6} = n$

$3\frac{2}{3} - \frac{1}{4} = n$

$3\frac{4}{6} - 1\frac{2}{3} = n$, etc.

Use horizontal and vertical format. Check solutions by using the principle of inverse operations.

**

41. Multiplication of Whole Numbers

Reinforce multiplying multiples of ten by multiples of ten.

Develop multiplication of multiples of ten and multiples of one hundred, e.g.,

| a. | b. | c. |
|---|---|---|
| 10 x 100 = n | 10 x 100 = n | 10 x 100 = n |
| 10 x 200 = n | 20 x 100 = n | 20 x 200 = n |
| 10 x 300 = n | 30 x 100 = n | 30 x 300 = n |
| etc. | etc. | etc. |

Use horizontal and vertical format.

**

42. Division of Whole Numbers

Continue to develop procedures for recording division in vertical format.

Divisors: Numbers in the tens greater than 50
Dividends: Numbers in the hundreds

Adapt procedure suggested in Unit 33.

***

43. Measurement: Linear

Use projects involving experiences to develop understanding of relationships among inches, feet, yards, miles, e.g., $\frac{1}{3}$ yd. = 1 ft.; $\frac{1}{3}$ ft. = 4 in.

Develop skill in changing to different units of measure.

**

44. Multiplication of Whole Numbers

Reinforce multiplication of multiples of ten and multiples of one hundred.

Use the distributive property of multiplication over addition and develop multiplication algorithms when one factor is named by a 3-digit numeral and the other factor is named by a 2-digit numeral. Use horizontal and vertical format, e.g., 23 x 321 = n.

a. Rename 23 as 20 + 3.

| 321 | 321 | 321 | 6420 |
|---|---|---|---|
| x 23 | x 20 | x 3 | + 963 |
| | 6420 | 963 | 7383 |

b.

| | | | |
|---|---|---|---|
| 321 | | 321 | |
| x 23 | | x 23 | |
| 6420 | (20x321) | 963 | ( 3x321) |
| 963 | ( 3x321) | 6420 | (20x321) |
| 7383 | (23x321) | 7383 | (23x321) |

c. Rename 23 as 20 + 3, then

$$\begin{aligned} 23 \times 321 &= (20 \times 321) + (3 \times 321) \\ &= 6420 + 963 \\ &= 7383 \end{aligned}$$

**

45. Division of Whole Numbers

Continue to develop procedures for recording division in vertical format.

Divisors: Numbers in the tens less than 50
Dividends: Numbers in the thousands

Reinforce multiplying multiples of one hundred and numbers in the tens; multiplying multiples of ten and numbers in the tens.

An illustrative teaching procedure follows:

$31\overline{)7679}$ $\qquad$ $n \times 31 = 7679$ $\qquad$ $7679 \div 31 = n$

a. Estimate:

| | |
|---|---|
| $100 \times 31 = 3100$ | $3100 < 7679$ |
| $200 \times 31 = 6200$ | $6200 < 7679$ |
| $300 \times 31 = 9300$ | $9300 > 7679$ |

Therefore, $n > 200$ and $n < 300$

b. Possible ways to arrive at solution:

| | R. 22 |
|---|---|
| 247 | |
| 31)7679 | |
| -6200 | 200 |
| 1479 | |
| - 310 | 10 |
| 1169 | |
| - 620 | 20 |
| 549 | |
| - 310 | 10 |
| 239 | |
| - 217 | 7 |
| 22 | 247 |

| | R. 22 |
|---|---|
| 247 | |
| 31)7679 | |
| -6200 | 200 |
| 1479 | |
| -1240 | 40 |
| 239 | |
| - 217 | 7 |
| 22 | 247 |

c. Compare quotient with estimate.

d. Check by multiplication.

***

46. Measurement: Time

Use projects involving experiences to develop:

understanding of relationships among months and year; seconds, hours, and minutes; days, weeks, months; etc. For example:

$\frac{1}{3}$ of a year = 4 months

skill in changing to different units of measure

Develop understanding of decade and century.

**

47. Operations: Multiplication of Fractional Numbers

Reinforce procedures for naming a mixed numeral as an improper fraction.

Use generalization developed in Unit 14 to develop procedures for computing the product of two numbers each named by a mixed numeral.

$3\frac{1}{8} \times 2\frac{1}{2} = n$

$\frac{10}{3} \times \frac{5}{2} = \frac{50}{6}$ or $8\frac{2}{6} = 8\frac{1}{3}$

Products may also be computed by using the distributive property of multiplication over addition in horizontal and/or vertical format.

*

48. Number; Numeration: Decimals

Use objects, diagrams, number lines to reinforce understanding of tenths and hundredths as parts of a whole.

Adapt procedure indicated in Unit 30 to develop understanding of hundredths in decimal form.

**

49. Operations: Addition and Subtraction of Decimals

Reinforce: reading and writing decimal numerals in hundredths
counting forward and backward by hundredths (decimal form)
procedures for computing sums and differences using tenths

Adapt suggestions in Unit 31 to develop procedures for computing sums and differences using hundredths; horizontal and vertical format.

**

50. Operations: Addition of Fractional Numbers

Adapt suggestions indicated in Unit 39 to develop procedures for adding more than two fractional numbers with unlike denominators.

**

51. Operations: Subtraction of Fractional Numbers - Unlike Denominators

Reinforce: renaming numbers such as: $2 = \frac{4}{2}$; $2 = 1\frac{2}{2}$; $2\frac{1}{4} = 1\frac{5}{4}$

finding common denominator and least common denominator
expressing fractional numbers in simplest form

Continue to develop subtraction of fractional numbers with unlike denominators involving renaming.

An illustrative teaching sequence continues from Unit 40.

$$5\frac{1}{3} - 2\frac{4}{5} = n$$

$$27\frac{1}{3} - 13\frac{2}{3} = n, \text{ etc.}$$

Use horizontal and vertical format.
Check solutions by using the principle of inverse operations.

*

⊙ 52. Numeration: Base 5

Reinforce characteristics of the decimal system of numeration:
names of symbols (digits)
number of symbols
place value chart (grouping by ten)

Use materials such as multibase blocks, squared materials, discs, etc. to develop an understanding of the base 5 system of numeration.

Include: grouping by fives
use of numerals to name the groupings
basic symbols in base five (0,1,2,3,4)
place value chart for base five
interpret numerals in base five by means of place value chart
compare numerals in base five with numerals in base ten

***

53. Geometry: Circle

Reinforce relationships among radius, chord, diameter, circumference.

Develop concept of central angle and its arc.

**

54. Multiplication of Whole Numbers

Reinforce multiplication of multiples of ten and multiples of one hundred.

Develop multiplication of multiples of ten and multiples of one thousand, e.g.,

| a. | b. | c. |
|---|---|---|
| 10 x 1000 = n | 10 x 1000 = n | 10 x 1000 = n |
| 10 x 2000 = n | 20 x 1000 = n | 20 x 2000 = n |
| 10 x 3000 = n | 30 x 1000 = n | 30 x 3000 = n |
| etc. | etc. | etc. |

**

55. Division of Whole Numbers

Continue to develop procedures for recording division in vertical format.

Divisors: Numbers in the tens greater than 50
Dividends: Numbers in the thousands and beyond

Adapt procedures suggested in Unit 45.

**

56. Multiplication of Whole Numbers

Reinforce multiplication of multiples of ten and multiples of one thousand.

Use the distributive property of multiplication over addition and develop multiplication algorithms when one factor is named by a 4-digit numeral and the other is named by a 2-digit numeral. Use vertical format, e.g., 23 x 2356 = n.

Rename 23 as 20 + 3.

| a. 2356 | 2356 | 2356 | 47,120 |
|---|---|---|---|
| x 23 | x 3 | x 20 | + 7,068 |
| | 7068 | 47120 | 54,188 |

b.
$$\begin{array}{r} 2356 \\ \times\ 23 \\ \hline 7068 \\ 47120 \\ \hline 54188 \end{array}$$

Continue to develop multiplication algorithms when one factor is a number in the ten-thousands and beyond, and the other factor is a number less than one hundred.

***

57. Measurement: Liquid and Dry Measure

Use projects involving experiences to develop:
understanding of relationships among liquid measures - ounces, cups, pints, quarts, gallons

skill in changing to different units of liquid measures

Develop understanding of dry measure.

*

58. Numeration: Decimal System of Notation and Dollars and Cents

Reinforce: understanding of dollars and cents system
understanding of tenths and hundredths in the decimal system of notation

Use experience situations to develop understanding of the relationship between money values of penny, dime, quarter, half-dollar, dollar, and tenths and hundredths.

****

59. Statistics: Median

Reinforce understanding that:
the mode of a set of scores is the score that occurs most frequently

the mean (average) is the sum of all the scores divided by the total number of scores

Use projects involving experiences to develop the understanding that the median is the middle score when the set of scores is arranged according to size. Limit consideration to an odd number of scores.

*

60. Number; Numeration: Decimals

Use objects, diagrams, number lines to reinforce understanding of tenths and hundredths in the decimal system of numeration.

Adapt procedure indicated in Unit 30 to develop understanding of thousandths in decimal form.

**

61. Operations: Addition and Subtraction of Decimals

Reinforce: reading and writing decimal numerals in thousandths
counting forward and backward by thousandths (decimal form)
procedures for computing sums and differences using tenths and hundredths

Adapt suggestions in Unit 31 to develop procedures for computing sums and differences using thousandths; horizontal and vertical format.

***

62. Measurement: Scale Drawing

Use rulers to reinforce understanding of fractional parts of an inch: halves, fourths, eighths, sixteenths.

Develop understanding of tenths of an inch.

Use materials such as maps, diagrams, photographs, etc. to develop procedures for interpreting drawings made to scale.

****

63. Algebraic Concepts

Reinforce understanding of:
the graph of a number on a number line
the graph of a solution set of an open sentence with one place holder on a number line

Use experiences to develop:
concept of ordered number pair
procedures for associating an ordered number pair with a point in a plane (limit to first quadrant)
procedures for graphing in the first quadrant some solutions of an equation with two variables, $\square + \nabla = 8$

****

64. Probability

Experiment with tossing one coin to recall expressing probability of a favorable outcome as a fraction.

Experiment with tossing two coins. List the possible combinations (sample events) and express the probability of a favorable outcome as a fraction (probability ratio).

Continue to experiment with tossing two coins. Tally, tabulate, and graph results after six throws; after ten throws; etc. Compare actual results with predicted possibilities.

*

⊙ 65. Sets; Number; Numeration: Modulo 12

Reinforce understanding of "clock arithmetic" (12-hour clock). Observe properties of operation.

Develop understanding of:
- modular number system
- addition in modulo 12 system
- properties of addition in modulo 12 system

***

⊙ 66. Geometry: Sphere

Reinforce: concepts of space
space figures
sphere

Use models to develop understanding of some characteristics of a sphere and apply learnings to geography of the earth.