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| AUTHOR | Meyer, Donald |
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ABSTRACT

One of six summaries of workshop sessions (See TM 000 130), designed to strengthen the evaluation of costly programs and their effects, this handbook presents an analysis of both random and nonrandom sampling errors by application of the Bayesian model. This model attempts to formalize the process and procedures of inference from data through explicit consideration of the researcher's beliefs, insofar as they bear upon the study underway. Thus, if he feels that nonresponse error, for example, is likely to be large, this feeling is incorporated into the model. Major error sources for each parameter are sampled and values of all the variables are determined to show how these parameters define the sampling problem. Another method involves formulation of error ratios in terms of certain established values. An example of the application of the model to a survey using mailed questionnaires is given. (PR)

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# PROGRAM EVALUATORS HANDBOOK

## Sampling Model

ONE OF THE TRAINING
AND REVIEW SERIES
IN
E.S.E.A. TITLE III

The University of the State of New York
THE STATE EDUCATION DEPARTMENT
Albany, New York 12224

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## FOREWORD

The increased competition for the tax dollar has caused and will continue to cause more rigorous evaluations in all fields of education, particularly at the Federal level. Increasingly, legislators and their constituent taxpayers are demanding hard data which will indicate whether a costly program is achieving that which it has purported to achieve. Under these conditions, evaluation at all levels must satisfy the criteria elements of significance, credibility, and timeliness. Within this framework evaluative techniques must be strengthened.

Appropriate departmental personnel believed that strengthening the evaluative effort of the State might start with the Elementary and Secondary Education Act (ESEA) in general and Title III of that Act in particular. Further, it was believed that the 16 existing Regional Centers contained evaluators who might be in a strategic position to disseminate information gained through a workshop approach to the problem on the State level.

Leo D. Doherty, Supervisor of Education Research, of the Division of Evaluation, was asked to organize some review and training sessions appropriate for the task. He selected people from within the State to prepare and conduct formal lessons accompanied by simulated experiences and related materials. This document is one in a series of six summaries of sessions completed in a 3-month period terminating in February 1969.

While the sessions were paid for out of Title III funds, the contents are appropriate for use with other Titles such as 1, or other large program evaluative problems such as those encountered in N.D.E.A. Title III, Urban Education, or the like.

This document on Sampling Model was prepared by Donald Meyer, Syracuse University.



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CONTENTS

## SAMPLING MODEL

### Analysis of Errors in Sampling

Two basic kinds of errors:

1. Random sampling errors
2. Nonrandom sampling errors

Usual procedure:

Take a sample. Obtain a value for the relevant statistic. Use this as a point estimate of the population value and/or construct a confidence interval.

Example: Suppose we want to estimate proportion of parents favoring a particular kind of reorganization of the schools. If we define the family as the sampling unit, we take a random sample from some list of families and ask them the question. Observed statistics are $N$ = number of families in sample and $p$ = proportion saying they are favorable.

Let $\pi$ be the population proportion. From theory we know that the mean and variance of the sample proportion is:

$$E(p) = \pi$$

$$V(p) = \pi(1-\pi)/N$$

Therefore, a point estimate of $\pi$ is $p$ and a large sample confidence interval for $\pi$ is:

$$C\left[p - z_{\alpha/2}\sqrt{V(p)} \leq \pi \leq p + z_{\alpha/2}\sqrt{V(p)}\right] = 1 - \alpha$$

where $z_{\alpha/2}$ is the upper $\alpha/2$ point of the normal distribution. The difficulty is that the above confidence interval considers only the random sampling error. If the nonrandom sampling errors are large, such as the error from nonresponse, then our belief that the obtained confidence interval includes the true value could be quite small. What is required is a model which would enable us to also consider nonrandom sampling errors and incorporate any knowledge we have about them into our analysis. The advantage of doing this is twofold:

1. Our final reported interval would be a more adequate statement of reality;
2. To aid us in the design of future studies.

In connection with this last point note that the usual methods give little guidance as to design. The preferred study is the one with the larger sample size if we only use the length of the confidence interval as the criterion. Implicitly, the assumption is that studies are always performed "properly" and the sample is random. In actual practice this is seldom the case.

The Bayesian model, using subjective probability as a cornerstone, is an attempt to formalize the process and procedures of inference from data with explicit consideration of the beliefs of the researcher, insofar as they bear on the present study. If the researcher thinks that nonresponse error, for example, is likely to be large, then he incorporates these feelings into the model. The choice of design will then be made conditional on this concern. Of course, good researchers have always done this in an informal way. The only difference is that the Bayesian model makes it a formal process. The advantage is that all decisions of any kind are now explicit, and one can "research" the model itself for questions such as sensitivity, inter-relationships of decisions, etc.

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## Major Error Sources (Sampling)

1. Frame error

This source of error may exist if the frame from which the desired sample is drawn does not include all of the target population.

e.g. If one used the telephone book as the frame for residents in a city, then those not owning telephones are automatically excluded.

2. Selection error

This error may occur if certain elements in the frame have a greater chance of being included in the sample than others. (This is sometimes dealt with by the use of weighting factors.)

e.g. Some people's names may appear on the frame list more than once as, for example, parents with more than one child in school.

3. Nonresponse error

This error occurs if responses are not obtained from all the units in the planned sample and if those not responding differ on the variable under study from those responding. (It may be convenient to decompose this error into refusals and noncontacts.)

4. Measurement error

This error occurs if we do not obtain accurate answers. This can depend on the way the question is asked, the order of presentation, interviewer variables, etc.

5. Random sampling errors

This is the error usually assessed by use of the standard statistical formulae.

## Specification of Parameters

$\pi_y$ = proportion in population with attribute, y.

$\pi_{fd|y}$ = proportion of those having y included in <u>defined frame</u>.

$\pi_{fa|y}$ = probability of selecting a unit with y in the sampled frame.

$\pi_{r|f,y}$ = proportion of those in sampled frame having y who would <u>respond</u> if sampled.

$\pi_{s|r,y}$ = proportion of those responding who have y and would <u>say</u> they have y.

All subscripts with "bars" over them mean the negation, e.g.

$\pi_{\bar{r}|f,y}$ = proportion of those who have y and would <u>not respond</u>.

## <u>Sample Data</u>

$\hat{\pi}_r$ = proportion of planned sample who respond.

$\hat{\pi}_{s|r}$ = proportion of sample members who say they have y among those who respond. (This is the usual statistic reported.)

If the values of these parameters, together with the corresponding parameters for the not - y's, are known, then the model for the sampling design is completely specified.

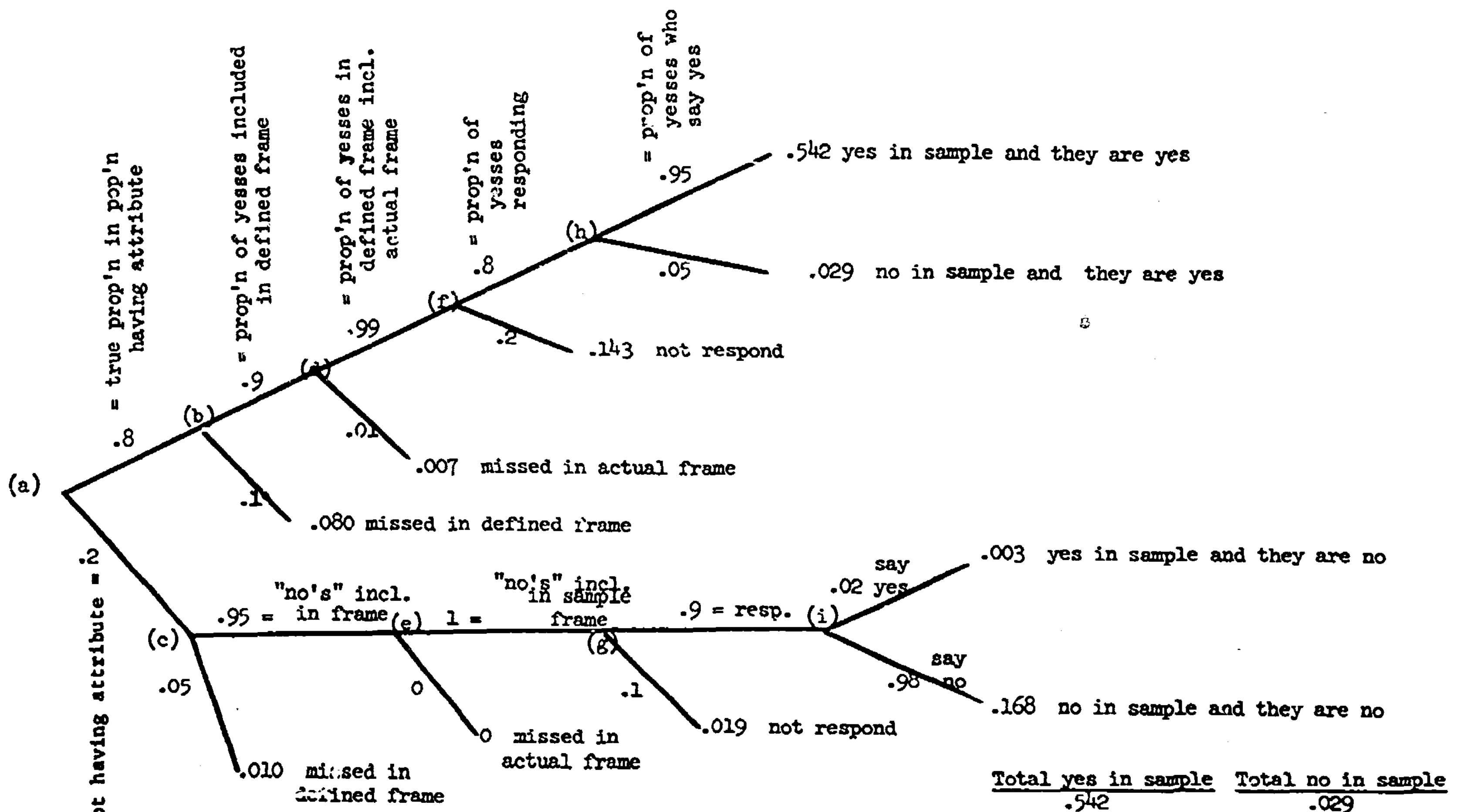
To elaborate on these concepts, we will perform a population analysis assuming we know the values of the parameters. In practice we will be uncertain as to the exact values of the parameters, but using our "best" guesses can be helpful since the implications for the sampling design will become clear. If a large bias is indicated, then even though we may be uncertain as to the actual magnitude of the bias, we may be confident about its direction, and we may be motivated to think about alternative designs. This will serve as a preliminary step for a more formal analysis.

Suppose we are interested in the proportion of families who are positive with respect to a certain attitude. (The head of the household defines the family for the purpose of the study.) We will sample using a list of residents from the city directory which is almost one year old, and we will send a questionnaire to those falling in our random sample. Let us assume that from our past knowledge we think that those who are positive towards the issue are also those who are more mobile in terms of residence, and therefore, are more likely to be missed by using a city directory this old.

The tree diagram shown in Figure 1 is one possible representation of the population and associated sampling design. Starting from point (a) the two branches show the values $\pi_y$ and $\pi_{\bar{y}}$ to be .8 and .2, respectively. That is, our best estimate of the proportion in the population who are positive is .8 and the estimate of the proportion negative is, therefore, .2. The branches at points (b) and (c) show the sources of frame error. By using the directory, we miss some of the members of the population. The tree shows that our estimate of the proportion of those favorable (saying "yes") who are included in the frame

Figure 1

Representation of Population Composition




| Total yes in sample | Total no in sample |
|---|---|
| .542 | .029 |
| .003 | .168 |
| $\boxed{.545}$ | $\boxed{.197}$ |
| **Non-response** | **Missed** |
| .143 | .007 |
| .019 | .090 |
| $\boxed{.162}$ | $\boxed{097}$ |

Total = 1.001

% Yes reported = $\frac{.545}{.742}$ = .73

% No reported = = .27

(directory) is .9 or $\pi_{fd|y}$ = .9. Then, $\pi_{\overline{fd}|y}$ = .1. Similarly, the proportion of those unfavorable (saying "no") included in the frame is .95 or $\pi_{fd|\bar{y}}$ = .95. Then, $\pi_{\overline{fd}|\bar{y}}$ = .05. Because of our feelings that those favorable are more mobile, and therefore, would have more likelihood of being missed, we have set $\pi_{\overline{fd}|\bar{y}} > \pi_{fd|y}$.

The branches at points (d) and (e) show the sources of selection error. We may send the questionnaires to the addresses listed in the directory, but they may not be delivered to the sampling units because of "no forwarding address" etc. One could lump this type of selection error with frame error, but it may be convenient to keep them separate for this problem. Similarly, if we were calling the sample units on the telephone and we couldn't reach them, we would find it convenient to think of this as a source of selection error. The difference is that frame error refers to the possession or nonpossession of the unit's name, and selection error refers to the likelihood of the unit to be given a chance to respond. The figure shows that selection error is assessed to be small since $\pi_{fa|y}$ = .99 and $\pi_{\overline{fa}|\bar{y}}$ = .01 while $\pi_{fa|\bar{y}}$ = 1. That is, no selection error for those who are unfavorable.

The branches at points (f) and (g) show the sources of nonresponse error. This refers to the nonreturning of the mailed questionnaires or the refusal to respond over the telephone. For this example we are assuming that those who are favorable will be less likely to return the questionnaire than those unfavorable : might be derived from a general theory that says the favorable group is more active in community affairs, busier, etc. and therefore, more likely to ignore our request to return the questionnaire. The unfavorable group may be more interested in making their views known, etc. Consequently, $\pi_{r|f,y}$ = .8 and $\pi_{\bar{r}|f,y}$ = .2 or the proportion of those favorable, and who are contacted, who will respond is .8 while the proportion not responding is .2. For the unfavorable group, $\pi_{r|f,\bar{y}}$ = .9 and $\pi_{\bar{r}|f,\bar{y}}$ = .1.

Finally, points (h) and (i) show the sources of measurement error. Again from the "busy" hypothesis, the favorable group may be more likely to misread the question and say "no" when their true attitude is "yes". Or it may be that some think it socially unacceptable to indicate a favorable attitude so they deliberately lie. For this problem, $\pi_{s|r,y} = .95$ and $\pi_{\bar{s}|r,y} = .05$ or 95 percent of those favorable and who return their questionnaires say they are favorable while 5 percent say they are unfavorable. Similarly, $\pi_{s|r,\bar{y}} = .98$ and $\pi_{\bar{s}|r,\bar{y}} = .02$.

By multiplying the numbers along any path to an end-point, we obtain the percentages for all possible events. The product of the numbers on the top path is .542 which means that, apart from random sampling error, 54.2 percent of the total population would be people who are favorable and would be so classified in the sample. Another .3 percent would be classified as favorable because of "unfavorable" people changing their response. The total percentage of "yesses" in the sample is then, .545. Similarly, the total percentage of those indicating "no" would be .197. This would mean that of the .742 responding (.545 + .197), the percentage of favorable is 73 percent. Since the true value is .8, we would have about a 9 percent underestimate.

The chart also shows that 9 percent of the population are not included in the defined frame and that .007 is the likelihood of a "favorable person" not being included in the actual sampled frame (selection error). The nonresponse rate is 16.2 percent. If the planned sample size is larger than about 150, then the obtained sample would be larger than 125 and the .95 confidence interval shown earlier would be, on the average, less than seven percentage points about .73, so that the interval would fail to cover the true value of .80!

We are not claiming that one would actually know these parameters as precisely as we have indicated on the chart. Our purpose is to show how these parameters define the sampling problem. The five parameters on the top branch and the four on the branch emanating from point (c) completely specify the population.

If a joint prior distribution could be assigned to these nine parameters, then the sample data consisting of the proportion of "yesses", $\hat{\pi}_{s|r}$, and the response rate, $\hat{\pi}_r$, could be combined with the prior using Bayes' theorem to obtain the joint posterior distribution. The marginal posterior distribution of $\pi_y$ could then be obtained by summing over the eight other parameters. This marginal would summarize our current state of knowledge about $\pi_y$ and point and/or interval estimates could be obtained if desired.

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## Formulation in Terms of Error Ratios

The assessment of the joint prior distribution of the nine parameters is difficult partially due to dependencies which may (probably) exist among them. One approach which has been suggested is to formulate new parameters which are ratios of the original nine (or functions of them).[1/] It is hoped that the dependencies are, thereby, eliminated. Error ratio model:

$$\frac{\pi_y}{\hat{\pi}_{s|r}} = \underset{(a)}{\frac{\pi_y}{\pi_{y|fd}}} \cdot \underset{(b)}{\frac{\pi_{y|fd}}{\pi_{y|fa}}} \cdot \underset{(c)}{\frac{\pi_{y|fa}}{\hat{\pi}_y}} \cdot \underset{(d)}{\frac{\hat{\pi}_y}{\hat{\pi}_{y|r}}} \cdot \underset{(e)}{\frac{\hat{\pi}_{y|r}}{\hat{\pi}_{s|r}}}$$

The left hand side is the ratio of the true proportion of "yesses" in the population and the observed proportion of those responding who say "yes" (or are favorable).

### Frame Error

Ratio (a) is a measure of frame error since it is the ratio of the true proportion in the population and the proportion of "yesses" in the defined frame. If its value is one, there is no frame error while a value greater than one would indicate that the proportion of "yesses" in the frame is less than that in the population. A value less than one would indicate the reverse. $\pi_{y|fd}$ is related to $\pi_{fd|y}$ used in the tree diagram by:

$$\pi_{y|fd} = \frac{\pi_y \pi_{fd|y}}{\pi_y \pi_{fd|y} + \pi_{\bar{y}} \pi_{fd|\bar{y}}}$$

Using the numbers from the example:

$$\pi_{y|fd} = \frac{(.8)(.9)}{(.8)(.9) + (.2)(.95)} = \frac{.72}{.91} = .791$$

1/ R.V. Brown, "Credence Analysis: The Appraisal of Research Error," unpublished doctoral dissertation, Harvard University, 1968.

This means that of the total population, 91 percent are included in the frame. Of this group 79.1 percent are "people who are favorable." The frame error ratio is:

$$\frac{.80}{.791} = 1.0101$$

In thinking about frame error it may be convenient to think of the ratio of proportion of "yesses" in the "uncovered" population to the proportion of "yesses" in the covered population since:

$$\frac{\pi_y}{\pi_{y|fd}} = \pi_{\overline{fd}}\left(\frac{\pi_{y|\overline{fd}}}{\pi_{y|fd}}\right) + \pi_{fd}$$

where $\pi_{\overline{fd}}$ is the proportion of population not included in the frame and $\pi_{fd}$ is the proportion included in the frame. Using the figures in the tree:

$$(.09)\left(\frac{.08/.09}{.72/.91}\right) = .91 = (.09)\left(\frac{.8888}{.7912}\right) + .91$$

$$= (.09)(1.12) + .91 = 1.0101$$

With 12 percent more "yesses" in the uncovered portion and 91 percent of everybody included, it is seen again that frame error is 1.0101.

## Selection Error

Ratio (b) measures selection error. It is the ratio of the proportion of y's included in the frame to the proportion of y's in the actual sampled frame (or probability of selecting a "y" in the frame).

The parameter, $\pi_{y|fa}$, is related to $\pi_{fa|y,fd}$ used in the tree by:

$$\pi_{y|fa} = \frac{\pi_y\ \pi_{fd|y}\ \pi_{fa|y,fd}}{\pi_y\ \pi_{fd|y}\ \pi_{fa|y,fd} + \pi_{\bar{y}}\ \pi_{fd|\bar{y}}\ \pi_{fa|\bar{y},fd}}$$

$$= \frac{\pi_{y|fd}\ \pi_{fa|y,fd}}{\pi_{y|fd}\ \pi_{fa|y,fd} + \pi_{\bar{y}|fd}\ \pi_{fa|\bar{y},fd}}$$

For our example:

$$\pi_{y|fa} = \frac{(.791)(.99)}{(.791)(.99) + (.209)\ (1)} = \frac{.783}{.992} = .7895$$

Selection error is $\frac{.7912}{.7895} = 1.002$

Random Sampling Error

Term (c) measures random sampling error. It is the ratio of the proportion of "yesses" in the actual sampled frame to the true average value in the planned sample. For most situations we will argue that the expected value of this ratio is one.

Nonresponse Error

Term (d) measures nonresponse error. It is the ratio of the true average value in the planned sample to the true average value in the achieved sample.

$\pi_{y|r}$ is related to $\pi_{r|y}$ by:

$$\pi_{y|r} = \frac{\pi_{y|fa}\ \pi_{r|y}}{\pi_{y|fa}\ \pi_{r|y} + \pi_{\bar{y}|fa}\ \pi_{r|\bar{y}}}$$

For our example:

$$\pi_{y|r} = \frac{(.7895)(.8)}{(.7895)(.8) + (.2105)(.9)}$$

$$= \frac{.6316}{.8211} = .7692$$

In our example, there was no random sampling error so we will use

$\hat{\pi}_y = \pi_{y|fa} = .7895$

The nonresponse error ratio is: $\frac{.7895}{.7692} = 1.0264$

It may be helpful to the assessor to think of the ratio, $\hat{\pi}_{y|\bar{r}}/\hat{\pi}_{y|r}$, which is the ratio of proportion of "yesses" in the nonresponse group to the proportion of "yesses" in the response group and use:

$$\frac{\hat{\pi}_y}{\hat{\pi}_{y|r}} = \pi_{\bar{r}}\left(\frac{\hat{\pi}_{y|\bar{r}}}{\hat{\pi}_{y|r}}\right) + \pi_r$$

For the example, $\pi_{\bar{r}} = .162/.903 = .179$

$$\frac{\hat{\pi}_y}{\hat{\pi}_{y|r}} = .179\left(\frac{.143/.162}{.769}\right) + (.821)$$

$$= .179\left(\frac{.883}{.769}\right) + .821 = 1.0267$$

Measurement Error

Ratio (e) measures measurement error. It is the ratio of the true average value in the achieved sample to the obtained value. That is, $\hat{\pi}_{s|r}$ is the act[illegible] proportion of those responding who say they are "yes." In the planning phase, this would be the expected value to be obtained.

For our example, $\pi_{s|r} = .73$

or $\frac{\hat{\pi}_{y|r}}{\hat{\pi}_{s|r}} = \frac{.769}{.73} = 1.053$

Total Decomposition

For the example the model is decomposed as:

$$\frac{.80}{.73} = \left(\frac{.80}{.7912}\right)\left(\frac{.7912}{.7895}\right)\left(\frac{.7895}{.7895}\right)\left(\frac{.7895}{.769}\right)\left(\frac{.769}{.73}\right)$$

1.10 = (1.0101)(1.002)(1)(1.026)(1.053) = 1.12 (round-off error)

This is a case where all error ratios were greater than one which would result in the underestimate, .73. It is possible, in general, for some of the ratios to "cancel" each other resulting in no net error.

Example of Use of Model

R. V. Brown [2/] has given an example of a survey using mailed questionnaires. The problem was to estimate the demand for parking space if meters were introduced in the town of Camford, England.

The sampling frame consisted of a list of 10,000 owners of registered cars. Questionnaires were mailed to 1000 owners requesting them to show on bar charts the hours between 8 A.M. and 6 P.M. on three specific days they would be parked if ample metered space were available. Nine hundred usable replies were received and 90 said they would be parked at peak hours.

The error rate decomposition is:

$$\frac{T}{N\hat{\pi}_{s|r}} = \underset{(a)}{\left(\frac{T}{N\pi_{y|fa}}\right)} \underset{(b)}{\left(\frac{\pi_{y|fa}}{\hat{\pi}_y}\right)} \underset{(c)}{\left(\frac{\hat{\pi}_y}{\hat{\pi}_{y|r}}\right)} \underset{(d)}{\left(\frac{\hat{\pi}_{y|r}}{\hat{\pi}_{s|r}}\right)}$$

where $T$ = total spaces required

$\hat{\pi}_{s|r}$ = proportion of respondents who say they need space

$N = 10,000$

Note that the left hand side is a ratio of numbers of spaces, not proportions as before.

For each ratio we assess the expected value and variance of our distribution of belief concerning the ratio. However, since it is difficult to assess the variance we will use the 95 percent credible interval of the distribution. If the natural logarithm of the ratio is normally distributed, then the range of the .95 C.I. is approximately four standard deviations. Furthermore, it is convenient to use the rel-variance which is the variance divided by the square of the mean.

Then,

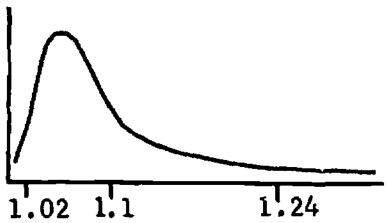
$$\text{rel-variance} = v \simeq \left[\frac{\text{range of .95 C.I.}}{4 \text{ mean}}\right]^2$$

2/ R. V. Brown, "Evaluation of Total Survey Error", Journal of Marketing Research, Vol. IV, May, 1967.

Assessment of Frame Error

$T/N\pi_{y|fa}$ is the ratio of total spaces required to the number required by the 10,000 owners on the list. All potential parkers are not included in the frame since out-of-town parkers will use some space.

The assessor thinks total demand will be about 1.1 of that demanded by in-town parkers. Out-of-town parkers will use about 10 percent of the spaces required by in-town parkers, so the mean is assessed at 1.1. The .95 C.I. ranges from .02 to .24.




(a)

The distribution is positively skewed since it is quite unlikely for (a) to be less than one. Note that the upper limit of the .95 credible interval specifies some probability that out-of-town parkers could possibly use 24 percent or more of the spaces used by in-town parkers.

$$v = \left[\frac{.22}{4(1.1)}\right]^2 = \left[.05\right]^2 = .0025$$

or $$E((a)) = 1.1, \quad v = .003.$$

Assessment of Selection Error:

It is thought that no selection error has occurred. Therefore, the expected value of the selection error ratio is one with rel-variance of zero.

Measurement Error

In assessing $\frac{\hat{\pi}_{y|r}}{\hat{\pi}_{s|r}}$ the assessor knows people are unreliable in what they say about hypothetical future actions. He feels people will tend to underestimate their parking needs. He sets the expected value at 1.2 which means he feels that the true percentage is 20 percent higher than that

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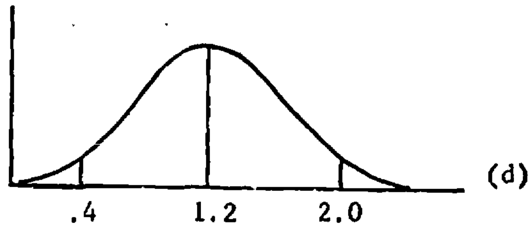
reported. He is somewhat vague, however, so he sets the .95 credible interval to be .4 to 2. His prior distribution appears as follows: (Exact shape is not critical.)




Then, rel-variance $= v = \left[\frac{1.6}{4(1.2)}\right]^2 = .111,$

or $E((d)) = 1.2$ with $v = .111.$

Nonresponse Error

In assessing $\hat{\pi}_y / \hat{\pi}_{y|r}$, the assessor feels that those not responding would be people not especially interested in parking in Camford. Therefore, the "parking rate" for nonresponders would be less than that for the responders.

However, he knows that

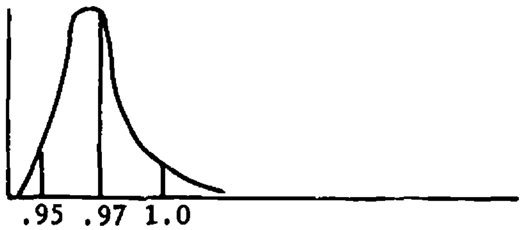
$$\frac{\hat{\pi}_y}{\hat{\pi}_{y|r}} = \pi_{\bar{r}}\left(\frac{\hat{\pi}_{y|\bar{r}}}{\hat{\pi}_{y|r}}\right) + \pi_r$$

and since $\pi_r = .90$ and $\hat{\pi}_{y|r} = .10$, then

$$\frac{\hat{\pi}_y}{.10} = .10\left(\frac{\hat{\pi}_{y|\bar{r}}}{.10}\right) + .90 = \hat{\pi}_{y|\bar{r}} + .90$$

which means $\hat{\pi}_y / \hat{\pi}_{y|r} \geq .90$ since $\hat{\pi}_{y|\bar{r}} \geq 0$. If $\hat{\pi}_{y|\bar{r}} = .07$, then $\hat{\pi}_y / \hat{\pi}_{y|r} = .97$

The distribution of the ratio is, therefore, assessed as shown below:




(c)

The $E\left[\hat{\pi}_y/\hat{\pi}_{y|r}\right] = .97$ with the .95 credible interval ranging from .95 to 1.0. The rel-variance, $v = \left[\frac{.05}{4(.97)}\right]^2 = .0002$

<u>Sampling Error</u>

The ratio, $\pi_{y|fa}/\hat{\pi}_y$, is the ratio of the true proportion of "yesses" in the sampled frame to the true average value in the planned sample. The ratio would differ from unity only because of sampling error.

From theory for binomial sampling:

$$E(\pi_{y|fa}/\hat{\pi}_y) \approx 1$$

$$v(\pi_{y|fa}/\hat{\pi}_y) \approx \frac{\frac{1}{E(\pi_{y|fa})} - 1}{n}$$

Also, if the ratios are independent.

$$E(\pi_{y|fa}) \approx E(\hat{\pi}_y) = E(\hat{\pi}_{s|r}) \times E(\hat{\pi}_y/\hat{\pi}_{y|r}) \times E(\hat{\pi}_{y|r}/\hat{\pi}_{s|r})$$

$\hat{\pi}_{s|r}$ was observed to be .10, so

$$E(\hat{\pi}_y) = (.10)(.97)(1.2) = .116$$

$$v(\hat{\pi}_{y|fa}/\hat{\pi}_y) \approx \frac{\frac{1}{.116} - 1}{1000} = .0076$$

Note that we use the planned sample size of n = 1000 since the nonresponse error, resulting in the actual sample of 900, has been separately expressed.

ERIC

Total Error Analysis

If the error ratios in the model decomposition are independent, then

$$E\left(\frac{T}{N\hat{\pi}_{s|r}}\right) = E\left(\frac{T}{N\hat{\pi}_{y|fa}}\right) E\left(\frac{\pi_{y|fa}}{\hat{\pi}_y}\right) E\left(\frac{\hat{\pi}_y}{\hat{\pi}_{y|r}}\right) E\left(\frac{\hat{\pi}_{y|r}}{\hat{\pi}_{s|r}}\right)$$

and the rel-variance, $v\left(\frac{\pi_{y|fa}}{\hat{\pi}_{s|r}}\right)$, is the sum of the rel-variances of the error ratios plus a product correction term consisting of all products two at a time and three at a time. The correction term will be small if all or all but one of the rel-variances are small.

Summary Table

| Source of Error | ratio | mean | rel-variance | |
|---|---|---|---|---|
| random | $\pi_{y\|fa}/\hat{\pi}_y$ | 1 | .0076 | |
| nonresponse | $\hat{\pi}_y/\hat{\pi}_{y\|r}$ | .97 | .0002 | |
| measurement | $\hat{\pi}_{y\|r}/\hat{\pi}_{s\|r}$ | 1.2 | .1111 | |
| frame | $T/N\pi_{y\|fa}$ | 1.1 | .0025 | product correction |
| | | | .0011 | |
| Total | $T/N\hat{\pi}_{s\|r}$ | 1.28 | .1225 | |

Since N = 10,000 and $\hat{\pi}_{s|r}$ = .10, then the expected number of spaces T, is:

$$E(T) = (10{,}000)(.10)(1.28) = 1280$$

with a rel-variance of .1225. The variance is $V(T) = (1280)^2 (.1225) = 200704$ and $\sqrt{V(T)} = 448$.

The .95 credible interval is:

$$C.I. = 4(448) = 1792$$

The distribution is not symmetrical since this would assign probability to negative values of T. We will assume that the ratio, $T/N\hat{\pi}_{s|r}$, is distributed log-normal which would follow if the component ratios were log-normal (or numerous by the central limit theorem).

If this is so, then a relationship between the .95 credible interval and the mean is:

$$\frac{\text{upper limit}}{\text{mean}} = \frac{\text{mean}}{\text{lower limit}}$$

Then: $$\frac{\mu}{1280} = \frac{1280}{\ell}$$

$$\mu - \ell = 1792$$

These two equations give $\mu = 2459$ and $\ell = 667$.

This means that

$P(667 \leq T \leq 2459) \leq .95$ with an expected value of 1280 spaces.

The usual .95 confidence interval for $\pi_y$ would be:

$$C\left[.10 - 2\sqrt{\frac{(.10)(.90)}{900}} < \pi_y < .10 + 2\sqrt{\frac{(.10)(.90)}{900}}\right] = .95$$

$$C\left[.08 < \pi_y < .12\right] = .95$$

or in terms of number of spaces:

$$C\left[800 < T < 1200\right] = .95$$

Note that the .95 credible interval using error ratio analysis is more than four times as large as the .95 confidence interval and the expected value of 1280 spaces is 28 percent higher than the point estimate, 1000.