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ABSTRACT

Reported are the results of an experiment designed to test the effectiveness of computer use by junior college students in a calculus class. The primary objective involved the replacing of pencil and paper computations by a computer program output. The topical areas of investigation were limits, extrema, functional evaluation, and integration. There was no significant difference in achievement between fifteen students using the computer as a calculator and fifteen students using pencil and paper. (RS)

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June 10, 1970

U.S. DEPARTMENT OF
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SUMMARY

The objectives of this project were: (1) to identify specific conceptual difficulties students have in introductory calculus caused by their inability to perform certain necessary numerical computations by pencil and paper methods, particularly in the topical areas of limits, extrema, functional evaluation and integration; and (2) to prepare and evaluate computer programs to be used by students to do the requisite calculations to gain a basic understanding of these topics. To facilitate evaluation, behavioral objectives have been written which specify for each selected topical area the observable behaviors desired, conditions under which they will be observed, and appropriate performance criteria levels. Thus a careful experiment has been designed and carried out to test the effectiveness of computer use by students on their performance relative to these four mathematical concepts. The details of these objectives and computer oriented problems, as well as the tests to evaluate them, are described. Student participants were quite receptive to the project, and there are indications that this particular use of the computer may have improved their understanding of the topical areas covered.

INTRODUCTION

The recent increase in the number of students enrolling in two-year colleges and technical institutes has caused great concern for educators about the teaching of calculus in two-year curricula. Because of the wide range in ability and the vastly different needs of the students in these curricula, the "standard" calculus course may no longer be appropriate for many of the students now enrolled in the course.

A number of fundamental problems face the student who first learns calculus. One of the most serious of these problems is the matter of finding the value of a function. Since paper and pencil methods are usually quite slow, only the simplest and often times most unrealistic, examples can be used. If some special computational tool were readily available routinely so that students could make rapid calculations of values of functions, not only could more be evaluated, but more realistic and complicated functions could be considered in the presentation of calculus.

Probably more important in the study of calculus, the concept of limits can take on meaning for a student only when he can see a large number of calculations revealing a limiting process. To do this task by pencil and paper computation is again slow and tedious, so that only a few simple cases can be given to the students in a conventional class.

In integral calculus, integrals are evaluated by using formulas which students memorize. If the process could be carried out in detail by numerical methods, students might acquire a greater understanding of the process, therefore, the essential problem is not just to be able to do rapid computation for purposes of applying calculus, it is to do these computations to learn the calculus more profoundly.

The problem under consideration in this project has been brought to the surface over the years that the author has been teaching calculus in the two year college. Evaluation of students' performance has, over those years, indicated deficiencies in the area of limits, functional evaluation and limit and extremum concepts. Students seemed unable to feel any specificity about the topics. They simply lacked computational experience to form the conceptual elements of these topics.

This present project was undertaken as an attempt to contribute to the teaching of calculus by demonstrating that the computer can be integrated into a conventional course to enhance students' understanding by permitting rapid calculations at points when it is most needed. It is expected that the software developed and evaluated will suggest important guidelines for the development of similar such materials for an entire three course sequence in introductory calculus.

INFORMATION SHEETS

One of the aims of instruction in calculus, apart from its goal of teaching the student techniques for the solution of various important classes of problems, is education of the student in the nature of mathematics as an edifice of logic. In order to encourage students to reflect on the fundamental concepts of calculus and on the theoretical development of the subject, an information sheet was placed at the beginning of each project item in hopes of bridging the gap between the logical structure of calculus and its use in problem solving. The format that was used for each information sheet included a number of specific items that would be directly related to a behavioral objective on a computer solved problem. Another basic aim of this information sheet was to tell the student exactly what items were being considered as important in his quest for mathematical growth.

BEHAVIORAL OBJECTIVES

An objective is an intent communicated by a statement describing a proposed change in a learner - a statement of what the learner is to be like when he has successfully completed a learning experience. To facilitate evaluation of the project, behavioral objectives were written, specifying for each selected topical area observable behaviors desired; conditions under which they would be observed and appropriate performance criteria levels. Thus, a careful experiment was designed and carried out to determine the effectiveness of the use of the computer by students on their performance relative to these mathematical concepts. There were two major project objectives. One was to write behavioral objectives for mathematical abilities in the topical areas of functional evaluation, limits of functions, extrema and integration. The second major project objective was to write behavioral objectives for the Calculus I course that involve problem solving abilities that are not usually acquired by students without considerable numerical computation. Pencil and paper computation may be so slow as to preclude such numerical experience with these topics (the question under investigation in this study). If students can be provided with this computational ability through the computer, their mathematical problem solving abilities with respect to these topics may be enhanced. An objective of this project was to determine the relative effectiveness of conventional assignments versus the student's use of the computer in acquiring abilities specified by the behavioral objectives of the first major project objective. Behavioral objectives were distributed to each student enrolled in the authors' class at the same time they received an information sheet over the topic being discussed. Each behavioral objective written was directly related to the information sheet distributed. In summary, one must remember that a meaningful objective is one that succeeds in communicating to the student the authors' instructional intent.

COMPUTER PROGRAMS

A set of 10 computer-solved problem assignment sheets were developed over the topical or unit areas indicated below. The problem assignments corresponded to the unit topics covered during the semester. The problem assignments were:

1. Functional Notation, Operations on Functions and their Graphs.

2. Types of Functions and some Special Functions.

3. The Limit of a Function, Theorems on Limits of a Function.

4. One-sided Limits.

5. Limits Involving Infinity.

6. Maximum and Minimum Values of a Function.

7. Applications Involving an Absolute Extremum.

8. Increasing and Decreasing Functions.

9. The Definite Integral.

10. Properties of the Definite Integral.

The format that was used for each assignment included problems directly related to the behavioral objectives for that assignment. The distribution of computer related assignment sheets was made as follows: A group of 15 students were selected at random from each of two classes (of 30 each), both taught by the author. Students were assigned numbers by the division secretary on an "even" and "odd" basis. The author had no information on which students were "odd" and which were "even" until the conclusion of the experiment.

Even numbered students worked problems from the assignment sheet using the computer as a teaching tool. Computer print-outs were used to plot graphs and examine values of the functions. Odd numbered students worked assignment sheets the conventional way by paper and pencil method. No use of the computer was made by this group of students. Following each of the four major topical areas of study, criterion tests were administered.

The effectiveness of the computer related assignment sheets and the two methods of solution were measured by means of a post-test designed in terms of behavioral objectives.

METHOD

The Analytical Geometry and Calculus I Course taught by the author at Florissant Valley Community College was divided into modular or weekly units, a list of which is shown in (Appendix A). The project was not in effect for all weekly units, but only for those topical areas directly related to Functions, Limits, Extrema, and Integration. Information sheets, behavioral objectives, and problem assignments (Samples shown in Appendix B, C and D) were an integral part of each unit covered by the project. At the end of all but a few of the weekly units a 20 minute quiz was given to the students. A sample quiz which followed unit four and five, the limit of a function and one-sided limits, is shown in Appendix E. No evaluation of this weekly unit quiz was attempted; however, students comments were favorable. The major portion of this project involved the selection of specific items covering the areas of functional evaluation, limits, extrema, and integration, together with writing behavioral objectives for these selected items and preparing computer programs (Samples shown in Appendix F) for the problems listed in each assignment sheet.

During the Fall term of 1969, two classes of introductory calculus were taught by the author. Because of the difficulty of selection, no attempt was made to match the two classes. Instead, a pre-test was given to each class, attempting to measure initial abilities in terms of stated objectives. One group used the computer output routinely to make necessary computations to utilize previously prepared instructional materials on functional evaluation, limits, extrema, and integration. The other group studied the same topics using conventional pencil and paper methods.

For an evaluation of the computer materials and methods, a post-test was administered to each class. The materials used in this project, as well as the evaluation tests, are described in detail in the following section.

RESULTS

The primary method of evaluation for the project was by means of the scores made on post-tests. Because many of the students had little exposure or previous experiences in the area of mathematics dealing with limits, extrema, and integration, the decision reached was that the administering of a pre-test in these areas would have been of little value. Therefore, only a post-test was used in these areas. The scores shown in the following tables represent the mean scores for each topical area covered. Also shown are the standard deviations (S.D.) and the sample numbers (N).

TABLE I

FUNCTIONAL EVALUATION

GROUP TEST SCORES

	NON-PROJECT	PROJECT
Mean	47.43	53.34
S.D.	10.55	8.27
N	30	29

$t = 2.39$
$df = 57$
$.01 < p < .05$

TABLE II

LIMITS OF A FUNCTION

GROUP TEST SCORES

	NON-PROJECT	PROJECT
Mean	50.03	50.83
S.D.	20.07	16.73
N	29	29

$t = .16$
$df = 56$
$.8 < p < .9$

TABLE III

EXTREMA

GROUP TEST SCORES

	NON-PROJECT	PROJECT
Mean	72.81	79.74
S.D.	19.98	17.56
N	27	27

$t = 1.35$
$df = 52$
$.1 < p < .2$

TABLE IV

INTEGRATION

GROUP TEST SCORES

	NON-PROJECT	PROJECT
Mean	3.64	4.52
S.D.	3.94	4.18
N	28	27

$t = .80$
$df = 53$
$.4 < p < .5$

As indicated in the tables, the mean scores of the project group did increase over the mean of the non-project group. However, the relatively large standard deviation indicates that this increase in mean scores would not be statistically significant. A standard t-test for comparison of the means of the two different samples was computed using as samples the post-test scores for both project and non-project groups. In each case, except for functional evaluation, mean differences could have occurred by chance alone. The project was also evaluated by means of the questionnaire shown in (Appendix G). Each of the questions on it have five levels of response. In order to find the overall response by the project group, a graduated weighting scale was used ranging from +2 for strongly agree, +1 for mildly agree, zero for not sure, -1 for mildly disagree to -2 for strongly disagree. The results are shown in (Table V).

TABLE V

RESULTS OF QUESTIONNAIRE

QUESTION	WEIGHTED RESPONSE
Do you feel the information sheets were a help to you in understanding the calculus?	.94
Do you feel the behavioral objectives as stated, help guide you in your study of calculus?	1.1
Do you feel the computer information was an aid in problem solving?	.31
Do you feel the problems contained in the assignment sheets were of help in understanding the calculus?	.45
If you were given a choice, would you choose this method to study calculus?	.13
Would you like to see this project extended to all the areas covered in Calculus I?	.21

Of the comments which were made at the bottom of the questionnaire, the general consensus of the project group was favorable. Of all the responses to question two on the questionnaire, not one had a weight less than zero (not sure).

CONCLUSION

The fact that the mean scores on the post-tests were higher for the project group; with the means obtained from the area of functions having a greater significance than the three remaining areas, might indicate that this project, would improve a students' understanding of the course material as well as his problem solving ability. The results of the questionnaire by itself would indicate that certain parts of the project were successful. The lack of statistical significance in some mean comparisons, indicates that for these students both methods of instruction are equally effective. In concluding, I would like to say that by using the computer as a unique teaching tool offers the instructor a most interesting and relevant option for the teaching of calculus, an option which will lead to new skills, new attitudes and new interests. The material used in this project will be available at Florissant Valley Community College as part of the calculus sequence.

APPENDIX A

COURSE OUTLINE

UNIT	TOPIC	CHAPTER*
1.	Some properties of real numbers and introduction to analytic geometry.	1, 2
2.	Function notation, operations on functions and their graphs.	3
3.	Types of functions and some special functions.	3
4.	The limit of a function, theorems on limits of a function.	4
5.	One-sided limits.	4
6.	Limits involving infinity, horizontal and vertical asymptotes.	4
7.	Continuity of a function at a number.	5
8.	Continuity of a function on an interval and theorems on continuity.	5
9.	The derivative.	6
10.	Differentiation of algebraic functions.	7
11.	Maximum and minimum values of a function.	8
12.	Applications involving an absolute extremum.	8
13.	Increasing and decreasing functions.	8
14.	Concavity and points of inflection.	8
15.	Antidifferentiation.	9
16.	The definite integral	10
17.	Properties of the definite integral fundamental theorem of integral calculus.	10

APPENDIX A - (Continued)

* Chapter number in The Calculus with Analytic Geometry, Louis Leithold, Harper and Row, New York, Evanston and London, 1968.

APPENDIX B

Information Sheet

Functions

Various fields of human endeavor have to do with relationships that exist between one collection of objects and another. Graphs, charts, curves, tables, and formulas are familiar to everyone who reads the newspapers. These are merely devices for describing special relations in a quantitative fashion. Mathematicians refer to certain types of these relations as functions. The following are a few examples of these functions:

(a) A function whose range consists of a single number is called a constant function, $f(x) = 3$ is one example.

(b) A function $f(x)$ defined for all real x by a formula of the form $f(x) = mx + b$ is called a linear function because its graph is a straight line. Several examples are: $f(x) = 3x + 7$ and $f(x) = x$.

(c) For a fixed positive integer n, let f be defined by the power function $f(x) = x^n$, where x is some real number. When $n = 1$, this is called the identity function. For $n = 2$, the graph is a parabola. For $n = 3$, the graph is a cubic curve.

(d) A polynomial function $P(x)$ is one defined for all real x by an equation of the form

$$P(x) = C_0 + C_1x^1 + C_2x^2 + \cdots + C_nx^n = \sum_{K=0}^{n} C_Kx^K$$

The numbers $C_0, C_1, C_2, \cdots, C_n$ are called the coefficients of the polynomials, and the non-negative integer n is called its degree (if $C_n \neq 0$). They include the constant functions and the power function as special cases. Polynomials of degree 2, 3, and 4 are called quadratic, cubic, and quartic polynomials respectively.

(e) Suppose we return to the Cartesian equation of a circle, which we will refer to as a radical function or algebraic function, $x^2 + y^2 = r^2$ and solve this equation for y in terms of x. There are two solutions given by $y = \sqrt{r^2 - x^2}$ and $y = -\sqrt{r^2 - x^2}$. There was a time when mathematicians would say that y is a double-valued function of x given by $y = \sqrt{r^2 - x^2}$. However, the more modern point of view does not admit "double-valuedness" as a property of functions. The definition of functions requires that for each x in the domain, there corresponds one and only one y in the range. Geometrically, this means that vertical lines which intersect the graph do so at exactly one point. Therefore, to make this example fit the theory, we say that the two solutions for y define two functions, say f(x) and g(x), where

$$f(x) = \sqrt{r^2 - x^2} \quad \text{and } g(x) = -\sqrt{r^2 - x^2}$$

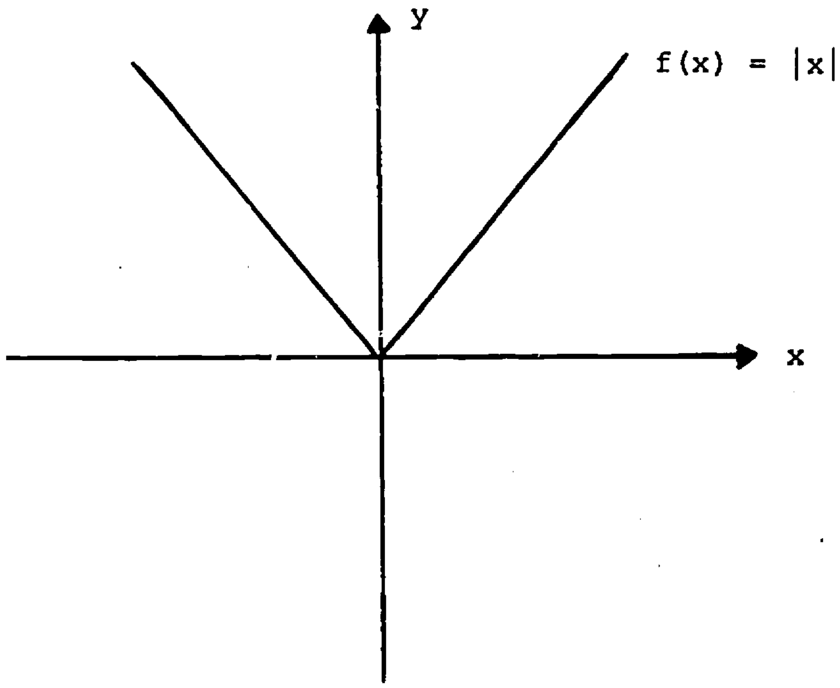
for each x satisfying $-r \leq x \leq r$.

(f) Let f(x) and g(x) be two real functions having the same domain D. We can construct new functions from f(x) and g(x) by adding, multiplying, or dividing the function values. The function u(x) defined by the equation $u(x) = f(x) + g(x)$ if $x \varepsilon D$ is called the sum of f(x) and g(x) and is denoted by $f(x) + g(x)$. Similarly, the product $v(x) = f(x) \cdot g(x)$ and the quotient $w(x) = f(x)/g(x)$ are the functions defined by the respective formulas $v(x) = f(x) \cdot g(x)$ if $x \varepsilon D$, and $w(x) = f(x)/g(x)$ if $x \varepsilon D$ and $g(x) \neq 0$. We will consider the quotient function to be most significant.

(g) A function which consists of a combination of distinct equations will also be studied.

$$y = \begin{cases} -3x - 4, & \text{if } x < -1 \\ x, & \text{if } -1 \leq x \leq 1 \\ (x - 2)^2, & \text{if } 1 < x \end{cases}$$

(h) A function which assigns to each real number x the non-negative number x is called the absolute-valued function. A portion of its graph is shown below.



A graphical solution of f(x) = |x| is shown above.

APPENDIX C

Behavioral Objectives

50.125 Analytic Geometry & Calculus I

Topic:	Functions and their graphs
Objective I:	Given the constant function f(x) the student should be able to calculate value(s) of f(x) for each real number x, draw a sketch of the graph of f(x), determine the domain and range of f(x) as well as find the set of all x satisfying the condition that f(x) will be bounded by a specific set of values in a period of ______________ minutes.

Topic:	Functions and their graphs
Objective II:	Given the absolute function $\|ax + b\|$ where a and b are constant values, the student should be able to calculate values of f(x) for each real number x, draw a sketch of the graph of f(x), calculate the domain and range of f(x) as well as find the set of all x satisfying the condition that f(x) will be bounded by a specific set of values in a period of ______________ minutes.

Topic:	Functions and their graphs
Objective III:	Given the linear function ax + b where a and b are constant values, the student should be able to calculate values of f(x) for each real number x, draw a sketch of the graph of f(x), calculate the domain and range of f(x) as well as find the set of all x satisfying the condition that f(x) will be bounded by a specific set of values in a period of ______________ minutes.

Topic: Functions and their graphs

Objective IV: Given the power function x^n, where n = 1, 2, 3, ... the student should be able to calculate values of f(x) for each real number x, draw a sketch of the graph of f(x), calculate the domain and range of f(x) as well as find the set of all x satisfying the condition that f(x) will be bounded by a specific set of values in a period of ____________ minutes.

Topic: Functions and their graphs

Objective V: Given the polynomial function $C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$, the student should be able to calculate values of f(x) for each real number x, draw a sketch of the graph of f(x), calculate the domain and range of f(x) as well as find the set of all x satisfying the condition that f(x) will be bounded by a specific set of values in a period of __________ minutes.

Topic: Functions and their graphs

Objective VI: Given the radical function $\sqrt{r^2 - x^2}$, the student should be able to calculate values of f(x) for each real number x, draw a sketch of the graph of f(x), calculate the domain and range of f(x) as well as find the set of all x satisfying the condition that f(x) will be bounded by a specific set of values in a period of ____________ minutes.

APPENDIX C - (Continued)

Topic: Functions and their graphs

Objective VII: Given the sum, products, and quotient functions the student should be able to calculate values of f(x) for each real number x, draw a sketch of the graph of f(x), calculate the domain and range of f(x) as well as find the set of all x satisfying the condition that f(x) will be bounded by a specific set of values in a period of ______________ minutes.

Topic: Functions and their graphs

Objective VIII: Given a function consisting of several formulas, the student should be able to calculate values of f(x) for each real number x, draw a sketch of the graph of f(x), calculate the domain and range of f(x) as well as find the set of all x satisfying the condition that f(x) will be bounded by a specific set of values in a period of ______________ minutes.

i.e. Given the student $f(x) = \begin{cases} -2x - 4, & \text{if } x < -1 \\ x \quad , & \text{if } -1 \leq x \leq 1 \\ (x-2)^2, & \text{if } 1 < x \end{cases}$

(a) calculate f(x) when x takes on the values of -4, -1.5, -.5, .5, 1.5, 2, and 4.

(b) draw a sketch of the graph of f(x) and determine the domain and range of f(x).

APPENDIX D

Assignment Sheet 1

Course: Analytical Geometry & Calculus I   10 Exercises

Topical Area - Functional Evaluation

Name ______________________________

One of the basic concepts of modern mathematical thought is that of the function, which is defined as follows: A function is a set of ordered pairs of numbers (x, y), in which no two distinct ordered pairs have the same first number. The totality of all possible values of x is called the domain of the function, and the totality of all possible values of y is called the range of the function.

Using the above information answer the following questions to the best of your knowledge.

Exercise 1: Let f be defined by the linear function $4x + 1$.

(a) What is the value of f(x) when x = -3? When x = -1? When x = 1? When x = 4?
(b) Draw a sketch of the graph of f(x).
(c) What is the domain of f?
(d) What is the range of f?
(e) Find the set of all x satisfying the condition that f(x) is greater than or equal to -2 and less than or equal to 4. i.e. $-2 \leq f(x) \leq 4$.

Exercise 2: Let f be defined by the absolute function $|3x + 2|$.

(a) What is the value of f(x) when x = -3? When x = -1/3? When x = 0? When x = 1/3? When x = 3.5?
(b) Draw a sketch of the graph of f(x).
(c) What is the domain of f(x)?
(d) What is the range of f(x)?

Exercise 3: Let f be defined by the polynomial function $2x^2 - 1$.

(a) What is the value of f(x) when x = -5? When x = -3? When x = 2? When x = 4? When x = 7?
(b) Draw a sketch of the graph of f(x).
(c) What is the domain of f(x)?
(d) What is the range of f(x)?
(e) Find the set of all x satisfying the condition that f(x) is greater than or equal to -1 and less than or equal to 17. i.e. $-1 \leq f(x) \leq 17$.

Exercise 4: Let f be defined by the polynomial function $x^3 - 9x^2 + 11x + 21$.

(a) What is the value of f(x) when x takes on integer values between -10 and a +10. i.e. $-10 < x < 10$.
(b) Draw a sketch of the graph of f(x).
(c) What is the domain of f(x)?
(d) What is the range of f(x)?
(e) Find the set of all x satisfying the condition that f(x) is greater than 21.

Exercise 5: Let f be defined by the radical function $f(x) = \sqrt{x^3 - 4}$.

(a) What is the value of f(x) when x = -2? When x = 2? When x = 9? When x = 12.125?
(b) Draw a sketch of the graph of f(x).
(c) What is the domain of f(x)?
(d) What is the range of f(x)?
(e) Find the set of all x satisfying the condition that f(x) is equal to 2. i.e. f(x) = 2.

Exercise 6: Let f be defined by the product function $f(x) = (x^2 + 1)(x^3 - 2x)$.

(a) What is the value of f(x) when x = -1? When x = 0? When x = .3? When x = .2? When x = .1?
(b) Draw a sketch of the graph of f(x).
(c) What is the domain of f(x)?
(d) What is the range of f(x)?
(e) Find the set of all x satisfying the condition that f(x) is greater than 10.

Exercise 7: Let f be defined by the quotient function $f(x) = (x - 2)/(x + 4)$.

(a) What is the value of f(x) when x = -9? When x = -3? When x = 2? When x = 3.5? When x = 7? When x = 10.5?
(b) Draw a sketch of the graph of f(x).
(c) What is the domain of f?
(d) What is the range of f?
(e) Find the set of all x satisfying the condition that f(x) is greater than or equal to -1/2 and less than or equal to 1.

Exercise 8: Let f be defined by the quotient function

$$f(x) = \frac{x^4 - 3x^3 - 11x^2 + 23x + 6}{x^2 + x - 6}$$

(a) What is the value of f(x) when x lies between the values of -10 and 11. i.e. $-10 < x < 11$.
(b) Draw a sketch of the graph of f.
(c) What is the domain of f?
(d) What is the range of f?

Exercise 9: Let f be defined by the function

$$f(x) = \begin{cases} x^2 - 4 & \text{if } x < 3 \\ 2x - 1 & \text{if } 3 \leq x \end{cases}$$

(a) What is the value of f(x) when x = -7? When x = 3.5? When x = 5? When x = 8.5? When x = 9.9?
(b) Draw a sketch of the graph of f(x).
(c) What is the domain of f?
(d) What is the range of f?

Exercise 10: Let f be defined by the function $f(x) = (-x)^{-3/2} + (x - 1)^2$.

(a) What is the value of f(x) when x = -4? When x = -1.11? When x = 5? When x = 7.75?
(b) Draw a sketch of the graph of f(x).
(c) What is the domain of f?
(d) What is the range of f?

APPENDIX E

Course: Math 50.125, Calculus I

Topical Areas a. Right hand and left hand limits
b. Limits involving infinity
c. Horizontal and vertical asymptotes

Length of time for the following question to be answered is 20 minutes.

Name: ______________________________

I. For the following functions find the indicated limit if it exists; if the limit does not exist, give the reason. Take one of the functions and draw a sketch of its graph.

a. $f(x) = 3 + 2x - 4$

Find:

1. $\lim_{x \to 2^+} f(x)$ 2. $\lim_{x \to 2^-} f(x)$ 3. $\lim_{x \to 2} f(x)$

b. $g(x) = \begin{cases} x^2, & \text{if } x \leq 2 \\ -2x + 8, & \text{if } 2 < x \end{cases}$

Find:

1. $\lim_{x \to 2^+} g(x)$ 2. $\lim_{x \to 2^-} g(x)$ 3. $\lim_{x \to 2} g(x)$

II. In the following problem, find the indicated limit.

a. $\lim_{x \to +\infty} \sqrt{x^2 + 1} - x$

III. Find the horizontal and vertical asymptotes of the graph of the given function.

a. $f(x) = \dfrac{x^2}{4 - x^2}$

APPENDIX F

Assignment 1; Ex-3

```
      DIMENSION A(50)
      DIMENSION X(50)
      WRITE (3,90)
 90   FORMAT(///T26,'IN THE FOLLOWING EXERCISE, THE
     FUNCTION IS THE SET  OF ALL ORDERED'/T31,'PAIRS
     (X,Y) SATISFYING THE GIVEN EQUATION 2X**2 - 1')
      WRITE (3,91)
 91   FORMAT(///T26,"EXERCISE 3.  LET F BE DEFINED BY THE
     POLYNOMIAL 2X**2 - 1')
      WRITE (3,100)
 100  FORMAT(///T30,'THE INFORMATION SHOWN IS THE SOLU-
     TION OF THE POLYNOMIAL FUNCTION (2*X**2 - 1)')
      WRITE (3,101)
 101  FORMAT(///T30,'THE VALUE OF X WAS CHANGED EACH
     TIME A VALUE OF THE   FUNCTION WAS COMPUTED')
C
C     THE FUNCTION TO BE EVALUATED IS 2*X**2 - 1
C
      READ (2,50) N, AO, (A(I), I = 1, N)
 50   FORMAT(I5/(2F15.7))
      WRITE (3,102) N
 102  FORMAT(///T45,'THIS IS A FUNCTION OF DEGREE', I2)
      READ (2, 51) M, (X(J), J = 1, M)
 51   FORMAT( I5/(9F7.3))
      DO 11 J = 1, M
      P = A(N)
      DO 10 I = 2, N
      N1 = N + 1 - I
 10   P = P * X(J) + A(N1)
      P = P * X(J) + AO
      WRITE (3,103) X(J), P
 103  FORMAT(//T24,'X = ',E15.5,10X 'F(X) = ',E15.5)
 11   CONTINUE
      WRITE (3,92)
 92   FORMAT(///T20,'AFTER EXAMINING THE NUMERICAL
     VALUES GIVEN ABOVE PLEASE ANSWER THE FOLLOWING
     QUESTIONS.'///T30,'(A)  WHAT IS THE VALUE OF F(X)
     WHEN X = -5, WHEN X = -3,'/T36,'WHEN X = 2, WHEN
     X = 4, WHEN X = 7.'///T30,'(B)  DRAW A SKETCH OF
     THE GRAPH OF F(X).'///T30,'(C) WHAT IS THE DOMAIN
     OF F(X).'///T30,'(D)  WHAT IS THE RANGE OF F(X),')
      WRITE (3,93)
 93   FORMAT(///T30,'(E)  FIND THE SET OF ALL X SATISFY-
     ING THE CONDITION THAT F(X) IS GREATER THAN OR'/
     T30,'EQUAL TO -1 AND LESS THAN OR EQUAL TO 17.')
      STOP
      END
```

Assignment 1; Ex-3

IN THE FOLLOWING EXERCISE, THE FUNCTION IS THE SET OF ALL ORDERED PAIRS (X,Y) SATISFYING THE GIVEN EQUATION 2X**2 - 1

EXERCISE 3. LET F BE DEFINED BY THE POLYNOMIAL 2X**2 - 1

THE INFORMATION SHOWN IS THE SOLUTION OF THE POLYNOMIAL FUNCTION (2*X**2 - 1)

THE VALUE OF X WAS CHANGED EACH TIME A VALUE OF THE FUNCTION WAS COMPUTED

THIS IS A FUNCTION OF DEGREE 1

X = -0.50000E 01	F(X) = 0.49000E 02
X = -0.40000E 01	F(X) = 0.31000E 02
X = -0.30000E 01	F(X) = 0.17000E 02
X = -0.10000E 01	F(X) = 0.10000E 01
X = 0.00000E 00	F(X) = -0.10000E 01
X = 0.20000E 01	F(X) = 0.70000E 01
X = 0.40000E 01	F(X) = 0.31000E 02
X = 0.50000E 01	F(X) = 0.49000E 02
X = 0.70000E 01	F(X) = 0.97000E 02

AFTER EXAMINING THE NUMERICAL VALUES GIVEN ABOVE PLEASE ANSWER THE FOLLOWING QUESTIONS.

(A) WHAT IS THE VALUE OF F(X) WHEN X = -5, WHEN X = -3, WHEN X = 2, WHEN X = 4, WHEN X = 7.
(B) DRAW A SKETCH OF THE GRAPH OF F(X).
(C) WHAT IS THE DOMAIN OF F(X).
(D) WHAT IS THE RANGE OF F(X).
(E) FIND THE SET OF ALL X SATISFYING THE CONDITION THAT F(X) IS GREATER THAN OR EQUAL TO -1 AND LESS THAN OR EQUAL TO 17.

APPENDIX F

Assignment 1; Ex-6

```
      DIMENSION A(50)
      DIMENSION X(50)
      DIMENSION B(50)
      WRITE(3,90)
 90   FORMAT(///T26,'IN THE FOLLOWING EXERCISE, THE
     FUNCTION IS THE SET OF ALL ORDERED'/T31,'PAIRS
     (X, &) SATISFYING THE GIVEN EQUATION (X**2 + 1)
     (X**3 - 2*X)')
      WRITE(3,91)
 91   FORMAT(///T26,'EXERCISE 6.  LET F BE DEFINED BY
     THE PRODUCT FUNCTION (X**2 + 1)(X**3 - 2X)')
      WRITE(3,100)
 100  FORMAT(///T30,'THE INFORMATION SHOWN IS THE
     SOLUTION OF THE PRODUCT FUNCTION (X**2 + 1.)
     (X**3 - 2.*X)')
      READ(2,50) N, AO, (A(I), I = 1, N)
 50   FORMAT(I5/3F15.5))
      READ(2,51) M, (X(J), J = 1, M)
 51   FORMAT(I5/(10F5.2))
      READ(2,52) L, BO, (B(K), K = 1, L)
 52   FORMAT(I5/(5F15.5))
C
      DO 11J = 1, M
      P = A(N)
      DO 10 I = 2, N
      N1 = N + 1 - I
 10   P = P *X(J) + A(N1)
      P = P *X(J) + AO
C
      Q = B(L)
      DO 9 K= 2, L
      L1 = L + 1 - K
 9    Q = Q * X(J) + B(L1)
      Q = Q * X(J) + BO
      F = P * Q
      WRITE(3,104) X(J), F
 104  FORMAT(///T22,'X = ', E15.5,10X,'F(X) = ',E15.5)
 11   CONTINUE
      WRITE(3,92)
 92   FORMAT(///T20,'AFTER EXAMINING THE NUMERICAL VALUES
     GIVEN ABOVE PLEASE ANSWER THE FOLLOWING QUESTIONS.
     '///T30,'(A) WHAT IS THE VALUE OF F(X) WHEN X = -6,
     WHEN X = -2, WHEN X = 4.5,'/T36,'WHEN X = 7, WHEN
     X = 10.'///T30,'(B) DRAW A SKETCH OF THE GRAPH OF
     F(X).'///T30,'(C) WHAT IS THE DOMAIN OF F(X).'///
```

Assignment 1; Ex-6

```
      T30,'(D)  WHAT IS THE RANGE OF F(X),')
       WRITE(3,93)
 93    FORMAT(///T30,'(E)  FIND THE SET OF ALL X SATISFY-
      ING THE CONDITION THAT F(X) IS GREATER THAN 10.')
       STOP
       END
```

IN THE FOLLOWING EXERCISE, THE FUNCTION IS THE SET OF ALL ORDERED PAIRS (X,Y) SATISFYING THE FIVEN EQUATION (X**2 + 1)(X**3 - 2*X)

EXERCISE 6. LET F BE DEFINED BY THE PRODUCT FUNCTION (X**2 + 1)(X**3 - 2X)

THE INFORMATION SHOWN IS THE SOLUTION OF THE PRODUCT FUNCTION (X**2 + 1.)(X**3 - 2.*X)

X = -0.60000E 01	F(X) = -0.75480E 04
X = -0.41000E 01	F(X) = -0.10814E 04
X = -0.20000E 01	F(X) = -0.20000E 02
X = 0.00000E 00	F(X) = 0.00000E 00
X = 0.20000E 01	F(X) = 0.20000E 02
X = 0.40000E 01	F(X) = 0.95200E 03
X = 0.45000E 01	F(X) = 0.17451E 04
X = 0.50000E 01	F(X) = 0.29900E 04
X = 0.70000E 01	F(X) = 0.16450E 05
X = 0.10000E 02	F(X) = 0.98980E 05

AFTER EXAMING THE NUMERICAL VALUES GIVEN ABOVE PLEASE ANSWER THE FOLLOWING QUESTIONS.

(A) WHAT IS THE VALUE OF F(X) WHEN X = -6, WHEN X = -2, WHEN X = 4.5, WHEN X = 7, WHEN X = 10.

APPENDIX F - (Continued)

Assignment 1; Ex-6

(B) DRAW A SKETCH OF THE GRAPH OF F(X).
(C) WHAT IS THE DOMAIN OF F(X).
(D) WHAT IS THE RANGE OF F(X).
(E) FIND THE SET OF ALL X SATISFYING THE CONDITION THAT F(X) IS GREATER THAN 10.

// *END OF DATA

APPENDIX F

Assignment 1; Ex-8

```
      DIMENSION A(50)
      DIMENSION X(50)
      DIMENSION B(50)
      WRITE(3,90)
 90   FORMAT(///T26,'IN THE FOLLOWING EXERCISE, THE
     FUNCTION IS THE SET OF ALL ORDERED'/T31,'PAIRS
     (X,Y) SATISFYING THE GIVEN EQUATION'/T31,'
     X**4 - 3X**3 - 11X**2 + 10X + 6)/(X**2 + X - 6)')
      WRITE(3,91)
 91   FORMAT(///T26,'EXERCISE 8. LET F BE DEFINED BY THE
     QUOTIENT FUNCTION'/T31,'(X**4 - 3X**3 - 11X**2 +
     23X + 6)/(X**2 + X - 6)')
      WRITE(3,100)
 100  FORMAT(///T30,'THE INFORMATION SHOWN IS THE
     SOLUTION OF THE QUOTIENT FUNCTION (X**4 - 3*X**3 -
     11*X**2 + 23*X + 6)/(X**2 + X - 6)')
C
C     THE FUNCTION TO BE EVALUATED IS A QUOTIENT FUNCTION
C
      READ(2,50) N, AO, (A(I), I = 1, N)
 50   FORMAT(I5/5F10.5))
      WRITE(3,102) N
 102  FORMAT(///T45,'THIS IS A FUNCTION OF DEGREE', I2)
      READ(2,51) M, (X(J), J = 1, M)
 51   FORMAT(I5/(7F10.5)
      READ(2,52) L, BO, (B(K), K = 1, L)
 52   FORMAT(I5/(3F15.5))
C
      DO 11 J = 1, M
      P = A(N)
      DO 10I = 2, N
      N1 = N + 1 - I
 10   P = P * X(J) + A(N1)
      P = P * X(J) + AO
C
      Q = B(L)
      DO 9 K = 2, L
      L1 = L + 1 - K
 9    Q = Q * X(J) + B(L1)
      Q = Q * X(J) + BO
      F = P/Q
      WRITE(3,104) X(J), F
 104  FORMAT(///T22,' X = ', E15.5,10X,'F(X) = ',E15.5)
 11   CONTINUE
```

Assignment 1; Ex-8

```
      WRITE(3,92)
 92   FORMAT(///T20,'AFTER EXAMINING THE NUMERICAL VALUES
     GIVEN ABOVE PLEASE ANSWER THE FOLLOWING QUESTIONS.
     '///T30,'(A)  WHAT IS THE VALUE OF F(X) WHEN X = -7,
     WHEN X = -5.25, WHEN X = 3.67,'/T36,' WHEN  X = 8.25,
     WHEN X = 10.11.'///T30,'(B)  DRAW A SKETCH OF THE
     GRAPH  OF F(X).'///T30,'(C)  WHAT IS THE DOMAIN OF
     F(X).'///T30,'(D)  WHAT IS THE RANGE OF F(X).')
     STOP
     END
```

IN THE FOLLOWING EXERCISE, THE FUNCTION IS THE SET OF ALL ORDERED PAIRS (X,Y) SATISFYING THE GIVEN EQUATION X**4 - 3X**3 - 11X**2 + 10X + 6)/(X**2 + X - 6)

EXERCISE 8. LET F BE DEFINED BY THE QUOTIENT FUNCTION (X**4 - 3X**3 - 11X**2 + 23X + 6)/(X**2 + X - 6)

THIS IS A FUNCTION OF DEGREE 4

X = -0.70000E 01	F(X) = 0.76000E 02
X = -0.52500E 01	F(X) = 0.47562E 02
X = -0.30000E 01	F(X) = 0.00000E 00
X = 0.00000E 00	F(X) = -0.10000E 01
X = 0.36700E 01	F(X) = -0.22111E 01
X = 0.82500E 01	F(X) = 0.34062E 02
X = 0.10110E 02	F(X) = 0.60772E 02

AFTER EXAMINING THE NUMERICAL VALUES GIVEN ABOVE PLEASE ANSWER THE FOLLOWING QUESTIONS.

(A) WHAT IS THE VALUE OF F(X) WHEN X = -7, WHEN X = -5.25, WHEN X = 3.67, WHEN X = 8.25, WHEN X = 10.11.
(B) DRAW A SKETCH OF THE GRAPH OF F(X).
(C) WHAT IS THE DOMAIN OF F(X).
(D) WHAT IS THE RANGE OF F(X).

// *END OF DATA

APPENDIX G

QUESTIONNAIRE

	Strongly Agree	Mildly Agree	Not Sure	Mildly Disagree	Strongly Disagree
Do you feel the information sheets were a help to you in understanding the calculus?	☐	☐	☐	☐	☐
Do you feel the behavioral objectives as stated, help guide you in your study of calculus?	☐	☐	☐	☐	☐
Do you feel the computer information was an aid in problem solving?	☐	☐	☐	☐	☐
Do you feel the problems contained in the assignment sheets were of help in understanding the calculus?	☐	☐	☐	☐	☐
If you were given a choice, would you choose this method to study calculus?	☐	☐	☐	☐	☐
Would you like to see this project extended to all the areas covered in Calculus I?	☐	☐	☐	☐	☐

Please make any comments you care to make about the program. ______________________________

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