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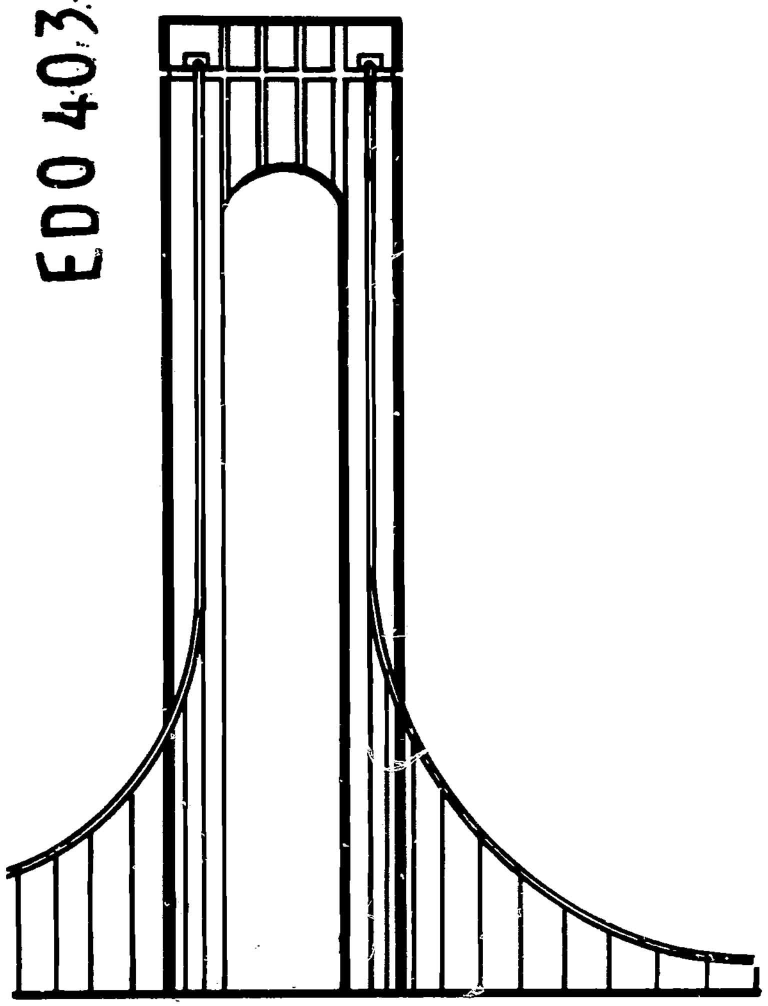
ABSTRACT

This mathematics curriculum resource handbook provides background information and techniques of instruction designed for instructors helping students to prepare themselves for the General Educational Development Tests. It consists largely of fundamental concepts which high school graduates are expected to retain, together with some techniques which may be of use in developing these concepts. Two specific although not "new," approaches to the presentation of mathematics characterize this program. The first is the importance placed on the language of mathematics as a unifying concept. The second approach is the use of manipulative devices. Wherever possible, it is desirable to use paper constructions, models, and movable figures as teaching methods. Emphasis is placed on the general area of problem solving. An annotated list of instructional materials (textbooks, workbooks, and review books) and the addresses of the publishers are included. (Author/NL)



HIGH SCHOOL EQUIVALENCY

Mathematics

PART II: CURRICULUM RESOURCE HANDBOOK

THE UNIVERSITY OF THE STATE OF NEW YORK/THE STATE EDUCATION DEPARTMENT
BUREAU OF CONTINUING EDUCATION CURRICULUM DEVELOPMENT/ALBANY

HIGH SCHOOL EQUIVALENCY

PART II:

Curriculum Resource Handbook

MATHEMATICS

The University of the State of New York
THE STATE EDUCATION DEPARTMENT
Bureau of Continuing Education Curriculum Development
Albany, 1970

Foreword

This mathematics handbook represents a further step by the Department toward the goal of providing adults with realistic personal achievement and its concomitant benefits. Competency in the mathematical methods and with the concepts described herein should facilitate the earning of a high school equivalency diploma, an accomplishment which will assume ever greater importance in our increasingly demanding society.

A field test edition of this manual was distributed to a representative sampling of schools for critical comment. More than two-thirds of the responses indicated that the publication was very helpful. A majority of the remaining comments judged the materials adequate. Hopefully, this final version reflects the constructive criticism received. Further, it is a continuing responsibility of this Bureau to maintain the currency of, and provide supplementary materials for, the high school equivalency program.

The Bureau expresses appreciation to Anthony Prindle, Chairman, Mathematics Department, Linton High School, Schenectady, who prepared the original draft of these materials, and to Margaret Farrell, Associate Professor, State University of New York at Albany, who contributed to the planning and design of the project. R. Allan Sholtes, Guilderland Central Public Schools, continued his work as general writer for the high school equivalency materials.

Department personnel who assisted in the planning and review of the manuscript include: Frank Hawthorne, Chief, Bureau of Mathematics Education; Fredric Paul, Associate, Bureau of Mathematics Education, who carefully reviewed the manuscript and made pertinent suggestions for its modification; John P. McGuire, Chief, and John Rajczewski, Assistant, Bureau of Higher and Professional Educational Testing, who actively assisted the project through their analysis of the field test results in relation to the high school equivalency examination. William Jonas, formerly an Associate in this Bureau, and now with the Bureau of General Continuing Education, helped coordinate the project and revised portions of the manuscript. Barry Jamason, Associate, Bureau of Continuing Education Curriculum designed and prepared the manuscript for publication.

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Message to the Instructor

High school equivalency preparation programs have posed serious problems for those concerned with the development of effective instructional methods in this area. The Department's recent publication, *High School Equivalency Part I: Theory and Design of the Program*, was the first in a series of publications designed to help instructors and administrators in their efforts to develop educationally sound programs of high quality. It provides valuable information concerning the G.E.D.T., program suggestions, and some initial direction for such efforts.

This mathematics curriculum resource handbook provides background information and techniques of instruction designed for instructors helping students to prepare themselves for the G.E.D.T. in general mathematical ability. It consists largely of fundamental concepts which high school graduates are expected to retain, together with some techniques which may be of use in developing these concepts.

In general, topics are presented in this publication in the order of their importance. It is anticipated, therefore, that instructors will:

- Survey the strengths and weaknesses of students in relation to their computational skills
- Group students for instructional purposes
- Establish priorities for each group
- Select topics from this publication for presentation in accordance with these priorities

It should be clearly understood that this publication is not intended to serve as a course of study or curriculum. Most students in these programs already understand many of the concepts presented herein. Furthermore, it is usually not necessary for students to understand all of these concepts in order to succeed in achieving their minimal goals. Nonetheless, it is desirable for students to master as many of them as possible. It is hoped that instructors will use this material to evaluate their current programs and improve the quality of their programs wherever and whenever possible.

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Bureau of General Continuing Education

MONROE C. NEFF, *Director*
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READING CHARTS (Con.)

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READING SKILLS IN MATHEMATICS

Nature of text materials

The successful reading of any kind of material depends upon the reader's ability to relate new data to previous experience. This skill is particularly applicable in mathematics because concepts are developed sequentially.

In addition, mathematics employs a symbolic system which consists of figures, signs, formulas, and equations of the English language. Therefore, the approach to learning the mathematic symbolization should proceed from actual experience with the thing or concept to the language symbol, and not from the language symbol to the thing or concept.

Suggested procedure:

- Show that a symbol, like a vocabulary word, may represent a single item, as in the use of Δ to denote a triangle.
- Show that a symbol may represent an entire concept. For example, a mathematical symbol may involve the understanding of an operation as in the use of the *plus* symbol: $8 + 3$.
- Develop an understanding of a symbol such as π and how it is arrived at before presenting the symbol or any subsequent applications in problem situations.
- Relate the structure of the mathematics material to the structure of English material. Show that word phrases are elements of the English sentence as number phrases are elements of the mathematical sentences.

Examples of phrases:

Fifteen divided by some number

$15 \div y$ or $\frac{15}{y}$

Four times some number

$4 \cdot y$ or $4y$

- Illustrate how word phrases used in combination with a linking verb produce a complete thought and a complete sentence. Show that the verb *is* often performs the same function as the equal sign (=).

Examples of sentences:

Mary's age is four times Jack's age.
Mary is 12 years old.
$4 \cdot y = 12$.

Mary's age plus three years is the same as John's age plus five years.
$12 + 3 = x + 5$.

- Reinforce the concept that an algebraic sentence is called an *equation*, and point out the common root of the words *equal* and *equation*.

Improving comprehension

Since all mathematical understanding is based upon previously learned concepts and skills, the instructor should begin by determining the extent of the knowledge possessed by his students. He should, through review activities, utilize this data in the introduction of new materials. This review should involve particularly the precise meanings of the technical terms used.

The study of each new unit should be preceded by a preparatory phase during which teacher and students explore new concepts through developmental discussion, demonstrations, and visual and manipulative experiences. During this phase, new words and mathematical symbols become meaningful, and each student develops a background of experience which will aid in the comprehension of written material.

In reading mathematics textbooks and other materials, the student must be able to grasp meanings which are sharply defined, clear, and unambiguous. Subsequently, he learns to read passages which include statements of principles and generalizations, explanations of processes, and problems for solution.

Since these materials require slow, careful reading and a high degree of mental concentration, pupils may need instruction and practice. They should learn to keep pencil and paper at hand for making notes or constructing diagrams.

The instructor should place special emphasis on the reading of verbal problems for mathematical solution since these problems are written in a brief, highly compact style and often use technical words.

Suggested procedure:

- Ask the student to restate the problem in his own words. This forces him to organize his thoughts, and he may, during the process, discover weaknesses in his understanding or reveal such weaknesses to the instructor.

- Ask the adult student to diagram the problem solution. The diagram helps to clarify relationships and to keep the facts available.

- Ask adult students to dramatize a condition involving people in a problem situation.

The instructor should remember that reading is a mental process and that problem solving requires both thought and know-how. He should aid students in developing ways of thinking about problems which will enable him to visualize situations, see relationships, grasp problems, and take the necessary steps toward solution. His goal is to help students develop patterns of attack so that they can work independently.

The instructor should give practice in attacking unstructured problems as well as those which are preformulated. He should use a variety of types of problems including those with insufficient data, problems having unnecessary data, problems involving spatial visualization, logic, and problems which do not present a question and which must be completed by the pupil.

Vocabulary acquisition

The expression "knowledge of word meanings" indicates a knowledge of the concepts expressed by the words rather than a mere verbalization or parroting of words, definitions, formulas, and the like. The teacher should give high priority to the acquisition of word meanings during the reading preparation phase and in all other phases of study. He should stress not only strictly mathematical terms, but also certain other word categories such as:

- Words in general use which are frequently encountered in mathematics textbooks

- Words whose mathematical meanings differ from their general meanings or their meanings in other subject areas

- Words whose mathematical meanings are more precise than their general meanings

Use of structural analysis

The instructor should aid the students in becoming familiar with the meanings of commonly used prefixes, roots, or suffixes. He should include these word parts in classroom treatment of vocabulary acquisition and review this data at every opportunity. This reinforcement of new terms will facilitate spelling and the learning of new terms which employ the same roots or affixes, since the prefixes, base words, and suffixes have specific meanings. Inasmuch as the mathematical word parts often indicate quantities, measurement, or geometric figures, the instructor can bring out the meanings of those parts. For instance, with the word *binomial*, the teacher might ask the students to recall the meaning of the prefix as used in *bicycle* and *biweekly*, and also bring out the meaning of the root, *-nomial*. He might then introduce the related terms *mononomial*, *trinomial*, and *polynomial*. Gradually, as word parts are met again in additional combinations, as in *triangle*, *polygon*, the meanings of the parts are reinforced. Such word study assists the student reader in visualizing concepts, since knowing the meaning of the parts of the words *quadrilateral* and *pentagon* should bring to mind the geometric figures.

In subsequent lessons, the teacher may reinforce the instruction by asking students to dissect such words as *triangle*, *pentagon*, *quadrilateral*, and to draw diagrams of the words under discussion.

Some additional examples are:

- Base words plus prefixes: *polynomial*, *equidistant*

- Base words plus suffixes and/or inflectional endings: *trapezoidal*, *rhombuses*, *equalities*, *radii*

Summary of reading skills for mathematics

The skills list that follows may be used as a checklist during the course. It sets forth skills that are particularly important to full comprehension of mathematical materials. The student should learn to:

- Comprehend factual materials

  Recognize the main idea
  - sense problems
  - define problems

  Recognize details
  - select relevant facts
  - see relationships

  Organize and classify facts

  Note sequence

  Adjust his reading rate to his purpose

- Increase his vocabulary

  Recognize and understand technical terms
  - understand and select exact meanings
  - suit meaning to context

  Use the dictionary, textbook aids, and reference materials

- Understand graphic materials

  Read graphs and diagrams

  Read charts

- Follow directions

Mathematics

in the

High School Equivalency Program

The Mathematics Program

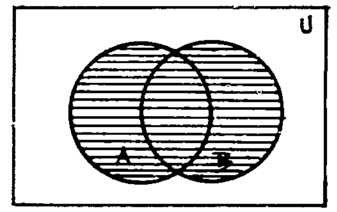
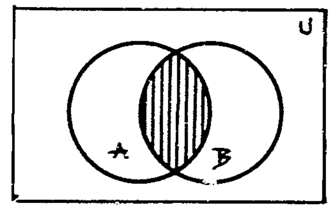
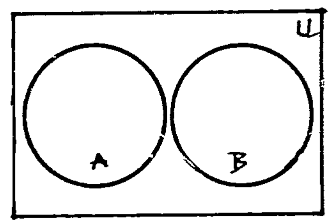
Two specific, although not "new," approaches to the presentation of mathematics characterize this program. The first is the importance placed on the language of mathematics as a unifying concept. Without the language, the presentation of mathematics becomes structureless and mechanical.

The second approach is the use of manipulative devices. Wherever possible, it is desirable to use paper constructions, models, and movable figures as teaching methods. Because most or all of the students in the equivalency program will be facing totally new concepts, it is strongly suggested that these concepts be approached in a constructive rather than a formal and mechanical way. If so desired, this constructive approach may be followed with a more formal presentation of the concept.

Emphasis is placed on the general area of problem solving. Every effort has been made to present problems suitable to the needs of the students.

Improvement of reading skills is the most vital aspect of all phases in the high school equivalency program. Pages vi and vii list the basic reading skills with a cross reference to each of the four handbooks on adult reading developed by the Department. Special consideration should be focused upon the area entitled Reading in Mathematics.

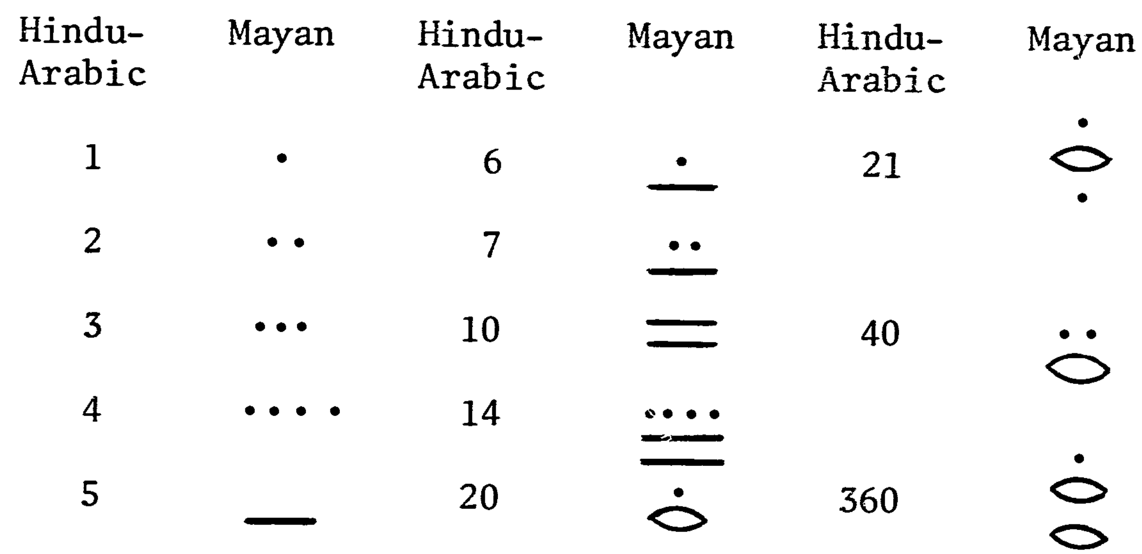
TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
I. Arithmetic A. Set concepts 1. Set	A set is any collection of objects having something in common.	Illustrate with examples such as the following: • Set of rivers in the United States • Set of odd numbers • Set of students in your class • Set of red-haired movie actresses • Set of single-digit prime numbers
2. Cardinal number of a set	The cardinal number of a set is the number of elements in the set.	The set {a,b,c} has a cardinal number of 3. (What is the cardinal number of the set of rivers in the U.S.? The set of students in this class?)
3. Finite set	A finite set has a countable number of elements.	Can we count all the rivers in the U.S.? The number of students in this class?
4. Infinite set	An infinite set is one which has an uncountable number of elements.	Can we count the number of odd numbers? Further examples of infinite sets would include: • The set of all counting numbers {1,2,3....} • The set of fractions between 0 and 1
5. Set operations a. Union	The union of two sets is a set consisting of all of the elements of both sets.	The union of two sets is associated with the word "or." A ∪ B means the union of two sets {A <u>or</u> B}. Let A = {a,b,c,d} and B = {c,d,e,f} then A ∪ B = {a,b,c,d,e,f}
b. Intersection	The intersection of two sets is the set of elements included in both sets at the same time.	The intersection of two sets is associated with the word "and." A ∩ B means the intersection of two sets {A <u>and</u> B}. Let A = {a,b,c,d} and B = {c,d,e,f} then A ∩ B = {c,d}

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
c. Representation	Sets may be represented pictorially by the use of a rectangle to represent the universal set with circles to represent subsets of the universal sets.	The following illustrations may help to clarify these concepts: A ∪ B (union)  (includes all points in either A *or* B or both) A ∩ B (intersection)  (includes only those points in both A *and* B)
6. The null (or empty) set	The null (or empty) set is the set which contains no elements.	The null set is symbolized by ∅ or { }.
7. Disjoint sets	Disjoint sets are sets with no common elements or whose intersection is empty.	The sets {1,2,3} and {4,5,6} are disjoint.  Sets A and B are disjoint.
8. Subsets	One set is a subset of a second if each element belonging to the first also belongs to the second.	If a set A = {a,b,c}, each of the following is a subset of A: { }, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}. The null set is a subset of every set, and each set is a subset of itself.
	The number of subsets of a given set depends upon the number of elements in the set.	Consider {a}. There are two subsets, namely { }, {a}. For {a,b}, there are four subsets, namely { }, {a}, {b}, {a,b}. For {a,b,c} there are eight subsets as

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
		previously indicated. Thus the number of subsets can be determined if the cardinal number of the set is known.
		The total number of subsets in a set consisting of one element is therefore two. A set consisting of two elements will have four possible subsets. A set consisting of three elements will have eight subsets. An examination of this pattern shows that the total number of possible subsets of a set that contains n elements is 2^n.
9. Equivalent sets	Two sets are equivalent if they have the same cardinal number, that is, if they contain the same number of elements.	The distinction is made here between "equal" sets (sets which are identical), and equivalent sets. If set A = {a,b,c} set B = {1,2,3} A and B are equivalent, but not equal. If A = {a,b,c} and C = {a,b,c}, sets A and C are equivalent and equal.
B. Set of whole numbers		
1. Place value	The Hindu-Arabic number system is based upon position or place value, and the position of a digit in a number determines its value.	In the number 3,333 each "3" has a different value.
2. Expanded notation	Expanded notation is a method of writing a number in such a way as to reveal the place value of each digit.	The student must first understand the idea of an exponent as a shorthand for writing multiplication. $3 \cdot 3 = 3^2$, or 3 used as a factor twice. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5$, or 10 used as a factor 5 times.
a. Historical development	Early systems of writing numerals were based on tally marks.	Examine the historical development of other systems and point out the advantages of the Hindu-Arabic system.
b. Mayan system		An additive system based on 20 and 360 was formulated in this hemisphere by the Mayans. Its distinctive feature was that it incorporated the concept of "zero"

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES

and hinted at a positional system of numeration. The table below shows the Mayan system.



A really large number would be very difficult to write out.

The Hindu-Arabic system provides a more concise and practical method of writing numerals and performing operations than earlier systems.

Returning to the decimal system, with Hindu-Arabic numerals, it is easy to see the value of shorthand notation. 1,234 then means 4 units, 3 tens, 2 hundreds (or 10^2), and one thousand (or 10^3). Simplified this becomes $4 + (3 \times 10^1) \div (2 \times 10^2) + (1 \times 10^3)$.

A further illustration: 75,234 is $(4 \times 1) + (3 \times 10^1) + (2 \times 10^2) + (5 \times 10^3) + (7 \times 10^4)$.

To add now is a matter of proper grouping.

$$\begin{aligned} 334 &= 4 \times 1 + 3 \times 10^1 + 3 \times 10^2 \\ \underline{567} &= \underline{7 \times 1 + 6 \times 10^1 + 5 \times 10^2} \\ & \quad 11 \times 1 + 9 \times 10^1 + 8 \times 10^2 \\ 11 \times 1 &= \underline{1 \times 1 + 1 \times 10^1} \\ &= 1 \times 1 + 10 \times 10^1 + 8 \times 10^2 \\ &= 1 \times 1 + 0 \times 10^1 + 9 \times 10^2 \end{aligned}$$

This becomes 9 hundreds + one, or 901.

3. Fundamental operations
   a. Addition

Addition is the first fundamental operation and is a binary operation

The basic addition facts should be reviewed and reinforced here.

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	because only two elements can be combined under addition at one time.	
b. Multiplication	Multiplication is a binary operation and is treated as repeated addition.	There are many ways of indicating multiplication such as 3 x 5, 3•5, (3)(5), or, in the case of letters, ab means a times b. The basic multiplication facts are reviewed through the nine-times table. 3 x 7 = 7 + 7 + 7 or 3 + 3 + 3 + 3 + 3 + 3 + 3.
4. Three principal laws of operation		
a. Commutative law	The commutative law states that the order in which multiplication or addition is performed is not important.	It's easy to see that 2 + 3 = 3 + 2 and 5 x 4 = 4 x 5. Subtraction and division are noncommutative for 5 - 3 does not equal 3 - 5 and 4 ÷ 2 is not the same as 2 ÷ 4.
b. Associative law	The associative law concerns grouping when adding and multiplying three or more elements.	Associativity tells us that 2 + (3 + 5) = (2 + 3) + 5, and 2•(3 x 5) = (2 x 3)•5. An interesting insight into nonassociative addition is (5 - 3) + 2 ≠ 5 - (3 + 2).
c. Distributive law	The distributive law states that multiplication is distributed over addition.	This is exemplified by 2 x (3 + 5) = 2 x 3 + 2 x 5 or 2 x 8 = 6 + 10 where 16 = 16. In general a x (b + c) = a x b + a x c.
5. Identity elements		
a. For addition, zero	The identity element is the number which, when added to a second number, leaves it unchanged; zero has this property.	Clearly a + 0 = 0 + a = a. Zero is unique, since if any other number is added to a, the sum is different from a.
b. For multiplication, one	The number one has the property that any number multiplied by one remains the same.	Obviously 6 x 1 = 6, 1 x 7 = 7 and in general a x 1 = a for any a. It should be noted that one may be written in many different ways. For example $\frac{7}{7}$ is equal to one as is $\frac{5}{5}$, $\frac{37}{37}$ and in fact $\frac{a}{a}$ so long as $a \neq 0$. Thus, $\frac{3}{4}\cdot\frac{5}{5} = \frac{3}{4}$ or $\frac{3}{4} = \frac{15}{20}$.
6. Inverses		
a. Additive	The additive inverse for any number is a new number which, when added	The notion of an inverse hinges closely upon that of the identity. If we consider a number "x", its

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
	to the first, yields the additive identity, zero.	*additive* inverse is a number z such that $x + z = 0$. This is a complicated way of saying that the opposite of 3 is -3, for $3 + (-3) = 0$.
b. Multiplicative	The multiplicative inverse for any number is a new number which when multiplied by the first yields 1, the multiplicative identity.	Thus, for multiplication the inverse for a number x is a second number y such that $x \cdot y = 1$. Again, this is a complicated way to say that the inverse for 5 is $\frac{1}{5}$, for $5 \cdot \frac{1}{5} = 1$. Also, the inverse for $\frac{2}{3}$ is $\frac{3}{2}$. It should be noted that every number has an additive inverse, but that the number "zero" has no multiplicative inverse since there is no number "z" such that $0 \cdot z = 1$.
7. Inverse operations a. Subtraction	Subtraction is the inverse operation for addition.	*Caution:* Students may have been taught any of four methods of subtraction. It would be unwise to teach one specific method of subtraction if the student obtains the correct result in a different way. The mathematical definition of subtraction hinges on the understanding of an inverse and $5 - 3 = 5 + (-3)$ or verbally "5 minus 3 is the same as 5 plus the opposite or negative of 3." More generally, $a - b = a + (-b)$.
b. Division	Division is the inverse operation for multiplication and is learned as repeated subtraction.	The mathematical definition of division is formed in a manner similar to subtraction: $\frac{a}{b} = a \div b = a \times \frac{1}{b}$. Verbally we say that "a divided by b means a times the inverse of b." Clearly, division by zero is impossible since zero has no inverse. 25 ÷ 5 basically says "How many times can 5 be subtracted from 25?" A useful technique is using the letter R for remainder as in the example, $7\overline{)25}$ with quotient 3 R 4.
8. Factoring	A factor is the same as the divisor (no remainder).	Factors of 6 are 2 and 3. Factors of 27 are 9 and 3, or 3, 3, and 3.
a. Prime number	A prime number is a number greater than one which can be divided evenly by only itself and one.	Examples are 2, 3, 5, 7, 11, 13 ....

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
b. Composite number	A composite number contains divisors other than itself and one.	...4, 6, 8, 9, 10, 12 .... are all composite numbers. *Note:* 0 is a special number, neither prime nor composite.
c. Even numbers	Even numbers are those divisible by 2.	...0, 2, 4, 6, 8 .... are even numbers.
d. Odd numbers	Odd numbers leave a remainder of 1 when divided by 2.	...1, 3, 5, 7, 9 .... are odd numbers.
e. Divisibility	A number is divisible by a second if the quotient of the first divided by the second is an integer.	...27 is divisible by 9 since $27 \div 9 = 3$, but not by 7 since the quotient is $3\frac{6}{7}$ (not an integer).
f. Greatest common divisor	The greatest common divisor of two or more numbers is the largest positive integer which will divide each of them evenly.	The abbreviation G.C.D. is used for greatest common divisor. For instance: the G.C.D. of 24 and 36 is 12, of 35 and 49 is 7, of 119 and 289 is 17. This principle is useful in reducing fractions.

One interesting way of finding the G.C.D. of two numbers is by Euclid's rule. This is optional, but has motivational significance.

Consider 24 and 36. Compute the G.C.D.

- Divide the smaller into the larger, $24\overline{)36}$ = 1 R 12.
- Divide the remainder into the divisor. $12\overline{)24}$ = 2 R 0.

The final divisor 12 is the G.C.D.

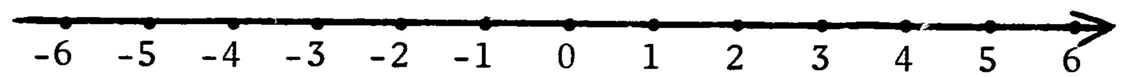
A second example will further illustrate Euclid's rule. Consider 119 and 289. At first glance it appears that one cannot divide both by the same number.

But a trial yields 17. $119\overline{)289}$ = 2 R 51

$51\overline{)119}$ = 2 R 17

$17\overline{)51}$ = 3 R 0

Thus 17 is the G.C.D.

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
g. Least common multiple principle	The least common multiple (L.C.M.) of two integers a and b is the smallest number which is evenly divisible by a and b.	This abstract concept is best explained by examples. • L.C.M. of 3 and 5 is 15 since 15 is the smallest number divisible by 3 and 5. Note that in this case 15 = 3•5. • L.C.M. of 4 and 6 is 12 since both 4 and 6 will divide 12 and there is no smaller number divisible by both. In this case we note that the L.C.M. is *not* simply the product of the two numbers. • The L.C.M. of 18 and 27 is 54, not so easy to see. But again there is a device to arrive at the L.C.M. First factor each number. 18 = 2•3•3 27 = 3•3•3 The product of the common factors (3 and 3) and the remaining factors (2 and 3) gives 54, the required L.C.M.
C. The integers	The integers include the counting numbers, zero, and the negatives of the counting numbers.	The number line, either vertical or more commonly horizontal, is the best tool to use in considering the integers. The analogy to a thermometer shows the necessity for the existence of negative numbers. Number Line  The number line should be used at each application of the use of negative numbers until such time as the position on a line is firmly implanted.
1. Trichotomy law	The trichotomy law states that of two integers a and b, either a is less than b, equal to b, or greater than b.	The symbols used are a < b, meaning a is less than b; a = b; a > b, meaning a is greater than b. (Also, a ≠ b means a is not equal to b.) Simply stated, the trichotomy law tells us that one of the properties of the integers is that of two integers one can determine which is the larger.

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2. Open sentence	An open sentence is a statement which may be either true or false.	For example "Something plus 3 = 5" is an open sentence since it is true if something is 2, but false otherwise. A second example is $\square - 3 = 17$, which is true if the box (really a variable) represents 20, and false otherwise. The algebraic principle exemplified by the box serving as a variable is most useful in solving elementary equations without resorting to the more abstract use of the letter x. It would be well at this point to solve each of the four simple types of equations to facilitate an algebraic approach to arithmetic: (1) $\square + 3 = 5$, (2) $\square - 2 = 7$, (3) $3 \times \square = 12$, and (4) $\frac{\square}{5} = 10$, in each case indicating to the student that he must fill the box to make the statement true. "He was the President of the United States," is an open sentence, for "he" is really a variable. If you replace "he" with Abraham Lincoln it is true, but if you choose Daniel Webster, it is false.
3. Closed sentence	A closed sentence is a statement which is true or one which is false with no ambiguity.	Some examples of closed sentences are: 2 + 3 = 5 (True) 7 x 8 = 63 (False) Charles De Gaulle was a King of England. (False)
D. Rational numbers	A rational number is the quotient of two integers x and y with $y \neq 0$.	Mathematically, this is a valid definition of a rational number. More realistically, it is better to examine the first 5 letters of the word rational, for a rational number is indeed a ratio or a fraction. Examples of rational numbers are $\frac{2}{3}$, $\frac{5}{2}$, $2\frac{1}{2}$, $-3\frac{1}{3}$, 3.1 (for this may be expressed as $\frac{31}{10}$), and 7 (since seven may be written as $\frac{7}{1}$).
1. Reducing fractions	Dividing the numerator and denominator of a fraction by their greatest common factor reduces it to its lowest terms.	Use the "1" principle and the G.C.D. to reduce fractions. For example: $\frac{48}{72} = \frac{2\cdot2\cdot2\cdot2\cdot3}{2\cdot2\cdot2\cdot3\cdot3} = \frac{2}{3}$ Euclid's rule may be used as a motivational device in reducing a fraction to its lowest terms.

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		Example: Reduce $\frac{133}{209}$ $133\overline{)209}$ = 1, remainder 76; $76\overline{)133}$ = 1, remainder 57; $57\overline{)76}$ = 1, remainder 19; $19\overline{)57}$ = 3, remainder 0 Hence, 19 is the G.C.D. of 133 and 209; $\frac{133}{209} = \frac{19 \times 7}{19 \times 11}$, therefore $\frac{133}{209} = \frac{7}{11}$.
2. Multiplying fractions	To multiply two fractions, multiply the numerators together and multiply the denominators together.	The basic principle here is that $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$. For example, $\frac{3}{7} \times \frac{5}{9} = \frac{15}{63}$, or $\frac{5}{21}$. We recall that $\frac{a}{b} \times 1 = \frac{a}{b}$, and that "1" may assume disguises such as $\frac{3}{3}$, $\frac{7}{7}$, $\frac{-11}{-11}$, a useful tool to be used in addition.
a. Cancellation	Cancellation consists of dividing the numerator and denominator of a fraction by the same number.	Many prefer not to use the word cancellation, but it is an easy shorthand device in reducing fractions and is likely to be part of the background and vocabulary of the student. The cancellation principle is best explained by example. $\frac{15}{14} \times \frac{21}{5} = \frac{15 \cdot 21}{14 \cdot 5}$ $= \frac{5 \cdot 3 \cdot 7 \cdot 3}{7 \cdot 2 \cdot 5}$ $= \frac{5 \cdot 7 \cdot 3 \cdot 3}{5 \cdot 7 \cdot 2}$ $= \frac{3 \cdot 3}{2} \times \frac{5 \cdot 7}{5 \cdot 7}$ $\frac{5 \cdot 7}{5 \cdot 7}$ is really 1, so our result is $\frac{3 \cdot 3}{2} \times 1 = \frac{9}{2} = 4\frac{1}{2}$. Simplifying in the original example $\frac{15}{14} \times \frac{21}{5}$, we see that 7 will divide both numerator and denominator, so we "cancel" 7 into each. The terminology is not as important as a careful explanation of the process.

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3. Complex fractions	To simplify complex fractions, multiply by "1" using the form of the least common multiple of the denominators divided by itself.	The best way to simplify the result of $\frac{a}{b} \div \frac{c}{d}$ is to use again the "1" property (a x 1 = a). Here, $1 = \frac{bd}{bd}$ is the choice we make, since bd is the product of the two denominators. Then, $\frac{\frac{a}{\not b}}{\frac{c}{\not d}} \times \frac{\not bd}{b\not d} = \frac{ad}{bc}$. The foregoing is the general principle upon which we base our simplification of complex fractions and leads up to the division of fractions. For example, $\frac{\frac{3}{7}}{\frac{5}{8}}$ is simplified by using $1 = \frac{7 \cdot 8}{7 \cdot 8}$ as a multiplier to obtain $\frac{\frac{3}{\not 7}}{\frac{5}{\not 8}} \times \frac{\not 7 \cdot 8}{7 \cdot \not 8} = \frac{3 \cdot 8}{5 \cdot 7} = \frac{24}{35}$.
4. Division of fractions	To divide two fractions, invert the divisor and multiply.	Much justification is necessary for this principle. Complex fractions lead to division of fractions as follows: $\frac{3}{7} \div \frac{2}{3}$ may be written as $\frac{\frac{3}{7}}{\frac{2}{3}}$. Now we choose $1 = \frac{3 \cdot 7}{3 \cdot 7}$ and multiply to obtain $\frac{\frac{3}{7}}{\frac{2}{3}} \times \frac{3 \cdot 7}{3 \cdot 7}$ to get $\frac{3 \cdot 3}{2 \cdot 7} = \frac{9}{14}$. Several numerical examples lead us to the generalization that $\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{\not b}}{\frac{c}{\not d}} \cdot \frac{\not bd}{b\not d} = \frac{ad}{bc}$. The definition of multiplication shows us that $\frac{ad}{bc} = \frac{a}{b} \times \frac{d}{c}$. Thus $\frac{a}{b} \div \frac{c}{d}$ is simplified to $\frac{a}{b} \times \frac{d}{c}$. This is the justification for inverting a divisor in division.

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5. Proper and improper fractions	The distinction between proper and improper fractions is that $\frac{a}{b}$ is a proper fraction if a < b, and $\frac{a}{b}$ is improper if a = b or if a > b.	Illustrate $\frac{7}{10}$, $\frac{9}{11}$, $\frac{24}{48}$ as proper fractions; $\frac{5}{3}$, $\frac{4}{2}$, $\frac{3}{3}$ as improper fractions.
6. Addition and subtraction of fractions	The least common multiple principle is utilized to add or subtract fractions.	If we start with fractions having the same denominator, addition is simplified as in $\frac{3}{4} + \frac{5}{4} = \frac{8}{4} = 2$. For any other addition, make use of the least common multiple generalization to first find the least common denominator. *Example 1:* $\frac{3}{5} + \frac{1}{4} = ?$ The L.C.M. of 5 and 4 is 20, thus each fraction is converted to a fraction with denominator 20. $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$ $\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$ $\frac{12}{20} + \frac{5}{20} = \frac{17}{20}$. *Example 2:* $\frac{5}{12} + \frac{7}{16} = ?$ $12 = 2 \cdot 2 \cdot 3$ $16 = 2 \cdot 2 \cdot 2 \cdot 2$ The L.C.M. is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 48$. $\frac{5}{12} \times \frac{4}{4} = \frac{20}{48}$ $\frac{7}{16} \times \frac{3}{3} = \frac{21}{48}$ $\frac{20}{48} + \frac{21}{48} = \frac{41}{48}$
7. Decimal fractions	A decimal fraction is a common fraction with a denominator which is a power of ten.	Examination with expanded notation is helpful here. .7 is read $\frac{7}{10}$ and may be so written.

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		.76 is $\frac{76}{100}$, or $\frac{76}{10^2}$, or $\frac{7}{10^1} + \frac{6}{10^2}$ .765 is $\frac{765}{1000}$, or $\frac{765}{10^3}$, or $\frac{7}{10^1} + \frac{6}{10^2} + \frac{5}{10^3}$ .7654 is $\frac{7654}{10000}$, or $\frac{7654}{10^4}$, or $\frac{7}{10^1} + \frac{6}{10^2} + \frac{5}{10^3} + \frac{4}{10^4}$
a. Terminating decimals	Terminating decimals are those having a finite number of digits.	A terminating decimal fraction can be converted to a common fraction by using the appropriate power of ten. A handy rule is to count the number of decimal places and use this number as the power of ten *or* the number of zeros to follow the "1" in the denominator. Several examples should be presented. .075 has three decimal places, so the equivalent common fraction is $\frac{75}{10^3}$ or $\frac{75}{1000}$.
b. Repeating decimals	Repeating decimals are nonterminating and have a certain sequence of digits which repeat; they are equivalent to common fractions.	The student should recognize the simplest repeating decimals such as .333.... = $\frac{1}{3}$ and .666.... = $\frac{2}{3}$.
c. Conversion from common fractions to decimals	Division is used to convert a common fraction to a decimal equivalent.	The first step is to rewrite the fraction in terms of division, properly placing the decimal point. $\frac{3}{4} = 4\overline{)3}$. Then annex any number of zeros for desired accuracy, and divide. $4\overline{)3.00}$ = .75 or $\frac{5}{7} = 7\overline{)5.000000}$ = .714285 (which then repeats).
d. Approximating decimal fractions	In converting common fractions to decimal equivalents, nonterminating decimals occur frequently and rounding off must be used.	If the fraction $\frac{1}{3}$ is converted to a decimal, it is conventional to write the result as .333.... or $.\overline{3}$ to indicate that it repeats, but for practical purposes in problem solving, an approximation is used. Thus $\frac{1}{3} \approx$ .3 or .33 (≈ meaning *approximately equal to*). In teaching rounding off, the important principle to keep in mind is the degree of accuracy required. .348 when rounded to the nearest tenth is .3 (.348 is closer to .3 than to .4). All that is necessary in rounding off is to consider the next digit to the right of the re-

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		quired approximation. If it is less than 5, it is dropped along with all successors; if it is 5 or more, round off upward by 1. *Examples:* • Round off .726 to the nearest tenth. Solution: Since the hundredths place is occupied by 2, it is less than 5 so the approximation is $.726 \approx .7$ • Round off .726 to the nearest hundredth. Solution: Consider the hundredths place. The digit to the right is 6 in the thousandths place so $.726 \approx .73$ to the nearest hundredth. (.726 is closer to .73 than to .72)
e. Basic operations with decimals	Basic operations with decimals involve meaningful manipulation of the decimal point.	
(1) Addition	To add decimal fractions, make sure that the decimal points fall directly under each other.	Indicate here the relation to common fractions. $.7 + .24 = \frac{7}{10} + \frac{24}{100}$ $= \frac{70}{100} + \frac{24}{100}$ $= \frac{94}{100} = .94$ but $.7 = .70$ $\begin{array}{r} .70 \\ +\ \underline{.24} \\ .94 \end{array}$, thus giving evidence of the need for positioning the decimal point.
(2) Multiplication	To multiply two decimal fractions, multiply the numbers without regard for the decimal point; then position the decimal point in the product according to the sum of the places in the multipliers.	$.7 \times .2 = \frac{7}{10} \times \frac{2}{10} = \frac{14}{100}$ $= .14$ $.03 \times .27 = \frac{3}{100} \times \frac{27}{100} = \frac{81}{10000}$ $= .0081$

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		In the foregoing, several illustrations clarify the concept .7 x .2 = .14. It is evident that we multiply the numerals, obtaining 14, and count the number of decimal places (2) starting from the right. Thus, .03 x .27 is 81 with four decimal places, giving us .0081.
(3) Division	To divide one decimal fraction by a second, use the principle of multiplying by a form of "1" to make the denominator an integer.	$.25 \div .5 = \frac{.25}{.5} \times \frac{10}{10}$ $= \frac{2.5}{5} = 5\overline{)2.5}$ with quotient .5 Several examples of this type will lead the student to accept the conventional method. 1.221 ÷ .37 (1) $.37\overline{)1.221}$ (2) $37\overline{)122.1}$ (The decimal point is moved two places to the right in each case, in reality multiplying numerator and denominator by 100.) (3) $37\overline{)122.1}$ = 3.3; 111; 111; 111 (The decimal point is placed in the quotient as shown.) (4) 1.221 ÷ .37 = 3.3
f. Common decimal equivalents	Decimal equivalents of common fractions are useful in solving certain types of problems.	Each should be verified by converting to common fractions and reducing to lowest terms. Problems should then be introduced to apply these principles. Money problems, discount, business, general investment, and averaging lend themselves to this topic.
7. Percent a. Definition	Percent means hundredths.	$7\% = \frac{7}{100}$, $37\% = \frac{37}{100}$, $\frac{1}{2}\% = \frac{1}{200}$.

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b. Conversion to common fraction	To convert from percent to common fraction, multiply the number of percent by $\frac{1}{100}$.	Several examples should be used, with particular emphasis on the more difficult cases of percents greater than 100 and less than 1. $125\% = 125 \times \frac{1}{100} = \frac{125}{100} = \frac{5}{4}$ $\frac{1}{3}\% = \frac{1}{3} \times \frac{1}{100} = \frac{1}{300}$ $3\frac{1}{3}\% = 3\frac{1}{3} \times \frac{1}{100} = \frac{10}{3} \times \frac{1}{100} = \frac{10}{300} = \frac{1}{30}$
c. Conversion to decimal fraction	To convert from percent to decimal fraction, multiply the numerical percent by $\frac{1}{100}$.	Shortcuts are available for conversion from percent to decimal fraction. Since percent means hundredths, an example suggests that we remove the percent sign and move the decimal point two places to the left. Avoid memorization of such a rule without sufficient justification. *Examples:* $37\% = 37 \times \frac{1}{100} = \frac{37}{100} = .37$ $8.7\% = 8.7 \times \frac{1}{100} = \frac{8.7}{100} = \frac{87}{1000} = .087$.
d. Conversion from common fraction to percent	To convert from common fraction to percent, divide numerator by denominator resulting in a two-place decimal and then multiply by 100.	For this purpose only a simple intuitive approach is necessary, as in a fraction equal to something divided by 100. All percent problems can be described as ratios. Take for example, the common fraction $\frac{3}{5}$. If we wish to convert to percent, we think of the ratio "n out of 100." Hence $\frac{3}{5} = \frac{n}{100}$. Then $5n = 300$, $n = 60$. All problems involving percent hinge on this technique.
e. The three fraction equivalents	Common fraction, decimal fraction, and percent equivalents occur in families frequently used.	The families of common fractions, decimal fractions, and percent equivalents should be developed here with emphasis on those with denominators 2, 3, 4, 5, 6, 8, and 10. Common elements and similar properties should be noted to facilitate learning and association.

Common Fraction	Decimal Equivalent	Percent Equivalent
$\frac{1}{3} = \frac{2}{6}$	$.33\frac{1}{3}$	$33\frac{1}{3}\%$
$\frac{2}{3} = \frac{4}{6}$	$.66\frac{2}{3}$	$66\frac{2}{3}\%$

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f. Using percent (1) Finding a percent of a number	To find a percent of a number, use a proportion in which the desired percent will be the first two terms and the unknown and known numbers occupy the third and fourth terms.	In using percent, a basic proportion will carry through to all types of problems. There are three types of problems, the first of which is finding a percent of a number. An example illustrates the proper technique. Find 17% of $130. Thinking verbally, we realize "17 *out of* 100 must be the same as n *out of* $130." This suggests a proportion $\frac{17}{100} = \frac{n}{130}$. Using the idea that the product of the means is equal to the product of the extremes, $100n = 17 \times 130$ $100n = 2210$ $n = \frac{2210}{100}$ $n = \$22.10$
(2) Finding what percent one number is of another	To compute what percent one number is of another, use a proportion whose first ratio is the first number to the second, and whose second ratio is the variable percent compared to 100.	Again, illustrations are superior to the memorization of rules. A student took a test consisting of 80 items and had 64 correct. What percent of correct answers did he have? Again we think "64 out of 80" is the same as "what out of 100?" Thus $\frac{64}{80} = \frac{n}{100}$ $80n = 6400$ $n = 80\%$
(3) Finding a number when a percent is known	To find a number when a percent is known, use a proportion in which the unknown is the last term.	Typically more students have difficulty with this concept of percent than the other two. Utilizing a proportion helps to unify all three cases. As before, an example is the best teaching device. A savings account paying 5% interest per year was found to gain $42 interest after one year. How much was invested?

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		We rephrase this to say, "The ratio of 5 to 100 (5%) must be the same as \$42 to the total principal." Thus $\frac{5}{100} = \frac{42}{n}$ $5n = 4200$ $n = \$840$
	Ratios and proportions are useful in solving a variety of mathematical problems.	The use of ratio and proportion tends to generalize the handling of percent, instead of compartmentalizing it. With these concepts reinforced by sufficient drill, the techniques are more likely to make sense and be retained. Suggested problems are: • Percent of increase and decrease • Interest and investment • Profit and loss
E. The set of irrational numbers	On the number line, the irrational numbers complete a set of real numbers.	Point out that if all rational numbers are placed on a number line, there are more "holes" than places filled. The irrational numbers fill in the "holes."
1. Square root defined	The square root of a number N is the number "a" such that a x a = N. $a = \sqrt{N}$ is read "a is the square root of N."	Simple illustrations suffice initially. $\sqrt{1} = 1, \quad \sqrt{4} = 2, \quad \sqrt{9} = 3, \quad \sqrt{121} = 11$ A square root is often termed a "radical" expression, as are third, fourth, and higher order roots. The number beneath the radical sign is termed the radicand.
2. Multiplying square roots	The product of the square root of a and the square root of b is the square root of ab.	Illustrations should demonstrate the multiplication of radicals. $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}, \quad \sqrt{7} \cdot \sqrt{91} = \sqrt{637}$ Also $\sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$
3. Simplifying radicals	To simplify a radical expression, first factor using the multiplication principle and then reduce the perfect squares.	Simplify this rule to read "Whenever a factor occurs twice beneath a radical sign, it may be removed and placed outside the radical."

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		$\sqrt{48} = \sqrt{2\cdot2\cdot2\cdot2\cdot3}$ There are two groups of 2's, so $\sqrt{48} = 2\cdot2\cdot\sqrt{3} = 4\sqrt{3}$ $\sqrt{72} = \sqrt{2\cdot2\cdot2\cdot3\cdot3} = 2\cdot3\cdot\sqrt{2} = 6\sqrt{2}$
4. Division of radicals	Division of radical numbers obeys the same law as multiplication.	$\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{4} = 2$
5. Computing square root by estimation	An approximation of square root is obtained by a process involving dividing and averaging. (It is suggested that square root tables be used since the two methods herein described are only for those interested in the mathematical exercise.)	There is a general rule for computing square root, but this should be avoided since it is too complicated to remember. A better way to find square root is by dividing and averaging as illustrated below. Compute the square root of 178 ($\sqrt{178}$) to the nearest tenth. *Step 1.* Find a whole number which when squared will be closest to 178, but not more than 178. In this case $13^2 = 169$. *Step 2.* Divide this number (13) into the desired number (178). $\begin{array}{r} 13.69 \\ 13\overline{)178.00} \\ \underline{13} \\ 48 \\ \underline{39} \\ 90 \\ \underline{78} \\ 120 \\ \underline{117} \end{array}$ *Step 3.* The divisor and quotient are now averaged. $\frac{13 + 13.69}{2} = 13.345$ *Step 4.* Repeat the process using 13.345 as the new divisor. $\frac{178}{13.345} = 13.338$

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		Averaging: $\frac{13.338 + 13.345}{2} = 13.342$
		We obtain 13.342. Actually, this result is accurate to the nearest thousandth, but since only the nearest tenth was required we say $\sqrt{178} \approx 13.3$

```
          1 3. 3 4 1 6
        √178.00000000
          1
23      | 78
          69
263     | 900
          789
2664    |11100
         10656
26681   | 44400
          26681
266826  |1771900
         1600956
          170944
```

(At the left is the square root rule, which may be used to demonstrate the foregoing. However, it should not be taught as a method of extracting square roots because of the degree of difficulty.)

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II. Basic Structure of Algebra A. Terminology	The basic concept is that algebra is a language, and to use it, one must be familiar with its vocabulary.	
1. Variable	A variable is a place holder usually denoted by x, y, z, but often by a question mark, square, or triangle.	*Examples:* 3 + ? = 7 $\square + \Delta = 7$ (Both $\square$ and Δ are variables.) $2x + 5 = 9$ $x + y + z = 7$
2. Open sentence	An open sentence is a statement that may be either true or false depending upon the replacement for the variable.	The statements above are open sentences. (3 + ? = 7 is true when ? = 4, but false otherwise.)
3. Replacement set	A replacement set is a set of values that may be used in place of a variable.	Consider the open sentence "He is eligible to be President of the United States." A replacement set could be the set of U.S. citizens.

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4. Solution set	A solution set is a set of values from the replacement set which makes a statement true.	A solution set for the previous sentence could be the set of U.S. citizens over 35 years of age.
5. Algebraic phrase	An algebraic phrase is a group of words and/or symbols which have a meaning.	$3 + 5 \cdot x$ is an algebraic phrase and is (in fact) an open phrase, since it has a variable. $7 - (2 \div 3)$ is a closed phrase.
6. Monomial	A monomial is an algebraic phrase with only one term (no use of + or - signs).	x, $3y$, $7y^2$, $\frac{3x}{y}$ are all monomials.
7. Binomial	A binomial is an algebraic phrase with two terms, separated by a + or - sign.	$x + y$, $3x - 5z$, $x^2 - 5x$ are all binomials.
8. Polynomial	A polynomial is any algebraic phrase having more than one term.	$x^2 - 5x + 6$, $x + y$, $3a + b = 7c + d$ are all polynomials. (Note that a binomial is a polynomial, but that the converse is not necessarily true.)
9. Function	Mathematically, a function consists of three parts, a set called the domain, a set called the range, and a rule which associates to each element in the domain a unique element in the range. A function is a dependency relationship in an open sentence.	If we look at $y = 2x$, we see that y "depends" upon x, for whenever a value is inserted for x, the resulting y value is known (if the open sentence is to be true).
10. Exponent	An exponent is a shorthand for repeated multiplication.	4^3 means $4 \cdot 4 \cdot 4$. The exponent indicates how many times the base occurs as a factor.
11. Radical	The radical sign ($\sqrt{\ }$) indicates the reverse of an exponent.	For example, $\sqrt{9}$ means the square root of 9, or the number which when multiplied by itself will be 9. $\sqrt[3]{64}$ means the cube root of 64, or that number which when used as a factor 3 times, will be 64. For example, $5^2 = 25$ $\sqrt{25} = 5$ $4^3 = 64$ $\sqrt[3]{64} = 4$ $2^4 = 32$ $\sqrt[4]{32} = 2$

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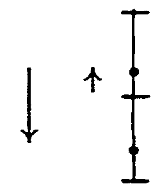
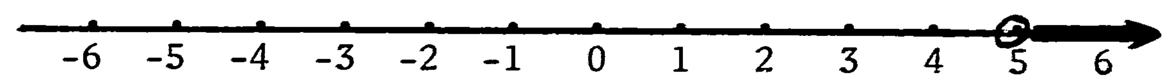
12. Quadratic	A quadratic phrase is a polynomial involving a single variable where the highest power is 2.	*Example:* x^2, $2x^2 + 3$, $x^2 - 5x + 6$ are quadratic phrases.
B. Algebraic operations	The fundamental algebraic operations are addition, subtraction, multiplication, division, and root-taking.	In approaching the fundamental operations it is necessary to translate English phrases into algebraic phrases. Key words must be reemphasized. English phrase — Algebraic phrase "The sum of x and y" — $x + y$ or $y + x$ "The difference between x and y" — $x - y$ or $y - x$ "The product of x and y" — $x \cdot y$ or $y \cdot x$ "The quotient of x and y" — $\frac{x}{y}$ or $\frac{y}{x}$
1. Order of operation	In all areas, multiplication and division take precedence over addition and subtraction.	$2 + 3 \cdot 4$ means: $2 + (3 \cdot 4) = 2 + 12 = 14$. $15 - 9 \div 3$ means: $15 - 3$ or 12.
2. Simplifying algebraic expressions	Following the laws of operation, simplify by adding or subtracting as indicated.	For illustrative purposes, use examples such as 3 apples + 2 pears which cannot be simplified, while 3 apples + 2 apples = 5 apples. In this concrete fashion, we see that it leads easily to $2x + 3x = 5x$, while $2x + 3y$ cannot be simplified. Similarly $x^2 + 3x + 5x - 7x + 7x^2 - 5$ is simplified to $8x^2 + x - 5$.
3. Evaluation of algebraic expressions	To evaluate an algebraic expression, substitute a value for a variable.	Make the illustrative problem concrete as follows: • If one apple costs 8¢, the cost of 7 apples is □. $7 \cdot 8¢ = 56¢$ • If $x = 8$, find the value of $7x$. $7x = 7 \cdot 8 = 56$ The use of symbols such as □ and Δ, facilitates learning the replacement process. $\square + 2 \cdot \Delta = ?$ If $\square = 7$ and $\Delta = 5$, we have $7 + 2 \cdot 5 = 7 + 10 = 17$.

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
		This is carried through to such examples as: To find $2x^2 + 7x$, when $x = 5$ $2x^2$ means $2 \cdot x \cdot x$ $2 \cdot 5 \cdot 5 + 7 \cdot 5 - 7 = 50 + 35 - 7 = 78$
4. Signed or directed numbers	Signed, or directed numbers, have two properties, magnitude and direction.	Again, the idea is to keep the presentation simple, with a minimum of rigor. The notion of a thermometer is a useful tool in approaching addition of signed numbers.
a. Addition	To add signed numbers with like signs, add and retain the sign of each; with unlike signs, subtract and retain the sign of the number with the larger magnitude.	 If the temperature is $^{+}5°$ and cools 15°, we may write $[^{+}5 + (^{-}15)]$ and the algebraic sum is $^{-}10°$. Subtraction, being an inverse operation, should be considered a reversal of direction on the number line.
b. Multiplication	When multiplying two signed numbers, multiply their magnitudes and prefix the product with "plus" if the original signs were the same and "minus" if they were different.	Multiplication, since it may be thought of as continuous addition, can be approached as follows: $+3 \cdot -5$ is equivalent to saying -5 or adding -5 -5 -5 ? -5 to -5 to -5, and the product logically is -15 when the number line concept is used. To multiply $-3 \cdot -5$, we may be forced to assume the result as +15, but here is one means of justification. *Suppose* money saved is "+" money spent is "−" time future is "+" time past is "-" Now logically, if one is spending (-) \$5 per week, 3 weeks ago (-), he was \$15 better off or $^{+}15$. This does not constitute a proof, but merely provides some justification for the idea that the product of two negative numbers is positive.

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
C. Algebraic sentences 1. Simple equations	The solution of first degree equations and inequalities includes the estimation and checking of answers.	The basic approach to equation solving is by use of fundamental axioms. Consider 3x + 2 = 17. The solution is best arrived at by the use of inverse operations. In this instance, the indicated operation is addition, +, and its inverse is subtraction. Hence, subtract 2 from each member of the equation, preserving the balance. This leaves 3x = 15. Now the indicated operation is multiplication, and its inverse is division. Thus, we divide each member by 3 obtaining a solution {5}. No solution is complete without a check. In this instance, replace the variable x with 5 to verify accuracy.
2. Simple inequalities		If we change the equation to an inequality, 3x + 2 > 17, the same principles apply. By subtracting and dividing, we arrive at a simplified sentence, x > 5. This is best illustrated graphically on a number line. The number line would show:  The solution set is that collection of points indicated by the line. The open circle at 5 indicates that that point is *not* to be included in the solution set.
3. No solution	There are equations which do not have solutions.	Sentences joined by "and" or "or" lead to solution of higher degree equations. There are simple equations which have no solution (whose solutions must be the null set), and the student should be aware of this possibility. *Examples:* x = x + 2 (There is no value of *x* that will make this true.) y = y + 5 (There is no value of *y* that will make this true.)

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		Students should have an opportunity to solve a variety of simple equations such as the following: $x + 5 = 7$ $\frac{x}{2} = \frac{1}{5}$ $a - 4 = 6$ $2m + 11 = 33$ $\frac{b}{3} = 12$ $8y - 17 = 29$
D. Verbal problems 1. Key words	Common English words have algebraic equivalents which facilitate the formation of equations taken from verbal problems.	In approaching verbal problems, greatest stress must be placed on translation of key words. Among these are:

Key Words	Algebraic Translation
is, are, was, were, shall, will	=
of (as in $\frac{1}{2}$ of), product	x
minus, difference, less than, diminished by, decreased by	-
sum, increased by, more than	+
divided by, quotient of, ratio of	÷

2. Solution test	All solutions should be examined for degree of reasonableness.	It would be well to start with several instances of straight translation such as "4 more than a certain number" and "the quotient of two numbers." After setting up an equation based on a verbal problem and solving the problem, the task is to examine the solution for feasibility. An answer must be reasonable. The solution should be checked, not in the equation formed, but by referring back to the original question.

Example:

Four *times* the *sum* of a *number* and 3 *is* the same as 5 *times* the *difference* between the number and 6.

number	x
times	•
sum	+
is	=
times	•
difference	−

Caution: Four times the sum means 4•(? + ?) *not* 4•? + ?. The sum is a distinct entity.

4•(x + 3) = 5•(x - 6)

(1) 4x + 12 = 5x - 30 (applying the distributive law)
  -4x = -4x

(2) + 12 = x - 30 (subtracting 4x from each member)
  + 30 = + 30

(3) 42 = x (adding 30 to each side)

(4) The check consists of returning to the original problem and ensuring that our answer is consistent with the conditions provided.

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E. Algebraic techniques 1. Multiplication	Repeated use of the distributive law applies to the multiplication of algebraic expressions.	Monomial times binomial a•(b + c) = ab + ac Binomial times binomial (a + b)•(c + d) = (a + b)•c + (a + b)•d = ac + bc + ad + bd These utilize the distributive law twice to arrive at the product.
2. Special products	Certain special products occur frequently and are treated separately to lead to factoring.	The special products to be considered are threefold. First, the square of a binomial. It is best to use numerical examples first to lead to the generalization.

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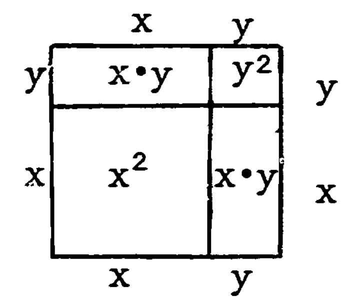
		$(x + 3)\cdot(x + 3) = (x + 3)\cdot x + (x + 3)\cdot 3 = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$ leading to $(x + y)\cdot(x + y) = (x + y)\cdot(x) + (x + y)\cdot y = x^2 + xy + xy + y^2 = x^2 + 2\cdot xy + y^2$
	To square a binomial, square the first term, add twice the product of the first and second terms, and add the square of the second term.	 An interpretation lends credence to the algebraic derivation. As indicated, the area of the square is $(x + y)^2$ while the sum of the areas of the components is $x^2 + 2\cdot xy + y^2$.
		It is clear that the length of each side of the large square is x + y. The instructor should recognize that the area concept has not yet been developed, and should introduce the elementary concept that the area of a rectangle is equal to the product of its base and altitude.
	The sum and difference of two numbers when multiplied resolve to the square of the first minus the square of the second.	Illustrate with numerical examples prior to generalizing. $(x + 3)\cdot(x - 3) = (x + 3)\cdot x + (x + 3)\cdot(-3) = x^2 + 3x + (-3x)\cdot(-9) = x^2 - 9$ $(x + y)\cdot(x - y) = (x + y)\cdot x + (x + y)\cdot(-y) = x^2 + xy - xy - y^2 = x^2 - y^2$
3. Factoring	Factoring binomial and trinomial expressions hinges on the common monomial factor and the distributive law.	Factoring is treated simply as the opposite of multiplication. The concept of factoring is first taught with the natural numbers ($420 = 2^2\cdot 3\cdot 5\cdot 7$, the right side indicating all the prime factors of 420). With this background, the transition to algebraic phrases is simplified. $a\cdot(b + c) = ab + ac$ is the distributive law, and $ab + ac = a\cdot(b + c)$ is the same statement.

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		Examples of this type tend to reinforce the concept of the monomial factor. Binomial factors are approached similarly. $(x + 1)\cdot(x + 3) = (x + 1)\cdot x + (x + 1)\cdot 3$ $= x^2 + 1x + 3x + 3$ $= x^2 + 4x + 3$ Reverse the process and $x^2 + 4x + 3 = (x + 1)\cdot(x + 3)$.
	Factoring is performed by a sequential-step procedure.	*Note:* The technique for factoring should be confined to simple illustrations where the lead coefficient is one. The steps are provided in the following: $x^2 - x - 30$ (1) $= (\quad)\cdot(\quad)$ — *Step 1* is to simply write two parentheses. (2) $= (x \quad)\cdot(x \quad)$ — *Step 2* indicates the factors of x^2. (3) $= (x \quad 5)\cdot(x \quad 6)$ — *Step 3* is the vital step wherein the factors of the final term (-30) are tried. (4) $= (x + 5)\cdot(x - 6)$ — *Step 4*. Since the algebraic sign of the final term is negative, the two signs in the parentheses must be different. (5) $= (x + 5)\cdot x + (x + 5)\cdot(-6)$ $= x^2 + 5x - 6x - 30$ $= x^2 - x - 30$ — *Step 5*. No factoring exercise is truly complete until the multiplication has been carried out to verify accuracy.
4. The quadratic equation	To solve a quadratic equation, factor, setting each factor equal to zero, and solve by linear methods.	Initial procedure for solving quadratic equations involves the equation $A\cdot B = 0$. The student must first realize that this is equivalent to saying, "Either

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		A = 0 *or* B = 0 *or* both A and B = 0." It is a simple step to solve the uncomplicated quadratics written in the form $x^2 + px + q = 0$. Solve $x^2 - 2x - 63 = 0$. (1) $(x - 9)\cdot(x + 7) = 0$ (factoring the left member) (2) $x - 9 = 0$, or $x + 7 = 0$ (using the "or" principle) (3) $x = 9$, or $x = -7$ (simplifying step 2 by addition and subtraction) (4) $\{9, -7\}$ (the proper way to designate a solution set) (5) $9^2 - 2(9) - 63 = 0$ $81 - 18 - 63 = 0$ $0 = 0$ (checking each answer by substitution) $(-7)^2 - 2(-7) - 63 = 0$ $49 + 14 - 63 = 0$ $0 = 0$
5. Fractions	The fundamental operations on algebraic fractions closely parallel the arithmetic method.	
a. Multiplication	To multiply two algebraic fractions, multiply their numerators and multiply their denominators.	Illustrate with numerical examples and generalize to $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.
b. Addition or subtraction	To add or subtract, first find the least common denominator and apply the arithmetic technique.	Start again with numerical examples such as $\frac{3}{4} \pm \frac{5}{7}$ and proceed to $\frac{a}{b} \pm \frac{c}{d}$ where the common denominator is bd. Thus, we obtain $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$. Avoid involved fractions leading to sums and differences with quadratic denominators.

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c. Division	To divide one algebraic fraction by a second, invert the divisor and multiply.	Once again, the use of many arithmetic examples without variables facilitates mastery of the concept. $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} x \frac{d}{c} = \frac{ad}{bc}$$
d. Verbal problems	The solution of verbal problems involving fractions hinges on close adherence to format and careful attention to key words.	Algebraic problems involving fractions should be kept rather elementary. Certain business problems can best be handled algebraically, as illustrated in number 2 below.

(1) A certain fraction has a numerator 12 less than its denominator and *is* equivalent to $\frac{1}{5}$. Find the fraction.

Steps: Let x = the denominator

$x - 12$ = the numerator

$\frac{x - 12}{x}$ = the fraction

$\frac{x - 12}{x} = \frac{1}{5}$ (from the statement)

$5 \cdot (x - 12) = x \cdot 1$ (using the proportion rule)

$5x - 60 = x$

$5x = x + 60$

$4x = 60$ (using techniques for solving simple equations)

$x = 15$

$x - 12 = 3$

$\frac{3}{15}$ is the fraction. To check, observe that the numerator *is* less than the denominator and that the fraction is equivalent to $\frac{1}{5}$.

(2) An article sold for $20.70 after it had been discounted 10%. What was the original list price?

Steps: Let x = the original price

$\frac{1}{10}$ = discount rate

original price - discount = selling price

$x - \frac{1}{10}x = 20.70$

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		a: $x - \frac{1}{10}x = \frac{9}{10}x$ $\frac{9}{10}x = 20.70$ $x = (20.70)(\frac{10}{9})$ $x = \frac{207}{9} = \$23$
		b: $x - \frac{1}{10}x = 20.70$ $10\cdot(x - \frac{1}{10}x) = (10)\cdot(20.70)$ $10x - x = 207$ $9x = 207$ $x = \$23$ (Solution b uses the least common multiple.)
III. Arithmetic from the Algebraic Point of View		
A. Ratio	A ratio is a method of comparing two quantities by division; ratio and fraction are synonymous.	"Line a is 3 inches longer than b," is comparing by subtraction. "Line a is $\frac{2}{3}$ as long as line b" is really comparing by division and is a ratio. The best examples are measurements such as the ratio of 2 ounces to 3 pounds, or the ratio of 18 inches to 2 yards. It should be made clear that no unit of measurement is ever attached to a ratio.
B. Proportion	A proportion is the statement of two equal ratios.	$\frac{a}{b} = \frac{c}{d}$ is a proportion with the terms numbered consecutively (a is the first term, b the second, c the third, and d the fourth). By definition a and d are called the extremes and b and c (the second and third terms) are called the means. A fundamental rule should be developed justifying the fact that, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$, or in any proportion, the product of the means is equal to the product of the extremes.

| --- | --- | --- |
		$\frac{a}{b} = \frac{c}{d}$; $\frac{a}{\not b}\cdot(\not b d) = \frac{c}{\not d}\cdot(b\not d)$ (multiplication rule) ad = bc Among the many examples of the use of proportions, some of the best are found in map reading and scale drawings. *Example 1:* On a map, 1 inch represents 5 miles. What distance is represented by $3\frac{3}{4}$ inches? *Solution:* Let x = the distance $\frac{1''}{3\frac{3}{4}''} = \frac{5 \text{ mi.}}{x \text{ mi.}}$ Using the proportion rule, "The product of the means equals the product of the extremes," we obtain $x = 5 \cdot 3\frac{3}{4} = 18\frac{3}{4}$ miles. *Example 2:* A recipe for a dozen cupcakes requires 2 eggs. To make 66 cupcakes, how many eggs are necessary? *Solution:* Let x = the number of eggs required $\frac{2}{x} = \frac{12}{66}$ (setting up the proportion) 12x = 132 (product of means equals product of extremes) x = 11 eggs
C. Variation	The variation concept is a linear (straight line) relationship between two variables.	The best way to teach variation is by a practical illustration rather than by the abstract definition.
1. Direct variation	Direct variation between two variables x and y states that as y increases in value, so does x, and moreover by a proportionate amount.	Consider the distance, rate, time concept where $d = r\cdot t$. If an automobile is traveling at a rate of 30 m.p.h., it is obvious that the more time that elapses, the greater the distance covered. The relationship is $d = 30\cdot t$ and 30 is the constant of variation.

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2. Inverse variation	Inverse variation between two variables states that an increase in one variable is associated with a linear decrease in the other.	Similarly, if a specified distance is to be covered, say 100 miles, the relationship between rate and time is inverse, for the faster one travels, the smaller the amount of time required to cover the distance. Then, $100 = r \cdot t$, or $r = \frac{100}{t}$.
	Direct: $y = kx$, where k is called the constant of variation.	Water pressure varies directly as the depth of the water. If the pressure at 10 feet is 4.3 pounds per square inch, then the pressure at 27 feet would be how great?
	Indirect: $y = \frac{k}{x}$, where k again is called the constant of variation.	

Solution: Let p = pressure
d = depth
then $p = kd$
$4.3 = k \cdot 10$
$k = .43$ lbs./sq. in.
since $p = kd$
$p = .43d$
$p = (.43) \cdot (27)$
$p = 11.61$ lbs./sq. in.

Alternative solution:
P_1 = 1st pressure
P_2 = 2nd pressure
D_1 = 1st depth
D_2 = 2nd depth

$$\frac{P_1}{D_1} = \frac{P_2}{D_2}$$

$$\frac{4.3}{10} = \frac{P_2}{27}$$

$P_2 = 11.61$ lbs./sq. in.

Further illustrations

If it will take 4 men 5 days to do a specific job, how long will it take 9 men to do the job? (Assume a linear relationship.)

Solution: Let m = men working
d = days used
k = constant of variation
then $d = \frac{k}{m}$
$5 = \frac{k}{4}$
$k = 20$

$$\text{now } d = \frac{20}{m}$$

$$d = \frac{20}{9} = 2\tfrac{2}{9} \text{ days.}$$

An alternative solution is the use of a proportion:

$$\frac{4 \text{ men}}{9 \text{ men}} = \frac{d \text{ days}}{5 \text{ days}}$$

$$9d = 20, \text{ and } d = 2\tfrac{2}{9} \text{ days.}$$

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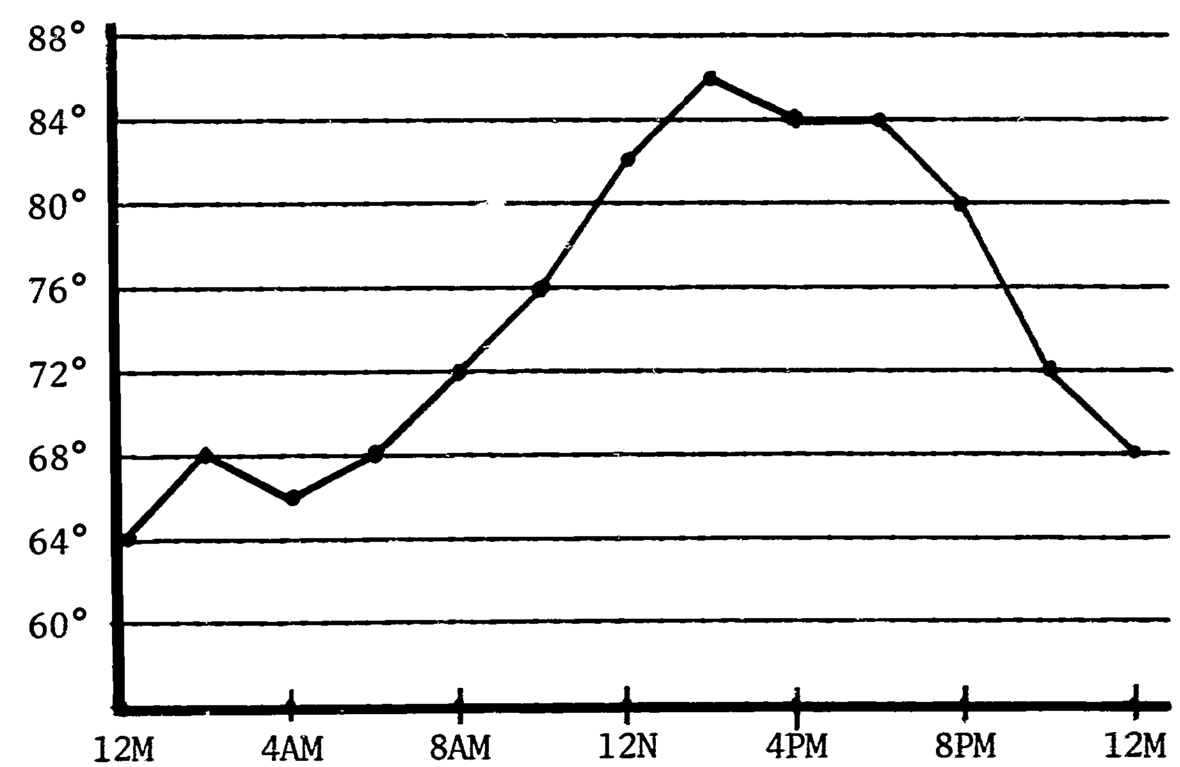
D. Graphs	A graph is a visual portrayal of data.	The main objective is to be able to read and interpret each kind, to apply the results to problem situations, and finally, to construct each variety.
1. Bar graph	The bar graph consists of vertical or horizontal bars of varying length to illustrate discrete data.	The bar graph has as its principal function the picturing of discrete data, while the line graph illustrates data which exists on a continuum. For example, temperature deviation at hourly intervals lend itself to a line interpretation, while automobile production in several countries can be shown best on a bar graph. On the basis of the line graph, certain statistical concepts can be illustrated.
2. Line graph	The line graph is a broken line designed to display continuous data, plotting one variable against a second.	
3. Circle graph	The circle graph divides a circle into segments proportional to parts of a whole.	



TOPICAL OUTLINE CONCEPTS AND UNDERSTANDINGS SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
		Questions to be asked: • What was the hottest temperature recorded, and at what time did it occur? (2 p.m.; 86°) • During what two-hour period was the temperature most nearly constant? (4 p.m. to 6 p.m.) • At what two times of the day was the temperature 76°? (10 a.m. and 9 p.m.)
IV. Nonmetric Geometry	Nonmetric geometry does not depend upon measurement, but involves the basic structure of deductive logic.	Although it is not within the scope of this course to develop demonstrative geometric techniques, an examination of the foundations of geometry is vital to a clear approach to problem solving.
A. Undefined terms	The undefined terms of geometry are point, line, and plane.	It should be pointed out that any structure has to have a starting point, and undefined terms provide us with such a basis.
B. Definitions	Definitions are constructed from undefined words, common nontechnical English words, and previously defined words.	
1. Line segment	A line segment is a portion of a line bounded by two points.	Intuitive discussion of types of lines such as straight, curved, broken, horizontal, vertical, and oblique should be introduced at this point.
2. Ray	A ray is a portion of a line extending from one point.	
3. Intersecting lines	Intersecting lines are lines which have a common point.	
4. Parallel lines	Two lines are parallel if they are in the same plane, have no common point, and do not meet no matter how far extended.	Here we might mention skew lines, lines neither intersecting nor parallel since they are not contained in the same plane.

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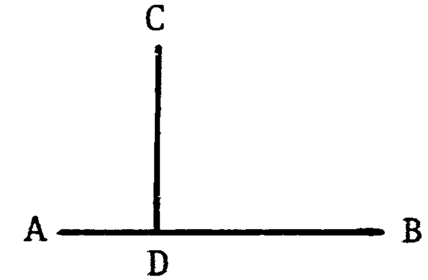
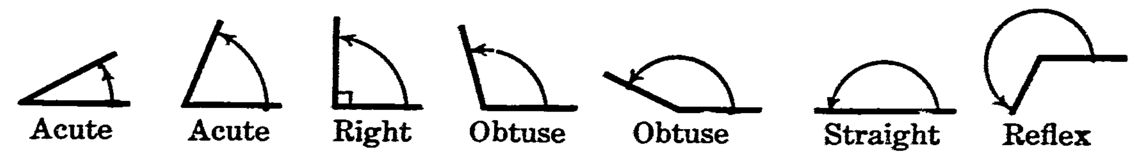
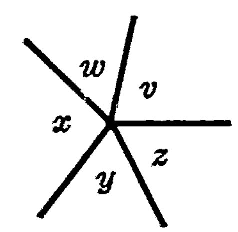
5. Perpendicular lines	Two lines are perpendicular if they form equal, adjacent angles with each other. (Alternatively, two lines are perpendicular if they meet to form a right angle.)	Angles ADC and BDC are equal and adjacent, hence lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are perpendicular (symbolized $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$). 
6. Angles a. Right	A right angle is one of the adjacent angles formed by perpendicular lines.	
b. Acute	An acute angle is less than a right angle.	
c. Straight	A straight angle is an angle formed by two rays extending in opposite directions from a given point.	Ask students to estimate the number of degrees in each angle in the figure at the right. 
d. Obtuse	An obtuse angle is greater than a right angle and less than a straight angle.	
e. Vertical	Vertical angles are the opposite angles formed by two intersecting straight lines.	
f. Complementary	Two angles are complementary if their sum is a right angle (90°).	Functional definitions are the most meaningful; that is, defining each in sequence and in a context.
g. Supplementary	Two angles are supplementary if their sum is a straight angle (180°).	Supplementary and complementary angles lend themselves to interesting algebraic applications. *Example:* Two angles are complementary and one is two less than three times the other. Find the angles. *Solution:* Let x = the first angle let 90 - x = the second angle x = 3 (90 - x) - 2 x = 270 - 3x - 2 x = 268 - 3x

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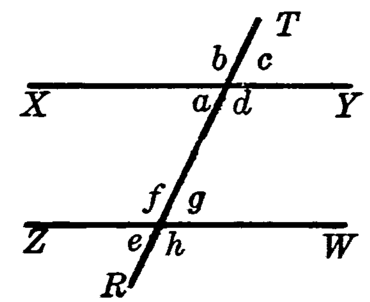
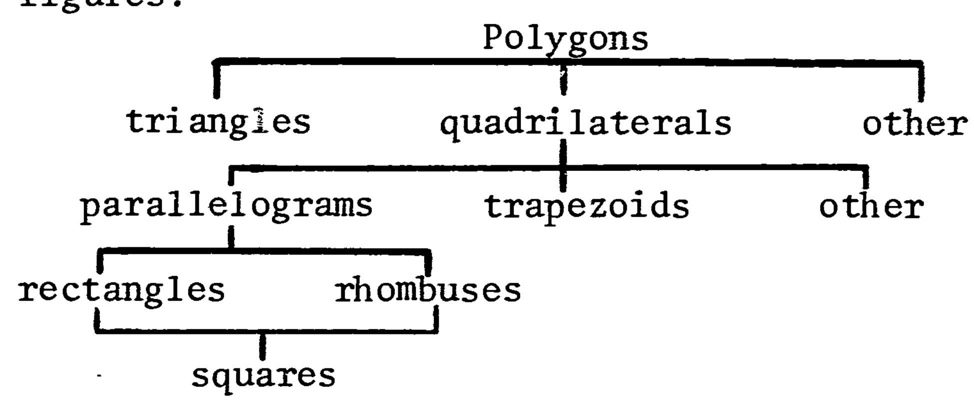
		$4x = 268$ $x = 67°$ $90 - x = 23°$
C. Geometric relationships 1. Parallel lines a. Transversal	A transversal is a line which intersects a system of parallel lines.	
b. Principles	When two parallel lines are cut by a transversal, the alternate interior angles are equal. When two parallel lines are cut by a transversal, the corresponding angles are equal.	At this point consider the relationships involving parallel lines cut by a transversal.  $\overleftrightarrow{XY} \parallel \overleftrightarrow{ZW}$. $\overleftrightarrow{TR}$ is a transversal.
	When two parallel lines are cut by a transversal, interior angles on the same side of a transversal are supplementary.	Angles *b*, *c*, *e*, *h* are exterior angles. Angles *a*, *d*, *f*, *g* are interior angles. Angles *d* and *f*, and *a* and *g* are alternate *and* interior angles. Angles *b* and *f*, *a* and *e*, *c* and *g*, *d* and *h* are corresponding angles.
2. Simple closed figures a. Polygon	A polygon is a closed broken line.	A diagram similar to the one that follows, will assist in the organization and classification of geometric figures.
b. Triangle	A triangle is a polygon having three sides.	
1. Acute	An acute triangle is one having three acute angles.	
2. Obtuse	An obtuse triangle is a triangle having one obtuse angle.	

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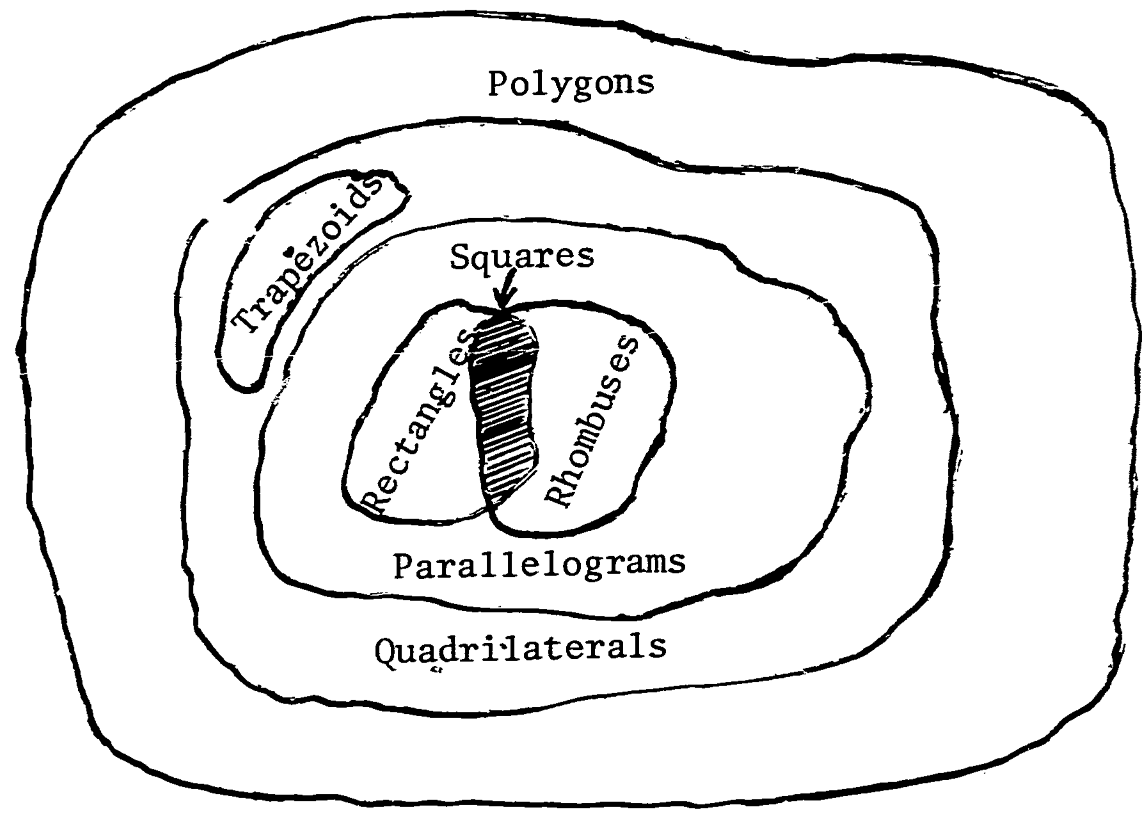
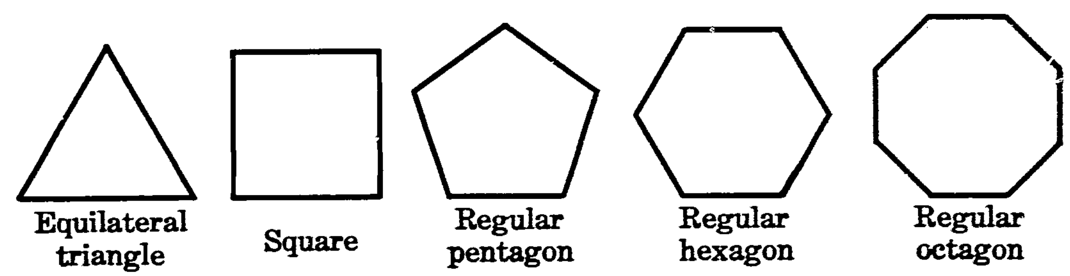
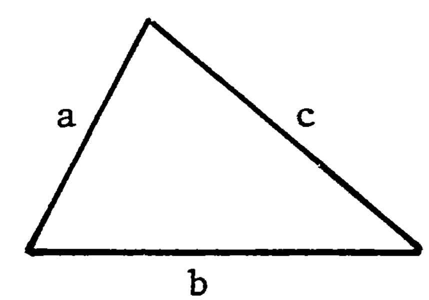
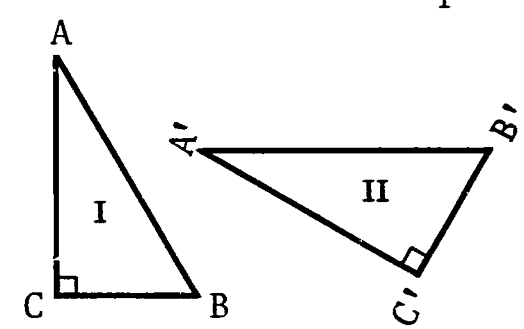
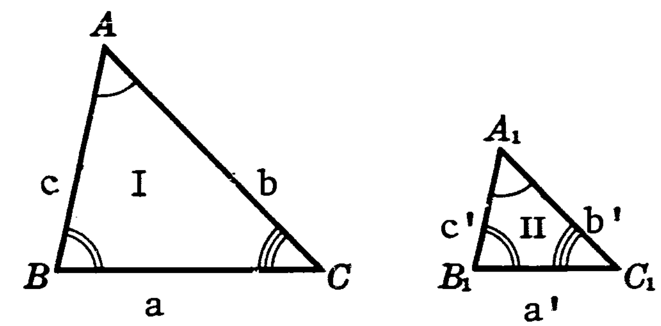
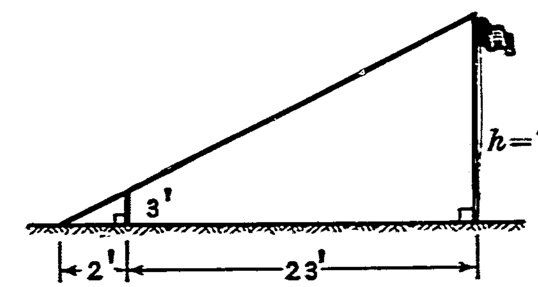
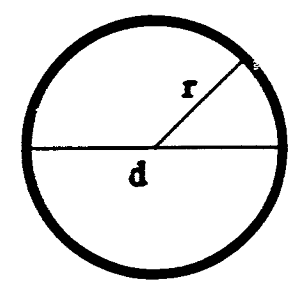
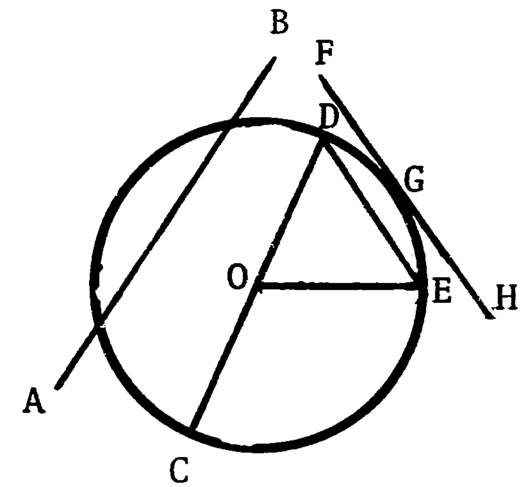
3. Right	A right triangle has one right angle.	The diagram below will serve to portray visually the relationships among polygons.
4. Isosceles	An isosceles triangle has two equal sides, and two equal angles.	
5. Scalene	A scalene triangle has no two sides the same length.	
6. Equilateral	An equilateral triangle has all three sides and angles equal.	
c. Quadrilateral	A quadrilateral is a four-sided polygon.	
1. Parallelogram	A parallelogram is a quadrilateral with opposite sides parallel.	
2. Rectangle	A rectangle is a parallelogram with four right angles.	
3. Square	A square is a rectangle having all sides equal.	
4. Rhombus	A rhombus is a parallelogram having all sides equal.	
5. Trapezoid	A trapezoid is a quadrilateral having only two sides parallel.	
d. Pentagon	A pentagon is a polygon having five equal sides.	
e. Hexagon	A hexagon is a polygon having six equal sides.	
f. Octagon	A octagon is a polygon having eight equal sides.	A polygon having equal sides is a regular polygon.
g. Decagon	A decagon is a polygon having ten equal sides.	Have students identify the common regular polygons shown above.

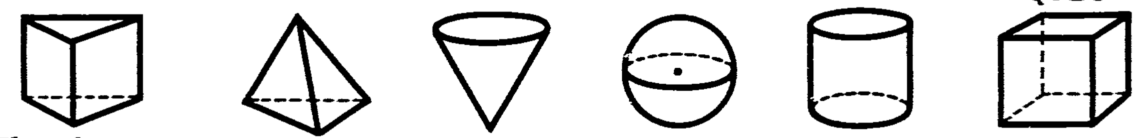
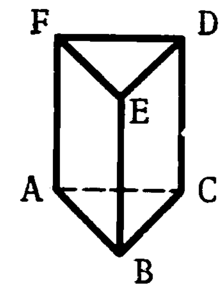
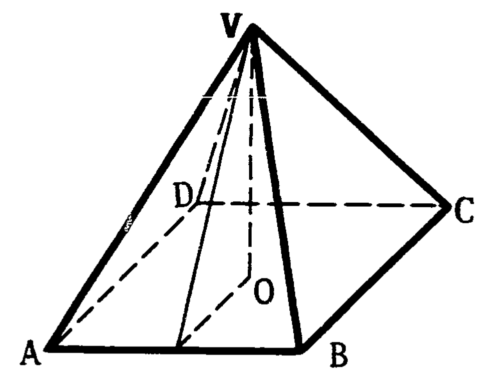
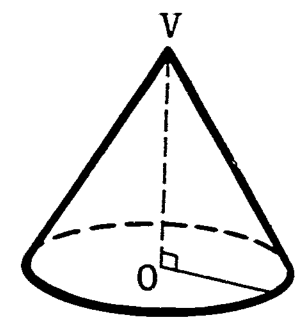
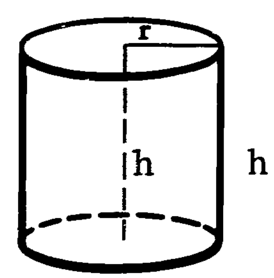


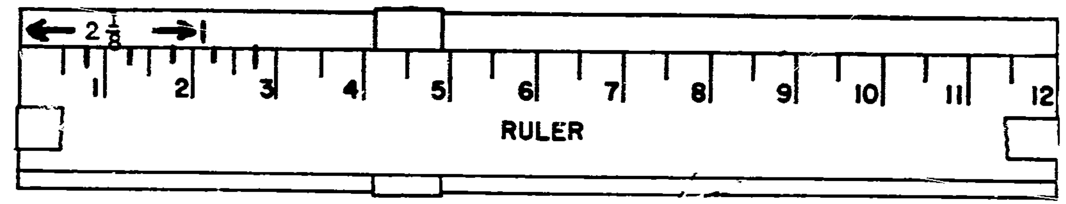
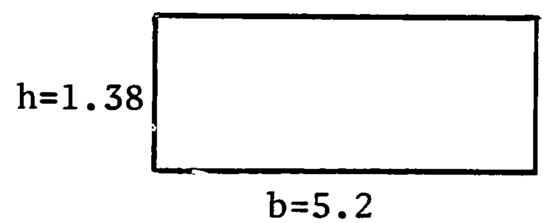


TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
3. Congruent triangles	Congruent figures are figures alike in size and shape; they can be made to coincide.	Congruence should be intuitively described as the ability to make figures coincide. Examples might start with circles, which are congruent if their radii are equal, or squares which are congruent if a side of one is the same length as a side of the other.
	Two triangles are congruent if:	Construction of a triangle, given the requirements of a, b, and c and pointing out that any other triangle constructed from the same data would be congruent to the original, is the best technique to reinforce the understanding.
a. Side-angle-side test	• Two sides and the included angle of one are equal to two sides and the included angle of the second.	Given a________ as the sides of a triangle b________ c________
b. Angle-side-angle test	• Two angles and the included side of one triangle are equal to the corresponding parts of the second.	
c. Side-side-side test	• Three sides of one triangle are equal to three sides of the second.	
d. Corresponding parts	If two triangles are congruent, their corresponding parts are equal.	At this point it would be appropriate to measure the angles of several triangles to arrive empirically at the relationship for the sum of the angles (180°).  $\Delta ABC \cong \Delta A'B'C'$ If $\angle C$ and $\angle C'$ are both right angles, AB and A'B' are corresponding, since they are opposite equal angles.
4. Similar triangles a. Definition	Similar triangles are triangles with all three angles of one equal to all three angles of the other and corresponding sides proportional.	The best approach is an illustration which demonstrates the fact that two similar figures are alike in shape, but not necessarily in size.

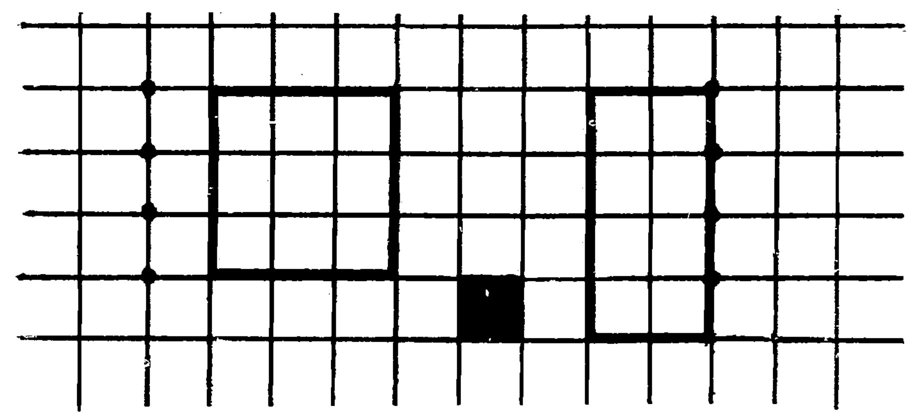
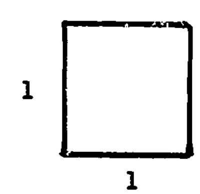
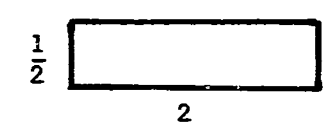
TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
b. Corresponding sides proportional	To say that corresponding sides are proportional means that they have the same ratio.	 If ΔI is similar to ΔII, then $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$.
c. Uses of similar triangles	One principal use of similar triangles is indirect measurement.	*Example:* A flagpole casts a shadow which measures 25 feet. At the same time a yardstick has a shadow 2 feet long. How tall is the flagpole?  (1) These represent similar triangles since all the corresponding angles are equal. (2) The proportionality concept yields the equation $\frac{?}{3} = \frac{25}{2}$; $2 \cdot ? = 25 \cdot 3$; $2 \cdot ? = 75$; $? = 37\frac{1}{2}$ feet.
5. Circles a. Definition	A circle is a plane, closed curve, all points on which are equidistant from a given point. The circumference and not the area inside constitutes the circle.	
b. Special line segments and lines	A radius is a line segment joining the center to any point on the circle.	These concepts are best introduced nonrigorously by means of a diagram. $\overline{OE}$ is a *radius*, drawn from the center to any point in the circumference. $\overline{OD}$ and $\overline{OC}$ are also radii.

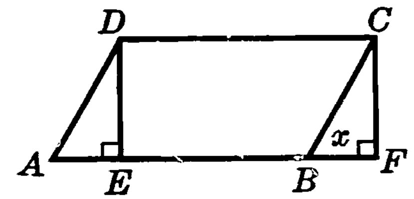
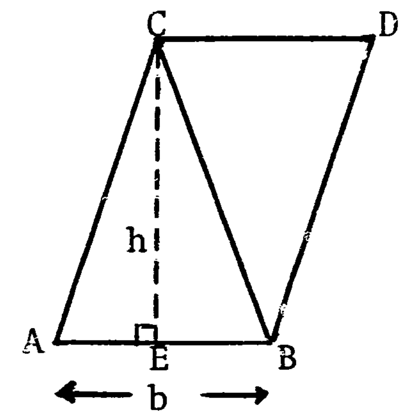
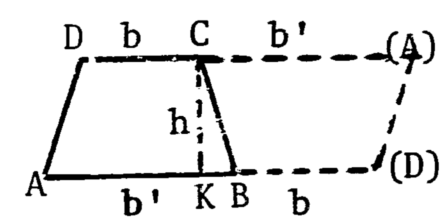
TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
	A chord joins any two points on the circle.	$\overline{DE}$ is a *chord*, a line segment joining any two points in the circumference.
	A diameter is a chord which passes through the center.	$\overline{CD}$ is also a chord, and since it passes through the center, it is a *diameter*.
	A secant is a line intersecting the circle in two points.	 $\overleftrightarrow{AB}$ is a *secant*, a line intersecting the circle in two points. *CE*, the curved portion, is a minor arc, while *CDG* is a major arc. *CD* is a *semicircle*.
	A tangent to a circle has exactly one point in common with the circle.	$\overleftrightarrow{FH}$ is a *tangent* to the circle at point *G*.
c. Central angle	A central angle is formed by two radii.	Angles *DOE* and *COE* are central angles.
d. Arcs	An arc of a circle is a portion of the circumference bounded by two points, and has the same measure as the central angle intercepting it.	In the above diagram, *CE* is an arc.
	A minor arc is less than a semicircle, and a major arc is greater than a semicircle.	In the same diagram, *CE* is a minor arc and *CDG* is a major arc (clockwise).
e. Concentric	Two concentric circles are circles with a common center.	
f. Circumference	The circumference of a circle is given by the formula $C = \pi d$, where C is the circumference, π is a constant approximately equal to $3\frac{1}{7}$ or 3.14, and d is the diameter of the circle.	Since circles are all similar figures, corresponding parts must have the same ratio. Thus, $\frac{C_1}{d_1} = \frac{C_2}{d_2} = \frac{C_3}{d_3}$ and the ratio of the circumference to diameter is the constant π (Greek letter). $\frac{C}{d} = \pi$, or $C = \pi d$, remembering that the exact value of π cannot be determined.

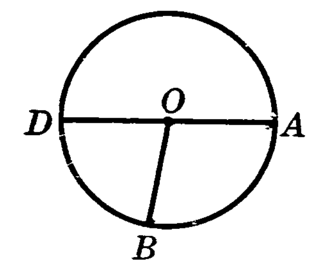
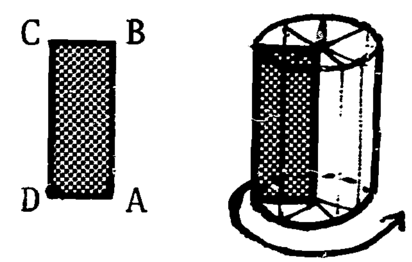
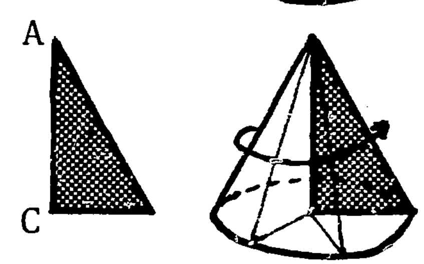
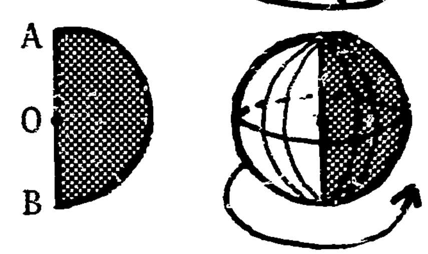
TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
6. Solid geometric figures		
a. Polyhedron	A polyhedron is a solid geometric figure with faces which are polygons.	The best way to consider solid geometric figures is visually. A diagram or even a room can be used for illustration.
b. Prism	A prism is a special polyhedron with lateral edges parallel, and with two congruent and parallel bases.	 This figure is a polyhedron, but also a triangular prism (triangular since its base is a triangle). Δ ABC and Δ FED are bases of the prism, and $\overline{CD}$ is an altitude.
c. Pyramid, cone	Pyramids and cones are similar in that both have a primary vertex, with elements to a geometric figure called the base. In the cone the base is a circle, and in the pyramid it may be any polygon.	 This figure is a rectangular based pyramid, with ABCD its base and $\overline{OV}$ as its altitude. (ABCD is a rectangle.)  This figure is a cone with O as its base and $\overline{OV}$ as its altitude.
d. Cylinder	A cylinder is a solid with two parallel bases, each a congruent circle, having elements joining the two.	 This figure is a cylinder with congruent circles for bases and altitude, the perpendicular distance between the bases.
7. Units of measurement (metric and English systems)	Two important systems of measurement are the metric and English systems.	

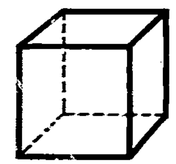
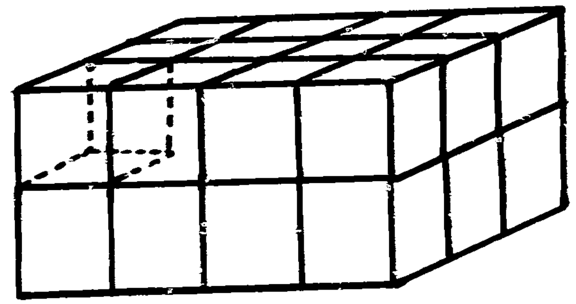
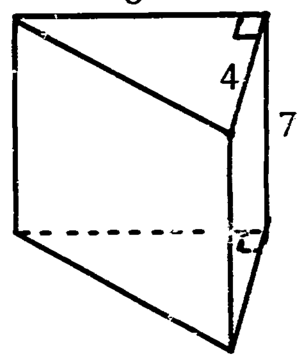
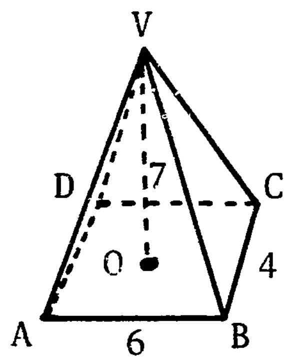
TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
a. Precision	Precision is the degree of closeness of estimation of measurement.	For example, if a line is found to measure $5\frac{1}{4}$", we mean that it is closer to $5\frac{1}{4}$" than to 5" or $5\frac{1}{2}$". In this case, the precision is $\frac{1}{4}$". If a measure is stated as 6.23', the meaning is that our real measure is between 6.225' and 6.235' and our precision is .01.
		
		From the ruler above, the arrows show a measure of $2\frac{1}{8}$". This is an estimation, meaning that the true measure is between 2" and $2\frac{1}{4}$", but closer to $2\frac{1}{8}$" than either. The degree of precision is $\frac{1}{8}$".
b. Accuracy	Accuracy is determined by the significant figures, neglecting zeroes at the left-hand end and occasionally at the right-hand end, but not when a result of measurement.	*Examples:* (1) The width of a thin sheet of steel .000375 inches has 3 significant figures. (2) The distance to the sun is 93,000,000 miles. This has 2 significant figures. (3) An area is 67.300 sq. ft. This contains 5 significant figures, since the number is a result of measurement.
	When multiplying approximate numbers, the product is as accurate as the least accurate factor.	An illustration of accuracy is to be found in an area problem. (See page 46 for concept of area.)
		 Since the base and altitude are each approximate numbers with the least accurate having 2 significant digits, the area can have no greater accuracy.
		$A = bh$, $A = (1.38)\cdot(5.2) = 7.176$ sq. in., but since h may be as low as 1.375 and as high as 1.384, and b is bounded by 5.15 and 5.24, our limits on area are 7.08125 and 7.25216. It is easy to see that 7.176 sq. in. is not accurate and should be rounded off to 7.2 sq. in.

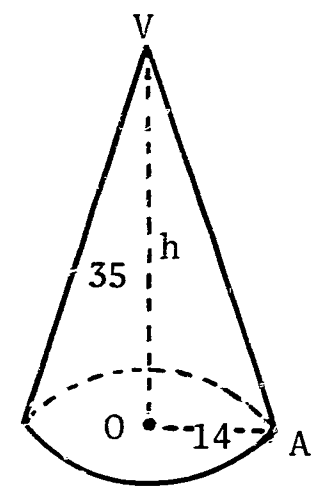
TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
c. English system	The English system of measurement is the one in most common use in the United States. • The basic units of linear measure are inch, foot, yard, rod, and mile. • The basic units of weight are ounce, pound, and ton. • The basic units of liquid measure are pint, quart, and gallon.	The instructor should review at this point the basic units in the English system. Use a series of examples to reinforce the necessary understandings.
d. Metric system	The metric system is based on the use of decimals and standard units of measure. • The standard unit of linear measure is the meter. • The standard unit of weight in the gram. • The standard unit of liquid measure is the liter.	The prefixes *milli* and *centi* refer to 1,000 and 100 respectively. Thus, the millimeter is easy to remember as $\frac{1}{1000}$ of a meter and a centimeter is $\frac{1}{100}$ of a meter. *Kilo* means 1,000. The students should be given exercises in reading and converting units within the metric system.
e. Metric-English conversions	Conversion between the metric and English systems is accomplished by the use of relationships. • One inch = 2.54 centimeters. • One mile = 1.609 kilometers. • One pound = 0.373 kilograms. • One quart = 0.946 liters.	The memorization of extensive conversion tables is not necessary. However, the student should remember basic conversion relationships. A proportion is an excellent method of converting one system to another. *Example:* An anti-aircraft cannon has an internal barrel diameter of 88 mm. Compute the number of inches in the diameter. (1) 88 mm. = 8.8 cm. since 1 cm. = 10 mm. (2) $\frac{x}{1} = \frac{8.8}{2.54}$ The number of inches in the barrel x is to 1 (the standard unit) as 8.8 (the number of cm.) is to 2.54, the number of cm. in 1 in. (3) $x = \frac{8.8}{2.54} = 3.46$ inches.

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
8. Areas a. Meaning b. Plane figures	Area, in a nonrigorous fashion, is expressed in terms of a square unit.	Unit is taken to mean any linear measure such as inch, foot, yard, mile, centimeter, meter, or kilometer.
(1) Rectangle	• The area of a rectangle is given by the formula A = bh, where A is the area given in square units, b the base measured in linear units, and h the altitude similarly measured. • The square is a special case where base and altitude are equal, hence the formula $A = s^2$, where s is the length of a side.	The square below is a rectangle where the base and altitude are both the same. This one is 3 units on a side with area = 3 x 3 = 9 square units. The rectangle below contains 4 rows and 2 columns and, by counting, 8 square units. Hence, A = bh, where b = base and h = altitude. 
	There is a difference between a unit square and a square unit.	Understanding the meaning of a unit is vital to an appreciation of metric geometry. The square unit, for example, can be illustrated by floor tiles (usually a 9-inch square) together with a discussion involving a *covering* concept. It should be made clear that there is a difference between a unit square and a square unit. Unit square:  Square unit:  In contrast, some such configuration as , or even  could possibly enclose one square unit of area.

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
(2) Parallelogram	• The area of a parallelogram is determined by the same formula as that of the rectangle, A = bh, where h is a perpendicular between the two bases.	 In parallelogram ABCD, either $\overline{AB}$ or $\overline{CD}$ is the base, DE is an altitude. If this figure were constructed of cardboard, and the triangle DAE snipped off and placed so that $\overline{AD}$ coincided with $\overline{BC}$, a rectangle would be formed. Since its area is given by A = bh, and it is obviously equivalent to the parallelogram, the area of parallelogram ABCD must be A = bh.
(3) Triangle	• A triangle has the property of being one half of a parallelogram and its area is given by the formula $A = \frac{1}{2}bh$.	 In Δ ABC, $\overline{AB}$ is a base b and CE is the altitude h. If the Δ ABC is "flipped over," a new triangle CDB is formed, which is congruent to Δ ABC. By the laws of congruence, $\overline{CD} \cong \overline{AB}$ and $\overline{BD} \cong \overline{AC}$ so ABCD is a parallelogram, its area A = bh. Since the triangle is $\frac{1}{2}$ of the parallelogram, its area $A = \frac{1}{2}bh$.
(4) Trapezoid	• The area of a trapezoid is expressed by the formula $A = \frac{1}{2}h(b + b')$, where h is the altitude and b and b' the two parallel bases.	Once again, demonstrate the same technique as used for the triangle applies as in the diagram above.  In trapezoid ABCD, bases $\overline{AB}$ and $\overline{CD}$ are denoted by b and b', while altitude $\overline{CK}$ is denoted by h. If we turn the trapezoid upside down, and move it over so that $\overline{CB}$ falls on $\overline{BC}$, the top base and bottom base have a length equal to b + b'. The other two sides of the figure are each $\overline{AD}$. Thus, the new figure constructed is a parallelogram and its area is given by A = (b + b')h.

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
		The original trapezoid is exactly half the area of this parallelogram. Thus, its area is given by $A = \frac{1}{2} h (b + b')$. In each of the previous three derivations, paper cutouts of the figures will facilitate instruction.
(5) Circle	• The circumference of a circle is given by the formula $C = \pi \cdot d$. • Since a diameter is equal to two radii, an alternate form is $C = \pi \cdot 2r$ or $C = 2\pi \cdot r$. • The area of a circle is given by the formula $A = \pi \cdot r^2$, where r is the radius, π the constant approximately equal to 3.14.	A derivation for the area and circumference of a circle is beyond the scope of this course. Demonstrate that π = the ratio between the circumference and the diameter; $\pi = \frac{c}{d} = \frac{22}{7}$ or 3.14.  $\overline{DA}$ = diameter $\overline{DO} \cong \overline{OA} \cong \overline{OB}$ = radius
9. Three-dimensional figures	Solids of revolution are formed by revolving certain geometric figures about an axis.	The following are illustrations of solids of revolution.  Revolve rectangle ABCD about $\overline{AB}$ as an axis to form a cylinder.  Revolution about $\overline{AC}$, the leg of a right triangle, produces a cone.  Revolution about $\overline{AB}$, the diameter of semicircle O, produces a sphere.

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
a. The unit cube	The concept of volume is predicated upon the cubic unit; the cubic unit being the amount of space enclosed by a cube, one unit each edge.	 Simply, one cubic unit is the amount of space enclosed by a cube, each edge of which is one unit.
b. Volume of a prism, cylinder	For all prisms and cylinders, the volume is given by the formula V = Bh, where B is the area of the base and h is the altitude.	The diagram at the right shows a rectangular prism (solid) with a base 4 units by 3 units and altitude 2. It is easy to see that there are really two layers, each containing 12 cubes for a total volume of 24 cubic units. 
		 In this triangular prism, the bases are right triangles with base 8 and altitude 4. The area of the base of prism $B = \frac{1}{2}\cdot 8\cdot 4$. Thus, B = 16, V = Bh, $V = 16\cdot 7 =$ 112 cubic units.
c. Volume of a cone, pyramid	The volume of a cone or a pyramid is $\frac{1}{3}$ the volume of the corresponding prism or cylinder and is given by the formula $V = \frac{1}{3}$ Bh, where B and h are the area of the base and the altitude respectively.	 In pyramid V-ABCD, the base is a rectangle ABCD and $\overline{VO}$ is a perpendicular drawn from V to the plane of ABCD. If AB = 6, BD = 4, and h = 7, the area of the base of rectangle ABCD = $6\cdot 4 = 24$ square units. The volume $= \frac{1}{3}$ Bh $= \frac{1}{3}\cdot 24\cdot 7 = 56$ cubic units.



In the cone, the base B is a circle with radius OA = 14. The altitude h (drawn perpendicular to the plane of the circle) is 35.

$B = \pi \cdot r^2$

$B = \frac{22}{7} \cdot 14 \cdot 14$

$B = 616$ square units

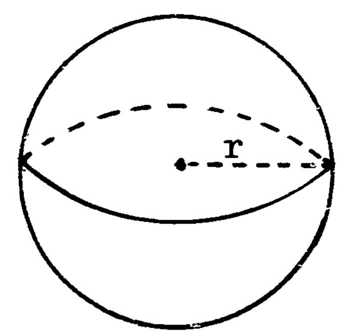
Now, $V = \frac{1}{3} \cdot Bh$

$= \frac{1}{3} \cdot 616 \cdot 35$

$= \frac{21560}{3} = 7186\frac{2}{3}$ cubic units.

d. Volume of a sphere

The volume of a sphere is determined by the relationship $V = \frac{4}{3} \cdot \pi \cdot r^3$, where r is the radius.



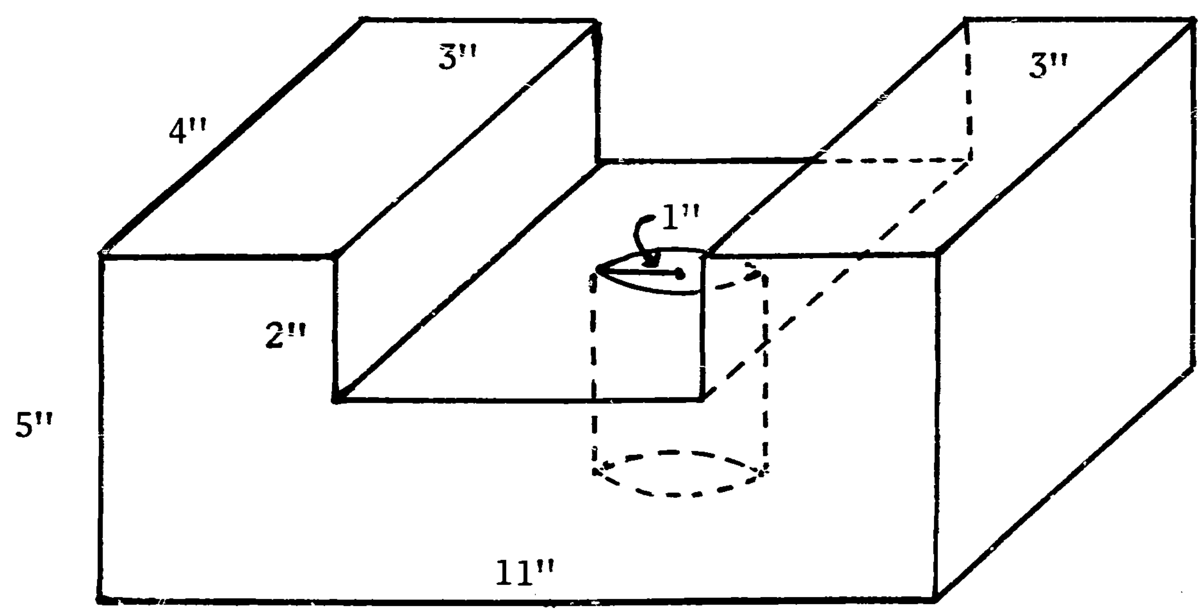
The volume of a sphere should be discussed, but its derivation omitted.

Example: If in a sphere r = 10, $\pi = 3.14$, then $V = \frac{4}{3} \cdot (3.14) \cdot (10)^3 = \frac{4}{3}(3.14) \cdot 1000 = 4186\frac{2}{3}$ cubic units.

e. Problems

Problems must be reduced to their components for systematic solution.

Example 1: In the drawing below, a hole of radius 1 inch has been drilled in the block. Compute the volume of metal necessary to produce the block.



In effect, the problem is to break the figure into two rectangular prisms and a cylinder. If there were no cutouts, the simple block would have a volume $V_1 = 11 \cdot 4 \cdot 5 = 220$ cubic inches. But we must subtract the portions which have been removed.

A rectangular solid $V_2 = 2 \cdot 4 \cdot 5 = 40$ cubic inches is the first cutout representing the "notch" in the top.

A cylinder $V_3 = \pi \cdot 1 \cdot 1 \cdot 3 = 3.14 \cdot 3 = 9.42$ cubic inches is the volume of the second cutout.

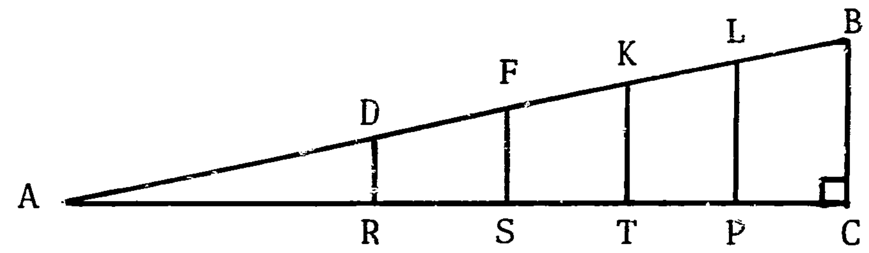
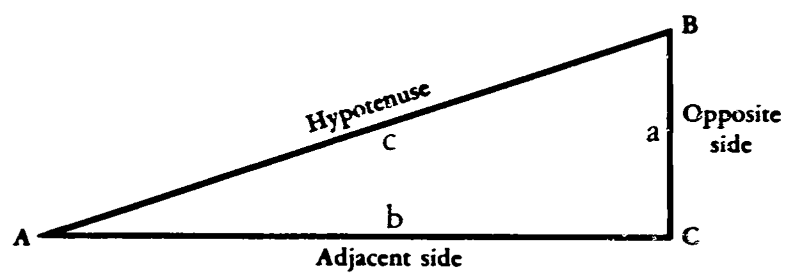
$V = V_1 - V_2 - V_3 = 220 - 40 - 9.42$
$V = 170.58$ cubic inches.

Example 2: A room is to be painted with a paint that costs $7.60 a gallon or $2.40 a quart. The paint covers 400 sq. ft. per gallon. The room is 26' x 14' and 8' high, but has 2 doors 3' x 7' and a picture window 9' x 7' not to be painted. Find the cost of the paint required to paint the walls and ceiling.

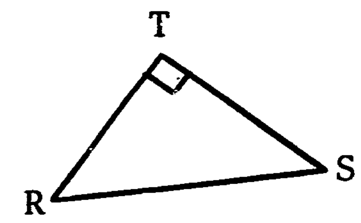
Solution: Surface area includes walls and ceiling only.
Ceiling = 26' x 14' = 364 sq. ft.
Walls = 8' x 26' x (2) + 8' x 14' x (2) = 640 sq. ft.
Subtractions are 2 doors = 42 sq. ft., one window 63 sq. ft., a total of 105 sq. ft.
Total area to be painted is 899 sq. ft.
Since 1 qt. covers 100 sq. ft., nine qts. or 2 gallons, 1 qt. are necessary; or $15.20 + $2.40 = $17.60.

V. Trigonometry
A. Trigonometric functions

The three fundamental trigonometric ratios are sine, cosine, and tangent.

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
1. The sine	The sine of an acute angle of a right triangle is the ratio of the side opposite the angle to the hypotenuse.	 In the drawing, ABC is a triangle with right angle C. From points D, F, K, and L on $\overline{AB}$ perpendiculars are drawn to $\overline{AC}$. Now Δ ADR is similar to Δ ABC because they have angle A in common and each triangle has a right angle. Then the ratio $\frac{DR}{AD}$ = ratio $\frac{BC}{AB}$. In the same way Δ ASF is similar to Δ ACB and $\frac{FS}{FA} = \frac{BC}{AB}$, and we see that $\frac{DR}{DA} = \frac{FS}{FA} = \frac{KT}{KA} = \frac{LP}{LA} = \frac{BC}{BA}$, and in fact, no matter where the perpendicular is drawn, the ratio of the perpendicular to the hypotenuse is constant. This constant ratio is called the sine of the angle A. We note that the two sides involved are the opposite and the hypotenuse.  In Δ ABC, with reference to $\angle A$, $\overline{BC}$ is clearly opposite to $\angle A$, while $\overline{AC}$ and $\overline{BC}$ are both adjacent. By convention, we call $\overline{AC}$ the "adjacent" side and reserve the name hypotenuse for $\overline{AB}$, the longest side.
2. The cosine	The cosine of an acute angle of a right triangle is the ratio of the side adjacent to the angle to the hypotenuse.	Referring to the diagram at the top of this page, we see by similar triangles that $\frac{AR}{AD} = \frac{AS}{AF} = \frac{AT}{AK} = \frac{AP}{AL} = \frac{AC}{AB}$.

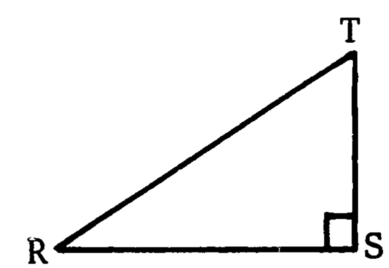
TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
		Once again, with a series of right triangles, the ratio is a constant if the angle A is kept constant. This ratio is called the cosine of ∠A. Our notation is cos∠A.



In Δ RST, $\cos\angle R = \frac{RT}{RS}$ and $\cos\angle S = \frac{ST}{SR}$.

In each case the cosine is the ratio of the adjacent side to the hypotenuse.

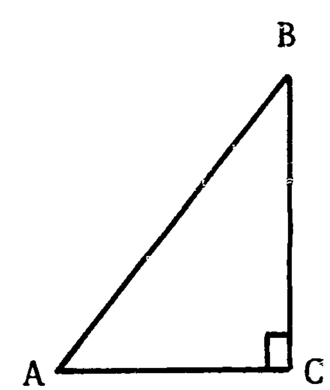
3. The tangent	The tangent function of an acute angle of a right triangle is the ratio of the opposite side to the adjacent side.	The third ratio, the tangent ratio (abbreviated tan), is defined by tangent of an angle = $\frac{\text{opposite}}{\text{adjacent}}$. Once again, by the diagram in Δ RST, $\tan\angle R = \frac{TS}{RS}$.



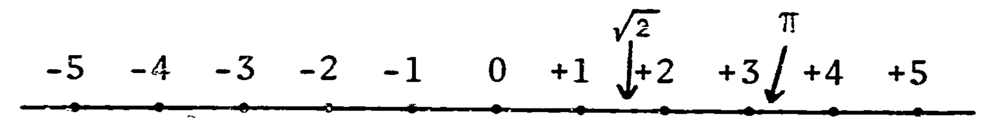
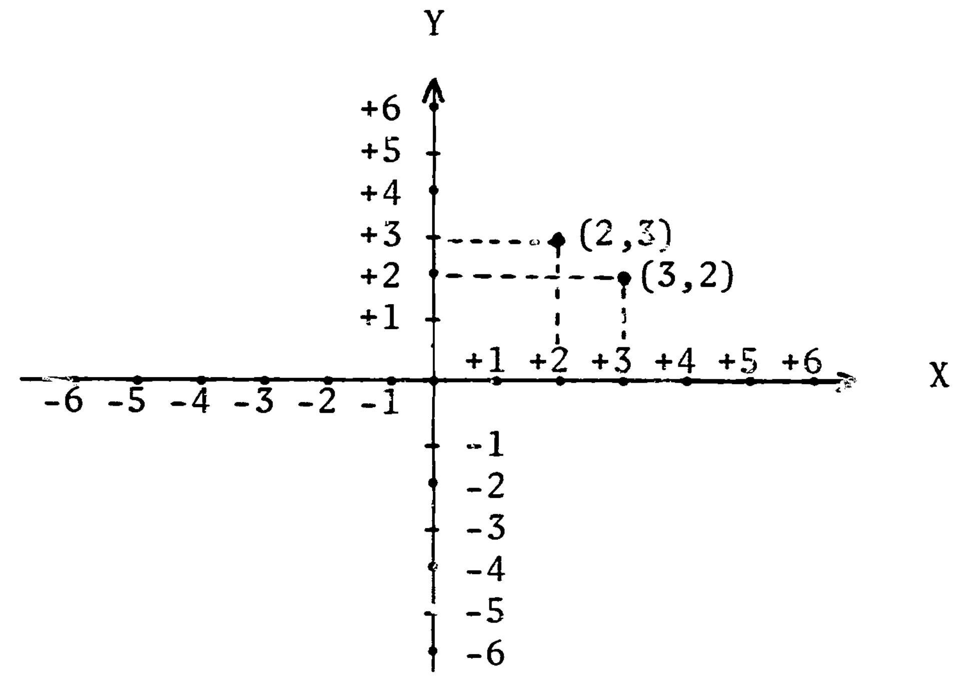
$\tan\angle R = \frac{TS}{RS}$ and $\tan\angle T = \frac{RS}{ST}$.

In each case we take the ratio of the opposite side to the adjacent side.

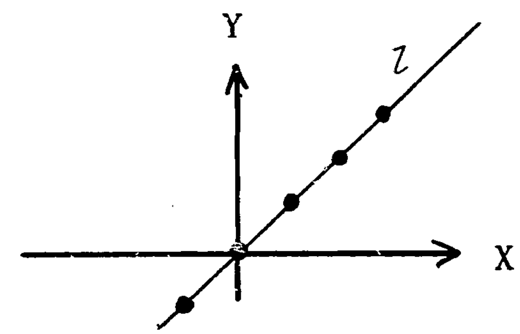
B. Problem solving using trigonometric relationships		A proper start, prior to actual problem solving, involves exercises to determine the function.



GIVEN DATA	REQUIRED TO FIND	FUNCTION TO USE
(1) AB, ∠A	BC	(sine)
(2) AB, ∠A	AC	(cosine)
(3) BC, ∠B	AC	(tangent)
(4) AC, BC	∠B	(tangent)
(5) AB, AC	∠B	(sine)

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
VI. Coordinate Geometry A. The Cartesian plane 1. The set of real numbers and the line	There is a one-to-one correspondence between the real numbers and the points on the number line.	For every real number there is a point on the line and for every point on the line there is a real number. 
2. The association of a pair of real numbers with a point in the plane	For each pair of real numbers, there is a unique point in the plane and for each point in the plane there is a unique pair of real numbers.	If we take two perpendicular number lines, they determine a plane, commonly termed the Cartesian plane after its originator, René Descartes.
a. Ordered pair	An ordered pair of numbers is so termed because the order of the numbers determines the location of the point.	
b. Abscissa	The abscissa is the x-value (horizontal) or first element of the ordered pair.	
c. Ordinate	The ordinate is the y-value (vertical) or second element of the ordered pair.	
d. Coordinates	Coordinates is another term for ordered pair used in locating a point.	
e. Axis	There are two axes, the X and Y, which are really two perpendicular number lines.	If one number locates a point on the number line, two numbers will locate it in the plane. Such a pair is called an ordered pair, since the pair (2,3) represents a different point than (3,2). The first number in the ordered pair is the x-distance, or abscissa, and the second number is the y-distance, or ordinate. The x-and y-number lines are called the X and Y-axes, and their intersection is the origin (0,0).
f. Origin	The origin is the intersection of the two axes and is the zero point on each axis (0,0).	

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
		The student should practice plotting many points, and conversely, given the point, he should find the ordered pair associated with the point.
B. The linear equation	The linear equation in the form $ax + by = c$ is so termed because its graph is a straight line.	
1. Graphing the linear equation	To graph a linear equation, it is sufficient to establish two ordered pairs of numbers which satisfy the equation, usually displayed in a tabular form.	If ordered pairs of numbers are represented by points, then the points on a line must have a special relationship.



Considering line l as a collection of points, the relationship seems to be that each x-value is exactly twice each y-value, or $x = 2y$.

For every line there is an equation in the form $ax + by = c$. The first problem is, given such an equation, plot the line that is associated with it.

Given: The equation $2x - 3y = 12$
Find: The graph of the equation

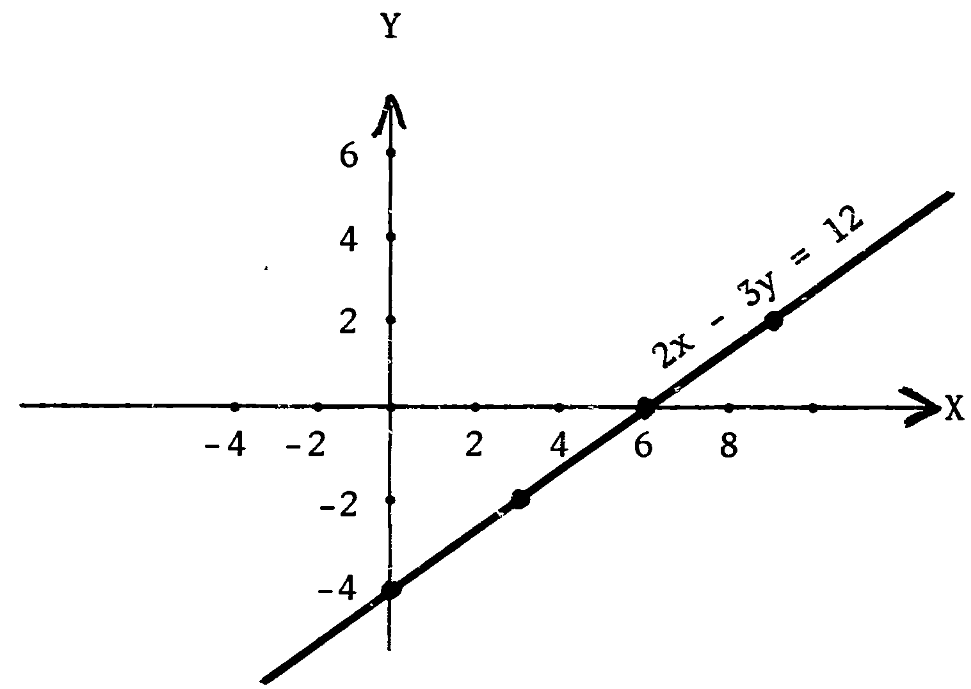
This resolves itself into finding several ordered pairs which will satisfy the equation. The best way is to solve the equation for one variable, as follows:

$$2x - 3y = 12$$
$$2x = 3y + 12$$
$$x = \frac{3y + 12}{2}$$

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
		At this point, arbitrary values are assigned to y, and the associated x-values are determined. A table forms the desirable relationships.

x	6	9	3
y	0	2	-2

Although two points are sufficient to determine a straight line, it is advisable to use a third for a check point.



TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
2. Solving systems of two equations in two variables graphically	To solve a system of two equations graphically, plot the straight line corresponding to each; the solution is the point of intersection of the two lines.	The second problem to consider is two equations of the form ax + by = c and to find a graphic solution, an ordered pair which satisfies each of them. This solution exists generally, but there are exceptions. Two lines may be parallel and have no point of intersection, or they may coincide and have an infinite number of points of intersection. Several examples of this type will bring the student to the point where he can solve two simultaneous linear equations graphically for x and y.

A linear equation is one whose graph is a straight line. Simultaneous equations occur at the same time. Thus, if we have two equations of the form $ax + by = c$, we seek values of x and y which will satisfy both at the same time.

Example:

(1) $2x - 3y = 4$
(2) $x + 2y = 9$

From (1):

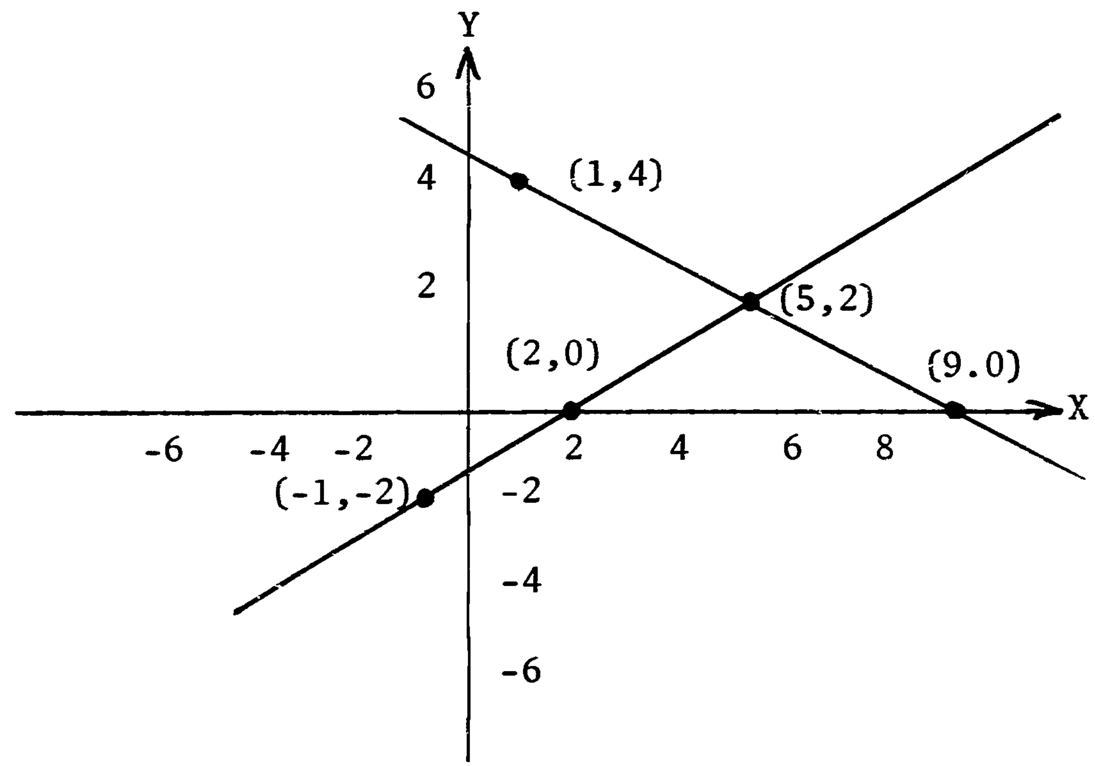
$$2x = 3y + 4$$
$$x = \frac{3y + 4}{2}$$

x	2	5	-1
y	0	2	-2

From (2):

$$x = -2y + 9$$

x	9	5	1
y	0	2	4



Clearly, the intersection of the two lines is at (5,2), and this point occurs in each table. The student should check the answer in both equations.

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
3. Solving systems of equations algebraically	There are two methods for solving simultaneous linear equations algebraically, eliminating a variable by multiplication and/or addition or substitution.	In either case, we avoid high level abstraction and confine the concept to the concrete.

Example: $3x - 4y = 7$

$5x + 3y = 31$

Solution: In either method, it is essential to eliminate one of the variables. One way to accomplish this is by substitution as follows:

(1) Select one equation and solve it for one of the variables.

$$3x - 4y = 7$$
$$3x - 4y + 4y = 4y + 7$$
$$3x = 4y + 7$$
$$\frac{3x}{3} = \frac{4y + 7}{3}$$
$$x = \frac{4y + 7}{3}$$

(2) Using the expression obtained, substitute for x in the second equation.

$$5x + 3y = 31$$
$$5 \cdot \left(\frac{4y + 7}{3}\right) + 3y = 31$$
$$\frac{20y + 35}{3} + 3y = 31$$

(3) Solve for y in the usual fashion. Common denominator is 3.

$$3 \cdot \left(\frac{20y + 35}{3}\right) + 3(3y) = 3(31)$$
$$20y + 35 + 9y = 93$$
$$29y = 58$$
$$y = 2$$

(4) Use this value of y in the first equation to find x.

$$x = \frac{4y + 7}{3} = \frac{8 + 7}{3} = \frac{15}{3} = 5$$

The solution is (5,2).

(5) *Check!* The solution must be checked in each equation.

$$\begin{aligned} 3x - 4y &= 7 \\ 3\cdot 5 - 4\cdot 2 &= 7 \\ 15 - 8 &= 7 \\ 7 &= 7\checkmark \end{aligned} \qquad \begin{aligned} 5x + 3y &= 31 \\ 5\cdot 5 + 3\cdot 2 &= 31 \\ 25 + 6 &= 31 \\ 31 &= 31\checkmark \end{aligned}$$

Alternatively, a method less prone to error, and avoiding fractions, is the addition-multiplication method. Using the preceding example:

$$3x - 4y = 7$$
$$5x + 3y = 31$$

Solution:

(1) In this method, the trick is to manipulate the two equations by multiplication so as to match up coefficients of one of the variables. We notice that the coefficient of y is -4 in the first equation and 3 in the second. To make them match, we multiply the first equation by 3 and the second by 4. Why?

$$\begin{aligned} 3(3x - 4y) &= 3\cdot 7 \\ 4(5x + 3y) &= 4\cdot 31 \\ 9x - 12y &= 21 \\ \underline{20x + 12y} &\underline{= 124} \end{aligned}$$

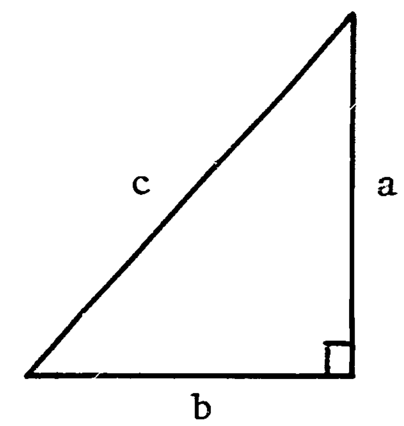
(2) Add the two resulting equations $29x = 145$ and solve to obtain $x = 5$.

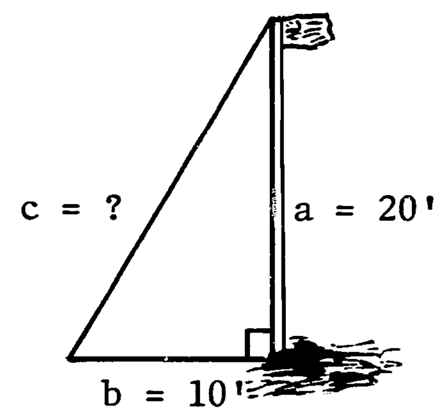
(3) Use either equation, substituting x to find y.

$$\begin{aligned} 5\cdot(5) + 3y &= 31 \\ 25 + 3y &= 31 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

(4) The solution is (5,2).

(5) Check in both equations!

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
4. Pythagoras' theorem	Pythagoras' theorem states that in a right triangle the square of the hypotenuse is equal to the sum of the squares of the two legs.	 In the drawing, $c^2 = a^2 + b^2$. Frequently, integers can be found which will form the sides of a right triangle. Some of the most frequently encountered of these Pythagorean triples are: (3, 4, 5) (5, 12, 13) (7, 24, 25) Any multiple of one of these triples such as (6, 8, 10) will also produce a right triangle.
	Pythagoras' theorem is used to measure indirectly.	*Example 1:* A flagpole 20' high is supported by a guy wire attached at the top and on the ground 10' from its base. How long is the wire?

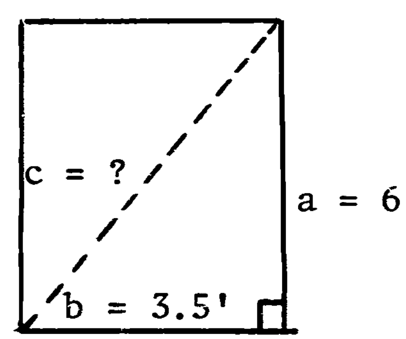


$c^2 = a^2 + b^2$
$c^2 = 20^2 + 10^2$
$c^2 = 400 + 100$
$c^2 = 500$
$c = \sqrt{500}$

Using the trial method, assume $c = 22'$. $\frac{22 + 22.64}{2} = 22.31$

$c \approx 22.3$

Example 2: Can a circular table top 7.3 ft. in diameter fit through a doorway 3.5 ft. wide and 6.5 ft. high?



a = 6.5
b = 3.5
c = ?

$c^2 = a^2 + b^2$
$c^2 = 42.25 + 12.25$
$c^2 = 54.50$
$c = \sqrt{54.50}$

Since $(7.3)^2 = 53.29$ we see that $7.3 < \sqrt{54.50}$ and therefore the table should just fit through the door.

TOPICAL OUTLINE	CONCEPTS AND UNDERSTANDINGS	SUPPORTING INFORMATION AND INSTRUCTIONAL TECHNIQUES
VII. Statistics A. Measures of central tendency	Central tendency is the trend of large groups of data to cluster about the middle point.	*Example:* Suppose five men are discussing salary. If one makes $7,000 per year, three make $8,000, and one makes $1,000,000 a year, the statistics could be deceiving.
1. The mean	The mean for a set of data is synonymous with average. To compute the mean, add all the elements in the set and divide by the number of elements.	The mean would be $\frac{7000 + 3(8000) + 1,000,000}{5}$ or $262,000. This would not give a clear picture of the group.
2. The median	The median for a set of data is that element of the set below and above which lie 50% of the elements of the set.	The median at $8,000 would be meaningful. The median tends to limit the effect of extremes values in assessing the typical score.
3. The mode	The mode is the score occurring most frequently in a set of data.	The mode here would also be $8,000, as the score that occurs most frequently. *Example:* Suppose your scores on a set of tests were 75, 75, 75, 80, 85, 90, 90, 95, and 91. The mean would be $\frac{756}{9} = 84$. The median would be 85, the score in the center, with four above it and four below it. The mode, however, would be 75 as the score that occurred most frequently. In this case, only the mean would be of any value, since the average or arithmetic mean would be recorded on the report.
B. The normal curve	The normal curve of probability is a graphic way of showing that a large group of measurement tends to cluster at the mean.	To understand the normal curve, illustrate with a line graph of a large set of data which, when smoothed, approaches the normal curve.

SAMPLE TEST QUESTIONS

Mathematics

Directions (1-42): Write on the answer sheet the *number* that identifies the correct answer to each question or problem.

1. The *mean* of 8, 12, and 22 is
   (1) 42 (2) 21 (3) 14 (4) 12

2. The population of a certain city is 328,637. If this number were expressed to the *nearest thousand*, it would be written
   (1) 300,000 (3) 328,000
   (2) 330,000 (4) 329,000

3. The number 8 multiplied by itself is 64. The number 8 is called the
   (1) dividend of 64 (3) product of 64
   (2) square root of 64 (4) square of 64

4. In the number 2495, the digit 9 is in the
   (1) units place (3) hundreds place
   (2) tens place (4) thousands place

5. A number is increased by one-half of itself. The old number is what fractional part of the new number?
   (1) $\frac{2}{3}$ (2) $\frac{1}{2}$ (3) $1\frac{1}{2}$ (4) 2

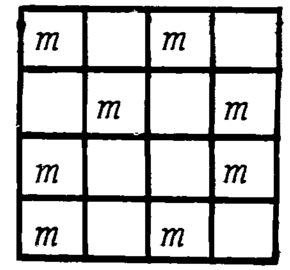
6. Forty minutes is what fractional part of an hour?
   (1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{5}{6}$ (4) $\frac{7}{8}$

7. Which fraction most accurately represents the part of the year remaining after August 31?
   (1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

8. The area of a desk top is given in square inches. To find the area in square feet,
   (1) divide by 12 (3) multiply by 12
   (2) divide by 144 (4) multiply by 144

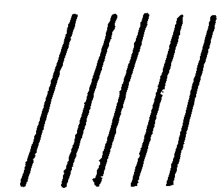
9. The ratio of $\frac{1}{2}$ to 100 may be expressed as
   (1) 1 to 200^2 (2) 2 to 100 (3) 5% (4) 50%

10. What percent of the small squares at the right contain the letter *m*?
    (1) 40
    (2) 50
    (3) 60
    (4) 80

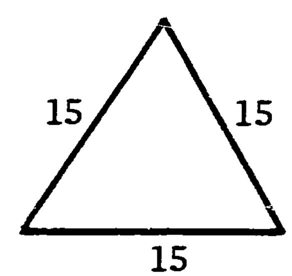


11. All houses that are exactly one mile from a certain supermarket are on
    (1) two intersecting lines
    (2) two parallel lines that are one mile from the market
    (3) a straight line that passes through the market
    (4) a circle with the market as its center and a radius of one mile

12. The lines at the right are
    (1) parallel
    (2) vertical
    (3) horizontal
    (4) perpendicular



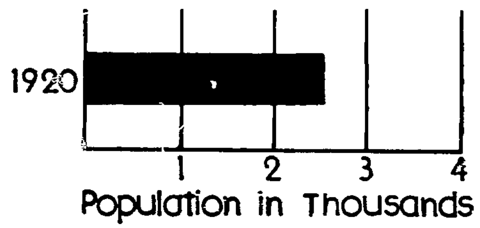
13. In the figure at the right, the number of degrees in each angle is
    (1) 180
    (2) 60
    (3) 45
    (4) 30



14. How many edges does a cube have?
(1) 4 (2) 8 (3) 12 (4) 16

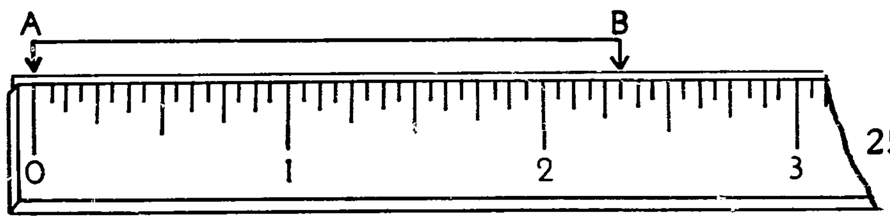
15. The radius of a circle is equal to the side of a square. The area of the circle is most nearly how many times greater than the area of the square?
(1) 2 (2) $2\frac{1}{2}$ (3) 3 (4) $3\frac{1}{7}$

16. The drawing at the right is part of a bar graph showing population trends in a village. How many people lived in the village in 1920?



(1) 2.5
(2) 25
(3) 250
(4) 2,500

17. What is the length of *AB* to the *nearest fourth of an inch?*



(1) $2\frac{1}{4}$ in.
(2) $2\frac{1}{2}$ in.
(3) $2\frac{3}{8}$ in.
(4) $2\frac{5}{16}$ in.

18. On a blueprint of a house, a line $\frac{1}{2}$ inch long represents 4 feet. The scale of the blueprint is
(1) $\frac{1}{2}$ in. to 1 ft. (3) $\frac{1}{8}$ in. to 1 ft.
(2) 2 in. to 1 ft. (4) $\frac{1}{4}$ in. to 1 ft.

19. Three times a number n increased by 4 may be expressed as
(1) $n + 12$ (2) $3n + 4$ (3) $3n - 4$ (4) $7n$

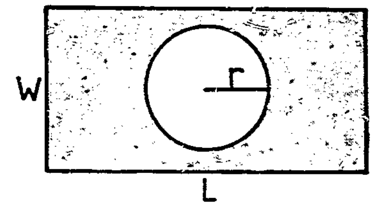
20. A newspaper press can print n newspapers in 30 minutes. How many papers can it print in one hour?
(1) $\frac{n}{2}$ (2) $2n$ (3) $30n$ (4) $60n$

21. If one pencil costs 5 cents, how many can be bought for n cents?
(1) $5n$ (2) $5 + n$ (3) $\frac{n}{5}$ (4) $n - 5$

22. If $n + 3 = 10$, then n equals
(1) 30 (2) 13 (3) $3\frac{1}{3}$ (4) 7

23. A strip of wood $(12f + 6)$ inches long was cut into 3 equal pieces. The length of each piece is
(1) $(4f + 2)$ in. (3) $(36f + 6)$ in.
(2) $(4f + 6)$ in. (4) $(36f + 18)$ in.

24. In the rectangle at the right, the area of the shaded portion is



(1) $2LW - \pi r^2$
(2) $2L + 2W - 2\pi r$
(3) $LW - \pi r^2$
(4) $LW - 2\pi r$

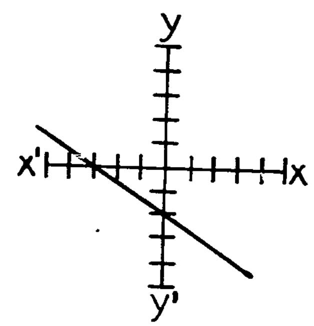
25. If $10 = \sqrt{n}$, the value of n is
(1) 10 (2) 20 (3) 100 (4) $\sqrt{10}$

26. Which is a correct step toward solving the equation $\frac{1}{2}x - 1 = 5$?
(1) $x - 1 = 10$ (3) $x - 2 = 10$
(2) $x - 2 = 5$ (4) $2x - 2 = 10$

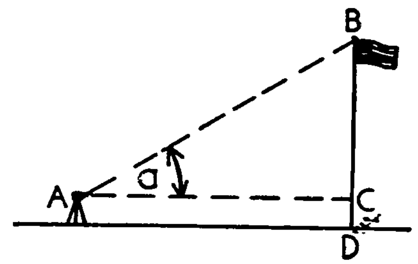
27. A car travels at the rate of 40 miles per hour for $(t - 2)$ hours. The distance the car travels in this time is represented by
(1) $\frac{40}{t - 2}$ (3) $40t - 80$
(2) $t + 38$ (4) $40t - 2$

28. Which is a root of the equation $x^2 - x - 12 = 0$?
(1) — 4 (3) + 3
(2) — 3 (4) + 4

29. In the graph at the right, what is the value of x when y is 0?
(1) 0
(2) — 2
(3) 3
(4) — 3



30. In the drawing at the right, the height of the flag pole was found by using a transit to measure angle A. Angle A is called the
(1) central angle
(2) angle of depression of B from A
(3) angle of elevation of B from A
(4) supplementary angle



31. Subtract: $30\frac{7}{8}$ — 14
(1) $15\frac{1}{8}$ (2) $16\frac{7}{8}$ (3) $17\frac{1}{8}$ (4) $17\frac{7}{8}$

32. Divide: $10\frac{1}{2} \div 1\frac{1}{2}$
(1) $\frac{1}{14}$ (2) $\frac{1}{7}$ (3) 7 (4) 14

33. Multiply: $2\frac{5}{4} \times 1\frac{1}{2}$
(1) $16\frac{1}{2}$ (2) $2\frac{1}{3}$ (3) $5\frac{1}{2}$ (4) $4\frac{1}{8}$

34. Express $\frac{24}{30}$ as a percent.
(1) 40% (2) 60% (3) 75% (4) 80%

35. Find the value of a: $2a - 3 = 15$
(1) 6 (2) 9 (3) $10\frac{1}{2}$ (4) 36

36. In the formula $r = \frac{s^2 + h^2}{2h}$, if $r = 5$ and $h = 1$, then s may equal
(1) 9 (2) 2 (3) 3 (4) 13

37. What are the prime factors of $3x^2 - 12$?
(1) $3(x^2 - 4)$ (3) $3(x - 4)(x + 4)$
(2) $3(x - 2)(x + 2)$ (4) $(3x + 6)(x - 2)$

38. A train travels 226.6 miles to reach New York City. When the Ellis family boarded the train to go to New York City, it had already traveled 90.6 miles of this distance. How many miles did they travel on the train?
(1) 90.6 (2) 136 (3) 226.6 (4) 317.2

39. The Sims' bill for gas and electricity was $8.40 for the month of June. By paying the bill within 10 days, they received a discount of $.42. What percent did they save by paying promptly?
(1) 5% (2) 2% (3) 20% (4) 42%

40. The price of a 3-pound can of vegetable shortening increased from 80 cents to 93 cents. What percent of increase was this?
(1) 1.6 (2) 14.0 (3) $16\frac{1}{4}$ (4) 86

41. A picture $4\frac{1}{2}$ inches wide and $5\frac{1}{2}$ inches long is to be enlarged so that its width will be 9 inches. How many inches long will it be?
(1) 1 (2) 8 (3) 10 (4) 11

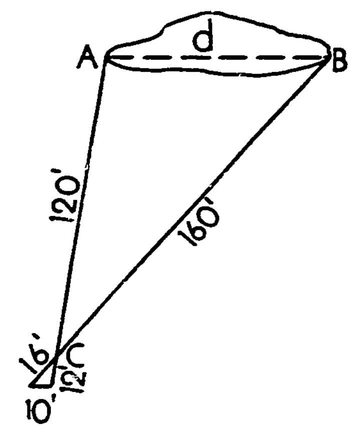
42. A group of Boy Scouts wanted to find the distance d across a pond. With the aid of a surveyor's tape they staked off the figure shown in the drawing. Which proportion enabled them to find d?

(1) $\frac{d}{120} = \frac{16}{10}$

(2) $\frac{160}{120} = \frac{d}{10}$

(3) $\frac{120}{12} = \frac{d}{10}$

(4) $\frac{10}{d} = \frac{120}{16}$



Directions (43-45): In each of the following questions, *do not solve the problem*, but choose the equation which, if worked out, will give the correct answer to the problem.

43. In a triangle, two of the angles are the same size and the third angle is three times as large as either of the other two. Find the number of degrees in each angle.

(1) $6x = 180$ (3) $x + x + 3x = 180$

(2) $x + 3x = 180$ (4) $x + x + 3x = 360$

44. A ship leaves a harbor at 12 miles per hour. Seven hours later a Coast Guard cutter leaves the same harbor at 20 miles per hour and follows the same course as the ship. How many hours, t, will it take the Coast Guard cutter to overtake the ship?

(1) $20t = 12\ (t + 7)$ (3) $20t + 12\ (t + 7) = 0$

(2) $20t = 12\ (t - 7)$ (4) $20t + 12\ (t - 7) = 0$

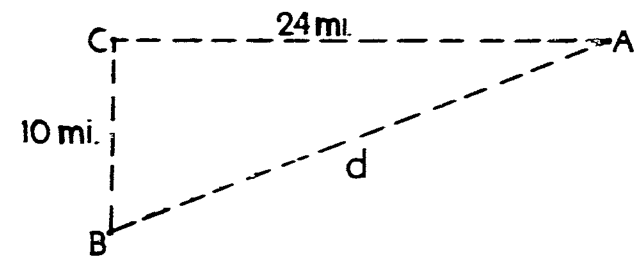
45. In the figure at the right, A, B, and C are the locations of three towns on a map. Town A is 24 miles due east of town C, and town B is 10 miles due south of town C. Find the shortest distance d between towns A and B.

(1) $10^2 + 24^2 = d^2$

(2) $10 + 24 = d$

(3) $\sqrt{10} + \sqrt{24} = d$

(4) $\frac{24}{d} = \cos 90°$



Directions (46-51): Write the correct answer to each of the following on the separate answer sheet.

46. If R = $\{a, b, c\}$, what is the total number of subsets of R which contain exactly two members?

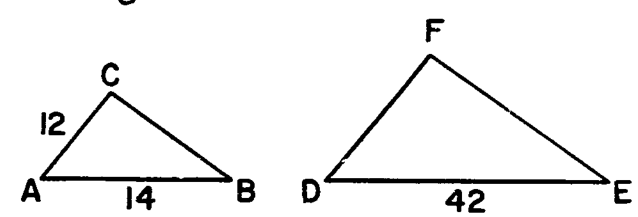
47. If x is a variable whose domain is {2, 3, 5}, what is the largest value that the expression $10 - x$ may have?

48. If y is a member of {1, 2, 3, 4, 5}, what is the solution set of the inequality $2y > 5$?

49. What number is the reciprocal of 5?

50. What is the product of $(x + y)$ and $(x - y)$ when $x = y$?

51. In the similar triangles ABC and DEF shown below, $\angle B = \angle E$, $\angle C = \angle F$, AC = 12, AB = 14, and DE = 42. What is the length of DF?



ANSWERS TO SAMPLE TEST QUESTIONS

Mathematics

1.	(3)	14.	(3)	27.	(3)	40.	(3)
2.	(4)	15.	(4)	28.	(2)	41.	(4)
3.	(2)	16.	(4)	29.	(4)	42.	(3)
4.	(2)	17.	(1)	30.	(3)	43.	(3)
5.	(1)	18.	(3)	31.	(2)	44.	(1)
6.	(2)	19.	(2)	32.	(3)	45.	(1)
7.	(3)	20.	(2)	33.	(4)	46.	(3)
8.	(2)	21.	(3)	34.	(4)	47.	8
9.	(1)	22.	(4)	35.	(2)	48.	{3, 4, 5}
10.	(2)	23.	(1)	36.	(3)	49.	$\frac{1}{5}$
11.	(4)	24.	(3)	37.	(2)	50.	0
12.	(1)	25.	(3)	38.	(2)	51.	36
13.	(2)	26.	(3)	39.	(1)		

NOTATIONS OF THE INSTRUCTOR

Useful Instructional Materials - Annotated

TEXTBOOKS, WORKBOOKS, AND REVIEW BOOKS

Listed is a supplemental collection of textbooks, workbooks, and review books which may be used for study and reference along with pamphlets and other learning devices suitable for adult use in the high school equivalency program. No specific endorsement is intended for any of the items listed. Many publishers are willing to supply examination copies for interested teachers or directors. Annotations give some information on content and possible usefulness.

Arithmetic, intermediate series. Holt. (Holt adult basic education)

Covers the basic elementary curriculum in arithmetic from initial number concepts through work with the four arithmetical operations.

Basic algebra, ninth year. Cambridge.

Includes some modern mathematics in this beginner's course. Illustrative examples help show how to do basic algebra.

Fundamental mathematics, advanced series. Holt. (Holt adult basic education)

Extends skill with fractions. Reviews basic computations with decimals. Extends understanding of percent. Covers the broad concepts of geometry.

General mathematical ability. Cowles. (High school equivalency examination preparation series)

Arithmetical systems and processes are explained with problem-solving methods and interpretation of word problems. Students may work on their own since answers to the practice exercises are explained. Designed specifically for use in studying for GED tests.

Introduction to modern mathematics, books 1 and 2, a modern-traditional approach. Cambridge.

The material is divided into three parts. The first part is a blend of modern and traditional. Fundamental arithmetic processes are presented in the exercises too. In part two, algebra is presented. Part three is a course in modern math from the introduction to sets through the number line and the cartesian coordinate system. Book two has basic algebra for beginners.

Mastering elementary algebra. Keystone.

Along with the current treatment of the fundamental facts and ideas, there is a chapter dealing with inequalities and their solution sets.

Mathematics, a basic course, books 1 and 2. Cambridge.

Book one is a complete basic course in math from simple arithmetic through geometry. Topics include using numbers, common and decimal fractions, percentage, common measurement scale drawing, graphs, geometry. Book two topics include simple arithmetic, social arithmetic, functional use of algebra and geometry. There are many exercises with illustrative problems.

Ninth year mathematics review. Amsco.

Designed to aid the student in understanding the basic concepts of elementary algebra and to help him acquire important algebraic skills. There are exercises, model problems, and instructional materials. Each chapter has a series of learning units for self study. The basic concepts of each unit use simple language and symbolism. Model problems have detailed explanations.

Review guide in preliminary mathematics, 8th year. Amsco.

Condensed version of Review Text in Preliminary Mathematics, 8th Year.

Review text in preliminary mathematics, 8th year. Amsco.

A comprehensive survey of arithmetic, geometry and algebra for grades 7-8. May also be used for general or practical math, grades 9-10. Many illustrative problems and graded exercises. Utilizes both traditional and modern math techniques.

PROGRAMED AND SELF-DIRECTED MATERIALS

Programed and self-directed materials may be particularly useful in High School Equivalency classes because they make it possible for the instructor to work efficiently with students of widely varying educational backgrounds and needs. The following is a partial listing of such materials that are currently available. No effort has been made by the Bureau to evaluate these materials. Inclusion here is not intended as an endorsement of any specific item on the list. Most publishers are willing to provide examination copies to interested directors and teachers upon request. The instructor will have to evaluate the materials he intends to use in the light of the particular needs of the individual students who are to use them. Annotations give some information on content and possible usefulness.

Addition of fractions. Graflex.

Adding fractions with like denominators. 65 frames.

Adventures in algebra: tutor text. Doubleday.

Begins with consideration of symbols in mathematics and goes on to the deeper understanding of the concept of numbers.

Arithmetic facts: practice program. Graflex.

Two books of 296 frames, each covering addition and subtraction, multiplication and division facts.

Arithmetic of the whole numbers: TEMAC programed learning. Encyclopedia Britannica Press.

Unit course for teaching the four basic arithmetic operations with whole numbers. The techniques are developed step-by-step from basic definitions. For grades 6-8. 1582 frames.

Basic mathematics: a problem-solving approach. Addison-Wesley.

Five programed texts to help guide slow students in principles of basic mathematics. Book one covers patterns, the pendulum, discovering numerical relationships. Book two is on equations. Book three is about equations, formulas. Book four develops fractions, multiplication and division of fractions. Book five is concerned with fractional equations, decimals, and percent.

Basic mathematics: TEMAC programed learing. Encyclopedia Britannica Press.

Five books constituting a full year course in general mathematics. Useful for a remedial course in pre-algebra math. 4992 frames. Useful in general 9th grade math and for review in 7th and 8th math.

Be a better reader series, books 1-6. Prentice-Hall.

The series by Nila Banton Smith helps in improving basic reading skills and in developing special skills needed in reading science, mathematics, social studies and literature in junior and senior high school. For effective reading in mathematics there is practice in reading concepts, signs, and geometric figures.

Decimals and percentage: tutor text. Doubleday.
Basic instruction in this area of everyday arithmetic. Each page of information has a question at the bottom. The reader selects the correct answer from a number of choices in order to proceed. If a mistake is made, the error is explained in detail and another try is in order. Thus, the reader understands what he studies.

Decimals and per cents. Allyn.
Book of 196 pages, 963 frames. A write-on program.

Fractions: tutor text. Doubleday.
The nature of fractions and practice in their use.

Guidebook to mathematics. Economy Co.
A useful review of mathematics fundamentals for teenagers and adults. Tries to develop ability in the use of mathematics in daily life with budgets, interest rates, and the use of bank accounts.

Introduction to verbal problems in algebra: TEMAC programed learning. Encyclopedia Britannica Press.
Provides the verbal strategy for solving word problems. 1024 frames.

Learning about fractions. Graflex.
78 frames give the function of the numerator and denominator. There is an accompanying flannel board.

Learning to learn. Harcourt.
Mature students may want to use the section on reading comprehension in mathematics.

Lessons for self-instruction in basic skills: arithmetic fundamentals--addition, division, multiplication, and subtraction. series E-F, gr. 7-8; series G, gr. 9+. California Test Bureau.
The series covers reading, arithmetic fundamentals, contemporary mathematics, and the language of mathematics for grades 3-9 and above. There are five different titles. Only those sections were suggested which are appropriate to the equivalency program. It is a multi-level program in reusable booklets with separate answer sheets useful for classroom activity and independent study. The lessons are intrinsic and not linear. There are record sheets to indicate progress for students weak in certain areas.

Lessons for self-instruction in basic skills: contemporary mathematics. California Test Bureau.
Deals with sets and set symbols, bases, properties of whole numbers, the operations of addition, subtraction, multiplication, and division. Considers modular arithmetic.

Preparing for algebra: TEMAC programed learning. Encyclopedia Britannica Press.
Focuses on the fundamental operations of arithmetic. Treats fractions, decimals, and exponents in the context of equation solution. 1809 frames.

Programed math. McGraw.
There are eight teaching skills books for remedial work in lower-track mathematics in this structured series. Word problem books, which take up basic addition, advanced addition, subtraction, multiplication, division, fractions, and decimals, accompany the teaching skills books. In addition, tests, placement exams, progress tests, teacher guides are available.

Ratios and proportions: TEMAC programed learning. Encyclopedia Britannica Press.
Teaches fundamental operations necessary in figuring ratios and proportions, stressing products of extremes. 1344 frames.

Understanding problems in arithmetic. Coronet.
For grades 4, 5, 6. 60 pages.

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