

DOCUMENT RESUME

ED 040 252

UD 010 209

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TITLE A New Model of School Effectiveness.  
INSTITUTION Stanford Univ., Calif. Stanford Center for Research and Development in Teaching.  
PUB DATE Feb 70  
NOTE 36p.; Paper presented at a conference sponsored by the Office of Education, Bureau of Educational Professions Development, February 1970

EDRS PRICE MF-\$0.25 HC-\$1.90  
DESCRIPTORS \*Academic Achievement, Effective Teaching, Elementary School Students, Elementary School Teachers, \*Individual Power, Models, \*Parent Attitudes, Predictive Measurement, School Role, School Surveys, Statistical Analysis, Student Attitudes, \*Student Motivation, \*Teacher Influence  
IDENTIFIERS Coleman Report

ABSTRACT

This study estimates a model of the schools reflecting what is known of the educational process. The model proposed represents the variables of achievement, students' sense of efficacy, motivation, and parents' attitudes as completely interdependent. Thus, the latter three variables are of interest because not only do they affect achievement levels but they themselves are affected by achievement. The data used to estimate this system were derived from the Equal Opportunity Survey on which the Coleman Report was based. The sample is composed of sixth grade students in a large eastern city who had attended only the school in which they were enrolled at the time of the survey, 1965-66. Teacher characteristics are based upon averages for all the teachers in each school who were teaching in grades three to five. Since both explanatory variables and those which are going to be explained are interdependent, their values must be solved simultaneously in order to obtain unbiased estimates of their effects. The findings indicate that educational programs focusing on student attitudes may be able to compensate for "disadvantages" in socioeconomic background. (JM)

2/19/70

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## A NEW MODEL OF SCHOOL EFFECTIVENESS

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February 1970

This paper was prepared for a conference on "How do Teachers Make a Difference?" sponsored by the Bureau of Educational Professions Development of the U.S. Office of Education. The analysis in the paper has been drawn from a larger study which is being authored jointly with Stephan Michelson. Research support has been provided by the Stanford Center for Research and Development in Teaching, and the editing of the data was done at the Brookings Institution. The Center for Educational Policy Research at Harvard supported some of the computational costs.

In addition to Stephan Michelson, the author is indebted to Randall Weiss for his research assistance and for his thoughtful contributions.

EDO 40252

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## A New Model of School Effectiveness

### Introduction

The subject of how schools affect the development of youngsters has been under intensive study for at least fifty years. In most cases the unit of analysis has been the classroom where attempts are made to relate differences in environmental and interaction variables to differences in student performance. The usual approach has been to set up experimental and control groups, to apply the "treatment" to the experimental one, and to look for significant differences in outcomes between the two groups. Unfortunately, the extensive research utilizing this methodology has not come up with a reasonably consistent and reproducible set of findings on how differences in schools create differences in human development.

Certainly one of the reasons for the inability of these experiments to provide useful conclusions is the assumption of ceteris paribus, all other things being equal between control and experimental groups. Rather, the complexity of the world within which education takes place suggests that observed similarities between control and experimental groups on one or two dimensions is not adequate for the ceteris paribus assumption. Many influences must be accounted for in seeking the determinants of scholastic achievement, attitude formation, and so on.

In the last decade a number of studies have attempted to go beyond the standard type of educational experiment by using large-scale multi-variate statistical models to account for many more variables than could be included in the typical control group/experimental group comparison. These studies have related the achievement of students to variables reflecting the student's race, socio-economic status, teacher and other school variables, as well as the characteristics of fellow students. The rather consistent set of findings emerging from these

studies suggests that three measured factors are significantly related to student academic achievement: race and socio-economic status of the student as well as the characteristics of his teachers.<sup>1</sup>

Generally these endeavors have utilized survey data on student achievement, socio-economic backgrounds, and school resources to explain variance in student achievement. Typically, their findings are based upon fitting a linear regression via the ordinary least-squares criterion for the following formulation:

$$A_{it} = F (B_{it}, S_{it}, O_{it})$$

where  $A_{it}$  is the standardized achievement score of the  $i$ th student at time  $t$ ;  $B_{it}$  represents a vector of family background characteristics at time  $t$ ;  $S_{it}$  represents a vector of school resources such as teacher characteristics, facilities, student environment created by peers, and so on at time  $t$ ; and  $O_{it}$  represents community and other characteristics that might affect achievement. These attempts might be conceived of broadly as attempts to estimate educational production functions. That is, studies of the educational production process are analogous to the econometric effort of estimating production processes in other industries.<sup>2</sup> While it is not the purpose of this study to review all of the properties of educational production functions and the problems encountered in estimating them, it is useful to discuss briefly a few of these.

<sup>1</sup>See the survey of these studies in James W. Guthrie, George B. Kleindorfer, Henry M. Levin, and Robert Stout, Schools and Inequality: A Study of the Relationships between Social Status, School Services, and Post-School Opportunity in the State of Michigan, a report prepared for the National Urban Coalition, Washington, D. C. (mimeo, September 1969).

<sup>2</sup>For a survey of econometric work on production functions see A. A. Walters, "Production and Cost Functions: An Econometric Survey," Econometrica, Vol. 31, Nos. 1-2 (January-April, 1963), pp. 1-66. The most comprehensive work on educational applications is Samuel S. Bowles, "Towards an Educational Production Function." A paper prepared for the Conference on Research in Income and Wealth (Madison, Wisconsin, November 1968), mimeo. The theory of production can be found in any basic text on microeconomics. See for example, William J. Baumol, Economic Theory and Operations Analysis (Englewood Cliffs, N. J.: Prentice Hall, 1963), Chap. 11.

### The Focus on a Single Output

Most studies of the educational production function have used standardized achievement scores as the output of the process. Yet, schools are expected to produce many outcomes in addition to increasing academic achievement.<sup>3</sup> The formation of a variety of attitudes and skills as well as many social externalities are attributed to the schools.<sup>4</sup> An empirical analysis of educational production that considers only one output ignores these other outcomes. Only if these other outcomes are produced in fixed proportion to the output under scrutiny does no problem arise in focusing on a single output such as standardized achievement.<sup>5</sup>

Ideally, the estimation of the educational production process should be based upon total educational output. That is, in some way we would want to weight the outputs produced by some common factor (utilities, votes, social values) in order to obtain a total index of output. Multi-product firms that sell their outputs in the marketplace are able to obtain such a measure by using prices as weights to obtain a monetary value for total product. Unfortunately, we can neither measure all of the outputs that schools are supposed to produce nor do we possess a yardstick or "numeraire" to put them into an index of output.

This focus on achievement scores as the single measure of school output creates at least two problems in measuring the educational production process.

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<sup>3</sup>For classifications of these, see Benjamin Bloom (ed.), Taxonomy of Educational Objectives, Handbook I: Cognitive Domain (New York: David McKay Co., Inc., 1956); and D. R. Krathwohl, B. S. Bloom, and B. B. Masia, Taxonomy of Educational Objectives (New York: David McKay Co., Inc., 1964).

<sup>4</sup>See Burton Weisbrod, External Benefits of Public Education, An Economic Analysis (Princeton, N. J.: Industrial Relations Section, Department of Economics, Princeton University, 1964).

<sup>5</sup>There is no empirical verification for this assumption.



First, the single focus on achievement limits the usefulness of educational production studies to providing insights for only one dimension of school output. The efficient ordering of inputs for producing achievement may be exceedingly inefficient for increasing student motivation, efficacy, imagination, and other desirable outcomes. This study will attempt to partly reconcile this problem by considering relationships among educational inputs and several outputs.

Second, estimates of the educational production process will underestimate the relation between any single output and school resources as long as priorities for that output vary among schools. To take an extreme case, academic high schools tend to emphasize language skills much more heavily than do vocational high schools. Accordingly, equal resources devoted to both groups of schools, ceteris paribus would likely have a greater impact on verbal achievement among the academic students than the vocational ones.

This relationship is further confounded if the priorities of schools vary according to the socio-economic composition of their student bodies. Certainly, the middle class schools are generally more academically oriented in a college-preparatory sense than are the lower class schools which seem to emphasize more heavily the general or job-oriented curricula. In such a case the socio-economic background variables of the students act as a proxy for the emphasis on academic skills relative to other school goals; and their statistical importance in "producing" academic achievement scores will be overstated while the impact of school resources will be understated.

#### Educational Production Theory and the Meaning of Production Data

Estimates of production functions in other industries are based upon the assumption that firms are maximizing output for any set of inputs; that is, firms are assumed to be technically efficient. Only under these conditions will estimates relating inputs to output reflect the most efficient way of producing that output.

In order to satisfy that assumption there are at least three general conditions that must be presumed. (1) The firm has knowledge of the relevant production set; (2) the firm has discretion over the way in which inputs are used; and (3) there is an effective incentive that spurs the firm to apply its knowledge of the production set and its ability to combine inputs into maximizing output for any set of physical inputs. Under these conditions the observed production data depict the production frontier, the largest output attainable for each set of inputs. Whether these are valid presumptions for private firms may be open to question, but they are clearly inappropriate ones for the schools.<sup>6</sup> There is no basis for asserting that educational decision-makers know their relevant production sets or that they have a great deal of discretion over how their inputs are used. The present organization of school inputs tends to be based on sacrosanct traditions rather than management discretion. Finally, the incentives of the marketplace that spur firms to be technically (and allocatively) efficient--profits, sales, and so on--are conspicuously absent from the educational scene. In particular, there is no evidence that educational firms such as schools and school districts maximize standardized achievement. Thus, at best the observations on inputs and outcomes represent average ones under the present state of operations, not maximum or technically efficient ones.

Moreover, the lack of knowledge on the relevant production set means that one cannot specify with reasonable accuracy the inputs germane to any particular output. Specification of the educational production model must depend more on intuition and hunch than on a body of well-developed behavioral theory. That is, there is no well-validated theory of learning on human development which can be used as a

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<sup>6</sup>For a discussion of their relevance to estimating production functions for industry, see Dennis Aigner and S. F. Chu, "On Estimating the Industry Production Function," American Economic Review (September 1968), No. 4, pp. 826-839.

guide in specifying inputs and the general functional relationships between inputs and outputs. In the absence of such a foundation, much of the early work in estimating educational production relations has necessarily involved a hunting expedition into the deep entangled forest of possible educational influences. The problem with such an expedition is that we have been like hunters shooting at anything that moved since we have had no clear picture of the animals that we wanted to collect.

A second and related problem is that even when we do know what kind of conceptual animal we wish to bag, we do not know how or where to capture it. Clearly, innate intelligence should be considered as an input when attempting to estimate the educational production function for achievement. Yet, like the mythical unicorn there has been much written about innate intelligence, but no one has ever seen one. That is, we have no way of measuring this important determinant of educational outcomes. Moreover, measures of teacher proficiency or other school inputs are not available. Rather we must use such conventional indicators as teacher experience, degree level, number of books in the library, and so on in the hope that we are capturing some of the actual influences of which we are unaware or which we are unable to measure adequately.

The result of both not knowing how education is produced and not being able to measure many of the inputs suggests a high probability of bias in the estimates of the production coefficients. The exclusion of variables that belong in the equation as well as the inclusion of erroneous variables all lead to such biases.<sup>7</sup> Moreover, the fitting of such data to a linear function can also result in specification biases in a world that is characterized by non-linearities. All of the

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<sup>7</sup>Henri Theil, "Specification Errors and the Estimation of Economic Relationships," Revere Institute Internationale de Statistique, Vol. 25 (January 1957), pp. 41-51.



empirical studies of the educational production process are prime candidates for such biases.

### Data Refinement

Perhaps it is useful to divide data problems into two types: intransigent and remediable. In actuality this dichotomy is a state of the art distinction rather than one which is in the stars. At a future time, intransigent difficulties may be alleviated by greater knowledge of the phenomenon or by better measurement techniques. Examples of the former problem are our inability to measure innate abilities. As we noted above, the omission of such a variable is likely to induce a bias in our estimates. In such a case it is important that we explore the biases from not including such a measure in the specification of our production model, but we can do little beyond this.

On the other hand, data deficiencies arise that are partly or fully remediable. For example: a needed item is sufficiently measureable, but it was omitted from the survey on which the production estimates will be made. In such a case, one can attempt to find a close proxy among the existing information source or one can resurvey to obtain the missing item. The latter alternative is time-consuming and costly, so it is often the former course of action that is taken. Yet, the use of a proxy or surrogate piece of information is subject to the vagaries of interpretation, and its use may create more problems than it solves.<sup>8</sup> In many cases it may be wise to acknowledge the omission and to speculate on the resulting bias rather than to use a questionable proxy.

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<sup>8</sup>As an illustration, Bowles uses the number of days that the school was in session as a proxy "...to represent the general level of community interest in and support of education." op. cit., p. 49. Yet, such an indicator is more likely to be governed by State mandate than by community educational interests, educational support, and political processes. That is, each State requires a minimum session in order for the school district to qualify for aid. Accordingly, the main variance in the measure is accounted for among States. For the national sample used by Bowles the mean for the "days-in-session" variable was 180 and the standard deviation was only 4.

Yet, in all too many instances data problems are remediable, and in those cases the information should be refined to more closely approximate the concept which they are expected to represent. Most studies examining the educational production process have used school data for each student whether the student had actually attended the school in the past or whether he hadn't. For example, the EEO survey was undertaken in September-October of the 1965-66 school year. Clearly the relevant school data for each child are those pertaining to the schools that he actually attended, and in many cases the school that he was attending in 1965-66 was different from those that he had previously attended. That is, the high rate of residential mobility is translated into school mobility, and present school factors may be erroneous measures for actual school characteristics unless some data refinement is attempted.<sup>9</sup>

To the degree that the school factors used in the analysis are spurious ones, the estimated effect of them on achievement will be biased downward.<sup>10</sup> Unfortunately, this problem pervades the EEO work as well as its reanalysis, and the problem is more serious among the analyses for blacks and other minorities than for whites because of the higher mobility factor among the former groups. One way of correcting for this source of error is to include in the sample only those students who had received all of their education in the schools which they were currently attending. That is the approach taken in this study. Another possibility is that of obtaining historical data on all of the schools that the students attended. Given the fact that much school mobility is among school districts and states, this task may be beyond the realm of practicality.

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<sup>9</sup>See S. Bowles and H. M. Levin, "The Determinants of Scholastic Achievement--An Appraisal of Some Recent Evidence," The Journal of Human Resources, Vol. III, No. 1 (Winter 1968), pp. 3-24.

<sup>10</sup>See John Kain and Eric Hanushek, "On the Value of Equality of Educational Opportunity as a Guide to Public Policy," Program on Regional and Urban Economics, Discussion Paper No. 36, Harvard University (May 1968).

Other data problems that are remediable are those resulting from missing observations of items for particular students. The EEO Survey suffered particularly from these hindrances.<sup>11</sup> There are many ways of handling this problem, but ignoring it is clearly not one of them.<sup>12</sup> A final difficulty that characterizes the data sets used for measuring educational production is the interdependence among the so-called explanatory variables. In general, a child's home background and his school are highly correlated in that higher socio-economic status children attend schools with greater resource endowments. This factor has prevented many studies from obtaining reliable estimates of the separate effects of school and background characteristics on achievement.<sup>13</sup> One way of circumventing this difficulty is to carry out the analysis for stratified subsamples of students with homogeneous socio-economic backgrounds.<sup>14</sup>

#### Purpose of This Study

While we have noted some of the problems that arise in applying econometric analysis of production to the schools, this study will not make the heroic claim of having avoided such pitfalls. Rather, this effort addresses itself to moving towards estimating a model of the schools that more nearly mirrors what we know of the educational process. Indeed, we will proceed in the following way: First, we will posit a model of the schools and compare it with the more traditional formulation; second, we will discuss the data that will be used to estimate the

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<sup>11</sup>See S. Bowles and H. M. Levin, op. cit., pp. 6-7.

<sup>12</sup>See Janet Elashoff and R. M. Elashoff, "On Regression Analysis with Missing Data," Computers, Data Bases, and the Social Sciences, Ralph Bisco (ed.), John Wiley & Sons, forthcoming.

<sup>13</sup>This has been discussed at length by Bowles and Levin in "The Determinants of Scholastic Achievement," and by the same authors in "More on Multicollinearity and the Effectiveness of Schools," The Journal of Human Resources (Summer 1968), pp. 393-400.

<sup>14</sup>This has been attempted in Herbert Kiesling, "Measuring a Local Government Service: A Study of School Districts in New York State," Review of Economics and Statistics (August 1967), pp. 356-367. Also see James W. Guthrie, et al., op. cit., pp. 135-144.

structure of the model; third, we will review the estimation procedure and results; and finally, we will discuss the implications.

### Specification of the Model

Most studies of educational production have not attempted to specify in a systematic way the particular formulation of how schools affect achievement. Rather, they have taken a set of school and student background factors and related them statistically to achievement without discussing the underlying behavioral assumptions implied by their work. One exception has been an important study by Eric Hanushek that did posit a more concrete model of achievement.<sup>15</sup> The following formulation is based upon Hanusek's foundation.

Assume that we wish to examine the determinants of student achievement at a point in time. Clearly, that achievement level is related not only to the present influences that operate on that student, but also to past ones. That is, from the time a child is conceived various environmental characteristics combine with his innate characteristics to mold his behavior. More specifically, a child's achievement performance is determined by the cumulative amounts of "capital" embodied in him by his family, his school, his community, and peers as well as his innate traits. The greater the amount and the quality of investment from each of these sources, the higher will be the student's achievement level. Thus, a student's academic performance is viewed to be a function of the amount of different kinds of capital embodied in him.

The general formulation of the capital embodiment model is as follows:

$$(1) A_{it} = g \left[ F_{i(t)}, S_{i(t)}, P_{i(t)}, O_{i(t)}, I_{it} \right]$$

where the  $i$  subscript refers to the  $i$ th student; the  $t$  subscript refers to time period  $t$ ; and the  $t$  subscript in parentheses ( $t$ ) refers to being cumulative to

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<sup>15</sup>See Eric Hanushek, The Education of Negroes and Whites (Unpublished Doctoral Dissertation, Massachusetts Institute of Technology, 1968).



time period  $t$ . Thus:

$A_{it}$  = a vector of educational outcomes for the  $i$ th student at time  $t$ .

$F_{i(t)}$  = a vector of individual and family background characteristics cumulative to time  $t$ .

$S_{i(t)}$  = a vector of school inputs relevant to the  $i$ th student cumulative to time  $t$ .

$P_{i(t)}$  = a vector of peer or fellow student characteristics cumulative to  $t$ .

$O_{i(t)}$  = a vector of other external influences (community, etc...) relevant to the  $i$ th student cumulative to  $t$ .

$I_{it}$  = a vector of initial or innate endowment of the  $i$ th student at  $t$ .

It is assumed that  $g'$  is positive for all these arguments or that the marginal product of additional capital embodiment from any one of the five sources has a positive effect on student educational outcome.<sup>16</sup>

This formulation reflects the well-accepted concept that a child receives his educational investment from several sources in addition to the school. For example, the family provides a material, intellectual, and emotional environment which contributes to the child's performance level. Likewise, the school, peer groups, and community affect both learning and emotional behavior of students. Yet, in order to estimate these effects, one must take this general formulation and make it more specific.

Suppose we wish to follow the examples of other researchers by estimating a production function for achievement. Again, we can view a student's level of achievement on a verbal test, for example, as a function of his capital embodiment from several sources as well as his innate traits. But, in addition to

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<sup>16</sup> Following the capital embodiment approach more strictly, Dennis Dugan has calculated the monetary value of parents' educational investment in their offspring by calculating the opportunity cost or market value of such services. The values of father's educational investment, mother's educational investment, and school investment (all measured in dollars) seem to have high combined predictive value in explaining achievement levels. See Dennis Dugan, "The Impact of Parental and Educational Investments Upon Student Achievement." Paper presented at 129th Annual Meeting of the American Statistical Association (New York City, August 21, 1969), mimeo.



these sources of capital embodiment, his educational achievement at a point in time is likely to be related to his educational attitudes and his parents' educational attitudes. More specifically, we might postulate that:

$$(2) \quad A_{1it} = g \left[ F_i(t), S_i(t), P_i(t), O_i(t), I_i(t), A_{2it}, A_{3it}, A_{4it} \right]$$

where

$A_{1it}$  = the achievement level of the  $i$ th student at  $t$ .

$F_i(t)$ ,  $S_i(t)$ ,  $P_i(t)$ , and  $O_i(t)$  are as previously defined;

$A_{2it}$  = a measure of the student's sense of efficacy or fate control at  $t$

$A_{3it}$  = a measure of educational motivation of the  $i$ th student at  $t$

$A_{4it}$  = parents' educational expectations for the  $i$ th student at  $t$

That is, we would expect student achievement to be higher the greater his sense of efficacy, his educational motivation, and his parents' expectations, ceteris paribus. By efficacy we refer to the student's feeling that he has a measure of control over his destiny, that it does not depend strictly on chance. Educational motivation refers to the desire to succeed in an educational sense (for example, the desire to get good grades and to attain additional schooling). Parents' educational expectations might be viewed as how well the parents expect the child to perform by educational criteria.

But these three variables are of more than passing interest because not only do they affect achievement levels, but they themselves are affected by achievement. This raises the question of whether a single equation is adequate for estimating educational production, even when one is concerned with only a single measure of output such as achievement. That is, the single equation model tacitly assumes that each of the explanatory variables is determined outside of the system; that is they are exogenous. In other words, the explanatory variables influence the level of student achievement, but student achievement is assumed not to influence the so-called explanatory variables.

An illustration of this assumption and its lack of realism in the present instance is useful. Let us start off with a very simple model of achievement where student efficacy is considered to be the only factor affecting student achievement, all other factors being held constant. We can present this simple paradigm by drawing an arrow showing the causal direction that is assumed:

Student Achievement ← Student Efficacy

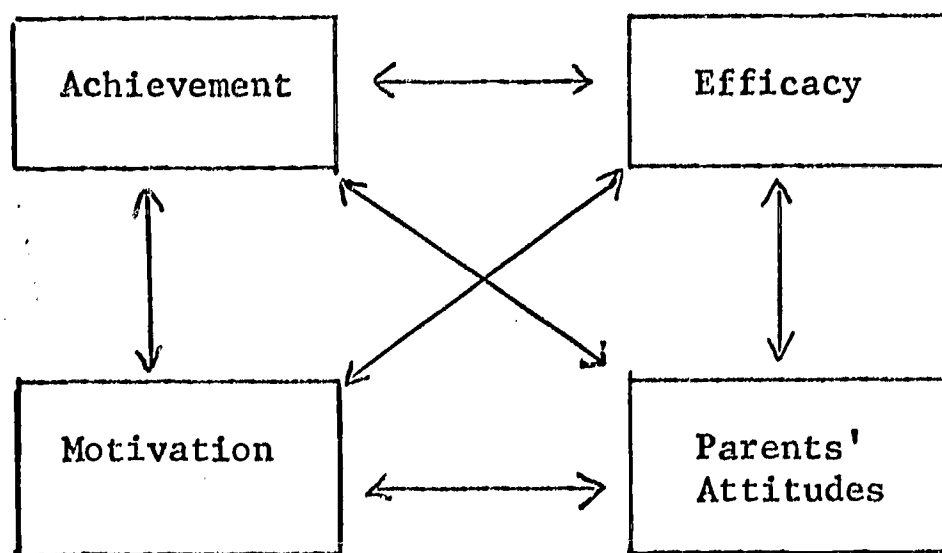
This simple depiction suggests that student achievement is greater the higher the level of efficacy. In process terms, students who believe that they have a measure of control over their achievement level are more likely to try to do well than students who believe that it all depends upon luck. But it is probably also true that the higher the level of his achievement, the higher the level of his efficacy. That is, by doing well his sense of fate control is enhanced or reinforced because his efforts can really make a difference in his achievement. Thus, achievement stimulates efficacy and efficacy stimulates achievement as depicted below:

Student Achievement ↔ Student Efficacy

Moreover, the other attitudinal variables that influence such school outputs as standardized achievement performance are also influenced themselves by student achievement and by each other. For example, parents' educational expectations for a student will affect the student's performance level; but the student's performance level will also affect the parent's educational aspirations for him. Most parents will expect less from a child who has consistently low test scores and grades than one who has higher levels of both attributes. The same is probably true of teacher expectations for pupil progress. In summary, many crucial variables in the educational process interact in such a way that we cannot take their levels as given

in order to predict other factors. Rather both explanatory variables and those which we wish to explain are interdependent, and their values must be solved simultaneously in order to obtain unbiased estimates of their effects.

That is the following relationship exists in concept. In this particular system, everything



depends upon everything else, so that complete simultaneity exists. Every one of the variables is linked by a double arrow to every other variable. In actuality the simultaneity may be complete or partial, but in either case the ordinary least-squares solution of equation (2) will lead to biased and inconsistent estimates.<sup>17</sup> Rather, we must estimate the full set of equations representing the simultaneous equations system.

<sup>17</sup> That is, the residual term is likely to be correlated with  $A_{2it}$ ,  $A_{3it}$ , and  $A_{4it}$ , and the direct application of the ordinary least squares estimator will not yield unbiased estimates of the structural parameters of (2). See J. Johnston, Econometric Methods (New York: McGraw Hill Book Company, 1963), Chap. 9.

The following formulation describes the simultaneous equation model:

$$(3) \quad A_{1it} = g_1 \left[ F_{1i(t)}, S_{1i(t)}, P_{1i(t)}, O_{1i(t)}, I_{1it}, A_{2it}, A_{3it}, A_{4it} \right]$$

$$(4) \quad A_{2it} = g_2 \left[ F_{2i(t)}, S_{2i(t)}, P_{2i(t)}, O_{2i(t)}, I_{2it}, A_{1it}, A_{3it}, A_{4it} \right]$$

$$(5) \quad A_{3it} = g_3 \left[ F_{3i(t)}, S_{3i(t)}, P_{3i(t)}, O_{3i(t)}, I_{3it}, A_{1it}, A_{2it}, A_{4it} \right]$$

$$(6) \quad A_{4it} = g_4 \left[ F_{4i(t)}, S_{4i(t)}, P_{4i(t)}, O_{4i(t)}, I_{4it}, A_{1it}, A_{2it}, A_{3it} \right]$$

In this system there exists an equation for each of the endogenous variables. Two characteristics of the system are of immediate importance. First, the solution of the system depends upon its identifiability. In general, proper identification requires that there be as many equations as endogenous variables and that all variables are not present in all relations.<sup>18</sup>

In this regard, it should be noted that the specification of each of the exogenous variables is unique in each relation. That is, it is reasonable to believe that different family factors, school factors, innate characteristics, and so on, affect achievement  $(A_{1it})$  than affect the other endogenous variables  $(A_{2it})$ ,  $(A_{3it})$ , and  $(A_{4it})$ . Accordingly,  $F_{1i(t)}$  is considered to be a different vector of family influences than  $F_{2i(t)}$ ,  $F_{3i(t)}$ , and  $F_{4i(t)}$ .<sup>19</sup>

<sup>18</sup>A description of the identification problem is found in J. Johnston, *op. cit.*, pp. 240-252. Also see Franklin Fisher, "Generalization of the Rank and Order Conditions for Identifiability," *Econometrica*, Vol. 27 (1959), pp. 431-447.

$$^{19} F_{i(t)} = \left[ f_{1i(t)}, f_{2i(t)}, \dots, f_{ni(t)} \right]$$

That is, there are  $n$  elements in the  $F_{i(t)}$  vector, but not all of them are germane to any particular equation.

The potential uniqueness of  $S_i(t)$  for each equation is also represented by the appropriate subscript as well as the uniqueness of the other vectors. It is particularly useful if we can distinguish between school characteristics that relate to achievement  $(A_{1it})$  and those that relate to student and parental attitudes.

A second characteristic of the system represented by equations (3), (4), (5), and (6) is that each of the endogenous variables represents an output of the educational process as well as an input into it. Just as schools are expected to increase achievement, they are also expected to contribute to such attitudes as efficacy and motivation. Thus, we can evaluate the system for each of several outcomes rather than restricting ourselves only to the analysis of student achievement.<sup>20</sup> The system of equations allows us to solve for student efficacy, student motivation and parents' educational expectations as well as student achievement.

#### Estimating the Equations

The data used to estimate this system were derived from the Equal Opportunity Survey on which the Coleman Report was based. The sample is composed of sixth-grade students in a large eastern city who had attended

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<sup>20</sup>The parents' attitude variable might be considered to be an intermediate output in that its social value is more a function of its effectiveness in producing other outputs rather than its use as an end in itself. In a similar vein the teachers' attitudes might be introduced into the model as an endogenous variable.



only the school in which they were enrolled at the time of the Survey, 1965-66.<sup>21</sup> Teacher characteristics are based upon averages for all of the teachers in each school who were teaching in grades 3-5. These averages were intended to reflect the teacher characteristics that had influenced student behavior up to the time of the survey. Since family background characteristics and other educational influences were measured only at a point in time, it is tacitly assumed that these measures bear a constant relation to the stock of capital embodied in each child from these sources. That is, it is assumed that the values of those inputs cumulative to time  $t$  bear a constant relation to the flow of inputs observed at time  $t$ .

While all of the equations specify innate traits as exogenous variables, we do not possess measures of  $I_{it}$ . That is, our statistical model does not include the  $I_{it}$  vectors despite the fact that they belong in the system, a priori. It is important to speculate on the expected bias in the estimates of the other parameters, if the students' innate traits are not included in the equations. In general, those variables that are correlated with the omitted one will be biased upwards.<sup>22</sup>

It is probably reasonable to assume that innate traits have at least

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<sup>21</sup>These data were derived jointly with Stephan Michelson at The Brookings Institution from magnetic tapes provided by Alexander Mood. The same set of data are used in the Michelson paper contained in the volume, "The Association of Teacher Resources with Children's Characteristics."

<sup>22</sup>See Henri Theil, op. cit.

some component that is reflected in the vector of family background characteristics.<sup>23</sup> Even if one minimizes the possible genetic relation between parental traits and the child's innate characteristics, there are other possible linkages. In particular, the child drawn from lower origins is a more likely candidate for prenatal protein starvation, a factor which may limit his innate potential.<sup>24</sup> The result of the probable association between family background characteristics and student's innate traits is that the effect of the  $F_i(t)$  vector on achievement (and perhaps on other outcomes) will be overstated. That is, family background characteristics will be biased upwards to the extent of their covariance with the missing variable, innate characteristics. In general, it is reasonable to conclude that all of the studies that have tried to explain the determinants of scholastic achievement have overstated the effects of family background by omitting measures of innate traits.

### Some Results

What follows are some estimates of a simultaneous equation system similar to that posited above. The particular sample in this analysis

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<sup>23</sup>For contrasting views on the extent to which innate traits are genetically determined with particular emphasis on "intelligence," see J. McV. Hunt, *Intelligence and Experience* (New York: Ronald Press, 1961); and Arthur R. Jensen, "How Much Can We Boost IQ and Scholastic Achievement?" *Harvard Educational Review*, Vol. 39, No. 1 (1969), pp. 1-23. See also Gerald Lesser and Susan S. Stodolsky, "Learning Patterns in the Disadvantaged," *Harvard Educational Review*, Vol. 37, No. 4 (1967), pp. 546-93.

<sup>24</sup>See Nevin S. Scrimshaw, "Infant Malnutrition and Adult Learning," *Saturday Review*, Vol. 51, No. 11 (March 16, 1968), pp. 64-66.

consists of almost 600 white students attending some 36 schools in Eastmet City. The basis on which particular variables were chosen to enter the relation was based partially on a priori judgment, partially on statistical tests of significance, and partially on the quality of the measures.

On the basis of over 100 items of information that we distilled from the original survey data, we chose those variables that might be expected, logically, to enter into each relation. As an example, the quality of library services as represented by library books per student might reasonably be expected to affect the student's achievement level; yet, one would be hard pressed to discern a direct relationship between student's and parents' attitudes and library books. Accordingly, the library measure was specified only in the achievement equation. Likewise, such information as teacher's salary is reflected in the teacher characteristics that the salaries purchase.<sup>25</sup>

Some items that were entered showed statistical relationships that were so nearly random that they were eliminated from subsequent equations. Whether the lack of a statistical association was due to their poor measurement or their misspecification cannot be determined a priori. What follows is a set of estimates that must be judged only for their heuristic values. That is, alternative specifications are equally plausible, and the grounds

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<sup>25</sup> For more information on this relationship see Henry M. Levin, Recruiting Teachers to be published by Charles E. Merrill. Also see H. Levin, "A Cost-Effectiveness Analysis of Teacher Selection," The Journal of Human Resources, Vol. V, No. 1 (Winter 1970), pp. 24-33.

for specification biases are substantial.<sup>26</sup> Further refinement of the

insert Tables 1-5

data and the specifications are undoubtedly necessary before firm policy influences can be drawn.

Table 1 shows the list of all variables included in the estimates; and Tables 2, 3, 4, and 5 show estimates of the equations for verbal score, student's attitude, grade aspiration, and parents' attitude, respectively. The sample comprises 597 white students in the sixth grade of Eastmet City in the Fall, 1965.

Before interpreting the results, it is important to note that the statistical model used here differs slightly from that shown in equations (3), (4), (5), and (6) in that only the first three equations are estimated simultaneously. That is, the fourth equation is estimated by

ordinary least-squares, and it bears a recursive relation to the rest of the model. The figure that follows illustrates this property as well as the simultaneous relationships estimated among the other

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<sup>26</sup>Under certain conditions the simultaneous equation estimates are subject to greater specification biases than the ordinary least squares ones. See Robert Summers, "A Capital Intensive Approach to the Small Sample Properties of Various Simultaneous Equations Estimators," Econometrica (January 1965) pp. 1-47. Also see Franklin M. Fisher, "The Relative Sensitivity to Specification Error of Different k-class Estimators," Journal of the American Statistical Association (June, 1966, Vol. 61, No. 314 Part 1, pp. 345-57. Stephan Michelson has shown results for alternative specifications of the single equation model in op. cit. published in this volume.

TABLE 1

## List of Variables in Simultaneous Equations System

<u>Name of Variable</u>	<u>Measure of</u>	<u>Coding</u>
Verbal Score	Student Performance	Raw Score
Student's Attitude	Efficacy	Index compiled from questions 33-40 in the Sixth Grade Student Questionnaire of the Equal Opportunity Survey. (e.g., I can do many things well. Yes No Not Sure I sometimes feel I just can't learn. Yes No The higher the value of the index, the greater the perceived efficacy of the student.)
Parents' Attitude	Educational Expectations of Parents	Index based upon three questions: (1) How good a student does your mother want you to be? (2) How good a student does your father want you to be? (3) Did anyone at home read to you when you were small, before you started school? (and how often?)
Grade Aspiration	Student Motivation	Grade Level the Student Wishes to Complete
Sex	Male-Female Differences	Male = 0 Female = 1
Age	Over-Age for Grade	Age 12 or over = 1 Less than 12 = 0
Possessions in Student's Home	Family Background (Socio-economic Status)	Index of possessions: Yes = 1 No = 0 for each; Index is sum. television telephone dictionary encyclopedia automobile daily newspaper record player refrigerator vacuum cleaner



TABLE 1 (Cont'd.)

<u>Name of Variable</u>	<u>Measure of</u>	<u>Coding</u>
Family Size	Family Background	Number of people in home
Identity of Person Serving as Mother	Family Background	Real Mother at Home = 0 Real Mother not Living at Home = 1 Surrogate Mother = 2
Identity of Person Serving as Father	Family Background	Real Father at Home = 0 Real Father not Living at Home = 1 Surrogate Father = 2
Father's Education	Family Background	Number of Years of School Attained
Mother's Employment Status	Family Background	Has Job = 1 No Job = 0
Attended Kindergarten	Family Background	Yes = 1 No = 0
Teacher's Verbal Score	Teacher Quality	Raw Score on Vocabulary Test
Teacher's Parents' Income	Teacher Socio-economic Status	Father's occupation scaled according to income (1000's of dollars)
Teacher Experience	Teacher Quality	Number of Years of Full-time Experience
Teacher's Undergraduate Institution	Teacher Quality	University or College = 3 Teacher Institution = 1
Satisfaction with Present School	Teacher's Attitude	Satisfied = 3 Maybe Prefers Another School = 2 Prefers Another School = 1
Percent of White Students	Student Body	Percentage estimated by teachers
Teacher Turnover	School	Proportion of teachers who left in previous year for reasons other than death or illness

TABLE 1 (Cont'd)

<u>Name of Variable</u>	<u>Measure of</u>	<u>Coding</u>
Library Volumes Per Student	School Facilities	Number of volumes divided by school enrollment

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Note: All data are taken from the Equal Opportunity Survey for Eastmet City. The survey instruments are found in James S. Coleman et al., Equality of Educational Opportunity (Washington, D. C.: U. S. Government Printing Office, 1966).

TABLE 2

Estimates of Verbal Score Equations  
for White Sixth Graders in Eastmet City  
(t values in parentheses)

	<u>Ordinary Least Squares</u>	<u>Two Stage Least Squares</u>	<u>Reduced Form</u>
Student's Attitude	0.641 (4.88)	2.649 (1.72)	
Grade Aspiration	0.921 (5.21)	0.591 (0.53)	
Parents' Attitude	0.605 (2.81)	0.873 (0.74)	
Sex	0.616 (1.06)	-0.571 (0.49)	0.817
Age	-6.099 (4.26)	-5.513 (2.78)	-6.010
Possessions	0.990 (3.84)	0.521 (1.05)	1.229
Family Size	-0.330 (2.14)	-0.036 (0.12)	-0.552
Identity of Mother	-----	-----	-0.433
Identity of Father	-----	-----	-0.327
Father's Education	0.243 (2.10)	0.026 (0.12)	0.273
Mother's Employment	-----	-----	-0.509
Attended Kindergarten	1.520 (1.73)	1.768 (1.32)	2.372
Teacher's Verbal Score	0.332 (1.61)	0.220 (0.84)	0.250
Teacher's Parent's Income	-----	-----	-0.118
Teacher Experience	0.751 (8.77)	0.694 (5.28)	0.787
Teacher Undergraduate Institution	6.547 (2.66)	5.833 (1.94)	6.525

TABLE 2 (Cont'd.)

	<u>Ordinary</u> <u>Least Squares</u>	<u>Two Stage</u> <u>Least Squares</u>	<u>Reduced</u> <u>Form</u>
Satisfaction w/ Present School	1.201 (0.90)	1.658 (0.86)	1.960
Percent of White Students	-----	-----	-0.047
Teacher Turnover	-0.054 (0.61)	0.044 (0.34)	-0.101
Library Volumes Per Student	0.562 (1.82)	0.498 (1.31)	0.565
Constant Term	-23.94	-29.75	-7.902
$\bar{R}^2$	.53	.34	

TABLE 3

Estimates of Student Attitude Equations  
for White Sixth Graders in Eastmet City  
(t values in parentheses)

	<u>Ordinary</u> <u>Least Squares</u>	<u>Two Stage</u> <u>Least Squares</u>	<u>Reduced</u> <u>Form</u>
Verbal Score	0.061 (5.54)	0.052 (2.03)	
Parents' Attitude	0.112 (1.69)	0.042 (0.15)	
Sex	0.560 (3.15)	0.557 (3.08)	0.577
Age	0.241 (0.54)	0.135 (0.27)	-0.015
Possessions	0.107 (1.39)	0.143 (1.29)	0.174
Family Size	-0.108 (2.30)	-0.124 (2.05)	-0.138
Identity of Mother	-----	-----	-0.011
Identity of Father	-0.082 (1.30)	-.092 (1.36)	-0.100
Father's Education	0.070 (2.02)	0.081 (1.88)	0.088
Mother's Employment	-0.318 (1.58)	-0.307 (1.44)	-0.320
Attended Kindergarten	-----	-----	0.059
Teacher's Verbal Score	-----	-----	0.006
Teacher's Parents' Income	-----	-----	-0.003
Teacher Experience	-----	-----	0.163
Teacher Undergraduate Institution	-----	-----	0.020



TABLE 3 (Cont'd.)

	<u>Ordinary</u> <u>Least Squares</u>	<u>Two Stage</u> <u>Least Squares</u>	<u>Reduced</u> <u>Form</u>
Satisfaction w/ Present School	-0.163 (0.42)	-0.129 (0.33)	-0.089
Percent of White Students	-----	-----	-0.001
Teacher Turnover	-0.047 (2.70)	-0.048 (2.73)	-0.051
Library Volumes Per Student	5.132	5.330	5.132
$\bar{R}^2$	.19	.19	

TABLE 4

Estimates of Grade Aspiration Equations  
for White Sixth Graders in Eastmet City  
(t values in parentheses)

	<u>Ordinary</u> <u>Least Squares</u>	<u>Two Stage</u> <u>Least Squares</u>	<u>Reduced</u> <u>Form</u>
Verbal Score	.0557 (6.75)	.0876 (4.18)	
Parents' Attitude	.0372 (0.75)	-0.391 (1.46)	
Sex	-0.111 (1.84)	-0.192 (1.30)	-0.077
Age	-0.351 (1.05)	-0.243 (0.63)	-0.772
Possessions	0.052 (0.87)	.074 (0.85)	0.092
Family Size	-0.057 (1.64)	-0.077 (1.62)	-0.079
Identity of Mother	-0.223 (2.35)	-0.310 (2.62)	-0.227
Identity of Father	-0.056 (1.11)	-0.560 (1.03)	-0.077
Father's Education	-----	-----	0.024
Mother's Employment	0.282 (1.89)	0.401 (2.34)	0.279
Attended Kindergarten	0.644 (3.20)	0.547 (2.47)	0.756
Teacher's Verbal Score	-----	-----	0.022
Teacher's Parents' Income	-0.0005 (0.38)	-0.176 (1.15)	-0.186
Teacher Experience	-----	-----	0.069
Teacher Undergraduate Institution	-0.460 (1.08)	-0.135 (0.28)	0.439

TABLE 4 (Cont'd.)

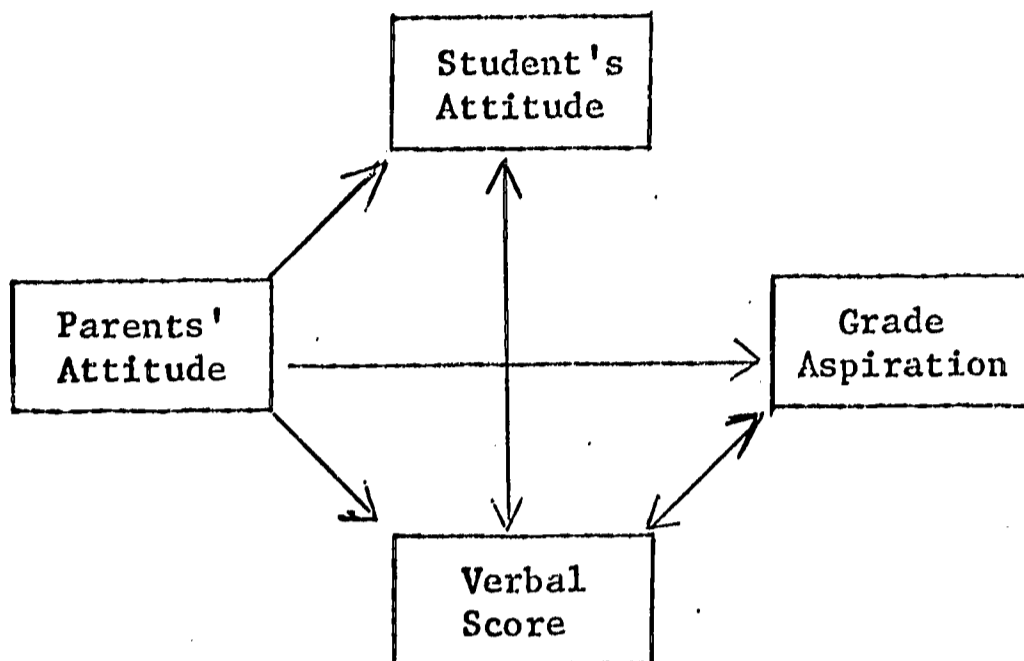
	<u>Ordinary Least Squares</u>	<u>Two Stage Least Squares</u>	<u>Reduced Form</u>
Satisfaction w/ Present School	0.785 (2.56)	0.693 (2.80)	0.866
Percent of White Students	-----	-----	0.021
Teacher Turnover	-----	-----	-0.005
Library Volumes Per Student	-----	-----	0.050
Constant Term	9.174	10.900	8.850
$\bar{R}^2$	.26	.15	

TABLE 5

Estimate of Parents' Attitude Equation  
for White Sixth Graders in Eastmet City  
(t values in parentheses)

	<u>Ordinary</u> <u>Least Squares</u>
Sex	-0.110 (1.00)
Possessions	0.218 (4.84)
Family Size	-0.119 (4.14)
Identity of Mother	-0.309 (4.36)
Identity of Father	-0.018 (0.44)
Mother's Employment	0.198 (1.59)
Percent of White Students	-0.065 (2.11)
Teacher's Turnover	-0.009 (.89)
Constant Term	3.465
$\bar{R}^2$	.13

equations. The system is overidentified a priori



because the endogenous variables are not common to each of the three simultaneous equations.

Two stage least-squares was used for the three simultaneous equations. Each of the Tables for the equations on verbal score, student's attitude, and grade aspiration show an ordinary least-squares estimate, a two-stage least-squares (simultaneous equations) estimate, and a reduced form. The latter is obtained by solving the simultaneous equations system via algebraic substitution.<sup>27</sup>

#### Some Interpretations

The interpretations that are given here are highly speculative. They are offered only as illustrations of the properties of the model. Further testing of the structure and improved data are necessary to confirm results

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<sup>27</sup>See J. Johnson, op. cit., pp. 231-236.



reported here. Accordingly, the interpretation of the findings is not an attempt to be exhaustive as much as it is an effort to show how this approach might be used ultimately to examine various hypotheses.

### Verbal Score

The variables entering the verbal score equation were selected as being representative of the different vectors in equation (3) with the obvious omission of innate traits. Such conventional teacher's characteristics as degree level showed no significant relation with student verbal score, although teacher's experience appears to be strongly related in this sample.

It is especially instructive to compare the ordinary least-squares estimates (which do not take account of the simultaneity) with the two stage estimates (which do take account of it). In this way we can note some of the biases in interpretation that might arise from the usual ordinary least squares estimates. In particular it appears that the direct effect of several family background characteristics on verbal achievement is overstated substantially in the single equation (OLS) estimate. For example, the coefficient for family size is only one-tenth as large in the TSLS estimate as the OLS one. This suggests that the large observed negative relation between family size and achievement in the ordinary least-squares formulation should not be interpreted as a direct effect, but one that works through an intervening variable, student's attitude. The much larger coefficient for student's attitude in the TSLS estimate in combination with the great decline in the family size coefficient in the simultaneous-equations formulation indicates that students from larger families probably

have lower verbal scores because of their poorer attitudes rather than because of an inextricable link between family size and other background characteristics on the one hand and achievement on the other. The existence of this phenomenon is also supported by the smaller coefficients in the TSLS estimate for such socio-economic factors as father's education and possessions.

The possible significance of these findings is that educational programs that focus on student attitudes may be able to compensate for "disadvantages" in socio-economic background. Indeed, this tentative interpretation argues against the simplistic observations of some social philosophers/ that educational programs cannot compensate for such background deficiencies as low socio-economic status. Since these background factors appear to have much of their direct effects on attitude and through attitudes, on achievement. Successful efforts to change student attitudes might be used to offset "deleterious" background conditions.

In this vein it is also interesting to note the reversal of sign for the sex variable between the OLS and TSLS estimates. In the OLS formulation females show higher verbal scores than males, while in the TSLS they show lower scores. Again, it appears that the higher verbal scores of females are more likely attributable to a higher sense of efficacy rather than to any direct sex-achievement effect. This is confirmed by the strong, positive coefficient for females in the student attitude equation in Table 3. It is also supported by the well established view that schools represent feminizing influences, receptive to girls and

hostile to boys. Under such conditions one would expect females to have greater efficacy and through efficacy, greater achievement.<sup>28</sup>

The reduced form equation shows all of the system's influences on verbal score--whether directly through the verbal score equation or indirectly through students' attitudes, grade aspiration, or parents' attitudes. On balance, sex is positively related to verbal score. Those variables that affect attitudes and grade aspiration directly are shown to affect verbal score because attitudes and grade aspiration affects verbal score. Thus, while the identity of the mother showed no significant direct relation with verbal achievement it does show a negative influence of a maternal substitute in the reduced form because of its direct negative relation on student grade aspiration. The same is true of father's identity which shows a direct negative effect of a father surrogate on student's attitude and thus on indirect effect in the reduced form on verbal score.

#### Other Equations

Table 3 presents comparable equations for student's attitude, and Table 4 shows them for grade aspiration. Because of the tentative nature of the findings at this stage of the art, we will not detail all of these results. Rather, we will focus on a pattern that is of general interest. In particular, it appears that when the mother has a job, the child's grade aspiration is higher (Table 4), but his efficacy or attitude is lower

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<sup>28</sup> See Patricia Sexton, Feminized Male: Classrooms, White Collars, and the Decline of Manliness (New York: Random House, 1969). As we might expect, females show lower grade aspirations. (Table Four).

(Table 3). Even in the reduced forms of these two equations, the differences in sign prevail, and in the reduced form on verbal score (Table 2) a child whose mother works shows a lower test performance ostensibly because of the effect of his mother's employment on his own efficacy.

The findings in these Tables are pregnant with suggestions, and it is interesting to speculate on their meaning. Yet we must caution against any final interpretation until improved measurement and replication of the model confirm the observed patterns. Accordingly, it is best to summarize where this excursion has taken us.

#### A Summary

In this paper an analogy between the economist's concept of an educational production function has been outlined. The problems of estimating the same have been emphasized. Despite these obstacles, the importance of knowing the production relationships in the educational sector has stimulated much recent research. The effort presented in this paper is an extension of this research by positing a simultaneous-equations approach for viewing the educational process. It appears that the properties of a simultaneous-equations system mirror the world more closely than the single-equation approaches that are presently being used. Further developments in this direction are proceeding, and it is hoped that before long, we can obtain a reasonably reliable set of estimates of school effectiveness by using this technique.