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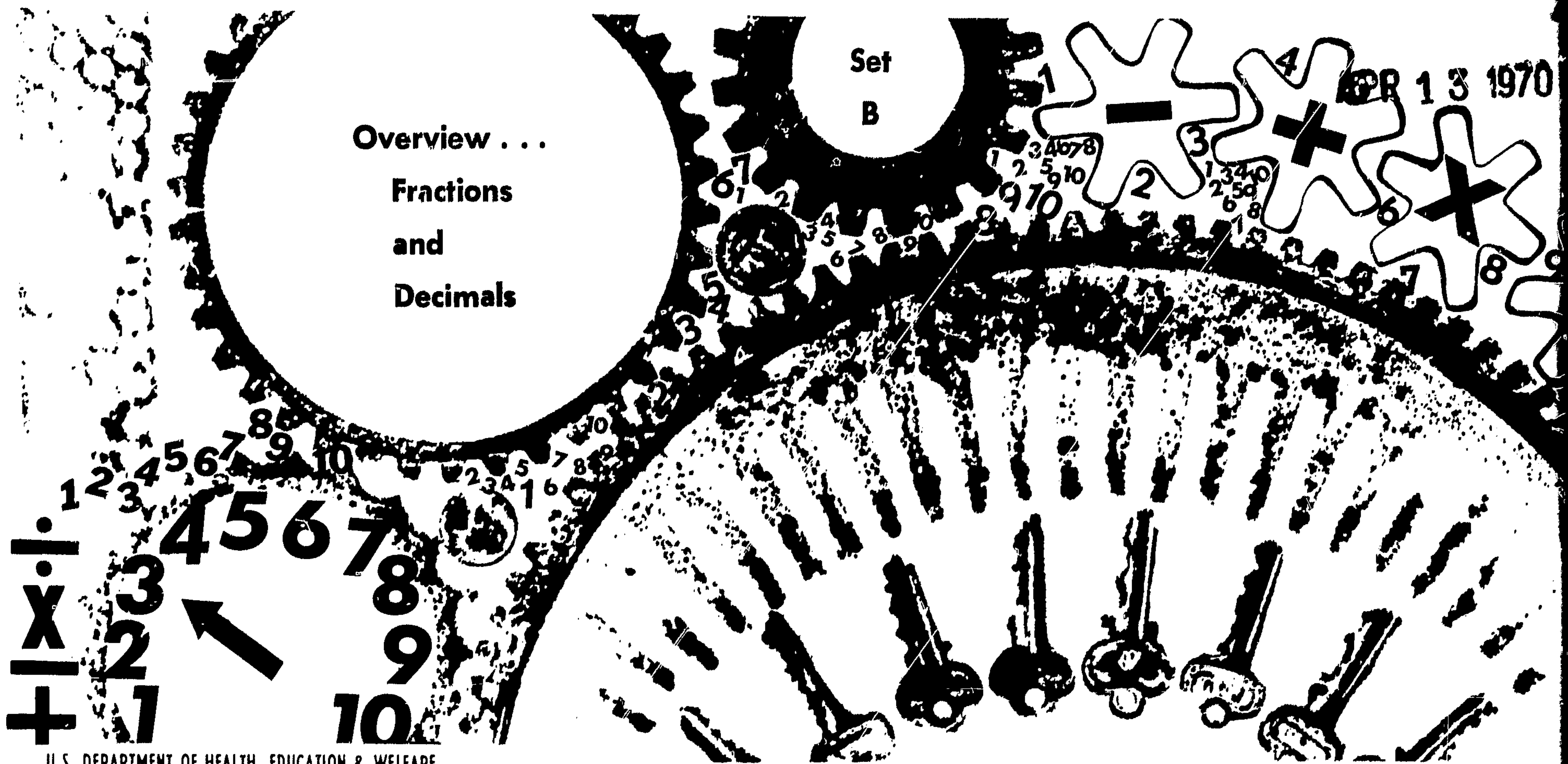
ABSTRACT

Reported are suggestions indicated by research studies to eight questions concerning the teaching of rational numbers. (1) Can young children learn fractional concepts? Answer: Manipulative materials are of value to learning in the primary grades. (2) How should the common denominator be determined? Answer: Both factoring and equivalent fractions are good. (3) Does error analysis help? Answer: Identification and correction of specific errors should yield greater achievement. (4) How should we teach multiplication? Answer: No conclusion. (5) How should we teach division? Answer: Inversion and reciprocal algorithms seem most effective. (6) What other things affect achievement? Answer: Manipulative materials, practice and meaningful teaching all help. (7) Should decimals, fractions, and place value be related? Answer: Yes, these concepts should be associated. (8) How should we teach placement of the decimal point when dividing? Answer: Greater accuracy results from the "multiplication by a power of ten" approach. (RS)



Using Research: A Key to Elementary School Mathematics

RATIONAL NUMBERS: FRACTIONS AND DECIMALS

Can young children learn fractional concepts?

We know that children come to school with some knowledge about fractions: at least 50 per cent can recognize halves, fourths, and thirds. They can extend this knowledge beginning in the primary grades, especially with a systematic program emphasizing the use of manipulative materials.

How should children find the common denominator for addition and subtraction with fractions?

The little research evidence on this question indicates that the procedures of (1) setting up rows of equivalent fractions and (2) factoring the denominators are both effective. That most errors are made by pupils when "reducing," when determining the numerator, and when adding needs to be considered as we plan lessons. We should also devote particular attention to examples in which pupils have the most difficulty, those in which the common denominator is not apparent.

Is it helpful to analyze errors pupils make with fractions?

In general, for all processes with fractions, we know that errors are most frequently caused by (1) difficulty with "reducing," (2) lack of comprehension of the process, and (3) computation. If we plan carefully to help pupils identify and correct their errors, greater achievement, with accuracy, should result.

Greater attention to regrouping and to "cancellation" might also help pupils to avoid errors when these two procedures are needed.

How can we most effectively rationalize the algorithm for multiplication with fractions?	There is little research evidence to answer this question. We know that for multiplication with fractions (as for other operations), use of programmed materials and of multi-level materials are effective. Using the inversion method to teach division of fractions may also increase achievement in multiplication with fractions.
What algorithm shall we use for division with fractions?	Most studies have indicated that use of either the inversion or the reciprocal algorithm is probably most effective for most types of examples requiring division with fractions. When pupils are taught why the inversion algorithm works (by using the reciprocal principle), retention seems to be improved. You might consider using the common denominator algorithm as an alternate procedure for pupils having difficulty, since it is most closely related to division with whole numbers.
What other things contribute to improved achievement with fractions?	Teaching about fractions and operations with fractions meaningfully has been found to be effective. Having pupils manipulate materials and providing practice are also helpful, of course.
Is it helpful to relate decimals with fractions or place value?	You should apparently place emphasis on both fractions and place value: when decimals are taught only in relation to place value, achievement and retention are not as high as when emphasis is placed on both numeration and the relationship to fractions. There is some evidence to suggest, however, that since computation with decimals seems to be more nearly like computation with whole numbers than like computation with fractions, reinforcement of whole number computational skills is provided when decimals are taught before fractions.
How should we teach children to place the decimal point in division with decimals?	Research indicates that to facilitate understanding we should teach children to locate the decimal point in the quotient by making the divisor a whole number by multiplying it by a power of 10, and then multiplying the dividend by the same number. Greater accuracy results than when children merely subtract the number of decimal places in the divisor from the number of places in the quotient.

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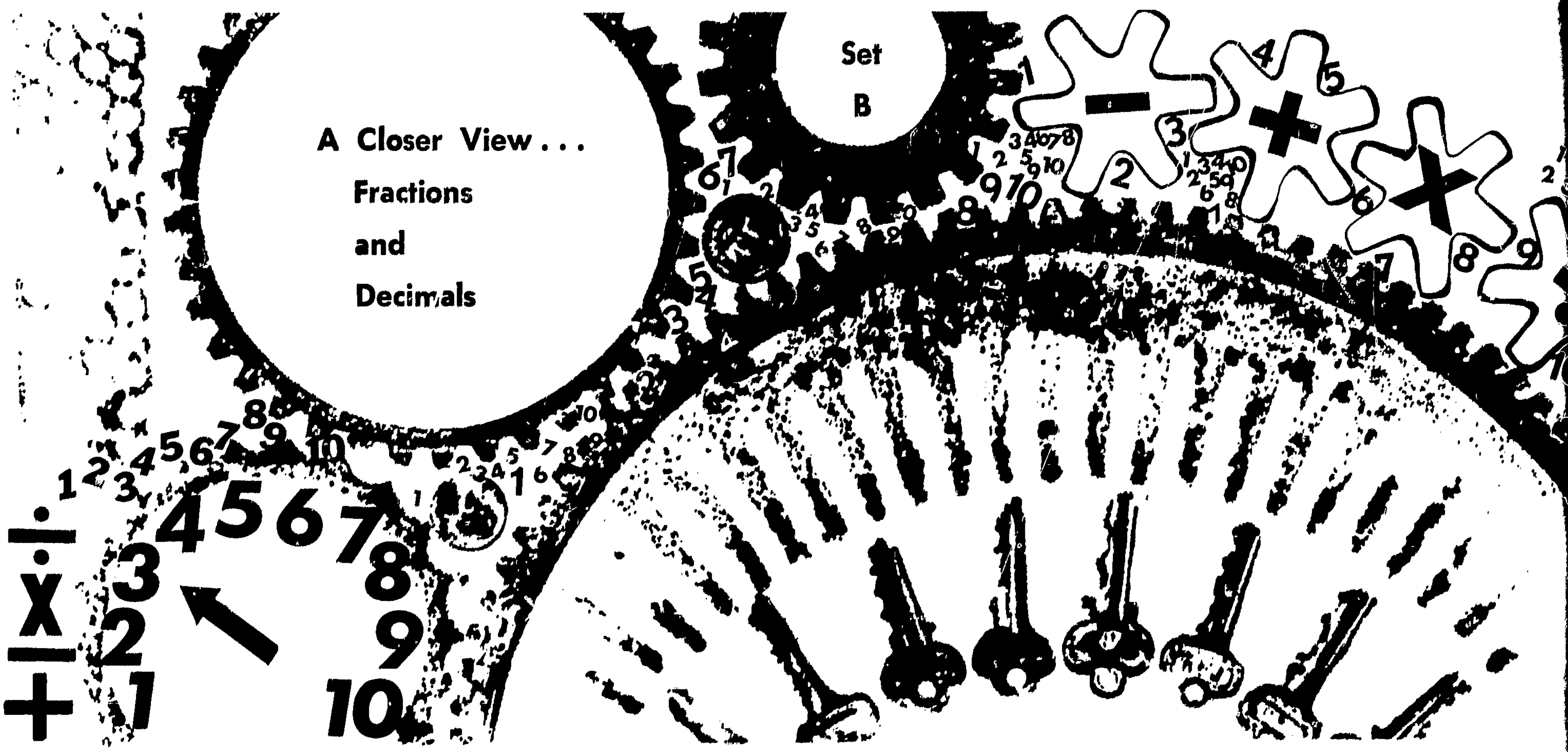
If you would like more information about the research whose findings are cited above, contact MARILYN N. SUYDAM, Project Director, at The Pennsylvania State University, University Park, Pennsylvania, 16802.



Using Research: A Key to Elementary School Mathematics

RATIONAL NUMBERS: FRACTIONS AND DECIMALS

Since several interpretations of the above words are possible, let's clarify how we're using them. We shall use the word <u>fraction</u> to refer to a <u>number</u>: a number that may be expressed in the form $\frac{a}{b}$, where $\underline{a}$ and $\underline{b}$ are whole numbers and $b \neq 0$. The word <u>decimal</u> will be used to refer to a particular kind of fraction: one that is expressed in our familiar positional place-value notation, with the denominator being some power of 10.

Can young children learn fractional concepts?

We have found from surveys of what children know about mathematics upon entering school that at least 50 per cent can recognize halves, fourths, and thirds, and have acquired some facility in using these fractions. Gunderson and Gunderson (1957) interviewed 22 second graders following their initial experience with a lesson on fractional parts of circles. The investigators concluded that fractions could be introduced at this grade level, with the use of manipulative materials and through oral work with no symbols used.

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The bulletin was prepared by MARILYN N. SUYDAM, The Pennsylvania State University, Project Director, and J. FRED WEAVER, The University of Wisconsin-Madison, Project Consultant. Art by Ed Saffell.

It should be noted that research is variable with respect to its quality; hence, the same degree of confidence cannot be placed in all findings. An attempt has been made to take this fact into consideration in preparing this bulletin.

A planned, systematic program for developing fractional ideas seems essential as readiness for work with symbols. Use of manipulative materials is vital in this preparation.

How should children find the common denominator for addition and subtraction with fractions?

There is little evidence on the effectiveness of procedures for finding the common denominator in addition with fractions, and even less for subtraction with fractions. Anderson (1966) analyzed errors made by 26 fifth grade classes using two procedures for finding the least common denominator when adding two "unlike" fractions: by setting up rows of equivalent fractions, and by factoring the denominators. There were no significant differences between the two procedures on tests of four kinds of addition with fractions examples. Furthermore, Anderson reported that errors connected with (1) "reducing," (2) determining the numerator, and (3) addition, occurred most frequently, with the greatest frequency of error in examples in which the least common denominator was not apparent.

Bat-haee (1969) compared 112 fifth graders who were taught (1) the factoring method or (2) the "inspection" method of a current textbook series. Those taught by the factoring method scored significantly higher on the experimental posttests.

Is it helpful to analyze errors pupils make with fractions?

Many earlier studies were concerned primarily with the specific errors children make. In general, it was found that, for all operations with fractions, the major errors were caused by (1) difficulty with "reducing," (2) lack of comprehension of the operation involved, and (3) computational errors (e.g., Brueckner, 1928a; Morton, 1924; Schane, 1938). Such findings frequently influenced the material included in textbooks.

Guiler (1936) was among those who reported success with a remedial program which provided practice on correcting errors which had been identified. Ramharter and Johnson (1949) had "good" and "poor" achievers think aloud while they attempted to correct errors in six examples involving subtraction with fractions. On subsequent tests, "good" achievers consistently corrected more errors, using a guidesheet effectively.

Aftreth (1958) had sixth grade pupils identify and correct errors imbedded in 19 completed sets of examples in addition and subtraction with fractions, while a control group worked the examples. No significant differences on either immediate or delayed recall tests were found for addition with fractions, while some significant differences favoring the group working the examples were found for subtraction with fractions. The author suggested that having pupils correct their <u>own</u> errors might be more effective than having them correct imbedded errors.

Fifth graders tested by Scott (1962) made more errors in subtraction with fractions involving regrouping than in subtraction with whole numbers involving regrouping. He suggested that current emphasis on the decimal system may reduce the "flexibility" which the child must have to deal successfully with subtraction with fractions when regrouping is necessary.

Romberg (1968) reported that among sixth graders who used a correct algorithm to multiply fractions, about twice as many pupils in "modern" programs as in "traditional" programs either did not express products in simplest form (as directed) or made errors in doing so. He attributed this difference to pupils' failure to "cancel," and suggested that the cancellation process is important -- even essential -- if efficiency in multiplication is one of the desired outcomes of instruction.

How can we most effectively develop the algorithm for multiplication with fractions?

There is little research evidence to answer this question. Recent research on multiplication with fractions has been primarily within the context of programmed instruction, where the purpose of the investigation was to compare various programming strategies, while fractions served merely as the content vehicle. For instance, Kyte and Fornwalt (1967) used programmed materials on multiplication with fractions to ascertain the rate of mastery by pupils at two IQ levels. While they found that pupils with superior IQ's were able to master identified types of examples more quickly than those with normal IQ's, the study says nothing about what procedures they used to teach the operation with fractions.

Miller (1964) found that significantly higher gains in multiplication with fractions were made by pupils using programmed practice materials, which provided immediate knowledge of answers, than by pupils using conventional textbook materials. In another investigation, higher achievement on the experimental posttest resulted when multiplication with fractions was taught with multi-level materials rather than with single textbooks (Triplett, 1963).

What algorithm shall we use for division with fractions?

Bergen (1966) prepared booklets designed to teach pupils by complex fraction, common denominator, or inversion algorithms. No significant differences were found between complex fraction and inversion algorithms, but each was significantly superior to the common denominator algorithm on most types of examples.

common denominator

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \div \frac{2}{4} = \frac{3 \div 2}{4 \div 4} = \frac{3 \div 2}{1} = 3 \div 2 = \frac{3}{2}$$

Sluser (1963) compared teaching the common denominator and inversion algorithms with and without explanation of the reciprocal principle as the rationale behind inversion. The group given the explanation scored lower on tests of division with fractions than a group merely taught to invert and multiply. He suggested that only above average pupils could *inversion*

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2}$$

understand the principle. However, a large percentage of errors occurred because pupils performed the wrong operation. Krich (1964) reported no significant differences on immediate posttests for pupils taught why the inversion procedure works, as compared with those merely taught the rule. On retention tests requiring recall, however, the group taught with meaning scored significantly higher.

complex fraction

$$\frac{3}{4} \div \frac{1}{2} = \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{\frac{3}{4} \times \frac{2}{1}}{\frac{1}{2} \times \frac{2}{1}} = \frac{\frac{6}{4}}{1} = \frac{6}{4} = \frac{3}{2}$$

In a study by Capps (1963) the effectiveness of the common denominator and inversion algorithms for division with fractions was compared. There were no significant differences in achievement on tests of addition, subtraction, and division with fractions, while pupils taught the inversion algorithm scored significantly higher on immediate posttests and on retention tests of multiplication with fractions than those taught the common denominator algorithm. This retroactive effect on multiplication was also reported by Bidwell (1968). He found that the inverse operation procedure was most effective, followed by complex fraction and common denominator procedures. The complex fraction procedure was better for retention, while the common denominator procedure was poorest.

What other things contribute to improved achievement with fractions?

Howard (1950) reported on a study with 15 classes of pupils in grades 5 and 6 who were taught addition of fractions by three methods differing in the amount of emphasis on meaning, use of materials, and practice. Pupils retained better when they learned fractional work through extensive use of materials and with considerable emphasis on meaning, plus provision for practice. Krich (1964), Shuster and Pigge (1965), Sebold (1946), and Feinstein (1952) also support the importance of using meaningful methods for work with fractions.

Many other investigations have been done in which fractions have served as the content. For example, Fincher and Fillmer (1965) were interested in exploring programmed instruction variables. They reported that programmed materials were more effective in teaching addition and subtraction with fractions than was conventional classroom instruction.

Is it helpful to relate decimals with fractions or place value?

Faires (1963) introduced some pupils to decimals through a sequence based on an orderly extension of place value, with no reference to common fraction equivalents, while others were taught fractions before decimals, as is usually done. Gains in computational achievement and at least as good an understanding of fraction concepts resulted. Faires indicated that "computation with decimals is [apparently] more nearly like computation with whole numbers than with fractions;" thus reinforcement of whole number computational skills is provided.

O'Brien (1968) reported that pupils taught decimals with an emphasis on the principles of numeration, with no mention of fractions, scored lower on tests of computation with decimals than those taught either (a) the relation between decimals and fractions, with secondary emphasis on principles of numeration, or (b) rules, with no mention of fractions or principles of numeration. On later retention measures, the numeration approach was significantly lower than use of the rules approach, but not significantly different from the fraction-numeration approach.

How should we teach children to place the decimal point in division with decimals?

Brueckner (1928b) and Grossnickle (1941) analyzed the difficulties with decimals which children have, citing misplacing of the decimal point in division as one of the major sources of error. Flournoy (1959) compared sixth grade classes taught to locate the decimal point in the quotient by (1) making the divisor a whole number by multiplying by a power of 10, and then multiplying the dividend by the same number, or (2) subtracting the number of decimal places in the divisor from the number of places in the dividend. Multiplying by a power of 10 resulted in greater accuracy, as Grossnickle (1941) had concluded earlier.

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