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ABSTRACT

This study reports on the position of programed learning in the teaching of college mathematics. A historical survey is given describing the following programing styles: Skinner, Crowder, eclectic, hybrid, Pressey, and problem. Conclusions based on research are given to claims about the benefits of programing including those of: providing for individual differences, tutorial function, controlling the learning process, providing for greater motivation, importance of overt responses, and the saving of teacher time. The summary states that the teacher and text will continue to be of greatest importance, with programing material used only as an auxiliary study aid. The report is based on studies made by the Committee on Educational Media of the Mathematical Association of America in 1963. (RS)

# Programed Learning and Mathematical Education

The Committee on Educational Media was established in 1962 by the Mathematical Association of America to investigate the use of new media in mathematical higher education. The major part of the Committee's work is carried on through four sub-projects: Calculus, Programed Learning, Individual Lectures, and Preservice Training of Elementary Teachers. The membership of the Committee in 1964 was:




The work of the Committee is supported by a grant from the NATIONAL SCIENCE FOUNDATION.

# Programed Learning

# and

# Mathematical Education

*A CEM Study*

This work is a CEM Study, prepared in 1964 at the request of the Programed Learning Panel. It represents the views of the author, and not necessarily those of the Programed Learning Panel, the Committee on Educational Media, or the Mathematical Association of America. In 1964, the membership of the Programed Learning Panel of CEM consisted of the following.

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# Programed Learning

# and

# Mathematical Education

**A CEM Study**

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## Preface

This report is based on studies made during 1963-1964 while I was serving as consultant on programed learning for the Committee on Educational Media of the Mathematical Association of America. However, the opinions expressed are my own and do not carry the endorsement of either the Committee or the Association.

I have benefited from service on the Panel on Programed Learning of the C. E. M., from comments on ten memoranda written for internal use, and from participation in the C. E. M. summer 1964 writing project. Publishers, organizations, and individuals have been generous with materials and helpful suggestions. It would be impossible to acknowledge them all. However, I wish to thank Edwin E. Moise, Leander W. Smith, Marshall H. Stone, and the S. M. S. G. algebra programing group for helpful comments on a first draft of this report.

It should be emphasized that this report is concerned with *programed learning in mathematics at the college level.* I discuss broader issues only in that context. It appears that effective educational methods are highly specific to subject matter and level, so that such a limitation would be desirable even if it had been possible to cover a wider area. Mathematics teachers at the high school and elementary level are particularly cautioned that my descriptions and opinions may or may not apply to their situation.

For the sake of informality and easy reading, I have abjured footnotes and cut citations to a minimum. The bibliography represents only a small percentage of the hundreds of items examined. However, it is sufficient to provide substantial further information for those interested, and leads to the literature in which can be found verification of the statements made. In accordance with C. E. M. usage I have spelled "programing" and "programed" with one "m," but the double letter is used where it appears in quotations and citations.

Kenneth O. May

Berkeley, California
October 1964

## A Definition of Educational Programing

Since the phrase "programed learning" has been applied to several kinds of instructional activity, a genetic approach may be helpful. The movement traces its ancestry to Sydney L. Pressey, who in 1926 observed that his multiple choice testing machine also performed a teaching function. Though he was well aware of the implications of this discovery, it had little impact in psychology or education until teacher shortages led industry and the military to experiment with teaching machines.

The current movement was launched by B. F. Skinner and associates, who in the fifties devised teaching machines inspired by "operant conditioning" experiments on animals. There followed a ten year boom and bust in teaching machines and a still continuing boom in printed materials whose form suggests their origin in teaching machine tapes.

Most programed materials follow the Skinner paradigm—a linear sequence of "frames," each consisting of a few words (stimulus), a blank to fill (response), and a correct answer (reinforcement). Error rate is kept low (about 5%), and all students go through the same sequence. We call this *Skinner programing*.

A second well known pattern is that of Norman Crowder. Here the frames are longer, consisting of an expository passage followed by multiple choice questions, and the student is "branched" to other frames according to his answers. This has been dubbed "intrinsic" or "scrambled," but we call it *Crowder programing*.

After some years of controversy over the merits of these two schools, there has emerged a tendency toward eclecticism. One finds a wide variation in styles and many of the $2^n$ possible combinations of particular programing devices. Some so-called programed books are hardly more than conventional texts with a few blanks inserted. Others look like flow charts for computer programing. But typically one now finds the Skinner pattern, modified by some branching and by a few of the amenities of text books that had to be abandoned for machine presentation. We call this *eclectic programing*.

Notable for their high quality are the programed texts produced by the University of Illinois Committee on School Mathematics and by the School Mathematics Study Group. The latter has published three programed versions of their ninth grade algebra, one in the Skinner mode (Form CR—constructed response), one in the Crowder mode (Form MC—multiple choice), and one combining the two with conventional exposition and problem sets (Form H—hybrid). We call this *hybrid programing*.

Finally, we mention that Pressey, the putative father of the movement, has remained a thoughtful critic of his offspring. He favors conventional exposition followed by a set of multiple choice questions designed to point up essentials. We call this pattern *Pressey programing.*

Publications that lay claim to being programed differ from previous educational materials primarily in the *degree* to which they control the student's learning activity. The contrast is most marked in areas, such as the social sciences, where text books consist almost entirely of exposition, with very little space devoted to questions or other devices to help the student. It is least marked in subjects, such as mathematics and foreign languages, where the material is presented in a form convenient for study and is accompanied by a variety of exercises. The reader will not be surprised to learn that the overwhelming majority of programs have been written in these latter fields.

The familiar college mathematics text book follows a pattern that might claim to be called programing. The "frame" is an expository section followed by a graded problem set with (partial) answers. By working through this unit the student is supposed to master the exposition, increase his skills, and make new discoveries. This mode we call *problem programing.* It enforces response and provides feedback, characteristics often used to define programing, but it accomplishes this with a much higher degree of student initiative than Skinner or Crowder programs. It can be varied so as to be similar to the Skinner, Crowder, or Pressey modes by choice of problems and accompanying instructions.

The above discussion suggests the following definition: *educational programing is the scheduling and control of student behavior in the learning process.* From this point of view a program specifies the steps in a learning process, and programed materials are those that include a detailed program. In game theoretic terms, a program is a strategy that gives the move to make at each point in the learning process. Programed materials provide both content and the strategy for learning it.

With this definition it follows that all educational materials are programed to some extent. At one extreme is the treatise, which vaguely suggests a learning strategy by its organization and calls for a response only implicitly by its content and the gaps in its reasoning. The reader does his own programing. Intermediate is the familiar text through which the student can follow an enormous number of paths. Here programing is shared by book, teacher, and student. At the other extreme is the Skinner program which specifies a unique path.

In this report we shall use "programing" in the sense of the above definition, though we are quite aware that it is more often defined so as to include some particular feature dear to the heart of the definer. We refer to specific modes wherever possible, *but in this report "programed materials" means "fully" programed books in the Skinner, Crowder, or eclectic modes.* Pressey and problem programing we consider in a different category because they assist the student by presenting him with helpful problems but leave him largely free to determine his own learning program. Hybrid programing is an integration of new and old devices.

The issue is not whether learning should be programed, but rather how, when, and by whom. In how great detail should learning be programed? (Should the learning strategy be pure or mixed?) To what extent should it be done in advance? What share of programing should be done by the student, the teacher, and the materials? What role should programing play in the teaching system as a whole? In order to answer the questions just posed, we need to have in mind the systems by which mathematics is now taught at the college level.

## Current Teaching Systems

In the typical system a teacher plays the essential and controlling role. He determines both the general outline (course plan) and much of the detail (daily assignments) of the learning program. He imparts information in lectures and employs a variety of other procedures, involving give and take with his class as a group and with individuals. The text book is the second main component of current teaching systems. It serves for exposition, review, and reference. Most important, it is a storehouse of problems. Its use is programed by student and teacher. Other components may be present: work book, reader, teaching assistant, and audiovisual aids.

In this system the student learns by multiple exposure and activity in a repeated cycle of listening, reading, problem solving, writing, getting feedback from answers and returned problem sets, etc. He is guided by the teacher's example, the assignments, and instructions in the book. But his detailed path is self-determined. We know that students learn in such systems. They even learn when possibilities are not fully exploited and when components are missing or of low quality. But we know also that many students do not learn and that the system at its best has some evident weaknesses.

The "straight lecture" has well known drawbacks. Even the best content is usually wasted through lack of student attention. Unless some interaction occurs, the lecturer might as well be talking to a multitude (in a large room, on film, or over TV), so that the procedure is wasteful of teacher time. The student could just as well (and often does) read the lecture in print. Of course we are not talking of the highly personalized lecture, in which the student sees "a mind at work," nor of the group interaction achieved by the master teacher. But, generally speaking, learning takes place rather inefficiently in existing lectures because of failure to involve the student.

Text books have somewhat similar faults. They are most effective when they involve the student, i.e. in doing problems. They are least efficient in exposition. Often the student does not read the text at all. He just does the problems and ignores the verbiage. How familiar is the plaintive "I understood the lecture and the book, but I don't see the connection with the problems." We say then that the student has *not* understood, and we undertake to lead him step by step until he can attack the problems on his own.

It seems that even when the familiar system functions well, the student at times needs additional detailed guidance (programing). And even in a complete system this guidance is often not available. When some components are inadequate or absent, the problem may become acute. The typical college student cannot learn from the typical college text on his own. Yet he is being put more and more on his own by the increasing teacher shortage. This suggests that programed materials may have a role to play even in the best existing systems, and that they, or some other device to replace the teacher's detailed guidance, are essential in a seriously understaffed system.

## The Claims For Programed Materials

The programing movement has a very broad appeal. It offers to psychologists a new means for controlled experimentation, to educational administrators a lifeline in a sea of students, to audiovisual specialists an additional medium, and to academic or commercial promoters exciting possibilities. Programed learning has become fashionable, and a new specialist—the programer—has been created. It is natural that the sober findings of experimenters have been submerged by the confused verbiage of sciolists, the careless assertions of enthusiasts, and the unprincipled cries of hucksters. Since these claims are widely publicized, we must deal with them here, if only to sort out those worthy of serious consideration. We generally omit references to the literature, but the original sources can easily be found through the bibliography. Needless to say we do not try to assign claims to the categories mentioned above. The reader is reminded that "programed materials" means those in the Skinner, Crowder, or eclectic modes and does not include hybrid or other types described above.

One essential fact has been established without a doubt: *Students do learn from programed materials.* On the other hand, *there is no conclusive evidence that students learn significantly more or with greater efficiency.* Programed materials have been used in a wide variety of situations and in comparison with an equal variety of so-called conventional methods. The typical result is n. s. d. (no significant difference).

Summarizing the results of using Skinner programed materials in place of texts in pre-calculus courses at the University of Buffalo, Sharpe [30]† writes: "To date, the experiment as it has been conducted indicates a probability that programed materials may do an equivalent job, but presents no evidence that programed materials are superior. . . . Students in this experiment had programs backed up by good instructors, yet no records were broken!"

Of course one would expect if programs were added to existing systems, that more learning would take place, since additional exposure could hardly decrease results [21]. But as substitutes for text-books, programs have not been impressive. This is not surprising. Programed materials offer more detailed guidance than texts, but they have few of the many features that make texts so handy for preview, summary, and review. Above all, they lack extended problems and connected exposition. The n. s. d. result, here as elsewhere, is due in part to a balancing of many variations in the total teaching system.

Moreover, it is evident that programed materials have inherent limitations as to the terminal behavior they can induce. Skinner and Crowder programing are compatible with objective testing and are not designed to elicit behavior that must be judged otherwise. Skinner programing gives no practice in reading or writing sustained discourse. Crowder programs provide slightly more extended reading, but limit student activity to checking multiple choice boxes, though this checking must sometimes be based on some scratch work. The following conclusion seems plausible: *Programed materials are incapable of eliciting the full range of behavior included in the objectives of college mathematics.* R. C. Buck has spoken eloquently to this point [11, 12], and we return to it below.

Clearly programed materials are no panacea, but it still may be that they will find a place in the teaching system. With this in mind, we examine some specific claims.

(a) *Do programed materials perform a tutorial function?* A common claim is that a program "has all the advantages of a private tutor." (*N. E. A. Journal*, November 1961, p. 18) It seems to be based on no experimental evidence or analysis of the tutorial function, but on the mere fact that both tutors and programs ask questions. Of the assertion that Skinner sequences are like the Socratic dialogues one can say that the great teacher-philosopher would not be flattered to have his intensely thoughtful dialogues compared to the operant conditioning of pigeons and humans. The similarity exists only if one ignores content.

† Numerals in square brackets refer to the bibliography beginning on page 22.

There is a grain of truth in the comparison between tutoring and Crowder programing, since some flexibility is introduced in the latter by branching. But it is only a grain, because programed materials lack the essential feature of the tutoring situation—the interaction of two human beings in all its intellectual and emotional complexity. Can one imagine a tutor who asks every student the identical sequence of questions and always gets the same answers (Skinner), or who offers the student only two or three pat explanations prepared in advance to correspond with anticipated answers to multiple choice questions (Crowder)? Comparison would be more apt with a coach or catechist, but even here the programed materials would make a very poor showing. We conclude that *programed materials do not perform the tutorial function, though they may perform the drill sometimes done in the name of tutoring.*

(b) *Do programed materials provide for individual differences?* Provision for individual differences is usually cited as a hallmark of programed materials. It is true as claimed that "the student can go at his own pace," but this is a characteristic of all printed materials for individual use. Self-pacing or external pacing is determined by the teacher, assignments, and schedules rather than by the kind of printed materials being used. The fact is that *Skinner programing removes all individualization except in pacing.* Crowder programing has a better claim to individualization because of branching, but the variety of paths is rather limited compared with that permitted by the usual text book. Indeed, sheer bulk limits branching, and the scrambled format prevents any significant departure from the alternatives anticipated in advance. We conclude that *programed materials* (Skinner, Crowder, eclectic) *are less adaptable to individual differences than are hybrid, problem, and Pressey programs.*

When stressing the importance of self-pacing, programing enthusiasts seem to be contrasting the individual studying a program with the group listening to a lecture or studying together. A comparison with an individual studying a text and doing problems would be more appropriate. But in any case numerous studies have indicated that self-pacing is not as helpful as one might imagine. As long as the group pace is not too far from the average, learning is not significantly impeded. In particular, group-paced work with Skinner programs yields good results. Advocates of self-pacing seem to have overlooked the advantages of group interaction. It appears that *pacing is not a very important issue and that self-pacing has no necessary or unique connection with programed materials.*

The real possibilities for using programed materials to cope with individual differences lie in different directions. One is the development of large libraries of brief units focused on narrow problems, beamed to specific student difficulties, and utilizing programing devices most appropriate to the audience and difficulty. Such libraries, stored in books, films, slides, tapes, and computer memories, could serve as valuable learning resources into which the student could branch on his own or teacher initiative. A second direction is the use of computers to exploit the branching idea to the point where individual differences are really accommodated. We discuss this below in the section on automation.

Individualization is a great rallying cry in secondary education, but little is said about it in college. No doubt this is partly due to the greater homogeneity of college students, but the college population is still quite heterogenous enough. A more likely explanation is the general opinion that college students are mature enough to provide individualization for themselves. It can be argued that the best way to individualize education for the college student is to let him do his own programing after providing him with a variety of materials and assuring him of help if needed. In this light, programed materials are an obstacle to individualization unless their use is optional and tailored to individual needs.

(c) *Does programing provide greater control of the learning process?* This claim is certainly justified. As we have seen, Skinner programing virtually determines every move. It allows variation only in pacing and is so designed that the student almost always gives the expected response. This has obvious advantages for research in the psychology of learning. It enables the programer to locate poor frames and to improve the program. It likewise helps the diagnostic work of the teacher. But it does not follow that it is best for the student. Programed materials force greater involvement in the learning process, but they do it in a predetermined inflexible pattern that excludes student responsibility for controlling the learning process. In contrast, hybrid and problem programing give the student wider latitude. He decides on his own pattern of reading and problem solving, and we know that the result is quite individual and complicated—usually involving a great deal of switching back and forth between reading, checking, calculating, thinking, and writing.

One of the goals of college mathematics is to teach the student to "work on his own"—to "write his own program" in the professional jargon. He will certainly not learn to do this if he is fed ideas intravenously drop by drop instead of having to get out and grub for them. Yet programed materials are explicitly designed to carry all students painlessly from ignorance to mastery, provided only that they follow directions.

One of the standard boasts of Skinner programers is "the student writes the program." By this they mean that the program is revised until the error rate is low. The perfect program is supposed to produce learning without error. Student failure is abolished. There is only program failure. The boast overlooks that one of our goals is to teach students that errors are in-

inevitable and instructive, to give them courage to take the chances without which great achievement is impossible. By eliminating sustained effort to overcome obstacles and to correct errors we would be failing to prepare the student for the adult world.

Crowder and Skinner supporters take sharp issue on the need for a low error rate. Skinner disciples consider errors dangerous because they may become fixed, while disciples of Crowder let them play a role in branching. Research studies show n. s. d. when the error rate is varied, but the whole controversy is beside the point. There are times in the educational process when errors are to be avoided, when accurate painstaking routine work is essential. There are other times when errors are to be permitted, when freewheeling experimentation is appropriate. Mathematics requires both following and breaking "the rules." *No fixed programing pattern can take care of the full range of objectives in mathematical education.*

One of the selling points for teaching machines was that they prevented "cheating," i.e., looking ahead or back in the program. Programed books try to approximate this control by "scrambling" in a random pattern, by sequencing frames on successive pages, by slightly removing the answers, or by admonishing the student to cover the answer. A number of studies showed, however, that students learn just as much when they look at answers before responding! This is partly due to the trivial nature of the answers called for in most programs. But it suggests that tight control of the student is less in his, than in the experimenter's interest. Of course, students will not learn much if they thoughtlessly copy answers, as illustrated by students who "go through" programs without learning anything. But it may be helpful to look at answers, to say nothing of reviewing or checking over previous material—a procedure virtually impossible in most programed materials. Our current text books allow for much flexibility on this. Some problems have complete answers, some partial, and some not at all. The student has practice in working at various levels of independence. Moreover, the problems are designed to teach him how to use the exposition, to which he is *supposed* to refer to for help. Perhaps the teacher and the book should provide more guidance in using these feedbacks, but preventing the student from getting information is not conducive to learning.

We conclude that *programed materials inhibit initiative, independence, and responsibility in the learning process, and do not contribute to the achievement of related educational objectives.*

To the above conclusion it may be objected that programed materials *could* accomplish such goals if only we could specify operationally the desired "terminal behavior." "You tell me what you want in concrete operational terms, and I'll program it!" We can accept this challenge, but the desired terminal behavior cannot be tested immediately or objectively, and the appropriate program would take the form of a guide to the study of unprogramed materials and the working of unprogramed problems. Someone should try to write such a program, one that would help the student use a text book effectively and then go beyond it. It should not follow any existing programing pattern.

(d) *Do programed materials provide greater motivation?* As is well known, conditioning experiments on animals depend on a close temporal linkage of desired responses with feedbacks of a rewarding or reinforcing character. In Skinner programing, immediate knowledge of the correctness of the response is supposed to function in this way for humans. Of course it is true that immediate feedback increases learning by quickly correcting errors, reducing anxiety, and giving encouragement. On the other hand, research studies show that students learn as well or better when they are "prompted" (supplied with the answer in advance) as when they are questioned and then reinforced. Animals are hungry for the pellets they get for correct responses. Apparently students are not as hungry for confirmation of an answer that is 95% sure to be right.

While students do get some satisfaction from always moving ahead, they typically get bored with programed materials. Perhaps this is because human motivation is more complicated than that of pigeons, especially in such an abstract and aesthetic activity as mathematics. Experience at the college level tells us that students have two prime drives in studying mathematics: their belief that mathematics will be useful to them, and the joy that comes from mathematical insight and accomplishment. The former propels them through even bad courses taught by incompetent teachers from miserable text books. The monotonous pat on the back of Skinner programing makes little difference. The second drive is an addiction that the teacher tries to establish by getting the student to do mathematics and to appreciate the mathematics done by others. Both these drives call for connected exposition and non-trivial problems, which are precluded by the Skinner and Crowder patterns. We infer that *programed materials cannot provide adequate motivation at the college level.*

(e) *Does programing lead to better specification of content and objectives?* One characteristic of programers has been their insistence on precise specification of objectives in behavioral and testable terms. Before writing, they define "terminal behavior" and prerequisites, not in vague generalities, but in detail. Then they write "readiness-tests" and "post-tests" that pin things down further. Since the material is to be presented piecemeal it must be analysed and ordered carefully. This is all to the good, an example for every writer and teacher to heed. But there is the danger that when goals are difficult to define they may be abandoned.

In mathematics we desire not merely rote responses on an objective examination, but the development of behavior patterns, both overt and covert, that are reflected fully only in substantial calculations, nontrivial expositions, originality, continued interest in mathematics, performance in later courses, and use of mathematics in later life. Above all we desire *unexpected* insightful responses, which, by definition, cannot be programed except by avoiding the overprograming that inhibits them.

Programers typically suggest that "conventional text books" are not as carefully planned as are programed materials, though no evidence is given to support such claims. In fact, mathematics texts are not very different from programs in this respect, depending in both cases on the competence, imagination, and industry of the authors. Most programs remind one of the "cook book" for which the reform movement in mathematics is trying to substitute more literate materials. The best mathematics texts include matter not related to any immediate behavioral goal, but nevertheless important for the education of the student. The claimed superior specification of programed materials covers an impoverishment resulting from the limitation of objectives.

One characteristic of programing is the fragmentation of subject matter. "Step size" has been a favorite topic of debate among the specialists, though all seem to agree that knowledge should be fed in small amounts. As usual, research shows n. s. d. with variation in step size, but it is obvious that people who learn solely from fragmented presentations will not learn to see the big picture, to read long passages, to analyze complex ideas without guidance, or to express themselves in an extensive way. An essential feature of mathematical thinking is to look at problems in both the small and the large, to master both detail and big ideas. Problem and hybrid programing provide practice in this; Skinner and Crowder programing do not.

Once again we find that programing has the faults of its virtues. The programing movement has made a contribution by emphasizing the importance of planning, but the Skinner and Crowder approaches have tended to narrow objectives and to fragment subject matter to the impoverishment of both goals and content. *The potential of programing for improving content has not been realized.*

(f) *Are programed materials better designed to achieve their objectives?* One of the boasts of programers is that by revising on the basis of testing they produce a product guaranteed to achieve results. After objectives have been specified, a draft program is prepared, tried out on individuals, and revised until it appears a good first approximation. Then it is "field-tested" and revised until appropriate error rates and post-test results are obtained. Programers seem to imagine that this process is something quite new and in sharp contrast to the offhand way in which conventional texts are supposedly dashed off. Actually the only innovations are large scale statistical testing and the trial and error method of writing and revising. Both these procedures were required by the fact that many programs were written by people unfamiliar with the subject matter and without experience teaching it. It is doubtful if large scale statistical testing can add much to the product of a competent mathematics teacher who has experimented, drafted, tried, revised, and benefited from the thoughtful comments of reviewers and colleagues who have used preliminary editions. The basic difficulty with "scientific" statistical methods of testing is that the meaning of the results depends so much on the original limitation of objectives, the testing procedures and content, the population of students, and the teaching system in which the program has been tried. An unsound mathematics program could easily make high scores if the testing were done "right." Moreover, as we have pointed out, tests cannot measure some of the most important features of a course. Actually, testing seems to have played a bigger role in promotion than in improving quality beyond what one could expect from competent and experienced writers. *There is no convincing evidence that better design is a concomitant of programed materials.*

(g) *What is the importance of overt response?* Overt versus covert response has been a bone of contention. As one might expect, research studies show no significant difference between programs requiring "overt response" (writing in a blank) and "covert response" (choosing an alternative). Pressey [27] condensed a Skinner type program in psychology into an expository passage followed by a few multiple choice questions. The result was better learning and an 80% saving in study time (and paper!). Students learn as much from Skinner programs with the blanks already filled in as they do from filling them in. All this is not surprising, since millennia of experience shows that what people learn depends on what happens in their minds, and that overt conduct is important only in so far as it impinges on the central nervous system.

On the other hand, we also know that certain kinds of overt action are required to condition the mind to produce certain overt acts. In particular we are convinced of the common sense idea that to learn any type of behavior we have to practice it. Programed materials cannot teach certain desirable overt responses because they give no opportunity for them to be practiced. Moreover, the kind of overt behavior we desire in mathematics cannot be manifest without very complex and extended covert activity (thinking—if the behaviorists will excuse the expression). It follows that we wish to elicit both covert and overt responses, even though we can only observe the latter. One of the great disadvantages of excessive programing is that it enables the student to "succeed" without extensive or intensive thought.

On this matter programing enthusiasts flagrantly overlook the actual role played by text books. For example, Markle wrote: "A program requires of the student more than does a text book. In a program the text is more incomplete. The student himself completes the text by filling in key sentences or answering significant questions." [23, p. iii] Anyone who compares a program with the typical mathematics text book will find this rather amusing. Our books are full of incomplete arguments, and the number of problems (to say nothing of the number of steps required to solve them) is often larger than the number of frames in a linear program supposedly covering the same material.

The point for college mathematics is that *programed materials cannot teach the full repertoire of covert and overt behavior that is desirable for effective work in mathematics.*

(h) *Is there an art of programing?* The early years of the programing boom were dominated by psychologists and others who took for granted that programing was an independent skill and that a programer need not know anything about the subject. Many programs have been written by programers who "followed" text books with little more knowledge of the subject matter than that provided by a consultant. It is amusing to read the early discussions of the qualifications of a good programer without seeing once any mention of knowledge of the subject or of experience in teaching it (to say nothing of experience in tutoring, the supposed model).

These early claims based on the alleged superiority of the expert programer as opposed to the amateur text book writer are seldom heard now. It is recognized that knowledge of subject matter helps. But the heresy remains that there is a special profession of programing, independent of subject matter competence. Psychologists and educators in the past have not written manuals on how to prepare textbooks, but they feel no inhibition in the new field of program writing. Their publications often include insights and specific examples that are illuminating, but they illustrate once again that teaching problems are specific to the subject matter and the student. Generalities are of little use, and the best results come from a combination of imaginative teaching and deep mastery of content. In so far as the techniques of programing are valuable, they should be utilized by writers of textual materials. But there is no more reason for the profession of programer than for that of text book writer. We opine that *there should be no profession of program writing separate from the general art of writing educational materials as part of the profession of teaching particular subjects.*

(i) *Does programing reduce educational costs?* In certain situations in industry and the military, where teachers are lacking and goals narrowly defined, programed materials and teaching machines have accomplished tasks not otherwise possible. But in the typical college situation there is no indication that programed learning is more economical. Existing programed materials are generally more costly per course than text books, in spite of much thinner content. For example, a recently published program on vectors takes about 100 pages (scrambled) to cover material normally dealt with in about six or seven pages of an elementary calculus book. It cost about thirty times as much per idea. Part of such high costs are due to depending on formal procedures rather than knowledge and experience. Nevertheless, there seems no reason to expect that programed materials could be produced at a lower cost per page than other printed matter, and detailed programing inevitably multiplies bulk by an order of magnitude. The S. M. S. G. ninth grade algebra in Crowder form required 2357 pages bound in six volumes. An entire calculus course with as rich a content as the familiar voluminous texts, if programed in detail, would require a large pocketbook for its purchase and a sturdy wheelbarrow for its transportation. Evidently *programed materials are more expensive than text books,* but they might still cut educational costs if they allowed economies elsewhere.

(j) *Do programed materials save teacher time?* There is no evidence that programed materials can take the place of a complete teaching system or perform the functions of a teacher. Serious trouble has been experienced by those who tried experiments of this kind. It is of course possible for individuals to learn from programs without assistance just as they can learn from text books on their own, but there is no evidence that they learn better, and our previous discussion suggests that what they learn is narrower in scope and thinner in depth. On the other hand programed materials might, just as texts now do, save class time by providing drill and practice. Here they would compete with work books and problem sets, but at higher cost. In so far as they take over routine classroom chores they may be used to allow larger classes or to shift the teacher's activity in the direction of more individual tutoring. *There is no reason to think that programed materials will displace teachers. As supplements to text books they may, however, bring about shifts in the teacher's role by taking over some routine drill.*

The above catalog of claims is fairly complete, though we have not included a number that are based on the fallacious attribution to programing of effects due to other causes. For example, writers and users of programs have benefited by their experiences, but they might have gained as much from *similar* experiences with unprogramed materials.

In summary, the advantages and disadvantages of programed materials are just what one might deduce from their characteristics. They have no magic, and their claims to universality or general superiority are quite unfounded. On the other hand they are effective within their limitations, and the programed learning movement has developed techniques and concepts that may be valuable in college mathematics. Before discussing these possibilities, we consider the related matter of teaching machines.

### Automation

Machines are usually designed to simulate existing operations or to mechanize the fabrication of existing products. Examples are the typewriter to simulate writing and the printing press for copying manuscripts. But as these examples illustrate, machines usually modify both process and output. Pressey's first machine was designed to give and score a multiple choice test. It actually changed the manner of giving such tests, taught while examining, and suggested the possibility of other teaching machines.

It sometimes happens, also, that machines are designed to produce a new product by a new process. The first Skinner teaching machines were built for the automatic operant conditioning of pigeons. For human use, they were redesigned to use verbal stimuli, responses, and rewards. The result was a new kind of learning. But the machine package soon appeared to be inessential. When printed stimuli were used on subjects capable of turning pages, the machine proved less efficient than the student in moving from frame to frame. What remained was the Skinner program, a machine tape in book form.

Machines designed to present Crowder programs had more to do. But branching increased the cost, and students appeared quite willing and able to accomplish the same thing by page flipping in a scrambled book. That most ancient teaching machine, the book, proved superior to machines for presenting simple branching programs.

The virtual demise of the teaching machine industry before it could even get in the black might have been anticipated. As long as material can conveniently be put in book form, a teaching machine presenting the same material is just a cumbersome, expensive (or unreliable), and tiresome page turner. There is no more reason to expect such machines to replace bound books for educational purposes than to expect people to buy equipment for presenting their other reading matter automatically. Books just aren't that inconvenient, and they don't get out of order! A teaching machine must either get old results at lower costs or do something that has not previously been possible. Moreover the gain must be substantial, since machines have disadvantages. For example, one educator has pointed out that they "place the habituation of the act of learning one step further from learning as it occurs in normal living situations." [24]

We have noted previously that Crowder programed materials have some claim to simulate a tutor because of the branching structure, which is too bulky in book form but quite well suited to an electronic computer. "Computer based instruction" clearly has possibilities worthy of exploration, and energetic experimentation is in progress. The main problems are communication with the machine and the actual design of a suitably rich and flexible branching program. So far the typewriter and visual displays are being tried for communication, and some programs have been developed. As one might expect, the computer behaves like a slightly deaf teacher with an enormous memory and little imagination, who has been coached by someone with quite a bit of knowledge and experience. Interestingly enough, the computer sometimes directs the student to read a book. At other times it asks questions, displays material, and comments on student responses. It can take into account all past performance of the student and all information about him that has been fed in, *provided* someone has written a program sufficiently complex to involve all these factors. In effect the student is learning under the guidance of the teacher who programed the machine. Instruction is individualized and mimics the tutor just in so far as the programer anticipates all individual differences.

Experience shows that students learn from computer based instruction. The use of a computer is justified, however, only in so far as it can do things not otherwise possible at comparable cost. A potential cost reducing factor is that the computer can tutor substantial numbers of students at the same time. Nevertheless, computers are expensive and awkward. Their advantages and costs have to be compared with those of other devices for achieving the same degree of individualization. It seems possible that the same effect could be achieved by libraries of the kind mentioned under (b) in the previous section.

One kind of teaching machine that has proved itself is the simulator of environments in which the student needs practice. Examples are the well known simulated space vehicles in which the cosmonaut can gain skill without the expense or risk of actual flight. Such teaching machines are very specialized. Their high cost is balanced by the still higher cost of the real thing. The language laboratory seems to fit in this category, since it simulates the expensive experience of conversing with someone fluent in the language. There does not appear to be much application for such machines in college mathematics, except possibly in the field of computer science.

The devices discussed so far are all for automating individual instruction.

They are directly in line with the first invention of this sort—writing. They compete with text books, work books, problem assignments, and individual personal instruction. What are the possibilities for automating group instruction, that prehistoric move to meet the teacher shortage?

Radio, T.V., films, and tapes immediately come to mind. The first two mainly increase the size of the audience. The last two, like books, permit unlimited duplication at other times and places. They simulate personal communication more closely than does a book. But it does not follow that they will replace books. On the contrary, for most subjects, and certainly for college mathematics, verbal communication is slower, less effective, and more costly than reading. Automating the lecture does not change this. It simply permits the lecturer to reach more people at the cost of losing audience feedback. Clearly these media compete with the live lecture, not with books or the small class. (Of course, we are talking of "canned" lectures, not of the use of films to present visual materials uniquely possible in the medium, e.g. animation. Such films justify themselves by presenting something not previously possible.) There seems little likelihood that these media will replace the teacher of college mathematics, because he spends or ought to spend, only a small part of his time in "straight lecture." But such materials could be very useful to present the lecture portion of a class or to otherwise supplement live teaching.

Would it be possible to simulate the typical small mathematics classroom with its two way communication and student participation? The first step in this direction was taken in connection with T.V. lectures. Equipment was developed to permit listeners to question the lecturer, with the entire exchange audible to all viewers. The result was not very different from a lecture in an enormous hall with provision for a few questions. Perhaps the effect is better because the equipment overcomes acoustical and visual problems. Still it is far from the live small classroom.

It is certain that the real live teacher (as opposed to a lecturer) cannot be *fully* simulated by a machine, because the teacher is able to respond to *unanticipated* events. On the other hand, one could simulate much of the small class activity. Classroom communication systems exist for presenting films, film strips, slides, and sound tapes individually or in combination under the control of a teacher's console. Students can communicate instantly from individual push button stations and have their answers evaluated and individually recorded. The console will also display frequencies of different responses to the teacher. The teacher can record the entire class presentation, including his own participation. Then, without any teacher, the console can reproduce everything, including questions to the class, delays for response, etc. The cost is relatively low. With such equipment a gifted teacher might extend to a very large audience some of the values of a small class. More important, his presentations recorded on tape could be reproduced and used without limitation.

Because of its unlimited duplicability and cheapness, the book has for many centuries been the main medium of individually paced learning. The taped classroom presentation described above may possibly play a similar role for group instruction. Of course, a mechanized classroom is not the same as a live one. Neither is a book the same as individual personal instruction by the author. Yet books have some distinct advantages. First they make high quality content widely available. Second they communicate more quickly. Thirdly they are very portable and flexible in use. Finally, they can be adapted by the individual to his own needs. Packaged classroom presentations seem to have some of these advantages, and they might take over a large part of group instruction.

The historical record suggests that improvements in educational technology increase the number of learners and the amount learned without in any way diminishing the need for human instruction. This is not surprising. Students need the personal touch of the teacher, group interaction with their peers, the unexpected, the humorous, as well as the routine. No matter how much students *could* learn on their own with the aid of various devices, their demand for human guidance and example will remain. A shortage of teachers may force the student-teacher ratio up and lower the quality of education, but the loss cannot be fully repaired by automation. Of course, students may still learn without much attention from other human beings, and they may even learn as much in terms of narrowly conceived criteria, but their education will nevertheless have been impoverished in ways not easy to measure. Speculations about machines replacing teachers are based on arguments that would apply equally well to books or phonograph records. We may expect students to continue to demand live teachers, and the most likely effect of educational automation is a shift of teacher activity away from routine tasks and toward the essentially human aspects of the teaching job.

Automation clearly is not essential to programed instruction. On the other hand, automation requires detailed advanced programing whereever it does not provide for human control on the spot. We are not speaking here of programed materials but simply of the obvious fact that a nonhuman presentation must be laid out in advance, whether it be a book, a computer controlled tutorial, or a taped class.

## Programing College Mathematics

If the previous analysis has any merit it follows that programing and automation are not alternatives to familiar ways of teaching but rather two related aspects of any educational process. The questions posed on page 3 cannot be given useful general answers. We have to examine our present system with a view to specific weaknesses and possible improvements.

We look first at a classroom small enough to allow questions and discussion. We know that even in such classes, attention lags and students get "lost." Often teachers force participation, reinforce ideas, and check understanding by verbal questions—the Pressey programing pattern. Feedback to the teacher consists of a few verbal responses and/or more subtle clues. Certainly a more complete response would be better. One way of achieving it is to have students write brief responses that are collected. The writer can testify that this yields attention and encourages regular outside preparation, but the teacher does not see the results until later. A classroom communicator of the kind described in the previous section would seem to meet the need completely, except that it is limited to multiple choice questions. Would it really help and is it worth the cost? Only experiment can tell.

Next we turn to the large class, where intimate personal acquaintance is lost and the teacher has to talk to the group as a whole with only token individual feedback. This is probably the typical situation in college mathematics today, since "small" classes have for years been too large for genuine small group interaction. As we have pointed out, such a class is not essentially different from a TV presentation which includes provision for questions from the viewers. Unless the lecturer is very unusual, such classes are largely a waste of time. Students cut when they can, or they come and think about other matters. Here a classroom communication system would permit a gifted live lecturer to use a variety of media, to get instantaneous observable feedback, and to record student participation. Some of the values of small classes could be incorporated in very large ones, and there would be advantages not present in even the best small class. Moreover, high quality presentations of this kind could be reproduced from tapes without a live teacher present. Automated presentations would not be sensitive to student reaction, but then neither is a lecturer in a large hall. And the teachers using such automated presentations could provide for flexibility by their own direct intervention as well as by modifying the tape. I suspect that learning would be substantially increased because of student involvement, even if content were not improved.

Packaged classroom presentations are certainly still in the future, but courses of filmed lectures are already available. Their main weakness is student boredom. This might be changed dramatically if they were accompanied by carefully planned questions to be answered by the student on the spot. These could be included in the film, or (more cheaply) be presented by other media with the film stopped. Answers could be written or given through a communication system. The combination might be better than many small classes. It is certainly worth trying.

Now we turn to the primary locus of learning college mathematics—the student studying alone. He is reading and doing problems. Often his trouble is that he can't read the book with understanding or do problems that vary from the worked examples. Hopefully, the discussion so far has convinced the reader, if he was not already convinced, that *the solution is not to throw away the book but rather to teach the student to use it effectively.* Only by using books will the student be able to learn what he must in our present society.

Imagine him reading the exposition. He finds some definitions, axioms, and theorems more or less motivated and explained. He is offered no help in mastering them, the idea being that he will do so by repeated reading, by working problems, and by whatever devices he has picked up from past experience. Perhaps he does, but more often he makes little headway and gives up. Why not provide auxiliary materials to help the student; not just more exposition, not just general advice on how to study, but a program of activities to master the topic? The student could use such materials as much or as little as required. True, if he "knows how to study," he "ought" not to need such help. But he does need it, and it is part of our job to see that he gets it. Individual personal help is impossible on a continuing basis, and printed substitutes would be better than nothing.

Now the student attacks the problems. But he gets stuck, and besides his work habits and exposition are regrettable. He needs help and guidance, but he does not get it until much later in class or when his sloppy problem set is returned with justifiably caustic comments by the reader. Why not provide him with more detailed guidance in solving the problems, something between the completely worked example in the book and the problem with only a final answer for checking? Such programed study aids might begin with completely worked problems and gradually require the student to do more himself. They could inculcate good form. They would be designed to teach the student how to attack problems on his own, even when he did not "know how to do them." They might be called programed work books.

Sometimes our student meets another difficulty. He finds himself ignorant of something the author and teacher have assumed as a prerequisite. The solution is, of course, to study this material in another book. But since there is no teacher for this project, the best book might be one that is highly programed.

Should not a good text book give the kind of help just described? To do so in sufficient detail for all students would lead to impossible bulk. The text book would no longer be as useful for the big picture, for review, and for reference. At the college level the hybrid style runs the risk of falling between two stools.

Does the student really need such detailed guidance, and is it good for him? It depends on the student of course, and it is bad to give him any more or any less than needed for his maximum development. The guidance need not be limited to telling him what to do in each case. It may provide him general rules and guidelines. An example is Polya's *How to Solve it,* which presents general rules for attacking problems. Students might gain from auxiliary materials that guided them in applying such strategies to particular problems. Could not such programs lead the student by the hand through heuristic thinking, always permitting him to branch out on his own, but leading him as much as necessary in the process of finding results new to him? Such a program would be more than a "hint" and less than a solution. It could not follow any of the standard programing styles.

A famous way of programing mathematical teaching is connected with the name of R. L. Moore. Roughly speaking, the teacher supplies the axioms, theorems, and some intuitive material. The students supply the proofs without benefit of books or other aids. The teacher and students act as critics. This method, or variations of it, has been used with great success to produce mathematicians. The teacher plays the central role, as important for what he refrains from doing as for what he does. Could such a procedure be programed for the individual student? Of course it can! Many mathematicians have programed their own study in this way by simply not looking at the proof of a theorem until they have worked out their own. Some new proofs have been found this way. Any student could try to follow this pattern. But perhaps we might produce a program designed especially to help such a student.

In sum, we need a wider variety and larger quantity of auxiliary materials to assist students in their individual study outside of class. These auxiliaries need not be programed according to any existing style, but they must offer more programing than the usual text. They should all have a double purpose: to help the student master the material at hand, and to help him learn to master such material with less outside assistance. Programs should program themselves out of the student's life. Moreover, the amount of help should continue, as at present, to be tapered off as the student advances, until he can program his own learning from straight mathematical exposition. In order to accomplish this, the student must always do some things on his own. There should always be some material to be studied with no guidance and some problems without any hints and answers. There should be gentle but firm pressure on the student to work on his own.

## Existing Programed Materials

Of well over one hundred programs in mathematics [8], only a handful are usable at the college level. None cover a full course of college mathematics, though some claim to deal with high school courses often taught in college. The evaluation of programed materials was very difficult for a time because of a failure to follow familiar standards and format. It was hard to review a book that lacked a table of contents, chapters, subheadings, index, or even pagination! (The writer has found one useful dodge: just look at the answers and tests; then spot check a few frames.) But these faults are being corrected, and reviews of programs are beginning to appear in journals. Meanwhile, one can say in general that existing programs are much thinner in content, of a substantially lower quality, and much higher in cost than corresponding text books. Even if the student were to master everything in a typical programed text book now on the market, he would get only a part of a respectable college course. Available programs should be used only for additional exposure, independent study, or remedial work.

## Summary and Conclusions

The problem of education is not to decide whether one procedure is superior to another in isolation, but rather to build effective teaching systems. A teaching system for college mathematics must take into account the full range of objectives, many of which cannot be tested easily or objectively. It must reflect an apparent contradiction in mathematics itself: the fact that mathematics requires, on the one hand, accurate and even automatic application of existing rules, algorithms, and theories, and on the other hand, insight, imagination, originality, trial and error. These twin aspects are present at every level of mathematical education and practice. Our problem is to find ways to teach both in the context of increasing demands and a growing teacher shortage. The solution is not to narrow our goals to those compatible with some instructional device, but to experiment with a variety of devices without abandoning those that have proved themselves capable in the past. In particular:

1. Teacher and text book should and will remain the central components of college teaching of mathematics.

2. College mathematics text books might well incorporate some devices developed by the programing movement, but they should maintain the present exposition-problem pattern.

3. Programing (the scheduling and control of student behavior in the learning process) should remain primarily in the hands of the individual

teacher and student, with the responsibility being shifted increasingly toward the student as he gets older.

4. Printed programs should be used only as auxiliary study aids, not in place of text books or teachers.

5. Adherence to any one style of programing should be avoided in favor of eclectic and hybrid styles determined by the particular teaching task. We should look for new patterns.

6. Programs in mathematics should be written by mathematicians on the same basis as other materials, judged by the same standards, and sold at comparable prices.

7. Vigorous experimentation should be undertaken in writing and using a variety of special purpose auxiliary materials.

8. Experiments in automation should concentrate on devices that maximize individualization for the single student (e.g. computer based instruction) and, for group instruction, that extend the teacher's range and the degree of student involvement (e.g., multi-media presentation systems and classroom communicators).

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