

## DOCUMENT RESUME

ED 035 953

24

CG 005 017

AUTHOR BOURNE, LYLE E., JR.; AND OTHERS  
TITLE DEVELOPMENT OF CONCEPTUAL SKILLS: SOME PRELIMINARY FINDINGS. TECHNICAL REPORT NO. 81.  
INSTITUTION COLORADO UNIV., BOULDER.; WISCONSIN UNIV., MADISON. RESEARCH AND DEVELOPMENT CENTER FOR COGNITIVE LEARNING.  
SPONS AGENCY OFFICE OF EDUCATION (DHEW), WASHINGTON, D.C. BUREAU OF RESEARCH.  
REPORT NO TR-81  
BUREAU NO BR-5-0216  
PUB DATE MAR 69  
CONTRACT OEC-5-10-154  
NOTE 26P.

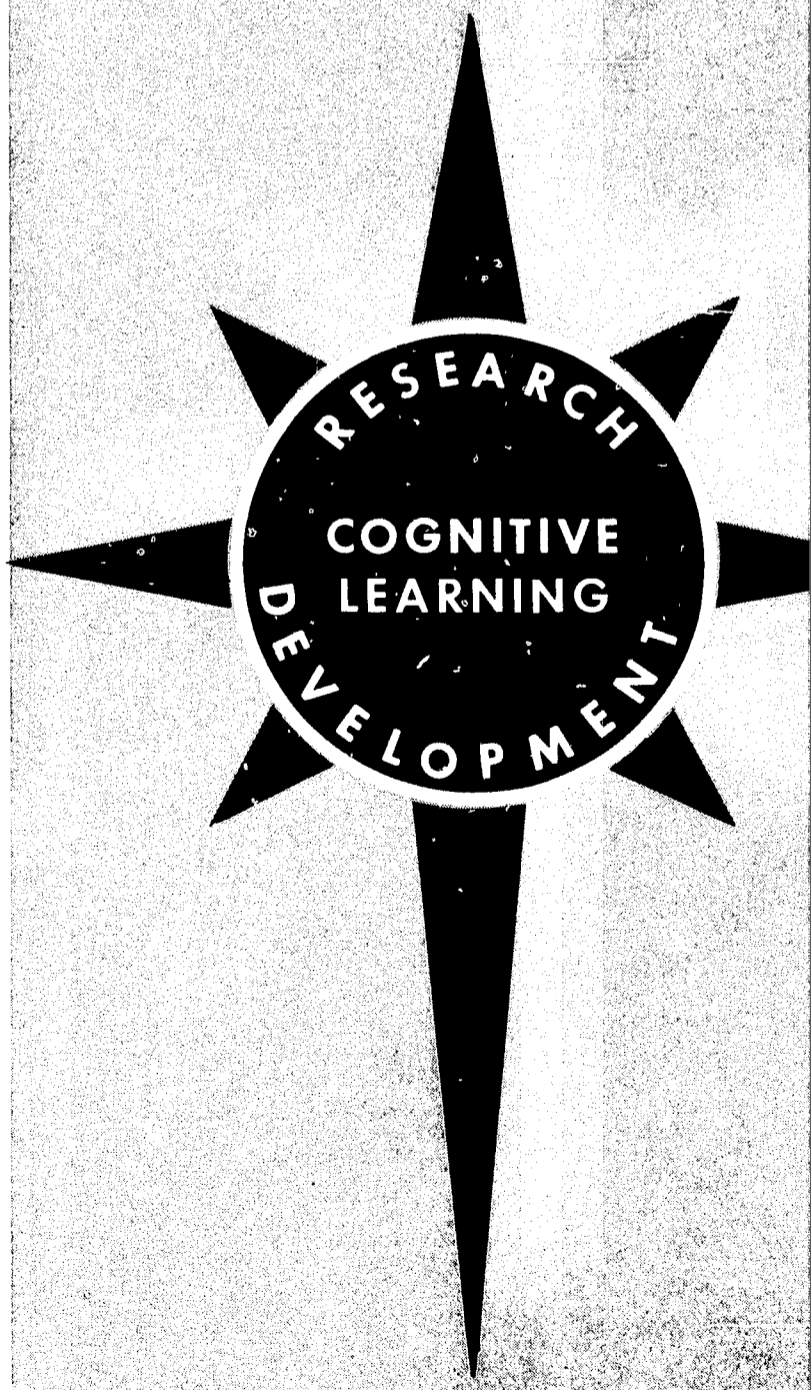
EDRS PRICE MF-\$0.25 HC-\$1.40  
DESCRIPTORS ADULT LEARNING, CHILDREN, \*CONCEPT FORMATION, EDUCATIONAL RESEARCH, \*LEARNING PROCESSES, \*NONVERBAL LEARNING, PATTERNED RESPONSES, \*PROBLEM SOLVING, \*ROTE LEARNING

## ABSTRACT

A SERIES OF EXPLORATORY STUDIES AND THREE EXPERIMENTS DEALING WITH CONCEPTUAL RULE LEARNING ARE REPORTED IN THIS PAPER. DISCUSSANTS RELATED THE RESULTS TO SUBJECT MATTER FIELDS AND TO EDUCATIONAL RESEARCH AND DEVELOPMENT. FOUR GROUPS OF SUBJECTS, FIVE TO TWELVE YEARS OLD, SOLVED SIX RULE LEARNING PROBLEMS. IT WAS CONCLUDED THAT YOUNGER CHILDREN SOLVE RULE LEARNING PROBLEMS IN A ROTE FASHION, WHILE ADULTS USE A STRATEGY BASED ON MEDIATED STIMULUS GROUPINGS. OLDER CHILDREN CAN LEARN THIS STRATEGY FROM INDIRECT EXPERIENCES, WHILE YOUNGER CHILDREN REQUIRE DIRECT TRAINING OF COMPONENT SKILLS. (AUTHOR)

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# DEVELOPMENT OF CONCEPTUAL SKILLS: SOME PRELIMINARY FINDINGS



WISCONSIN RESEARCH AND DEVELOPMENT

**CENTER FOR  
COGNITIVE LEARNING**

CG 005017



May 1969

Errata Sheet for Technical Report No. 81:

Please substitute corrected Tables 1 and 2 for those appearing on Page 3.

Table 1. Conceptual Rules Describing Binary Partitions of a Stimulus Population

Name	Primary Rule		Name	Complementary Rule	
	Symbolic Description <sup>a</sup>	Verbal Description		Symbolic Description	Verbal Description
Affirmation	R	All red patterns are examples of the concept	Negation	$\bar{R}$	All patterns which are not red are examples
Conjunction	$R \cap S$	All patterns which are red and square are examples	Alternative denial	$R \bar{S}$ $[\bar{R} \cup \bar{S}]$	All patterns which are either not red or not square are examples
Inclusive disjunction	$R \cup S$	All patterns which are red or square or both are examples	Joint denial	$R \bar{S}$ $[\bar{R} \cap \bar{S}]$	All patterns which are neither red nor square are examples
Condition	$R \rightarrow S$ $[\bar{R} \cup S]$	If a pattern is red then it must be square to be an example	Exclusion	$R \cap \bar{S}$	All patterns which are red and not square are examples
Bicondition	$R \leftrightarrow S$ $[(R \cap S) \cup (\bar{R} \cap \bar{S})]$	Red patterns are examples if and only if they are square	Exclusive disjunction	$R \bar{S}$ $[(R \cap \bar{S}) \cup (\bar{R} \cap S)]$	All patterns which are red or square but not both are examples

<sup>a</sup>R and S stand for red and square (relevant attributes), respectively. Symbolic descriptions using only three basic operations— $\cap$ ,  $\cup$ , and negation— $\bar{\phantom{x}}$ —are given in brackets.

Table 2. Primary Bidimensional Rules

Name	Symbolic Description <sup>a</sup>	Verbal Description
Conjunctive	$R \cap S$ "and"	All patterns which are red and square are examples.
Inclusive disjunctive	$R \cup S$ "and/or"	All patterns which are red or square or both are examples.
Conditional	$R \rightarrow S$ "if, then"	If a pattern is red, then it must also be square to be an example.
Biconditional	$R \leftrightarrow S$ "if and only if"	Red patterns are examples if and only if they are square.

<sup>a</sup>R and S stand for redness and squareness, the relevant attributes.



May 1969

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Please substitute corrected Figure 4 for that appearing on Page 5.

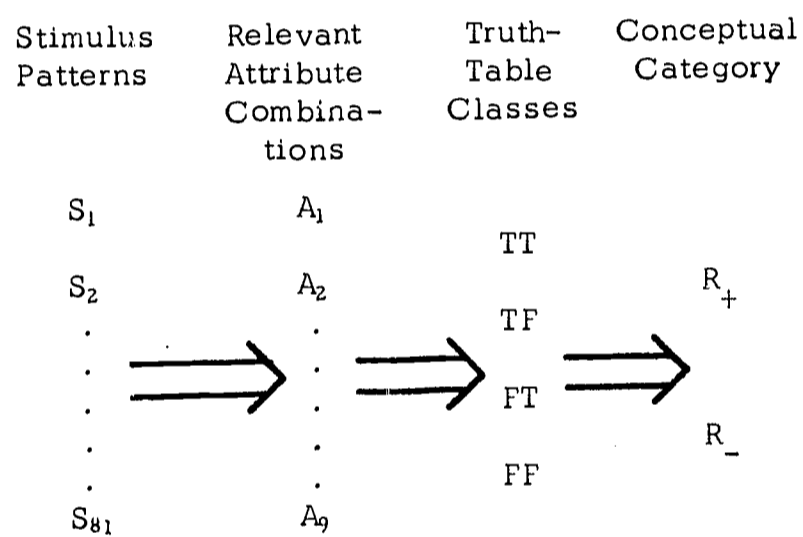


Figure 4. A logical analysis of the steps involved in mastering the truth-table strategy.



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May 1969

Errata Sheet for Technical Report No. 81:

Please substitute corrected Table 4 for that appearing on Page 6.

Table 4. Assignments of Stimulus Classes to Response Categories (+ and -) Under the Four Primary Bidimensional Rules

Stimulus Class	General Notation	Stimulus Set <sup>a</sup>	Conjunctive ( $R \cap S$ )	Disjunctive ( $R \cup S$ )	Conditional ( $R \rightarrow S$ )	Biconditional ( $R \leftrightarrow S$ )
RS	TT	RS	+	+	+	+
$R\bar{S}$	TF	RTr, RC	-	+	-	-
$\bar{R}S$	FT	GS, BS	-	+	+	-
$\bar{R}\bar{S}$	FF	GTr, GC, BTr, BC	-	-	+	+

<sup>a</sup>The following abbreviations are used: T, true (or present); F, false (or absent); R, red; G, green, B, blue; S, square; Tr, triangle; C, circle.





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$R\bar{S}$	TF	RTr, RC	-	+	-	-
$\bar{R}S$	FT	GS, BS	-	+	+	-
$\bar{R}\bar{S}$	FF	GTr, GC, BTr, BC	-	-	+	+

<sup>a</sup>The following abbreviations are used: T, true (or present); F, false (or absent); R, red; G, green, B, blue; S, square; Tr, triangle; C, circle.

BR-5-0216  
TR 81  
PA-24  
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Technical Report No. 81

DEVELOPMENT OF CONCEPTUAL SKILLS:  
SOME PRELIMINARY FINDINGS

By Lyle E. Bourne, Jr.  
University of Colorado

Discussants Thomas A. Romberg and Dorothy A. Frayer  
Wisconsin R & D Center

Report from the Situational Variables and Efficiency of Concept Learning Project  
Herbert J. Klausmeier and Robert E. Davidson, Principal Investigators

Wisconsin Research and Development  
Center for Cognitive Learning  
The University of Wisconsin  
Madison, Wisconsin

March 1969

This paper was presented at a special colloquium of the R & D Center. Preparation of the report was performed pursuant to a contract with the United States Office of Education, Department of Health, Education, and Welfare, under the provisions of the Cooperative Research Program. The opinions expressed in this publication do not necessarily reflect the position or policy of the Office of Education and no official endorsement by the Office of Education should be inferred.

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## STATEMENT OF FOCUS

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from the Situational Variables and Efficiency of Concept Learning Project in Program 1. General objectives of the Program are to generate new knowledge about concept learning and cognitive skills, to synthesize existing knowledge, and to develop educational materials suggested by the prior activities. Contributing to these Program objectives, the Concept Learning Project has the following five objectives: to identify the conditions that facilitate concept learning in the school setting and to describe their management, to develop and validate a schema for evaluating the student's level of concept understanding, to develop and validate a model of cognitive processes in concept learning, to generate knowledge concerning the semantic components of concept learning, and to identify conditions associated with motivation for school learning and to describe their management.

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## ABSTRACT

The main paper reports a series of exploratory studies and three experiments dealing with conceptual rule learning. Discussants relate the results to subject-matter fields and to educational research and development.

The term "conceptual rule" is defined and relevant research reviewed. Exploratory studies yielded the following results: (1) After six problems based on the same rule (conjunctive, disjunctive, conditional, or biconditional), all subjects reached errorless performance regardless of initial rule difficulty. (2) Decrease in mean errors to criterion over a series of twelve successive rule learning problems, three for each of four types of rules, indicated interrule transfer. Analysis of responses showed increased percentages of subjects solving while making no more than one error per truth-table class. (3) Four groups of subjects, 5- to 6-year-olds, 8- to 9-year-olds, and 11- to 12-year-olds solved six rule learning problems, three conjunctive and three disjunctive. Younger children showed more errors per truth-table class than older children and adults. In Experiment I, 5- to 5 1/2-year-olds, 6- to 6 1/2-year-olds, 7- to 7 1/2-year-olds, and adults solved problems based on four rule types. Performance on a truth-table sorting task following multiple rule learning increased as a function of age. In Experiment II, 5-, 6-, and 7-year-old children were taught either truth-table sorting or dimensional sorting, then tested on rule learning problems. Mean trials to criterion were less for truth-table pretrained than dimensional pretrained subjects at all age levels, with older children showing the greatest difference. Experiment III employed 5- and 7-year-old subjects. First, half of the subjects were taught each truth-table class, the other half dimensional sorting. All subjects were then given three truth-table sorting tasks and three rule learning problems. Transfer from truth-table problems to rule learning problems was observed for 5-year-olds only when it had been preceded by class training; for 7-year-olds, class training had no effect on rule learning. It was concluded that younger children solve rule learning problems in a rote fashion, while adults use a strategy based on mediated stimulus groupings. Older children can learn this strategy from indirect experiences, while younger children require direct training of component skills.

The idea that strategies are hierarchical and can be taught is of importance to subject-matter specialists in education. Basic research plays a crucial role in research and development by serving as a source of ideas and direction for educational development.

## DEVELOPMENT OF CONCEPTUAL SKILLS: SOME PRELIMINARY FINDINGS<sup>1</sup>

Lyle E. Bourne, Jr.  
Institute for the Study of Intellectual Behavior  
University of Colorado

The purpose of this report is to summarize some recent empirical and theoretical research on conceptual problem solving. What is new and interesting about this work is not the general topic, which is a relatively mature one in modern psychology, but rather the methodological approach, which we think has some promising new wrinkles. While the experiments so far have been largely exploratory, the results seem to have a few significant implications for an understanding of the development of human conceptual skills.

### CONCEPTS AND CONCEPTUAL TASKS

We begin the discussion with some definitions and procedures which are used throughout the paper. The experiments focus on the way in which people solve problems based on *class concepts*. A class concept is a principle which describes some partitioning of a stimulus population. In the simplest case, the

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<sup>1</sup>Some of the research reported here was conducted in the Institute for the Study of Intellectual Behavior, University of Colorado. Its preparation was supported by a training grant, 5-T01-MH-10427, and a research grant, MH-14314, from the National Institutes of Mental Health; a research grant, GB-3404, from the National Science Foundation; and a grant-in-aid from the University of Colorado Council on Research and Creative Work.

Many of my students and colleagues have contributed materially to this work. I am particularly indebted to W. Buchanan, D. Darnell, D. Dodd, D. Guy, R. Haygood, and W. King.

This paper was first presented as an Invited Address to Division 3 of the American Psychological Association at the 1968 convention in San Francisco, California.

partition is binary and the resulting groups are called the *positive* and the *negative instances* of the concept. In any problem used in the studies to be described, the subject has to learn how to make such a partition on a finite, well defined, and clearly dimensionalized population. The principle involved has the form of a relationship (*R*) among attributes (*x, y, ...*) of the stimulus population, and can be written:

$$C \equiv R(x, y, \dots)$$

Problems used in our experiments are based exclusively on bidimensional concepts, in which case, we can conveniently rewrite the description:

$$C \equiv x R y$$

The expression involves two kinds of information: The *relevant attributes* of the concept and the *rule* (or relationship) which integrates those attributes. Two types of experimental tasks are defined on these two components, the difference between them depending upon what information the subject needs to discover in order to solve the problem.

The more conventional and familiar conceptual problem is called the *attribute identification* (or attribute learning) problem and it may be described:

$$C \equiv ? R ?$$

The rule, *R*, or general form of solution is given and the task for the subject is to identify the two attributes which enter into the definition of the concept by observing a sequence of positive and negative instances. The complementary task is called *rule learning* and it may be described:

$$C \equiv x ? y$$

In this case the relevant attributes are given and the rule is an unknown. The rule must be



learned or identified by the subject, here again by observing a series of positive and negative instances. It is this task—rule learning—with which the present paper is primarily concerned.

Most of the rules that are used in the research to be reported, while familiar, are not immediately obvious to the subject. He can be expected to make errors in the course of solving for the unknown component of the problem. But, if the subject is exposed to a series of problems, each based on the same rule, he becomes quite proficient. In fact, as we will see, with multiple problem experience, most subjects will use any of the rules involved here errorlessly.

### Limits

There are two constraints that need to be noted before considering the experimental results. First of all, the stimulus material from which problems are constructed is a population of geometrical designs, varying at most along four dimensions, such as size and form. Secondly, the rules are four simple operations from symbolic logic which generate binary—that is to say, positive vs. negative instance—partitions of the population. These partitions are based on the presence or absence of two relevant attributes within stimulus patterns. Whether our data are reproducible with other, more meaningful materials or with more complex conceptual rule-systems is a question to which no easy answer can be given.

### Conceptual Rules

A detailed description of the rules used has been presented elsewhere (Neisser & Weene, 1962; Haygood & Bourne, 1965; Bourne, 1967) and need not be repeated here. It should be noted, however, that the rules were selected from a simple logic system which includes a total of ten unique and nontrivial ways of partitioning a stimulus population. These ten rules can be reduced to five pairs, each pair consisting of a primary and a complementary stimulus partition. The basis of this pairing is that any instance which is positive under one rule is negative under its complement. The rules are described in Table 1. The primaries appear on the left-hand side and are labeled, for convenience, the affirmative, the conjunctive, the disjunctive, the conditional, and the biconditional rules. Affirmation, a unidimensional rule, and all the complementary rules were excluded in the research to be reported: Our studies were

concerned only with the learning and the application of four *primary bidimensional* rules which are redescribed in Table 2 as the conjunctive, "and"; the disjunctive, "and/or"; the conditional, "if, then"; and the biconditional, "if and only if," rules.

### Some Examples

In any experiment the subject typically was given a series of problems to solve. The experimenter supplied preliminary instructions indicating the nature of the stimuli (geometrical designs), the kind of responses required (assigning the stimuli to one of two categories, "Positive" or "Negative") and the type of feedback to be provided about the correctness of responses. In rule learning problems, the two relevant attributes of the concept were named before the problem began, and the subject was instructed to find out in as few instances (or trials) as possible the correct relationship between these two attributes. For the subject, the idea was to discover how to sort the stimulus patterns properly into the positive and the negative instance categories, using the two named attributes.

For sake of example, suppose color and form are the two relevant dimensions, and that redness and squareness are the two relevant attributes on those dimensions. Suppose each stimulus dimension has three attributes. There are then nine different attribute combinations based on the two relevant dimensions. These are given in Table 3, along with the mappings of these combinations into the positive and negative categories prescribed by each of the four rules. Figure 1 gives a pictorial display of the same example. In any problem, the subject had to learn one of these four arrangements, based on two given relevant attributes.

### INITIAL EXPERIMENTS

Our early investigations of conceptual rule learning were unabashedly empirical. One purpose was merely to develop an overall picture of performance in different types of conceptual problems. We were concerned with how subjects learned rules as generic principles and how they used these rules in subsequent transfer problems. It is clear that there are no differences among the four rules in the number of unique stimuli or in the number of stimulus-response assignments to be learned. Yet, not surprisingly, for untrained subjects, the rules differ in difficulty. The hardest rule is biconditional, the next condi-

Table 1. Conceptual Rules Describing Binary Partitions of a Stimulus Population

Name	Primary Rule		Name	Complementary Rule	
	Symbolic Description <sup>a</sup>	Verbal Description		Symbolic Description	Verbal Description
Affirmation	R	All red patterns are examples of the concept	Negation	$\bar{R}$	All patterns which are not red are examples
Conjunction	R S	All patterns which are red and square are examples	Alternative denial	$R S$ $(\bar{R} \quad \bar{S})$	All patterns which are either not red or not square are examples
Inclusive disjunction	R S	All patterns which are red or square or both are examples	Joint denial	$R \downarrow S$ $(\bar{R} \quad \bar{S})$	All patterns which are neither red nor square are examples
Condition	$R \rightarrow S$ $(\bar{R} \quad S)$	If a pattern is red then it must be square to be an example	Exclusion	R $\bar{S}$	All patterns which are red and not square are examples
Bicondition	$R \leftrightarrow S$ $(\bar{R} \quad \bar{S})$	Red patterns are examples if and only if they are square	Exclusive disjunction	R S $(\bar{R} \quad \bar{S})$	All patterns which are red or square but not both are examples

<sup>a</sup>R and S stand for red and square (relevant attributes), respectively. Symbolic descriptions using only three basic operations— , and negation—are given in brackets.

Table 2. Primary Bidimensional Rules

Name	Symbolic Description <sup>a</sup>	Verbal Description
Conjunctive	R S "and"	All patterns which are red and square are examples.
Inclusive disjunctive	R S "and/or"	All patterns which are red or square or both are examples.
Conditional	$R \rightarrow S$ "if, then"	If a pattern is red, then it must also be square to be an example.
Biconditional	$R \leftrightarrow S$ "if and only if"	Red patterns are examples if and only if they are square.

<sup>a</sup>R and S stand for redness and squareness, the relevant attributes.

Table 3. Category Assignment, Positive or Negative, of Each Combination of Attributes from Two Dimensions (here, Color and Form) Known to be Relevant to a Concept

Stimulus Patterns	Rule			
	Conjunctive	Disjunctive	Conditional	Biconditional
Red squares	+	+	+	+
Red triangles	-	+	-	-
Red circles	-	+	-	-
Green squares	-	+	+	-
Green triangles	-	-	+	+
Green circles	-	-	+	+
Blue squares	-	+	+	-
Blue triangles	-	-	+	+
Blue circles	-	-	+	+

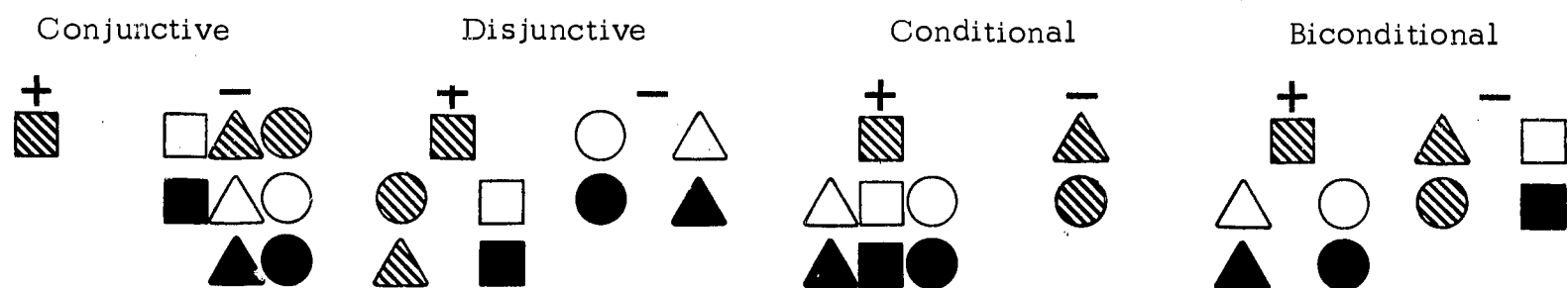


Figure 1. Partitions of a stimulus population illustrating all four primary bidimensional rules. Redness (cross hatching) and squareness are relevant attributes in all cases.

tional, the third disjunctive, and the easiest conjunctive, an ordering which has been reproduced in at least six different experiments (e.g., Bourne 1967). These differences are transient, however. Figure 2 shows what happens when a series of problems all based on the same rule are solved by a subject. In this experiment each of four separate groups of subjects worked on a different rule. After six problems on the same rule each subject achieved the level of errorless performance, no matter how difficult his rule was at the outset of the problem series.

There is also a considerable amount of interruler transfer. That is, practice on one rule generally has a positive effect on performance with a different rule, although there are important exceptions (Dodd, 1967). In another experiment, subjects solved thirteen successive rule learning problems—attributes given, rule unknown. For present purposes, the first twelve can be considered training problems; three were based on each of the four bidimensional rules. Problems on any

particular rule were consecutive, and the order of rules was counterbalanced over subjects. In a final, thirteenth, problem subjects were asked to identify in as few instances as possible which of the four rules just learned was the solution. A measure of overall performance taken in the first twelve problems is shown in Figure 3 to illustrate the form and the degree of interruler transfer.

#### The Truth-Table Strategy

Some of our data indicate that interruler transfer is traceable to the acquisition by subjects of a simple yet powerful problem-solving strategy. In the course of multiple rule learning, subjects acquire a mode of responding which is best described as an intuitive version of the logical truth-table. In some sense, the subject learns to mediate the stimulus pattern to response category assignments by collapsing and coding the entire stimulus population into four classes—the

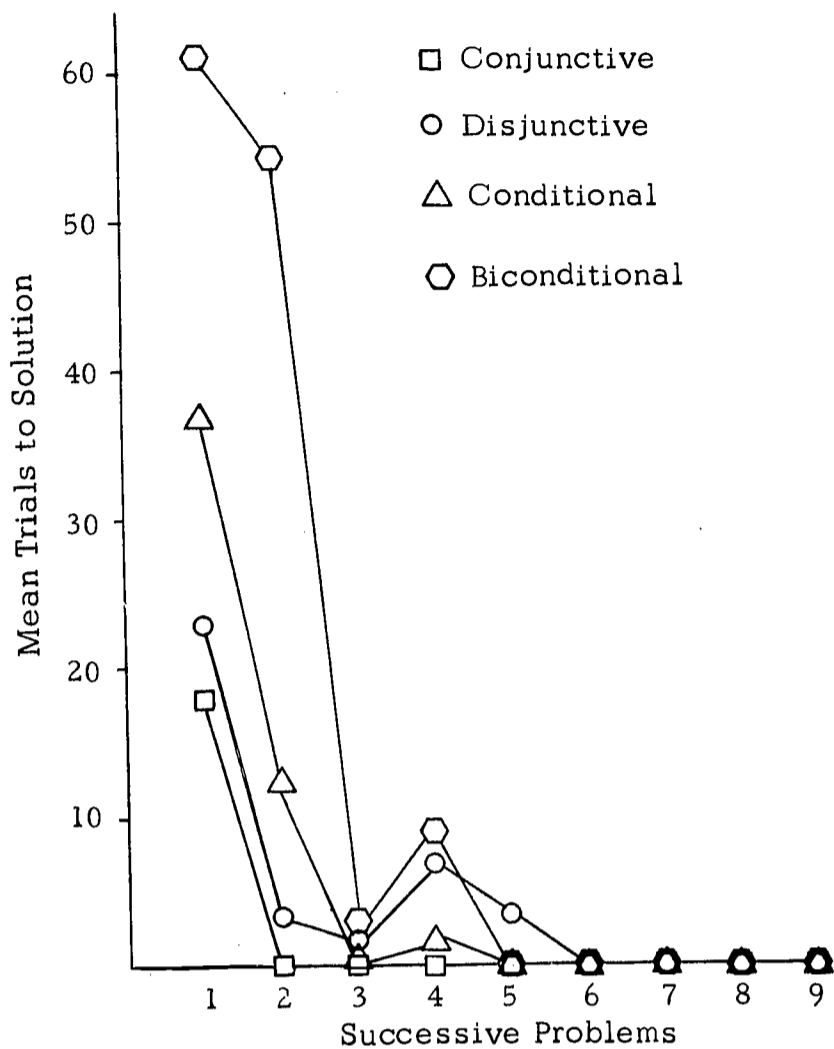


Figure 2. Performance on successive rule learning problems. For each group of subjects, all problems are based on the same rule.

$x$ - $y$ , the  $x$ -not  $y$ , the not  $x$ - $y$ , and the not  $x$ -not  $y$  classes—based on the given relevant attributes  $x$  and  $y$ . We can redescribe these groups as the true-true (TT), true-false (TF), false-true (FT), and false-false (FF) classes to correspond with conventional truth-table terminology. Once the coding process has been completed, each new rule learning problem is solved simply by learning the connections between the four coded classes of patterns and the two response categories.

There are several steps in arriving at this level of skill. For one thing, the subject must learn to attend, on instruction, only to dimensions exemplified by the given relevant attributes. He must, in other words, be able to collapse over irrelevant variations in the stimuli. (There are two irrelevant dimensions in the stimulus population used in our studies.) Next, the subject must associate the attribute combinations (nine in our population), derived by collapsing over irrelevant dimensions, with their respective truth-table classes. Only when the truth-table has been fully mastered can the subject solve rule learning

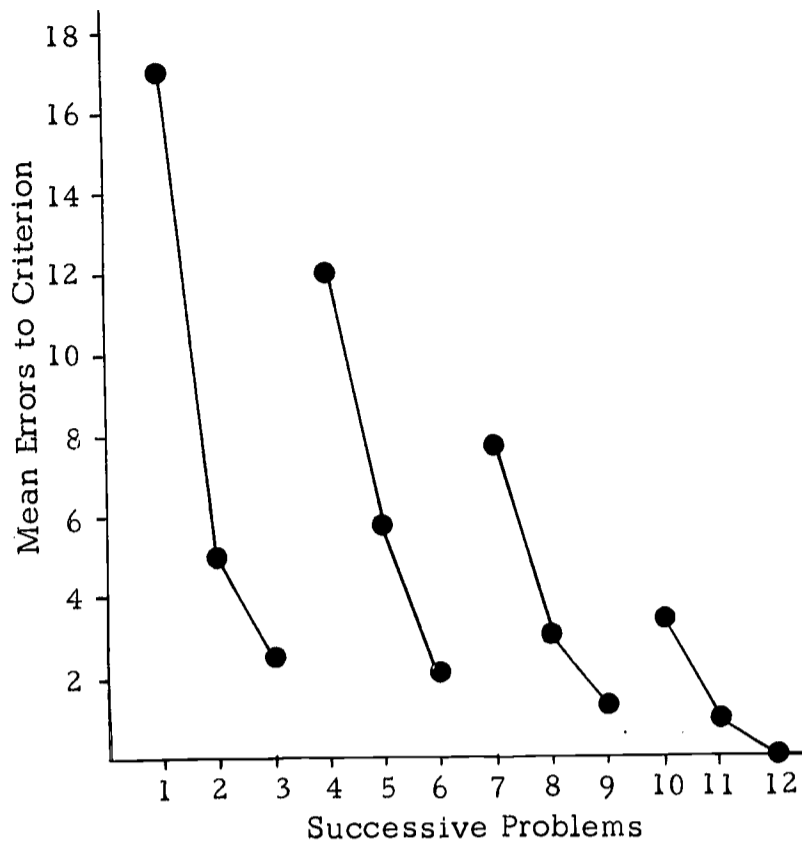


Figure 3. Performance on a series of rule learning problems. Each set of three successive problems is based on the same rule. All four primary rules are represented, with the order counterbalanced across subjects.

problems as 4:2 paired associates tasks. The steps involved in achieving the strategy are portrayed in Figure 4.

Stimulus Patterns	Relevant Attribute Combinations	Truth-Table Classes	Conceptual Category
$S_1$	$A_1$	TT	$R_+$
$S_2$	$A_2$	TF	
.	.	FT	$R_-$
.	.		
.	.	FF	
$S_{81}$	$A_9$		

Figure 4. A logical analysis of the steps involved in mastering the truth-table strategy.



Table 4. Assignments of Stimulus Classes to Response Categories (+ and -) Under the Four Primary Bidimensional Rules

Stimulus Class	General Notation	Stimulus Set <sup>a</sup>	Conjunctive (R S)	Disjunctive (R S)	Conditional (R → S)	Biconditional (R ↔ S)
RS	TT	RS	+	+	+	+
R $\bar{S}$	TF	RTr, RC	-	+	-	-
$\bar{R}S$	FT	GS, BS	-	+	+	-
$\bar{R}\bar{S}$	FF	GTr, GC, BTr, BC	-	-	+	+

<sup>a</sup>The following abbreviations are used: T, true (or present); F, false (or absent); R, red; G, green, B, blue; S, square; Tr, triangle; C, circle.

### Evidence of the Strategy

An observation that we take as strong evidence of truth-table performance is the ability of a subject to solve a rule learning problem with at most one error on instances of the four different truth-table stimulus types. As is shown in Table 4, each rule makes a unique assignment of the truth-table classes to response categories. Knowing where one member of any class is assigned, Positive or Negative, is tantamount to knowing where all members of that class are assigned. A subject who understands the truth-table might make a category error on the first instance of any class, say TF, in a new problem (or a new rule), but should perform errorlessly on instances of that class thereafter.

The data from the multiple rule learning experiment described above indicate that no subject showed any signs of truth-table performance on Problem 1 of the 13-problem series. All subjects required more than one instance of each of the four truth-table categories before solving that problem. By the time the second rule was introduced (Problem 4), however, 12% of the subjects gave such evidence. That is, 12% of the subjects solved the first problem on the second rule while making at most only one error per truth-table class. On the third rule (Problem 7), 27% of the subjects were truth-table solvers. On the fourth rule (Problem 10), 51%. Then, on the thirteenth problem—the rule identification problem—83% of the subjects solved while making no more than one error per truth-table class.

Table 5 shows the mean number of trials to last error on instances of the four truth-table classes on the first and the thirteenth problems. Before training subjects take a

Table 5. Mean Trials to Last Error on Instances of the Four Stimulus Classes (Rule learning (RL) problems for training were based on all four primary rules. Pre- and Posttest data are averaged over all rules.)

Stimulus Class	Before RL Training	After RL Training
TT	1.75	.00
TF	6.25	.61
FT	5.50	.53
FF	8.68	.67

fairly large mean number of trials to learn the correct assignment of each of the four truth-table categories. They are, from all appearances, learning to assign the stimulus instances within each category in a rote fashion and somewhat independently. After truth-table training the subjects require an average of less than one trial to learn the assignment of stimulus patterns to truth-table categories. Thus, one example is enough for the sophisticated subject to determine the correct placement of a sizeable subset of patterns.

### DEVELOPMENT OF CONCEPTUAL SKILLS

There are other data along these lines that might be reported, but I want to devote the remainder of this discussion to a special, related issue which is of considerable interest and is the focus of most of our current research effort. The results reported so far are taken from college subjects, 16 to 30 years of age.



While we were collecting them, a student interested in developmental problems conducted an exploratory comparison of the performance of children and adults on rule learning tasks. He used groups of 5- to 6-year-olds, 8- to 9-year-olds, 11- to 12-year-olds and adults. Each subject solved six rule learning problems—attributes given, rule unknown—three based on the conjunctive and three on the disjunctive rule. A seventh problem, rule identification having one of these two rules as its solution, was given at the end of the training series. Not unexpectedly, performance on all problems improved with age. But now, looking at the data in the light of the preliminary work discussed earlier, we can see that logically it would take a "truth-table" subject only one instance to solve the rule identification problem. Remember that the solution can be only a conjunction or a disjunction. These rules differ only in the assignment of TF and FT ( $x$ -not  $y$  and not  $x$ - $y$ ) instances, which are positive for a disjunction, negative for a conjunction. An example of either class is sufficient to know the solution. All adults and eighteen of twenty 11- to 12-year-olds performed that way. This is suggestive of adult-like truth-table performance on the part of most 11- to 12-year-olds. Eight- to 9-year-olds showed some tendency to make errors on both the TF and the FT categories. These subjects seemed to be able to code the stimuli into classes but failed to develop a complete understanding of the truth-table strategy, insofar as it allows for interclass inferences, from the training problems. Finally, 5- to 6-year-olds made errors on instances of all classes, especially the TF and FT classes. They gave little or no evidence of coding or of the development of a truth-table strategy.

### Experiment I

Conclusions drawn from the foregoing exploratory study are quite tenuous. The analyses were strictly ad hoc and there were severe technical limitations, the main ones being that only two rules were used and only three problems were given on each rule. The experiment was not properly designed nor was it intended to reveal the development of a truth-table strategy. For these reasons, a followup experiment was undertaken to test the differential abilities of children. We were interested primarily in the lower age range and therefore used three groups: 5- to 5 1/2-year-olds, 6- to 6 1/2-year olds, and 7 to 7 1/2-year-olds. These subjects were

required to solve problems based on all four primary bidimensional rules. Because biconditional and conditional rules are difficult, even for adult subjects, the problems were simplified in two ways. First, the experimenter helped the subject through the first problem on each rule, and, secondly, problems were constructed on a reduced two-dimensional (instead of four-dimensional) stimulus population. Each subject solved three problems per day for four days. The three problems on any particular day were all based on the same rule. On the fifth day the subject was tested for his knowledge of the truth-table with a specially constructed four-category sorting problem. To solve this problem the subject had to learn that the categories correspond to the four classes of patterns prescribed by a two-dimensional truth-table constructed on two given attributes. As in the case of rule learning, patterns were presented to the subject one at a time for sorting. The categories were unlabeled, but a correction procedure was used which allowed the subject to observe one correctly sorted instance of each category. It is important to remember that the two attributes on which the truth-table sort was to be made were named for the subject at the outset.

We shall skip over the rule learning data though they are interesting in their own right. Briefly, we observed the same relative rule difficulty as was seen earlier with adults. Also, performance improved over problems, and there were interrule transfer effects, though they were less for young children than for adults.

The primary consideration is performance on the truth-table problem before and after the rule learning experience. To appreciate these data, it is helpful first to look at the performance of adults on a similar task. Table 6 shows that with no prior rule learning experience adults require some 24 trials to master the

Table 6. Mean Trials to Last Error on a Truth-Table Sorting Problem After Twelve RL Problems Based on Varying Numbers of Rules

Number of Rules	Trials
0	23.96
1	14.59
2	16.31
3	9.04
4	5.58

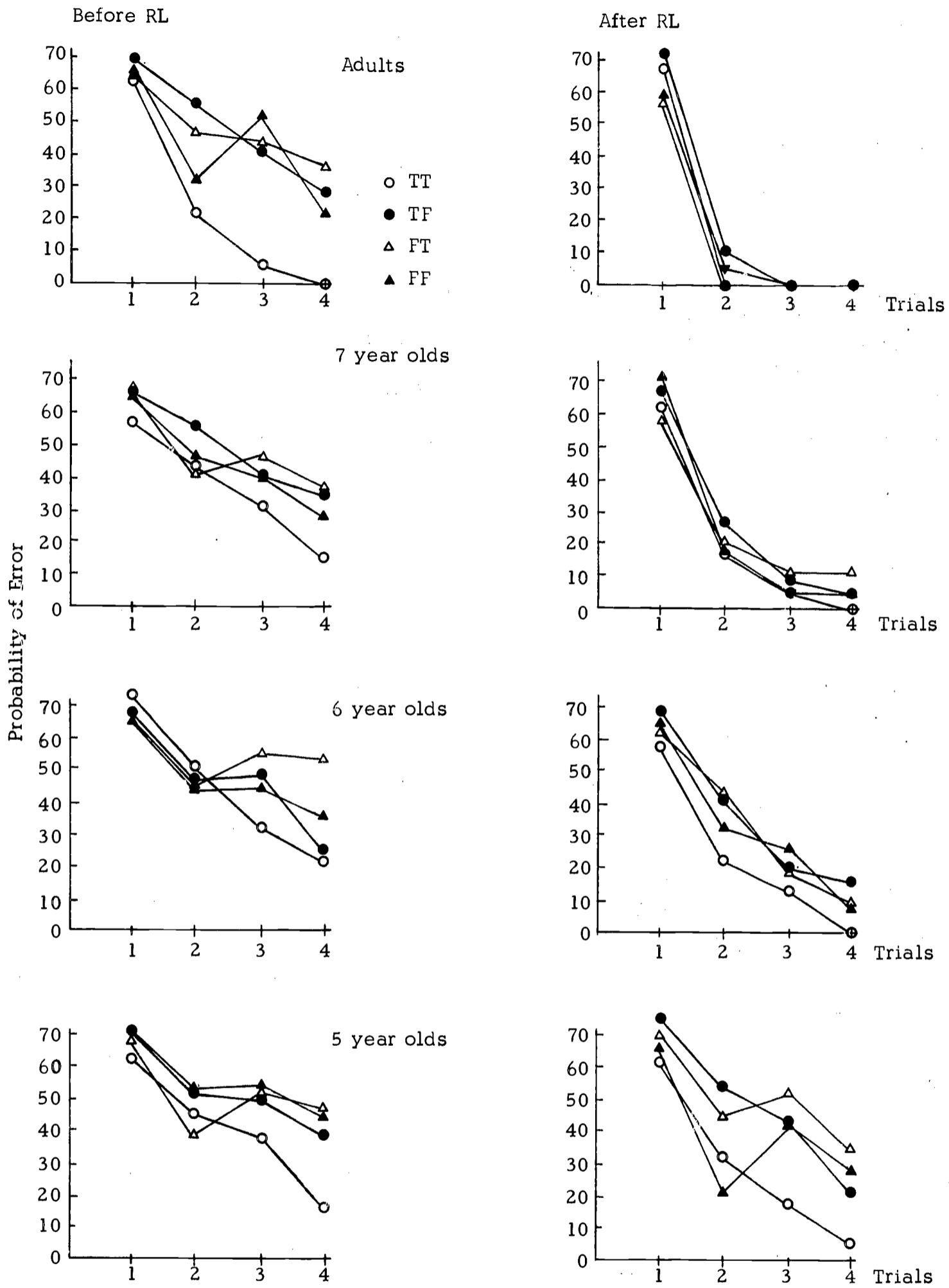


Figure 5. Probability of error for the first four instances of all four truth-table categories. Data are taken from the four category truth-table tasks, before and after solving twelve rule learning (RL) problems. For all subjects, three of the RL problems were based on each of the four primary rules.

truth-table sort. The remaining data in this table were provided by adult subjects who, like the children in this experiment, solved twelve successive rule learning problems prior to the truth-table task. For some subjects all twelve problems were based on the same rule, the rule differing from subject to subject. Another group of subjects solved six problems based on each of two rules. A third group solved four problems based on each of three rules and a fourth group solved three problems based on each of the four primary bidimensional rules. There was complete balancing of rules and rule orders within groups. It is evident from the data that performance on the truth-table task does indeed improve with variety and amount of preceding rule learning experience, and we take these data to mean that adult subjects do evolve the truth-table approach through multiple rule learning experience.

Figure 5 shows even a finer breakdown of performance both by adult subjects and by the 5-, 6-, and 7-year-old children on the truth-table task. These axes show the probability of making an error on the first four presentations of instances of each of the four truth-table categories. The results are complex, but it can be seen that probability of error generally declines over the course of four instances for all truth-table categories. Considering adults only, the rate of decline is much more marked *after* rule learning experience than it is *before* rule learning experience. Before rule learning, the drop is gradual for all the stimulus classes. After rule learning the drop is abrupt, moving from near chance to zero between the first and the second instance of each of the four categories. One instance per class is sufficient for rule-trained adults to solve the problem. Once again we take this as evidence of the acquisition of a truth-table strategy by adults through multiple rule-learning experience.

Now consider the performance of children on the truth-table task. Before rule learning, there is some similarity between the data of children and adults; but it is also reasonably clear that as our subjects age, mature, and become more experienced they are able to master the truth-table problem in fewer and fewer trials. The right hand side of Figure 5 shows the performance of children *after* multiple rule learning experience. Here the pattern is quite unlike that of adults. Even the oldest children require several instances of each of the truth-table categories before they cease to make errors. The 7-year-olds are

more adult-like than the 5-year-olds, however. The 5-year-olds indeed show little improvement on truth-table problems as a consequence of multiple rule learning experience.

If we count the number of subjects at each age who did master the truth-table problem with at most one error on each of the four classes, we find that only one of twelve 5-year-olds, three of twelve 6-year-olds, and six of twelve 7-year-olds achieve this level of performance, while 21 of 24 rule-trained adults solved with at most one error per category.

The effect of interpolated rule learning experience on truth-table performance is clearly more marked as age increases. Younger subjects take the stimulus patterns individually and appear to learn their assignment to response categories by rote. Moreover, they show less positive transfer from one task to the next than adults do. The very youngest children give practically no evidence of positive transfer from rule learning to the truth-table task.

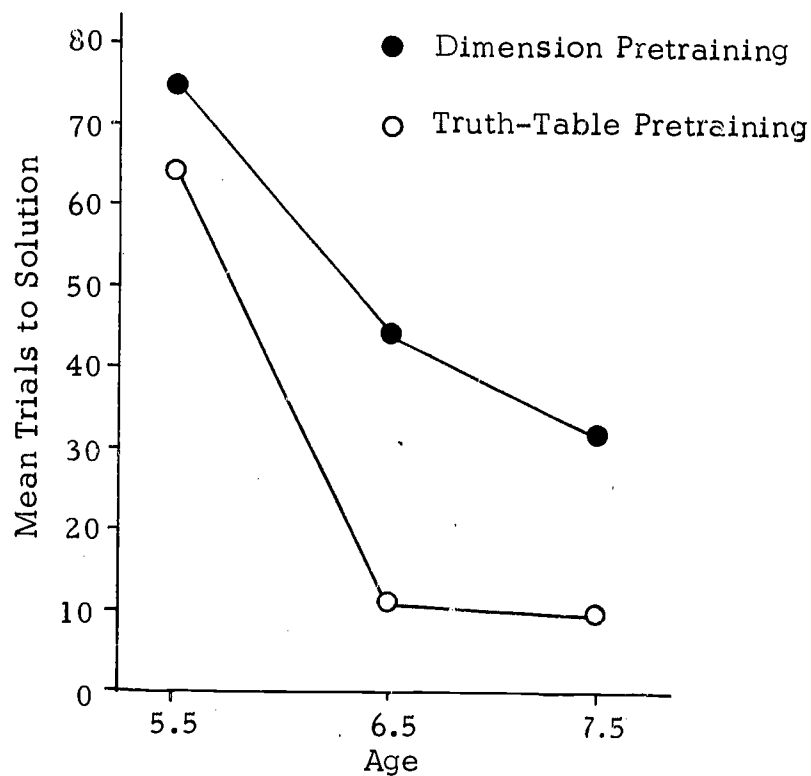
## Experiment II

Our next experiment was the converse of Experiment I. Here we attempted to teach the truth-table first and to observe the transfer effects on subsequent rule learning tasks. Once again we used 5-, 6-, and 7-year-old children. The experiment began by having half of the children work through four truth-table problems, identical to those used in the last experiment. The attributes, as always, were named at the outset. The first problem was a demonstration by the experimenter, and the following three were solved by the child. An equal number of children were exposed to four unidimensional sorting tasks, designed to provide familiarity with the stimulus population but no truth-table practice. Immediately thereafter all children solved three rule learning problems, based on either the conjunctive or the disjunctive rule.

Figure 6 portrays the mean number of trials to solution for the first and for the average of all three rule learning problems, as a function of the child's age and of the nature of his pretraining. These data show that positive transfer effects from truth-table pretraining become increasingly larger with age. There is only a 9% difference in trials to solve between logic pretrained and nonlogic pretrained children at age five. That percentage increases to 78% at age six and to 85% at age seven.



PROBLEM 1 ONLY



PROBLEM 1-3

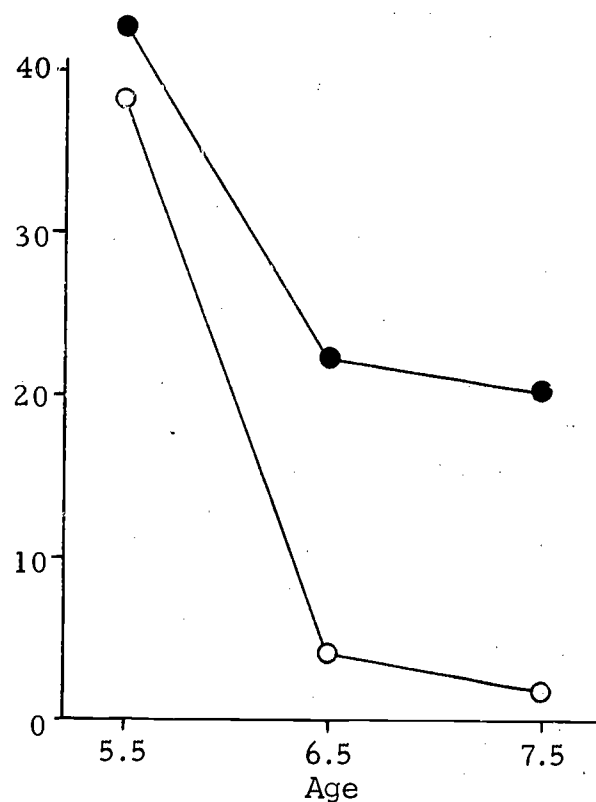


Figure 6. Combined performance on conjunctive and disjunctive RL problems after pretraining on the truth-table or on the stimulus attributes.

### Discussion

The older the child, the better he performs, obviously. It is more important to know what systematic properties of behavior change with age to produce this trend. The evidence suggests that the younger the child, the less likely he is to display the type of strategic behavior that these simple logical problems require and, of course, that adults very quickly come to exhibit.

Can we say anything about the conditions implicated in this developmental progression? There are several possibilities. First, the youngest children in the foregoing studies were preschoolers while the older ones were in kindergarten or first grade. Better performance and greater transfer exemplified by older children could be a product of the expansion of social or intellectual experiences provided by formal schooling. A more interesting possibility is that the younger children are simply maturationally incapable of coding and performing the other necessary logical operations for solving these problems in an adult-like fashion.

A third possibility is that the youngest children, for reasons of limited training, lacked the basic, underlying skills that are implicated in this logical system. The relatively complicated experiences provided by multiple rule learning and four-category truth-

table sorting tasks might have been too overwhelming or too indirect to provide meaningful training for preschoolers on these component skills.

We have some informal evidence to support the latter possibility. Note that to use the truth-table, the subject must be able to code and to work with several sets of stimulus patterns simultaneously. Coding involves, among other things, the logical operation of negation. The subject must understand, in a certain way, the concept "not" as in "not red," in order to reduce the stimulus population to manageable size. In interviews with the children after the preceding experiments, we asked questions about the similarities and differences among the stimuli used in these problems. For example, we might ask how otherwise identical green and blue blocks were alike in contrast to a red block. With probing, adults and some older children give the answer "not red." But we were able to elicit this response from very few of the youngest children. They would respond more commonly to the effect that green and blue blocks are not alike, that there is no similarity between them, and that they should be treated separately. This observation suggested to us that the failure of younger children to perform in an optimal fashion, even after prolonged rule learning and direct truth-table experience, is in part connected with their inability to understand the concept of

negation or how to take the complement of a set. And, of course, these notions are basic to the truth-table logic and strategy.

Adults and older children apparently can evolve a useful problem-solving strategy in the context of the rule learning problems themselves. But that strategy rests on simpler skills and on having available certain constituent bits of information. It might be that 5-year-olds simply have not mastered the necessary subparts or performance substructures for proceeding to the requisite, more complicated form of behavior.

### Experiment III

Our last experiment was designed to investigate this possibility. Its purpose was to teach some of the underlying components of the truth-table strategy to preschoolers and to first graders. From the foregoing argument, our expectation was that the younger children would benefit more from constituent pretraining than the older children, on the assumption that the constituent skills are naturally available to 7-year-olds but not to 5-year-olds.

The experiment was conducted in three phases. In Phase I half the children were given eight successive sorting problems which, as a whole, will be referred to as class or constituent training. The sorting problems were of four types, and there were two problems of each type. The first problem on any type was a demonstration by the experimenter; the second was a real problem solved by the subject. The four types of problems were based on the four classes of the truth-table. That is, the subject learned separately and successively to sort stimulus patterns into the TT, the TF, the FT, and the FF categories. The order of training on these categories varied from subject to subject. As in rule learning, two attributes were named and the subject used the presence or absence of either attribute as a basis of sorting. We hoped that this training would require the subject to recognize and to understand the concept of negation, i.e., "not  $x$ "; that the necessity to construct the TF (or FT or FF) category, given two attributes in positive or TT form, would induce the subject to recognize the concept of not  $x$  and to sort objects with properties  $y$ ,  $z$ , and so on into that category. The emphasis throughout was on the absence of an attribute as a basis for classifying patterns together and on the operation "not."

The other subjects received training on stimulus dimensions. Once again eight sort-

ing problems were administered; two sorts on each of the four dimensions of the population. The subject was required to sort stimulus objects into three categories corresponding to the three levels of a named dimension. The first sort was a demonstration, the second was real.

In the second phase all subjects were given three truth-table problems, each based on a different pair of given relevant attributes. The first problem was a demonstration, the next two were real problems solved by the subject. In the last phase, three rule learning problems were administered to all subjects. For half the subjects the problems were disjunctive, for the other subjects the problems were conjunctive.

The hypothesis under investigation was that older children, as in the earlier experiments, would develop the truth-table strategy easily from the truth-table problems. Five-year-olds, however, were presumed to need more basic training, particularly in the operation of negation in order to benefit from truth-table experience. Figure 7 portrays the results. We observe transfer from truth-table practice in 5-year-olds only when that practice has been preceded by class training. If truth-table practice is preceded by dimension training, then these problems are slow to be mastered and there is little or no transfer to later rule learning. Whether or not the truth-table task is preceded by basic training is relatively inconsequential for 7-year-olds.

In summary, then, the truth-table task is difficult for 5-year-old children who have not first built up the necessary component bits of knowledge. Not much is learned from dimension training to affect either truth-table or rule learning performance. For 7-year-olds, however, the truth-table tasks help the subject in the rule learning problems without any extra benefit derived from preliminary, component training.

### SUMMARY AND CONCLUSIONS

This is where the data run out. There are no revolutionary new findings or conclusions to be drawn and we are clearly not satisfied with our present state of knowledge. Obviously there are details to be worked out in future research. But rather than speculate on these future developments let us reconsider the results and what they might mean.

We have studied the performance of human beings of various ages in a type of conceptual problem and paradigm called rule learning. It is trivial, of course, to note that there are



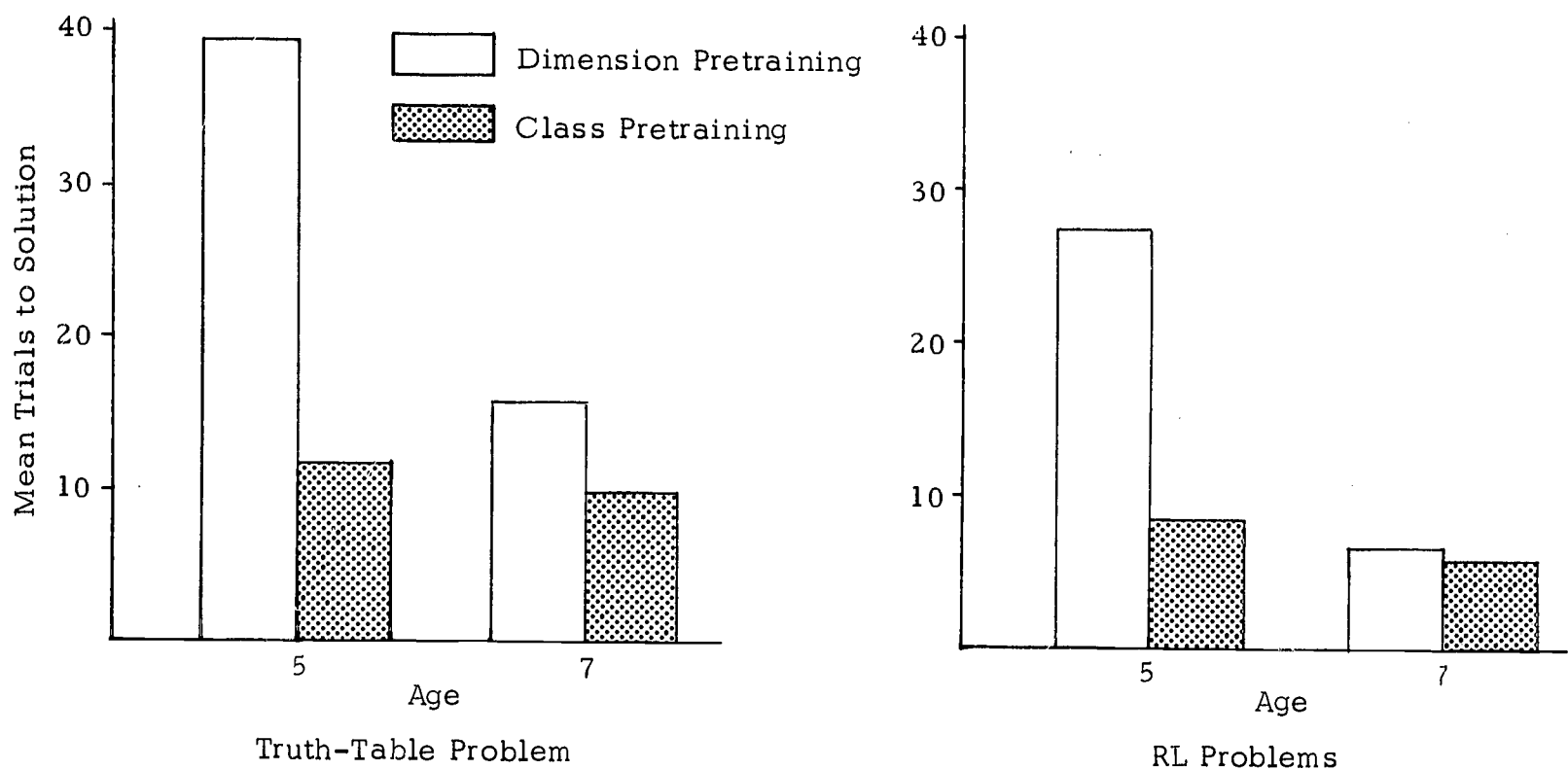


Figure 7. Performance on the truth-table problem and on combined conjunctive and disjunctive RL problems after pretraining on the separate classes of the truth-table or on the stimulus attributes.

overall changes in the quality of performance with age. It is, however, of some interest to note that the way in which solutions are achieved changes with the years and with training, from an apparent one-object-at-a-time rote performance to a highly sophisticated strategic approach based on mediated stimulus groupings.

The basic skills which underlie this strategy appear to develop naturally between the ages of 5 and 12. Among other things, what are required are a kind of acquired stimulus equivalence, the concept of negation, and an ability to deal with several sets of stimulus objects concurrently. Of course even with these rudiments, the truth-table strategy has to be learned; witness the learning-to-learn data of adult subjects. But given these constituents it can be learned in the context of relatively complex experiences such as a series of rule learning tasks or training on the four-category truth-table problem. The characteristic feature of this strategy is that it reduces a highly variable stimulus population to a small number of coded classes; in the present case a  $3 \times 3$ , or in general an  $m \times n$ , matrix of possibilities to a  $2 \times 2$  arrangement. Once the strategy is mastered, rule learning problems are solved as near trivial 4:2 paired associate tasks, and the initially present differences in difficulty among conceptual rules are essentially eliminated.

Five-year-old children fail to acquire the strategy from relatively complex and/or indirect experiences such as multiple rule learning. As a consequence, they appear deficient, particularly on transfer measures. However, a stepwise approach which involves training on the component skills can bring out the strategy and a form of terminal performance on rule learning problems comparable to that of adults.

I recognize that these findings are in a general sense little more than what other experimenters have shown. That is, they imply that behavior is better described as a hierarchical in contrast to, say, a linear structure—a fact that may of course be represented theoretically in a variety of ways—and that any particular behavior depends on the more elementary units of which it is composed. A person cannot proceed efficiently to complex performance without well integrated performance substructures. If there are any usefully unique features of our work so far they are, first, that it documents these principles in the context of uncommonly complex tasks and, second, that it shows that an integration of substructures that is normally, and might even be said to be inextricably, connected with developmental variables can derive as well from an adequately designed program of experience and training.

We intend to proceed now to more detailed studies of the development of component

knowledge and skills connected with logical conceptual problems. We hope further to determine whether the knowledge we teach to young children in these problems has all the performance properties which are character-

istic of older children and adults in terms of, for example, the range of achievements or degree of transfer it allows. I am uncertain about what to expect, but these questions seem to me to deserve empirical study.

#### REFERENCES

Bourne, L. E., Jr. Learning and utilization of conceptual rules. In B. Kleinmuntz (Ed.), *Concepts and the Structure of Memory*. New York: Wiley & Sons, 1967.

Dodd, D. H. Transfer effects from rule learning to logical problems. University of Colorado, Cognitive Processes Report No. 104, 1967.

Haygood, R. C., & Bourne, L. E., Jr. Attribute- and rule-learning aspects of conceptual behavior. *Psychological Review*, 1965, 72, 179-195.

Neisser, U., & Weene, P. Hierarchies in concept attainment. *Journal of Experimental Psychology*, 1962, 64, 640-645.

## IMPLICATIONS FOR SUBJECT-MATTER FIELDS

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Although the following comments are intended to be general, they are bound to be colored by the mathematical background of this writer. Thus, the examples used to illustrate points are drawn from mathematics and from this author's work in that field at the R & D Center.

Not being a psychologist, I have found the following paradigm (Figure 8) convenient to classify research and theorizing in educational psychology. Solving a problem involves the perception of the stimulus situation (identification of the problem); the establishment of some overall plan, procedure, process, or strategy in order to answer the particular problem (selection of a general intellectual process to be used to solve this problem); the coding of the information from the stimulus situation (identifying the dimensions upon which the problem is to be solved and noting the relative values of objects in that dimension on those dimensions); memory (storage and/or retrieval) of the coded data; and imposing transformations (formal algorithms or mediational elaborations) upon the encoded data.

With this paradigm in mind, and with my familiarity with Professor Bourne's previous work, I assumed this paper would deal solely with coding, the sorting and classification of stimuli, and not other cognitive operations. Although I agree with the importance of the identification of dimensions, the identification of values or attributes on those dimensions, and the use of rules to form class concepts, I was ready to question the paper based on previous arguments on the irrelevance of such coding to the learning of complex concepts in mathematics and had ready such questions as the following: (1) Why are only obvious perceptual dimensions used? Most "real" concepts deal with constructural not perceptual dimensions. (2) What about verbal labeling? Most "real" concepts are

taught using verbal labels. (3) Why are only bidimensional problems used? Most "real" concepts have many dimensions. (4) Why not use negation of irrelevant dimensions as E. J. Martin<sup>1</sup> has argued? (5) Why not consider structural concepts where dimensions are previously learned? (6) Why in the theoretical formulation  $C \equiv R(x, y, \dots)$  is it not made clear whether  $x$  and  $y$  are attributes on the same dimension or different dimensions, or does it matter?

Questions such as these have previously been discussed by Professor Bourne,<sup>2</sup> and in this paper he cautions the reader about the reproducibility of the findings of concept or rule learning experiments for more complex conceptual systems. But still I was not prepared for his introduction of the truth-table strategy and his description of research on its use. Suddenly the frame of reference became planning rather than coding; that is, the interrule transfer demonstrated in Bourne's studies seemed to be traceable to subjects' acquisition of a simple yet powerful problem-solving strategy. A general intellectual process was identified, one which adults appear to use in the solution of simple types of problems. Even in such simple situations there seem to be several steps at arriving at the strategy—note that the truth-table strategy was not directly taught to adults.

The identification of such a strategy from a simple problem setting is an efficient method of discovering such processes. Perhaps the approach used by Bourne is more efficient than content analysis such as we have been

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<sup>1</sup>Martin, Edwin J. "Formation of Concepts." In B. Kleinmütz, *Concepts and the Structure of Memory*. New York: John Wiley, 1967.

<sup>2</sup>Bourne, Lyle E., Jr. *Human Conceptual Behavior*. Boston: Allyn & Bacon, 1966.

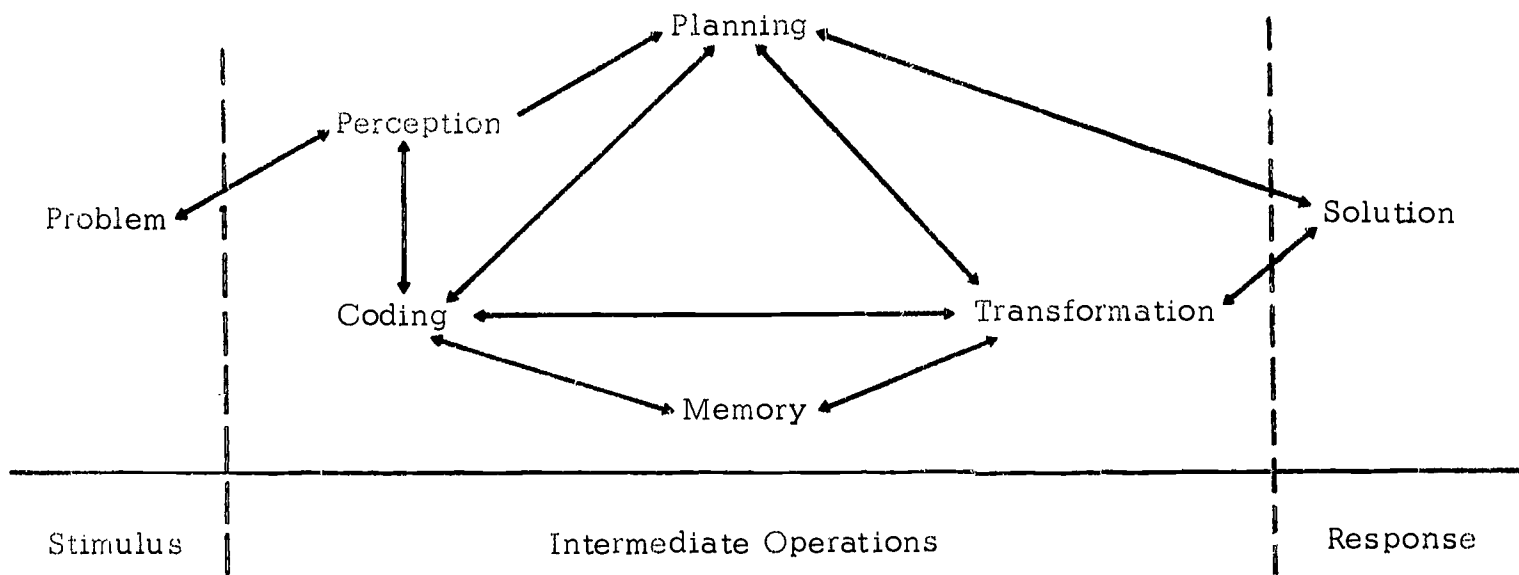


Figure 8. Types of Cognitive Operations

This figure is a liberal translation of a discussion by Jerome Kagan in *Developmental Studies in Reflection and Analysis*, Fels Research Institute Report, 1963.

doing<sup>3</sup> or the analysis of problem-solving protocols such as Newell, Shaw, and Simon have done.<sup>4</sup> Furthermore, the truth-table strategy itself was interesting to me because of its connection to mathematics; it can be described by logical rules. (It should be noted that the distinction between conditional and unconditional relations is always hard to teach in logic.)

Staff of the mathematics instruction project at the Center has been concerned with the identification of strategies or general intellectual processes which appear necessary for children to learn in order to solve mathematics problems. Such processes as comparing, classifying, contrasting, ordering, equalizing, putting with, taking from, and locating have been identified with certain kinds of mathematics problems. It would appear quite likely that such strategies are generalizable to other stimulus problems.

The truth-table strategy discussed here can be viewed as one general intellectual process or strategy used by students to sort the relevant information of a problem into dimensions and attributes and perform the

<sup>3</sup>Romberg, Thomas A., Fletcher, Harold J., and Scott, Joseph A. "A Measurement Approach to Elementary Mathematics Instruction." *Working Paper No. 12*. Madison: Wisconsin R & D Center, 1968.

<sup>4</sup>Newell, A., Shaw, J. C., and Simon, H. "Report on a General Problem-Solving Program." *Proceedings of the International Conference on Information Processing*. Paris: UNESCO House, 1959.

appropriate transformation of this information in order to make a correct response to the problem. Not to slight the discovery or identification of this strategy—identifying processes by which we sort information from the existing world into something that can be worked with is extremely important—even more important is what Professor Bourne chose to do with this discovery. Recognizing that strategies are learned, he demonstrated empirically that adults and young children do not use the same strategies on the same kind of problem are significant. Too often we have assumed that young children learn subject content in a manner similar to that of adults. Rarely have psychologists demonstrated in as convincing a manner as Professor Bourne has done here that adults and young children approach problems in quite different ways. Professor Bourne did not stop there; in addition, he actually tried to teach students to use the particular strategy.

Too often educators have concluded from status studies which describe differences between adults and young children that young children are maturationally incapable of coding and performing the logical operations for solving problems like an adult. Although I am in agreement with the vast amount of status data (particularly Piaget's) about how students actually do attack problems, none of this implies that students cannot be taught to use an adult process. Professor Bourne's observation that "the youngest children, for reasons of limited training, lack the basic underlying skills that are implicated in this logical system" seems to be the most reasonable explanation of young children's performance.



The tentative conclusions emerging from the Center's work support this point of view. In a series of exploratory studies dealing with the long observed but little understood problems of conservation of numerousness<sup>5</sup> and conservation of length,<sup>6</sup> we assumed that students lacked the underlying skills necessary for problem solution. For example, a conservation of numerousness problem asks that the child compare the numerousness of one configuration of a set with that of a transformed configuration of the same set. One explanation of children's difficulty with conservation problems is simply that children are maturationally unable to solve them. Another is that they do not have the appropriate skills; i.e., they attend not to the dimension numerousness but to other dimensions which are truly transformed. In our exploratory studies dimensional and process training have changed young children's ability to solve conservation problems. Again, I was excited to discover Professor Bourne training young children to use a process.

Professor Bourne was not willing to speculate on future developments. Yet, if his work

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<sup>5</sup>Scott, Joseph A. "The Effect of Selected Training Experiences on Performance on a Test of Conservation of Numerousness." *Technical Report*. Madison: Wisconsin R & D Center, in press.

<sup>6</sup>Gilbert, Lynn. "An Introduction of Length Concepts to Kindergarten Children." *Technical Report*. Madison: Wisconsin R & D Center, in press.

continues in the directions this paper indicates, it is these future developments that are likely to have most implications for the teaching of subject matter.

Adult problem-solving strategies such as the one identified here appear to develop naturally between the years of 5 and 12. *Naturally* means through the environment which includes schooling and instructional experience. The notion that such strategies are hierarchical and can be taught is the most important idea in this paper for subject-matter specialists. One can only agree with Professor Bourne when he says that these findings are in a general sense little more than what others have stated (behavior is best described as hierarchical, and any particular behavior that you would like to produce depends on more elementary units of which it is composed). However, few researchers have identified such strategies or investigated the use of the strategies. Only as the kinds of strategies people use to solve problems are identified, as the subsequent parts of those strategies are specified, and as efforts to teach young children the components of these strategies are begun will generalized information be produced which can be used in the study of many subject areas.

Thus, the implication of Bourne's paper for subject-matter specialists is not that the stimulus information given in the experiments is generalizable to subject areas but that the way in which information is processed is very likely generalizable. The identification of general intellectual processes has implications for the teaching of subject matter.

## IMPLICATIONS FOR RESEARCH AND DEVELOPMENT IN EDUCATION

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The major focus of the Wisconsin R & D Center is the improvement of education through a better understanding of cognitive learning. Both concept learning and acquisition of cognitive skills are primary areas of research. In its study of cognitive learning, the Center engages in a wide range of activities, from basic research similar to that which has been reported by Professor Bourne to development-based research in the school setting. A unique aspect of an R & D Center is the oppor-

tunity for a continuous interplay between these divergent activities. This interplay is shown in Figure 9. Activities which are typically carried out under Program 1, *Conditions and Processes of Learning*, are indicated by the broken borders; activities which are typically carried out under Program 2, *Processes and Programs of Instruction*, by the solid borders. Dr. Bourne's research on conceptual skills is typical of the kind of basic research executed under Program 1.

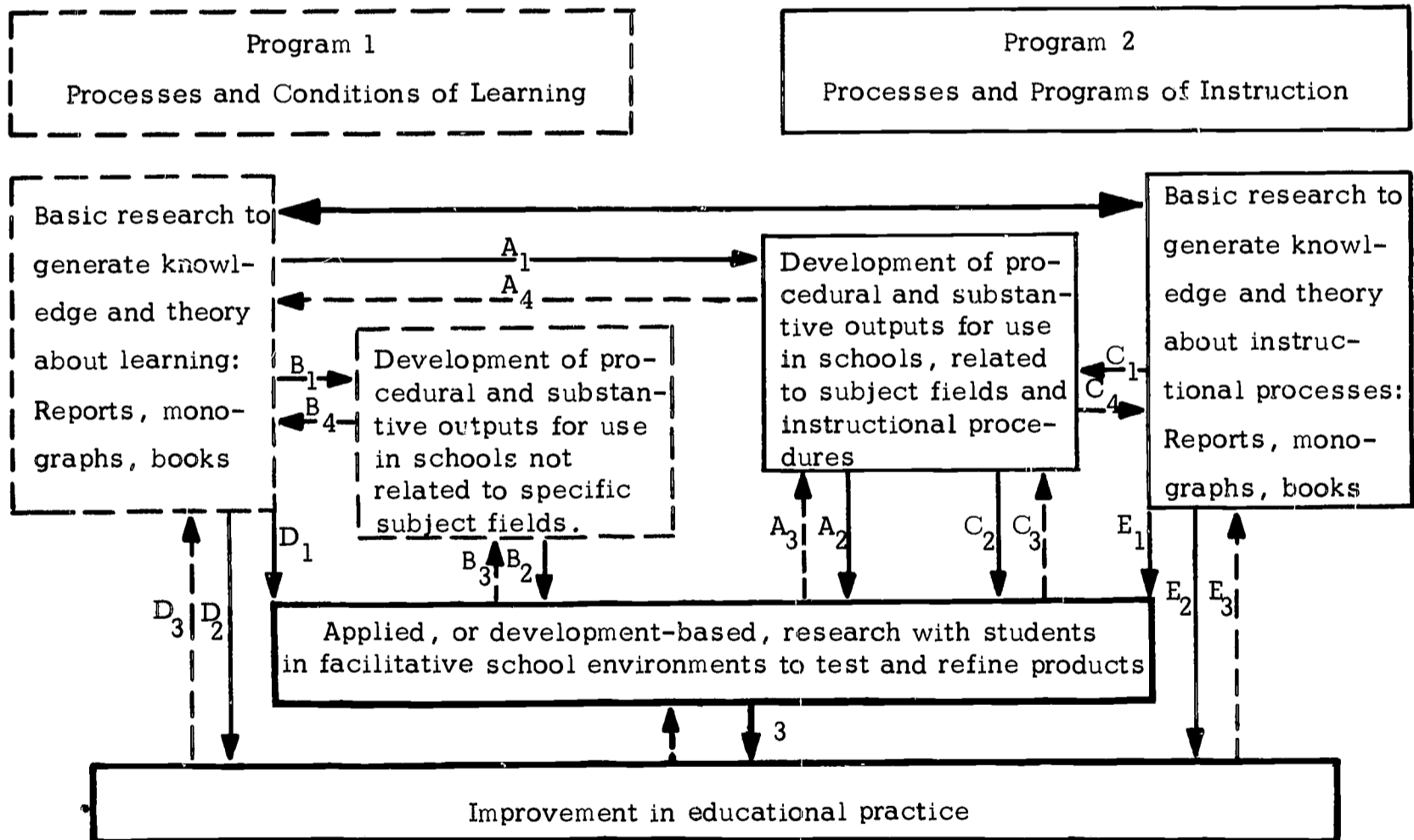


Figure 9. Relationships among research and development activities at the Wisconsin R & D Center for Cognitive Learning.

Basic research plays a crucial role in educational research and development. This research establishes a firm base of knowledge which can serve as a source of ideas and direction for educational development. The substantive results of Dr. Bourne's research extend knowledge in areas highly relevant to the focus of the Wisconsin R & D Center. First, new information is supplied concerning rule learning—a largely neglected aspect of concept learning. The role of attending to stimulus attributes, coding, and learning sub-concepts is pointed out. Second, techniques for inducing an effective cognitive strategy are shown. Both rule learning and strategies are central to the problem of facilitating cognitive learning.

Turning from the substantive results outlined in Dr. Bourne's paper, we may profitably take a closer look at the methodology employed. Controlled experimentation was employed to clarify the manner in which cognitive development proceeds. The original observation, that performance on rule learning improves with age, is similar to that noted for many types of cognitive tasks. The observation, however, is relatively unenlightening since it does not clarify the factors which account for the improvement. Maturational factors, general social or intellectual experiences, or development of specific prerequisite skills may be crucial. To ascertain whether the performance of the younger child can be improved and what techniques could effect this improvement, the causal factors must be determined. In the case of rule learning, some of these factors have been isolated. Let us review the progression of experimentation which clarified these factors.

First, it was noted that performance on rule learning improved with age. Data were examined to see what fine differences in response patterns existed among age levels. Preliminary work and logical analysis suggested that high performance might be related to the coding of stimuli into truth-table classes; therefore, the number of errors per truth-table class was calculated for each age group. Clear differences emerged.

Second, children of three age groups solved from one to four rule learning problems

and were then explicitly tested for ability to sort stimuli into truth-table categories. Results indicated that the youngest children exhibited practically no evidence of positive transfer from rule learning to the truth-table task. Older children exhibited somewhat greater transfer, but still less than that of adults.

A third step in the process of determining the factors that account for improvement of rule learning with age was to teach truth-table categorizing first and to observe the effects on subsequent rule learning tasks. Positive transfer effects were small for the youngest children but became increasingly larger with age. At this point, since the youngest children failed to profit from pre-training, it might appear that they were maturationally incapable of performing the cognitive processes necessary to solve these problems in an efficient manner. Informal evidence from interviews with children, however, suggested an alternative—that failure to profit from pre-training resulted from inability to understand the concept of negation.

A final step, then, was to provide class pretraining designed to help the children recognize and understand the concept of negation prior to truth-table training and rule learning. With this class pretraining, the youngest children nearly equalled the performance of the older children on rule learning.

To summarize, data from the original experiment were examined for differences in response patterns. The existence of these differences was explicitly tested in a second experiment. Training for these patterns was attempted, and when this failed to have significant effect, training for the components of the pattern was undertaken. Admittedly, this is only one possible approach for teasing out the factors which lead to changes in performance. Since the problem of age-related differences in cognitive performance is a recurrent one in education, it is worthwhile to analyze approaches such as Dr. Bourne's which have yielded information concerning such factors.

From the preceding comments it is clear that both the substantive results and the methodology of Dr. Bourne's experimentation are valuable contributions to research and development in education.