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ABSTRACT

This document is a reprint of the geometry section from the 1954 syllabus Mathematics 10-11-12, An Integrated Sequence for the Senior High School Grades. The text outlines a tenth year geometry course integrated with arithmetic, algebra, and trigonometry, which progresses from an informal to a formal level. The topics suggested include definition, axioms, congruence, constructions, lines, angles, parallelograms, loci, circles, angle measure, similarity, numerical trigonometry, area, coordinate geometry and regular polygons. A suggested time schedule and teaching sequence is included. The content represents the minimum amount required by the Tenth Year Regents examination in New York. (RS)

Tenth Year Mathematics

1968 Reprint from 1954 Syllabus

THE UNIVERSITY OF THE STATE OF NEW YORK/THE STATE EDUCATION DEPARTMENT
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TENTH YEAR MATHEMATICS

Reprint from the syllabus, *Mathematics 10-11-12*

1969

The University of the State of New York
The State Education Department
Bureau of Secondary Curriculum Development
Albany, New York 12224

FOREWORD

This publication is a reprint of the Tenth Year Mathematics section of the syllabus, *Mathematics 10-11-12, An Integrated Sequence for the Senior High School Grades*. It presents the minimum material for which students are responsible on the Tenth Year Regents examination.

A revision of the tenth year course is underway to accompany the separate publications *Ninth Year Mathematics-Course 1, Algebra (1965)*, *Eleventh Year Mathematics (1968)*, and *Experimental 12th Year Mathematics (1960)*.

Additional suggestions for teaching various topics may be found in the *Mathematics Handbook, A Handbook of Resource Material to Accompany the Course of Study in Mathematics 10 (1962)*.

Copies of the above publications may be ordered by the principal from the Publications Distribution Unit, State Education Department, Albany, New York 12224.

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DEFINITION

While no immediate changes in the scope, format, or coverage of the Tenth Year Mathematics examinations are contemplated, certain changes of notation and language will be made beginning with the June 1969 examination and continuing until further notice. This does not involve a change in the present syllabus which will be followed in regard to topics tested until a formal revision is prepared and distributed.

The following notation and language will be used:

(1) Line Representation

$\overline{AB}$ means line segment AB

AB means the length of line segment AB

$\overleftrightarrow{AB}$ means the line determined by A and B

$\overline{ABC}$ means the line segment AC with point B between A and C

(2) Measure

m $\angle$ ABC means the measure of $\angle$ ABC in degrees

m $\overset{\frown}{AB}$ means the measure of arc AB in degrees

(3) Congruence

The symbol $\cong$ will be used to designate line segments of equal lengths, angles of equal measure, circles of equal radii, arcs of equal measure on circles of equal radii, as well as congruent polygons.

The symbol = as applied to such figures means they are identical.

The language and notation of the Ninth Year Mathematics syllabus may, of course, also be used where appropriate.

The aforementioned changes are intended to make the questions on the examination more nearly precise and to make the language of the examination consistent with modern usage employed in many schools. Students will be expected to recognize this symbolism but need not follow it in their answers.

It is emphasized that this is a change of language and notation and is not to be construed as a change in the basic syllabus for the course. These changes in language and notation are being made with the advice and concurrence of the committee of teachers composing the Tenth Year Mathematics Regents Examinations.

TENTH YEAR MATHEMATICS

For many years courses in plane geometry have consisted largely of a body of somewhat unrelated geometric theorems, the performing of simple geometric constructions, the solution of original exercises, an introduction to the study of locus and a brief review of the numerical trigonometry of the ninth grade. Little or no attention has been given to the use or to the extension of the basic principles of arithmetic and algebra. As a result, many of these concepts and skills are lost during the tenth year from lack of use. One purpose of the present syllabus is to remedy this situation by attempting to integrate plane geometry with arithmetic, algebra and numerical trigonometry in so far as such integration is possible and desirable. Some of the ways in which this may be brought about are:

Greater use of common and decimal fractions and per cents in mensuration problems

An introduction to the meaning and use of approximate number

Increased emphasis on numerical trigonometry

The use of algebraic symbolism and algebraic proof wherever this is desirable

The use of algebraic equations in the solution of geometric problems

An introduction to coordinate geometry

Another major change in this syllabus is the reduction of the number of required geometric theorems. Concentration on a few groups of closely related theorems may be expected to result in a clearer understanding of the nature of proof and the meaning of sequential thinking. Furthermore, under former syllabuses there has always been the tendency to memorize theorems and to drill on certain types of original exercises. It is hoped that reducing the number of theorems required for examination purposes will discourage this.

While it is generally agreed that formal logic should not be a part of the tenth grade work, it is possible to develop some of the simple ideas related to this subject. Accordingly, the syllabus stresses certain points which may contribute to the pupil's ability to think critically in nonmathematical as well as mathematical situations. Topics which can be related to affairs of everyday living are the significance of definition, the necessity for certain assumptions in any argument, the nature of indirect proof, the recognition of converse and inverse theorems and the meaning of circular reasoning.

Perhaps the most conspicuous feature of the new syllabus is the addition of a unit of coordinate geometry. In addition to the intrinsic value of the unit as a part of the mathematical program, it also makes possible an ideal integration of algebra and geometry and lends variety and interest to the work of the tenth grade. This work is a continuation of that done in the ninth grade and provides preparation for work of greater difficulty and of greater value in the eleventh grade.

A suggested time schedule and teaching sequence accompanies this syllabus, but this does not imply that either must necessarily be followed. These are merely suggestions which may be of value to those who have had little or no experience in teaching the course.

Scope of Content

I The transition from informal to formal geometry

A Definition and use of basic terms and concepts (1)*
B The use of axioms and postulates (2)
C Fundamental theorems (3)
D Congruence (4)
E Fundamental constructions (5)

II Formal geometry

A Basic propositions

In view of the large number of propositions that may be taught profitably, it can not be expected that pupils reproduce the proofs of all. It is suggested that from the following list those numbered 1, 2, 3, 10, 11, 12, 13, 14, 30, 31, 35, 36, 43, 44, 50, 54, 59, 66 be accepted without proof. The other propositions given in this section should be proved in class, but only those marked with an asterisk may be called for on the examination. (6)

Triangles

1 Two triangles are congruent if two sides and the included angle of one are equal to the corresponding parts of the other.

* Numbers in parenthesis refer to Suggestions for Teaching on pages 19–36.

2 Two triangles are congruent if two angles and the included side of one are equal to the corresponding parts of the other.

3 Two triangles are congruent if the three sides of one are equal to the three sides of the other.

4 When two straight lines intersect, the vertical angles are equal.

*5 If two sides of a triangle are equal, the angles opposite these sides are equal.

*6 If two angles of a triangle are equal, the sides opposite these angles are equal.

*7 Two right triangles are congruent if the hypotenuse and a leg of one are equal to the corresponding parts of the other.

Inequality

8 If two sides of a triangle are unequal, the angles opposite these sides are unequal and the greater angle lies opposite the greater side.

9 If two angles of a triangle are unequal, the sides opposite these angles are unequal and the greater side lies opposite the greater angle.

10 An exterior angle of a triangle is greater than either nonadjacent interior angle.

Parallelism and Perpendicularity

11 The perpendicular is the shortest line that can be drawn from a given point to a given line.

12 If two lines are parallel to the same line, they are parallel to each other.

13 When two lines are cut by a transversal and a pair of alternate-interior (or corresponding) angles are equal, the two lines are parallel.

14 When two parallel lines are cut by a transversal, the alternate-interior (or corresponding) angles are equal.

15 The opposite sides of a parallelogram are equal.

16 The diagonals of a parallelogram bisect each other.

17 If the opposite sides of a quadrilateral are equal, the figure is a parallelogram.

18 If two sides of a quadrilateral are equal and parallel, the figure is a parallelogram.

19 If the diagonals of a quadrilateral bisect each other, the figure is a parallelogram.

20 If three or more parallel lines cut off equal segments on one transversal, they cut off equal segments on any transversal.

21 The line segment that joins the midpoints of two sides of a triangle is parallel to the third side and equal to one-half the third side.

Angle Sum

*22 The sum of the angles of a triangle is equal to a straight angle.

23 The exterior angle of a triangle is equal to the sum of the two nonadjacent interior angles.

24 The sum of the interior angles of a polygon of n sides is equal to $(n - 2)$ straight angles.

25 The sum of the exterior angles of a polygon formed by extending each of its sides in succession is equal to two straight angles.

Locus

26 The locus of points equidistant from two given points is the perpendicular bisector of the line segment joining the two points.

27 The locus of points within an angle and equidistant from the sides of the angle is the bisector of that angle.

28 The perpendicular bisectors of the sides of a triangle meet in a point which is equidistant from the three vertices.

29 The bisectors of the angles of a triangle meet in a point which is equidistant from the sides of the triangle.

Circles

30 If in the same or equal circles two arcs are equal, their chords are equal.

31 If in the same or equal circles two chords are equal, their arcs are equal.

*32 A diameter perpendicular to a chord of a circle bisects the chord and its arcs.

33 If in the same or in equal circles two chords are equal, they are equidistant from the center.

34 If in the same or equal circles two chords are equidistant from the center, the chords are equal.

35 A line perpendicular to a radius at its outer extremity is tangent to the circle.

36 A tangent to a circle is perpendicular to the radius drawn to the point of contact.

37 Tangents drawn to a circle from an external point are equal.

38 Two parallel lines intercept equal arcs on a circle.

Angle Measurement

*39 An angle inscribed in a circle is measured by one-half its intercepted arc.

40 An angle formed by a tangent and a chord drawn from the point of contact is measured by one-half the intercepted arc.

*41 An angle formed by two chords intersecting inside the circle is measured by one-half the sum of the intercepted arcs.

*42 An angle formed by two secants, a tangent and a secant or two tangents is measured by one-half the difference of the intercepted arcs.

Similarity

43 A line parallel to one side of a triangle and intersecting the other two sides divides those sides proportionally.

44 If a line divides two sides of a triangle proportionally, it is parallel to the third side.

*45 If the three angles of one triangle are equal to the three angles of another triangle, the triangles are similar.

46 If an angle of one triangle is equal to an angle of another triangle and the sides including these angles are in proportion the triangles are similar.

*47 If in a right triangle the altitude is drawn upon the hypotenuse,

(*a*) The two triangles thus formed are similar to the given triangle and similar to each other.

(*b*) Each leg of the given triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse.

48 If in a right triangle the altitude is drawn upon the hypotenuse, the altitude is the mean proportional between the segments of the hypotenuse.

*49 The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

50 If the square of one side of a triangle is equal to the sum of the squares of the other two sides, the triangle is a right triangle.

51 If two chords intersect within a circle, the product of the segments of one is equal to the product of the segments of the other.

52 If from a point outside a circle a tangent and a secant are drawn the tangent is the mean proportional between the secant and its external segment.

53 The perimeters of two similar polygons have the same ratio as any two corresponding sides.

54 If two polygons are similar, they can be divided into the same number of triangles similar each to each and similarly placed.

Areas

*55 The area of a parallelogram is equal to the product of one side and the altitude drawn to that side.

*56 The area of a triangle is equal to one-half the product of a side and the altitude drawn to that side.

*57 The area of a trapezoid is equal to one-half the product of the altitude and the sum of the bases.

58 The areas of two similar triangles are to each other as the squares of any two corresponding sides.

59 The areas of two similar polygons are to each other as the squares of any two corresponding sides.

Regular Polygons and the Measurement of the Circle

60 A circle can be circumscribed about, or inscribed in, any regular polygon.

61 Regular polygons of the same number of sides are similar.

*62 The area of a regular polygon is equal to one-half the product of its perimeter and its apothem.

63 The circumferences of two circles are to each other as their radii.

64 The ratio of the circumference of any circle to its diameter is constant. The constant is denoted by π and hence $c = 2\pi r$.
65 The area of a circle is equal to one-half the product of its circumference and its radius. Hence $A = \pi r^2$
66 The areas of two circles are to each other as the squares of their radii.

B Fundamental constructions (7)

1 To bisect a line segment
2 To bisect an angle
3 To construct a line perpendicular to a given line through a given point on or outside the line
4 To construct a line parallel to a given line through a given point
5 To divide a line into any number of equal parts
6 To construct a line tangent to a given circle through a given point on or outside the circle
7 To locate the center of a given circle
8 To construct a circle inscribed in or circumscribed about a given triangle
9 To construct a triangle similar to a given triangle on a given line segment as base
10 To inscribe an equilateral triangle, a square and a regular hexagon in a given circle

C Formulas (8)

In addition to the relationships derived from listed theorems, such as, 24, 40, 48, 51, 53 and 58, certain mensuration formulas are to be taught intensively. Among these are the following:

Lines

1 Right triangle ($\angle C = rt\ \angle$)... $c^2 = a^2 + b^2$
2 Equilateral triangle .... $h = \frac{s}{2}\sqrt{3}$
3 Square .... $d = s\sqrt{2}$
4 Circle .... $c = 2\pi r$ or $c = \pi d$,
$l = \frac{n}{360} \times 2\pi r$

Areas

1 Rectangle .... $K = bh$
2 Square .... $K = s^2$
3 Parallelogram .... $K = bh$
4 Triangle .... $K = \frac{1}{2} bh$
5 Rhombus .... $K = \frac{1}{2} dd'$
6 Equilateral triangle .... $K = \frac{s^2}{4}\sqrt{3}$
7 Trapezoid .... $K = \frac{h}{2}(b + b')$
8 Regular polygon .... $K = \frac{1}{2} ap$
9 Circle .... $K = \pi r^2$
10 Sector of a circle......... $K = \frac{n}{360} \times \pi r^2$

D Proofs of original exercises (9)

E Locus and construction (10)

III Arithmetic

A Use of integers, common and decimal fractions, and percents in mensuration problems (11)

B Introduction to the meaning and use of approximate numbers (12)

IV Algebra

A The use of signed numbers and the fundamental processes of algebra as they occur in the applications of geometry

B Evaluation, transformation, and interpretation of formulas relating to geometric figures

C Review of radicals to cover the techniques needed for geometric applications at this level

D Square root. Ability to use a table of square roots and an understanding of the square root process.

E Ratio and proportion. Review and extension to cover all geometric applications at this level.

F Solution of equations. Review and extension to include the solution of the complete quadratic by factoring. (13)

G The use of algebraic symbolism in geometric proof (14)

H Algebraic proof (15)

V Trigonometry

A The meaning of the sine, cosine, and tangent ratios resulting from the study of similar triangles

B The use of a four-place table of sines, cosines and tangents. Interpolation not required.

C Solution of the right triangle

D Problems in indirect measurement involving the use of more than one right triangle

E Problems involving the regular polygon

F Development and use of the formulas $K = ab \sin C$ and $K = \frac{1}{2} ab \sin C$ for the area of the parallelogram and triangle respectively (*optional*) (16)

VI Introduction to coordinate geometry (17)

A Meaning and use of the terms: coordinate system, axes of reference, origin, abscissa, ordinate

B Coordinates of points used in connection with problems involving loci and area (18)

C Midpoint of a line segment (19)

D Distance between two points (20)

E Formula for the slope of a straight line (*optional*) (21)

F Equations of straight lines (*optional*) (22)

VII Geometry in nonmathematical settings (23)

Suggestions for Teaching

1 Many of the basic terms and concepts of geometry have been considered informally in grades 7–9 and certain fundamental relationships have been derived informally. The pupil should now be made to realize that such conclusions were reached largely through *measurement* and *observation* and hence are lacking in generality. Moreover, there is no evidence of relationship among theorems reached by the experimental method alone. At this point there is need for consideration of the nature of deductive proof illustrating the way in which some propositions follow inevitably from others. This permits the organization of many seemingly unrelated ideas into a logical organization called a postulational system.

In making the transition from informal geometry to formal geometry, the pupil should be made to realize the importance both of understanding clearly the basic concepts of geometry and of explaining these concepts by means of verbal statements. Later these definitions play an important part in demonstrative geometry. In general, it is good practice to introduce only those terms and concepts that can be used immediately. At this stage it might be well to confine the discussion to those that are directly associated with points, lines, angles, polygons and circles. This work should include simple geometric exercises of an experimental nature involving important relationships which are to be demonstrated later.

It is particularly important that the pupil appreciate that the making of a definition consists of two parts, first, the placing of the concept in a larger class of concepts (previously defined) and second, the assigning to this concept those particular characteristics which distinguish it from all other members of that class. Such practice easily brings about a realization of the importance of sequence in definition and naturally leads to an understanding of the significance of propositional sequence. This is one of the main objectives of tenth grade mathematics. Likewise, the need for undefined terms should be made clear and this, in turn, associated with the necessity of assumptions (axioms and postulates) in formal geometry. Attention also should be given to the meaning of redundancy in definition and to the fact that while redundancy is not always an undesirable feature of a definition, it is better, generally speaking, to include in a definition only the descriptive characteristics which are essential. This idea will be clarified later when the student is able to appreciate the difference between a definition and a proposition, for example, the definition of a parallelogram as contrasted with the statements of propositions 17, 18, 19. Important, too, is the matter of the reversibility of a definition. This point, of course, can not be made clear until the pupil learns how to use definitions in formal proofs. The reversibility of a definition, however, will be considerably clarified when the meaning of a converse proposition is understood.

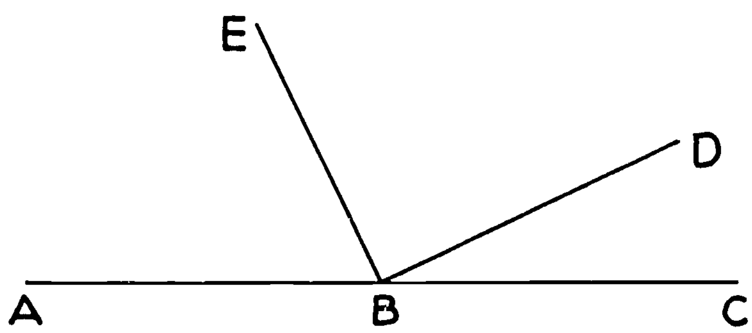
The following exercises illustrate types that may be used as an aid in making the transition from informal to formal geometry.*

* The inclusion of certain illustrative exercises throughout this syllabus must not be construed to mean that other exercises of greater difficulty or of different kind may not appear on the final examination.

a **Algebraic**

(1) The sum of the complement and the supplement of a certain angle is 160°. Find the number of degrees in the angle.

(2) In the accompanying figure, BE is perpendicular to BD, $\angle ABE = x + 2°$, $\angle EBD = 2x - 8°$, $\angle DBC = x - 9°$. Is ABC a straight line?



b **Geometric**

(1) Draw a triangle having three unequal sides. Using the protractor measure the angles of the triangle. What conclusion concerning the sides and the opposite angles seems warranted?

(2) Draw an isosceles triangle having an acute angle for the vertex angle. Measure the other two angles. Repeat this with an obtuse angle and also with a right angle. What conclusion seems to be warranted concerning the other two angles in each triangle?

c **General**

(1) Define each of the following terms and arrange them in proper sequential order: Perpendicular lines, right angle, angle, adjacent angles.

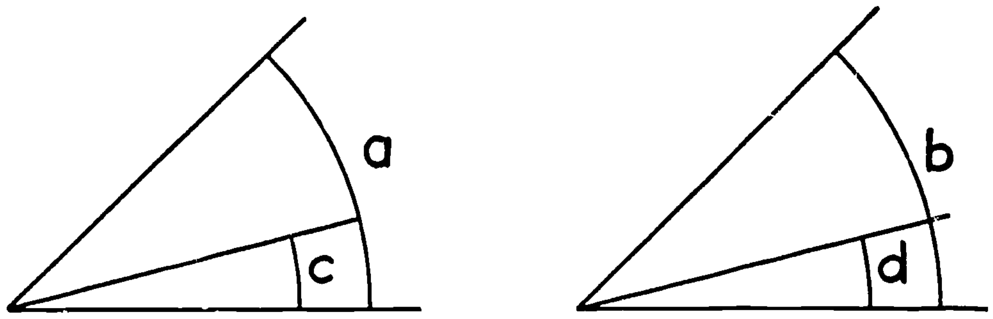
(2) In the case of each of the following explain why the statement does not constitute a good definition:

(*a*) A triangle is a polygon having three sides and three angles.

(*b*) A straight angle is an angle whose sides lie in the same straight line.

(*c*) A diagonal of a quadrilateral divides the quadrilateral into two triangles.

2 In the transition from informal to formal geometry the work should be done slowly and carefully. Pupils should be made to appreciate the need for basic assumptions in any kind of argument, whether mathematical or nonmathematical. Care should be taken to select only those axioms and postulates of which immediate use can be made. It is good practice to introduce the work by showing the applications of the axioms of equality to the solution of simple equations. Sufficient pains should be taken to insure that students understand the use of axioms in geometric settings. For example: If $\angle a = \angle b$ and $\angle c = \angle d$, what angle in the figure represents $\angle a - \angle c$? $\angle b - \angle d$? Why is $\angle a - \angle c$ equal to $\angle b - \angle d$?



This work must include a sufficient number of simple geometric proofs based on the use of definitions, axioms, and postulates. The work should begin with proofs having only one step, followed by those having two, and then three or more steps.

3 After the nature of the basic assumptions is thoroughly understood and the student is able to prove these simple geometric exercises, certain preliminary theorems may well be considered. These theorems may be prescribed by the textbook or by the teacher. The proofs of some of them may be assumed if it seems wise to do so; others perhaps, should be demonstrated as class exercises. For example:

a Complements (supplements) of the same or of equal angles are equal.

b If two adjacent angles have their exterior sides in the same straight line, the angles are supplementary.

c If two straight lines intersect, the vertical angles are equal.

In any case these preliminary theorems, together with the definitions, axioms, and postulates considered thus far, should be set apart as constituting a list of accepted authorities which the student may use in proving original exercises. The distinction between what the student may use and may not use as *reasons* in proving originals always causes difficulty and, therefore, should be settled at this early stage (see note 9, page 27). This work, too, should be followed by appropriate exercises and due attention should be given to the manner in which proofs should be presented both orally and in written form.

4 The three fundamental laws of congruence (see propositions 1–3) may be developed informally as class exercises. This part of the work should include a number of simple geometric exercises in which the pupil learns how to prove that triangles are congruent and that line segments and angles are equal. For example:

a In isosceles triangle ABC, points R and S are the midpoints of legs AB and AC respectively. Line segments BS and CR are drawn. Prove triangle ABS congruent to triangle ACR.

b Triangles ACB and ADB have the same base AB and are on the same side of AB. Side AD intersects side BC at O. If $AC = BD$ and $BC = AD$, prove that $\angle CAD = \angle DBC$ and that $CO = DO$.

5 Emphasis here should be placed on the *proofs* of geometric constructions (see note 7, page 26). Since the proof of proposition 5 is usually based on the existence of the bisector of an angle, it may be desirable to include proposition 5 at this point.

For the reason that the construction of geometric figures exemplifies so well the meaning of the terms *determined, underdetermined,* and *overdetermined,* it is recommended that a discussion of these ideas be taken up at this time. For example, in constructing a line through a given point perpendicular to a given line, the pupil frequently locates two additional points thus overdetermining the required line. This work should be continued throughout other units as the necessary materials become available.

6 A proper study of geometry centers around certain broad topics such as congruence, construction, inequality, parallelism, angle sums, locus, angle measurement, similarity, measurement of plane figures etc. It is fitting that the syllabus stress certain propositions from these fields and also their logical interrelationships within each field. The concept of the nature of proof as well as the logical structure exhibited in a chain of propositions are essential objectives in the teaching of formal geometry.

There are many ways in which the concept of sequential thinking may be taught. A recognition of the definitions and assumptions on which the sequence is based, the logical order of the theorems which make up the sequence, the possible interchange of certain theorems in a sequence without destroying the validity of the reasoning are some of the points to be stressed in presenting this feature of geometry.

In order that these objectives may be more easily realized, four topics will be used to stress the concept of sequence.* The theorems found under each topic, taken in the order given, form a possible sequence. In these sequences, however, some theorems have proofs not suitable for examination purposes but are essential as links in a chain of theorems. Others have more suitable proofs and these are the ones that may be called for on the examination. The four topics are as follows:

a **Congruence and parallelism**

Definition: Parallel lines are lines which lie in the same plane and do not intersect, however far they are extended.

Assumed Theorem: Through a given point only one straight line can be constructed parallel to a given line.

Theorems: (1) When two parallel lines are cut by a transversal the alternate-interior (or corresponding) angles are equal.

(2) The sum of the angles of a triangle is equal to a straight angle.

(3) Two triangles are congruent if two angles and a side opposite one of them are equal to the corresponding parts of the other.

(4) Two right triangles are congruent if the hypotenuse and a leg of one are equal to the corresponding parts of the other.

(5) A diameter perpendicular to a chord of a circle bisects the chord.

NOTE. If in the proof of (4) the hypotenuses are placed together, then insert the theorems: If two sides of a triangle are equal the angles opposite these sides are equal; and if two angles of a triangle are equal the sides opposite these angles are equal.

* This does not mean that *only* these four topics shall be used to stress sequential thinking. Teachers may care to consider other "chains of propositions," such as

(1) Propositions 2, 14, 15, 16

(2) Propositions 2, 4, 10, 13, 14, 22, 24

b **Angle measurement**

Definition: A circle is a plane closed curve all points of which are equally distant from a fixed point.

Assumed Theorem: A central angle is measured by its intercepted arc.

Theorems: (1) If two sides of a triangle are equal the angles opposite these sides are equal.

(2) An exterior angle of a triangle is equal to the sum of the two nonadjacent interior angles.

(3) An angle inscribed in a circle is measured by one-half its intercepted arc.

a Case I — where the center of the circle is on one side of the angle.

b Cases II and III — where the center of the circle is inside and where it is outside the angle.

(4*a*) An angle formed by two chords intersecting inside the circle is measured by one-half the sum of the intercepted arcs.

(4*b*) An angle formed by two secants intersecting outside the circle is measured by one-half the difference of the intercepted arcs.

NOTE. This sequence may end with either of two different theorems, (4*a*) or (4*b*).

c **Similarity**

Definition: Similar polygons are polygons which have their corresponding angles equal and their corresponding sides proportional.

Assumed Theorem: A line parallel to one side of a triangle and intersecting the other two sides divides these sides proportionally.

Theorems: (1) If two angles of one triangle are equal to two angles of another triangle, the triangles are similar.

(2) If in a right triangle the altitude is drawn upon the hypotenuse

(a) The two triangles thus formed are similar to the given triangle and similar to each other.

(b) Each leg of the given triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse.

(3) The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

NOTE. While (1) is the theorem needed in this sequence, proposition 45 (see page 15) of which this should be considered a corollary, will be the required theorem.

d **Area**

Definition: The area of any plane surface is the number of square units of a given kind which it contains.

Assumed Theorem: The area of a rectangle is equal to the product of its base and its altitude.

Theorems: (1) The area of a parallelogram is equal to the product of one side and the altitude drawn to that side.

(2) The area of a triangle is equal to one-half the product of a side and the altitude drawn to that side.

(3a) The area of a regular polygon is equal to one-half the product of its perimeter and its apothem.

(3b) The area of a trapezoid is equal to one-half the product of the altitude and the sum of the bases.

(4) The area of a circle is equal to one-half the product of its circumference and its radius.

NOTE. This sequence may end with either (3a) and (4), or (3b).

7 Although construction methods have been taught in previous years the teacher should review them. In the tenth year the pupil should be able to explain why the construction result is valid (see note 5, page 23).

8 Emphasis should be placed on the derivation of the more important mensuration formulas and on their use. Numerical exercises from various fields and, if possible, of a practical nature should be included. The pupil should be able to use competently formulas such as those given in this list.

9 As in the past one or more original exercises will be called for on the final examination. It is, of course, impossible to prescribe all the statements that may be used as authorities to support proofs of original exercises. Such a list depends entirely on the text in use. In general the reasons cited for various steps in a proof should be restricted to the definitions, assumptions, theorems and corollaries which are formally set forth in the textbook.

10 Here the emphasis should be placed not so much on the ability to reproduce the proof of a locus theorem as on the understanding of its meaning and use. In addition to the two locus theorems (see propositions *26, 27*) other fundamental locus theorems should be considered informally and should be used to determine the position of points by means of intersecting loci. If time permits, the study should be extended to include the construction of simple geometric figures by means of intersecting loci. Simple problems having to do with loci expressed algebraically should be included (see illustration given in notes 18–22).

Some teachers may wish to extend the work to three dimensional loci. If so, the illustrations chosen should be those that follow naturally from the corresponding ideas in plane geometry, for example: the locus of points in space at a given distance from a given point; equidistant from two given points.

11 In order that the basic skills of arithmetic be kept in review, it is recommended that mensuration problems be selected which require, occasionally at least, the use of numbers other than simple integers. Furthermore, it should not always be expected that the final answer be integral. Care should be taken to see that the data given in numerical problems are consistent and that the accuracy or precision expected in the final answer is stated.

12 A detailed discussion of approximate numbers and of standard practice with such numbers should not be undertaken at this level. However, if teachers feel that the character of the class warrants an extension of this topic beyond that suggested for the ninth grade, they may wish to consider the following points:

a When, for example, it is given that a length is 8 feet 4 inches, it is understood that the true length is not exactly that amount, but rather that it is greater than 8 feet 3½ inches and less than 8 feet 4½ inches. That is to say, the ***apparent error*** (greatest possible error) in this measured length is ½ of an inch. Similarly, if a weight is given as 6.4 pounds, it is understood that the true weight is between 6.35 pounds and 6.45 pounds and that the apparent error is .05 pounds.

b Measurements ***given in terms of the same unit*** are said to have the same precision if they have the same apparent error.

c If numbers representing measurements of different precision are to be added (or subtracted), good practice is to round off whatever numbers are necessary so that the resulting numbers have at most one decimal place more than the least precise measurement and then round off the sum (or difference) to the same precision as the least precise number. For example:

(1) Add 3.492 ft, 7.7 ft, and 6.884 ft

$$\begin{array}{r} 3.49 \\ 7.7 \\ 6.88 \\ \hline 18.07 \end{array} = 18.1 \text{ ft}$$

(2) Subtract 7.8 in. from 12.437 in.

$$\begin{array}{r} 12.44 \\ 7.8 \\ \hline 4.64 \end{array} = 4.6 \text{ in.}$$

d The digits 1 through 9 are called significant figures. Also the digit 0 may or may not be significant. It is not significant when it is used merely to locate the position of a decimal point. For example:

(1) In the number .0024, the 2 and the 4 are significant figures but the two zeros are not.

(2) In the number 34.08, all the digits including the zero are significant.

(3) In the number 2400, the 2 and the 4 are significant figures but we have no way of telling whether the zeros are or are not significant. (If the number were written in the form 2.4×10^3, we would then know that the zeros are not significant.)

e If a number has *n* significant figures, it is said to have *n-place accuracy.* Thus, .437 has three-place accuracy; .00437 also has three-place accuracy; 0.2057 has four-place accuracy; 24000 has two place accuracy and may have three, four, or even five-place accuracy.

f If two numbers are to be multiplied (or divided), it is good practice to round off the more accurate number and then round off the product (or quotient) to the same accuracy as the less accurate number. For example:

(1) Multiply .02754 by 43

```
  .0275
     43
  -----
    825
   1100
 ------
 1.1825 = 1.2
```

(2) Divide 729.864 by 135

```
          5.406 = 5.41
     ------------
 135)729.9
     675
     -----
      54.9
      54.0
      -----
        .900
        .810
```

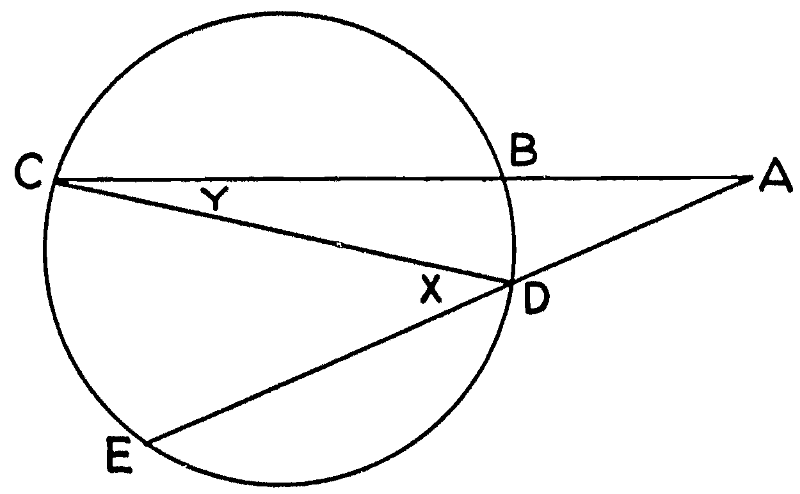
g The following rule should be emphasized throughout this work. " All numerical results, before they are stated in final form, should be obtained with at least one more digit than the number of significant digits allowed by the approximate data. Then this last digit is rounded off." (Quoted from the Twelfth Yearbook of the National Council of Teachers of Mathematics, " Approximate Computation," Aaron Bakst, 1937, page 54.)

13 In accelerated classes it may be desirable to extend this work to include the solution of the quadratic equation by completing the square and by means of the quadratic formula.

14 Whenever possible algebraic symbolism should be used as a means to simplify and clarify the proofs of certain propositions. This is especially desirable in connection with angle sum and measurement of angles in a circle.

To illustrate:

ABC and *ADE* are secants with chord *CD* drawn. Let the number of degrees in $\angle CDE$ be represented by x and in $\angle DCA$ by y.



Then the number of degrees in arc *CE* is $2x$ and in arc *DB* is $2y$. But the number of degrees in $\angle A$ is $(x - y)$. Therefore an angle formed by two secants intersecting outside the circle is measured by one half the difference of the intercepted arcs.

15 In addition to the wide variety of original exercises in geometry which can be proved algebraically there are certain geometric theorems in which algebraic proof can be used to advantage. See propositions 49, 50, 56, 57, 58, 59, 62, 63, 64, 66.

16 Although this item is optional, it is suggested, if time permits that it be included in the required work of the tenth grade. It is the only entirely new trigonometric idea proposed for this level and, in addition, exemplifies the integration of geometry, algebra and trigonometry in the field of area.

17 Coordinate geometry furnishes a new technique whereby it is possible to establish simple geometric relationships and to prove certain geometric theorems in a much easier manner than that of the usual synthetic method. Special care must be exercised, however, to make sure that proofs by means of coordinate geometry are not circular.

18 Exercises such as the following should be considered:

a Write as an equation the locus of points (1) whose ordinates are equal to 4, (2) whose abscissas and ordinates are equal, (3) such that the sum (or difference) of the coordinates is a given constant. Represent these loci graphically.

b Find the area of the triangle whose vertices are $A(3,2)$, $B(8,3)$, $C(4,10)$. Suggestion: Draw the ordinates of A, B and C and use the formula for the area of a trapezoid. (This method of finding areas should be extended to include other rectilinear figures.)

19 The formulas $x = \frac{x_1 + x_2}{2}$ and $y = \frac{y_1 + y_2}{2}$ should be derived in class and used in exercises such as:

a Given the points $A(1,1)$, $B(10,3)$, $C(12,10)$ and $D(3,8)$, show that the line segments AC and BD bisect each other. What kind of quadrilateral is $ABCD$?

b Find the lengths of the medians of the triangle whose vertices are $(2,8)$, $(10,12)$, and $(16,0)$. (See note 20.)

c Find the coordinates of the point on the x-axis which is equidistant from the points $(2,3)$ and $(-10,3)$

20 The distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ should be derived in class for the case in which the points are in Quadrant I, and used in exercises such as:

a Find the perimeter of the triangle whose vertices are $(-1,-2)$, $(3,5)$, and $(0,1)$.

b Show that the triangle whose vertices are $(-3,0)$, $(1,-2)$, and $(5,6)$ is a right triangle.

c Show that the circle whose center is the point $(3,2)$ and which passes through the point $(5,2)$ will pass through the points $(3,4)$, and $(3,0)$.

d Show that the locus of points at a given distance r from the origin is given by the equation $x^2 + y^2 = r^2$.

e Show that the following points lie on a straight line: $(-1,-1)$, $(1,3)$, $(2,5)$

21 The meaning of *slope* should be made clear and the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$ should be derived in class and used in a variety of exercises. Pupils should know that parallel lines have the same slope and, conversely, lines having the same slope are parallel. Exercises such as the following are suggested:

a Plot the points $A(2, 8)$, $B(6, 4)$, $C(3, -2)$, and $D(-1, 2)$. Find the slope of line AB, of line DC, of line AD and of line CB. Why is $ABCD$ a parallelogram? Prove that $ABCD$ is a parallelogram by showing (1) that the opposite sides are equal, (2) that the diagonals bisect each other.

b Using the formula for the slope of a line, prove that the following points lie on the same straight line:

(1) $(0, 0)$, $(2, 3)$, $(4, 6)$

(2) $(3, 1)$, $(-4, 1)$, $(6, 1)$

22 This work should include a study of the families of lines represented by the equations $y = mx$ and $y = x + b$. Exercises such as the following should be considered:

a Find the equation of the straight line whose slope is 3 and which passes through a point on the y-axis four units above the origin.

b Where does the line whose equation is $5x - 2y = 20$ cross the x-axis? the y-axis?

c Express algebraically the locus of points

(1) Equidistant from the points $(3, 5)$ and $(-1, 5)$

(2) Equidistant from the points $(-1, 6)$ and $(-1, 4)$

What point satisfies both conditions given in (1) and (2)?

d Express algebraically the locus of points which are four units from the line whose equation is $y = 2$; three units from the line whose equation is $x = 3$.

e The coordinates of the vertices of a quadrilateral are $(0, 0)$, $(12, 0)$, $(12, 5)$ and $(0, 5)$. Write the equations of the sides of the quadrilateral; of the diagonals of the quadrilateral.

f The vertices of quadrilateral $ABCD$ are $A(-2, -3)$, $B(10, 7)$, $C(2, 9)$ and $D(-4, 5)$. By using the formula for the slope of a straight line show that the line joining the midpoints of AB and BC is parallel to the line joining the midpoints of CD and DA.

23 In addition to an understanding and use of the ideas, concepts and principles of geometry there are other outcomes of a more general nature which should be recognized as an indispensable part of the tenth grade work. Frequently the study of geometry is justified on the ground that, perhaps more than any other subject, it contributes to critical thinking and sound reasoning. This may be so but it does not follow automatically. Without a continual and persistent

effort on the part of the teacher and the student to correlate the thinking in geometry with that in nonmathematical fields very little, if any, improvement in the ability to think soundly and critically is likely to result.

Accordingly teachers are urged to return to discussions of this sort again and again in order to make certain that the essential ideas as suggested are reasonably well understood. Ways and means of presenting this material will, of course, vary with the individual teacher. Some teachers may prefer to extend this work to include other ideas not specifically mentioned here. It is not the intent to prescribe exactly what shall be done along these lines. This will be determined somewhat by the time element and certainly by the teacher. Also it is not easy to test the progress that pupils make along such lines. The attempt, however, should be made and questions pertaining to such work will be given on the final examination.

Some of the points which may well be stressed are given in the paragraphs that follow:

a The significance of definition. Pupils should take part in class discussions which show the necessity of a clear understanding of the meaning of terms as a basic requirement in any intelligent argument. For example, what terms in the following statements require definition if the meaning of the statement is to become clear?

(1) Eligibility for membership on a school team is dependent on a satisfactory scholastic standing.

(2) High school fraternities are undemocratic.

(3) Argentina is a fascist state.

b The significance of assumptions. Pupils should appreciate the necessity of assumptions as the basis of any argument and should realize that conclusions reached can not be relied on unless the truth of the assumption is accepted. A suggestion from the teacher that each pupil bring to class an example showing how a certain theory, belief or doctrine is based on one or more unproved propositions or assumptions will frequently yield most satisfactory results. The following are offered as suggestions:

(1) The Nazi race theory and its results

(2) The United States protective tariff and its consequences

(3) Isolationism in the United States after World War I and its effect on the League of Nations

c Converse and inverse propositions. Pupils should understand the meaning of such propositions and should realize that the converse or the inverse of a proposition is not always true simply because the direct proposition is true.

To illustrate:

Direct: In a Bestrite pen the ink flows freely.

Converse: If the ink in a pen flows freely, it is a Bestrite.

Inverse: If a pen is not a Bestrite, the ink does not flow freely.

Is the converse true? The inverse? Is it safe to reason from the inverse that it is unwise to buy a pen that is not a Bestrite?

Reasoning from the converse and from the inverse is a trick of the demogogue and the propagandist. An effective outcome of the teaching of geometry should be a realization of the need to be watchful for fallacious reasoning of this kind when heard on the radio or from the platform or when found in editorials and advertisements.

d Indirect reasoning. Frequently it is found that indirect proof is difficult for beginners in geometry; consequently some teachers may feel inclined not to teach it. It should not, however, be entirely neglected. The mere fact that it is used so commonly in everyday life justifies giving time and thought to the matter. Whether or not indirect proof is completely rigorous need not concern us here. In fact, striving for complete rigor throughout the study of geometry frequently does more harm than good. If pupils in dealing with indirect proof recognize the necessity of considering all the possibilities that can exist in a given situation and of eliminating all such possibilities except the one which they wish to establish as true, the objective of teaching indirect proof has, in large part, been accomplished.

Propositions 8 and 9, either of which may be proved by the indirect method, and many original exercises such, for example, as *the diagonals of a trapezoid can not bisect* *each other,* illustrate the fact that at times the indirect method of proof is to be preferred. Much of its value, however, will be lost if the idea is not carried over into life situations.

e Circular reasoning. By giving due attention to sequence in geometry it is likely that the meaning of circular reasoning can be made clear. Instances from geometry in which the reasoning is circular should be cited and the pupil should be made to see clearly why conclusions resulting from circular reasoning are invalid. Point out, for example, the circular reasoning involved in proving both propositions 8 and 9 by the indirect method in which each is made to depend on the other. Encourage pupils to bring to class instances in daily life in which circular reasoning is used.

f Use of the terms "determined," "underdetermined" and "overdetermined." These mathematical ideas have their counterpart in every day affairs and should be emphasized. One of the most common mistakes made by beginners in geometry is the use of overdetermined lines (See note 5). Class discussions such as those listed below tend to clarify these ideas.

(1) In which of the following cases are the values of x and y determined, underdetermined, or overdetermined?

(*a*) $x - y = 8$

(*b*) $x + y = 8$, $2x - 3y = 1$, $3x + 2y = 21$

(*c*) $x - y = 8$, $2x - 2y = 13$

(*d*) $x - y = 8$, $2x - 3y = 1$

(2) A telephone number is Exeter 5094 R. Which part of the number denotes the exchange? The line? The party on the line? Does this number represent a situation which is determined, overdetermined, or underdetermined?

g Irrelevancy. Frequently an argument heard on the platform or in the courtroom, or in any place where it is the purpose to persuade, irrelevant statements are deliberately injected in order to confuse the issue. Extravagant advertising also often makes use of this device. If pupils are encouraged to look for this same feature in their problems in geometry it may rightly be expected that they may become more critical of what they see and hear. Problems such as the following may help.

In which of the following statements is there more information given in the hypothesis than is necessary to reach the conclusion?

(1) Line segments joining the midpoints of the opposite sides of a parallelogram bisect each other.

(2) If a parallelogram is circumscribed about a circle, the parallelogram is a rhombus.

(3) The altitude upon the base of an isosceles right triangle bisects the vertex angle.

(4) If a piece of metal is pure iron, a magnet will attract it.

SUGGESTED TIME SCHEDULE AND TEACHING SEQUENCE

Unit	Topics	Time Allotment in Days
I	Understanding and use of basic terms and concepts; significance of definition in mathematical and nonmathematical settings.	12–14
II	Understanding and use of axioms and postulates; meaning and importance of assumptions in nonmathematical situations.	8–10
III	Congruence. See propositions 1–3.	6–7
IV	Fundamental constructions; meaning of the terms determined, underdetermined, and overdetermined in both geometric and nongeometric fields.	6–7
V	Parallel and perpendicular lines. See propositions 11–14; meaning and use of indirect reasoning in both geometry and everyday life.	8–10
VI	Angle sum. See proposition 22–25. Converse and inverse propositions.	4–5
VII	Parallelograms. See proposition 15–21. Sequential thinking; circular reasoning.	9–11
VIII	Loci and construction; coordinate geometry. See page 19, VI, *A–C*.	15–17
IX	Circles. See proposition 30–38.	8–9
X	Measurement of angles in a circle. See propositions 39–42.	6–7
XI	Similarity. See propositions 43–54.	20–22
XII	Numerical trigonometry. See page 19, V, *A–D*	6–7
XIII	Area. See propositions 55–59. Coordinate geometry. See page 19, VI, *D–F*. Numerical trigonometry. See page 19, V, *F*.	20–22
XIV	Regular polygons and the measurement of the circle. See propositions 60–66. Numerical trigonometry. See page 19, V, *E*.	6–7
	Total number of days:	134–156