DOCUMENT RESUME

ED 031 823 40 EC 003 028

By-Cawley, John F.; Goodman, John O.

Arithmetical Problem Solving: A Program Demonstration by Teachers of the Mentally Handicapped.

Connecticut Univ., Storrs. School of Education.

Spons Agency-Office of Education (DHEW), Washington, D.C.

Bureau No-BR-5-0390

Pub Date 67

Grant-OEG-32-16-0270-5009

Note-78p.

EDRS Price MF-$0.50 HC-$4.00

Descriptors-Academic Achievement, Arithmetic Curriculum, Educable Mentally Handicapped, *Exceptional Child Research, Intelligence Factors, Item Analysis, *Mathematics, *Mentally Handicapped, *Teaching Methods, Test Construction, *Testing, Test Reliability, Test Validity

The purposes of the study were to investigate the effects of the combination of a trained teacher and a planned program on the problem solving abilities of mentally handicapped children, to develop measures of verbal problem solving (IDES) and arithmetic understanding (PUT), and to analyze the interrelationships among primary mental abilities and various combinations of achievement tests. A teacher training workshop presented for 10 evenings focused on the organization and use of 86 lessons in nine units for 18 weeks. Teaching methods concentrated on developing understanding through the solution of problems which originated in the classroom. Trained teachers used the prepared program with 161 retarded children; untrained teachers used the program with 58 retarded children; and there were 132 retarded and 89 average-ability controls. The results showed that the IDES and the PUT appear to be stable and consistent measures of the arithmetic performance of mentally handicapped children, and that the teacher training program and the prepared program of arithmetic curriculum were effective. Measures of primary mental abilities and academic achievement were highly interrelated. The study supports the contention that problem solving and concept development among the mentally handicapped can be influenced by education. (LE)

BR-5-0390
PA-40

ARITHMETICAL PROBLEM SOLVING: A PROGRAM DEMONSTRATION BY TEACHERS OF THE MENTALLY HANDICAPPED

John F. Cawley

John O. Goodman

School of Education
The University of Connecticut
1967

The research reported herein was conducted pursuant with a grant with the U. S. Office of Education. Project No. 5-0390-4-11-3

OEG-32-16-0270-5009

Acknowledgments

Appreciation is gratefully extended to those teachers and pupils who participated in this project. We acknowledge, with thanks, the permissions of superintendents of schools and directors of special and elementary programs who permitted us to enter their school systems and work with the teachers and pupils.

Mr. Charles Robinson, Stamford Public School System, Stamford, Connecticut served as research assistant during the initial year of this study. Miss Barbara Donnelly, Bridgeport Public School System, Bridgeport, Connecticut served as a research assistant. Mrs. Melicent Mortlock also served as a research assistant. Mr. Robinson and Miss Donnelly developed the tests which were used in the study. Mrs. Mortlock wrote a substantial part of the teaching program, along with Mr. Robinson.

Mr. Richard Fawcett, Montville Public School System, Montville, Connecticut conducted the testing, organized the workshop and guided the project through its second year.

Dr. Ross Armstrong, University of Minnesota, was an invaluable consultant.

Mrs. Evi Indars served as project secretary throughout its tenure.

We are indebted to all those who helped.

JFC

JOG

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

In recent years research workers have undertaken a variety of investigations relative to the developmental, intellectual and learning characteristics of the mentally handicapped. A substantial share of these activities might be categorized as laboratory and/or clinical research. As data and information have become available from the aforementioned, there has been a greater realization of the need to broaden our research programs to include the study of educational methodology, instructional materials and curricula. The accent on the latter is evident in the emergence of applied research through demonstration programs.

Demonstration programs generally focus upon the implementation of propositions which are logically or experimentally derived. These propositions often manifest the ingredients of previous research, which, through the demonstration programs, are translated into practice. In essence, this was the orientation of the project reported herein—a project dealing with arithmetical problem-solving among the mentally handicapped.

The development of extended arithmetical programs for the mentally handicapped has been inhibited by a number of factors. Among these has been the fact that a considerable portion of the information upon which we base our programs is acquired after the influence of instruction. Characteristics of various samples are sought and identified. These traits should serve as the basis for program innovations. Instead, they are frequently interpreted as the rationale underlying the elimination of certain content from the curriculum. To illustrate, Cruickshank (1948) found that mentally handicapped children experience difficulty in solving problems containing extraneous numbers. This implies a difficulty in the elicitation of relevant cues from the verbal elements of the problem and confusion with respect to the selection and organization of stimulus materials. An examination of classroom arithmetic lessons, curriculum guides and discussions with teachers indicates a paucity of teacher-directed arithmetical experiences dealing with problems containing superfluous material. The lack of activities concerned with these types of problems is often based upon the fact that research has shown that mentally handicapped children experience difficulty with them. Surely, education is sufficiently sophisticated to create instructional processes which could stress the modification of these curriculum related traits.

Another factor which must be cautiously assessed in the arithmetic program for the mentally handicapped is "concreteness." We are informed (Burns, 1961) that retarded children are superior at solving problems of a concrete nature in contrast to the abstract. Two issues seem relevant. The first is that many teachers confuse the terms "concrete" and "meaningful." In many instances, mentally handicapped children manipulate objects, but they do this <u>without</u> <u>meaning</u>, <u>without</u> <u>understanding</u>. The second issue deals with the contrasts in which concrete materials are used in programs for mentally handicapped, average and above average children. A major difference between instruction for the retarded and the non-retarded is that "concrete learnings" are the products with the retarded, whereas they are usually part of the <u>process</u> with the non-retarded. The concrete approach has greater value in the development of arithmetical understandings and principles when it is part of the process of learning. As a process, it functions as a mediator for higher learning. For every concrete activity with which the child is confronted, there should be one or two specific understandings or principles which the teacher will assist him to attain. The concrete, or manipulative activity, should work toward this end, rather than becoming an end in itself (Cawley and Pappanikou, 1967).

A third factor is the source of instruction. In the classroom, the teacher is considered to be the prime motivator for the conditions for learning. Beyond this, the teacher has considerable influence over the content which is selected for presentation in the classroom and the goals which this content is expected to achieve.

In experimental situations (Klausmeier and Check, 1962; Klausmeier and Feldhusen, 1959; Klausmeier and Loughlin, 1961; Wilson, 1964; Callahan, 1962) the primary source of instruction is frequently a specialist in a particular area. Investigations which employ the regular classroom teacher as the main instructional figure (Lerch and Kelly, 1966; Gibney, 1962) are not as common. Demonstration programs appear to have the potential for a much greater grass root impact if they use the regular classroom teacher, than if they utilize a specialist. Accordingly, it was the intent of the present project to demonstrate that trained teachers, employing a well planned and described arithmetic program, could effect a significant improvement in the problem-solving attainments of mentally handicapped children.

Objectives

The purposes of the present study are to:

1. demonstrate that the verbal problem solving achievement and levels of understanding of arithmetical principles among mentally handicapped children can be significantly improved.
2. develop a verbal and a non-verbal test which will assess problem solving attainments of the mentally handicapped.
3. study the relationships among selected achievement characteristics and mental abilities of the mentally handicapped.

Related Literature

Background Studies

In spite of the fact that arithmetic is generally a regular part of the academic program for mentally handicapped children, the literature in this field is meager. Approximately twenty years ago Cruickshank (1946; 1948a, b;) compared retarded and average children on a variety of psychological processes and arithmetic problem-solving abilities. His research demonstrated a tendency for mentally retarded boys to suggest one operation to solve a problem, but to employ a different operation in their attempt to arrive at a solution. He also found that mentally retarded children experienced difficulty in solving problems which contain extraneous numbers. Cruickshank also noted a tendency for retarded children to be less capable in multiplication and division than in addition and subtraction. Their vocabulary and their ability to define arithmetical terms was also inadequately developed.

Attainment in arithmetic, as is the case in most areas, is influenced by a number of variables. In a study of psychological and sociological characteristics of sixth-grade children, Cleveland and Bosworth (1967) identified three groups of subjects with IQ ranges of 75-89, 90-110 and 111-125. The top and bottom quartile in each group were contrasted. Achievement differences between high and low socioeconomic subjects were noted. High achievement was related to more favorable social adjustment and personality traits.

Esther Unkel (1966) has shown that subjects of equal abilities and varying socioeconomic status demonstrate differences in achievement. Low socioeconomic subjects performed less satisfactorily on arithmetic reasoning, arithmetic fundamentals and total arithmetic scores.

Rose and Rose (1961) have shown that a greater percentage of high socioeconomic status children below 90 IQ achieved at or above their class median than did low socioeconomic subjects. Large percentages of the mentally handicapped reside in low socioeconomic neighborhoods. The combination of mental retardation and low socioeconomic status forms a union which appears to adversely effect arithmetic achievement.

Building from the work of Piaget, Woodward (1961) studied the development of number concepts in retarded children and adults. Her areas of investigation were:

1. one-to-one correspondence and equivalency of corresponding sets. One tact used a row of counters which were spread out after the original presentation. Subjects were asked to indicate whether or not there was a change in the number of counters.

2. equalizing unequal groups. This task required subjects to make equal in number, two unequal groups of counters, by moving some from the larger to the smaller group.

3. seriation. This involved the placing of sticks of varying length in a specific order. An inquiry into cardinal and ordinal number concepts was conducted.

4. conservation of continuous quantity, or the constancy of an amount when it undergoes a change in shape.

Most subjects performed at either the concrete operation level or the intuitive level, although there were differences within the four experiments. The retarded subjects seemed to perform at a level similar to an average child of four-to-seven years. Median CA for retarded adults was 19; Median CA was 12.9 for retarded children.

Holt (1963) notes the possible value of incorporating the psychological basis of mathematical concepts into programs for the mentally handicapped. He suggests that this can be accomplished by the development of projects within the classroom. On one project dealing with the study of maps, Holt proposed a series of six stages. They are: (1) topological concepts, (2) equating distance with number, (3) conservation of quantity, (4) prospective and Euclidean concepts, (5) conservation of length and (6) measurement.

Aline Furman (1967) has also given consideration to Piaget's work in arithmetical programming for the mentally handicapped. Figure 1 contains one of approximately fifty lessons dealing with various components of quantity.

Methodological Studies:

Costello (1941) examined the effectiveness of three methods of teaching arithmetic to the mentally retarded. They were the socialization approach, in which the subject engaged in active experiences; the sensorization approach, which emphasized the concrete mode of presentation, and verbalization, or telling. Socialization proved to be the most effective.

The "concrete" notion was also tested by Finley (1962) who explored concrete, pictorial and symbolic presentation of arithmetical materials to mentally retarded and average children of equal mental ages. There were no significant differences between retarded and non-retarded subjects on concrete and pictorial approaches; the concrete approach tended to be the least effective. Problems relative to mode of testing—group versus individual administration—tend to limit the degree to which we might generalize these results.

In a series of studies among children of high I.Q. (120-146), average I.Q. (90-110) and low I.Q. (50-80), Klausmeier and others investigated a variety of characteristics in problem-solving situations. In one of these, (Klausmeier and Loughlin, 1961), the research workers adapted tasks appropriate to the level of ability of the child rather than the frequently used control

Figure 1

THE CONSERVATION OF QUANTITIES

AIM: To develop an understanding of the conservation of discontinuous quantities.

PURPOSE: To develop an understanding of the invariance of quantity, varying the form and dimensions of the containers.

MATERIALS: Two glasses - one narrow and one wide (vary just the width of the glasses, keeping the heights equal) identical marbles.

PROCEDURE:

1. Bring out the two glasses and ask if the two glasses are the same. What makes them not the same? Put the two glasses up against one another and see the differences.
2. Then place the same number of marbles in each glass. Do not make any verbal statement as to the equality.
3. Caution- do not ask children if both glasses have the same number of marbles before you ask how many marbles are in each- it might encourage a set- if child says "no" then he might continue to base further answers on this wrong response.
4. Ask the child to count the marbles in each glass. Ask if he counted the same number of marbles in glass A as in glass B.
5. Then replace the marbles and ask- Do both glasses have the same number of marbles in each?
6. Does the number of marbles in glass A equal the number of marbles in glass B?
7. If you don't understand repeat same procedure with the same number of marbles.
8. If don't fully grasp it after the repetition, follow the same procedure with fewer marbles, emphasizing the multiplication of the relations between the glasses.

OUTCOME: The beginnings of an understanding of the invariance of quantity.

of mental age. Although a range of differences existed within each group, bright children demonstrated a greater tendency to note and correct mistakes independently, to verify solutions and to utilize logical approaches, whereas children of low intelligence were non-persistent, offered incorrect solutions and employed randomized approaches.

Klausmeier and Check (1960) evaluated retention and transfer in arithmetic. Three levels of problems were developed, one for each group. The low I.Q. group dealt with the compilation of a specific amount of money with the fewest number of coins, while the other two samples were required to use a larger number of coins to equal a certain amount. Average and above-average children used paper and pencils to arrive at a solution, whereas the children in the low I.Q. group manipulated coins. Subjects were assisted in the problem solving experience for a period of 15 minutes. Two samples of 60 each, 20 from each I.Q. group, were assigned to retention and transfer treatments.

After a period of 5 minutes, the retention group returned to solve the original problems and the transfer group was confronted with new problems; a similar procedure was presented after 7 weeks. The average time to criterion was not significantly different among the groups, nor were there significant differences between retention and transfer groups after periods of 5 minutes and 7 weeks. It appears that low I.Q. children are able to retain and transfer arithmetic problem solving abilities when the task is appropriate to the group.

Klausmeier and Feldhusen (1959) examined arithmetic learning and retention as related to school instruction. Subjects were presented with tasks involving counting and addition and were taught those arithmetic facts which they did not know, for a period of 19 minutes. Retention was then measured at 5 minute and 6 week intervals. There were no significant differences in the interval acquisition of unknown facts or in the retention of facts at the 5 minute or 6 week period.

Smith and Quackenbush (1960) felt that teaching machines are useful components of the arithmetic program. They contrasted the year long performance of subjects who used teaching machines with the previous year's attainments without the machines. Achievement in arithmetic was greater during the period in which teaching machines were used. In another study with teaching machines (Blackman and Capobianco, 1965), no overall differences in reading or arithmetic were observed, although the experimental and control groups demonstrated significant pre-to-post test gains.

Callahan and Jacobson (1967) conducted a three week instructional program for mentally handicapped children in which the Cuisenaire Rods functioned on the primary instructional device. There are indications that individualized instruction of this sort aids in the acquisition of number facts and in the retention of arithmetical concepts.

Concern for the below average youngster is further indicated in studies of slow-learner children by Lerch and Kelly (1966) and Gibney (1962). In the former, seven topics, ranging from whole numbers to ratios and percents, were taught to seventh-grade subjects. Experimental subjects gained more than controls. The second study was a crash program to teach meaningful multiplication in eight days. There were no significant differences in the immediate post testing, but differences did emerge on a delayed recall test four weeks after the program ended.

The available literature does not justify the exclusion of problem-solving from the arithmetic experience of the mentally handicapped. On the contrary, it is obvious that (1) very little is known regarding the problem-solving strategies used by these youngsters, (2) the evaluation and assessment of arithmetic skills and abilities is an area of needed research, (3) stop gap measures, such as short term remedial programs, need to be expanded into longitudinal studies, and (4) greater emphasis must be placed upon research in this area.

Chapter II

RESEARCH AND TEACHING PROGRAM

The plan utilized in the present project provided an opportunity to assess the progress of mentally handicapped children under two experimental conditions and to view these with samples of mentally handicapped and non-mentally handicapped children who served as controls.

Definition

1. The term "mentally handicapped" refers to those subjects whose intelligence quotients were within a range of 50 to 80.

2. The term "average" refers to subjects whose intelligence quotients were within a range of 90 to 109.

3. The term "Experimental A" refers to mentally handicapped participants whose teachers were given special training in the use of the technique and the program.

4. The term "Experimental B" refers to mentally handicapped participants whose teachers were not given any special training in the use of the technique or the program.

5. The term "Control A" refers to mentally handicapped children who functioned as comparison subjects.

6. The term "Control B" refers to children of average intelligence who functioned as comparison subjects.

Descriptive data relative to the population of the present project are contained in Table 1.

Table 1

CHARACTERISTICS OF SUBJECTS: CHRONOLOGICAL AGES AND INTELLIGENCE QUOTIENTS

	C. A. in Months	I. Q.
Control A		
$\bar{X}$	172.52	66.01
S. D.	19.80	7.93
Control B		
$\bar{X}$	127.74	100.13
S. D.	4.53	5.82
Experimental A		
$\bar{X}$	167.48	67.93
S. D.	10.15	8.21
Experimental B		
$\bar{X}$	159.23	67.65
S. D.	18.83	7.49

The original research plan called for the administration of the California Test of Mental Maturity to all subjects in order to arrive at an approximation of mental age levels. It was suggested that interruptions to the classroom could be minimized if we could use test scores which were part of the regular testing program. We demurred! Thus, the intelligence quotients for the average children are based upon the Otis Quick Scoring Test.* The WISC was employed with retarded subjects.

The Demonstration Program

The Workshop

A major experimental consideration extended to the Experimental A sample was the training of the teachers who would conduct the program. This training program focused upon the organization of the 86 lessons and the incorporation of these lessons into the teaching procedure.

* Beta

The workshop was conducted late in the afternoon and early in the evening for a ten day period. Teachers were paid for attending, but no remuneration was extended for their participation in the actual project.

The Teaching Method

The teaching method outlined below is designed to help mentally handicapped pupils to understand mathematical principles and operations and to apply these understandings to the solution of real and described problems. The method is the result of the work of John O. Goodman, one of the principal investigators, It has been elaborated on in a recent special education publication (Cawley and Pappanikou, 1967).

The teaching procedure focuses upon the development and application of understanding through the solution of real problems which originate in the classroom. Children use manipulative materials and objectify solutions and generalizations with concrete and pictorial devices. The procedure is outlined as follows:

1. Appraising Readiness. Appraise pupils' understanding of principles and operations basic to the new topic to be taught. This will be done through observation, paper-pencil-tests, teacher question-pupil answer techniques.

2. Real Problem. Plan a situation related to an activity actually carried out in the classroom from which real problems may be drawn and which can be solved by applying the operation or fact to be taught. Discuss with pupils the situation and the problem.

3. Solving a Real Problem. Have the pupils use manipulative devices to work out a solution to the real problem as originated from the classroom situation described in 2 above. The purpose is solution of the problem by whatever method pupils use to yield correct solutions. Do not insist on using the operation or algorism to be taught.

4. Abstracting Generalizations and Principles. Have pupils generalize principles evident in methods used to solve the problem. Record generalizations in abstract form.

5. Objectify. Have pupils objectify the generalizations abstracted through use of representative materials, pictures, charts, or/and numerical symbols.

6. Relate to Conventional Algorism. Write operations in numerals used by pupils to solve the real problem which most nearly represent the conventional algorism.

7. Verify the Conventional Algorism. Help pupils verify the conventional algorism by using generalizations developed in step 4, by previously developed operations and facts and with manipulative materials, or picture operations which verify the algorism.

8. Appraise Understanding. The purpose of appraisal at this point is to identify pupils who need additional experience with manipulative and visual materials in order to understand the operation or fact developed, and to derive valid generalizations; pupils who have developed a good understanding but need additional experience to reinforce the understanding, and pupils who experience no difficulty in understanding and recall can be identified. Close observation of pupils' work as they progress through steps 2 to 7 may be an adequate base for appraisal. It is some times advisable to give a formal test of understanding.

9. Grouping. Group children, according to judgment, of the need for experience to assure mastery. Three groups may be needed, but most often two will be adequate. The low achievers will be given additional experiences with manipulative materials, objectifying and verifying the principles to be developed. The high achievers will derive other properties of the operation taught.

10. Practice. Provide a variety of practice experiences with the conventional form, derived and verified in problems. Prove computation.

11. Evaluate. Evaluate for mastery through observation, paper-pencil tests, progress charts, and drill excercises.

12. Making Use of the Understanding and Skill Developed. Solve real problems from the classroom situation, textbook problems, and pupil-made problems.

The Teaching Program

Approaches to problem-solving in children's arithmetic are generally placed in three categories. The most frequently used approach in programs for the mentally hardicapped is practice. In practice programs, the child is shown how to perform an operation and then given numerous opportunities to utilize his own devices in inculcating the problem-solving procedure into his academic repertoire.

The action-sequence-approach to verbal problem solving is a main ingredient in the Scott-Foresman Program (Hartung, 1961, et al) and the wanted-given procedure is advocated by Clark and Eads (1954).

The essential elements of these programs may be designated as follows:

Action-Sequence:

In the action-sequence program the child is trained to:

1. "see" or recognize the real or imagined action-sequence structure of a problem.

2. express the action-sequence in an equation

3. compute using the operation indicated by a direct equation; or imagine an appropriate second action if the first equation is indirect. Express the imagined action-sequence in an equation and then compute.

4. check by re-writing the equation.

Wanted-Given:

In the wanted-given program the child is trained to:

1. recognize the wanted-given relationship imbedded in a problem.

2. express the wanted-given relationship in an equation.

3. compute by using the operation directly indicated by the equation.

The curriculum planned for this demonstration is comprised of a combination of the action-sequence and wanted-given procedures. The reduced emphasis on the structuring of the problems in the form of equations and the organization of lessons, which independently and collectively utilize the above techniques, represent the major points of demarcation from the programs proposed by their originators.

The teaching program consisted of twelve units which included a total of one hundred thirty-five lessons. The number of lessons per unit was based upon the amount of effort needed to accomplish the unit, rather than some artificially equalized sum. The units were:

I. The Numerical Value of Zero

II. Place Value

III. Addition

IV. Recognition of Symbols

V. Fractions

VI. Subtraction

VII. Equivalence (Money Values)

VIII. Advanced Addition - Carrying (Omitted)

IX. Translation (Omitted)

X. Odd and Even Numbers

XI. Understandings of Geometric Forms

XII. Equivalent Fractions (Review of Reducing Fractions) (Omitted)

A sample lesson is contained in Figure 2: Step 3 of Unit I, Lesson III.

In preparing the teaching program, an excess number of units and lessons were prepared in order that an adequate amount of material would be available. It was necessary to reduce this amount to coincide with the eighteen week teaching program.

The teachers and the project directors negotiated the modifications and reduction of the original program so as to teach the agreed upon number of eighty-six lessons. Units VIII, IX and XII were dropped from the experimental program. None of the lessons in remaining units were eliminated. Rather, the teachers felt that two or three could be put together and completed in one day. This was the procedure used to arrive at the teaching format.

Teachers in Experimental B were given a one hour introduction to the program. The materials were distributed at this session and no further contact was made. These teachers agreed to use the teaching program, fully aware of the fact that another group of teachers had been trained in a workshop.

The teachers in each of the experimental groups administered the pre- and post-tests, whereas the control subjects were tested by research assistants.

Development of the IDES and PUT

One of the basic problems confronting users of standardized tests with the mentally handicapped is the lack of consideration for this sample in the development and standardization of commercial tests. Narrative data gives proportionate representation to the mentally handicapped, but validity and reliability data are seldom reported for this group. For this reason, it was decided to develop a verbal problem solving test and a test of principles and understanding.

Figure 2

THE NUMERICAL VALUE OF ZERO

AIM: The place of zero in a number sequence.

PROCEDURE: Illustrate the writing of numbers greater than nine as a combination of any of the digits 0-9.

LESSON PLAN:

Material - *number tags to represent any and all digits, 0-9; one tag per student - *unlined paper plus common pins.

Draw on the chalkboard a set of objects to represent the number ten. Ask children to count the number of objects in the set and to describe this set with the proper numeral. Draw on the chalkboard several other sets of objects to represent numbers greater than nine. Encourage class participation in describing these sets with the proper numeral.

Illustrate to children the necessity of using a combination of the digits from 0-9 to describe any set containing a number of objects greater than nine.

Proceed with the following game to:

1. illustrate that any two-digit number is a combination of any of two digits in the sequence 0-9
2. introduce place value

Have ten children sit in a circle. Provide each child with a number tag so that all digits, 0-9, are represented. (If more than ten children are present in class, have class members take turns.)

Draw on the chalkboard the number pattern the children are illustrating. Show children that zero is both first and last digit in a revolving number series. Again explain to children that with these ten digits, any number can be written; illustrate by drawing several one-, two-, and three-digit numbers on the chalkboard.

Instruct children that when you call out a number, the two children who represent the digits in this number must stand up and show how they can make two numbers (reversing the digits). Write on the board the numbers illustrated. Call out various two-digit numbers, making sure each child has several chances to participate. Keep a list of the numbers illustrated. (To be used in the following lesson.)

OUTCOME: An understanding of the formation and writing of numbers greater than nine; orientation to place value.

This test of verbal problem solving included Indirect, Direct and Extraneous number problems (IDES). See Appendix A. The following are illustrative of problem types:

1. Indirect Problems are problems in which the action-sequence structures do not directly indicate which operation is to be employed.

    Joe had 3 marbles left after he had given 4 to John. How many did he have in all?

2. Direct Problems, or problems in which the nature of the problem indicates the nature of the operation to be used.

    Joe has 3 red marbles and 6 black marbles. How many marbles does he have altogether?

3. Extraneous Number Problems are problems which contain superfluous stimuli.

    Joe had 3 red marbles, 6 black marbles and 4 jacks. How many marbles does he have?

The Principles and Understandings test (PUT) began as a test of specific arithmetic principles. See Appendix B. Originally, it required a great deal of reading which proved to be somewhat difficult, as can be noted from the following examples:

Principle

The idea that addition is a process of combining groups, and that the combined group is an equivalent of the sum of the parts.

Sample Item

When adding this problem:

$$\begin{array}{r}63\\41\\62\end{array}$$

1. When two groups are combined, the answer is greater than the sum.
2. When three groups are combined, the answer is greater than the sum.
3. When the three groups are combined, the answer is equal to the sum.

The Law of Commutation which indicates that the order of the addends can be rearranged without changing the sum.

Sample Item

When adding, the groups which are added:

1. can be arranged in any order without changing the answer
2. must be arranged in the same order or the answer will change
3. must have the largest number placed at the bottom

After considerable trial and error, the PUT was modified and in its final form it requires very little reading. Our problem with the PUT was an interesting one and one which supports the notion that the assessment of the arithmetical reasoning abilities of the mentally retarded is in need of further research. Identifying an arithmetical principle which the youngster could manage was difficult because it frequently required reading skills which he did not manifest. When the reading level was changed, the ability to assess the particular principle was lost. The situation became circular in nature and it was finally decided to develop a test which was minimal in its reading requirements.

Item Analysis

The procedures employed in the development of the experimental tests were similar to standard procedures. The first step consisted of an identification of the areas which were to be assessed and the content which had to be included. Substantial numbers of items were organized into an item pool. These items were placed into test form and trials initiated. The pilot tests were about sixty items in length. The IDES and the PUT were administered to over five hundred retarded and non-retarded children. Each administration was subjected to a comprehensive item analysis. The item analysis technique described by Garber (1966) was utilized. A listing for each item, similar to that contained in Figure 3 was obtained.

Figure 3

Summary Figure for Item Analysis

	A	B	C	D	E	WRONG	RIGHT	R+W	DIFF	DISC	R-BIS	R-P.BIS
UPPER 27	6	1	0	0	0							
LOWER 27	2	3	2	1	0							
TOTAL N	15*	7	2	1	0	10	15	25	0.40	0.61	0.69	0.55
MEAN	19.9	17.9	16.5	15.0	0.0	17.3	19.9	18.9				
BLOCK A									B		C	

Figure 3 shows an example of the analysis for a single test item. Block A comprise the response option summary. In the example, 15 students selected the correct reponse while ten students selected wrong response options. Compared to the lower 27% of students, those in the upper group more often tended to select the right answer option. The complete description of this program is contained in Appendix C

Emphasis was given to the difficulty levels of each item and its discrimination capabilities. Tate (1955) suggests, other things being equal, that tests composed of items of 50 percent difficulty yield maximum reliability. Items included in the final form ranged from forty to sixty percent in difficulty and above sixty percent in discrimination capability. In an effort to obtain further information relative to the instrumentation and on the experimental program, item analyses were conducted on the pre- and post-tests for each of the four samples.

The pre- and post-test difficulty and discrimination ratios for the thirty item IDES are combined in Table 2 and those for the thirty-five item PUT are contained in Table 3. In each table the difficulty is expressed as the numerator and the discrimination as the denominator. Thus, the 44/29 for item 1 for pre-test in Experimental A represents the difficulty/discrimination ratios for that item for that sample. A review of Table 2 suggests that there is considerable similarity in the mentally handicapped children, but that the items are comparatively easy for the children of average

Table 2
PRE AND POST TEST DIFFICULTY
AND DISCRIMINATION RATIOS: IDEAS

	N=161 Experimental A		N=58 Experimental B		N=132 Control A		N=89 Control B	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1.	44/29	31/56	56/38	62/85	52/50	47/37	12/24	8/16
2.	76/33	59/60	88/6	87/5	76/46	77/44	45/80	22/54
3.	43/62	26/53	46/42	46/50	31/39	37/38	11/28	7/15
4.	74/38	58/69	67/47	65/40	60/20	64/17	39/77	20/47
5.	59/52	53/67	70/27	65/20	69/44	66/44	19/41	15/47
6.	65/58	47/67	58/35	69/50	52/56	62/65	32/69	22/59
7.	51/74	38/75	52/45	51/80	43/55	43/68	18/48	7/36
8.	68/53	45/49	69/51	60/30	56/49	48/29	30/66	20/43
9.	26/56	18/38	35/39	27/80	23/36	25/59	6/14	5/12
10.	59/60	48/75	69/55	63/60	55/65	55/59	25/56	15/50
11.	28/56	28/44	25/42	29/55	19/56	23/38	14/48	7/38
12.	80/-2	71/38	85/-1	86/5	78/19	83/16	63/24	49/57
13.	72/47	65/67	85/21	85/30	80/23	80/42	54/41	29/47
14.	50/35	37/56	61/38	55/50	40/44	54/58	22/45	20/66
15.	62/48	59/62	66/55	52/75	51/55	59/26	32/39	25/63

Table 2 (Con't)

IDES

	Experimental A		Experimental B.		Control A		Control B	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
16.	52/62	39/69	64/55	47/50	44/59	42/48	24/42	23/74
17.	62/61	46/76	61/70	64/50	47/74	48/70	29/86	18/69
18.	69/38	62/65	77/25	61/40	54/51	67/38	51/61	34/45
19.	67/41	59/67	72/40	73/20	69/35	76/19	50/77	34/63
20.	58/31	50/53	66/58	63/65	57/39	63/56	34/49	25/69
21.	39/47	32/45	47/30	36/55	44/41	38/34	13/28	12/36
22.	49/71	31/71	51/46	51/75	40/45	47/68	18/52	17/39
23.	45/68	38/75	36/64	41/75	38/46	31/66	25/46	21/52
24.	28/66	18/65	30/71	26/75	13/46	15/43	10/34	6/24
25.	63/54	44/75	61/62	65/65	53/68	58/61	38/45	26/37
26.	41/76	34/76	53/61	44/85	29/66	34/59	18/59	16/39
27.	39/54	28/67	38/61	33/70	29/57	27/54	14/38	10/54
28.	53/54	51/65	60/11	58/30	46/53	61/35	32/66	19/54
29.	61/63	41/84	72/35	60/55	53/59	46/54	33/63	22/70
30.	44/71	30/75	35/48	42/80	29/51	32/57	20/62	9/38

Table 3

PRE AND POST TEST DIFFICULTY
AND DISCRIMINATION RATIOS: PUT

	N=161 Experimental A		N=58 Experimental B		N=132 Control A		N=89 Control B	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1.	11/30	9/24	12/27	12/27	10/47	6/13	6/12	0/6
2.	55/30	49/51	58/27	52/53	38/30	57/42	22/58	18/26
3.	13/50	7/29	24/57	11/63	8/37	6/40	2/9	0/13
4.	56/37	47/43	69/25	49/11	49/49	47/52	24/42	21/24
5.	7/31	5/27	3/17	0/47	6/32	3/25	0/3	0/16
6.	14/49	7/36	5/26	6/68	10/36	6/30	2/9	1/16
7.	67/56	45/69	53/56	63/47	56/21	51/47	13/36	10/42
8.	18/49	5/22	5/13	6/53	13/36	7/27	1/12	1/35
9.	30/61	19/56	23/44	10/89	23/52	23/50	3/8	1/23
10.	33/54	23/58	40/33	30/58	36/38	28/32	13/42	7/33
11.	54/59	40/32	54/43	50/47	50/41	38/44	15/33	12/37
12.	82/17	71/35	88/-2	77/-16	79/29	83/23	27/51	20/36
13.	71/21	67/36	64/53	82/0	77/4	78/16	60/56	63/34
14.	74/39	56/52	64/16	58/32	77/16	65/34	27/45	19/20
15.	49/61	26/50	41/58	38/63	39/63	33/55	9/21	8/17
16.	32/61	18/46	24/24	21/53	37/55	23/50	13/33	7/33
17.	17/49	11/27	8/22	8/58	22/56	12/45	1/6	0/13

Table 3 (Con't)

PUT

	Experimental A		Experimental B		Control A		Control B	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
18.	43/76	24/51	35/72	35/68	43/55	33/57	3/15	2/32
19.	49/64	35/65	48/52	33/79	46/63	24/57	28/54	4/52
20.	59/56	44/56	55/39	47/42	58/47	47/42	11/33	8/52
21.	56/26	61/15	46/61	54/47	53/44	56/21	48/22	42/36
22.	73/46	63/46	81/10	91/0	76/27	73/36	45/66	41/63
23.	46/64	27/53	52/31	51/37	47/53	46/47	16/24	9/43
24.	49/46	34/62	44/54	36/53	53/47	31/55	22/36	11/27
25.	26/55	15/31	28/40	28/26	19/60	20/50	8/24	1/39
26.	10/36	7/17	4/14	2/37	5/36	2/20	3/15	0/13
27.	3/20	1/10	0/9	4/63	6/42	2/20	1/9	0/16
28.	21/66	9/26	12/43	4/68	17/58	7/30	1/12	0/16
29.	17/49	11/24	16/44	17/58	16/58	9/32	5/15	3/17
30.	46/78	25/50	50/84	43/89	38/63	29/62	8/18	6/16
31.	21/58	14/41	22/53	20/58	23/66	11/42	7/24	1/29
32.	18/52	10/29	14/44	13/37	15/53	5/32	4/15	0/17
33.	24/56	15/28	23/40	14/68	23/56	20/50	3/12	2/42
34.	38/56	25/45	39/63	34/74	25/64	28/47	2/15	2/35

intelligence. There is a consistent pattern in the post test performance of the main experimental samples. This pattern is not as consistent among subjects in Experimental B and Control A.

A review of Table 3 indicates that the majority of items were not particularly difficult. The items turned out to be of lesser difficulty for these samples than for the pilot samples on which the items were developed. This item analysis suggests that selected arithmetical concepts can be incorporated into the behavioral systems of the mentally handicapped.

Reliability

Reliability may be defined as the consistency or the stability of measurement by a test. Reliability estimates of the IDES and PUT were determined at four different administrations, under a variety of classroom conditions.

Two test-retest reliability estimates were computed for each test. Data for the first estimate is contained in Table 4. The PUT and the IDES were administered

Table 4

TEST-RETEST CORRELATIONS FOR TWO INDEPENDENT SAMPLES OF MENTALLY HANDICAPPED CHILDREN (Time Lapse = 7 days)

	PUT			IDES	
	Pre	Post		Pre	Post
$\bar{X}$	25.14	26.24	$\bar{X}$	16.68	16.90
SD	5.46	3.43	SD	7.62	7.66
N	69	69		72	72
	r = .86*			r = .95*	

* Significant beyond .01 level

to separate samples of mentally handicapped children, with a time lapse of seven days. The r of .86 for the PUT and r of .95 for the IDES are indicative of favorable temporal stability.

Product-Moment Correlations, Table 5, were also calculated between the pre- and post-tests for each of the samples in the project. These correlations are

Table 5

PRE-POST TEST CORRELATIONS FOR FOUR GROUPS ON IDES AND PUT
(Time Lapse = 18 weeks)

Group	N	IDES Pre-Post	PUT Pre-Post
Control A	132	0.557*	0.517*
Control B	89	0.492*	0.251
Experimental A	161	0.580*	0.465*
Experimental B	58	0.280*	0.332*

* Significant beyond .01 level

based upon a time lapse of eighteen weeks. This is a lengthy period for a test-retest analysis, in that we may be measuring the stability of the trait within the student, rather than the stability of the student's performance (Adams, 1964). None-the-less six of the eight correlations are significant, although there is considerable error variance.

The available data was also subjected to internal consistency analysis. In one instance, as can be seen in Table 6, internal consistency was assessed by

Table 6

COEFFICIENTS OF INTERNAL CONSISTENCY FOR PUT AND IDES*

	PUT		IDES	
	Split-Half	Odd-Even	Split-Half	Odd-Even
Control A	.81	.78	.83	.87
Control B	.73	.91	.81	.83
Experimental A	.86	.83	.78	.83
Experimental B	.75	.90	.73	.93

* All r's are significant beyond the .01 level

tabulating a correlation coefficient based upon a comparison of odd versus even items; in the second instance comparisons were made between performance on the first and second halves of each test. The IDES and PUT are not timed tests. Accordingly, it does not appear that the correlation coefficients are based upon measures of rate of work. Correction for the reduced size of each test was accomplished by use of Spearman-Brown formula: $r_{xx} = \frac{2r_{hh}}{1+r_{hh}}$

The IDES and the PUT maintained reasonable temporal stability and internal consistency for the present project.

Validity

Validity information indicates the degree to which the test is capable of certain aims (APA, 1966). Content validity and criterion related validity were given primary consideration in the present project.

The universe of items used in the present project emerged from an analysis of classroom texts and workbooks and texts used by professional educators. They maintain the qualities of the definitions proposed in professional texts and curriculum guides. In this regard, they appear to satisfy the requirements for content validity.

In order to obtain some comparison with external variables, two studies of criterion related validity were undertaken. Table 7 contains validity coefficients

Table 7

VALIDITY COEFFICIENTS BETWEEN IDES, PUT AND SELECTED ACHIEVEMENT TESTS[1)]

	IDES	PUT	$\bar{X}$	S. D.	N
	r				
IDES		.45*	12.67	8.10	69
PUT			18.54	28.08	46
STAT	.72*	.37*	19.09	13.21	73
CAR	.19	.39*	2.92	3.02	38
CAF	.08	.24	8.84	8.9	64

1) Raw Scores

* Significant beyond .01 level

STAT: Seeing Through Arithmetic Test
CAR: California Arithmetic Reasoning
CAF: California Arithmetic Fundamentals

between the IDES, PUT, the Seeing Through Arithmetic Test (Hartung, et al, 1961) and the California Arithmetic Reasoning and Fundamentals Test (Tiegs and Clark, 1957). With the exception of the r of .72 between the IDES and the STAT, the remaining coefficients suggest only minimal overlap in the content of the various tests. The CAF test, which is basically a test of computation, does not appear to be measuring the same traits as the IDES or PUT.

The second validity study was conducted with samples of mentally retarded children of varying developmental levels. There are modest r's, Table 8, between the IDES and PUT and selected ability measures. The only glaring exception to this pattern is the r of -.21 between the PUT and the Quantity subtest of the Primary Mental Abilities Test. An examination of the remainder of the table connotes the existence of criterion related validity. The only r which fails to attain significance, at the .01 level, exists between the PUT and the Reasoning section of the SRA (multilevel) Arithmetic Test. The magnitude of these coefficients raises some question as to excessive overlap in the content of the various tests. The style of the PUT is unquestionably different from that of the SRA series. The IDES, although not as balanced as the investigators desire, does contain a definite array of types of verbal problems which are suitable for use with the mentally handicapped. The suggestion is tendered that the IDES and PUT are valid and consistent measures of arithmetic attainment for samples of mentally retarded children who are comparable to those who participated in these preliminary studies.

Table 8

VALIDITY COEFFICIENTS AMONG IDES, PUT
AND SELECTED ABILITY AND ACHIEVEMENT TESTS

	IDES r	PUT r	X̄	S. D.	N of r
WISC FS I. Q.	.24	.33*	66.59	7.24	261
WISC Arithmetic	.40*	.38*	4.98	1.75	247
PMA(5-7) Quantity	.35*	-.21	16.85	7.49	115
PMA(7-11) Number	.52*	.49*	26.63	18.57	137
SRA(1-2) Arithmetic[1]					
Concepts	.75*	.46*	20.33	7.32	102
Reasoning	.58*	.57*	25.51	11.42	102
Computation	.53*	.42*	35.87	15.97	105
SRA(2-4) Arithmetic[1]					
Concepts	.66*	.46*	19.91	6.88	145
Reasoning	.74*	.52*	11.76	6.22	143
Computation	.57*	.49*	23.61	11.41	142
SRA(Multilevel) Arithmetic[1]					
Concepts	.69*	.65*	10.97	6.42	59
Reasoning	.37*	.24	12.19	5.10	48
Computation	.43*	.46*	13.56	6.51	57

* Significant beyond .01 level

1) raw scores

CHAPTER III

Results and Discussion

The results and discussion phase of this project is divided into two sections. The first section focuses upon the study proper, a review and evaluation of the gains made by participating subjects. The second section deals with the interrelationships among arithmetic and achievement in the language arts and reading, and the primary mental abilities.

The Demonstration Program

Two main elements were of primary concern in the empirical analysis of the demonstration program. Of first importance was the question concerning the magnitude of change, the impact as qualitatively measured by the IDES and PUT. One determination of the effectiveness of the demonstration procedure was an analysis of the simple differences which prevailed between pre-and-post-testing. These means, Table 9, were subjected to "t" tests for correlated means. The primary experimental sample, Experimental A, which included the special program and the trained teachers, showed significant gains on both the IDES and the PUT. These gains, the magnitude of which exceeded the .01 level, amount to an increase of approximately twenty percent of the pre-test scores. Control A, mentally handicapped children, and Control B, children of average intellectual ability, registered significant gains in the PUT and IDES respectively.

Experimental B, the mentally handicapped children whose teachers were not trained, but who used the program, failed to demonstrate any significant progress; these pre-post comparisons actually showed a decrease on the PUT.

An inspection of the pre-test means indicated, rather clearly, the superiority of Control B. This superiority was greater than that which could be attributed to random errors alone. The sample means were adjusted

Table 9

COMPARISON OF MEAN CHANGE SCORES, PRE- AND POST-TEST MEANS AND STANDARD DEVIATIONS FOR FOUR GROUPS ON IDES AND PUT

		Test					
		IDES			PUT		
Group		Pre	Post	"t" Post-Pre Diff.	Pre	Post	"t" Post-Pre Diff.
Control A - N = 132	Mean	15.46	14.30	-2.41	19.78	22.09	4.33*
	S. D.	5.95	5.90	—	6.34	5.78	—
Control B - N = 89	Mean	20.51	22.52	2.99*	28.30	27.47	-1.38
	S. D.	6.57	5.87	—	4.10	5.09	—
Experimental A - N = 161	Mean	12.80	15.45	5.03*	19.75	23.46	6.76*
	S. D.	6.82	7.83	—	7.34	5.80	—
Experimental B - N = 58	Mean	10.43	11.00	0.61	19.69	17.10	-2.51
	S. D.	4.78	6.60	—	5.33	7.63	—

* Significant beyond .01 level

with analysis of covariance. The adjusted means and regression coefficients are located in Table 10.

Table 10

ADJUSTED MEANS AND REGRESSION COEFFICIENTS FOR FOUR GROUPS ON A VERBAL ARITHMETIC TEST (IDES) AND AN ARITHMETIC CONCEPTS TEST (PUT)

		Verbal		Concepts	
Group	N	Y Adj.	b	Y Adj.	b
Control A	132	10.83	0.55	23.50	0.83
Control B	89	20.25	0.40	27.76	0.32
Experimental A	161	16.85	0.67	23.53	0.37
Experimental B	58	12.59	0.36	18.02	0.46
TOTAL	440				

The F's in Tables 11 and 12 attain significance beyond the .01 level. Comparisons were not made among the adjusted means resulting from the covariance analysis. This decision was prompted by the ceiling effects manifested by the average children. A logical comparison seemed sufficient, especially in view of the likelihood that the largest mean was a very conservative estimate of the sample's performance.

The identification of significant gains among the subjects who were taught the prepared program by trained teachers is a favorable aspect of this project. It appears that the combination can effect changes in the achievement expectancy of the mentally handicapped. Another sample, taught by trained teachers without the prepared program, would have provided the basis for firmer generalizations and conclusions. This first stage, however, should provide the basis for a more comprehensive research in the area of arithmetic.

Table 11

ANALYSIS OF COVARIANCE FOR DATA FROM FOUR GROUPS ON AN ARITHMETIC CONCEPTS TEST (FUT)

Source of Variation	Degrees of Freedom	Sums of Squares and Products Σx^2	Σxy	Σy^2	Deviation About Regression $\Sigma y^2 - \frac{(\Sigma xy)^2}{x^2}$	Degrees of Freedom	Mean Square
Among Groups	3	17,128	6,822	15,595	—	—	—
Within	436	5,190	3,401	3,938	13,023	435	29.94
Total	439	22,318	10,223	19,533	14,851	438	—
Difference for testing among adjusted treatment means					1,828	3	609.33

$F = 609.33/29.94 = 20.35$ (d.f. = 3/436)

Significant beyond the .01 level

Table 12

ANALYSIS OF COVARIANCE FOR DATA FROM FOUR GROUPS ON A VERBAL ARITHMETIC TEST (IDES)

Source of Variance	Degrees of Freedom	Sums of Squares and Products Σx^2	Σxy	Σy^2	Deviation About Regression $\Sigma y^2 - \frac{(\Sigma xy)^2}{x^2}$	Degrees of Freedom	Mean Square
Among Groups	3	17,680	9,781	20,057	—	—	—
Within	436	4,802	5,092	6,188	14,462	435	33.25
Total	439	22,482	14,823	26,245	16,472	438	—
Difference for testing among adjusted treatment means					2,010	3	670.00

$F = 670.00/33.25 = 20.15$ (d.f. = 3/436)

Significant beyond the .01 level

Figure 4 contains profiles of the reading, language arts and arithmetic performance of 30 mentally handicapped children who were subjects in the demonstration program and 30 mentally handicapped children who were not participants. These data were acquired as a phase of the study of selected achievement and ability characteristics of mentally handicapped children. As such, it is not intended as support or criticism of the main study. The trends, however, are interesting.

There is considerable similarity in the overall performance of participants and non-participants. Reading performance is flat and tends to attain a grade equivalent level of approximately two years, four months. Arithmetic performance is notably higher, particularly in computation. Facility in computation is a recognized attainment among the mentally handicapped. Whether this is a function of (1) overlearning, as a result of practice, (2) the fact that computation is relatively easy in comparison with problem-solving or, (3) the curriculum and methodology through which instruction is presented is an issue that remains. The combined peaks in arithmeti computation and spelling lends credence to the notion that skill is not uncommon in special education classes. The fact that the participants and non-participants were approaching fourteen years of age, having had seven or eight years of school, raises a question about possible developmental increases in other areas if the curriculum was more problem-solving oriented.

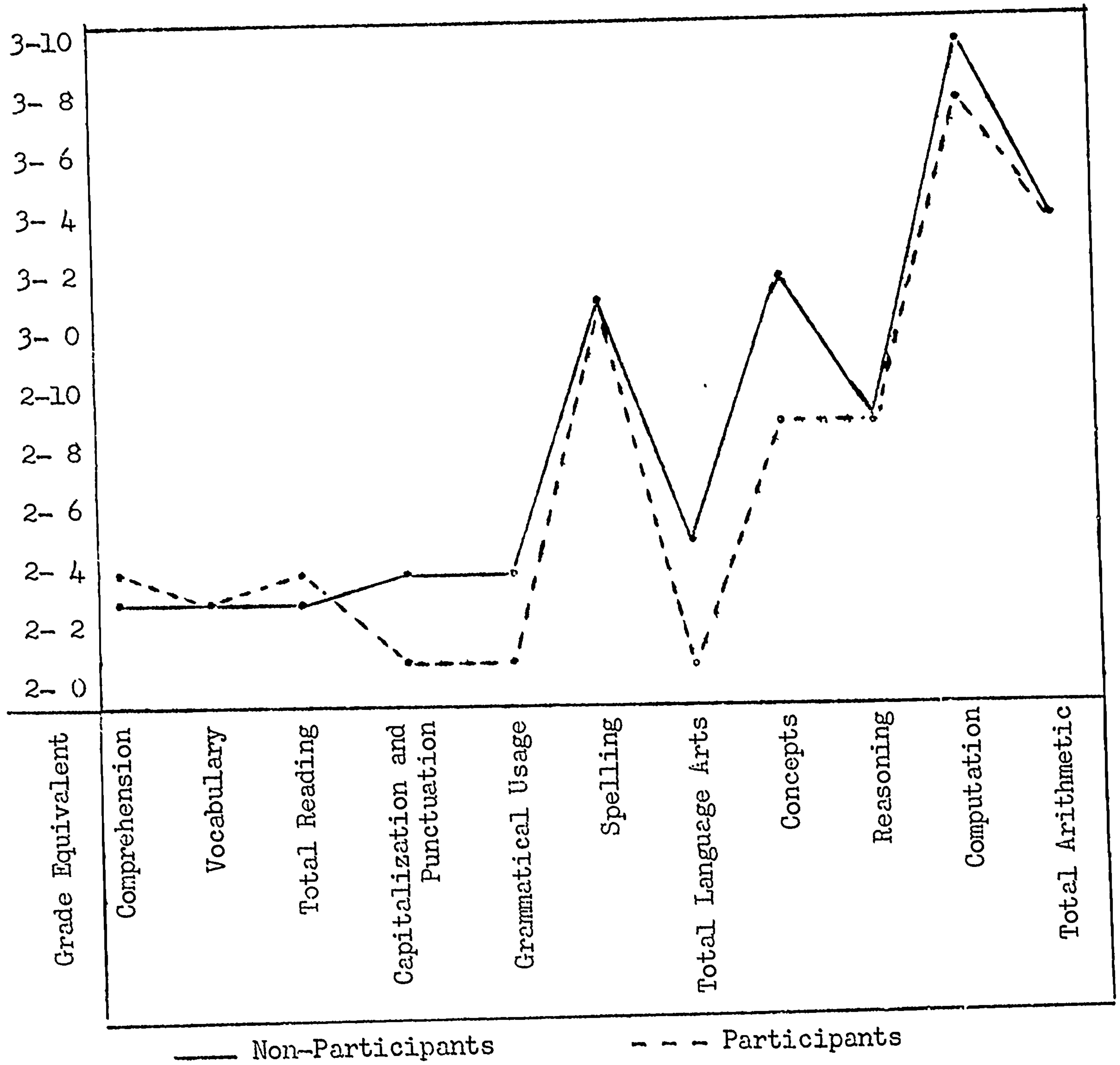
Interrelationships in Achievement and Ability[1]

This section is a report on the interrelationships among academic achievement and primary mental abilities, with the focus in arithmetic performance. Subjects represented all the available mentally retarded

[1]This phase of the report is not included in the original proposal. We were able to collect and analyze this data through the efforts of our excellent research assistants and through the cooperation of the teachers who administered the batteries.

Figure 4

ACHIEVEMENT PROFILES FOR PARTICIPANTS AND NON-PARTICIPANTS IN DEMONSTRATION STUDY



Participants

	$\overline{X}$	S. D.
Comprehension	22.03	9.17
Vocabulary	12.27	6.09
Total Reading	34.30	12.91
Capitalization and Punctuation	29.59	10.97
Grammatical Usage	18.96	6.78
Spelling	7.80	5.19
Total Language Arts	53.77	21.56
Concepts	19.63	5.32
Reasoning	12.07	5.72
Computation	22.30	10.78
Total Arithmetic	54.07	19.27

Non-Participants

	$\overline{X}$	S. D.
Comprehension	19.75	9.50
Vocabulary	11.68	8.74
Total Reading	31.36	17.47
Capitalization and Punctuation	32.66	9.57
Grammatical Usage	20.76	7.62
Spelling	8.24	6.31
Total Language Arts	61.66	20.28
Concepts	21.79	8.93
Reasoning	11.85	7.63
Computation	24.62	14.90
Total Arithmetic	56.55	29.94

children in the primary through junior high special classes in one eastern city. The SRA Primary Mental Ability Test and the SRA Achievement Series were administered by the special class teacher. Primary children, (CA Mean = 131.69; SD = 15.64) took the PMA 5-7 and the Achievement Series, Level 1-2; Older children (CA Mean = 158.14; SD = 17.26) took the PMA 7-11 and the Achievement series, level 2-4.

Tables 13, 14, 15 and 16 focus upon the younger children. Of the 45 r's contained in Table 13, only sixteen obtain significance at or beyond the .01 level. Total reading performance correlates significantly with the primary mental abilities, an exception being space. The .52 correlation coefficients between quantitative and arithmetic reasoning and quantitative and computation suggest a modest relationship between them.

The r's among the various arithmetic tests, Table 14, are all significant beyond the .01 level. There appears to be considerable overlap among these traits, as measured by this particular battery. The pattern of the interrelationships among these attained levels suggests that a reasonably well balanced approach to arithmetic is a valid consideration. Drill need not be the order of the day!

Achievement in arithmetic concepts and numerical reasoning, Table 15, appears related to the various reading skills. There are no significant r's between computation and any of the reading skills, at this level. One might speculate that the responsiveness to the skill of computation is more favorable because the lack of facility in reading does not inhibit the child's performance.

Table 16 is a summary table expressing the correlation coefficient among the primary mental abilities. Even with this homogenous sample, all r's are significant beyond the .01 level.

Table 13

COEFFICIENT OF CORRELATION AMONG PRIMARY MENTAL ABILITIES (CA 5-7), READING AND ARITHMETIC ACHIEVEMENT (Level 1-2)[1]

		Verbal	Perception	Quantitative	Motor	Space	$\overline{X}$	S. D.	N of r
Reading	Verbal-Pictorial Association	26	28	21	34*	13	20.34	13.97	54
	Language Perception	09	09	03	19	14	90.45	80.97	55*
	Comprehension	30	24	25	21	04	17.53	6.68	44
	Vocabulary	27	41*	11	35*	11	14.07	7.65	43
	Total Reading	41*	43*	53*	49*	26	121.55	50.79	51
Arithmetic	Concepts	41*	17	22	42*	21	20.33	7.32	68
	Reasoning	41*	20	52*	38*	32	25.51	11.42	68
	Computation	43*	25	52*	35*	40*	35.88	15.96	70
	Total Arithmetic	25	11	20	28	26	85.27	51.93	70
	$\overline{X}$	37.12	21.62	16.85	48.56	16.48			
	S. D.	7.81	7.66	7.49	16.44	8.98			

* Significant beyond .01 level

1) Raw scores

Table 14

INTERCORRELATIONS AMONG ARITHMETIC CONCEPTS, REASONING, COMPUTATION AND TOTAL ARITHMETIC SCORES (Level 1-2)[1]

	Concepts	Reasoning	Computation	Total Arithmetic	$\bar{X}$	S. D.	N of r
Concepts	X	61*	51*	66*	20.33	7.32	102
Reasoning		X	67*	63*	25.51	11.41	102
Computation			X	43*	35.88	15.96	105
Total Arithmetic				X	85.27	51.93	

* Significant beyond .01 level

1) Raw scores

Table 15

COEFFICIENTS OF CORRELATION AMONG
READING AND ARITHMETIC ACHIEVEMENT (Level 1-2)[1)]

		Concepts	Reasoning	Computation	Total Arithmetic	$\bar{X}$	S. D.	N of r
	Verbal-Pictorial Association	29*	23	13	32*	20.34	13.97	86
Reading	Language Perception	37*	48*	-05	82*	90.45	80.97	87
	Comprehension	49*	47*	27	13	17.53	6.68	67
	Vocabulary	46*	45*	12	24	14.07	7.65	66
	Total Reading	34*	61*	13	07	121.55	50.97	82
	$\bar{X}$	20.33	25.51	35.88	85.27			
	S. D.	7.32	11.41	15.96	51.93			

* Significant beyond .01 level

1) Raw scores

Table 16

INTERCORRELATIONS AMONG PRIMARY MENTAL ABILITIES (CA 5-7)[1]**

	Verbal	Perception	Quantitative	Motor	Space	$\bar{X}$	S. D.	N of r
Verbal	X	59*	70*	36*	51*	37.12	7.81	114
Perception		X	62*	56*	52	21.62	7.66	114
Quantitative			X	42*	58*	16.85	7.49	114
Motor				X	46*	48.56	21.19	115
Space					X	16.44	8.98	115

* Significant beyond .01 level

** test used approximated MA rather than CA of sample

1) Raw scores

The data in Tables 17 through 20 are somewhat more extensive than that which was available on the younger subjects. This is due to the addition of the language arts subtests in level 2-4 of the battery.

Only six of the fifty-five r's in Table 17 fail to attain significance at the .01 level. For this age group of mentally handicapped youngsters, acknowledging error variance, the relationship between mental abilities and academic achievement is firmly established. The extent to which one is predicated upon the other is an interesting question, particularly if the instructional program is geared toward the development of a limited capacity expectancy: "I'm of low ability, therefore, I cannot achieve; I'm a poor achiever, therefore, I am of low ability. He is of low ability, therefore, he cannot achieve; we will teach him accordingly; he will be a poor achiever."

Arithmetic abilities are highly interrelated among this sample. Correlation coefficients, Table 18, ranging from .61 to .90 are of exceptionally high order.

Reading and language arts, Table 19, correlate significantly with all the arithmetic scores. In contrast with the younger subjects where computation did not significantly correlate with reading skills, computation has a consistent relationship to reading and language arts skills.

Table 20 contains the correlation coefficients among the primary mental abilities. Significant r's are noted throughout.

Among the mentally retarded children who participated in this phase of the study, there can be little doubt about the measured interrelationships among the primary mental abilities and achievement in reading, arithmetic and the language arts. The tight circularity of this pattern suggests the need for measures which can identify discrete abilities, should they exist, at various developmental levels and a capacity to relate them to effective practices in curriculum and methodology.

Table 17

COEFFICIENTS OF CORRELATION AMONG PRIMARY MENTAL ABILITIES (CA 7-11) AND READING, LANGUAGE ARTS AND ARITHMETIC ACHIEVEMENT (Level 2-4)[1)]

		Verbal Meaning	Space	Reasoning	Perception	Number	$\overline{X}$	S. D.	N of r
Reading	Comprehension	48*	22	51*	46*	44*	22.82	9.33	105
	Vocabulary	41*	03	29*	29*	21	14.36	9.69	102
	Total Reading	54*	22	52*	42*	40*	36.37	14.91	105
Language Arts	Capitalization and Punctuation	35*	27*	48*	36*	32*	33.46	11.04	101
	Grammatical Usage	35*	30*	43*	21	34*	19.52	7.80	100
	Spelling	34*	10	35*	27*	33*	8.36	5.75	100
	Total Language Arts	45*	27*	52*	33*	38*	58.97	22.40	103
Arithmetic	Concepts	53*	48*	55*	59*	66*	19.91	6.88	100
	Reasoning	56*	37*	62*	53*	57*	11.76	6.21	96
	Computation	52*	38*	52*	48*	63*	23.61	11.41	95
	Total Arithmetic	60*	46*	62*	58*	70*	53.46	23.56	99
	$\overline{X}$	22.76	12.76	21.68	23.54	26.63			
	S. D.	10.88	5.92	10.20	15.85	18.57			

* Significant beyond .01 level
1) Raw scores

Table 18

INTERCORRELATIONS AMONG ARITHMETIC CONCEPTS, REASONING, COMPUTATION AND TOTAL ARITHMETIC SCORES (Level 2-4)[1)]

	Concepts	Reasoning	Computation	Total Arithmetic	$\overline{X}$	S. D.	N of r
Concepts	X	69*	73*	89*	19.91	6.88	140
Reasoning		X	61*	83*	11.76	6.21	141
Computation			X	90*	23.61	11.41	142
Total Arithmetic				X	53.46	23.47	

* Significant beyond .01 level

1) Raw scores

Table 19

COEFFICIENTS OF CORRELATION AMONG ARITHMETIC AND READING AND LANGUAGE ARTS ACHIEVEMENT SCORES (Level 2-4)[1)]

		Concepts	Reasoning	Computation	Total Arithmetic	$\bar{X}$	S. D.	N of r
Reading	Comprehension	45*	53*	50*	58*	22.82	9.33	144
	Vocabulary	34*	50*	39*	47*	14.36	9.69	142
	Total Reading	45*	56*	50*	58*	36.37	14.91	145
Language Arts	Capitalization and Punctuation	46*	46*	46*	53*	33.46	11.04	142
	Grammatical Usage	41*	43*	40*	47*	19.52	7.80	141
	Spelling	40*	32*	39*	44*	8.36	5.75	132
	Total Language Arts	52*	52*	53*	60*	58.97	22.40	146
	$\bar{X}$	19.91	11.76	23.61	53.46			
	S. D.	6.88	6.21	11.41	23.47			

* Significant beyond .01 level

1) Raw scores

Table 20

INTERCORRELATIONS AMONG PRIMARY MENTAL ABILITIES (CA 7-11)[1]**

	Verbal Meaning	Space	Reasoning	Perception	Number	$\overline{X}$	S. D.	N of r
Verbal Meaning	X	41*	58*	53*	48*	22.76	10.89	123
Space		X	58*	47*	33*	12.77	5.92	122
Reasoning			X	60*	56*	21.68	10.20	123
Perception				X	54*	23.54	15.84	122
Number					X	26.63	18.51	--

* Significant beyond .01 level

** Test used approximated MA rather than CA of sample

1) Raw scores

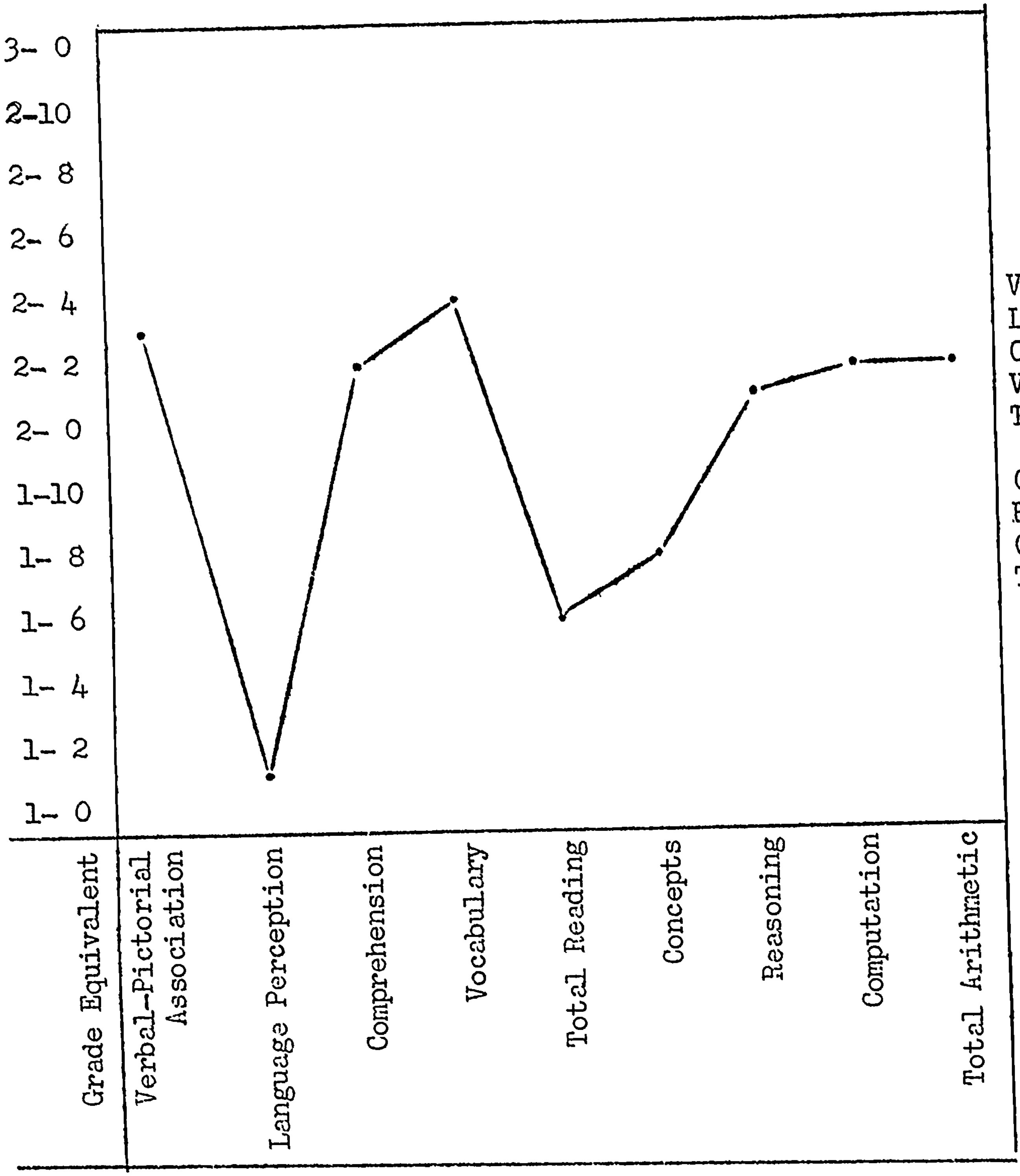
The final segment of this report is the presentation of the achievement profiles, Figures 5 and 6, of the intermediate, level 1-2, and the junior high, level 2-4, samples. There is no attempt at a direct quantitative comparison because of the differences in developmental levels.

Figure 5 depicts reading and arithmetic achievement. The two outstanding characteristics of the profile seem to be its relatively uneven composition and the obvious deficit in language perception. Language perception includes tasks of auditory discrimination, visual discrimination and the identification of printed words, when the stimulus is presented auditorally. This particular deficit is well recognized among mentally handicapped and culturally disadvantaged children.

In Figure 6, patterns of achievement differentiation become noticeable. Arithmetic computation and spelling, two practice oriented skills in special classes, have visable peaks. A longitudinal study of the achievement patterns of the younger children would provide the basis for interesting speculation if the profiles ultimately resembled those of the older children in the present study. Some determination of the reasons for this pattern should be discovered.

Figure 5

ACHIEVEMENT PROFILES FOR MENTALLY HANDICAPPED CHILDREN (SRA Level 1-2)



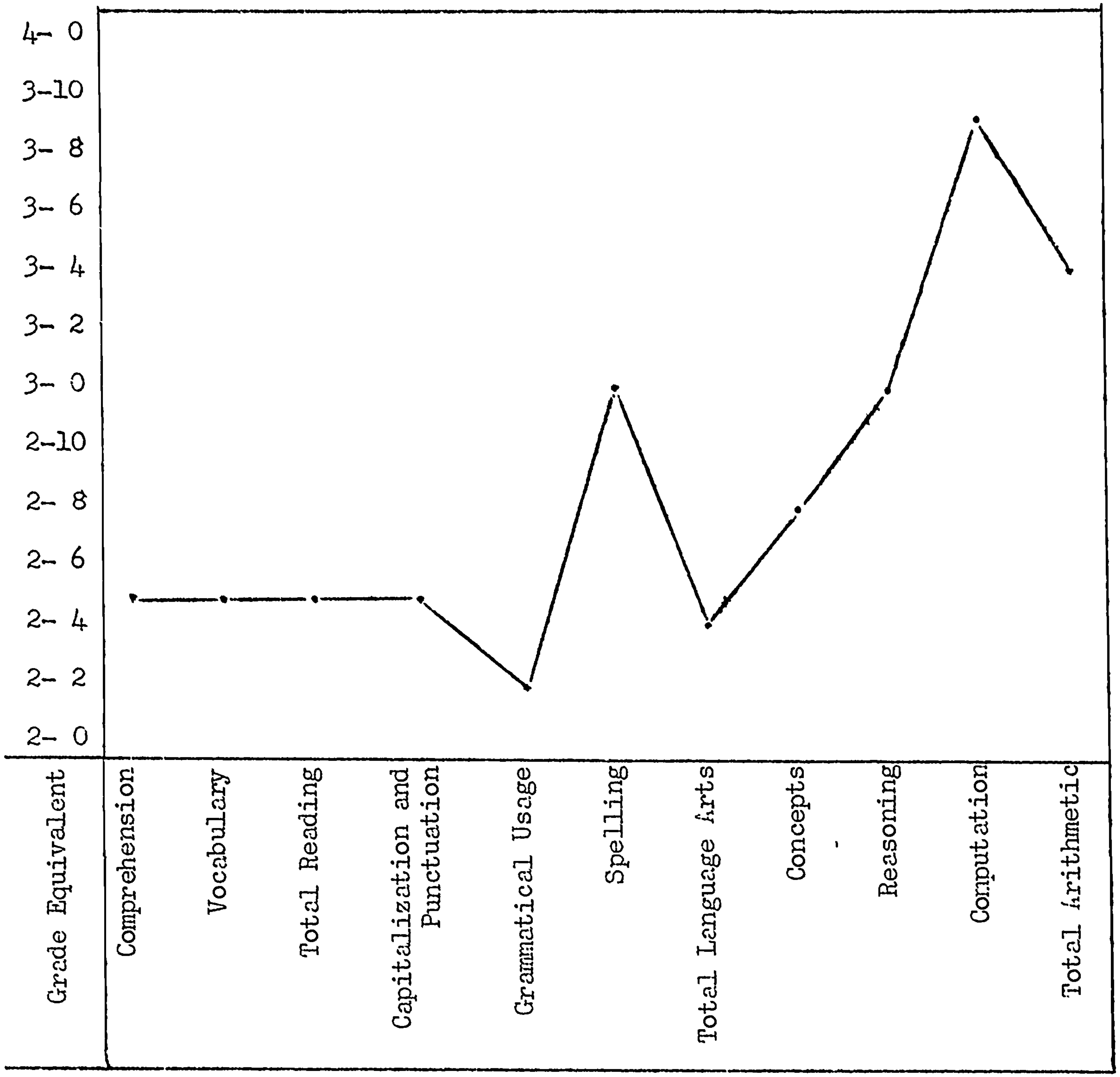
SRA

Achievement (level 1-2)

	$\overline{X}$	S. D.
Verbal-Pictorial Association	20.63	14.08
Language Perception	80.42	27.07
Comprehension	17.66	6.73
Vocabulary	14.51	7.62
Total Reading	121.07	48.30
Concepts	19.82	7.32
Reasoning	25.11	11.66
Computation	34.72	16.27
Total Arithmetic	83.59	54.14
Chronological Age (Months)	131.69	15.64

Figure 6

ACHIEVEMENT PROFILES FOR MENTALLY HANDICAPPED CHILDREN (SRA Level 2-4)



Level 2 - 4

	$\bar{X}$	S. D.
Comprehension	22.61	9.05
Vocabulary	13.85	7.22
Total Reading	36.33	15.20
Capitalization and Punctuation	33.85	10.88
Grammatical Usage	20.05	7.35
Spelling	8.41	5.78
Total Language Arts	60.01	21.57
Concepts	20.01	6.87
Reasoning	11.78	6.23
Computation	23.61	11.41
Total Arithmetic	53.46	23.47
Chronological Age in Months	158.14	17.26

CHAPTER IV

Conclusions and Recommendations

The major focal point of this study was an inquiry into the effects of the combination of a trained teacher and planned program in the problem-solving abilities of mentally handicapped children. A secondary concentration centered in the development of measures of verbal problem solving (IDES) and arithmetic understanding (PUT). A third segment dealt with an analysis of the interrelationships among primary mental abilities and various combinations of achievement tests.

Significant gains were noted between the pre- and post-test performance of the primary experimental sample in problem solving and in concept attainment. The mentally handicapped control sample demonstrated a significant gain in concept level, while the average control sample managed a significant increase in verbal problem solving. Quite likely, the lack of an increase in concept attainment among the average sample was a function of the test ceiling. The virtues of an average control sample are open to question. Unless we can effectively relate what we learn from the non-retarded to the retarded, or vice versa, there seems to be only limited value in utilizing the group for control purposes. There appears to be a great deal of merit in the use of heterogeneous samples in longitudinal studies or in clearly documented behavioral research.

The IDES and the PUT appear to be stable and consistent measures of arithmetical performance among similar samples of mentally handicapped children. Downward and upward extensions of these procedures have potential value. Aslo the downward and upward extension of the teacher training program and arithmetic curriculum appear to be justified.

There is an obvious need to conduct longitudinal and quasi-longitudinal research into the arithmetical abilities and achievements of the mentally retarded. Developmental studies, paralleling various curriculum and methodological innovations should be considered. Research should consider the techniques which retarded children employ in order to solve problems.

This study pointed out the interrelationships among various traits and accompanying performance levels. Characteristics of high and low achievers among the retarded should be investigated.

The nature of the classroom, its verbal interaction and language style during instruction is an exciting research possibility.

This study supports the contention that problem solving and concept development among the mentally handicapped can be influenced by education.

The data contained in this project suggest that verbal and non-verbal measures of arithmetical problem solving can be developed to assess these developments. The stability and validity of the procedures described herein are adequate for this purpose. As measures of arithmetical achievement, the IDES and PUT appear to constitute a satisfactory beginning in the field of mental retardation.

Bibliography

Adams, Georgia S. Measurement and Evaluation in Education, Psychology, and Guidance. New York: Holt, Rinehart and Winston, 1964. 654p.

American Psychological Association, Joint Committee on Standards for Educational and Psychological Tests and Manuals. Validity and Reliability Defined in Assessing Behavior: Readings in Educational and Psychological Measurement (eds. Flynn, John and Garber, Herbert) Reading, Mass.: Addison-Wesley Company, 1967.

Blackman, Leonard and Capobianco, Rudolph. An Evaluation of Programmed Instruction with the Mentally Retarded Utilizing Teaching Machines. American Journal of Mental Deficiency, 1965, 70, 262-269.

Burns, Paul C. Arithmetic Fundamentals for the Educable Mentally Retarded. American Journal of Mental Deficiency, 1961, 63, 57-61.

Burns, Paul C. Meaningful Mathematics for the High School Retarded. The High School Journal, 1964.

Callahan, John J. and Jacobson, Ruth. An Experiment With Retarded Children and Cuisenaire Rods. The Arithmetic Teacher, 1967, 14, 10-13.

Callahan, Leroy. Remedial Work with Underachieving Children. The Arithmetic Teacher, 1962, 9, 138-140.

Cawley, John and Pappanikou, A. J. The Educable Mentally Retarded in Methods in Special Education. (eds. Haring, N. G. and Schiefelbusch, R. L.) New York: McGraw-Hill Co., 1967. 430 p.

Clark, John and Eads, Laura, Guiding Arithmetic Learning. Yonkers-on-Hudson New York: World-Book Co., 1954.

Cleveland, Gerald and Bosworth, Dorothy. A Study of Certain Psychological and Sociological Characteristics as Related to Arithmetic Achievement. The Arithmetic Teacher, 1967, 14, 383-387.

Costello, Helen M. The Responses of Mentally Retarded Children to Specialized Learning Experiences in Arithmetic Doctor's Dissertation, University of Pennsylvania, Philadelphia, 1941.

Cruickshank, William M. Arithmetic Ability of Mentally Retarded Children. I. Ability to Differentiate Extraneous Materials from Needed Arithmetic Facts. Journal of Educational Research, 1948, 42, 161-170. (a).

__________________. Arithmetic Ability of Mentally Retarded Children. II. Understanding Arithmetic Processes. Journal of Educational Research, 1948, 42, 279-288.

__________________. Arithmetic Vocabulary of Mentally Retarded Boys. Exceptional Children, 1946, 13, 65-69.

________________. Arithmetic Work Habits of Mentally Retarded Boys. American Journal of Mental Deficiency, 1948, 52, 318-330. (b).

________________. Review of "Studies of Reading and Arithmetic in Mentally Retarded Boys." Exceptional Children, 1956, 23, 120-122.

Finley, Carmen J. Arithmetic Achievement in Mentally Retarded Children: The Effects of Presenting the Problem in Different Contexts. American Journal of Mental Deficiency, 1962, 67, 281-286.

Furman, Aline. Lesson Plans Adapted from the Experimental Procedures of Jean Piaget. Unpublished paper, University of Connecticut, 1967.

Garber, Herbert. Improving Professor-Made Multiple Choice Items Through Computer Scoring and Automatic Item Analysis. Mimeographed paper. University of Connecticut, 1966.

Gibney, Thomas. Multiplication for the Slow Learner. The Arithmetic Teacher, 1962, 9, 74-76.

Hartung, H. L., Van Engen, H., and Knowled, Lois. Teaching Guide: Seeing Through Arithmetic. Chicago: Scott, Foresman Co., 1961.

Holt, John. A Project for Presenting Mathematical Concepts to Educable Retarded Students. Mental Retardation, 1963, 1, 281-286.

Kaliski, Lottle. Arithmetic and the Brain-Injured Child. The Arithmetic Teacher, 1962, 9, 245-251.

Klausmeier, H. J., and Check, J. Retention and Transfer in Children of Low, Average, and High Intelligence. Journal of Educational Research, 1962, 55, 319-322.

________________., and Feldhusen, John F. Retention in Arithmetic Among Children of Low, Average and High Intelligence at 117 Months of Age, Journal of Educational Psychology, 1959, 50, 88-92.

________________., and Loughlin, Leo J. Behaviors During Problem Solving Among Children of Low, Average, and High Intelligence. Journal of Educational Psychology, 1961, 52, 148-152.

Lerch, Harold, and Kelly, Francis. A Mathematics Program for Slow Learners at the Junior High Level. The Arithmetic Teacher, 1966, 13, 232-236.

Lyda, W. J., and Duncan, Frances. Quantitative Vocabulary and Problem Solving. The Arithmetic Teacher, 1967, 14, 289-291.

Rose, Alvin and Rose, Helen. Intelligence, Sibling Position and Sociocultural Background as Factors in Arithmetic Performance. The Arithmetic Teacher, 1961, 8, 50-56.

Smith, Edgar, and Quackenbush, Jack. Devereux Teaching Aids Employed in Presenting Elementary Mathematics in a Special Education Setting. Psychological Reports, 1960, 7, 333-336.

Spicker, Howard. Research Implications: Selected Factors Influencing Learning and Retention Abilities of the Mentally Retarded. Education and Training of the Mentally Retarded, 1966, 1, 92-95.

Tate, Merle. Statictics in Education. New York: The Macmillan Co., 1955. 597p.

Tregs, Ernest, and Clark, Willis. California Achievement Tests. Hollywood: California Test Bureau, 1957.

Unkel, Esther. A Study of the Interaction of Socioeconomic Groups and Sex Factors with the Discrepancy Between Anticipated and Actual Achievement in Elementary School Mathematics. The Arithmetic Teacher, 1966, 13, 662-670.

Wilson, John. The Role of Structure in Verbal Problem-Solving in Arithmetic: An Analytical and Experimental Comparison of Three Problem-Solving Programs. Unpublished doctoral dissertation. Syracuse University, 1964.

Woodward, Mary. Concepts of Number of the Mentally Subnormal Studied by Piaget's Method. Journal of Child Psychology and Psychiatry, 1961, 2, 249-259.

LIST OF APPENDICES

* Under separate cover

APPENDIX A

IDES

The Indirect, Direct and Extraneous (IDES) number test is a thirty item test of verbal problem solving.

The procedure for scoring is to tabulate the frequency of correct responses. One point is awarded for each correct response. No norms are provided. Statistical data, which include, item analysis, reliability and validity coefficients and means and standard deviations for mentally handicapped and average children are contained in the body of this report.

1. A teacher put a large pile of drawing paper on the sheld. She said that there were 5 pieces for each student in her room. How many pieces of paper are on the shelf if there are 28 students in the room?

2. At the grocery store, Mother found that cherries cost 29 cents per pound; peaches cost 10 pounds for 90 cents, and apples were 10 pounds for 79 cents. The peaches cost how much per pound?

3. A group of boys came to the school to play basketball. The boys were divided into 5 man teams. If we had enough for 6 teams, how many boys came to play ball?

4. The teacher told Peggy to bring up three boxes of chalk from the basement. Each box contained the same number of pieces. She told Tim to bring up ten erasers. Peggy brought up 36 pieces of chalk. How many pieces of chalk are there in each box?

5. Jim had a box of candy. He divided it equally among 8 firends. He found that each of them received 13 pieces. How many pieces were in the box to begin with?

6. Mr. Green had to drive 300 miles. He drove 150 miles in 3 hours. How many miles did he average in one hour?

7. Susan had a bag of jacks. She gave away all of her jacks to seven of her friends. She found that she could give each child 5 jacks. How many did she have in her bag to begin with?

8. Joe has 3 puzzles, 5 trucks, and some card games. If 15 friends came to see him and all wanted to play with the puzzles, how many would share each puzzle?

9. The child divided all the money they made from a puppet show. Each of the 4 friends had 5 cents. How much money had the children made from the show?

10. In our room are 12 children and 15 desks. Into how many equal rows of 5 may the desks be placed?

11. Half of the 14 boys at the birthday party had red straws. The rest had green straws. How many had green straws?

12. Judy wanted to take the pulse rate of her friend Sharon. Judy's pulse rate was 63 beats each minute. Sharon's pulse was 17 beats in 15 seconds. What was Sharon's pulse rate in one minute?

13. Joe had several boxes of gum. There were 8 pieces of gum in each box. How many boxes of gum did Joe have if he had just enough to give a piece to 24 friends?

14. Mike paid 25 cents for a quarter pound of peanuts and 30 cents for a half pound of candy. Ho much would a pound of candy cost?

15. Betty has an equal number of marbles in each hand. The total number of marbles in both hands is 18. How many marbles does she have in each hand?

16. Tom bound 3 candy bars and 6 pieces of bubble gum. The candy bars cost 5 cents each and the gum cost 2 cents each. How much did he spend on gum?

17. Jim is building a house with boards. He needs 18 boards. He hauls 6 boards in his wagon at a time. How many trips must he make to haul the boards?

18. Jerry has 23 books and 12 magazines in his library at home. The school has 10 times as many books as Jerry has. How many books does the school have?

19. Bill can fire 7 shots into a target before he must reload his gun. When he examines his target he finds 112 holes. If all of his shots went into the target, how many times did he reload his gun?

20. Tommy delivered groceries for 6 weeks. He received 15 cents for each box of groceries he delivered. How much did he earn for delivering 83 boxes of groceries?

21. Sue had many pets. She had 2 spotted kittens, 3 gray kittens, a brown puppy, and a pretty green bird. Sue had how many kittens?

22. John had 25 baseball cards. Bill had 16 baseball cards. John gave 9 of his cards to Bill. How many baseball cards does John have left?

23. Rick has 9 red marbles, 6 blue marbels, and 13 green marbels. John gave him 5 red marbels and 3 yellow marbles. How many red marbels does Rick now have?

24. Nine girls and 11 boys came to the party. Three boys wore ties. How many boys did not wear ties?

25. Bill sold 23 tickets for the school play. Jane sold 28, Ann sold 15, and Betty sold 18. How many tickets did the girls sell?

26. Pete had 16 apples and 14 pears. He gave away 9 apples. How many apples did he have left?

27. The team scored 13 points in the first game, 17 in the second, and 23 in the last. How many were scored in the first two games?

28. John's uncle sent him 325 European stamps. 275 of the stamps were usable but John used only 135. How many can he still use?

29. David got 3 picture books, 4 story books, and 5 puzzles for his birthday. How many books did he get for his birthday?

30. Betty and Sara took 8 sandwiches and 12 cookies on their picnic. They ate all but 2 sandwiches and 3 cookies. How many sandwiches did they eat?

APPENDIX B

PUT

The Principles and Understanding Test (PUT) is a thirty-five item test of arithmetical concepts.

The procedure for scoring is to tabulate the frequency of correct responses. One point is awarded for each correct response. No norms are provided. Statistical data, which include item analysis, reliability and validity coefficients and means and standard deviations for mentally handicapped and average children are contained in the body of this report.

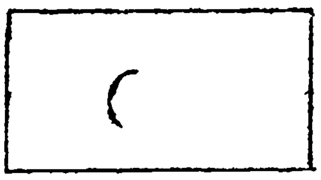
ARITHMETIC CONCEPTS TEST

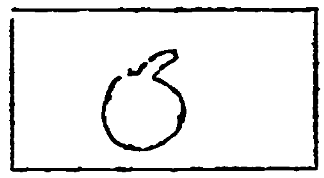
NAME ____________________

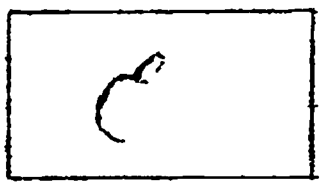
SCHOOL ____________________

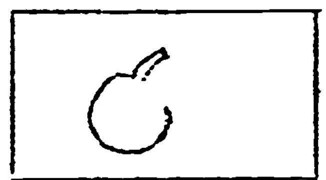
Directions: Draw a circle around the correct answer

(1) Here is one apple  Find $\frac{1}{2}$ of the apple.

A. 

C. 

B. 

D. 

(2) To <u>combine</u> four 7's quickly you

A. Add 4 + 7

C. Subtract 7 – 4

B. Multiply 4 X 7

D. Divide $4\overline{)\,7}$

(3) Find the one equal to 7.

A. 3 + 4

C. 4 + 4

B. 5 + 3

D. 3 + 2

(4) The number is 10. One part of it is 4. Which is the best way to find the other part?

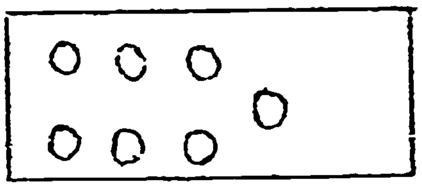
A. 10 – 4 = 6

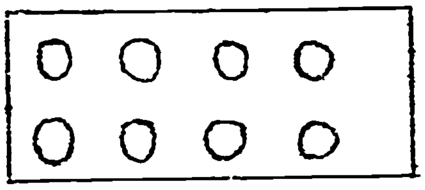
C. 10 + 4 = 14

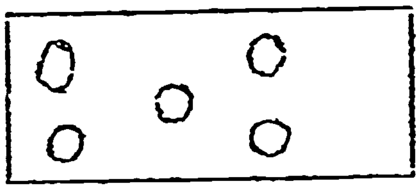
B. 10 – 5 = 5

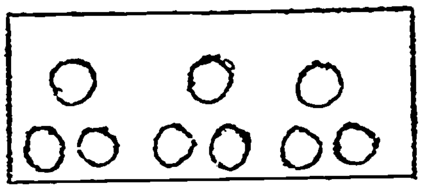
D. 10 – 3 = 7

(5) Find the 9 group

A. 

B. 

C. 

D. 

(6) Find the missing number. 6, 7, ____, 9, 10.

A. 5

B. 10

C. 8

D. 11

(7) When one is <u>added</u> to any number the answer is

A. the same number

B. the next number

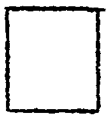
C. 1

D. 0

(8) Find the square

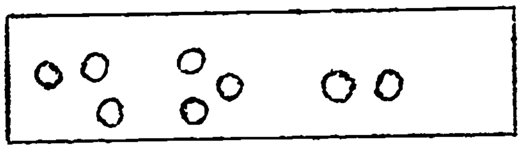
A. 

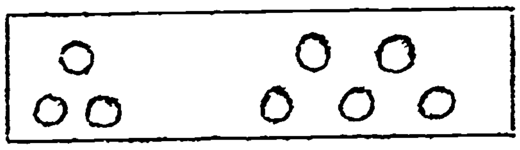
B. 

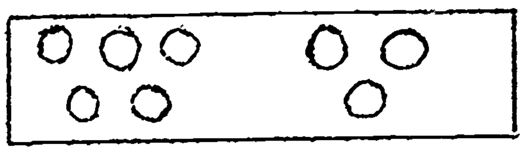
C. 

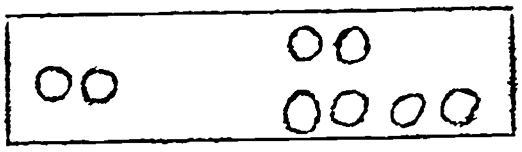
D. 

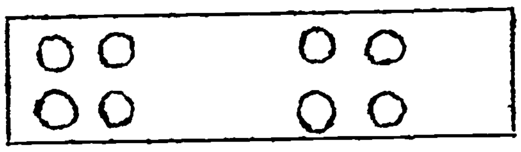
(9) <u>Separate</u> into <u>equal</u> groups



A. 

B. 

C. 

D. 

(10) Which one of these equals 1?

A. $\frac{1}{3} + \frac{1}{3}$

C. $\frac{1}{2} + \frac{1}{2}$

B. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

D. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

(11) 0 times a number equals

A. the number

C. 0

B. 1

D. you can't say what the number will be

(12) Zero (0) + a number equals

A. 0

C. 1

B. the number

D. one more than the number

(13) If you take away the <u>ones</u> from 19, what is left?

A. 0

C. 10

B. 1

D. 9

(14) You have two pies of equal size, If you divide one pie into 5 pieces and the other pie into 6 pieces, which pieces will be bigger?

A. from the pie with 5 pieces

C. they will be the same size

B. from the pie with 6 pieces

D. you can't say which will be bigger.

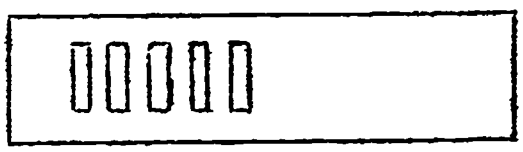
(15) If you divide an apple in half, the two pieces are

A. equal

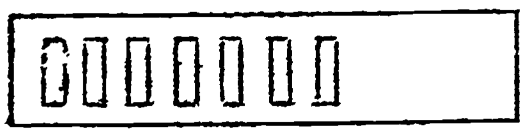
C. unequal

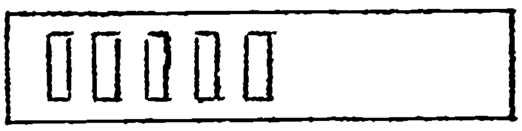
B. of different sizes

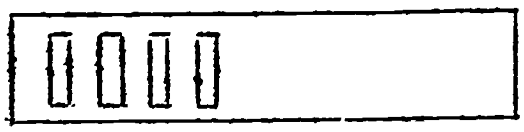
D. almost the same size

(16) Which group is the same as this one  ?

A. 

C. 

B. 

D. 

(17) The number which means <u>none</u> is

A. 2 C. 5

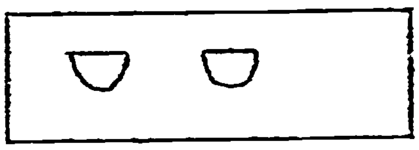
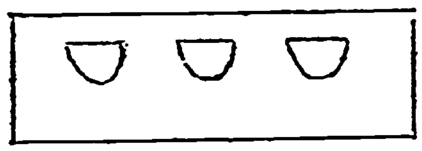
B. 0 D. 3

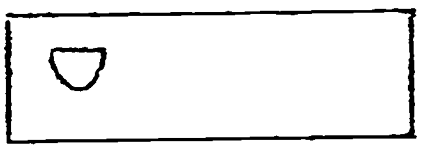
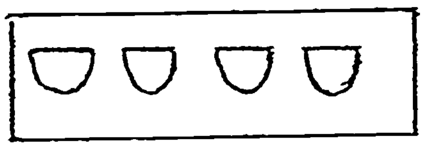
(18) $\frac{1}{4}$ says an object was divided into _____ equal parts

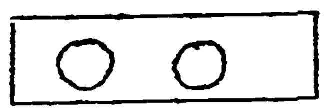
A. 5 C. 6

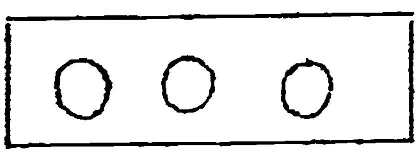
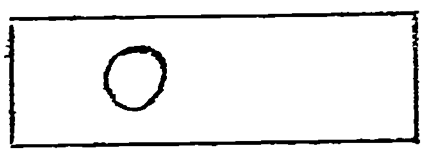
B. 9 D. 4

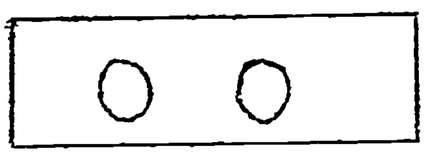
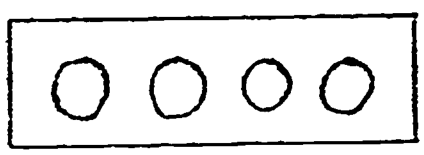
(19) We need 5 cups. We have  Which group would make 5?

A.  C. 

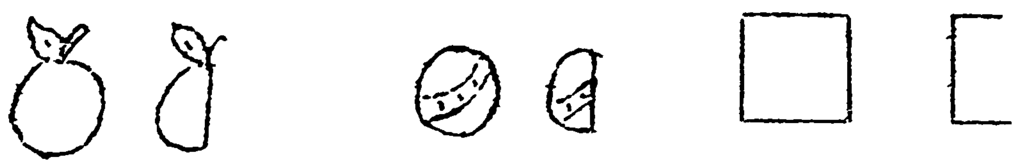
B.  D. 

(20) This is the group we have . How many must we add to make this group the same size as the group we have? 

A.  C. 

B.  D. 

(21) Halves of different things



A. are always the same size C. are almost always the same size

B. are always the same shape D. may be different in size

(22) To find out how many groups of 2 there are in the number 10, we

A. Add

B. Subtract

C. Divide

D. Multiply

(23) All the numbers 10, 14, 15, 18, 19 have

A. 5 tens

B. 2 tens

C. 0 tens

D. 1 ten

(24) <u>All</u> <u>the</u> <u>group</u> means

A. Everything

B. 1 part

C. most of the group

D. $\frac{1}{2}$ of the group

(25) Find the <u>fraction</u>

A. 2

B. $\frac{1}{4}$

C. 4

D. 15

(26) Find the group with 7

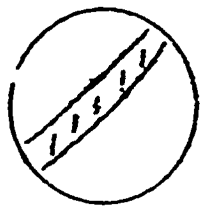
A. 0 0

B. 0 0 0 0 0 0

C. 0 0 0 0 0 0
0 0 0

D. 0 0 0 0 0

(27) Which is the biggest ball?

A. 

C. 

B. 

D. 

(28) Find the missing number 10, 20, ______, 40, 50

A. 15

C. 25

B. 60

D. 30

(29) Which one says "two times three?"

A. 2 X 3

C. 2 - 3

B. 2 + 3

D. $2\overline{)3}$

(30) How do you write "3 dollars and 5 cents?"

A. $ 3.5

C. $ .35

B. $ 3.50

D. $ 3.05

(31) How many quarters are there in 1 dollar?

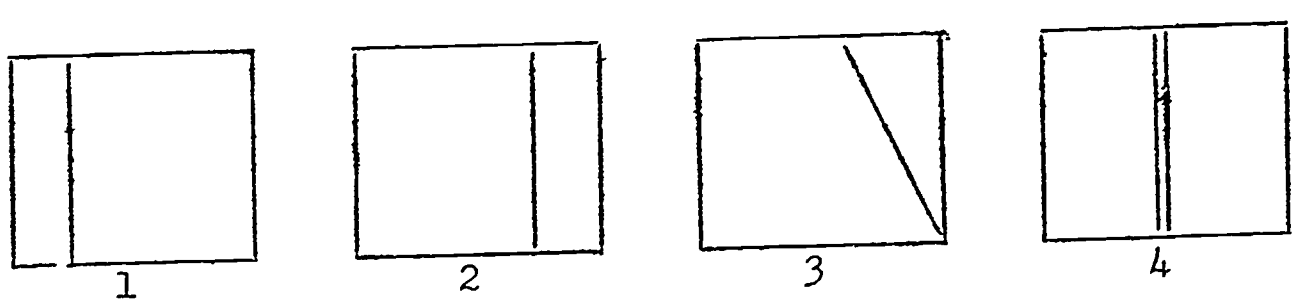
A. 1

C. 10

B. 4

D. 2

(32) Which one is divided in half ($\frac{1}{2}$) ?



A. 1

C. 3

B. 2

D. 4

(33) $\boxed{\text{H I J K L}}$

Which letter is <u>second</u>?

A. H   C. J

B. I   D. K

(34) In the number $ 3.06, which number tells how many dollars?

A. 6   C. 0

B. 3   D. none

APPENDIX C

THE ITEM ANALYSIS SERVICE

The Item Analysis Service

To aid in evaluating the power of each test question to distinguish between poorly prepared and knowledgeable students several summary figures and statistical indices are caluclated and then printed on the item analysis list.

A. Response Option Summaries:

Next to the item number are four row headings: UPPER 27, LOWER 27, TOTAL N, MEAN. (see Figure 3) The numbers following the first heading show how many students among those in the upper 27% of the class (on the test) choose each one of the answer options headed A B C D E. Similarly, the numbers following the second row heading show the choices made by the lower 27% of the class.

Following TOTAL N will be a series of numbers which indicates the classwide choice among the options including the "correct" asterisked response. And, finally, MEAN heads a five number series comprised of the average score on the entire test obtained by the groups of students who chose each option.

The two numbers under the column heading WRONG show how many students chose some option other than the keyed "correct" one and also what their average score on the total test was. The two numbers under RIGHT serve a similar purpose for those students who did choose the correct response.

Under the column heading R + W (Right plus Wrong) is shown the total number of students and the average test score for those who marked any response option for the question. By checking the N indicated here, you can tell quickly how many students omitted the question. If items near the end of the test were not attempted by a significantly large number of students, you might wish to allow a longer working time for future test administrations or reduce the number of items by eliminating some of the faulty ones (see DISCRIMINATION, below).

B. Difficulty Index:

Just as its name suggests, the difficulty index shows that proportion of the class which answered the question incorrectly ($1 - \frac{\text{RIGHT}}{\text{WRONG}}$). If all students were fortunate enough to choose the keyed option a—0.00 index will be printed. Please ignore the minus sign. It is meaningless in such cases. An index of 1.00 means that no one chose the correct option, hence the item did not contribute at all to the goal of discriminating more able from less able students. Everyone failed it, indiscriminantly.

Professional test contructors aim for an average difficulty of .50. They work very hard to improve any items which go outside the approximate limits of say .30 and .70. But remember, they have huge numbers of students with which to work and that makes their task somewhat less complex than is that of the typical university teacher. You ought not to gnash your teeth over a few, say one quarter of the items, which stray under .25 and over .75 difficulty.

C. Indices of Discrimination:

The capacity of a test question to categorize students into able and less able groups is a most desirable attribute.

Three indices of discrimination are provided in the item analysis service. They are:

Discrimination:

Under the column heading DISCRIMN will be found a number indicating the difference between two proportions (Figure 3). Values can range from -1.00 to +1.00. The two ratios are the proportion of students in the upper 27% ($D = \frac{RU}{Tu} - \frac{Rl}{Tl}$). A negative discrimination means simply that the lower portion of the class was relatively more successful in answering the question correctly than the upper portion was. In such cases, it is advisable to check the answer key to see if inadvertently an incorrect response option was designated "correct." If the key is correct, re-read the question looking for possible ambiguities in language.

Bi Serial Correlation:

The next item analysis index, under R-BIS, is a coefficient of correlation called the biserial correlation. It is an estimate of the well-known Pearson product moment correlation. This number shows the extent to which scores on the total test would tend to covary with an assumed normally distributed set of scores for the item in question. Of course, there is not a normal distribution of scores available for the item. Only two values are present, "Right" (1) and "Wrong" (0). If you have some good evidence which would lead you to accept the hypothesis that the amount of ignorance or knowledge concerning the test item content was normally distributed (that is, most students having an average amount of knowledge and the rest having smaller and greater amounts of knowledge in fewer and fewer numbers) then it would be appropriate to use the biserial correlation as an index of discriminating power.

The formula used for computing R-BIS is $r_b = \frac{M_p - M_q}{\sigma x}\left(\frac{pq}{Y_n}\right)$ where $\bar{X}p$= average score on total test for those who were correct, $\bar{X}q$ = average score on total test for the "wrong" students, p = proportion of class who were "correct," q = proportion "incorrect,"   = standard deviation of all scores on test, y = the ordinate of the normal probability curve at either the point p or q.

Please notice that R-BIS takes the whole class into consideration and not merely the upper and lower 27%. Because of this and the assumption of normality, there will be some strange differences between DISCRIMN and R-BIS occasionally. More often than not, R-BIS will be larger than DISCRIMN.

Point Biserial Correlation

The last index of discrimination provided is found under the heading R-P.BIS. It also is an estimate of the Pearson product moment correlation. Point biserial does not make an assumption of normality for the distribution of item scores and, therefore, will always be a more conservative index than is the biserial coefficient. In fact, when the proportions of pass and fail students for an item are equal, biserial will be almost exactly 25% larger than point biserial. As proportionality diverges from equality, the percent difference between the two indices becomes ever greater.

The formula for point biserial is $r_{pb} = \frac{M_p - M_q \sqrt{pq}}{\sigma x}$ all symbols here having the same meaning as in the formula for biserial correlation.