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Mathematical model building for educational planning in this country has been heavily influenced by the USOE DYNAMOD Model, a computerized Markov-type or input-output model. However, the input-output method is structurally inadequate to reflect the true behavior of the educational system. To introduce some elements of decision making into the model, some investigators have attempted to apply optimal control theory. Application of optimal control theory involves the addition of control variables, which are constrained in their values and thus reflect political or policy limits, to a general mathematical model consisting of equations defining the interdependence of sets of variables characterizing the educational system. Control theory models are theoretically attractive planning devices because they allow for the specification of a system's initial states and certain desired targets while providing for the selection of a policy which achieves these targets at a minimum cost while satisfying existing constraints. Although barriers to practical implementation exist, this approach promises to aid in revealing the values of a systems approach to social and economic problems. (TT)

AN OVERVIEW OF OPTIMAL CONTROL THEORY APPLIED TO EDUCATIONAL PLANNING

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An Overview of Optimal Control Theory Applied to Educational Planning

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The following paragraph, taken from a review by the economist Mark Blaug [3] provides an appropriate opening for this paper, and I quote as follows:

> "Educational planners are nowadays expected to have a firm grasp of educational psychology, to know a good deal about the sociology and economics of education, and to be perfectly at home with quantitative data and statistical results. But even all this is no longer enough: they will soon have to add mathematics to their other accomplishments, if Mathematical Models in Educational Planning [2] is anything to go by. Optimal control theory, systems analysis, operations research, mathematical programming, indeed, mathematical models of all kinds, are clearly becoming part of the tool-knit of educational planners. Perhaps not tomorrow, but certainly the day after tomorrow! One can imagine the outcry that this expansion of intellectual efforts will engender in educational circles, hints of which are to be found in an essay from that same book by Michigan State University Professor Paul Dressel."

A useful way to begin a discussion of mathematical model-building in educational planning is to distinguish between micro-models and macro-models. Micro-models refer to the educational system itself: stocks of students, teachers, equipment and buildings within the different sectors of the system, the flows between these parts, and, of course, the flows into and out of the entire system. Then we have macro-models which refer to the relationships between the educational systems and the economic system or the society in general. Reference [2] provides a detailed account of these kinds of models.

Here, in the United States, a lot of research has gone into the development of the Office of Education DYNAMOD Model, a computerized, Markov-type demographic flow model which calculates the number of students and teachers in 140 distinct population groups over selected intervals of time.

As a matter of fact, the proceedings in [11] will provide the best single description of the activities that fall under the heading of "Operations Analysis". This Symposium, arranged through the USOE, brought together for the first time systems analysts and school administrators in an attempt to bridge the so-called "communications gap". In the words of David Stoller, and I quote:

> "In a very real sense, the Operations Analysis program is a typical staff function of the Office of Education intended to supply background information for all the decision-makers throughout the educational system who manage and create policy for education."

A close analysis of the Markov-type or so-called input-output models will reveal that there is little to adjust or manipulate in order to achieve better results. In other words, the input-output method is structurally inadequate to reflect the true behaviour of the educational system. In order to introduce some elements of decision-making into the model, some investigators have attempted to apply optimal control theory.

The optimal control problem can be formulated in the following manner: there exists a mathematical model, that is, there exists a set of equations, expressed as differential or difference equations, which defines the interdependence of the sets of complementary variables that characterize the educational system being studied. These variables are referred to as state variables and control variables. A state variable, for example, might depict the total number of students enrolled in the sophomore year in engineering at U.C.L.A. A control variable could denote the number of assistantships or the amount of money made available in order to steer or attract additional students or out-of-state students. In addition, these variables can be, and usually are, constrained in their values. For example, a constraint on the state variable implies that only so many places are available in that course; a constraint on the control variable implies limited funds for financial aid to the out-of-state students and/or limited funds for research, or the effect of some comparable political decision.

The optimization problem, then, is to determine the values of the control variables such that some objective function, appropriately chosen to represent the costs of attaining that objective, is minimized over the time period of interest.

Note again the elements that enter into the formulation of an optimal control problem: (1) state variable, (2) control variable, (3) admissible state space, (4) admissible control set, (5) dynamic system state equations, (6) objective function, (7) time interval under consideration.

Historically, the first application of control theory to educational planning can be found in Alper's paper [2]; his contribution was mainly philosophical. The only other application can be found also in [2]; it is a highly mathematical exercise which treats the problem of determining the proportion of a citizen's lifetime which should be spent in school in order that the state's income be maximized. The most recent work is that by Köenig, Zemach, et al [9] at Michigan State University who have attempted to set up a resource allocation model of an institution of higher education by using state space concepts.

It is obvious that control theory models are <u>theoretically</u> attractive planning devices because they allow for the specification of certain initial states of a system, certain desired targets, and provide for the selection of a policy which achieves these targets at a minimum cost while satisfying existing constraints.

Formal application of control theory to management and planning for a university is particularly challenging. In a model of an institution of higher education, optimal control theory could address itself to such questions as: (1) given the input control vectors, determine within a given set of admissible policy parameters the set or sets of production policies (i.e. allocations on limited input resources) that result in minimum cost of education, or (2) given a particular set of production policies, determine within an admissible set the time sequence of inputs and controls that will produce a given change in the output at minimum total cost over a period of N years.

While the structure of the optimal control formulation is attractive, practical implementation is complicated for the following reasons:

1. the problem of optimizing a socio-economic system is difficult because, as yet, we cannot define to our satisfaction a suitable objective function or performance criterion.

2. the parameters that enter in the description of an educational process, say enrollment, are measured in years, and not in hours or days as in the operation of an industrial process.

3. furthermore, the educational process is a non-stationary process; this means that past responses to previous stimuli may be completely misleading

as to what present-day responses would be to identical stimuli. (e.g. student riots)

4. educational decisions made by the central decision-maker can be ignored or effectively modified by line personnel.

5. data requirements to implement the model are enormous. The system of equations describing an educational process requires a data base set up in terms of "flow data", and not "stock data", which is the form of present educational statistics.

6. Another problem for the educational planner who wishes to undertake model building is the generally negative reaction to such activities from school people who regard these approaches to administrative decisions as evidence of the dehumanization of educational systems.

Given these limitations and problems, where do we go from here?

We must recognize that any new concept or approach to the study of the problems of education must be introduced in stages. Initially, we should be concerned with the "spirit" of the approach. Such mathematical techniques, even if not immediately useful in solving some of our most pressing problems, do point up deficiencies in our present operations--for example, one of the questions to be considered is what kind of data do we need to collect and how? (We find that this demands a shift from purely empirical observations of gross numbers to flow data or individualized data concepts.) Secondly, the "spirit" of the new approach will force us to consider our systems from a different angle and on a different scale (i.e. a total systems approach). Finally, we are led to consider the appropriateness of the techniques to particular problems.

An understanding of control theory can lead to a judicious use of techniques developed in mathematics and engineering, and reveal the advantages and limitations of the systems approach to social and economic problems.

Finally, I would like to close with a warning, also quoted from Mark Blaug's review:

> "To ask mathematicians to produce blood from stones is to misunderstand the function of mathematics. The most we can expect from mathematical models is that known relationships will be restated more precisely and their consequence deduced more rigorously, as a result of which more questions will come to be asked."

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Educational Program Management Center
The Ohio State University
College of Education
1945 North High Street
Columbus, Ohio 43210

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NOTE:

References to earlier works of above authors will be contained within references listed at back of individual papers.