DOCUMENT RESUME

ED 028 545 EC 002 943

By-Vogelli, Bruce Ramon

Soviet Secondary Schools for the Mathematically Talented.

National Council of Teachers of Mathematics, Inc., Washington, D.C.

Spons Agency-Bowling Green State Univ., Ohio.; Kappa Delta Phi, West Lafayette, Ind.

Pub Date 68

Note-106p.

Available from-National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington, D.C. 20036

EDRS Price MF-$0.50 HC Not Available from EDRS.

Descriptors-Administration, Computer Oriented Programs, Course Content, Curriculum, *Educational Programs, *Exceptional Child Education, *Gifted, *Mathematics, Program Development, Program Evaluation, Scheduling, Secondary Grades, Special Schools, Talented Students, Teacher Education

Identifiers-Soviet Union

Efforts of Soviet educators to identify and develop mathematics talent through the establishment of secondary schools offering specialization in computer programing and mathematics are reported. The following programs are described: organization and results of the experimental class which began in September 1959 to offer a specialization in mathematics and computer programing; the goals, curriculum, and special features of current computer-programer secondary schools; teacher education programs for computer programing; mathematics and physics boarding schools; and part-time study programs in mathematics. Results of the development of these special mathematics programs indicated are that they are worthwhile although their immediate contribution is negligible compared to their potential. Appendixes list syllabi used in the programs. (SP)

BRUCE RAMON VOGELI

Soviet Secondary Schools for the Mathematically Talented

Soviet Secondary Schools for the Mathematically Talented

Soviet Secondary Schools for the Mathematically Talented

BRUCE RAMON VOGELI

Department of Mathematical Education
Teachers College, Columbia University

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
1201 Sixteenth Street, N.W., Washington, D.C. 20036

Library of Congress Catalog Card Number: 68–30961

Printed in the United States of America

Preface

SOVIET programs for mathematically talented pupils recently have attracted the attention of American mathematicians and mathematics educators. The success of the annual Soviet mathematics Olympiads has stimulated development of similar contests in the United States. The work of the Soviet school mathematics circles or clubs has been of sufficient interest to American teachers to encourage translation and publication of a number of enrichment booklets prepared by Soviet mathematicians for club use. The latest and most ambitious effort by Soviet mathematics educators to identify and develop young mathematical talent is the establishment of secondary schools offering specialization in mathematics. It is the programs of these mathematics secondary schools with which this report is concerned.

The report is based not only upon syllabi and documents pertaining to such schools but also upon the writer's experiences as an observer, lecturer, and teacher in mathematics secondary schools and related teacher-education facilities in Moscow and Leningrad. It is *not* a comparison of Soviet mathematics secondary schools and American counterparts, because counterparts do not exist. With the exception of an initial review of historical aspects of Russian mathematics education, the report does not attempt to describe the usual Soviet school mathematics curriculum. Occasional references to, and comparisons with, Soviet and American secondary school mathematics curricula are made for emphasis alone.

The first chapter is a historical survey of Russian and Soviet school programs in mathematics from 1701 to the present with emphasis on conditions leading to the establishment of special secondary schools for mathematically talented pupils. The second chapter outlines the organization and development of the initial computer-programmer experiment in one Moscow school. The goals, the curriculum, and the special features of computer-programmer secondary schools as established by a Ministry of Education commission are the subjects of Chapter III. Chapter IV discusses implications of the 1964 and 1966 school reorganization edicts for secondary schools with specialization in computer programming. Teacher education for computer-programmer schools is outlined in Chapter V, while Chapter VI is devoted to the curricula of two schools of the second type—the Moscow Mathematics Boarding School and the Novosibirsk Mathematics-Physics Boarding School. Part-time study programs in mathematics are discussed in Chapter VII. The final chapter summarizes the content of the preceding ones and emphasizes the potential impact of mathematics secondary schools upon Soviet science and technology. Translations of relevant Soviet school documents appear as appendixes to the text.

The travel and research necessary for preparation of the report were supported jointly by Kappa Delta Pi and Bowling Green University. Without the financial support of these agencies and the cooperation and assistance of Soviet colleagues in the Ministry of Education, the Academy of Pedagogical Sciences, and the Lenin Pedagogical Institute, the report could not have been prepared.

Contents

Tables

I

Introduction

Mathematics Education in Imperial Russia

SCHOOLS with strong mathematics curricula are not new to Russian education. The Moscow School of Mathematical and Navigational Sciences, founded in 1701 by Peter the Great, was unique in Europe. Its curriculum included arithmetic, geometry, the elements of trigonometry, astronomy, and mathematical geography.[1] The Petersburg gymnasium (1726) and the Smolny Institute for "young ladies of noble birth" (1764) offered instruction in arithmetic and elementary geometry.[2]

School regulations enacted by the tsarist government in 1804 established two types of primary schools: one-year parish schools and two-year district schools. The parish school curriculum included reading, writing, and the four basic arithmetic operations. Pupils continued the study of arithmetic in district schools, and, in addition, began the study of geometry.[3] During the first half of the nineteenth century, people's primary schools with three- and five-year programs replaced parish schools. The program of district schools was lengthened to six years. Mathematics instruction in both people's primary and district schools, however, was limited to arithmetic and the elements of geometry.

Of the sixty-one gymnasia in operation in the major cities of imperial Russia in 1850, sixteen were "modern gymnasia" with strong mathematics curricula.[5] Mathematics instruction in modern gymnasia totaled six to seven and one-half hours per week and made up 20 percent of the instructional program.[6] Unfortunately, the school law of 1871 replaced the curricula of the modern gymnasia with a classical curriculum that allocated three to four hours weekly (or approximately 13 percent of the total program) to instruction in mathematics.[7] Forty percent of this classical curriculum was ancient languages.[8] *Academic Plan for Mathematics in Gymnasia* and *A Sample Syllabus in Mathematics*, published by the imperial government in 1890, remained in force in practically their initial forms until 1917. Table 1 illustrates the distribution of class time by mathematical discipline and grade specified in the 1890 plan. Textbooks that reflected the recommendations contained in the sample

TABLE 1[9]

TIME DISTRIBUTION AMONG MATHEMATICAL DISCIPLINES FOR PREREVOLUTIONARY RUSSIAN GYMNASIA

Subject	Hours per Week According to Grade							
	1	2	3	4	5	6	7	8
Arithmetic	4	4	1	0	0	0	0	0
Algebra	0	0	2	2	2	2	1.5	3
Geometry	0	0	0	2	2	2	0	0
Trigonometry	0	0	0	0	0	0	1.5	0
Total	4	4	3	4	4	4	3	3

syllabus were prepared and published in considerable variety. A. P. Kiselev's manuals were particularly popular and soon became the principal determinants of Russian school mathematics programs.[10]

Methodological practice in Russian schools progressed more slowly than curricular reform. Although the 1804 school regulations specified that "the teacher should strive rather to form and cultivate the pupil's intellect than to charge and exercise his memory," [11] few teachers took this advice seriously. Arithmetic instruction in early schools was formal to the extreme. The writings of Pestalozzi (first Russian editions 1806–7) had only minor effect upon the teaching of arithmetic in Russian schools. Gymnasia syllabi paid lip service to "the development of the mental capacities of pupils." [12] However, in practice, mathematics instruction differed little from instruction in the linguistic forms and syntax of Greek and Latin.

A few progressive Russian educators spoke out against formalism in mathematics instruction. In 1865, K. D. Ushinski stressed the role of visualization and intuition in the teaching of mathematics and the need to consider ways in which the mathematics program could be "related to life." [13] From 1880 onward, various proposals for the reorganization of the school algebra course around the concept of function were championed unsuccessfully by progressive Russian educators. "If all mathematics is essentially the study of functions," V. S. Sheremetevski wrote in 1896, "it is clear that even the elementary course should be centered around the basic concept of functional dependence." [14] Later, S. I.

Shokhor-Trotskii advocated the use of carefully chosen problems to introduce new ideas; the integration of arithmetic, algebra, and geometry; and the use of a laboratory together with inductive teaching methods in mathematics—but with little success.[15]

Early Soviet School Mathematics Programs

With the coming of the October Revolution, the structure of education in Russia was altered significantly. Private schools were unilaterally eliminated. In November 1917 the various types of Russian public schools in existence were transformed into "unified labor schools" with nine-year courses of study. The elementary (*nachal'naia*) school consisted of Grades 1–4 and the secondary school of Grades 5–9. Children entered Grade 1 at age eight and completed Grade 9 at age seventeen.

Not only was the organization of Russian schools altered; curricula and teaching practices were reformed as well. According to Marx's dictates, the curriculum of the unified labor schools was "polytechnic"—that is, it was intimately concerned with the principles of the chief branches of science, industry, and agriculture. In mathematics, polytechnism implied emphasis upon practical applications of mathematics. Methodological reforms enacted by Communist educators included substitution of study "projects" or "complexes" determined, planned, and executed by self-governing student "brigades" for the traditional Russian *uroyli* (lecture-lesson). Course work in all areas was motivated by the project selected. Textbooks were all but abolished in favor of learning-by-doing procedures.

The first postrevolutionary mathematics syllabus for unified labor schools (1921) not only emphasized polytechnic principles but also included many mathematical innovations.[16] The arithmetic course was shortened from the six-year program of prerevolutionary schools to four years. Plane and solid geometry were partially integrated. The study of trigonometric functions as functions was added to the trigonometry syllabus. In Grades 6 and 7, the algebra course was assigned to relatively modern objectives of (1) extension of the concept of number, (2) development of the concept of function, and (3) familiarity with algorithms for solving equations and inequalities. The syllabus for Grades 8 and 9 included elements of analytic geometry and an introduction to the calculus (limits, derivatives, integrals, series, and simple differential equations). In practice, the 1921 syllabus proved too difficult—too extensive—for general use. A less demanding syllabus—"the syllabus minimum"—replaced the 1921 document in most schools.

Methodologically, the 1921 syllabus emphasized the value of creative activity in the teaching of mathematics, the need to broaden pupils' mathematical background, and the desirability of relating mathematics to life. The syllabus urged that problems be used not only to illustrate application of mathematical theory to practical situations but also to motivate new theoretical topics. Under the influence of these methodological doctrines and the "brigade-project" organization, the mathematics program gradually lost much of its theoretical content. Pupils studied mathematical "recipes" applicable in specific practical situations, often without consideration of their theoretical bases.

By 1931, the lack of knowledge displayed by unified-labor-school graduates had so alarmed the revolutionary government that on September 5 of that year, the Central Committee of the Bolshevik Communist Party decreed a complete reorganization of the school system. On August 25, 1932, a second decree called for immediate correction of "errors" in the academic program of the unified labor school.[17] The mathematical weakness of graduates was among the principal arguments voiced by labor school critics. Complex and project methods were abandoned immediately. School time once again was divided among distinct disciplines. All schools were required to provide six hours of mathematics instruction each week in all grades. The course of study was expanded to ten years and the matriculation age lowered to seven years. New syllabi in mathematics that defined *exactly* the content and sequence of each subject (arithmetic, algebra, geometry, and trigonometry) were prepared and implemented. Analytic geometry, the calculus, and other advanced topics were dropped from the mathematics programs of the few schools that had succeeded in using the 1921 syllabus. Prerevolutionary mathematics texts were resurrected, revised, and made the official standard.

The syllabi and texts issued in response to the 1931–32 decrees remained in force from 1934 until 1955 essentially unchanged. The Soviet school mathematics program of this period differed little in form and method from the programs of prerevolutionary gymnasia. Table 2 illustrates the division of time among mathematical disciplines during this period. Soviet pupils received approximately two thousand hours of instruction in mathematics, representing 20 percent of the total ten-year school program. Pupils in prerevolutionary gymnasia had approximately one thousand hours of instruction in mathematics, constituting 13 percent of the total eight-year program.

The polytechnic emphasis of the unified-labor-school curriculum was not found in the academically oriented ten-year school. In order to meet the practical demands of the "Five-Year Plans," special secondary schools

TABLE 2

TIME DISTRIBUTION AMONG MATHEMATICAL DISCIPLINES IN THE SOVIET TEN-YEAR SCHOOL [18]

Subject	Hours per Week According to Grade									
	1	2	3	4	5	6	7	8	9	10
Arithmetic	6	7	6	7	7	2	0	0	0	0
Algebra	0	0	0	0	0	3	3	3	2	1.5
Geometry	0	0	0	0	0	2	2	2.5	2	2.5
Trigonometry	0	0	0	0	0	0	0	0	1.5	2
Review	0	0	0	0	0	0	1	.5	.5	1
Total	6	7	6	7	7	7	6	6	6	6

and technicums with curricula designed to serve specific trades and industries were established. The mathematics curricula of these schools invariably were less extensive than that of the ten-year school.

Recent Revisions in the Soviet School Mathematics Program

Signs of dissatisfaction with the program of the ten-year school appeared following World War II. Criticism was frequently leveled at the academic orientation of the curriculum. In 1952, the Nineteenth Congress of the Communist Party of the Soviet Union issued the following directive, designed to strengthen the practical significance of the ten-year-school curriculum through reintroduction of polytechnic instruction.

> Toward the goal of further raising the socialist educational significance of the school of general education and toward insuring conditions for the free selection of professions by secondary-school graduates, we must set about realizing polytechnic instruction in the secondary school and carrying through measures necessary for the transition to universal polytechnic instruction.[19]

In 1953, the Ministry of Education of the Russian Republic, the largest and most influential of the fifteen republics that make up the Soviet Union, announced that polytechnic instruction in mathematics could not be "fully realized" within the existing syllabus.[20] In response, work was begun on the preparation of a new syllabus and a number of new mathematics textbooks. The syllabus was disappointingly similar to its predecessor. Several new texts, however, were substantial improvements upon their prerevolutionary counterparts.[21]

Progress toward the goal of universal polytechnic instruction was slower than anticipated. In a series of speeches during the fall of 1958,

N. S. Khrushchev proposed organizational changes in the Soviet school system designed to expedite the return to polytechnic education by rendering the Soviet school program "more practical, more closely connected to life." In response to Khrushchev's proposals, the academically oriented ten-year school was replaced by a compulsory eight-year program in which instruction emphasized the polytechnic aspects of mathematics. The eight-year school was followed by three years of non-compulsory schooling combined with work experience.

Pupils entered the eight-year school at age seven. The matriculation age for three-year schools varied with the individual and with the type of school, but in no case was it less than fifteen.

Only persons who completed successfully both the eight-year school and the three-year program were permitted to apply for admission to institutions of higher learning. Those with a record of actual professional employment were admitted first.

Students enrolled in the eight-year program studied only arithmetic, elementary algebra, plane geometry, and the rudiments of metric solid geometry and trigonometry.[23] Table 3 illustrates the distribution of time among mathematical subjects within the eight-year curriculum. In all, 1,615 class hours (or about 22 percent of the total curriculum) were devoted to the study of mathematics.

TABLE 3

TIME DISTRIBUTION AMONG MATHEMATICAL DISCIPLINES IN THE SOVIET EIGHT-YEAR SCHOOL

Subject	Hours per Week According to Grade							
	1	2	3	4	5	6	7	8
Arithmetic	6	6	6	6	6	4/0*	0	0
Algebra	0	0	0	0	0	0/4	4	2/3
Geometry (with Plane, Solid, and Trigonometric Topics)	0	0	0	0	0	2	2	3/2
Total	6	6	6	6	6	6	6	5

* Numerals separated by a diagonal indicate hours per week in the first and second semesters.

Pupils completing the eight-year school could choose to enter industry or agriculture, to continue their education by attending full-time three-year schools offering instruction in both academic and polytechnic subjects, or to combine labor and learning by attending part-time evening or seasonal schools of general education.

Pupils enrolled in full-time polytechnic schools devoted four hours each week to mathematics. Part-time pupils studied mathematics only three hours each week. Pupils completing both the eight-year school and the full-time three-year program spent a maximum of 2,067 hours in the study of mathematics, representing approximately 18 percent of all hours in all subjects. Graduates of the eight-year school and the part-time three-year school received 2,020 hours of instruction in mathematics.

The time spent in the study of mathematics in full-time three-year schools was but four hours per week—two hours per week less than the eight-year-school standard. This reduction was necessary to provide time for production practice and/or work experience. Although the number of class hours devoted to mathematics instruction in the new curriculum remained high, the proportion of time devoted to mathematics during the crucial secondary years was reduced from 22 percent in the eight-year school to 11 percent in the three-year school. This reduction is particularly serious in view of the fact that most three-year pupils expected to pursue a higher education.

Special Schools for Mathematically Talented Pupils

Members of the Soviet mathematical community expressed immediate concern that the proposed protraction and dilution of the secondary school mathematics program could adversely affect the supply of young mathematicians. In a letter to *Pravda,* Academicians Y. B. Zeldovich and A. D. Sakharov urged that courses in theoretical mathematics be offered in "special schools for the brightest potential scientists and mathematicians among youngsters of high school age."[24] Academician A. I. Markushevich tactfully asserted that under the proposed reorganization, "it still *may* be possible to prepare pupils for careers in mathematics"[25] (*italics mine*).

Perhaps the most significant and certainly the most surprising result of the concern for the future of Soviet mathematics education expressed by Zeldovich and others was that the Ministry of Education initiated limited experiments with special schools and classes for mathematically talented pupils. Secondary schools for Soviet pupils with musical and artistic talents have existed for many years. The reluctance of Soviet officials to recognize individual differences in intellectual capacity and the related fear of reestablishing a class structure (based this time upon educational rather than economic disparities) prevented concurrent development of schools for pupils with special academic talents. Apparently these barriers were insufficient to withstand assault from the Soviet mathematical community. Indeed, Khrushchev himself suggested that such schools might be "necessary" under the new orientation.[26]

Three types of experimental schools with mathematical specialization were authorized initially:

1. Secondary day schools with specialization in computer programming
2. Secondary boarding schools with specialization in mathematics and physics
3. Part-time schools offering supplementary instruction in mathematics only

Schools of the first type were organized within the framework of the 1958 law concerning polytechnic education. Their polytechnic goal was the preparation of computer clerks and programmers for Soviet industry.[27] Only later was official recognition given to the goal of providing an intensified mathematical education for talented pupils with interest in mathematics.

Schools of the second type were organized under the auspices of universities and branches of the Soviet Academy of Sciences and hence were relatively free from polytechnic influence. The principal goals of these schools were and still are the recognition and development of young mathematical talent.[28]

Schools of the third type were supplements to, rather than replacements for, ordinary secondary schools. Their purpose was a dual one: the provision of opportunities for pupils with interests in mathematics and the preparation of mathematics specialists for the Soviet economy.[29]

Schools of the first type have expanded rapidly. In Moscow alone, twenty-one secondary schools with specialization in computer programming were in operation at the beginning of the 1966/67 academic year.[30] Since an urban school enrolls 200 to 250 pupils, perhaps 5,000 pupils in the Moscow district are receiving an intensified mathematical education. Soviet educators estimate that as many as one hundred such schools are in operation in the whole of the Soviet Union. Thus, up to 25,000 young Soviet mathematicians and scientists could be in training at present. Schools of the second type have been established in Moscow, Leningrad, Kiev, the science city, Novosibirsk, and T'bilisi. Enrollment of the Moscow school is approximately 360,[31] and Novosibirsk enrollment in 1963 was 318.[32] Schools of the third type also are sponsored by institutions and hence are in operation only in selected communities with higher educational facilities.

The programs of mathematics boarding schools and of part-time schools vary greatly from school to school and from year to year within a given school. Syllabi, academic plans, and other documents pertaining to these schools usually are unavailable through official channels. In contrast, syllabi and academic plans for secondary schools with specialization in computer programming have been prepared and implemented in all these schools. For this reason, a major portion of this study is devoted to the more extensive and better coordinated secondary schools with specialization in computer programming.

REFERENCES

1. *Pedagogicheskii Slovar'* (Moscow: IAPN, 1960), II, 697.
2. *Ibid.*, p. 357.
3. A. D. Semushin, "Prepodavaniia Matematika v Srednei Shkole SSSR" (typewritten report prepared for UNESCO, 1963).
4. *Ibid.*
5. *Ibid.*
6. *Ibid.*
7. *Pedagogicheskii Slovar'*, I, 262.
8. *Ibid.*
9. Semushin, *op. cit.*
10. See B. R. Vogeli, *The Mathematics Program of the Soviet Secondary School—Its Status and Innovations* (Ann Arbor, Mich.: University Microfilms, 1959) for a discussion of the Kiselev texts.
11. Semushin, *op. cit.*
12. *Ibid.*
13. *Ibid.*
14. *Ibid.*
15. *Pedagogicheskii Slovar'*, II, 714.
16. Data concerning 1921 syllabus from R. S. Cherkasov, "The Development of the Teaching of Mathematics in Soviet Schools," trans. B. R. Vogeli, *The Mathematics Teacher*, XVIII (December 1965), 715–19.
17. V. M. Bradis, *Metodika Prepodavaniia Matematiki v Srednei Shkole* (Moscow: Uchpedgiz, 1954), p. 38.
18. E. N. Medynskii, *Narodnoe Obrazovanie v SSSR* (Moscow: IAPN, 1952), pp. 74, 86.
19. *Directivy XXX C'ezda KPSS* (Moscow: Gosudarstvennoe Izdatel'stvo Politicheskoi Literatury, 1952), pp. 27–28.
20. P. A. Larichev, "Prepodavaniia Matematika v Srednei Shkole v Sviazi s Vvedeniem Novoi Programmy," *Matematika v Shkole* (July-August 1957), pp. 16–23.
21. See B. R. Vogeli, *op. cit.*, for a critique of changes in Soviet texts.
22. "Novyi Etap v Razvitii Sovietskoi Shkoly," *Matematika v Shkole* (November-December 1958), pp. 1–3.
23. See B. R. Vogeli, "Recent Revisions in Soviet Mathematics Education," *School Science and Mathematics* (June 1960), pp. 425–38, for a detailed discussion of the post-1958 program.
24. *Pravda*, December 6, 1958.

25. Statement by A. I. Markushevich, interview by A. J. Lohwater at University of Michigan, November 1958.

26. "Novyi Etap," *loc. cit.*

27. *Programmy po Matematicheskim Distziplinam dlia Srednikh Shkol s Proizvodstvennym Obucheniem po Professii "Vychislitel'-Programmist"* (Moscow: IAPN, 1963).

28. S. I. Literat, "O Prepodavanii Matematiki v Fiziko-Matematicheskoi Shkole-internate," *Matematika v Shkole* (November-December 1963), p. 446.

29. S. V. Smirnov *et al.*, "O Novyka Formakh Vneklassnoi Raboty," *Matematika v Shkole* (March-April 1960). See also articles in *Matematika v Shkole,* No. 5, 1962; Nos. 1, 3, 1963; and No. 1, 1964.

30. Statement by S. I. Shvartzburd, interview by B. R. Vogeli, April 1964.

31. B. R. Vainberg, E. G. Glagoleva, and N. Kn. Rozov, "Zaochnaia Matematicheskaia Shkola pri Moskovskom Universitets," *Matematika v Shkole* (September-October 1964), pp. 59–64.

32. Literat, *op. cit.*

II

The Initial Experiment

Organization of the First Class

AUTHORIZATION for experimental secondary school classes with specialization in mathematics and computer programming was given by the Ministry of Education late in the summer of 1959. Work was begun in September of that year with a single ninth-grade class in School 425 in the Pervomaiski district of Moscow.[1] Recruitment of pupils for the initial class was undertaken hurriedly after the beginning of the school year. Within a period of three or four days, twenty-nine pupils from the district had been enrolled.[2] In the recruitment process, some resistance was encountered from parents not convinced that it was possible or desirable to prepare computer-programmers at the secondary school level.[3] Others were concerned that a highly specialized secondary education would hinder admission to institutions of higher education.

Because of the delay in securing Ministry approval, prerequisites for admission to the initial class were minimal. Any interested pupil from the Pervomaiski district eligible for admission to a secondary school was admitted to the experimental class if he received a grade of 4 or 5 (5 is the highest grade awarded) in mathematics in Grade 8.[4]

Experimental Plan and Syllabus

Syllabi, an academic plan, and teaching materials for the experimental program were developed by S. I. Shvartzburd, a mathematics teacher at School 425, and V. G. Ashkinuze, a scientific worker in the mathematics section of the Academy of Pedagogical Sciences.[5] E. A. Morozova and I. V. Girsanov of Moscow State University and A. A. Abramov, K. V. Kim, and E. N. Kapmazina of the Central Computing Center of the Soviet Academy of Science cooperated with Shvartzburd and Ashkinuze in the preparation of programming materials.[6]

The academic plan for computer-programmer trainees at School 425 was patterned after the second variant of the official 1959/60 academic plan for urban secondary schools with production training.[7] This variation permitted increased concentration on production training in the specialty during the tenth and eleventh year rather than equal emphasis in Grades 9, 10, and 11.

The initial plan for the experimental group is given in Table 4. Com-

TABLE 4

ACADEMIC PLAN FOR THE INITIAL COMPUTER-PROGRAMMER CLASS AT MOSCOW SCHOOL 425 [8]

Subject	Hours per Week According to Grade			Total Hours in All Three Grades	
	Grade 9	Grade 10	Grade 11	Per Week	Per Week
Academic Subject					
Literature	3	3	3	9	339
Mathematics	7	7	4	18	686
History	3	3	4	10	374
Constitution of the USSR	0	0	2	2	70
Economic Geography	3	2/0*	0	4	149
Physics	5	5	3	13	495
Astronomy	0	1	0	1	39
Chemistry	2	3	2	7	265
Biology	0	2	0	2	78
Drawing	2	0	0	2	78
Foreign Language	3	2	3	8	300
Physical Education	2	2	2	6	226
Total Hours in Academic Subjects	30	29	23	72	3,099
Computing Subject					
Approximate Computation	4	4/0	0	6	234
Theory of Mathematical Machines and Programming	0	4	4	8	312
Practical Work on Small Machines	2	2/0	0	3	117
Practical Work on Large Machines	0	0	9	9	315
Total Hours in Computing Subjects	6	6	13	26	978
Total Hours in All Subjects	36	36	36	98	4,071
Faculty Activity	3	2	2	0	304

* Numerals separated by a diagonal indicate hours per week in the first and second semesters.

parison with the academic plan for ordinary secondary schools indicates that, over the three-year period, computer-programmer trainees would receive 1,332 more class hours of instruction in the mathematical-physical disciplines than the usual secondary school pupil. If the time allocated "faculty activity" (a seminar or special course determined by the school faculty) is considered, the excess is even greater. Of the total excess, 978 class hours are in computer operation, 234 hours in special mathematics courses, and 120 hours in physics and electronics.[9]

Like the mathematics syllabus for ordinary secondary schools, the experimental syllabus for School 425 was subdivided according to subject. Subsyllabi were prepared for the courses (1) algebra and elementary functions, (2) geometry, and (3) trigonometry. Time allocations in syllabi, both for old and new topics, were intentionally generous due to "the lack of appropriate textbooks and the newness of the program." [10] In its initial form, the mathematics curriculum for computer-programmer trainees was significantly broader and deeper than the usual program. Topics contained in experimental syllabi but *not* found in the 1959/60 mathematics syllabus for ordinary secondary schools include (1) mathematical induction, (2) the generalized concept of function, (3) the derivative, (4) the indefinite and the definite integral, (5) series, (6) elements of the theory of probability, (7) elements of analytic geometry, (8) geometric transformations, (9) linear algebra and matrices, (10) nomography, (11) numerical methods, (12) differential equations, and (13) all work with machines and programming.[11] The experimental syllabi prepared by Shvartzburd and Ashkinuze are given as Appendix C.

Results of the Initial Experiment

Eleven of the twenty-nine pupils enrolled succeeded in completing the year "without receiving 3's in mathematics." [12] A grade of 3, although officially satisfactory, is not considered indicative of either ability or application. Twenty-eight of the original twenty-nine continued computer-programmer specialization in Grade 10. In addition, a new group was chosen to begin the program in Grade 9. This time, selection was based, not on willingness and marks alone, but also upon the pupil's interest in mathematics as a career, his participation in extracurricular activities in science and mathematics (circles, Olympiads, etc.), and the recommendation of former teachers.[13] Each applicant was interviewed by Shvartzburd or another member of the experimental staff.

As the program developed, so did its reputation. Other schools expressed interest in becoming secondary schools with specialization in computer programming. Two other Moscow schools joined School 425 in the computer-programmer experiment in 1960.[14] The programs of these schools differed slightly from the 425 program. In one school, Grade 9 pupils studied four mathematical subjects rather than three as at School 425: (1) mathematics (general course), (2) analysis, (3) numerical methods, and (4) a problem practicum. At the third school, pupils in all three grades enrolled simultaneously for (1) mathematics (general course), (2) elements of higher mathematics, (3) mathematical machines, (4) programming, (5) practical work with machines.[15]

Support for continuation and extension of special programs in mathematics was given publicly by leading scientists.[16] Encouraged by the success of early experiments and the favorable response from the academic community, the Minister of Education of the RSFSR, E. I. Afanasenko, appointed a commission to consider further the "question of mathematics secondary schools."[17] Afanasenko named as commission chairman A. I. Markushevich, professor of mathematics at Moscow State University and vice-president of the Academy of Pedagogical Sciences. Work was begun on the preparation of official directives governing the training of computer programmers. In the interim, existing schools with specialization in computer programming operated according to revised versions of the Shvartzburd-Ashkinuze plan and syllabi.[18] A conference in November 1962, attended by 160 "scientific workers" concerned with mathematics secondary schools, served as a sounding board for commission proposals.[19] With the publication and approval of the recommendations of the Ministry commission in the spring of 1963, Soviet secondary schools with specialization in computer programming emerged officially as an important segment of Soviet mathematics education.

REFERENCES

1. S. I. Shvartzburd, *Matematicheskaia Spetzializatziia Uchashchikhsia Srednei Shkoly* (Moscow: IAPN, 1963), p. 12.

2. S. I. Shvartzburd, "Iz Opyta Raboty s Uchashchimisia ikh Klassa, Ovladevaiushchimi Spetzial'nost'iu Laborantov Programmistov," *Matematika v Shkole* (September-October 1960), p. 11.

3. Shvartzburd, *Matematicheskaia Spetzializatziia, loc. cit.*

4. Shvartzburd, "Iz Opyta," *loc. cit.*

5. *Ibid.*

6. Shvartzburd, *Matematicheskaia Spetzializatziia, loc. cit.*

7. Appendix A.

8. Shvartzburd, "Iz Opyta," p. 10.

9. Shvartzburd, *Matematicheskaia Spetzializatziia*, p. 24.

10. Shvartzburd, "Iz Opyta," p. 11.

11. Appendix B.

12. Shvartzburd, "Iz Opyta," p. 15.

13. *Ibid.*

14. S. I. Shvartzburd, "O Podgotovke Programmistov v Srednei Obshcheobrazovatel'noi Politekhnicheskoi Shkole," *Matematika v Shkole* (March-April 1961), p. 22.

15. *Ibid.*

16. *Pravda,* December 31, 1960.

17. Memorandum dated May 26, 1961, cited in Shvartzburd, *Matematicheskaia Spetzializatziia,* p. 13.

18. Appendix D.

19. V. G. Ashkinuze and S. I. Shvartzburd, "Shkoly s Matematicheskoi Spetzializatziei," *Matematika v Shkole* (March-April 1963), p. 81.

III

The Recommendations of the Ministry Commission

The Tasks of the Commission

THE commission on mathematics secondary schools appointed by the Ministry of Education was to prepare and secure official approval of three documents: (1) a statement of qualification characteristics, (2) a mathematics syllabus, and (3) an academic plan.

Qualification characteristics for secondary school trainees in a given vocation are those technical skills and abilities that are desirable or essential for employment in the vocation.[1] There are the specific goals that determine the scope and content of any secondary training program.

The Soviet school syllabus in a particular subject defines "the content and scope of the knowledge, skills, and habits to be imparted to pupils in the course of the entire school program in this subject as well as in the limits of each grade."[2] Syllabi are official state documents and hence are compulsory for all schools and pupils.

The academic plan for Soviet schools prescribes the school calendar *in toto*, defines the sequence of the study, and gives the number of hours per week to be allotted to study in each subject-matter area. The plan is mandatory for every pupil enrolled in the type of school for which it was prepared.[3]

Qualification Characteristics

Qualification characteristics for the vocation "computer-programmer" are given at three levels:[4] (1) computer—first level, (2) computer programmer—second level, and (3) computer programmer—third level. Level 3 requirements were most stringent. Each level was further divided into (1) characteristic work, (2) required knowledge, and (3) samples of work.

Qualification characteristics for computer—first level are minimal: [5]

Characteristic work.—Simple calculation using a desk calculator. Construction of graphs. Work on the key punch for a specific computing machine with program given.

Required knowledge.—Rules for handling a desk calculator. How to carry out operations on desk calculators including operations with negative numbers. Rules for rounding off the results of operations. Rules for rounding off when using tables. The use of tables where linear interpolation is required. Extracting square root using a desk calculator.

Samples of work.—Tabulation according to given programs with the use of tables. Construction of simple programs for calculations by means of formulas. Sketching graphs in a given scale. Checking results of calculation by means of tests for smoothness (first differences, etc.).

Level 2 characteristics are significantly more demanding: [6]

Characteristic work.—Preparing programs of intermediate difficulty for one of the [standard] electronic computers according to a given algorithm. Breakdown of programs into simple blocks. Preparation of programs for work on a computer and carrying out calculations on a computer. Carrying out control computation on a desk calculator.

Required knowledge.—The basic physical and technical operating principles of an automatic electronic computer. Rules of safety for work on external equipment and on the machine itself. The control console of one of the standard automatic electronic computers and methods of working on it. External equipment of the machine and methods of working on it.

The command system of one automatic high-speed electronic computer, the basic code of the machine, the library of standard programs and the rules for its use. The output circuit of the machine, representation of numbers in the output circuit, translation of numbers from one numeration system to another.

Basic method of constructing programs utilizing single and double cycles. Methods of terminating programs containing cycles.

Basic numerical methods for solving important mathematical problems: the solution of systems of linear equations, the solution of algebraic and transcendental equations, calculation of integrals, integration of differential equations and systems of differential equations.

Samples of work.—Independent construction of programs (with the use of available standard programs) for problems of the following types:

solution of systems of linear equations (given the method), calculation of integrals (with given steps), etc.

Control solution of the same problems with the aid of a desk calculator.

Qualifications for "computer programmer—third level" include all requirements for Level 2 and, in addition, familiarity with automatic control and programming techniques.[7]

The majority of pupils completing the three-year curriculum are expected to qualify as computer programmers—second level. A limited number of superior pupils may be awarded Level 3 classifications, while Level 1 is reserved for those whose work is questionable.

Qualification at one of the three levels is awarded by a committee of teachers and computer specialists on the basis of an oral examination. This examination includes a critique of the programming problems solved by the pupils during the "practical experience" portion of the program, questions relating to the methods utilized, and the theory of machine computation in general.[8]

The first qualifying examination was given in April 1962 to the twenty-eight pupils remaining in the original experimental class at School 425.[9] Seven of the twenty-eight examinees received a rating of "computer programmer—third level" while the remaining twenty-one were classified as "computer programmer—second level." In 1963, the second group to take the qualification examination compiled a record of twenty-six Level 3 classifications and eight Level 2's.[10]

The Mathematics Syllabus

The syllabus prepared by the Ministry commission differed significantly in purpose and content from both the original and the revised Shvartzburd-Ashkinuze documents. While Shvartzburd and Ashkinuze justified the establishment of mathematics secondary schools on strictly polytechnic grounds—the preparation of computer programmers for the Soviet economy—the commission syllabus asserted that "the preparation of computer programmers in secondary school not only is directed toward the solution of the economic problem of preparing personnel with intermediate qualifications for operating computing and control equipment, but also provides opportunities for pupils with interest in mathematics to obtain an intensified mathematical education." [11] Official recognition of the need for secondary schools with programs suitable for pupils with specific academic interests represented a departure from former policy that has far-reaching implications for Soviet education in general.

Like the Shvartzburd-Ashkinuze documents, the commission's syllabus divided the mathematics program into three major parts: (1) the general course in mathematics, (2) the special mathematical disciplines, and (3) practical work with computing machines. Activities in each subdivision could be under way simultaneously. Thus, pupils in Grade 11 were enrolled in both general and special mathematical subjects and at the same time gained practical experience with a computer. The first two major divisions—the general course and the special disciplines—were subdivided again by individual courses. Subsyllabi were included for each course.

The General Course in Mathematics

The general course in mathematics occupies a central position in the preparation of computer programmers. Indeed, the topics included in the general course are of value "not only in the vocation of computer programmer but in any mathematical profession." [12] According to the commission's report, the general course in mathematics includes material necessary for (*a*) mastery of the profession of computer programmer, (*b*) continuation of a mathematical education at an institution of higher education, and (*c*) development of the pupil's mathematical outlook.[13]

Three subjects or subcourses constitute the general course: (1) algebra and elementary functions, (2) mathematical analysis, and (3) geometry. Trigonometry as a distinct subject was eliminated by the commission. Pupils enroll in all three subcourses at the same time. Each of the subcourses includes "a certain amount of traditional material" and in addition topics from "higher mathematics." [14] The individual syllabi are so constructed that subcourses are not divided into two parts—one elementary, the other part higher—but rather each course is a unified, continuous whole. The teacher is charged to support this unity by pointing out applications of elementary topics in advanced work and by underscoring implications of advanced topics for previous materials and methods.

Table 5 of this report is an abridged translation of the commission's subsyllabus for the course Algebra and Elementary Functions arranged by grade and topics within grade. The numeral following each topic entry represents the number of class hours recommended by the commission for this topic. The report indicates, however, that these are recommendations only and are not obligatory, as in the usual syllabus.

A significant portion of the first topic of the course Algebra and Elementary Functions is intended to be review. Review is included to provide for possible differences in the backgrounds of pupils from various eight-year schools.[16] Teachers are instructed to review the principal topics of the eight-year program at this time and simultaneously to strengthen and develop previously learned skills and concepts.[17] Questions relating to the solution of equations, systems of equations, and inequalities of the first and second degree are considered along with the review of linear and quadratic functions.

The second topic occupies a "central position in the extension of the pupil's concept of exponents."[18] Here pupils receive a careful and de-

TABLE 5

ALGEBRA AND ELEMENTARY FUNCTIONS SUBSYLLABUS FOR SCHOOLS WITH SPECIALIZATION IN COMPUTER PROGRAMMING[15]

GRADE 9

(Five Hours per Week, First Semester; Three Hours per Week, Second Semester)

Topic	Total Hours
1. Linear and quadratic functions; inequalities	15
2. Powers with rational exponents	26
3. Trigonometric functions of an arbitrary angle	15
4. Relations between trigonometric functions	13
5. Trigonometric reduction formulas and their corollaries	10
6. Trigonometric addition theorems and their corollaries	25
7. Exponential and logarithmic functions	36
8. Review	11
Total in all topics	151

GRADE 10

(Two Hours per Week, First Semester; Three Hours per Week, Second Semester)

Topic	Total Hours
9. Linear algebra and elements of linear programming	50
10. Complex numbers	14
11. Polynomials; equations of higher degree	22
12. Review	14
Total in all topics	100

GRADE 11

(Two Hours per Week)

Topic	Total Hours
13. Transcendental equations	18
14. Combinations and elements of the theory of probability	22
15. Review of the course Algebra and Elementary Functions and some questions from the mathematical analysis course	30
Total in all topics	70

tailed explanation of how the concept of exponent is extended to include rational numbers together with an understanding of the need for proof of theorems governing operations with powers having rational exponents. Many exercises involving identity transformations on rational and irrational expressions are recommended. Along with extension of exponent, pupils study power functions of the type $y = x^q$, where q is a rational number.

The next four topics in the revised curriculum constitute a basic treatment of trigonometry. Emphasis is upon the trigonometric functions as functions. A vector apparatus developed in the geometry course is used throughout. The trigonometric functions are discussed again in several courses at various levels. The topics "complex numbers" and "transcendental equations," which appear later in this same course, include work with trigonometric functions. The trigonometric functions are mentioned in certain sections of the analysis courses and in connection with the solution of triangles in geometry.[19] The special mathematical disciplines also include references to trigonometric function. The commission strongly recommended that trigonometry be distributed and integrated as described and that each trigonometric implication be grasped as an opportunity for "deepening [the pupils'] knowledge of trigonometric functions."[20]

The concluding topic of Grade 9, "exponential and logarithmic functions," is both important and extensive. Here the number e is introduced geometrically as that value of the base a for which the graph of the exponential function $y = a^x$ has at its point of intersection with the y-axis a tangent that intersects the x-axis in an angle of 45°.[21] In the study of logarithms as in trigonometry, the emphasis is upon logarithmic functions. The computational aspects of logarithms are relegated to the special mathematical discipline of numerical methods.

"Linear algebra and elements of linear programming" is the first topic in Grade 10. Questions connected with determinants and matrices are considered in close connection with the solution of linear equations. The concept of an n-dimensional vector is introduced and is followed by operations with vectors and discussion of linear independence for a set of vectors. Each general concept is illustrated and clarified by work in two and three dimensions. Examples of problems in linear programming are introduced as an application of the basic concepts of linear algebra.

The syllabus suggests that pupils be acquainted with the general problem of extension of the number concept before introduction of complex numbers in Grade 10.[22] An ordered-pair approach is recommended but not required. Complex numbers are applied in the final topic of Grade 10 in connection with the fundamental theorem of algebra. An interesting innovation at this level is a required proof of the uniqueness of a polynomial of degree n, given $n + 1$ of its values.

Grade 11 begins with the topic "transcendental equations." Work with transcendental equations includes discussion of approximate as well as exact methods of solution. In particular, graphical and iterative methods are emphasized.[23] This topic appears to serve as a useful review, since it includes many concepts and methods from previous work.

The final topic of the three-year sequence in algebra and elementary functions consists principally of an introduction to probability theory. The sample-space approach is mandatory.[24] Both independent and dependent events are considered, and addition and multiplication of probabilities are defined. The course closes with a rather extensive review in preparation for the qualifying and graduation examinations.

An abridged translation of the subsyllabus for the mathematical analysis portion of the general course in mathematics appears in Table 6. The analysis course is of two rather than three years' duration. Thus,

TABLE 6

MATHEMATICAL ANALYSIS SUBSYLLABUS FOR SCHOOLS WITH SPECIALIZATION IN COMPUTER PROGRAMMING[25]

GRADE 9

(Four Hours per Week, First Semester; Three Hours per Week, Second Semester)

Topic	Total Hours
1. Measures of segments; real numbers	8
2. Numerical sequences and their limits	26
3. The general concept of function	24
4. The derivative and its application	64
5. Review	12
Total in all topics	134

GRADE 10

(Three Hours per Week, First Semester; Two Hours per Week, Second Semester)

Topic	Total Hours
6. The indefinite integral	20
7. The definite integral	25
8. Simple differential equations	12
9. Series	26
10. Review	12
Total in all topics	95

in the general course in mathematics, pupils study algebra and elementary functions, mathematical analysis, and geometry in Grades 9 and 10. In Grade 11, the general course reduces to algebra and elementary functions and geometry alone.

Although the analysis course cannot be considered the equal of a first-rate American or Soviet university course in the calculus, it is of substantial quality. The course begins with an introduction to the concept of real number that is based upon geometric considerations. Emphasis is given to the correspondence between the real numbers and the points of a line. The fact that this correspondence is one-to-one is taken as a postulate.

The study of the system of real numbers leads naturally to the study of sequences of real numbers and limits. The syllabus suggests that physical analogies be used to "clarify the limit concept." [26] Proofs of limit theorems based upon the epsilon-delta definition are recommended but not required. Convergence of bounded monotonic sequences is proved, using "infinite decimal" as the definition of "real number." [27] Iterative methods are given special attention.

A general concept of function is developed in the next section. Monotonicity, boundedness, and the limit of a function are considered and applied in the further investigation of familiar functions. The relation of the graph of a function and its image under various linear transformations is considered.

The second half of the Grade 9 analysis course is devoted to differential calculus. According to the syllabus "excessive passion for the formal-foundational aspects is inappropriate in this course." [28] "Proofs" of theorems are given using geometric or mechanical analogies. The syllabus states, however, that "the formal-logical level of the course must be sufficiently high to provide a firm foundation for work in physics and for further work in mathematics." [29]

Integral calculus is introduced in Grade 10. The existence of the definite integral for any continuous function is accepted without proof. The use of the definite integral in expressing areas and volumes is given special attention.[30]

Work with differential equations in Grade 10 is introductory only. Topics discussed include (1) types of differential equations, (2) general and particular solutions, and (3) initial and boundary conditions. Again special emphasis is given to geometric and physical significance. Pupils are expected to learn enough about differential equations so that "in case it is necessary, they can study corresponding supplementary questions about differential equations independently." [31]

The final topic of the mathematical analysis course, "Series," includes (1) the concept of convergence and tests for convergence, (2) power series, (3) expansion of functions in power series, (4) Taylor's series, and (5) the binomial formula. Teachers are encouraged "to underscore the role of series as computational aids."[32]

Table 7 indicates that the geometry course in computer-programming schools, like the course Algebra and Elementary Functions, spans all three years of the general course in mathematics. Geometry begins in

TABLE 7

GEOMETRY SUBSYLLABUS FOR SCHOOLS WITH SPECIALIZATION IN COMPUTER PROGRAMMING[33]

GRADE 9

(Two Hours per Week, First Semester; Four Hours per Week, Second Semester)

Topic	Total Hours
1. Vectors	14
2. Analytic geometry	40
3. Metric relations in a triangle and solution of triangles	20
4. Geometric transformations	36
5. Review	12
Total in all topics	122

GRADE 10

(Two Hours per Week)

Topic	Total Hours
6. Axioms for solid geometry	3
7. Parallelism in space	14
8. Perpendicularity in space	25
9. Analytic geometry in three dimensions	12
10. Polyhedra	24
11. Review	6
Total in all topics	78 [*sic*]

GRADE 11

(Two Hours per Week)

Topic	Total Hours
12. Solids of revolution	20
13. Elements of mathematical logic	20
14. Review and problem solving	30
Total in all topics	70

Grade 9 with the introduction of vectors. Discussion is limited at first to two dimensions, with scalar multiplication defined as the product of the modulus of one vector and the projections of the other.[34] Later a similar definition is employed in connection with the topic "linear algebra" in the course Algebra and Elementary Functions. Vector methods are used repeatedly in the geometry course as well as in the special mathematical disciplines.

The topic "geometric transformations" is an interesting and valuable one. It is intended to serve as a unifying concept in the Soviet geometry program. The emphasis is upon different types of geometric transformations (symmetric, parallel, homothetic, and similarity transformations) and invariants under each. Although no connection with the section on linear algebra in Grade 10 is indicated in the syllabus, certainly one topic reinforces the other.[35]

Major portions of the geometry program in Grades 10 and 11, although designated as solid geometry, differ substantially in content from older Soviet and American courses. The concept of geometric transformation is extended to three dimensions, as are the vector and analytic methods introduced in Grade 9. Linear independence of sets of vectors in space is an obligatory subtopic. Area and volume formulas for polyhedra and other solids are developed by means of integration. The geometry course serves to "unite the general course in mathematics" through the use of concepts and methods from both algebra and analysis.[36]

The introduction to mathematical logic that concludes the course in geometry is relatively brief. Conjunction, disjunction, and negation are discussed and related to set operations. The nature of axioms and undefined terms and the origin of mathematical concepts in general are obligatory subtopics. Groups, rings, and fields are discussed briefly. The course closes with an explanation of the group-theoretic approach to geometry.

The Special Mathematical Disciplines

The special mathematical disciplines include material of a "special character, necessary for the successful mastery by the pupil of the vocation computer programmer and further practical work in this vocation."[37] Two subcourses are classified as "special disciplines"—Numerical Methods and Mathematical Machines and Programming.

Table 8 includes abridged translations of the content of both courses.

The course Numerical Methods begins in the second semester of Grade 9. In this grade, topics relating to manual calculation are empha-

TABLE 8

SYLLABUS FOR THE SPECIAL MATHEMATICAL DISCIPLINES FOR SCHOOLS WITH SPECIALIZATION IN COMPUTER PROGRAMMING [38]

Numerical Methods	Mathematical Machines and Programming
GRADE 9	
2 hours per week, second semester 44 hours altogether	
1. Calculation according to a given form (24)	
2. Using logarithms for calculation (20)	
GRADE 10	
3 hours per week, first semester 2 hours per week, second semester 95 hours altogether	4 hours per week, first semester 68 hours altogether
3. Numerical solution of equations (23)	1. Electronic digital computing machines (4)
4. Systems of equations (20)	2. Arithmetic bases of programming (10)
5. Interpolation and extrapolation (20)	3. Basic introduction to programming (36)
6. Numerical integration (14)	4. Translation of programs into machine language (26)
7. Numerical solution of differential equations (10)	5. Organization of programs (12)
8. Review (8)	
GRADE 11	
	4 hours per week, second semester 88 hours altogether
	6. Standard programs Automatic programming (26)
	7. Control methods (11)
	8. General characteristics of mathematical machines (24)
	9. Review (8)

sized. Rules for approximate computation, iterative methods of determining the zeros of functions, the use of tables, and calculations with logarithms are discussed. Where possible, work in the numerical methods course is coordinated with the pupils' practical work with computing machines.

The work in the tenth grade includes more varied numerical methods for both manual and machine use. Solution of equations by the methods of Lanczos and Lin and of linear systems by the Gauss-Seidel method are discussed. Euler's method is used to obtain approximate solutions of differential equations and systems of such equations. In each case some attention is paid to the problem of programming these methods for the computing equipment at hand.[39]

Because of variations in computing equipment available to various groups, the content of the programming course as described in the commission's syllabus is quite flexible. In practice, all or a majority of the work specified will be with a particular type or model of computer—that available to the class. Thus pupils completing the course probably would have relatively limited operational facility. Principles and procedures common to many machines are stressed, but actual operational competence is demanded on only one machine.

Practical Work on Computing Machines

An extensive course of practical instruction and practice in the use of computing equipment accompanies the general course in mathematics and the special mathematical disciplines. Pupils engage in practical work with desk calculators in Grades 9 and 10 and with a specific electronic computer in Grade 11.[40] At each level the practical work continues throughout the academic year.

During the first semester of Grade 9 and in the second semester of Grade 10, one two-hour practical-work period is provided every two weeks.[41] Practical work during the second semester of Grade 9 and all of Grade 10 is closely coordinated with the contents of the numerical methods course. This phase of the pupil's practical work is essentially a laboratory for the numerical methods course. At other times a two-hour laboratory period is scheduled each week.

Each pupil in Grade 11 is required to complete two or three major computational projects as a part of his practical work.[42] These projects must involve every operation necessary for solution of the problem on a computer: (1) preparing the program and readying it for introduction into the machine, (2) carrying out control calculation, (3) terminating the program, and (4) running the calculation on an available computer. The nature of the projects and the actual content of the pupils' practical work with electronic computers depend upon the equipment available and the interests of the computing-center personnel supervising the pupils' work.

The Academic Plan

The Ministry commission recommended that the academic plan for secondary schools with specialization in computer programming be based upon the first rather than the second variant of the standard academic plan for secondary schools with production practice.[43] Because of the specialization, the total time allocated by the standard plan for instruction in mathematics, physics, and practical training (approximately twenty hours per week) was redistributed by the commission among the general and special mathematical disciplines, physics and electronics, and laboratory work with computing machines. Table 9 illustrates this distribution.

TABLE 9

DISTRIBUTION OF TIME AMONG THE PHYSICAL-MATHEMATICAL DISCIPLINES FOR THE PROFESSION "COMPUTER PROGRAMMER"[44]

Subject	Grade 9		Grade 10		Grade 11		Total	
	Hours per Week	Hours per Year	Hours per Week	Hours per Year	Hours per Week	Hours per Year	Hours per Week	Hours per Year
Algebra and Elementary Functions	5/3*	151	2/3	100	2	70	8.5	321
Mathematical Analysis	4/3	134	3/2	95	0	0	6	229
Geometry	2/4	122	2	78	2	70	7	270
Numerical Methods	0/2	44	3/2	95	0	0	3.5	139
Mathematical Machines and Programming	0	0	0/4	88	4/0	68	4	156
Physics	7/5	229	4	156	3	105	13	490
Electronics and Radio Electronics	0	0	3	117	2/0	34	4	151
Practical Work with Small Machines	1/2	61	2/1	56	0	0	3	117
Practical Work with Large Machines	0	0	0	0	6/12	318	9	318
Total	19/19	741	19/21	785	19/19	665	58	2,191

* Numerals separated by a diagonal indicate hours per week in the first and second semesters.

Pupils enrolled in the computer-programmer training sequence receive a total of 2,191 hours of instruction and practice in their specialty over a three-year period. Of this total, 641 hours are in physics and electronics and 1,550 hours in mathematics. Pupils enrolled in ordinary Soviet secondary schools receive only 452 hours of instruction in mathematics and 382 hours in physics, a total of 834 class hours.[45] An American high school pupil completing four years of mathematics and one year of physics accumulates approximately 720 class hours in mathematics and 180 in physics.[46] Since the four-year American mathematics program includes at least 250 hours in elementary algebra and geometry that Soviet pupils receive *before* matriculating at a three-year school, the curriculum of Soviet secondary schools with specialization in computer programming contains approximately three and one-half times as much work in mathematics as the usual Soviet or American secondary school curricula.

Examination and Testing Procedures

In addition to preparing qualification characteristics, syllabi, and an academic plan for secondary schools with specialization in computer programming, the Ministry commission also commented on the problem of evaluation in such schools. For evaluation purposes, the three-year program was partitioned into two levels, the first including Grades 9 and 10, and the second, Grade 11. Two examinations were specified at the first level and a final qualifying examination at the conclusion of the second. First-level examinations at the termination of Grades 9 and 10 include material from both the general and special mathematical disciplines. Examinations emphasize "the most significant ideas" from the sequences rather than obscure concepts and unimportant techniques.[47]

These examinations, like most Soviet school examinations, are administered orally by "tickets." Each pupil draws an examination "ticket" from a collection of 20–22 tickets. There are two theoretical questions and one problem on each ticket. Usually these questions refer to quite different portions of the mathematics program. After a period of time (20–30 minutes), each pupil presents a 15-minute critique of his solutions to the questions of his ticket before a faculty committee. At least one supplementary question is directed at each pupil by the committee to test his knowledge further.[48] If a pupil fails an examination of this type, he may be reexamined a few days later, or a second examination may be administered the following fall.[49] Repeated failure can result in dismissal from the school.

Tickets for the Grade 9 examination normally include theoretical questions pertaining to the general course and computational exercises relating to the special disciplines. In Grade 10, each ticket includes one theoretical question from the general course and one from the special disciplines and, of course, an exercise.[50]

Theoretical questions included in all tickets are available to the pupils two months before the examination. As might be expected, regurgitation of memorized responses is a feature of Soviet oral examinations.[51] Problems, exercises, and supplementary questions are not released in advance, however.

The final qualifying examination given at the conclusion of Grade 11 is oral but does not involve selection of an examination ticket. Examinations at this level are restricted to the computing projects completed by each pupil as a part of the practical-work program. Each pupil is required to give an oral critique of his own project, including explanation of the practical significance of the problem, its mathematical content, the algorithms used in its solution, the basic programming principles applicable, and, of course, an explanation of the result obtained. The committee evaluates the pupil's knowledge upon the basis of his critique, the pupil's response to questions raised by committee members, and the recommendations of the pupil's laboratory supervisor. According to the commission's report, Level 3—the highest level of vocational classification—should be awarded only to pupils with a record of *independent* work of both mathematical and practical significance.[52]

Appendixes F and G are composed of questions and problems taken from examination tickets prepared for computer-programmer trainees.

REFERENCES

1. S. I. Shvartzburd, *Matematicheskaia Spetzializatziia Uchashchikhsia Srednei Shkoly* (Moscow: IAPN, 1963), p. 13.
2. S. E. Liapin, *Metodika Prepodavaniia Matematiki* (Leningrad: Uchpedgiz, 1955), I, 40.
3. E. N. Medynskii, *Narodnoe Obrazovanie v SSSR* (Moscow: IAPN, 1952), p. 70.
4. *Programmy po Matematicheskim Distziplinam dlia Srednikh Shkol s Proizvodstvennym Obucheniem po Professii "Vychislitel'-Programmist,"* pp. 3–5.
5. *Ibid.*
6. *Ibid.*
7. *Ibid.*
8. Shvartzburd, *op. cit.*, pp. 31–55.
9. *Ibid.*
10. *Ibid.*
11. *Programmy po Matematicheskim Distziplinam*, pp. 6–7.
12. *Ibid.*
13. *Ibid.*
14. *Ibid.*
15. *Ibid.*, p. 9.
16. *Ibid.*
17. *Ibid.*

18. *Ibid.*
19. *Ibid.*, pp. 23, 25, 26.
20. *Ibid.*, p. 10.
21. *Ibid.;* see also Shvartzburd, *op. cit.*, pp. 106–11.
22. *Programmy po Matematicheskin Distziplinam*, p. 11.
23. Shvartzburd, *op. cit.*, pp. 112–19.
24. *Programmy po Matematicheskim Distziplinam*, p. 22.
25. *Ibid.*, pp. 22–25.
26. *Ibid.*
27. Shvartzburd, *op. cit.*, pp. 62–83.
28. *Programmy po Matematicheskim Distziplinam*, p. 13.
29. *Ibid.*
30. See Shvartzburd, *op. cit.*, pp. 122–27, for a discussion of the approach to the derivative and integral used in schools with mathematical specialization.
31. *Programmy po Matematicheskim Distziplinam*, pp. 13–14.
32. *Ibid.*, p. 14.
33. *Ibid.*, pp. 25–28.
34. *Ibid.*, p. 14.
35. For examples of current Soviet emphasis in school geometry, see I. M. Iag'lom, *Geometric Transformations.* ("New Mathematical Library" [Syracuse, N.Y.: L. W. Singer Co., 1962]).
36. *Programmy po Matematicheskim Distziplinam*, p. 15.
37. *Ibid.*, p. 16.
38. *Ibid.*, pp. 29–34.
39. *Ibid.*, p. 16.
40. *Ibid.*, p. 17.
41. *Ibid.*
42. *Ibid.;* see also Shvartzburd, *op. cit.*, pp. 51–58, for description of pupil projects.
43. *Programmy po Matematicheskim Distziplinam*, p. 36; see also Appendix E.
44. *Programmy po Matematicheskim Distziplinam*, p. 35.
45. See Appendix E.
46. Based upon one Carnegie unit per 180 hours of instruction.
47. *Programmy po Matematicheskim Distziplinam*, p. 38.
48. *Ibid.*, pp. 38–42.
49. *Ibid.*, p. 39.
50. *Ibid.*
51. *Ibid.*
52. *Ibid.*, p. 40.

IV

The 1964 Reorganizations and Computer-Programmer Schools

A New Reform

DESPITE all-out efforts by Soviet educators to comply with demands made by Chairman Khrushchev in the fall of 1958 for secondary school programs that combined academic experience with production practice, trouble existed from the first. Administrative and logistic difficulties were manifold. Shop facilities were inadequate in most schools, and qualified teachers were in short supply. In practice, evening and seasonal schools proved more successful than day schools in achieving "the unity of theory and practice." Since part-time pupils usually were fully employed in industry or in agriculture, evening and seasonal schools could concentrate upon academic matters without depriving pupils of "polytechnic experiences."

In August 1964 the Minister of Education of the Russian Republic suggested that the period of secondary education could be reduced significantly if production practice were separated from academic instruction.

On August 13, 1964, the Central Committee of the Communist Party of the Soviet Union responded with an edict "concerning a change in the duration of the educational program in secondary general labor polytechnic schools with production training."[1] This edict demanded an immediate reduction of the secondary school program from three years to two years, to be accomplished by transfer of a major portion of the pupils' practical training to the summer months. In response, the Ministry of Education altered the 1964/65 school calendar immediately so that in Grade 9 the school year consisted of thirty-nine weeks. Furthermore, each pupil was required to enroll for production practice from

TABLE 10

INTERIM ACADEMIC PLAN FOR TWO-YEAR SECONDARY SCHOOLS WITH SPECIALIZATION IN COMPUTER PROGRAMMING [5]

Subject	Grade 9: Hours per Week, Semester 1 (17 Weeks)	Grade 9: Hours per Week, Semester 2 (18 Weeks)	Grade 9: Hours per Week, 4-Week Session	Grade 9: Hours per Year	Grade 10: Hours per Week, Semester 1 (17 Weeks)	Grade 10: Hours per Week, Semester 2 (18 Weeks)	Grade 10: Hours per Year	Total Hours for Two-Year Program
Nonmathematical Subject								
Literature	4	4	4	156	4	3	126	282
History	3	3	3	117	3	4	123	240
Sociology	0	0	0	0	2	2	70	70
Geography	0	0	0	0	2	2	70	70
Physics	5	5	5	195	5	5	175	370
Astronomy	0	0	0	0	1	1	35	35
Chemistry	4	3	3	134	3	3	105	239
Biology	2	2	0	70	0	0	0	70
Foreign Language	2	2	0	70	2	2	70	140
Physical Education	2	2	0	70	2	2	70	140
Drawing	1	1	0	35	0	0	0	35
Total Nonmathematical Hours				847			844	1,691
Mathematical Subject								
Algebra and Elementary Functions	4	3	3	134	4	5	158	292
Mathematical Analysis	4	4	4	156	4	0	68	224
Geometry	3	2	2	95	2	5	124	219
Numerical Methods, Programming, and Practice with Calculators and Computers	2	5	12	172	2	2	70	242
Total Hours in Mathematics				557			420	977
Total in All Categories	36	36	36	1,404	36	36	1,264	2,668
Faculty Activity	2	2	0	70	2	2	70	140
Two-Week Period Devoted to Computers and Programming	0	0	6 per day	72	6 per day	0	72	144

June 1 through July 15. A similar plan for Grade 10 was implemented in 1965/66.[2]

Implications for Mathematics Secondary Schools

The effects of this edict upon secondary schools with specialization in computer programming could have been catastrophic. Academically, the curriculum of these schools was already overcrowded. To compress the mathematical and programming content alone into two thirty-five-week school years would have been difficult. The Ministry of Education announced on August 18, 1964, that "it is not necessary to give up preparation of computer programmers in secondary schools," but rather, "in the case of preparation of pupils in a series of complex specialties, it is possible to increase the time allocated for theoretical activities by correspondingly decreasing the time allocated for practical instruction."[3] Ten days after this initial announcement, the *Teachers' Gazette* carried a rather detailed "consultation" for two-year secondary schools with specialization in computer programming, implementing the Ministry's suggestions.[4]

Unlike other two-year schools, computer-programming schools continued to operate thirty-nine weeks each year. The thirty-nine-week academic year is followed by a mandatory two-week period of "practical and theoretical instruction in numerical methods, programming, and work on machines at a computing center." The academic plan of computer-programming schools was altered accordingly.

The Current Plan

Table 10 is a translation of the interim academic plan for two-year schools with specialization in computer programming.

Under the interim Grade 9 plan for two-year computer-programmer schools, literature is allocated 4 hours, history 3 hours, physics 5 hours, and chemistry 3½ hours each week in comparison with 4½ hours, 3½ hours, 5 hours, and 4 hours respectively for other two-year schools. The 7½-hour weekly savings achieved by these academic adjustments and by deletion of the "production experience" requirements are absorbed by the mathematics program. Computer-programmer trainees receive an average of 3½ hours of instruction each week in Algebra and Elementary Functions, 2½ hours in Geometry, and 4 hours in Mathematical Analysis in comparison with a total of 6 hours of mathematics instruction each week in the usual ninth grade. In addition, computer-programmer trainees receive 2 hours of instruction in a special computer-oriented mathematics course during each week of the first semester. This is increased to 5 hours per week in the second semester and to 12 hours per week from June 1 to June 28. Time is provided for this special program by terminating course work in biology, foreign language, drawing, and physical education on May 31. Up to that time, biology, foreign language, and physical education all are allocated 2 hours per week and drawing 1 hour, unchanged from the usual program. Thus, in Grade 9, computer-programmer trainees accumulate a total of 557 class hours of instruction in Algebra and Elementary Functions, Mathematical Analysis, and Geometry, and computer-oriented mathematics, not including the two-week period of summer service at a computing center. This 557-hour total compares with 210 hours of instruction in mathematics in the usual ninth-grade class and 451 hours in Grade 9 of the superseded three-year secondary schools with specialization in computer programming.

In Grade 10 of computer-programming schools and also of ordinary two-year schools, literature and history are each allocated 3½ hours per week, and sociology and geography are each allocated 2 hours. Physics receives 5 hours per week, chemistry 3 hours, and foreign language and physical education 2 hours; these are also the same number of hours as the usual school program. The course Algebra and Elementary Functions receives an average of 4½ hours per week, Mathematical Analysis 2 hours, Geometry 3½ hours, and Numerical Methods and Programming 2 hours, for a Grade 10 mathematics total of 420 hours of instruction. The mathematics total for the two-year program is 977 hours, not including 140 hours allocated for faculty work or the two-week practical-work period. Total time allocated mathematics instruction, excluding practical work in the former three-year computer-programmer training program, is 1,085 hours—108 more than the current two-year allotment. Thus, the one-year compression seems to have detracted slightly from the overall mathematics program. However, the position of mathematics within the computer-programmer curriculum has improved from 25 percent to a present 36 percent of the total two-year curriculum. The computer-programmer curriculum still includes 557 more hours of instruction in mathematics than the curriculum of the ordinary two-year school and 507 more hours than the program of a four-year mathematics pupil in an American high school.

The Current Syllabus

Redistribution of mathematics topics within the Grade 9 program was recommended according to the following revised syllabus: [6]

Algebra and Elementary Functions

Linear and quadratic functions (15, 15)*
Powers with rational exponents (26, 26)
Trigonometric functions of an arbitrary angle (15, 15)
Relations between trigonometric functions (13, 13)
Conversion formulas and their corollaries (10, 10)
Trigonometric addition theorems and their corollaries (25, 25)
Exponential and logarithmic functions (30, 36)

Mathematical Analysis

Measure of a segment; real numbers (8, 8)
Numerical sequences and their limits (26, 26)
General concept of function (24, 24,
The derivative and its applications (64, 64)
The indefinite integral (24, 20)
Review (10, 12)

Geometry

Vectors (14, 14)
Analytic geometry (30, 40)
Metric relations in a triangle and the solution of triangles (12, 10)
Geometric transformations (18, 36)
Axioms for solid geometry and their corollaries (2, 3)
Parallelism in space (14, 14)
Review (5, 6)

Significant decreases in emphasis in comparison with the Grade 9 portion of the older three-year plan occur only in the area of geometry. Reductions of 25 percent and 50 percent respectively in the time spent on analytic geometry and geometric transformations are particularly interesting because of the significance of both topics for pupils preparing to continue the study of pure mathematics. Whether these reductions are expedients alone or whether they represent increased polytechnic emphasis in schools with specialization in computer programming is not clear.

Redistribution of mathematical topics within the Grade 10 program was undertaken in 1965/66 according to the following plan: [7]

Algebra and Elementary Functions

Linear algebra and elementary linear programming (50, 50)
Complex numbers (14, 14)
Polynomials, equations of higher degree (22, 22)
Transcendental equations (18, 18)
Combinations; elements of the theory of probability (22, 22)
Review (32, 30)

* The first number in parentheses following each topic denotes the hours per week allocated to that topic according to the new program, while the second number is the weekly allocation under the superseded three-year program.

Mathematical Analysis

The definite integral (25, 25)
Simple differential equations (12, 12)
Series (26, 26)
Review (5, 12)

Geometry

Perpendicularity in space (22, 25)
Systems of coordinates in space (12, 12)
Polyhedra (24, 24)
Solids of revolution (20, 20)
Elements of mathematical logic (18, 20)
Review and the solution of problems (28, 30)

The required 72-hour "practical experience" period was scheduled for January 11 to January 25, rather than during the summer, as was the case in Grade 9.[8] Early completion of this portion of the program permitted the important "qualification" examinaticns to be given in March or April. A note in *Matematika v Shkole* indicates that "some of the qualification characteristics require a second look—certain forms of work will have to be deleted and reductions made in the theoretical knowledge required of pupils."[9] Soviet educators apparently recognize that the two-year sequence is not quite the equal of the former program.

REFERENCES

1. *Uchitel'skaia Gazeta,* August 13, 1964.

2. See *Uchitel'skaia Gazeta,* August 15, 18, 20, 22, 25, and 29, 1964, for explanation of the reorganization edict and clarification of alterations to be made in various disciplines and special schools.

3. *Ibid.,* August 18, 1964.

4. *Ibid.,* August 29, 1964.

5. *Ibid.;* also "O Podgotovke Vychislitelei-Programmistov v Desiatiletnei Politekhnicheskoi Srednei Shkole s Proizvodstvennym Obucheniem," *Matematika v Shkole,* (November-December 1964), pp. 45–46.

6. "O Podgotovke Vychislitelei-Programmistov," p. 46.

7. *Ibid.,* p. 45.

8. *Il id.*

9. *Ibid.,* p. 46.

V

The Preparation of Teachers for Secondary Schools with Specialization in Computer Programming

The Need for Special Teacher-Education Programs

THE rapid increase in the number of secondary schools offering specialization in computer programming, together with the mathematical quality and technical demands of the official syllabus, have created a need for teachers with special qualifications. Although a teacher in the general course in mathematics may never be assigned to teach, say, Mathematical Machines and Programming, it is desirable that he have some knowledge of it so that appropriate emphasis can be given to related general topics. In 1963, the Ministry of Education initiated a teacher-education program at the Lenin Pedagogical Institute in Moscow with dual specialization in mathematics and programming.[1] One hundred nineteen entering students were selected for the program.[2]

The Lenin Institute Program for Prospective Mathematics-programming Teachers

Like the school curriculum it is designed to serve, the program for training teachers of computer programming is both intensive and of high mathematical quality. Its duration is five years, in contrast to four years for programs without dual specialization. Teachers for secondary schools with specialization in computer programming receive a total of 4,388 hours of classroom and laboratory instruction. In contrast, graduates of four-year American colleges or universities receive about 2,000 hours of classroom instruction and laboratory work. Of the 4,388-hour Soviet total, 2,730 hours are in mathematics and 450 hours in physics and electronics. These subtotals represent 62.2 percent and 10.3 percent of the total 4,388-hour program. Again in contrast, a "typical" mathematics major in an American four-year college or university program receives approximately 640 hours of instruction in mathematics and physics, constituting 30 percent of his total program. Clearly the Soviet program is an intensive one.

The scope and breadth (or rather lack of breadth) of the program can be determined by examination of the special academic plan prepared by the Ministry of Education (see Table 11).[3] Five courses included in the

TABLE 11

ACADEMIC PLAN FOR PEDAGOGICAL INSTITUTES 1963/64[5]

(Specialty in Mathematics-Programming)

A. Hours, Tests, and Examinations in Standard Subjects

Subject	Semesters in Which Examinations Are Given	Semesters in Which Credit Tests Are Given	Hours According to Type of Work: Lectures	Lab	Practical Work	Hours in All Types of Work	Hours According to Semester: 1 (18 Weeks)	2 (18 Weeks)	3 (18 Weeks)	4 (18 Weeks)	5 (18 Weeks)	6 (18 Weeks)	7A (7 Weeks)	7B (12 Weeks)	8 (18 Weeks)	9 (17 Weeks)	10 (12 Weeks)
History of the Communist Party of the Soviet Union ..	2/3	1	90	0	100	190	3	4	4	0	0	0	0	0	0	0	0
Political Economy	6/8	7	56	0	74	130	0	0	0	0	0	2	0	3	3	0	0
Marxist-Leninist Philosophy	4/5	—	0	0	0	134	0	0	0	4	4	0	0	0	0	0	0
Foundations of Scientific Communism ..	9	—	42	0	28	70	0	0	0	0	0	0	0	0	0	4	0
Psychology	2	—	0	0	0	74	2	2	0	0	0	0	0	0	0	0	0
Pedagogy and History of Pedagogy .	4/5	—	80	0	60	140	0	0	2	3	3	0	0	0	0	0	0
School Hygiene	—	3	20	0	0	20	0	0	1	0	0	0	0	0	0	0	0
Physical Education	—	1, 2, 3, 4	0	0	140	140	2	2	2	2	0	0	0	0	0	0	0

Subject	Semesters in Which Examinations Are Given	Semesters in Which Credit Tests Are Given	Hours According to Type of Work: Lectures	Lab	Practical Work	Hours in All Types of Work	Hours According to Semester: 1 (18 Weeks)	2 (18 Weeks)	3 (18 Weeks)	4 (18 Weeks)	5 (18 Weeks)	6 (18 Weeks)	7A (7 Weeks)	7B (12 Weeks)	8 (18 Weeks)	9 (17 Weeks)	10 (12 Weeks)
Foreign Language	3	1, 2, 3	0	0	0	240	4	4	3	2	0	0	0	0	0	0	0
Mathematical Analysis	1, 2, 3, 4	1, 2, 3, 4	212	0	268	480	6	7	6	7	0	0	0	0	0	0	0
Higher Algebra	1, 2, 3	2, 3	110	0	102	212	4	4	4	0	0	0	0	0	0	0	0
Analytic Geometry	1, 2	1, 2	90	0	86	176	6	4	0	0	0	0	0	0	0	0	0
General Physics	4, 5	3, 4, 5	108	72	106	286	0	0	6	5	5	0	0	0	0	0	0
Mathematical Logic	5	—	50	0	18	68	0	0	0	0	4	0	0	0	0	0	0
Theory of Functions of Real Variables	6	6	54	0	18	72	0	0	0	0	0	4	0	0	0	0	0
Theory of Functions of Complex Variables	7, 7	—	60	0	24	84	0	0	0	0	0	0	2	6	0	0	0
Theory of Probability	9	9	50	0	34	84	0	0	0	0	0	0	0	0	0	5	0
Theory of Algorithms	8	8	54	0	18	72	0	0	0	0	0	0	0	0	4	0	0
Theory of Numbers and Foundations of Arithmetic	6	5	70	0	30	100	0	0	0	0	3	3	0	0	0	0	0
Higher Geometry	4	4	54	0	36	90	0	0	0	5	0	0	0	0	0	0	0
Foundations of Geometry	9	—	0	0	0	84	0	0	0	0	0	0	0	0	0	5	0
Numerical Methods	7	5, 6	60	0	100	160	0	0	0	0	3	4	0	3	0	0	0
Machines and Programming	9	7, 8, 9	60	0	140	200	0	0	0	0	0	0	0	4	5	4	0
Basic Electronics	—	6	24	0	36	60	0	0	0	0	6/2	0	0	0	0	0	0
Practicum in Elementary Mathematics	—	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	0	0	380	380	3	3	2	2	2	2	2	3	2	0	4
Mathematical Methods	8	6, 7, 10	48	0	92	140	0	0	0	0	0	3	2	2	3	0	0
Visual Aids	—	8	0	0	50	50	0	0	0	0	0	0	0	0	3	0	0
Special Course	6, 7, 8	—	200	0	0	200	0	0	0	0	2	0/4	0	4	4	0	0

TABLE 11 (*Cont.*)

Subject	Semesters in Which Examinations Are Given	Semesters in Which Credit Tests Are Given	Hours According to Type of Work: Lectures	Lab	Practical Work	Hours in All Types of Work	Hours According to Semester: 1 (18 Weeks)	2 (18 Weeks)	3 (18 Weeks)	4 (18 Weeks)	5 (18 Weeks)	6 (18 Weeks)	7A (7 Weeks)	7B (12 Weeks)	8 (18 Weeks)	9 (17 Weeks)	10 (12 Weeks)
Special Seminar	—	5, 6, 7, 8, 9	0	0	198	198	0	0	0	0	2	2	0	2	2	4	0
Astronomy	9	—	52	18	0	70	0	0	0	0	0	0	0	0	0	4	0
Total Hours in All Subjects for Each Semester ..							30	30	30	30	20	26	6	27	26	26	4
Number of Examinations in Each Semester							3	5	4	5	4	4		3	4	4	0
Number of Credit Tests in Each Semester							6	6	6	6	5	6		6	5	5	1

B. Other Required Work

Area	Semester	Weeks	Dates
Pioneer Practice	6	4	June 8–August 12
Assisting Teacher	7	7	September 1–October 26
Practice on Mathematical Machine	10	4	February 1–February 28
Practice Teaching	10	12	March 1–May 23

academic plan are designed especially for students pursuing dual specialization in mathematics and programming, while the remainder are conducted according to official syllabi obligatory for all pedagogical institutes preparing mathematics teachers. Of these five special courses, four are in mathematics and one is in electronics.

Course work in mathematical logic and the theory of algorithms is based upon experimental syllabi prepared by Academician P. S. Novikov, a distinguished member of the Lenin Institute faculty.[4] Texts recommended for the courses include the Russian editions of *Foundations of*

Theoretical Logic, by David Hilbert and W. Ackermann, and *Introduction to Metamathematics,* by S. C. Kleene.

Conjunctions, disjunctions, negations, implications, and logical equivalence are introduced at the outset of the logic course. Truth tables are used to clarify logical operations and relations among operations. Both the absolute disjunctive normal form (ADNF) and the disjunctive normal form (DNF) of logical functions are discussed with respect to existence, uniqueness, and translation from DNF and ADNF. Dual and self-dual functions are introduced before consideration of conjunctive and absolute conjunctive normal forms. The logical apparatus developed is applied to circuitry problems of a nontrivial nature. A rather extensive discussion of the theory of algorithms based upon Turing machines and finite automata, together with a unit on the logic of predicates, makes up the second course. Reports from students to whom these courses were offered experimentally were quite favorable. The instructor was P. S. Novikov.

The course Numerical Methods, like Mathematical Logic and The Theory of Algorithms, was designed especially for the programming sequence. The course is partitioned into three principal subsections: (1) elements of functional analysis, (2) approximation of functions by polynomials, and (3) numerical integration.[5] Included in the first section are the concepts of metric space and complete space, iterative methods for solving transcendental equations in one unknown, and Euler's method for solving systems of *n* equations in *n* unknowns. Matrix methods for solution of systems also are discussed.

The second unit begins with a discussion of interpolation and the Lagrange polynomial. The method of finite differences is introduced, and Newton's two interpolation procedures are discussed. Finite and integral methods of least squares conclude the section.

Greatest emphasis is given to the section on numerical integration. The trapezoidal rule and Simpson's rule are introduced immediately. More sophisticated numerical procedures include the Euler, the Adams-Krylov, and the Runge-Kutta integration methods.

The experimental syllabus for the special course Computing Machines and Programming was prepared by the former dean of the Lenin Institute mathematics faculty, E. A. Shchegol'kov.[6] The course is divided into four units and a laboratory experience. The first unit deals with the arithmetic bases of machine calculation, numeration systems, codes, and coding for machine consumption. The second unit is concerned with the basic operational aspect of computing equipment, including memory, arithmetic, control, and input-output mechanisms. The third unit deals with the principles of programming, with applications to four current Soviet electronic computers. The fourth and final unit is a discussion of the circuitry and digital mechanism of a typical computer. Associated laboratory work includes work with numeration systems, familiarization with one type of computer, and preparation of programs.

All other mathematics courses listed in the academic plan follow official syllabi prepared for other than the special programming sequence. Abstracts of these syllabi make up Appendix H.

Like all Soviet students preparing to be secondary school teachers, students pursuing dual specialization in mathematics and programming are required to complete a rather extensive practice-teaching experience and to take a series of government qualifying examinations.[7] Practical work with children is undertaken in three stages. The first stage is a four-week period as a counselor in a Young Pioneer camp during the summer following the student's third year of study.[8] The second stage is part-time service for seven weeks at the beginning of the seventh semester as a "teacher's assistant" in a local school. The student continues to attend institute classes during this experience. A twelve-week practice-teaching experience during the student's tenth semester concludes the pedagogical practice sequence for the mathematics-programming specialist. Students seeking certification in mathematics alone are required to complete a practice-teaching experience of eighteen weeks rather than twelve weeks. The reduction in practice-teaching requirements for mathematics-programming students is necessary to provide time for the mandatory four-week computer practice in an actual computing installation.

Examinations required of students completing the mathematics-programming sequence include a government examination covering arithmetic, algebra, geometry, mathematical analysis, and the theory of functions, as well as qualifying examinations in the areas of programming and computer operation. The first examination, or "general examination," will be analogous to the government examination for graduates in the single specialty of mathematics. Since the first group of students with dual specialization in mathematics-programming do not graduate until 1968, content of the second examination had not been determined at the time of this writing.

The aim of the general state graduation examination in mathematics is to verify that the graduate is qualified to teach mathematics in a Soviet secondary school. In particular, the examination should "(1) establish the level of the graduate's mathematical development and the degree of his mathematical culture; (2) determine the strength of [his] knowledge of the basic elements of science and [his] ability to apply mathematical methods in science and technology; and (3) determine the degree of preparation for solving those problems from elementary mathematics that require knowledge of the ideas and methods of higher mathematics." [9]

Like the majority of Soviet examinations, the general graduation examination in mathematics is conducted using tickets. The examination syllabus instructs that each ticket should contain two or three questions from different areas, but it does not specify individual ticket content, as does the secondary school syllabus.[10] The examination syllabus does indicate that each examinee must be asked a supplementary question pertaining to some area not covered by the question on his ticket.[11] The responses of students not only must be complete and free of error but also must include derivation or proof of at least one formula or theorem employed in the solution. An abridged translation of the syllabus for the general government graduation examination in mathematics is included in Appendix I.

Comparison of the Soviet Curriculum and MAA Recommendations

It may be informative to compare the Soviet program for prospective teachers in mathematics secondary schools with the recommendations for the training of mathematics teachers published by the Mathematical Association of America. Level III of four levels of competence established by the MAA is for teachers of high school mathematics, who are "qualified to teach a modern high school mathematics sequence in Grades 9 through 12." Minimum requirements for Level III are as follows:

1. Three courses in analysis
2. Two courses in abstract algebra
3. Two courses in geometry beyond analytic geometry
4. Two courses in probability and statistics
5. Two upper-class electives, e.g., introduction to real variables or number theory

   (One of these courses should contain an introduction to the language of logic and sets.) [12]

Approximate translation of Soviet "class hours" into semester-hour courses was accomplished by considering sixteen class hours equivalent to one semester hour. MAA courses are understood to be three-semester-hour courses.[13]

Despite imprecision of translation, it is apparent from Table 9 that the Soviet curriculum for prospective teachers in mathematics secondary schools exceeds MAA Level III requirements in all areas except statistics. The Soviet deficit in statistics is the equivalent of a mere semester hour.

Table 12 does not include the 380 course hours allocated for the

TABLE 12

COMPARISON OF MAA LEVEL III RECOMMENDATIONS AND THE SOVIET CURRICULUM FOR TEACHERS IN COMPUTER-PROGRAMMER SCHOOLS

MAA Level III		Soviet Curriculum	
Subject	Number of Courses	Subject	Number of Courses
Analysis	3	Analysis	10 (three semester hours each)
Abstract Algebra	2	Higher Algebra	4 (three semester hours each)
Geometry (beyond Analytic)	2	Higher Geometry	1 (five semester hours)
		Foundations of Geometry	1 (five semester hours)
Probability and Statistics	2	Theory of Probability	1 (five semester hours)
Upper-Level Electives	2	Mathematical Logic	1 (four semester hours)
		Real Variables	1 (four semester hours)
		Complex Variables	1 (five semester hours)
		Theory of Numbers	2 (three semester hours each)
		Numerical Analysis and courses in computer-related Mathematics	10 (three semester hours each)

practicum in elementary mathematics. This practicum, an exhaustive recapitulation of secondary school mathematics from an advanced point of view, is considered by Soviet educators to be one of the most important portions of the teacher-training curricula in mathematics.

MAA Level IV (teacher of the elements of calculus, linear algebra, probability, etc.) seems more appropriate for teachers in Soviet mathematics secondary schools than Level III. Level IV requirements include "a master's degree with at least two-thirds of the courses being in mathematics and for which an undergraduate program at least as strong as Level III is a prerequisite."[14] If it is assumed that the average master's program in an American university is composed of thirty-six semester hours of coursework, then, according to MAA recommendations, a minimum of twenty-four semester hours of mathematics (or eight three-hour courses) is required if Level IV standards are to be met. If the practicum in elementary mathematics and special courses and seminars in mathematics are included, the Soviet program exceeds Level III requirements by the equivalent of forty three-semester-hour courses. This excess certainly more than equals the eight three-semester-hour graduate courses necessary for the minimum Level IV master's degree in an American university. There can be little doubt that Soviet teachers trained for service in mathematics secondary schools will be prepared to handle adequately the advanced curricula of these schools.

REFERENCES AND NOTES

1. V. G. Ashkinuze and S. I. Shvartzburd, "Shkoly s Matematicheskoi Spetzializatziei," *Matematika v Shkole* (March-April 1963), p. 83.

2. Enrollment and other data obtained from E. A. Shchegol'kov, dean of the mathematics faculty, Lenin Pedagogical Institute, Moscow, during frequent interviews by the author in 1964.

3. Copied from official documents provided by E. A. Shchegol'kov.

4. P. S. Novikov, "Programma po Kursu Matematicheskaia Logik i Algoritmy dlia Studentov III Kursa na 1963/64 Uch. G." (typewritten).

5. "Programma Kursa Metody Vychislenii'" (typewritten; MGPI im. V. I. Lenina Matematicheskii Facultet, 1963).

6. E. A. Shchegol'kov, "Programma po Kursa Vychislitel'nye Mashiny i Programmirovania" (typewritten; MGPI im. V. I. Lenina Matematicheskii Facultet, 1963).

7. See Table 8.

8. Young Pioneers are the Soviet counterpart of Boy and Girl Scout organizations.

9. *Programmy Pedagogicheskish Institutov—Gosudarstvennye Ekzameny po Matematike* (Moscow: Uchpedgiz, 1958), p. 6.

10. *Ibid.*

11. *Ibid.*

12. "Recommendations of the Mathematical Association of America for the Training of Mathematics Teachers," *American Mathematical Monthly*, LCIII (December 1960), 982–91.

13. *Ibid.*

14. *Ibid.*

VI

Soviet Boarding Schools Offering Specialization in Mathematics

The Moscow School

In an earlier chapter, a distinction was made between secondary day schools with specialization in computer programming and secondary boarding schools with specialization in mathematics. Mathematics boarding schools are sponsored by universities or branches of the Academy of Sciences while computer-programmer schools are under the sole jurisdiction of the Ministry of Education. There are five mathematics boarding schools in the Soviet Union—at Moscow, Leningrad, Kiev, Novosibirsk, and T'bilisi—but perhaps one hundred computer-programmer schools. The development of young talent is the principal purpose of the mathematics boarding schools, while officially, at least, this is only a partial goal of the computer-programmer school. Curricula of mathematics boarding schools vary from school to school, in contrast to the rigid curriculum obligatory for computer-programmer schools.

Boarding School 18, located in a Moscow suburb, is one of the four mathematics boarding schools now in operation. The school, known as the Moscow School of Mathematics and Physics, was organized in 1963 under the auspices of Moscow State University.[1] The school's principal sponsor is Academician A. N. Kolmogorov, the famed mathematician. Kolmogorov manages to teach at the Moscow school three days each week.[2] In addition to Kolmogorov, the Moscow teaching staff includes Academician I. K. Kikoin (physics) and Doctors of Science Ia. A. Smorodinski (physics) and V. I. Arnol'd (mathematics).

The Moscow school is slightly larger than the usual computer-programming school, enrolling approximately 360 pupils.[3] More than 90 percent of the enrollees are boys.[4] Pupils are selected after a personal interview with Professor Kolmogorov or another senior staff member. Selection criteria include past performance in mathematics classes, interest in mathematics as a career, and results of participation in mathematics Olympiads, with the latter of principal importance. A recent eleventh-year class at the Moscow school included twenty of the forty national Olympiad contest winners in this age group.

The mathematics curriculum, designed primarily by Kolmogorov, differs significantly from the curriculum prescribed by the Ministry of Education for secondary schools with specialization in programming. Although programming specialization is available in the eleventh year, it is an option only. A pupil may specialize in any suitable area of mathematics or physics in which he has interest and ability.

The general curriculum of the Moscow school is outlined in Table 13, while the mathematics-physics portion of the general curriculum for Grades 10 and 11 is given in detail in Table 14. The mathematics-physics curriculum for Grade 9 and the first quarter of Grade 10 is essentially a compression and acceleration of the usual secondary school mathematics-physics curriculum. Departure from the usual syllabus begins with the second quarter of Grade 10.

Table 13 indicates that the overall curriculum of the Moscow school is even more mathematically oriented than the curriculum of computer-programmer secondary day schools. Computer-programmer trainees receive a two-year total of 2,952 hours of instruction, of which approximately 50 percent is in mathematics and physics. Boarding school pupils

TABLE 13

MOSCOW SCHOOL ACADEMIC PLAN

Subject	Grade 9		Grade 10		Grade 11		Total	
	Hours per Week	Hours per Year	Hours per Week	Hours per Year	Hours per Week	Hours per Year	Hours per Week	Hours per Year
Russian Language and Literature	3	117	3	117	3	117	9	351
History and Social Studies	3	117	3	117	4	156	10	390
Geography	2	78	2	78	0	0	4	156
Biology	2	78	0	0	0	0	2	78
Chemistry	2	78	2	78	2	78	6	234
Foreign Language	3	117	3	117	3	117	9	351
Physical Education	3	117	3	117	3	117	9	351
Mathematics and Physics	16	624	17	663	18	702	51	1,989
Total	34	1,326	33	1,287	33	1,287		3,900

TABLE 14

MOSCOW SCHOOL MATHEMATICS-PHYSICS CURRICULUM

Subject	Hours According to Grade and Quarter						
	10			11			
	Quarter 2	Quarter 3	Quarter 4	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Algebra and Analysis	7	6	5	0	0	4	4
Geometry	4	5	5	0	0	3	3
Linear Algebra	0	0	0	3	3	0	0
Discrete Mathematics	0	0	2	3	3	0	0
Probability Theory	0	0	0	0	0	3	0
Problem Seminar	0	0	2	4	4	0	0
Physics	6	6	6	6	6	6	6
Total	17	17	20	16	16	16	13

accumulate 3,900 class hours over the three-year period, with 1,989 hours, or 55 percent, in mathematics and physics. A smaller portion of the boarding school total (97 hours) is allocated to practical work, while the programming curriculum includes 144 hours of computer laboratory experience. Because the Moscow school is a boarding school, Kolmogorov is able to stimulate individual pupil research and evening discussions relating to mathematics to a far greater extent than is possible in day schools. This feature adds significantly to the intensity and impact of the boarding school mathematics program.

Emphasis in the Moscow school curriculum is much more theoretical than in the computer-programmer sequence. Pupils receive considerably more instruction in analysis and discrete mathematics but less in numerical methods than computer-programming pupils. Boarding-school pupils study functions of a complex variable in analysis and the elements of group theory in algebra in some detail, while computer-programmer trainees barely touch upon these topics.

The method of presentation used in the Moscow school departs radically from the usual Soviet school practice. Lectures of forty-five or ninety minutes are given to an entire age group or sometimes even to two age groups, followed by work in much smaller groups (thirty or fewer pupils). These small groups are often split into subgroups with an instructor responsible for each. On occasion, team-teaching procedures are employed, several instructors working simultaneously with a group.

Since appropriate texts are unavailable, Kolmogorov has undertaken the preparation of a textbook for his school. In the interim, notes, university texts, and enrichment booklets prepared for circle use are employed.

The Novosibirsk School

The Novosibirsk Mathematics-Physics Boarding School was opened in January 1963 under the joint sponsorship of the Siberian section of the Soviet Academy of Sciences and Novosibirsk State University.[5] Its purpose was and is the preparation of future mathematicians. Initially, 116 pupils were selected for admission to the school on the basis of their previous success in mathematics circles and Olympiads. Enrollment increased to 318 in 1963/64 and to more than 350 by 1966/67.

Unlike the program of the Moscow school, the Novosibirsk curriculum includes mandatory computer-programming preparation in Grade 11. The academic plan for the Novosibirsk school allocates eight hours each week to formal instruction in mathematics in Grades 9 and 10 and twenty-one hours weekly in Grade 11. The unusual concentration in Grade 11 is due to the school's computer requirement. Novosibirsk pupils accumulate 1,443 hours of instruction in mathematics over the three-year period, not including required extracurricular activities.

Mathematics classes in Grades 9 and 10 at the Novosibirsk school are conducted in an interesting fashion. Pupils attend but one two-hour mathematics lecture each week. At other times, they meet with an instructor in small groups (usually about fifteen) for a weekly total of six hours of problem work. Problem sessions are devoted to explanation, application, and extension of the material presented in the lecture. Lecturers are senior members of the mathematics faculty at Novosibirsk State University or important scientific workers in the Siberian section of the Academy of Sciences. Problem sessions are conducted by junior faculty members and scientific workers. Few "regular" teachers participate in the mathematics program of the Novosibirsk school.

In order to maintain high standards of scholarship, school officials administer examinations in mathematics and physics at the close of each semester. Pupils failing to complete either examination satisfactorily are dismissed at once. Semester examinations are usually partially oral and partially written. According to published reports, both the curricular plan and the examination procedure have been "entirely satisfactory" thus far.

A series of special texts and problem books have been prepared and published in small quantities for Novosibirsk pupils. Textbook preparation was supervised by Professors A. A. Kiapunov, A. S. Kosikhin, S. L. Sobolev, and S. I. Literat.

An unusual feature of the Novosibirsk program is the requirement that all pupils participate actively in one or more scientific circles. Four hours a week are set aside in each pupil's schedule for circle participation, in addition to allocations for "daily study of scientific literature." If the hours devoted to circle work and research are added to instructional hours in mathematics, the Novosibirsk total increases to 2,028 over the three-year period, in contrast to 1,989 hours at the Moscow school. Circles regularly in operation include the following:

1. Theory of sets
2. Cybernetics
3. Modern algebra
4. Theory of numbers
5. Mathematical logic
6. Modern geometry

The work of these circles is composed of lectures by visitors and pupils, exposition of appropriate material, and solution of problems. The purpose of the circles is "to permit pupils to determine their interests and, possibly, the direction of their future scientific work." In addition to circle participation, Novosibirsk pupils participate in their own "mathematical society," with programs devoted to lectures, papers by members, and other activities analogous to those of societies of mature mathematicians.

The mathematics syllabus of the Novosibirsk school differs from the syllabi of the Moscow school and day schools with specialization in computer programming. A translation of the Novosibirsk mathematics syllabus is reproduced as Appendix J. Like the Moscow curriculum, the Novosibirsk mathematics program is more theoretically oriented than the curriculum of secondary day schools with specialization in computer programming. Unlike the Moscow school, however, the Novosibirsk school offers more work with numerical methods and computing hardware than secondary day schools. The Novosibirsk program appears to contain slightly more analysis than the Moscow curriculum, and the emphasis upon applied mathematics certainly is greater. In general, the technological orientation of the Novosibirsk community is reflected in the school's curriculum. It is probable that the Novosibirsk school is the most technologically oriented of the five existing Soviet mathematics boarding schools.

The effect of the 1964 reorganization edicts upon Soviet mathematics boarding schools is not entirely clear. Their programs, too, may be reduced by one year. Indeed, Kolmogorov indicated in the spring of 1964 that eventually the program of his school would be of two years' duration. Expansion of mathematics boarding schools also is uncertain. Although always experimental, mathematics boarding schools once were viewed as the principal means of providing intensified mathematical education for talented rural pupils.[6] Now another potential solution to this problem has emerged, thus diminishing the probability of rapid proliferation of these schools.

REFERENCES AND NOTES

1. B. R. Vainberg, E. G. Glagoleva, and N. Kn. Rozov, "Zaochnaia Matematicheskaia Shkola pri Moskovskom Universitets," *Matematika v Shkole* (September-October 1964), p. 60.
2. Statement by A. N. Kolmogorov, reported to Moscow Mathematical Society, March 1964.
3. Vainberg, Glagoleva, and Rozov, *op. cit.*, p. 60.
4. The statement by Kolmogorov cited above is the source of information in this and following paragraphs of this section, also of the information shown in Tables 13 and 14.
5. Most of the information in this section is from S. I. Literat, "O Prepodavanii Matematiki v Fiziko-Matematicheskoi Shkole-internate," *Matematika v Shkole* (November-December 1963), pp. 44–46.
6. Statement by Kolmogorov cited above.

VII

Part-Time Schools for Mathematically Talented Pupils

The Ivanovo Youth Mathematics School

THE development of a third form of special school for mathematically talented pupils paralleled the development of secondary day and boarding schools with mathematical specialization. A Youth Mathematics School or School for Young Mathematicians (abbreviated YMS) was opened in September 1959 in Ivanovo, a textile center northwest of Moscow.[1] The opening of the Ivanovo school coincided with the beginning of computer-programming classes at School 425 in Moscow. Unlike School 425, the Ivanovo school was a part-time school offering work in mathematics alone. Pupils attending the YMS were enrolled also in a regular secondary school where they studied the usual mathematics program. The YMS program was a supplement to, and not a replacement for, the three-year school program.

The Ivanovo YMS was conceived and directed by S. V. Smirnov, a member of the mathematics faculty at Ivanovo Pedagogical Institute. One purpose of the YMS was "to familiarize pupils as far as is possible with modern mathematics."[2] Unlike mathematics boarding schools, enrollment in Youth Mathematics Schools was not restricted to especially gifted children: "All pupils who display interest and pass entrance tests and interviews can attend the YMS."[3]

The Ivanovo YMS was organized into three classes, corresponding roughly to classes in the ordinary secondary polytechnic school. During the initial academic year, each of the three classes was composed of two groups of from six to twelve pupils.[4] Total enrollment was approximately fifty.[5] YMS classes met twice each week for two- or three-hour periods outside regular school hours. This arrangement permitted pupils to attend regular secondary school classes without interruption. Attendance at the YMS was compulsory and discipline for failure to attend severe. Repeated violation of attendance regulations led to expulsion.

The teaching staff included the department head and teachers of mathematics from the experimental school attached to the Ivanovo Pedagogical Institute. The composition of the teaching staff required approval by the academic council of the Institute, since teaching in the Youth Mathematics School was included in the basic teaching load of the department of mathematics and physics.

To gain admission to the Youth Mathematics School, an applicant was required to present a transcript from his secondary school attesting to his achievement and disposition. Since a large number of students submitted applications for the first YMS, not all could be accepted, so personal interviews were held with each entrant "to discover his motivations, his interest in the YMS, his general level of mathematical intelligence, and his interest in mathematics." [6] Included in the criteria for admission were the pupils' previous school grades, which were said to be necessary to judge their potentialities.[7] Thus, while the YMS was open to all students, those selected were either "honor pupils or well advanced in all subjects, and, as a rule, honor pupils in mathematics." [8] The period of study in the Ivanovo YMS was three years. Under the ten-year-school organization only pupils in Grades 8, 9, and 10 attended. Eleven-year-school pupils in Grades 9, 10, and 11 were admitted to the YMS after polytechnic reorganization of the school program. Pupils in the first two classes attended the YMS four hours per week, while third-year YMS pupils attended five hours per week.[9] The YMS graduate accumulated 507 hours of instruction in mathematics *in addition to* the mathematics included in the usual secondary school program.

Since one of the purposes of the YMS was to develop the pupil's "mathematical culture—in particular to develop ability to solve non-standard problems," much of the work of the YMS consisted of problem solving.[10] Some sections of the curriculum, particularly those concerned with review of the usual school program, were made up almost entirely of challenging problems.

The YMS curriculum also emphasized modern mathematics, especially applied mathematics. Programming, machine computation, and graphical methods were important examples. The theoretical portions of the curriculum supplemented the usual school courses by providing an introduction to higher mathematics. (An abridged version of the Ivanovo syllabus appears in Table 15.) The Ivanovo syllabus should be taken as a basis only. In practice, YMS activities were flexible and were often

TABLE 15

SYLLABUS—IVANOVO YOUTH MATHEMATICS SCHOOL [11]

FIRST YEAR

1. Review course in plane geometry (52 hours)
   a) Basic theorems in absolute geometry
   b) Theory of parallels
   c) Circles and regular polygons
   d) Theory of similarity
   e) Study of area
2. Arithmetic (38 hours)
   a) Numeration systems
   b) Theory of divisibility
   c) Rational numbers
   d) Concept of irrational number
3. Algebra (38 hours)
   a) Algebra of polynomials (divisibility, etc.)
   b) Linear equations (with an introduction to the theory of determinants)
   c) Numerical methods of solution of equations
4. Numerical methods (26 hours)
   a) Familiarity with desk calculators
   b) Basic methods of approximate computation
   c) Graphs of functions, graphical solution of equations

SECOND YEAR

1. Algebra (26 hours)
   a) Complex numbers
   b) Equations of higher degree, systems
2. Introduction to mathematical analysis (38 hours)
   a) Concept of function, exponential functions and logarithms, trigonometric functions
   b) Concept of limit, limit of a function, continuity
   c) Derivative and differential
   d) Fundamental theorem about differentials, Taylor's formula and its use
   e) Geometric applications, the tangent and normal to a plane curve, curvature
3. Analytic geometry (26 hours)
   a) Method of coordinates, transformation of a Cartesian-coordinate system, polar coordinates
   b) Geometric interpretation of equations
   c) Linear problems
   d) Canonical form of equations of second-order curves and the basic properties of these curves

TABLE 15 (*Cont.*)

4. Practicum on the solution of problems (38 hours)

THIRD YEAR

1. Mathematical analysis (38 hours)
   a) Basic concepts of the theory of sets, clarification of the concept of real number, limits
   b) Basic properties of continuous functions
   c) Integration
   d) Geometric applications of integral calculus
2. Mathematical logic (38 hours)
   a) Basic concepts and theorems of statement calculus
   b) Introduction to predicate calculus
   c) The concept of algorithm
3. Practicum on the solution of problems, including trigonometry (38 hours)
4. Numerical practicum and basic programming (38 hours)
5. Electronic laboratory (one hour per week—38 hours)

geared to the interests and talents of a particular group rather than being bound to an official study plan.

In connection with the study of theoretical logic in the third year, it was observed that students "easily coped with the material, although at first it seemed abstract." [12] Great stress was placed on examples and applications to geometry. At the second-year level, and to a lesser extent in the first year, algebra was presented from a modern point of view through the presentation of theorems with proofs. Some abstract concepts such as rings were introduced. Similarly, in the first year, emphasis was placed upon new principles of operation and the feasibility of operations. In the final lessons of the first year, pupils began to study functions graphically. "The very great importance of review lessons for all grades was discovered, since a preliminary check showed that students had forgotten trigonometric identities, solutions of trigonometric equations, and proofs of basic formulas." [13] A similar situation existed in geometry, with pupils "remembering theorems and their proofs poorly." [14]

The Moscow Schools for Young Mathematicians

In addition to the YMS at the Ivanovo Institute, similar part-time mathematics secondary schools were established in other cities. Five such schools, for Grades 9 and 10 only, were opened in Moscow under the sponsorship of the Moscow Mathematical Society and the Ministry of Education.[15] The Moscow project was directed by I. V. Girsanov and

E. A. Morazova. A "methodological group" composed of E. B. Dynkin, A. L. Onishchik, N. S. Bakhvalov, A. M. Il'in, A. G. Kostiuchenko, and I. M. Iag'lom, prepared the syllabus in use in all five Moscow schools.[16] In the first year (1959/60), 280 students were admitted to Grade 9 and 350 to Grade 10 from over 800 applicants. The quality of students admitted depended upon the number of applicants in each individual school district.

The courses in the Moscow YMS comprised algebra, analysis, and geometry. The algebra program for Grade 9 included divisibility theory, systems of numeration, combinatorial theory, elements of higher algebra, and systems of linear equations. In Grade 10 the courses in algebra dealt with the algebra of polynomials and mathematical induction. The ninth-grade geometry course included geometric transformations, coordinate geometry, and loci. In Grade 10 some connections between geometry, engineering, and physics were examined (the concept of center of gravity, etc.). Functions and their graphs, inequalities, and the theory of limits were studied in analysis in Grade 9. In Grade 10, functions again were investigated; continuity was studied; and methods of approximate computation, derivatives and their application, and the concept of integral were all covered in some detail.

Every class met two evenings a week for two hours. The instructors explained new terms and presented the solution of problems. At the end of each semester an inspector reviewed all the work of each student, and at the completion of the course each pupil was required to pass an examination.

Later, Professor E. B. Dynkin initiated a new kind of evening mathematics school in Moscow for seventh- and eighth-grade pupils. Classes meet once each week for a lecture of approximately 90 minutes followed by problem-solving sessions in small groups (15 pupils or less). At present 150 pupils are reported to be participating in Dynkin's school. Roughly half are matriculated pupils of the school, while the others are "guests." Guests may become "candidates" and then pupils, depending upon their performance in the work of the school. Topics of lectures from the school's program include mathematical logic and communication applications (Dynkin), theory of games (Weinberg), and non-Euclidean geometry (Yefremovich). Each lecturer also supplies the small problem groups with five competition problems relating to the lecture. Promotion of a guest to candidate and finally to pupil is determined by the individual's success with competition problems. The best pupils receive autographed mathematics books as awards. Pupils may, if they wish, present papers to the class in an effort to earn honor points.

Youth Mathematics Schools have been established in Kishinev, under the sponsorship of the Moldavian Mathematical Society; [17] in Iaroslavl; [18] in Irkutsk, under the sponsorship of Irkutsk State University; [19] in Kalinin; [20] and in other cities of the Soviet Union. Enrollment in Soviet Youth Mathematics Schools known to this writer exceeded five thousand in 1965. However, the rapid increase in the number of mathematics secondary schools offering specialization in computer programming has had a retarding effect upon the development of the YMS. It is doubtful, therefore, that many new Youth Mathematics Schools will be established henceforth.

The Mathematics Correspondence School

The success of YMS part-time study programs for pupils with interest and talent in mathematics led to a new and exciting approach to the problem of providing an "intensified mathematical education" for a minority of pupils. Although the YMS served a useful purpose, it failed to reach pupils residing in villages or small cities far from the centers of mathematical activity. The mathematics day schools, concentrated as they are in principal cities, also failed to reach large numbers of rural and village pupils of superior ability. Clearly, the five mathematics boarding schools, excellent though they may be, are unable to provide training for more than a small fraction of capable pupils. Moscow State University Professor I. M. Gel'fand, the renowned functional analyst, took the initiative for establishing at the university a "Republic Mathematics Correspondence School" (RZMSh) designed to serve talented school pupils *throughout* the Russian Republic.[21] Many of the features of the new correspondence school were drawn from the YMS conducted by Professor Dynkin of Moscow State University. The purpose of the new school is "to strive to reach the most distant corners [of the Russian Republic] in order to find those children who, with appropriate work, can develop their mathematical ability and later become good mathematicians, physicists, and engineers." [22]

The idea for a correspondence school was "supported enthusiastically by the rector of Moscow State University and the Ministry of Education." [23] At a meeting attended by Professors Gel'fand, Dynkin, Kirillov, Kashin, and Efimov and others, it was decided that the school should begin its work in September 1964.[24] Recruitment and enrollment of correspondence-school pupils was begun in March 1964. The school was to be open to pupils who had finished Grade 8. Pupils desiring admission to the correspondence school were required to take an entrance examination consisting of twelve problems. The examination, prepared by the scientific workers at the university, emphasized problems that would test the pupil's mathematical abilities, interests, and ingenuity, rather than his knowledge of one specific area or theorem from the school course. Problems finally chosen for the examination included rather complicated "circle-type" problems (referring to the work of mathematics clubs or circles) and problems related to the usual school syllabus but requiring unusual insight or nonstandard methods for solution.

The complete text for the entrance examination for 1964 follows:

1. Two people play the following game: the first names a one-digit number (that is, a number from 1 through 9); the second adds to it another one-digit number and states the sum. To this sum the first player adds still another one-digit number and states the sum, and so on. The winner is the one to name 66 first. How should one play this game in order to win? Who is the winner in a fair game of this sort, the beginning player or his opponent?

2. Factor:

   a) $x^8 + x^4 + 1$ (into 3 factors)

   b) $x^5 + x + 1$ (into 2 factors)

3. From the vertex B of the triangle ABC draw the median and the altitude. Assume that they divide $\angle ABC$ into three equal parts. Determine the measures of the angles of $\triangle ABC$.

4. Four children—A, B, C, and D—participate in a race. After the race, each asks the other in what position he finished. A answers, "I wasn't first and I wasn't last." B answers, "I wasn't last." C answers, "I was first." D answers, "I was last." Three of the children have answered honestly, but one has not. Which one has answered incorrectly? Who was first?

5. How many six-digit numbers are there, all digits of which are odd?

6. Prove that in an arbitrary triangle

   a) the sum of the lengths of the medians is less than the perimeter, and

   b) the sum of the lengths of the medians is more than ¾ of the perimeter.

7. On a table lie some books that must be wrapped. If they are wrapped four, five, or six to a package, then one extra book remains; but if they are wrapped seven to a package, then there are no extras. How many books could there be on the table?

8. Construct a triangle given two of its sides, a and b, if it is known that the angle opposite one of them is three times greater than the angle opposite the other.

9. *a*) Find all numbers satisfying the equation

$$x + y = xy.$$

   b) What kind of positive whole numbers could satisfy the equation

$$x + y + z = xyz?$$

10. A four-digit number is multiplied by the four-digit number obtained by writing its digits in opposite order. The eight-digit number obtained has zeros as its last three digits. Find all such four-digit numbers.

11. *a*) Construct the circle tangent to both a given circle at a given point and a given line.

b) Construct the circle tangent to both a given circle and a given line at a given point on the line.

12. a) How many roots does the following equation have?

$$x^2 - 3x + 1 = 0.$$

b) Sketch the graph of

$$y = x^2 - 3x + 1.$$ [25]

Packets of entrance examinations and brochures explaining the purpose and organization of the correspondence school were circulated by the Ministry of Education to all eighth-grade classes in selected districts of the Republic. Brigades of students in the mathematics-mechanics faculty of Moscow State University were recruited to evaluate the completed entrance examinations. More than 5,500 completed tests were received before the deadline. Some 500 were submitted later. In all, 1,429 pupils completed no fewer than seven or eight problems correctly—the minimum standard for admission to the new school. Of this number, 96 were excluded because they lived in regions where alternate opportunities for an "intensified mathematical education" existed. Of the 1,429 achieving passing marks on the examination, 813 were from villages and towns with less than 30,000 population, 310 from towns with population between 30,000 and 100,000, and the remaining 306 from urban centers.[26] These statistics indicate that the goal of reaching into "the most distant corners" for mathematical talent was achieved at least partially.

The aim of the organizers of the mathematics correspondence school "is to make it a real school—with systematic lessons, testing and evaluation procedures, and . . . active alumni."[27] Each lesson prepared and submitted by the pupil is to be carefully graded. More than 150 students and faculty members at Moscow State University have been recruited for this work. Each "teacher" is assigned approximately ten pupils for whose mathematical development he is personally responsible. Pupils receive at most ten formal lessons each year, some of them in two parts or on two levels. In addition, each pupil is expected to select one or two projects or areas of special study from a list of "faculty activities." These projects are to occupy the pupil's time between and after regularly assigned lessons. Pupils receive a collection of problems in mathematics as a part of their correspondence kits. This list contains not only problems pertaining to and required for the obligatory lessons but also eighty-two supplementary problems varied in both content and difficulty. Many of these problems were drawn from the materials used by Professor Dynkin in the Moscow State University Youth Mathematics School.[28]

The nature, content, and scope of the obligatory lessons distributed to correspondence pupils are still subject to experimentation. School planners are not striving to include a great many supplementary topics far beyond the scope of the regular school program, nor have they felt compelled to touch upon all aspects of the usual school course. For example, the topic "derivatives and integrals" is not a part of the experimental correspondence syllabus, although it is included in the usual school programs.

The first lesson in the correspondence sequence is entitled "The Method of Coordinates." [29] The first topic of this lesson duplicates material presented under a similar heading at ordinary secondary schools—coordinates on a line, the concept of absolute value, the distance between points. Two-dimensional coordinate systems are then introduced, and the formula for the distance between two points in a plane is developed. Various point sets within a plane are described algebraically. Many concepts are developed by means of problems. The first portion of the lesson concludes with a discussion of coordinate systems in space.

The second portion of the coordinate unit deals with the significance of coordinate methods in mathematics and physics and with the relationship between algebra and geometry. As a final topic, pupils are introduced to coordinate systems in four dimensions, the four-dimensional cube, and the extension of coordinate methods to spaces of higher dimension.

Usually the pupils' written work consists of a large number of problems and some supplementary proofs. Often pupils are expected to extend a particular problem, for example, "to suggest a method of choosing a coordinate system that would simplify the problem just solved." [30] Correspondence pupils from the same school or community are permitted to complete lessons collectively. In the opinion of school officials, "nothing but good can come from collective work—in the first place, such groups [of pupils who work collectively] can be formed by degrees into a first-class school circle; and, in the second place, at the present time in science basic strength rests, not with the individual, but with the group." [31]

An interesting feature is the fact that all correspondence lessons are to be published by the Science Publishing House for sale to the public. Thus lessons will be available to all interested persons rather than just to matriculated pupils. Selected problems from the work of the school are to be published regularly in the journal *Matematika v Shkole.*[32] Evaluation, however, will be available only to pupils formally enrolled in the correspondence school.

Since the Republic Mathematics Correspondence School is a new venture, it is not possible to gauge its effectiveness accurately or to predict its future course. Initial interest in the project has been high. Responses from pupils have been encouraging.[33] More than a thousand pupils were reported to have been promoted from the first to the second year in 1965[34] and again in 1966. If correspondence study proves a practicable supplement to the usual school mathematics program, expansion of the project to other regions and disciplines certainly will follow.

REFERENCES

1. S. V. Smirnov, "Pervye Shagi Iunosheskoi Matematicheskoi Shkoly," *Matematika v Shkole* (March-April 1960), pp. 145–57.

2. S. V. Smirnov, "Iunosheskaia Matematicheskaia Shkola pri Ivanovskom Pedinstitute," *Matematika v Shkole* (September-October 1962), p. 52.

3. "Proiekt Polozheniia o Matematicheskoi Iunosheskoi Shkole pri Ivanovskom Gosudarstvennom Pedagogicheskom Institute," *Matematika v Shkole* (March-April 1960), p. 47.

4. Smirnov, "Pervye Shagi," p. 45.

5. Smirnov, "Iunosheskaia Matematicheskaia Shkola," p. 52.

6. Smirnov, "Pervye Shagi," p. 45.

7. *Ibid.*

8. *Ibid.*

9. *Ibid.*

10. Smirnov, "Iunosheskaia Matematicheskaia Shkola," p. 55.

11. *Ibid.*, p. 56.

12. Smirnov, "Pervye Shagi," p. 46.

13. *Ibid.*

14. *Ibid.*

15. I. V. Girsanov and E. A. Morozova, "Shkoly Iunykh Matematikov," *Matematika v Shkole* (March-April 1960), pp. 47–49.

16. *Ibid.*

17. V. B. Zel'tzman, "Pervye Gody Raboty Iunosheskoi Matematicheskoi Shkoly," *Matematika v Shkole* (September-October 1962), pp. 58–59.

18. M. A. Svirelkin, "Iunosheskaia Matematicheskaia Shkola v Iaroslavle," *Matematika v Shkole*, (May-June 1963), pp. 84–85.

19. I. D. Stepanov, "Irkutskaia Iunosheskaia Matematicheskaia Shkola," *Matematika v Shkole* (May-June 1963), p. 85.

20. V. V. Ziablitzkii, "Shkola Iunykh Matematikov," *Matematika v Shkole* (January-February 1963), pp. 73–75.

21. Statement by I. M. Gel'fand, personal interview, May 1964.

22. B. R. Vainberg, E. G. Glagoleva, and N. Kn. Rozov, "Zaochnaia Matematicheskaia Shkola pri Moskovskom Universitets," *Matematika v Shkole* (September-October 1964), p. 60.

23. *Ibid.*, p. 61.

24. *Ibid.*

25. *Ibid.*, pp. 61–62.

26. *Ibid.*, p. 62.

27. *Ibid.*

28. *Ibid.*, p. 63.

29. *Ibid.*

30. *Ibid.*
31. *Ibid.*, pp. 63–64.
32. *Ibid.*, p. 64.
33. *Ibid.*, p. 62.
34. "Novyi Priem v Respublikanskovo Zaochnuiu Matematicheskuiu Shkoly," *Matematika v Shkole* (May-June 1965), p. 69.

VIII

Summary and Conclusions

RUSSIAN school mathematics programs have a long and interesting history. Tsarist school mathematics curricula were comparable to those of other European countries. Methodological practices in nineteenth-century Russian schools suffered from the usual disabilities of the period. Initial Soviet progressive alterations in the school mathematics program were superseded in 1934 by an academically oriented curriculum; in 1958 by a polytechnic program combining labor and learning; and, more recently, in 1964, by a return to a curriculum that does not include concurrent work experience.

The dilution of the mathematics program that resulted in 1958 from inclusion of "production practice" within the school program led to the founding of special schools with enriched mathematics curricula. Authorized in 1959 in response to requests from the mathematics community, more than 100 mathematics secondary schools were in operation at the beginning of the 1964/65 academic year. Secondary schools for mathematically talented pupils remained an established segment of the Soviet educational program following the 1964 reforms. All types of mathematics secondary schools—the day school for computer programmers, the mathematics-physics boarding schools and the various types of part-time mathematics schools—have as a principal goal the provision of "an intensified mathematical education for pupils with interests in mathematics." [1] The day schools have as their purpose "the preparation of personnel for operation of modern computing and control equipment." [2]

No effort appears to have been spared to expand and develop these schools and their curricula. A teacher-education program designed to prepare mathematics instructors for secondary day schools with specialization in computer programming was organized in 1963 and is now under way. Textbooks for special schools are in preparation. Eminent mathematicians are participating actively in the work of all types of mathematics secondary schools. The special status enjoyed by mathematics secondary schools is evidenced by their exemption from some aspects of the August 1964 regulations "concerning the shortening of the period of secondary education." [3] Secondary schools with specialization in mathematics were permitted to extend their school year by four weeks by deleting a portion of the normally mandatory summer work experience.

The curricula of both types of full-time schools can be characterized as intensive and of high mathematical quality. The intensity of these programs is manifested not only in the vast quantity of topics, concepts, and skills included but also in the high degree of concentration—of specialization—in mathematics. Part-time curricula, too, are impressive. The new Moscow State University Mathematics Correspondence School holds unusual promise for providing opportunities for talented rural youth.

Pupils attending mathematics day schools accumulate a minimum of 977 class hours of instruction in mathematics. The total for three-year mathematics boarding schools was approximately 2,000 hours. After reorganization as two-year boarding schools, the instructional total in mathematics should be approximately 1,333 class hours. Pupils matriculating at part-time mathematics secondary schools, as well as at regular secondary schools, accumulate a total of approximately 750 class hours of instruction in mathematics. Regardless of type, the amount of mathematics instruction in special mathematics secondary schools *far exceeds* the 420 class hours of instruction in mathematics required in ordinary two-year secondary schools.

Special mathematics secondary schools already appear to be proving their worth. More than 50 percent of all graduates of Soviet secondary schools offering an intensified mathematical education enter institutions of higher education as full-time students. The majority pursue studies in some area of mathematical specialization. Remaining graduates accept employment in mathematical specialties and attend evening classes. The immediate contribution of these schools to the Soviet economy is negligible, however, in comparison with their potential contribution to the mathematical community. What impact a potential annual production of ten thousand Soviet pupils with "intensified mathematical education" will have upon the future of world science will not be clear in this decade. It cannot fail to be significant.

REFERENCES

1. *Programmy po Matematicheskim Distziplinam dlia Srednik Shkol s Proizvodstvennym Obucheniem po Professii "Vychislitel'-Programmist,"* p. 6.

2. *Ibid.*

3. *Uchitel'skaia Gazeta,* August 13, 1964.

Appendixes

APPENDIX A

SECOND VARIANT 1959/60 ACADEMIC PLAN FOR SECONDARY SCHOOLS WITH PRODUCTION TRAINING

ACADEMIC SUBJECT	HOURS PER WEEK ACCORDING TO GRADE			TOTAL HOURS	
	Grade 9	Grade 10	Grade 11	Per Week	Per Year
Literature	3	3	3	9	339
Mathematics	4	4	4	12	452
History	2	3	4	9	335
Constitution of the U.S.S.R.	0	0	2	2	70
Economic Geography	0	2	2	4	148
Physics	4	4	2	10	382
Astronomy	0	1	0	1	39
Chemistry	2	3	2	7	265
Biology	3	0	0	3	117
Drawing	2	0	0	2	78
Foreign Language	2	2	3	7	261
Physical Education	2	2	2	6	226
Total Hours in Academic Subjects	24	24	24	72	2,712
General-technical subjects, production training (theoretical and practical), and productive labor	12	12	12	36	1,356
Total Hours in All Subjects	36	36	36	108	4,068
Faculty Activity	2	2	2	6	226

APPENDIX B

1959/60 Mathematics Syllabus for Grades 5–10 of the Soviet Ten-Year School

Grade 5 (Age 11–12)

Arithmetic (6 hours per week, 198 hours in all)

1. Whole numbers (20, 8)*
2. Divisibility of numbers (20, 8)
3. Common fractions (90, 36)
4. Decimal fractions (50, 20)
5. Practical work (6)
6. Review (12, 6)

Grade 6 (Age 12–13)

(6 hours per week, average, 198 hours in all)

Arithmetic (4 hours per week in first semester, 66 hours in all)

1. Percent (20, 10)
2. Proportion; directly and inversely proportional quantities (36, 16)
3. Review (14, 7)

Algebra (4 hours per week in the second semester, 66 hours in all)

1. Algebraic expressions; equations (16, 8)
2. Positive and negative numbers (20, 10)
3. Operations on integral algebraic expressions (30, 15)

Geometry (4 hours per week in the second semester, 66 hours in all)

1. Basic concepts (14, 7)
2. Parallelism (16, 8)
3. Triangles (32, 16)
4. Practical activity (4)

Grade 7 (Age 13–14)

(6 hours per week, 198 hours in all)

Algebra (4 hours per week, 132 hours in all)

1. Factorization of polynomials (36, 18)
2. Algebraic fractions (24, 12)
3. Equations of the first degree in one unknown (34, 17)
4. Equations in two unknowns, systems of equations (28, 14)
5. Review (10, 5)

Geometry (2 hours per week, 66 hours in all)

1. Quadrilaterals (26, 13)
2. Circles (34, 17)
3. Practical activity (6)

Grade 8 (Age 14–15)

(6 hours per week, 198 hours in all)

Algebra (4 hours per week in the first semester and 3 hours per week in the second, 116 hours in all)

1. Powers and roots (44, 22)
2. Equations of the second degree and those reducible to this form (42, 21)

* The first number in the parentheses indicates the hours spent in class; the second, the hours spent on homework.

3. Functions and graphs (12, 6)
4. Systems of equations of second degree (18, 9)

Geometry (2 hours per week in the first semester and 3 hours per week in the second semester, 82 hours in all)

1. Ratio and proportionality of segments (10, 5)
2. Homotheticity and similarity (18, 9)
3. Metric relations in a triangle and in a circle (36, 18)
4. Measurement of the areas of polygons (14, 7)
5. Practical activity (4)

Grade 9 (Age 15–16)
(6 hours per week, 198 hours in all)

Algebra (2 hours per week, 66 hours in all)

1. Limits (6, 3)
2. Progressions (14, 7)
3. Exponential and logarithmic functions, logarithms (40, 20)
4. Practical activity in calculating with the slide rule (6)

Geometry (2 hours per week, 66 hours in all)

1. Regular polygons (12, 6)
2. Length of circumference and area of a circle (10, 5)
3. Solid geometry (40, 20)
4. Practical activity (4)

Trigonometry (2 hours per week, 66 hours in all)

1. Trigonometric functions of an arbitrary angle (10, 5)
2. Algebraic relations among trigonometric functions of the same angle, conversion formulas (16, 8)
3. Trigonometric functions of a numerical argument (16, 8)
4. Addition theorems and their corollaries (24, 12)

Grade 10 (Age 16–17)
(6 hours per week, 198 hours in all)

Algebra (2 hours per week in the first semester and 3 hours per week in the second, 82 hours in all)

1. Permutations and Newton's binomial theorem (12, 6)
2. Complex numbers (12, 6)
3. Inequalities (30, 15)
4. Equations of higher degree (12, 6)
5. Review (16, 8)

Geometry (2 hours per week, 66 hours in all)

1. Polyhedra (28, 14)
2. Solids of revolution (20, 10)
3. Review and the solution of problems (18, 9)

Trigonometry (2 hours per week in the first semester and 1 hour per week in the second, 50 hours in all)

1. Solution of triangles (24, 12)
2. Practical activity in the field (6)
3. Review of trigonometry and the solution of problems in trigonometry and in geometry with application of trigonometry (20, 10)

APPENDIX C

Original Shvartzburd-Ashkinuze Computer-Programmer Syllabi

Grade 9

Algebra and Elementary Functions	*Geometry*	*Trigonometry*	*Approximate Computation*
3 hours per week 90 hours in all	2 hours per week 60 hours in all	2 hours per week 60 hours in all	96 hours in all
1. Numerical sequences (10) 2. Limits (12) 3. Mathematical induction (4) 4. Exponential and logarithmic functions (36) 5. General concept of function (12) 6. Inequalities (12) 7. Review (4)	1. Regular polygons (10) 2. Circumference and area of a circle (14) 3. Coordinate geometry (32) 4. Review (4)	1. Trigonometric functions of an arbitrary angle (15) 2. Relations among trigonometric functions (5) 3. Conversion formulas and their consequences (10) 4. Addition formulas and their consequences (30)	1. Concept of rounding off (6) 2. Solution of systems of linear algebraic equations (40) 3. Tables and interpolation (30) 4. The slide rule (10) 5. Graphical calculation and nomography (10)

Grade 10

Algebra and Elementary Functions	*Geometry*	*Trigonometry*	*Approximate Computation*
	3 hours per week 117 hours in all		45 hours in all
	5. Solution of triangles (15) 6. Geometric transformations (22)		6. Numerical solution of algebraic and transcendental equations (20)

Grade 10 (Cont.)

Algebra and Elementary Functions	Geometry	Trigonometry	Approximate Computation
4 hours per week 156 hours in all	3 hours per week 117 hours in all		45 hours in all
	7. Axioms for solid geometry (3)		7. Numerical integration (15)
8. The derivative and its applications (85)	8. Parallelism in space (15)		8. Numerical solution of differential equations (10)
9. The indefinite integral (15)	9. Perpendicularity in space (17)		
10. The definite integral (36)	10. Loci and transformations in space (14)		
11. Series (20)	11. Polyhedra (31)		

Grade 11

Algebra and Elementary Functions	Geometry	Trigonometry	Approximate Computation
2 hours per week 70 hours in all	2 hours per week 70 hours in all		
12. Complex number (16)	12. Solids of revolution (28)		
13. Equations of higher degree (12)	13. Review (32)		
14. Combinations and elementary probability theory (19)	14. Concluding remarks concerning the mathematics course (10)		
15. Review (23)			

APPENDIX D

Revised Experimental Plan and Syllabi for Secondary Schools with Specialization in Computer Programming

Academic Subject	Hours per Week According to Grade			Total Hours	
	Grade 9	Grade 10	Grade 11	Per Week	Per Year
Literature	3	3	3	9	339
Mathematics	8	7/6	4	18.5	703
History and Political Science	3	3	4/6	11	410
Economic Geography	2	2/0	0	3	112
Physics	6	4	3	13	495
Astronomy	0	0	2/0	1	34
Chemistry	2	2	3	7	261
Biology	0	2	0	2	78
Drawing	2	0	0	2	78
Foreign Language	3	3	3	9	339
Physical Education	2	2	2	6	226
Hours in All Academic Subjects	31	26.5	24	81.5	3,075
Technical Subject					
Electronics and Radio Technology	0	2/4	2/0	4	158
Approximate Computation	3	4/0	0	5	185
Mathematical Machines and Programming	0	0/4	3	5	193
Practical Work with Small Machines	2	2/3	0	4.5	178
Practical Work with Large Machines	0	0	7/9	8	281
Hours in All Technical Subjects	5	9.5	12	26.5	893
Totals	36	36	36	108	4,068
Faculty Activity	2	2	2	6	226

Mathematics Syllabus for Computer Schools

Grade 9

Algebra and Elementary Functions	*Geometry*	*Trigonometry*
4 hours per week 156 hours in all	2 hours per week 78 hours in all	2 hours per week 78 hours in all
1. Numerical sequences (12)	1. Regular polygons (12)	1. Vectors (10)
2. Limits (15)	2. Circumference and area of circles (16)	2. Trigonometric functions (15)
3. Exponential and logarithmic functions (36)	3. Coordinate geometry (42)	3. Relations among the trigonometric functions (10)

4. General concept of function (12)	4. Review (8)	4. Conversion formulas and their consequences (10)
5. Inequalities (14)		5. Addition formulas and their consequences (25)
6. The derivative and its application (61)		6. Review (8)
7. Review (6)		

Grade 10

Algebra and Elementary Functions	*Geometry*	*Trigonometry*
3 hours per week 117 hours in all	2 hours per week, first semester 3 hours per week, second semester 100 hours in all	2 hours per week, first semester only 34 hours in all
	5. Geometric transformations (20)	
	6. Axioms for solid geometry and their consequences (3)	
	7. Parallelism in space (14)	7. Applications of the derivative to investigation of the trigonometric functions, harmonic motion (10)
8. Indefinite integral, simple differential equations (30)	8. Perpendicularity in space (15)	8. Trigonometric equations (8)
9. The definite integral (40)	9. Loci and transformations in space (10)	9. Solving triangles (10)
10. Series (30)	10. Polyhedra (32)	10. Review (6)
11. Review (17)	11. Review (6)	

Grade 11

Algebra and Elementary Functions	*Geometry*	*Trigonometry*
2 hours per week 70 hours in all	2 hours per week 70 hours in all	
12. Complex numbers (16)	12. Solids of revolution (28)	
13. Equations of higher degree (12)	13. Review of plane and solid geometry (32)	

14. Combinatorial analysis and elements of probability theory (19)
15. Review and solution of problems (23)

14. Concluding remarks about the mathematics course (axioms, number and space, theory, and application) (10)

PROGRAMMING SYLLABUS

GRADE 9

Numerical Methods

3 hours per week
117 hours in all

1. The concept of error (10)
2. The elements of linear algebra, the solution of systems of linear algebraic equations (48)
3. Tables and interpolation (34)
4. The slide rule (10)
5. Graphical calculation and nomography (15)

Mathematical Machines and Programming

Practical Work with Machines (dependent upon facilities)

78 hours

The study of principles of a desk calculator and methods of carrying out operations on it

Tabulation of elementary functions

The solution of linear equations (various methods)

Finding the product of two matrices, inverse of a matrix

GRADE 10

Numerical Methods

4 hours per week, first semester
88 hours in all

6. Numerical solution of algebraic and transcendental equations (24)
7. Numerical integration (24)
8. Numerical solution of differential equations (Euler, Runge-Kutta, Adams-Shtermer methods) (20)

Mathematical Machines and Programming

4 hours per week, second semester
88 hours in all

1. General characteristics of mathematical machines (30)
2. UTsVM (general-purpose digital computer) (26)

Practical Work with Machines

100 hours

The solution of algebraic equations

Numerical integration of functions

Numerical integration of differential equations on desk calculators

	3. Numeration systems (12) 4. The BECM-2 (high-speed electronic computer type 2) (20)	

GRADE 11

Numerical Methods	*Mathematical Machines and Programming*	*Practical Work with Machines*
	3 hours per week 105 hours in all	281 hours
	5. The command system on the BECM-2 and principles of program preparation (40) 6. Flow diagrams (15) 7. Control methods (15) 8. Exercises in programming numerical problems (35)	Tabulation of functions, numerical integration of functions, numerical integration of differential equations and systems of differential equations on electronic computers Each pupil in Grade 11 should complete independently 2 or 3 projects involving all phases of the solution of problems on an ABTs-VM (automatic high-speed digital computer): preparation of programs, pilot calculations, running programs, etc.

APPENDIX E

FIRST VARIANT 1963/64 ACADEMIC PLAN FOR SECONDARY SCHOOLS WITH PRODUCTION TRAINING

ACADEMIC SUBJECT	HOURS PER WEEK ACCORDING TO GRADE		
	Grade 9	Grade 10	Grade 11
Literature	5, 4	3	3
Mathematics	6	4	4
History	3, 4	4	2, 4
Social Science	0	0	2
Geography	0	2	0
Biology	2	0	0
Physics	5	4	3
Astronomy	0	1	2, 0
Chemistry	4	2	2
Foreign Language	2	2	2
Drawing	1	0	0
Physical Education	2	2	1
Fundamentals of Production	0	0	2
Total Hours in Academic Subjects	30	24	24
General-technical subjects, production training (theoretical and practical), and productive labor	6	12	12
Total Hours in All Subjects	36	36	36
Production practice			
in days	36	0	0
in hours	216	0	0
Faculty Activity	2	2	2

APPENDIX F

Examination Tickets for Computer-Programmer Trainees

Grade 9 Tickets (1963)

Ticket 1

1. Numerical sequences. Arithmetic and geometric sequences. Formula for the nth term and the sum of the first n terms of a progression.
2. Example of the solution of equations by the method of iteration.

Ticket 2

1. Definitions of limit of a sequence and limit of a variable quantity. Examples.
2. Vectors of n dimension. Addition of n-dimensional vectors. Scalar multiplication and multiplication using matrices.

Ticket 3

1. Derivatives of exponential and logarithmic functions.
2. Matrices. Addition of matrices; multiplication of a matrix by a number. The identity matrix and the inverse of a matrix.

Ticket 4

1. Exponential functions. Examples.
2. Linear interpolation.

Ticket 5

1. Logarithmic functions. Examples.
2. Solution of systems of linear equations using determinants; Cramer's method.

Ticket 6

1. The number e.
2. Absolute error of an approximate number. Sum of approximate numbers. Difference of approximate numbers.

Ticket 7

1. Solution of systems of linear equations by the method of Gauss.
2. Definition of function. Monotonicity, evenness or oddness, and periodicity of functions.

Ticket 8

1. Circumference and area of a circle.
2. Multiplication of matrices.

Ticket 9

1. Scalar product of two vectors and its properties.
2. Logarithmic scale and the slide rule.

Ticket 10

1. Lagrange polynomial interpolation.
2. Regular polygons. Expressing the sides of a regular polygon (triangle, square, hexagon) in terms of the radius of the circumscribed circle.

Ticket 11

1. Solution of systems of linear equations by the method of simple iteration.
2. The functions $y = \sin x$ and $y = \cos x$ and their derivatives.

Ticket 12

1. The concept of derivative. The mechanical and geometric meanings of the derivative.
2. Norms of vectors and matrices.

Ticket 13

1. Linear functions.
2. Relative error of an approximate number. Relative errors of the product and ratio of approximate numbers.

Ticket 14
1. Solution of systems of linear equations by the method of Jordan.
2. Base-ten logarithms. Natural logarithms.

Ticket 15
1. Principles of determinants (third order).
2. Derivatives of sums, products, and quotients.

Ticket 16
1. Investigation of functions and sketching graphs using the derivative.
2. Trigonometric functions of double and half angles.

Ticket 17
1. Solution of systems of linear equations by the method of Seidel.
2. Triginometric addition theorems.

Ticket 18
1. Method of undetermined coefficients.
2. Algebraic relations between trigonometric functions of the same argument.

Ticket 19
1. Newton's interpolation formula.
2. Equations of parabolas.

Ticket 20
1. The functions $y = \tan x$ and $y = \operatorname{ctg} x$ and their derivatives.
2. Equations of the circles and the ellipse.

Problems for Grade 9 Examination Tickets (1962)

1. For which values of a does the system
$$3x + 2y = -3$$
$$ax + 3y = 7$$
have negative solutions?
2. Find the roots of the equation $10^x + x = 3$ with accuracy to .01.
3. Prove the inequality
$$\frac{a}{\sqrt{b}} + \frac{b}{\sqrt{a}} \geq \sqrt{a} + \sqrt{b} \text{ for } a > 0,\ b > 0.$$
4. Prove that $\triangle ABC$ is isosceles if $A(-5,2)$; $B(3,6)$; $C(4,-6)$.
5. Prove that quadrilateral $ABCD$ is a square if $A(-3,2)$. $B(1,4)$; $C(3,0)$; $D(-1,2)$.
6. Prove that $\frac{1}{2}$ is the limit of the sequence
$$\left(x_n = \frac{n+2}{2n+5}\right) \text{ as } n \to \infty.$$
7. Investigate the sign of the function
$$f(x) = \frac{3(x-1)(x^2+1)(x-3)}{(2-x)(x+1)}.$$
8. Determine the period of the function
$$y = \sin \frac{3}{4}x.$$
9. Prove that $\tan 3\alpha - \tan 2\alpha - \tan \alpha = \tan 3\alpha \tan 2\alpha \tan \alpha$.
10. Write the equation of the tangent to the hyperbola
$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$
at the point with ordinate $y = -1$.

11. Calculate with slide rule

$$x = \frac{2.71 \cdot 0.891}{13.24}.$$

12. Multiply the matrices

$$\begin{pmatrix} 2x & 2y & y^2 \\ x & -2y & z \end{pmatrix} \begin{pmatrix} 2x & x \\ 3x & z \\ y & 7 \end{pmatrix}$$

13. Prove that $8 \cos 10° \cos 50° \cos 70° = \sqrt{3}$.
14. Evaluate the determinant

$$\begin{vmatrix} 3 & 6 & 8 \\ -1 & 2 & 4 \\ 3 & 7 & -5 \end{vmatrix}$$

15. Invert the matrix

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 0 \\ -3 & -2 & 1 \end{pmatrix}$$

16. Solve by the method of Gauss

$$\begin{cases} 3x_1 + 2x_2 + x_3 = 10. \\ 2x_1 - x_2 - x_3 = -3. \\ x_1 + x_2 - 2x_3 = -3. \end{cases}$$

17. Determine the norm of matrix

$$A = \begin{pmatrix} 6 & 3 & -2 \\ 1 & 0 & .7 \\ 2 & 4 & 3 \end{pmatrix}$$

18. Use a table and linear interpolation to determine log 24.67283.
19. Use Lagrange's formula to find $f(0.27)$ for the function $y = f(x)$ if

x	.1	.2	.3	...
y	4	6	3	...

20. Calculate: $\cos 1°24' + \cos 182°$.

Grade 10 Tickets (1962)

Ticket 1

1. Derivatives of exponential and logarithmic functions.
2. Direct and supplementary methods for expressing numerals in the binary system of numeration. Expression of numerals in the binary and octal systems. Translation from decimal notation to binary and vice versa. Translation of numerals from the octal system to the binary and vice versa.

Ticket 2

1. Application of the derivative to the solution of maximum and minimum problems.
2. Expressing numerals, in decimal and binary systems with "fixed and floating" point. Writing numerals in normalized form. The range of representation in the machine BESM-2.

Ticket 3

1. The differential function and its application to the approximation of functions.
2. Volume of solids.

Ticket 4

1. Functions and the indefinite integral.
2. Basic conc pts of mathematical logic. Fundamental logical operations and their realizations in the UTsVM.

Ticket 5

1. The concept of ordinary equations. Initial conditions. Examples of physical problems solved with the aid of differential equations.
2. Arithmetic principles of the UTsVM.

Ticket 6

1. Block-plan of the UTsVM. Basic parts of the UTsVM, their significance, and their relationships. Operating principles of the UTsVM.
2. The definite integral as the limit of a sum.

Ticket 7

1. Newton's method.
2. Simple systems of automatic control and feedback. Simple functional arrangements.

Ticket 8

1. Convergence of numerical series. Necessary and sufficient conditions for convergence of numerical series. Examples.
2. Machines of continuous operation. Mathematical models. Hydraulic integrator.

Ticket 9

1. Concept of expansion of functions in series. Taylor's and Maclaurin's series.
2. Punch cards. Punch-card computing machines and their significance. Basic arrangement of the tabulator and its operating principles.

Ticket 10

1. Motions in geometry. Examples of motions.
2. Classifications of mathematical machines. Small digital machines and basic principles of their construction and operation.

Ticket 11

1. Bezout's theorem. Horner's method. The roots of a polynomial.
2. Volume of complete and truncated pyramids.

Ticket 12

1. Algebraic equations of nth degree. Roots. Newton's method for determining real roots.
2. Volumes of right and oblique parallelepiped.

Ticket 13

1. Calculating the roots of the equation $f(x) = 0$ with given accuracy. Methods of chords and tangents.
2. Volumes of rectangular parallelepipeds.

Ticket 14

1. Calculating the roots of the equation $f(x) = 0$ with given accuracy. Determining the roots of algebraic and transcendental equations by graphical methods. Method of halving segments. The conditions for the existence of a unique root of the equation $f(x) = 0$ on the segment $[a,b]$.
2. Laws of sines and cosines.

Ticket 15

1. Calculating the real roots of an algebraic equation by the method of Lobachevskii.
2. Properties of the plane angles of trihedral and polyhedral angles.

Ticket 16

1. Geometric meaning of the definite integral. Trapezoidal and rectangular rules.
2. Error. Conditions of parallelism for lines and planes.

Ticket 17

1. Numerical integration. Simpson's method. Error.
2. Angles of lines with planes.

Ticket 18

1. Numerical integration of differential equations by Euler's method.
2. Conditions for parallelism of planes.

Ticket 19

1. Numerical integration of differential equations by the method of Adams-Shtermer.
2. Theorem about two perpendiculars.

Ticket 20

1. Numerical integration of differential equations by the method of Runge-Kutta.
2. Conditions for perpendicularity of lines and planes.

Problems for Grade 10 Examination Tickets (1962)

1. If a pyramid is to have all dihedral angles at its base equal,
   a) what forms of triangles and what forms of parallelograms may serve as its base?
   b) what property should each quadrilateral serving as a base of such a pyramid have?
2. In a regular triangular pyramid the apothem is a. Does there exist a value for the altitude of the pyramid x for which the volume has its greatest value? If it exists, then calculate it and explain the peculiarities of the obtained pyramid.
3. Calculate
$$\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{2\,dx}{3\cos^2 2x}$$
4. Calculate using the trapezoidal rule
$$\int_1^3 (x^2 - 2x)dx \text{ for } h = .5\,.$$
5. A window has the form of a rectangle surmounted by an equilateral triangle. The perimeter of the window is $3m$. What should the base of the triangle be if the window is to be of maximum area?
6. Through a given point A construct a plane perpendicular to a given plane P.
7. Find the locus of points from which a given segment subtends less than a right angle.
8. The edge of a rectangular tetrahedron is a. Find its volume.
9. The volume of a right quadrilateral prism equals 8 dm^3. What should the side of the prism's base be in order that the surface area be the least?
10. Find the area of the figure bounded by the curve $y = 2x^2$, the x-axis, and the lines $x = 2$ and $x = 4$.
11. Calculate using Simpson's rule
$$\int_0^1 x^2dx; \qquad h = .25\,.$$
12. Find $f'(1)$ for $f(x) = 2\,ln(x + 1)$.
13. The base of a pyramid is a rhombus with side 6 cm. Two lateral faces of the

pyramid, perpendicular to the plane of the base, meet at an angle of 120°. The altitude of the pyramid is 3 cm. Find the lateral area of the pyramid.

14. The base of a pyramid is an isosceles trapezoid the bases of which are a and b. Each of the dihedral angles at the base of the pyramid is 45°. Find the volume of the pyramid.
15. Through a given line a, draw a plane parallel to a given line b.
16. Use Euler's method to find the solution of the differential equation $y' = y - 4x + 7$ if $x_0 = 0$, $y_0 = 1$, $h = .1$, and $x_k = .5$.
17. Calculate $\int e^{\cos x} \sin x \, dx$.
18. Calculate $\int \sin^3\phi \, d\phi$.
19. Approximate the roots of the equation $x^3 + 3x + 1 = 0$ by the method of chords and tangents on the segment $[^{-}1, {}^{-}.1]$.
20. The lateral edges of a pyramid are equal. What forms of triangles, parallelograms, and trapezoids may serve as the base of such a pyramid?

APPENDIX G

Supplementary Questions for Computer-Programmer Examinations

Sample Supplementary Questions for Grade 9

1. State the Pythagorean theorem and its converse.
2. Draw the graph of the equation $x + y = 1$.
3. Which is larger, a or b, if $\log_7 a > \log_7 b$?
4. Compute with slide rule $10^{.301}$.
5. Find the coordinates of the focus of the parabola $y = x^2$.
6. How can one obtain the equation of a circle from the equation of an ellipse?
7. What is an identity matrix?
8. What is the inverse of a matrix?
9. Which is larger: $\log_{1/2}5$ or $\log_{1/4}25$?
10. How can the value of π be calculated with arbitrary degree of accuracy using the double-angle formulas?
11. Does every matrix have an inverse?
12. For which values of a and b is
$$\sqrt{a+b} = \sqrt{a} + \sqrt{b}\ ?$$
13. Express $\frac{1}{2+x}$ as the sum of the terms of an infinite decreasing geometric sequence. For what values of x is this series convergent?
14. What is the period of the function
$$y = |\sin x|;\ y = \sin |x|;\ y = |\sin|x||?$$
15. How many axes of symmetry has the sine curve?
16. The base of an exponential function $a > 1$. In how many points does the line through the origin at an angle of 81° with the positive x-axis intersect the graph of the exponential function?
17. How does one find the period of the function $y = \sin x$?
18. Sketch the graph of $y = x^2 - bx$.
19. Is the function $y = \frac{\sin x}{x}$ odd or even?
20. What sort of graph does the function $y = \log_a(-x)$ have where $a > 0$?

Sample Supplementary Questions for Grade 10

1. What sort of figures are transformed into themselves by a rotation?
2. Given point O and line a beneath it. Into what is the line transformed by a rotation of 60° about O?
3. What is the influence of transformations on geometric theory?
4. What significance does the convergence of an iterative sequence have upon the design of some programs?
5. Does the function $y = 2x - \sin x$ have extrema?
6. Is the property of the plane angles of a tetrahedral angle true for polyhedral angles?
7. Why is the phrase "to two intersecting lines" required in the formulation of the principle of perpendicularity of lines to planes?
8. For what reason is the restriction $a > c$ made in the equation of the ellipse?
9. How does one prove that an iterative sequence converges?

10. At what point does the hyperbola

$$\frac{x^2}{9^2} - \frac{y^2}{b^2} = 1$$

intersect the x-axis?

11. How does one differentiate between the graphs of a parabola and one branch of a hyperbola?

12. How does one carry out calculations with numerals expressed in an arbitrary numeration system utilizing a "floating" decimal point?

13. Express the numeral 23 in base two.

14. What is an asymptote?

15. What sort of polygons can serve as the base of a pyramid in which all dihedral angles at the base are equal?

16. Given the numeral .1 in the binary system, what is the numeral in the decimal system?

11. How does one differentiate between the graphs of a parabola and one branch of two parallel planes in which these lines lie. Is it permissible to interpret this distance as the distance between two arbitrary points of these lines?

18. Is a continuous function necessarily differentiable?

19. Give an example of a necessary but not a sufficient condition for some mathematical fact.

20. Is there any way in which to cut a cube with a plane so that the cross section is an octagon?

APPENDIX H

Abstracts of Course Syllabi for Pedagogical Institutes Preparing Teachers for Computer-Programmer Schools

Mathematical Analysis

First semester

A. Introduction to analysis
   1. Functions
   2. Limits
   3. Continuity

B. Differential calculus for functions of one variable
   4. Derivative
   5. Differential

Second semester

   6. Properties of functions continuous on an interval
   7. Basic properties of the differentiable functions and their applications

C. Basic elementary functions
   8. Elementary algebraic functions
   9. Elementary transcendental functions

D. Integral calculus for functions of one variable
   10. Indefinite integral
   11. Definite integral
   12. Applications of the definite integral

Third semester

E. Series
   13. Numerical series
   14. Functional series
   15. Power series
   16. Expansion of functions in power series

F. Differential calculus for functions of several variables
   17. Functions, limits, continuity
   18. Partial derivatives and differentials
   19. Implicit functions
   20. Maxima and minima of functions of several variables

G. Integral calculus for functions of several variables
   21. Multiple integrals

Fourth semester

   22. Line integrals

H. Differential equations
   23. General concepts
   24. Equations of the first order
   25. Theorems on existence and uniqueness
   26. Equations not solvable for the derivative; reduction in order
   27. Linear equations of higher order

Higher Algebra

1. Systems of linear equations
2. Permutation, transposition, substitution of variables
3. nth-order determinants and their basic properties
4. System of n linear equations in n unknowns with determinants different from 0
5. n-dimensional vectors

6. Arbitrary systems of linear equations; criteria for consistency
7. Linear transformations and square matrices
8. Rings and fields
9. Complex numbers
10. Theory of divisibility in the ring of polynomials over a given field
11. Factorization of polynomials over a field into powers of linear factors $(x - a)$
12. Polynomial in one unknown with complex coefficients
13. Determining the complex roots of polynomials with real coefficients
14. Simple extension of numerical fields
15. Binomial equations, equations of higher degree
16. Polynomials in one unknown with real coefficients
17. Polynomials in one unknown with rational coefficients
18. Ring of polynomials
19. Resultant and its basic properties, Bezout's theorem
20. Groups

Analytic Geometry

1. Geometry on the line
   a) Coordinatization of the line
   b) Distance
2. Geometry in the plane
   a) Vectors and vector algebra
   b) Affine coordinates
   c) Transformation of coordinates
   d) Polar coordinates
   e) Equations of lines and curves
   f) Ellipse, hyperbola, parabola
   g) General theory of second-order curves
3. Geometry in space
   a) Coordinates in space
   b) Vectors in space, parallelism, and perpendicularity
   c) Parametric equation of planes
   d) Distance and angle between planes, perpendicularity of planes
   e) Equations of lines in space
   f) Distance from a point to a line, distance between two lines, angle between lines, perpendicularity
   g) Problems on lines and planes in space
   h) Surfaces and surfaces of revolution
   i) General equation of the second degree

Theory of Functions of a Real Variable

1. General theory of sets
   a) Set-theoretic operations
   b) Set equality
   c) Countable sets
2. Theory of point sets
   a) The real continuum, existence of transcendental numbers
   b) *n*-dimensional Euclidean space
   c) Neighborhoods, Bolzano-Weierstrass theorem
   d) Open and closed sets
   e) Cantor set
   f) Open coverings

3. Functions
   a) The general concept of function
   b) Monotonic functions
   c) Continuity, Peano and Jordan curves
4. Measure and integral
   a) Volume and area, measure and conditions for measurability (Jordan)
   b) Upper and lower integrals, Riemann integral, the integral as a measure, expressing measures as integrals

Theory of Functions of a Complex Variable

1. Fundamental concepts
2. Functions of a complex variable, derivatives and their geometric meaning, analytic functions
3. Elementary functions and their conformal maps
4. Series and integrals in the complex domain
5. Cauchy integral and Taylor series
6. Singular points and Laurent series
7. Concept of analytic continuation

Theory of Probability

1. Random events
   a) Definition and fundamental properties of probabilities
   b) An axiomatic approach to probability
2. Random variable
   a) Discrete random variables
   b) Binomial distribution
   c) Measures of central tendency, standard deviation
   d) Chebychev's inequality, Laplace theorems
   e) Continuous and discrete variables
   f) Normal distribution
   g) Law of large numbers, central limit theorem
   h) Coefficient of correlation
3. Random processes
   a) Radioactive decay and the process of Poisson
   b) Communication problems
   c) Markov chains
   d) Continuous random processes
4. Examples of more recent investigation
   a) Information theory
   b) Theory of matric games
   c) Monte Carlo methods

Theory of Numbers

1. Introduction
   a) Historical survey
   b) Russian contributions
2. Residue classes
   a) Complete system of residues
   b) The ring of residue classes
   c) Euler's function, Euler's and Fermat's theorems
3. Congruences with unknowns
   a) Solution of congruences of first degree
   b) Systems of congruences
   c) Congruences of higher degree

4. Quadratic residues
5. Arithmetic applications of the theory of congruences
   a) Divisibility theory
   b) Length of period of rational numbers
6. Approximation of irrational numbers with rationals
   a) Continued fractions
7. Algebraic and transcendental numbers
   a) Irrational numbers (π)
   b) Algebraic numbers, field of algebraic numbers
   c) Transcendental numbers, Gel'fand's result

Foundations of Arithmetic

1. Introduction
   a) The process of counting and the concept of number
   b) Axiomatic development of number systems
2. Natural numbers
   a) Axiomatic development (Peano)
3. Integers
   a) Isomorphism
   b) The problem of extending the concept of number
   c) Operations on integers
4. Rational numbers
   a) Construction of the field of rational numbers
   b) Operations on rational numbers
5. Real numbers
   a) Axioms for the real numbers
   b) Operations on real numbers
   c) The completeness property
6. Complex numbers
   a) Construction of the field of complex numbers
   b) Hypercomplex numbers
7. Conclusion
   a) The significance of the extension of the concept of number and the development of the number concept in the secondary school

Projective and Descriptive Geometry

A. Projective geometry
   1. Fundamental concepts of affine geometry in the plane
   2. Construction of projective space
   3. Elementary projective geometry
   4. Projective theory of curves of the second order
   5. Geometry and a group of transformations
   6. Conclusion
B. Descriptive geometry
   1. Method of projection and its properties
   2. Complete and incomplete representations and their applications in the pedagogical process

Differential Geometry

1. Curves in the plane
   a) The concept of line in differential geometry
   b) Tangents, degree of contact
   c) Length of arc
   d) Evolute and involute

2. Curves in space
   a) Concept of line in space
   b) Vectors as functions
   c) Tangents in space
   d) Taylor's formula, differentiation of vector functions
   e) Length of arc
   f) Normals and binormals
   g) Natural equations of curves
   h) Existence theorem
3. General theory of surfaces
   a) Vector functions of two scalar arguments
   b) Parametric representation
   c) First quadratic form of surfaces
   d) Normal curvature
   e) Second quadratic form of surfaces
   f) Geodesic curvature
   g) Historical survey

Foundations of Geometry

1. History of the development of the axiomatic method
2. The modern period in the foundations of geometry
3. Hilbert's axioms
4. Lobachevskian geometry
5. Riemannian geometry
6. Group-theoretic approach to geometry

Elementary Mathematics

First semester

Arithmetic of rational numbers

1. Whole numbers
2. Divisibility of natural numbers
3. Rational numbers
4. Continued fractions and indeterminate equations

Second semester

Theory and practice of computation

5. Exact computation
6. Elements of approximate computation
7. Aids to approximate computation
8. Graphical solution of equations and numerical methods of extracting roots

Third semester

Geometry

9. Plane figures
10. Similar figures
11. Geometric values (measures of length and angles)
12. Solid geometry
13. Geometric values (measures of area and volume)

Fourth semester

Geometric constructions

14. Constructions with ruler and compass
15. Basic loci in the plane and the use of locus in solving construction problems
16. Geometric transformations in the plane and applications to construction problems
17. Algebraic methods

18. Solvability of construction problems
19. Loci in space

Fifth semester

Algebra

20. Elementary methods of solution of algebraic equations with one unknown
21. Combinations
22. Binomial theorem, polynomial theorem
23. Polynomials in several variables
24. Nonlinear systems in several variables
25. Inequalities

Sixth semester

Irrational and transcendental equations

26. Irrational algebraic expressions in the real domain
27. Exponential and logarithmic functions in the real domain
28. Trigonometric equations in the real domain

Seventh semester

Trigonometry

29. Trigonometric functions of a real argument
30. Inverse trigonometric functions
31. Analytic representation of trigonometric functions
32. Applications of trigonometry to geometry
33. Elements of spherical geometry and trigonometry

Methods of Teaching Mathematics

1. General methods of teaching mathematics
2. Special methods of teaching mathematics in the eight-year school
   a) Methods of teaching arithmetic
   b) Methods of teaching algebra
   c) Methods of teaching geometry
3. Methods of teaching mathematics in the upper grades of the school of general education
   a) Peculiarities of teaching mathematics in the upper grades
   b) Methods of teaching algebra and elementary functions
   c) Methods of teaching geometry
   d) Practical activity
   e) Laboratory activity in the school

APPENDIX I

SYLLABUS FOR THE GOVERNMENT EXAMINATION IN MATHEMATICS FOR PEDAGOGICAL INSTITUTE GRADUATES

Arithmetic

1. Axiomatic construction of the set of natural numbers. The principle of complete mathematical induction and its significance in the secondary school course. Arithmetic operations.
2. The problem of extending the number concept. Consider one of the following questions:
   a) construction of the set of integers
   b) construction of the set of rational numbers
   c) construction of the set of complex numbers
3. Divisibility of integers. Greatest common divisor and least common multiple of several numbers. Fundamental theorem of arithmetic. Current status of questions on the distribution of primes.
4. Basic principles of congruences. Complete and reduced residue systems. Euler's and Fermat's theorems. Arithmetic applications of the theory of congruences.
5. Construction of the set of real numbers. Representation in the form of decimal fractions.
6. Elements of approximate computation. Simple methods of exact and approximate computation.

Algebra

1. Solution and investigation of systems of n linear equations with n unknowns. Elementary methods of solving linear systems. Systems of linear equations with two and three unknowns, their geometric interpretation. General criteria for consistency of an arbitrary system.
2. Questions of equivalence of algebraic equations and systems of equations.
3. Solution of algebraic inequalities and their geometric interpretations.
4. Number rings and fields. General theory of divisibility in a number field.
5. Reducible and irreducible polynomials over a number field. Factoring polynomials into products of irreducible factors.
6. Solution of binomial equations. Existence of roots of polynomials in the field of complex numbers, factoring polynomials into linear factors. Integral and rational roots of a polynomial.

Geometry

1. Mutual relations of lines and planes in space. Study of these questions by methods of elementary and analytic geometries.
2. Study of space curves by the methods of differential geometry.
3. Geometry and groups of transformations. Group of all collineations of the plane and important subgroups. Characteristics of the separate branches of geometry corresponding to these groups.
4. Basic concepts of geometry and their interrelations expressed in an axiom system. Axiom system for elementary geometry. Axiomatic method of constructing a geometry (give some theorems).
5. The concepts of consistency and independence for an axiom system. Analytic interpretation of an axiom system for Euclidean plane geometry.

6. Independence of the parallel axiom from the other axioms. Interpretation of Lobachevskian geometry.
7. Problems on constructions in the plane. Postulate system for ruler-and-compass constructions. Survey of basic methods of problem solving. Solvability criteria for ruler-and-compass construction problems.

Mathematical Analysis and the Theory of Functions

1. Sets and operations on them. The concept of set. Comparing sets. Countable sets and their basic properties. Nondenumerable continua.
2. Set of real numbers and its basic properties. The existence of upper and lower bounds for bounded sets. Limit points of numerical sets. Bolanzo-Weierstrass theorem.
3. Numerical sequences. Concept of limit of a numerical sequence and its properties. The existence of the limit of a monotonic sequence. Necessary and sufficient conditions for convergence of a sequence.
4. Concept of functional dependence. Definition of the limit of a function. Continuity of function at a point and classification of points of discontinuity. Properties of continuous functions on closed bounded sets.
5. Derivatives and differential of functions of one variable and their geometric and mechanical interpretations. Differentiability of functions of two variables. Conditions for differentiability.
6. Applications of differential calculus to the study of functions of one variable (increasing, decreasing, extrema).
7. Definite integral. Theorem on the existence of the definite integral. Concept of area of a plane figure and the length of arc. Their calculation with the aid of the definite integral.
8. Numerical series. Concept of convergence. Absolute and conditional convergence. Power series and their basic properties.
9. Definition of power. Power, exponential and logarithmic functions, and their properties.
10. Functions of a complex variable. Power series in the complex domain. The concept of analytic extension and the uniqueness theorem. Definition of the basic elementary functions with the aid of power series.
11. Ordinary differential equations, their solution, and integral curves. Initial conditions. Linear equations of the second order with constant coefficients and their application in the study of oscillation.

APPENDIX J

Mathematics Syllabus for the Novosibirsk Mathematics-Physics Boarding School

Grade 9

Elements of Differential and Integral Calculus

Topic 1 (lecture 4 hours, problem sessions 12 hours)

The concept of function and the graph of a function. Tangents. The derivative. Velocity. The area of a curvilinear trapezoid. The definite integral and its connection with the derivative. Determining paths from velocities. Examples of simple applications of the derivative and integral to problems in physics, chemistry, and mechanics.

Topic 2 (lecture 16 hours, problem sessions 48 hours)

Vectors and their projections. Addition of vectors, scalar multiplication. Projections of a broken line. Definitions of the sine and cosine by means of projections of the unit vector. Graphs of the sine and cosine. Definitions of the tangent, cotangent, secant, and cosecant functions and their graphs. Addition theorems and their corollaries. Trigonometric function with arguments nx. The derivative of the sine and cosine

$$\lim_{x \to 0} \frac{\sin x}{x}$$

The derivative of a sum, product, and quotient. The derivatives of tangent, cotangent, secant, and cosecant. Integration and its relation to the trigonometric functions. The equation of a pendulum without damping. The concepts of composite and inverse functions. The derivatives of composite and inverse functions. The inverse circular functions, their derivatives, and connection with simple integrals. Integral representation. Some properties of trigonometric polynomials. Orthogonality. Fourier's coefficient formula. Approximate expression of functions by means of trigonometric polynomials. Improvement of approximations. The role of Fourier polynomials. The formula for the mean-square deviation. Bessel's inequality. Perceval's equation for trigonometric polynomials. The order of Fourier coefficients. Krylov's effect. Integration by parts. Examples of calculating Fourier coefficients.

Topic 3 (lecture 16 hours, problem sessions 48 hours)

Algebraic polynomials, rational functions, and other algebraic functions. Integration and differentiation of the function $y - x^n$ for integral n. The function $y = ln\ x$. Properties of logarithms. The inverse function e^x. Derivatives of $ln\ x$ and e^x. Differential equations (for exponential functions). The concept of composite function. Derivatives of composite functions. Integration by substitution. The general power function, its derivative and integral. Algebraic polynomials, their norms, method of secants and tangents, differentiation of polynomials, factorization, binomial formula. Taylor's formula. Interpolation. Determining roots. Method of successive approximation. Strum's method. Rational functions. Factoring rational functions into elementary fractions. Various methods of determining coefficients. Integration of rational expressions. Hermite's method. Complex numbers and operations of them. Trigonometric form of a complex number. De Moivre's formula. Polynomials with complex domain. The theorem of Gauss. Estimation of roots. Conditions guaranteeing the existence of real roots.

Topic 4 (lecture 4 hours, problem sessions 12 hours)

Summary of the formulas of differential and integral calculus and methods of integration. Differentiation of sums, differences, and products of fractions and general powers. Differentiation of composite functions. Derivatives of the basic elementary functions. Tables of elementary integrals. Integration by means of decomposition, change of variables, and by parts. Information about classes of integrable function.

Topic 5 (lecture 16 hours, problem sessions 48 hours)

Elements of the theory of sets, combinatorial topics, mathematical logic. Elements and sets. Subsets. Basic operations—union, intersection, complementation. Principle of duality. Basic combination theory. Permutation, placement, combination with and without repetition.

Alphabets and words. Counting the number of permutations, placements and combinations and the number of words of a given length in a given alphabet. Basic logical algebra, expression and operation in it: conjunctions, disjunctions, negation. Normal forms of logical functions. Elements of the algebra of events. Combinatorial problems concerning probability. Boolean algebra. Realization of logical functions. The predicate calculus. Predicates and sets, logical operations on predicates.

The relation with operations on sets. Universal and existential quantifiers, various theorems on the principle of duality. The concepts of countable sets, equivalence, nondenumerability of the points of a segment, sets equivalent to the continuum, the magnitude of the sets of points of a square and of a cube.

Geometry

Topic 6 (lecture 30 hours, problem sessions 90 hours)

The coordinate method. The basic problems of analytic geometry (equations of lines, linear equations in the plane and in space). Determinants of second and third order. Mutual relationships of lines and planes in space on the basis of coordinate methods. Vectors. Scalar product of vectors. Linear dependence and independence. Cramer's rule in 3-space. The Kronecker-Kappel theorem. Curves of the second order, surfaces of the second order in canonical form and expressed as simple equations of second order.

Spaces of measure n, linear forms in n dimensions and the theory of determinants, quadratic forms in n dimensions, their canonical forms, and information about simplification of quadratic forms.

Grade 10

Topic 7 (lecture 30 hours, problem sessions 90 hours)

Numerical methods. Concept of exact computation. Interpolation problems, numerical integration, numerical solution of problems in linear algebra, numerical solution of differential equations. Elements of programming. The structure and function of electronic computers. Principle of recording numbers. Principle of utilizing commands. Examples of programs. Logical scheme of a program. Different forms of logical schemes. Programming using a logical scheme. Practice in programming and numerical methods.

Topic 8 (lecture 12 hours, problem sessions 36 hours)

Area and volume. Lebesgue measure. Expressing areas and volumes using integrals. Volumes of elementary geometric solids.

Grade 11

Syllabus for production training in the specialty "computer programming."

Mathematical Analysis (3 hours per week, 96 hours in all)

1. Real numbers. Theory of limits. (10)
2. Continuity. Uniform continuity. (10)
3. Series. (10)

4. Theoretical base of differential calculus of one variable and several variables. (16)
5. Theoretical base of integral calculus (including multiple and line integrals). (20)
6. Theory of point sets. (10)
7. Differential equations. (20)

Programming

1. Introduction. (30)
2. Elements of programming and general information about machines. The particulars of specific machines and techniques of programming for them.
3. Various methods of automatic and semiautomatic programming. (30)
4. Work on a whole problem in consultation with the "client." Analysis of the problem. Selection of a method of solution. Working out the algorithm for machine solution. Selection of the method of programming. Programming the problem and laying out the program. Carrying out the calculation on a machine. Working out and interpreting the results. Comparison with the problem in question. Practical derivation. (180)
5. Preparing a report.

Practical Experience

1. Basic methods of calculation. (40)
   (*a*) techniques of rounding off, (*b*) interpolation, (*c*) numerical integration, (*d*) numerical solution of differential equations.
2. Work on entire problems in consultation with the client. Analysis and simplification of the problem. Selection of a method of solution. Carrying out calculations and final interpretation of results.
   Comparison with the original problem. Practical derivation. (80)
3. Preparing a report. (8)

Higher Algebra (3 hours per week, 96 hours in all)

1. Linear algebra and quadratic forms (60)
2. Algebra of polynomials (36)

Mathematics Logic (3 hours per week, 96 hours in all)

1. Theory of sets. (20)
2. Statement calculus. (10)
3. Contact schemes. (10)
4. Predicate calculus. (30)
5. Elements of the theory of models. (26)

Bibliography

ASHKINUZE, V. G., and SHVARTZBURD, S. I. "Shkoly e Matematicheskoi Spetzializatziei," *Matematika v Shkole* (March-April 1963), p. 81.

BRADIS, V. M. *Metodika Prepodavaniia Matematiki v Srednei Shkole.* Moscow: Uchpedgiz, 1954.

CHERKASOV, R. S. "The Development of the Teaching of Mathematics in Soviet Schools," translated by B. R. VOGELI, *The Mathematics Teacher,* LVII (December 1965), 715–19.

Directivy XIX C'ezda KPSS. Moscow: Gosudarstvennoe Izdatel'stvo Politicheskoi Literatury, 1952.

GIRSANOV, I. V., and MOROZOVA, E. A. "Shkoly Iunykh Matematikov," *Matematika v Shkole* (March-April 1960), pp. 47–49.

LARICHEV, P. A. "Prepodavaniia Matematika v Srednei Shkole v Sviazi s Vvedeniem Novoi Programmy," *Matematika v Shkole* (July-August 1957).

LIAPIN, S. E. *Metodika Prepodavaniia Matematiki.* Leningrad: Uchpedgiz, 1955.

LITERAT, S. I. "O Prepodavanii Matematiki v Fiziko-Matematicheskoi Shkole-internate," *Matematika v Shkole* (November-December 1963).

MEDYNSKII, E. N. *Narodnoe Obrazovanii v SSSR.* Moscow: IAPN, 1952.

NOVIKOV, P. S. "Programma po Kursu Matematicheskaia Logik i Algoritmy dlia Studentov III Kursa na 1963/64 Uch. G." (typewritten).

"Novyi Etap v Razvitii Sovetskoi Shkoly," *Matematika v Shkole* (November-December 1958).

"Novyi Priem v Respublikanskovo Zaochnuiu Matematicheskuiu Shkoly," *Matematika v Shkole* (May-June 1965), p. 69.

Pedagogicheskii Slovar'. 2 vols. Moscow: IAPN, 1960.

Pravda, December 6, 1958.

"Proiekt Polozheniia o Matematicheskoi Iunosheskoi Shkole pri Ivanovskom Gosudarstvennom Pedagogicheskom Institute," *Matematika v Shkole* (March-April 1960), p. 47.

Programmy Pedagogicheskish Institutov. Moscow: Uchpedgiz.

Programmy Pedagogicheskish Institutov—Gosudarstvennye Ekzameny po Matematike. Moscow: Uchpedgiz, 1958.

Programmy po Matematicheskim Distziplinam dlia Srednikh Shkol s Proizvodstvennym Obucheniem po Professii "Vychislitel'-Programmist." Moscow: IAPN, 1963.

Programmy Srednei Shkoly NA 1959–60 Uchebnyi God, Matematika. Moscow: Uchpedgiz, 1959.

"Recommendations of the Mathematical Association of America for the Training of Mathematics Teachers," *American Mathematical Monthly,* LCIII (December 1960).

SEMUSHIN, A. D. "Prepodavaniia Matematika v Srednei Shkole SSSR" (typewritten report).

SHEHEGOL'KOV, E. A. "Programma po Kursa Vychisliteľnye Mashiny i Programmirovanie" (typewritten).

SHVARTZBURD, S. I. "Iz Opyta Raboty s Uchashchimisia ikh Klassa, Ovladevaiushchimi Spetzial'nost'iu Laborantov Programmistov," *Matematika v Shkole* (September-October 1960).

———. *Matematicheskaia Spetzializatziia Uchashchikhsia Srednei Shkoly.* Moscow: IAPN, 1963.

———. "O Podgotovke Programmistov v Srednei Obshcheobraz Ovatel'noi Politekhnicheskoi Shkole," *Matematika v Shkole* (March-April 1961).

SMIRNOV, S. V. "Iunosheskaia Matematicheskaia Shkola pri Ivanovsko Pedinstitute," *Matematika v Shkole* (September-October 1962).

———. "Pervye Shagi Iunosheskoi Matematicheskoi Shkoly," *Matematika v Shkole* (March-April 1960).

SMIRNOV, S. V., *et al.* "O Novyka Formakh Vneklassnoi Raboty," *Matematika v Shkole* (March-April 1960).

STEPANOV, I. D. "Irkutskaia Iunosheskaia Matematicheskaia Shkola," *Matematika v Shkole* (May-June 1963), p. 85.

SVIRELKIN, M. A. "Iunosheskaia Matematicheskaia Shkola v Iaroslavle," *Matematika v Shkole* (May-June 1963), pp. 84–85.

Uchitel'skaia Gazeta, August 13–29, 1964.

VAINBERG, B. R., *et al.* "Zaochnaia Matematicheskaia Shkola pri Moskovskom Universitets," *Matematika v Shkole* (September-October 1964).

VOGELI, B. R. "Recent Revisions in Soviet Mathematics Education," *School Science and Mathematics* (June 1960).

———. "The Mathematics Program of the Soviet Secondary School: Its Status and Innovations." Unpublished doctoral dissertation, University of Michigan, 1959. Ann Arbor, Mich.: University Microfilms, 1959.

ZEL'TZM N, V. B. "Pervye Gody Raboty Iunosheskoi Matematicheskoi Shkoly, *Matematika v Shkole* (September-October 1962), pp. 58–59.

ZIABLITZKII, V. V. "Shkola Iunykh Matematikov," *Matematika v Shkole* (January-February 1963), pp. 73–75.