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This report contains five papers which describe mathematical models of the educational system as it relates to economic growth. Experimental applications of the models tc ·articular educational systems are discussed. Three papers, by L. J. Emmerij, J. n, and G. Williams, discuss planning models for the calculation of educational requirements for economic development in Spain, Turkey, and Greece. Authors and es of the other two papers are (1) J. Tinbergen and H. C. Bos, "A Planning Model for the Educatio'al Requirements of Economic Development," and (2) J. Tinbergen and H. C. Bos, "Apprraisal of the Model and Results of Its Application." A related document is EA 001 764. (HW)

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econometric models of education

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econometric models of education

some applications

Papers by
JAN TINBERGEN and H.C. BOS

OECD Fellows Human Resource Development Programme

JAMES BLUM
LOUIS EMMERIJ
GARETH WILLIAMS

ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT

The Organisation for Economic Co-operation and Development was set up under a Convention signed in Paris on 14th December 1960 by the Member countries of the Organisation for European Economic Co-operation and by Canada and the United States. This Convention provides that the O.E.C.D. shall promote policies designed :

- *to achieve the highest sustainable economic growth and employment and a rising standard of living in Member countries, while maintaining financial stability, and thus to contribute to the world economy ;*
- *to contribute to sound economic expansion in Member as well as non-member countries in the process of economic development ;*
- *to contribute to the expansion of world trade on a multilateral, non-discriminatory basis in accordance with international obligations.*

The legal personality possessed by the Organisation for European Economic Co-operation continues in the O.E.C.D. which came into being on 30th September 1961.

The members of O.E.C.D. are Austria, Belgium, Canada, Denmark, France, the Federal Republic of Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States.

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PREFACE

The O.E.C.D. countries have become increasingly aware of the importance of education to the attainment of their social and economic goals. As a result, long-range educational planning is becoming an integral part of national economic planning.

This tendency has been facilitated by the development of new mathematical techniques of analysis. These techniques have been successfully applied to complex scientific, economic and industrial problems. They should be equally fruitful when applied to the equally complex processes of education.

The study contained in this volume is a first attempt to do so, to build a mathematical model of the educational system, based, in this case, on its relationship to economic growth. The study involves both the preparation of abstract models as well as their experimental application to particular educational systems. This experience has demonstrated the many potentialities of the mathematical model approach, and that further research is needed and warranted.

It is hoped that governments and research institutes will find the study useful guidance for developing these tools of analysis. The O.E.C.D., in its educational planning programme, is currently extending its investigation to the use of other models and techniques of Systems Analysis.

This volume is the product of the cooperative efforts of many facilities for educational planning which are available to the O.E.C.D. Member nations. These include the Mediterranean Regional Project, the Study Group in the Economics of Education, the Human Resource Development Fellowship programme, and the Secretariat of the O.E.C.D. Directorate for Scientific Affairs.

The original model, upon which this study was based, was the subject of a lecture given by Professor Jan Tinbergen at Frascati, Italy, as part of the 1962-63 training series of the O.E.C.D. Human Resource Development Fellowship Programme[1]. An additional seminar on the model was held in December 1962 at the Netherlands Economic Institute.

The Mediterranean Regional Project provided an excellent opportunity to test the model. The first stage of the M.R.P. involved the preparation of national studies assessing the long-term educational needs of six Mediterranean countries: Greece, Italy, Portugal, Spain, Turkey and Yugoslavia[2]. Thus, data were available for applying the model. Moreover, the results of these applications could be compared with those found by more conventional methods of analysis.

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1. H. S. Parnes (ed.) *Planning Education for Economic and Social Development*, O.E.C.D. Mediterranean Regional Project, Paris 1963.

2. The Mediterranean Regional Project, *Education for Economic and Social Development*, country reports for Greece, Italy, Portugal, Spain, Turkey and Yugoslavia, O.E.C.D. Paris 1965.

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Mr. Louis Emmerij, Mr. James Blum, and Mr. Gareth Williams have investigated the application of the model to the educational programmes of Spain, Turkey, and Greece. Mr. Emmerij, Mr. Blum and Mr. Williams were Fellowship holders in 1962-63, and are now members of the Secretariat of the O.E.C.D. Directorate for Scientific Affairs.

The May 1963 meeting on the Study Group in the Economics of Education considered revised versions of the model, as well as the first findings of the three Fellows. The theoretical treatment of the model, by Professors J. Tinbergen and H. C. Bos, the comments on it by Professors Thomas Balogh, Gottfried Bombach and Amatya Sen, as well as reports of the discussion are contained in the Study Group volume on *The Residual Factor and Economic Growth.*

The present volume has been prepared for publication by Mrs. L. Reifman, Secretary of the Study Group in the Economics of Education. It contains in Chapter I, the theoretical treatment of the models in their latest stage of development; in Chapters II, III, and IV, evaluations of the experiences of applying these models to three different countries; and in Chapter V, a review of these experiences.

It is significant to mention that the chapters submitted by Messrs. Blum, Emmerij, and Williams were in partial fulfilment of the requirements of their O.E.C.D. Fellowship in Human Resource Development. This programme had been established by the O.E.C.D. in 1961 in response to the need, expressed by the Member countries, to have training specially oriented toward producing the new policies and plans for education and science which would be consistent with the Organisation's economic growth objectives. As in this experience, the fellowship training consisted of a period of lectures and seminars led by prominent economists, sociologists, and educational planners; a period of research training with the national planning teams or with the Secretariat; and a period of preparing independent research reports.

It is anticipated that the current publication series, Education and Development, will contain some of the more significant contributions to the Fellowship programme, among these lectures and research reports, as well as other articles of importance to educational planners.

Alexander King
Director,
Directorate for Scientific Affairs

Part I

A PLANNING MODEL FOR THE EDUCATIONAL REQUIREMENTS OF ECONONIC DEVELOPMENT[1]

by

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Netherlands Economic Institute, Rotterdam

INTRODUCTION

This paper discusses a model for the planning of education. It includes various earlier, simpler versions as well as more elaborate, recent ones. The model is intended to represent the link between economic development and that of the educational system of a nation.

Educational development must show both qualitative and quantitative aspects; the former refer to changes in methods of teaching and the subject matter of teaching; the latter refer to changes in the dimensions and the composition of the educational system. This study disregards the qualitative aspects except when combined with the quantitative, e.g. in the numerical values of some coefficients.

FEATURES BROUGHT OUT BY ALL VERSIONS OF THE MODEL CONSIDERED

The simplest versions of the model, which for didactic reasons will be discussed first, may seem to some readers to be so oversimplified as to miss some of the essential features even of the quantitative problem of educational development. This objection should disappear when the more elaborate versions are discussed. It may be useful, however, to point out that these simplest versions bring out a scheme in which certain basic facts are taken into account which are characteristic of the relationship between economic development and education, e.g. :

1. economic life needs a stock of qualified manpower; the flow of new graduates from educational establishments represents a very small proportion of this stock in view of human longevity;
2. education often consists of a series of successive stages, each depending on the former for its supply of new recruits, e.g. expansion at university level would be impossible if sufficient secondary-level graduates were not available;

1. The original paper has been adapted partly as a result of the discussion at the May meeting and the comments made.

3. part of the stock of qualified manpower must be used in the education process itself—as seed is used in agriculture;
4. qualified manpower may be imported.

These basic facts are very important, particularly when rapid educational expansion is desired, and justify the place given to them in our models.

Usefulness of macro-models

We have preferred to start with highly simplified models in the scientific tradition which is also well known in economics. The Keynesian multiplier model is an outstanding example of this tradition. Its great advantage is the clarification it brings to some basic properties of the mechanisms. Our approach may be regarded as an attempt to apply the method of input-output analysis to some problems of education planning. It should not be overlooked that—although a high degree of aggregation may in some cases lead to erroneous results—macro-models often provide a first approximation to reality.

In the later sections of this chapter a number of breakdowns of the aggregates of production, manpower and the educational system will be discussed. The prior treatment of the macro-model will facilitate these discussions.

Models essentially planning models

Our models do not aim at a description of the "free" development of the educational system under the forces of supply and demand, and, therefore, at forecasting such a development. Their aim is to describe the demand flows for various types of qualified manpower to be expected from the organizers of production and of education. The purpose of the models is to aid in the process of planning for education and for labour-market policies, tacitly assuming that ways and means can be found to induce the population to seek the desirable education.

Only coefficients which can be estimated statistically have been introduced into the models. There is no point in introducing theoretically refined concepts and relationships if they cannot be translated into numerical estimates. In various versions of the models this eminently practical aspect limits the possibilities of refinement and is the reason for our adhering to simpler versions.

Alternative approaches invited

The theme under consideration has only recently attracted the attention of educationalists, economists and other social scientists. This contribution must be regarded as a tentative effort and we hope it will give rise to alternative and better models. It is relatively easy to sum up the deficiencies of the models chosen; it is more difficult to replace them by improved models which meet the practical conditions imposed. Alternative models must use measurable variables and coefficients and must have a possible solution.

The basic model

As a starting point let us consider a highly simplified model, published in "Kyklos"[1].

1. H. Correa and J. Tinbergen, "Quantitive Adaptation of Education to Accelerated Growth", *Kyklos* Vol. XV (1962), p. 776.

The model distinguishes two levels of educational activity — secondary and third — primary education being assumed to be no bottle-neck for the required expansion of secondary education and for production increases. The time unit chosen was 6 years, this being the supposed training period for secondary and third-level education respectively.

The following symbols are used, leaving out the time index t :

v , total volume of production (income) of the country;
N^2 , the labour force with a secondary education;
N^3 , the labour force with a third-level education;
m^2 , those who have entered the labour force N^2 within the previous 6 years;
m^3 , those who have entered the labour force N^3 within the previous 6 years;
n^2 , the number of students in secondary education;
n^3 , the number of students in third-level education.

The following relationships are assumed to hold between these variables :

(1) $N_t^2 = \nu^2 v_t$

(2) $N_t^2 = (1 - \lambda^2) N_{t-1}^2 + m_t^2$

(3) $m_t^2 = n_{t-1}^2 - n_t^3$

(4) $m_t^3 = n_{t-1}^3$

(5) $N_t^3 = (1 - \lambda^3) N_{t-1}^3 + m_t^3$

(6) $N_t^3 = \nu^3 v_t + \pi^2 n_t^2 + \pi^3 n_t^3$

These equations express the following relationships :

Equation (1). The labour force with a secondary education is used for production only and must develop proportionally with the volume of national production;

Equations (2) and (5). The labour force consists of those already in it one time unit earlier and those who have joined it during the previous 6 years. It is assumed that a proportion λ^2 and λ^3, respectively of those already in the labour force one time unit earlier has dropped out owing to death or retirement;

Equation (3). The number of newcomers to the labour force with a secondary education is equal to the number of students one time unit earlier minus the number of students now in third-level education;

Equation (4). The number of newcomers to the labour force with a third-level education is equal to the number of third-level students one time unit earlier;

Equation (6). The labour force with a third-level education consists of those employed in production, and is assumed to be proportional in numbers to the volume of production, and of those teaching at both levels of education and assumed to be proportional to the respective student numbers.

For numerical calculations the following values of the coefficients have been assumed : $\nu^2 = 0.20$; $\nu^3 = 0.02$; $\lambda^2 = \lambda^3 = 0.1$; $\pi^2 = 0.04$ and $\pi^3 = 0.08$. These last two coefficients imply student-teacher ratios of 25 and 12.5 respectively.

Measurement of the variables

The choice of a time unit of 6 years means that special attention must be paid to the measuring of the variables. Several ways of timing the variables are possible: they must however be consistent.

The N's are clearly stock variables : they refer to a certain point in time. The choice of which point in time this will be determines the timing of the other variables.

Suppose the stock variables N^2 and N^3 refer to the situation at the end of the 6-year period t. The gross increase in N^3 during these 6 years of period t is measured by m_t^3, which is the sum of the annual gross additions to the third-level labour force during these 6 years. Assuming that all students who enroll for the first time complete their study in 6 years and join the labour force the following year (i.e. they graduate on December 31st and join the labour-force on January 1st., to avoid the complication of the difference between the school and calendar years) the annual gross addition to N^3 in each of the years of period t equals the number of first-year students six years previously. The gross increase in N^3 during period t, measured by m_t^3, therefore equals the sum of the first enrolments in each of the 6 years of the period $t-1$. However, under the assumptions made, this sum is equal to the total number of students enrolled in the last year of the period $t-1$. The conclusion is that n_t^3 (or n_{t-1}^3) must be measured as the total number of third-level students enrolled in the last year of the period t (or period $t-1$), if N^3 refers to the situation in that year.

Similarly, we can deduce that n_t^2 (or n_{t-1}^2) must be measured as the total number of secondary-school students enrolled in the last year of the period t (or $t-1$) to be consistent with the chosen timing of N^2 at the end of the period. We may add that only in this case m_t^2 (and m_t^3) refer to the gross additions to the manpower stocks during the period t. An earlier timing of the stocks would mean that m_t would apply to additions to the stock in period t and in period $t-1$, which seems less elegant.

However, if this objection is not considered to be a very serious one, the best way of measuring the variables would be to take for the N's, n's and v the average of the annual figures for these variables for the 6 years of period t (or $t-1$). By taking the average annual figures the influences of special factors affecting the situation in the specific years is evened out.

SOME APPLICATIONS OF THE BASIC MODEL; GENERALIZATIONS TO BE CONSIDERED

PATTERNS OF BALANCED GROWTH (Problem I)

The ideal development of the educational system is one of regular growth parallel to the desired growth of the economy. If the economic variables develop with a constant rate of growth, it is possible to find one path of development of the educational variables showing the same rate of growth.

For the system to move along such lines, the initial circumstances (to be indicated by a suffix $t=0$) have to satisfy certains conditions. We shall establish these conditions for two different rates of development of the economy, represented by underlined and by barred symbols, respectively :

$$\text{(A)}\quad \underline{v}_t = \underline{v}_0\, 1.3^t \qquad \text{(B)}\quad \bar{v}_t = \bar{v}_0\, 1.4^t$$

These two paths are characterized by an increase in production of 30 per cent and 40 per cent per six-year period respectively.

In case A we introduce the assumptions :

$\underline{N}_t^2 = \underline{N}_0^2\, 1.3^t$; $\underline{N}_t^3 = \underline{N}_0^3\, 1.3^t$; $\underline{n}_t^2 = \underline{n}_0^2\, 1.3^t$; $\underline{n}_t^3 = \underline{n}_0^3\, 1.3^t$; $\underline{m}_t^2 = \underline{m}_0^2\, 1.3^t$ and $\underline{m}_t^3 = \underline{m}_0^3\, 1.3^t$ and substitute them in equations (1) to (6).

We shall find the following conditions fulfilled by the initial values : $N_0^2 = 0.2\, v_0$; $N_0^3 = 0.0245\, v_0$; $n_0^2 = 0.094\, v_0$; $n_0^3 = 0.0098\, v_0$; $m_0^2 = 0.062\, v_0$; $m_0^3 = 0.0076\, v_0$.

These relations determine the structure (i.e. the proportions) of the educational system as well as its absolute level. We can make the same calculations for case B and find other relations which must be fulfilled. The two sets of figures are compared in Table 1 where the initial volume of production v_0 has been taken as equal to 100; moreover, the development of the variables over the first few time units has been added.

TABLE 1. BALANCED GROWTH OF THE EDUCATIONAL SYSTEM FOR TWO GROWTH RATES: (A) 30 PER CENT AND (B) 40 PER CENT PER 6 YEARS

Production in billions of dollars; population in millions.

Symbols	Case	A: 30 per cent				B: 40 per cent			
	Time periods t	0	1	2	3	0	1	2	3
v	Volume of production	100	130	169	219	100	140	196	274
N^2	Manpower : with secondary education	20.0	26.0	33.7	43.7	20.0	28.0	39.2	54.8
N^3	Manpower : with third-level education	2.45	3.19	4.14	5.35	2.57	3.60	5.02	7.03
n^2	Students in sec. schools	9.4	12.2	15.8	20.5	12.0	16.8	23.5	32.9
n^3	Students in third level education	0.98	1.27	1.65	2.15	1.29	1.80	2.54	3.53
m^2	Manpower with sec. educ. and under 6 years' employment	6.2	8.0	10.5	13.6	7.2	10.1	14.1	19.8
m^3	Manpower with third-level educ. and under 6 years' employment	0.76	0.98	1.27	1.66	0.93	1.29	1.80	2.54

The table illustrates the relationship between the rate of growth and the structure of the educational system. While the total manpower with secondary education is the same for $t = 0$ in both cases, total manpower with third-level education must be higher in case B, since more teachers are needed. These are needed because the annual addition to trained manpower (of both levels) must now be larger.

TRANSITION PROBLEM—WITH FOREIGN AID (Problem II)

We now assume that a country wants to accelerate its economic development. Let it be at time $t = 0$ in the balanced position corresponding to the rate of growth A (30 per cent per 6 years) and let it aim at reaching the path **B**. This requires an increase in the numbers to be trained at levels 2 and 3, but in such a way as to fit into the new pattern of development. Temporarily, some of the equations cannot be fulfilled; for if they all were all the time, the economy would continue to move along path A. Mathematically, this means that either some new terms have to be added to some of the equations for some of the time or that some of the existing coefficients have temporarily to be changed. The former adjustment may be interpreted as foreign aid in the form of additional trained manpower becoming available. The latter may be regarded as a change in technology (either of the production process or the education process) equivalent to self-help of the country concerned. The former case will be discussed in this section.

An open question in both problems is the length of the transition or adaptation period. Generally speaking, it can be made shorter by applying more "instruments" simultaneously: but the structure of the equations sets limits to the possibilities. We shall inquire first, in both cases, whether the briefest period of transition, namely one time unit (of six years) is a possibility. This seems possible in the case of foreign aid, but not in that of self-help, since here the additional training takes time.

The method of analysis consists of writing down our system of equations for a series of consecutive time periods and of finding out how many unknowns are available and how many are needed in order to make the system determinate but not over-determinate (which would mean that inconsistencies slip in). For $t = 0$, all variables must be equal to the values shown in Table 1, Case A. If the transition period T is equal to 1, for $t = 2$ all variables must have the values shown in Table 1, Case B. It is only for $t = 1$ that we do not know the values of all the variables, We do know, however, the values of those variables for $t = 1$ which appear in the equations for $t = 2$. These have to be equal to the values on path B, otherwise the system would not, after $t = 2$, remain on that path.

The variables are N_1^2, appearing in equation (2) for $t = 2$; n_1^2, appearing in equation (3); n_1^3, appearing in equation (4); and N_1^3, appearing in equation (5). All these variables must have the "barred" values (meaning that they belong to path B) :

(7) $N_1^2 = \bar{N}_1^2$

(8) $n_1^2 = \bar{n}_1^2$

(9) $n_1^3 = \bar{n}_1^3$

(10) $N_1^3 = \bar{N}_1^3$

This leaves us with the following unknowns in the system of equations for $t = 1$: v_1, m_1^2, m_1^3, and the two additional terms to be added, indicating foreign supplies of trained manpower of 2nd and 3rd level. If we indicate imported manpower by i_1^2 and i_1^3 respectively, we obtain the following equations:

(1_1) $\bar{N}_1^2 = 0.2\, v_1$

(2_1) $\bar{N}_1^2 = 0.9\, \underline{N}_0^2 + m_1^2$

(3_1) $m_1^2 = \underline{n}_0^2 - \bar{n}_1^3 + i_1^2$

(4_1) $m_1^3 = n_0^3 + i_1^3$

(5_1) $\bar{N}_1^3 = 0.9\, \underline{N}_0^3 + m_1^3$

(6_1) $\bar{N}_1^3 = 0.02\, v_1 + 0.04\, \bar{n}_1^2 + 0.08\, \bar{n}_1^3$

The system happens to be determinate and not over-determinate for the particular reason that the value of v_1 satisfying equation (1_1), namely v_1, also satisfies equation (6_1); these two equations are dependent. The other 4 unknowns can be solved from equations (2_1) through (5_1). The solutions are :

$m_1^3 = \bar{N}_1^3 - 0.9\, \underline{N}_0^3 = 3.60 - 2.20 = 1.40$, a rise of 84 per cent.

$i_1^3 = m_1^3 - \underline{n}_0^3 = 1.40 - 0.98 = 0.42$, i.e. 30 per cent of m_1^3.

$m_1^2 = \bar{N}_1^2 - 0.9\, \underline{N}_0^2 = 28 - 18 = 10$, a rise of 61 per cent.

$i_1^2 = \bar{n}_1^3 - \underline{n}_0^2 + m_1^2 = 1.80 - 9.4 + 10 = 2.4$, i.e. 24 per cent of m_1^2.

It appears, therefore, that with foreign assistance amounting to 30 per cent of the newly-engaged third-level, and to 24 per cent of the newly-engaged second-level labour force, the transition period can be restricted to 6 years. It is assumed, however, that this foreign manpower will remain in the labour force.

TRANSITION PROBLEMS; WITHOUT FOREIGN AID (Problem III)

From equations (1_1) through (6_1) it can be seen that without foreign aid the transition problem cannot be solved in one time period only, at least if our model is adhered to. The possible new unknowns we may introduce are the coefficients appearing in equations (1_1) and (6_1) only; the coefficients of equations (2_1) and (5_1) can hardly be changed. Now, if we leave out i_1^2 and i_1^3 as variables, equations (2_1) and (3_1) contain only one unknown, and so do equations (4_1) and (5_1). Thus the problem cannot be solved. We must assume $T = 2$, implying that we must now consider as given the (underlined) values of the variables for $t = 0$ and the (barred) values of the variables for $t = 3$ (corresponding to path B) and use the equations for both $t = 1$ and $t = 2$. From the equations for $t = 3$ we can again derive that $N_2^2 = \overline{N}_2^2$; $N_2^3 = \overline{N}_2^3$; $n_2^2 = \overline{n}_2^2$; $n_2^3 = \overline{n}_2^3$. We are left with 12 equations :

(1) $N_1^2 = \nu_1^2 \upsilon_1$
(2) $N_1^2 = 0.9\, \underline{N}_0^2 + m_1^2$
(3) $m_1^2 = \underline{n}_0^2 - n_1^3$
(4) $m_1^3 = \underline{n}_0^3$
(5) $N_1^3 = 0.9\, \underline{N}_0^3 + m_1^3$
(6) $N_1^3 = \nu_1^3 \upsilon_1 + \pi_1^2 n_1^2 + \pi_1^3 n_1^3$

(11) $\overline{N}_2^2 = \nu_2^2 \upsilon_2$
(12) $\overline{N}_2^2 = 0.9\, N_1^2 + m_2^2$
(13) $m_2^2 = n_1^2 - \overline{n}_2^3$
(14) $m_2^3 = n_1^3$
(15) $\overline{N}_2^3 = 0.9\, N_1^3 + m_2^3$
(16) $\overline{N}_2^3 = \nu_2^3 \upsilon_2 + \pi_2^2 \overline{n}_2^2 + \pi_2^3 \overline{n}_2^3$

We have used Greek symbols for the coefficients, indicating that they may now have to be unknowns and, by adding a suffix 1 or 2, that their values may be different for the two transition periods.

It will be clear that there are now several possibilities, since the total number of possible unknowns—18—surpasses the number of equations. We shall try to find the simplest possible solution, keeping as many coefficients as possible equal to their normal values. Since there are several equations containing only one unknown, these may be solved first. With these solved other equations then contain only one unknown.

From (4) we find $m_1^3 = \underline{n}_0^3 = 0.98$

From (5) we find $N_1^3 = 0.9\, \underline{N}_0^3 + \underline{n}_0^3 = 3.19$

These results enable us to find, from (15) $m_3^2 = \overline{N}_2^3 - 0.9\, N_1^3 = 2.16$ and from (14) $n_1^3 = m_2^3 = 2.16$. Again we can now determine, with the aid of (3), $m_1^2 = \underline{n}_0^2 - n_1^3 = 7.24$, and from (2) $N_1^2 = m_1^2 + 0.9\, \underline{N}_0^2 = 25.24$. This then gives us, from (12), $m_2^2 = \overline{N}_2^2 - 0.9\, N_1^2 = 16.5$ and from (13) $n_1^2 = m_2^2 + \overline{n}_2^3 = 19.0$. We are now left with equations (11), (16), (1) and (6). The simplest possibility seems to take $\upsilon_2 = \overline{\upsilon}_2$ which satisfies (11) and (16) without any change in coefficients and also to leave ν_1^2 unchanged $= 0.2$. This makes $\upsilon_1 = \frac{N_1^2}{0.2} = 126.2$. The changes in coefficients must now be concentrated in equation (6); with the normal value of these coefficients, the right-hand side amounts to 3.45, while the left-hand side is 3.19; the difference being 0.26.

We must now reduce one or more of the coefficients v^3, π^2, and π^3. We cannot change π^3 only, because it would have to become negative. We can change π^2 only; this would imply a reduction in the ratio of $\frac{0.26}{0.76}$ or 34 per cent (0.76 being the value of the second term with the normal value of π^2). We could also change v^3, and the reduction ought to be only $\left(\frac{0.26}{2.52}\right)$ or 10 per cent. We may state therefore that the problem of transition can be solved without foreign assistance by either reducing the teacher-pupil ratio in secondary education by 34 per cent, or reducing the use of third-level manpower in production by 10 per cent, both during six years; another possibility would be reduction of 7.5 per cent $\left(\frac{0.26}{3.45}\right)$ in the teacher-pupil ratios in second and third level education or in the third-level manpower ratio in production. Weighted combinations of these alternatives are also possible. The values of the variables (in the restricted sense) in all these cases will be as indicated in Table 2.

TABLE 2. VALUES OF VARIABLES DURING PERIOD $0 < t < 3$

Units as before.

Symbols	Case	II With foreign assistance				III Without foreign assistance			
	Time:	$t = 0$	1	2	3	0	1	2	3
N^2	Manpower with secondary education .	20	28	39.2	54.8	20	25.2	39.2	54.8
N^3	Manpower with third-level education.	2.45	3.60	5.03	7.03	2.45	3.19	5.03	7.03
n^2	Students in secondary schools	9.4	16.8	23.5	32.9	9.4	19.0	23.5	32.9
n^3	Students in third level education.....	0.98	1.80	2.54	3.53	0.98	2.16	2.54	3.53
	"New" manpower								
m^2	with sec. education	6.2	10.0	14.1	19.8	6.2	7.2	16.5	19.8
m^3	with 3rd level education	0.76	1.40	1.80	2.54	0.76	0.98	2.16	2.54
v	Production	100	140	196	274	100	126	196	274

Because of the foreign assistance in manpower, Case II shows, at $t = 1$, a higher stock of manpower of both levels than Case III, reflected also in the "new" manpower, that is, the labour force with under 6 years employment. The number of students at both levels can be lower in Case II, because there will be less need for them when they are ready : the increase in the required stock of manpower of their type will then be less than in Case III. This is a consequence, however, of the implicit assumption in Case II that the foreign manpower will remain in the labour force until the age of retirement.

If we compare problem (III) with problem (II) we note that the number of unknowns surpasses the number of equations, leaving a number of degrees of freedom which have been used somewhat arbitrarily here. They could have been used, however, to find an optimum solution according to some criterion. This would then be called a programming problem. If the criterion is a linear function of the variables (and the restrictions are also linear, as is the case here) we have a problem of linear programming. This more satisfactory treatment will be found in the next example.

Introductory remarks on the "ization problem"[1] (Problem IV)

Description of example chosen

Former colonies usually start their independent life as a nation with a considerable number of foreigners occupying high places; for understandable reasons it is desired to replace them by citizens of the country concerned. This gives rise to such problems as the Indianization of India, the Egyptianization of Egypt, the Africanization of Africa, and so on, the problem often being referred to as the "ization problem". The problem is essentially one of education, but also has other aspects. It may take different forms in differing circumstances or as a result of customs continued from the past.

This section deals with one of the many ways in which the problem may be set and solved. Since our example does not fit all the circumstances in any particular country, it is to be considered as an example only, or as a starting point for discussion.

For convenience's sake, the problem and its treatment will be presented as one of africanization since such a problem came up during the preparation of a report for the Unesco Conference on Higher Education in Africa held in Tananarive, September 1962. The reader should be aware, however, of the lack of precise data on some of the relevant phenomena and therefore not apply the conclusions too readily to any specific country.

For the setting of our problem, we assume that the nation under consideration finds itself in a smooth path of development as defined in Problem I. At time $t = 0$ we assume there are foreigners, in manpower stocks N_t^2 and N_t^3 only. These are taken as equal to 22.5 and 2.5 times the unit of manpower (thousands or millions or anything else). We now assume that as from $t = 1$, all students will be nationals and that even larger numbers will be trained than would otherwise be necessary for a continuation of the even development of production. The additional numbers will be indicated by j_t^2 and j_t^3 for the 2nd and 3rd level respectively and for the time periods that will appear to be necessary. The additional numbers of students will make it possible for the expatriates in the labour force to return to their own countries. These "exports" will be indicated by E_t^2 and E_t^3 respectively. Consequently, the number of "newcomers" m_t will also be higher, and will be shown as k_t^2 and k_t^3 for the two levels, while J_t^3 indicates the excess for the stock of third-level manpower N_t^3. No such excess is needed for second-level manpower, since the production programme will not change and the total number of second-level manpower depends only on production. Additional men in the third-level stock are necessary because of the larger numbers of students.

Our problem will finally be to find an "optimum programme" of j and E figures (of additional students and of "exports" of qualified foreigners). The programme must lead to the departure of the total numbers of 22.5 and 2.5 mentioned; it must not interrupt the production process and it must satisfy certain bounds which we shall discuss later. Finally it must be as cheap as possible. This latter condition can be formulated algebraically; we assume that the expatriates are more expensive than nationals and that the extra cost is twice as high for third-level as for second-level people. It would be cheapest if all departures could take place in the time period $t = 1$; those who stay

1. The main portion of this section was published under this title in, *Zeitschrift für die gesamte Staatswissenschaft*, 119 (1963), p. 328.

longer will cost more. The additional cost is twice as high again for those who are staying two more time units than for those who are staying one additional time unit. Since it will be shown later that no more than three time periods have to be involved, the costs to be minimized are :

(7) $W = E_2^2 + 2E_3^2 + 2E_2^3 + 4E_3^3$

Non-interruption of the process of growth

We shall first inquire what are the conditions which must be met by our programme j_t^h, E_t^h ($h = 2, 3$; $t = 1, 2, 3$) to avoid interrupting the growth programme. This clearly requires that the enlarged education program satisfies the set of equations (1) to (6) for each of the time periods 1, 2 and 3 and that for $t = 0$ as well as for after $t = 3$ the normal education programme operates. For $t = 1, 2, 3$ this means that the additional terms in each equation introduced by the symbols j, k, J and E cancel out. In addition, there must not be such additional terms for $t = 0$ or for after $t = 3$. Because of the lagged variables appearing in equations (1) to (6) on the one hand, for $t = 1$ we have some terms which vanish (namely k_0^2, j_0^2 and J_0^3) and, on the other hand, some terms for $t = 3$ which must vanish because they also appear in the equations for $t = 4$. These are j_2^3, j_3^3 and J_3^3. All this leads to the following equations which must be met by our programme variables :

(12) $0 = k_1^2 - E_1^2$
(13) $k_1^2 = -j_1^3$
(14) $k_1^3 = 0$
(15) $J_1^3 = k_1^3 - E_1^3$
(16) $J_1^3 = 0.04 j_1^2 + 0.08 j_1^3$

(22) $0 = k_2^2 - E_2^2$
(23) $k_2^2 = j_1^2 - j_2^3$
(24) $k_2^3 = j_1^3$
(25) $J_2^3 = 0.9 J_1^3 + k_2^3 - E_2^3$
(26) $J_2^3 = 0.04 j_2^2 + 0.08 j_2^3$

(32) $0 = k_3^2 - E_3^2$
(33) $k_3^2 = j_2^2$
(34) $k_3^3 = j_2^3$
(35) $0 = 0.9 J_2^3 + k_3^3 - E_3^3$

In the numbers of the equations the first figure indicates the time period and the second figure the number of the equation in the original set (1) to (6). Equations (11), (21), (31) and (36) are identically fulfilled and hence disappear. Thus we have 14 equations. The number of unknowns is 18; but they must meet the additional conditions (these equations neglect the fact that during each time period 10 per cent of the stock of foreigners drop out as a result of death or retirement) :

(37) $E_1^2 + E_2^2 + E_3^2 = 22.5$
(38) $E_1^3 + E_2^3 + E_3^3 = 2.5$

There remain two degrees of freedom.

A similar process for two time periods only would have shown that no solution is possible in this case, and this is the reason why three time periods must be chosen for the programme.

Eliminating the variables k and J from the set of equations (12)-(35) we obtain the following equations to be satisfied by our programme variables [alongside (37) and (38)].

$$\left.\begin{array}{ll} -j_1^3 = E_1^2 & E_1^3 + 0.04 j_1^2 + 0.08 j_1^3 = 0 \\ E_2^2 = j_1^2 - j_2^2 & -0.9\,E_1^3 + j_1^3 - E_2^3 = 0.04 j_2^2 + 0.08 j_2^3 \\ E_3^2 = j_2^2 & 0.9\,(0.04\,j_2^2 + 0.08\,j_2^3) + j_2^3 = E_3^3 \end{array}\right\} \quad \text{A}$$

Calculations not given here show that the most elegant formulation of the programme is obtained when we chose two symbols ε^2 and ε^3 in which all programme variables can then be expressed. The programme is found to be :

$$\begin{array}{lll} j_1^2 = 25 + \varepsilon^2 + 0.01\,\varepsilon^3 & j_2^2 = -\,\varepsilon^2 & j_3^2 = 0 \\ j_1^3 = \varepsilon^3 & j_2^3 = 2.5 - \varepsilon^3 & j_3^3 = 0 \\ E_1^2 = -\,\varepsilon^3 & E_2^2 = 22.5 + \varepsilon^2 + \varepsilon^3 & E_3^2 = -\,\varepsilon^2 \\ E_1^3 = -1 - .04\,\varepsilon^2 - .07\,\varepsilon^3 & E_2^3 = .08 + .08\,\varepsilon^2 + 1.14\,\varepsilon^3 & E_3^3 = 2.7 - .04\,\varepsilon^2 - 1.06\,\varepsilon^3 \end{array}$$

It can be easily verified that this programme satisfies the equation group (A) as well as (37) and (38).

Bounds to be met

We shall now discuss a number of bounds which must be met by the programme variables, concerning the values of the E's. The upper bounds to be met are that total departures from the beginning of the programme cannot be higher than the numbers of expatriates present.

$$(39)\quad \left\{\begin{array}{ll} E_1^2 \leqslant 22.5 & E_1^3 \leqslant 2.5 \\ E_1^2 + E_2^2 \leqslant 22.5 & E_1^3 + E_2^3 \leqslant 2.5 \end{array}\right.$$

There may also be lower bounds due to "African impatience" and "foreign availability". By the first we mean that public opinion may not like to see a lot of new foreigners join the labour force, although our analysis shows that in the beginning this is necessary because of the increased number of students. The second means that, in addition, there may be a limit to the number of foreigners available. It seems reasonable to assume that the boundaries set to E_t^2 and the E_t^3 are roughly proportional to the numbers involved. For the sake of simplicity we therefore assume the boundary for E^2 to be ten times the boundary value for E^3 :

$$(40)\quad E_t^3 \geqslant -\eta \qquad E_t^2 \geqslant -10\,\eta.$$

Finally, there may also be limits to the values of j^3, especially for $t = 1$. We shall indicate this limit by ζ :

(41) $j_1^3 \leqslant \zeta$

where ζ is the number of secondary school leavers eligible for university education ("African availability").

Some conclusions and solutions

Our problem now takes the shape of a simple linear programming problem with parameters in some of the bounds. Very provisional estimates have given the impression that, for the time being, realistic values for the two parameters may be $\eta = 0.6$ for foreign availability and $\zeta = 0.4$ for African. African impatience might be represented by lower values of η, with the possibility even of $\eta < 0$, i.e. the immediate departure of a number of expatriates being desired.

Because of the existence of two degrees of freedom only a graphical representation in a plane can be used. The admissible regions for ε^2 and ε^3 will be found from the bounds solved for ε^3 as a function of ε^2 these appear to be :

$$\left.\begin{array}{rl} \text{(I)} & \varepsilon^3 \leqslant -12.5 - 0.5\,\varepsilon^2 + 12.5\,\eta \\ \text{(II)} & \varepsilon^3 \geqslant -0.7 - 0.07\,\varepsilon^2 - 0.88\,\eta \\ \text{(III)} & \varepsilon^3 \leqslant 2.55 - 0.04\,\varepsilon^2 + 0.94\,\eta \end{array}\right\} \text{derived from (40) for } E^3$$

$$\left.\begin{array}{rl} \text{(IV)} & \varepsilon^3 \leqslant 10\,\eta \\ \text{(V)} & \varepsilon^3 \geqslant -22.5 - \varepsilon^2 + 10\,\eta \\ \text{(VI)} & \varepsilon^2 \leqslant 10\,\eta \end{array}\right\} \text{derived from (40) for } E^2$$

$$\left.\begin{array}{rl} \text{(VII)} & \varepsilon^3 \geqslant -22.5 \\ \text{(VIII)} & \varepsilon^2 \leqslant 0 \end{array}\right\} \text{derived from (39) for } E^2$$

$$\left.\begin{array}{rl} \text{(IX)} & \varepsilon^3 \geqslant -43.8 - 0.5\,\varepsilon^2 \\ \text{(X)} & \varepsilon^3 \leqslant 2.4 - 0.04\,\varepsilon^2 \end{array}\right\} \text{derived from (39) for } E^3$$

$$\text{(XI)} \quad \varepsilon^3 \leqslant \zeta \quad \} \text{ derived from (41)}$$

The following conclusions may now be drawn :

Conclusion 1

For $\eta = 0$ (or < 0) no solution exists. This may be seen from (I) and (II) requiring that $\varepsilon^2 \leqslant -27.5$ which requires, with (V) that $\varepsilon^3 \geqslant 5$, whereas (IV) requires that $\varepsilon^3 \leqslant 0$.

Conclusion 2

The lowest η for which a solution exists is $\eta = 0.2$, which can be found if the point of intersection between I and II is required to lie on (V). We find : $\varepsilon^2 = -21$; $\varepsilon^3 = 0,5$ and $W = 55.4$.

Conclusion 3

For $\eta = 0.6$ we find an admissible region for (ε^2, ε^3) bounded by (I), (II) and (XI). The minimum value for W coincides with the point of intersection between (I) and (II), yielding, in this case : $\varepsilon^2 = -8.8$; $\varepsilon^3 = -0.6$;

$W = 44.3$, which is less than the value for $\eta = 0.2$. The boundary on j_1^3 is not active in this case.

Conclusion 4

A still cheaper solution will be found when we take $E_3^2 = E_3^3 = 0$, requiring $\varepsilon^2 = 0$ and $\varepsilon^3 = 2.5$, and making $W = 32.5$. But this "quickest" solution (since it is finished at the end of period 2) requires very large imports of foreign manpower to begin with :

$$E_1^2 = -2.5 \qquad\qquad E_1^3 = -1.2$$

and, moreover, a very high value of $j_1^3 = 2.5$.

Generalizations to be considered

After showing the uses of the basic model (and of several alternative versions) we shall discuss a number of possible generalizations, intended to counteract some of the drawbacks of over-simplification, e.g. such refinements as :

a) giving a more general form to the demand functions for manpower of various types;
b) disaggregating production into a number of sectors having different manpower requirements;
c) creating a method of dealing with drop-out in education;
d) providing an alternative method of treating retirement;
e) introducing more stages in the educational process;
f) introducing smaller time units into the time structure;
g) eliminating surplusses of certain types of manpower.

GENERALIZATION OF DEMAND FUNCTIONS

Some remarks on production functions

Equations (1) and (6) of the basic model, linking the number of employees having a given level of education to the volume of production, essentially represent or are derived from production functions. The two main types of production functions are, (a) those where the quantities of the production factors needed (such as labour) are expressed as functions of the volume of production, and (b) those where the volume of production is expressed as a function of the quantities of the factors. In (a) there are as many equations as there are production factors and no substitution between factors of production; in (b) there is only one equation and substitution is possible. As a rule the method for type (a) (without substitution) is assumed to reflect short-run relationships and type (b) longer-run adaptation possibilities. Even if no substitution possibility exists the relationship need not be one of proportionality as was assumed in our model.

There is little empirical evidence concerning substitution possibilities among the various types of labour, and so we have refrained from using them. Our generalizations therefore consist in introducing, between the volume of production and the quantity of labour, non-linear relationships reflecting the forces of increasing productivity and of the product's changing composition, both as a consequence of development.

SHAPE OF THE DEMAND FUNCTIONS

The proportionality relation between production and manpower used in our basic model is not too satisfactory and in the long run may easily lead to inconsistencies.

The first generalization of the model consists in assuming more general demand functions (1) and (6). Instead of the simple proportionality, with constant ν^2, between the manpower stock with secondary education N^2 and the level of production, the following relationship will be assumed :

(1a) $N_t^2 = \nu^{20} v_t^{\nu^{21}} \left(\frac{v_t}{a_t}\right)^{\nu^{22}}$

According to this relationship, N^2 is a function of the level of production v and the per capita income $\left(\frac{v}{a} \text{ if } a \text{ is total population}\right)$. The latter factor measures approximately the (negative) influence of increasing labour productivity on the demand for manpower. The per capita income can also be considered to represent the influence of changes in the composition of the national product.

Equation (1a) can be simplified if we know the time paths of v_t and a_t. Suppose these paths are given by :

(2.1) $v_t = v_0\, \omega^t$ or, approximately, $v_t = v_0\, e^{\omega' t}$, if $\omega' \doteq \omega - 1$.

and

(2.2) $a_t = a_0\, \alpha^t$ or, approximately, $a_t = a_0\, e^{\alpha' t}$, if $\alpha' = \alpha - 1$.

Substituting (2.1) and (2.2) in (1a), we find :

$$N_t^2 = w_0^2\, e^{\psi'' t} \text{ or, approximately, } N_t^2 = w_0^2\, \psi_2^t, \text{ if } \psi_2' = \psi_2 - 1.$$

where

$$w_0^2 = \nu^{20}\, v_0^{\nu^{21}+\nu^{22}}\, a_0^{-\nu^{22}}$$

and

$$\psi_2 = (\nu^{21} + \nu^{22})\, \omega' - \nu^{22}\, \alpha' + 1$$

Similarly, we assume the following demand function for manpower stock with third-level education :

(6a) $N_t^3 = \nu^{30} v_t^{\nu^{31}} \left(\frac{v_t}{a_t}\right)^{\nu^{32}} + \pi^2 n_t^2 + \pi^3 n_t^3$

Using the assumptions about the time paths of v_t and a_t, the first term at the right-hand side of this equation can be replaced by the expression :

$$w_0^3\, \psi_3^t$$

where

$$w_0^3 = \nu^{30}\, v_0^{\nu^{31}+\nu^{32}}\, a_0^{-\nu^{32}}$$

and

$$\psi_3 = (\nu^{31} + \nu^{32})\, \omega' - \nu^{32}\, \alpha' + 1$$

Generally speaking the ν coefficients will not be the same for second- and third-level manpower. The coefficients w_0^2 and ψ_2, therefore, will be different from w_0^3 and ψ_3, respectively.

DETERMINATION OF BALANCED GROWTH PATHS

The previous results suggest that the "smooth" movement of the variables which satisfy the equations (1a), (6a) and (2) to (5) can be described by equations of the following type :

$$z_t = z_{02}\, \psi_2^t + z_{03}\, \psi_3^t$$

where z_{02} and z_{03} are constants. Thus, we assume the following solutions: $n_t^2 = n_{02}^2 \psi_2^t + n_{03}^2 \psi_3^t$; $n_t^3 = n_{02}^3 \psi_2^t + n_{03}^3 \psi_3^t$ etc. for m_t^2, m_t^3, N_t^2 and N_t^3.

Substituting these expressions in the model, we can determine the values of the constants from the two sets of conditions they have to satisfy and which can be derived from the model. The first set is obtained by combining all terms with powers of ψ_2, the second set by combining all terms with powers of ψ_3. We find the following equations:

$N_{02}^2 = w_0^2$	$N_{03}^2 = 0$
$N_{02}^2 \psi_2 = (1-\lambda^2) N_{02}^2 + m_{02}^2 \psi_2$	$N_{03}^2 \psi_3 = (1-\lambda^2) N_{03}^2 + m_{03}^2 \psi_3$
$m_{02}^2 \psi_2 = n_{02}^2 - n_{02}^3 \psi_2$	$m_{03}^2 \psi_3 = n_{03}^2 - n_{03}^3 \psi_3$
$m_{02}^3 \psi_2 = n_{02}^3$	$m_{03}^3 \psi_3 = n_{03}^3$
$N_{02}^3 \psi_2 = (1-\lambda^3) N_{02}^3 + m_{02}^3 \psi_2$	$N_{03}^3 \psi_3 = (1-\lambda^3) N_{03}^3 + m_{03}^3 \psi_3$
$N_{02}^3 = \pi^2 n_{02}^2 + \pi^3 n_{02}^3$	$N_{03}^3 = w_0^3 + \pi^2 n_{03}^2 + \pi^3 n_{03}^3$

These equations are just sufficient to determine the values of all constants with given values for ψ_2 and ψ_3. Consequently, the "smooth" time paths of all variables of the model are determined.

SECTORAL DISAGGREGATION OF PRODUCTION AND ITS MANPOWER REQUIREMENTS

Reformulation of the model

Another elaboration of the model is to disaggregate the total national product into a few components, each with its own rate of development and manpower requirements; we shall limit it to the two sectors together comprising the whole economy.

The only reformulations of the model needed concern equations (1) and (6), and the addition of two definition equations. The new equations are:

(3.1a) $^1N_t^2 = {}^1\nu^2 v_t^1$	(3.6a) $^1N_t^3 = {}^1\nu^3 v_t^1$
(3.1b) $^2N_t^2 = {}^2\nu^2 v_t^2$	(3.6b) $^2N_t^3 = {}^2\nu^3 v_t^2 + \pi^2 n_t^2 + \pi^3 n_t^3$
(3.1c) $N_t^2 = {}^1N_t^2 + {}^2N_t^2$	(3.6c) $N_t^3 = {}^1N_t^3 + {}^2N_t^3$

The symbols v^1 and v^2 indicate the net outputs of sectors 1 and 2 respectively, $^1N^2$ measures the labour force with a secondary education in sector 1, $^2N^2$ the labour force with a secondary education in sector 2, etc. According to equation (3.6b), the teachers at both levels of education have been included in sector 2.

The development of the production in each of the sectors is given. Several alternative assumptions can be made to indicate that not all sectors are growing at the same rate. Here we assume that total national product is growing at a constant rate and that production in each sector grows according to $v_t^i = v_{01}^i \omega^t + v_{00}^i$ $(i = 1, 2)$. The constants v_{01}^i and v_{00}^i can be derived from the values of v_t^i for $t = 0$ and $t = 1$ in the following way.

For $t = 0$ we have	For $t = 1$ we have
$v_{01}^1 + v_{00}^1 = v_0^1$	$v_{01}^1 \omega + v_{00}^1 = v_1^1$
$v_{01}^2 + v_{00}^2 = v_0^2$	$v_{01}^2 \omega + v_{00}^2 = v_1^2$

Our assumption that total national product grows at a constant rate adds the condition: $v_{00}^1 + v_{00}^2 = 0$.

These 5 equations determine all constants v_{01}^1, v_{00}^1, v_{01}^2, v_{00}^2 and ω.

DETERMINATION OF BALANCED GROWTH PATHS

To determine the balanced growth values of the variables of the model, we assume that the trend values of the variables can be described by equations of the following type :

$z_t = z_{01}\,\omega^t + z_{00}$, where z_{01} and z_{00} are constants.

Thus, we assume

${}^1N_t^2 = {}^1N_{01}^2\,\omega^t + {}^1N_{00}^2$, ${}^2N_t^2 = {}^2N_{01}^2\,\omega^t + {}^2N_{00}^2$ etc.

for

${}^1N_t^3$, ${}^2N_t^3$, m_t^2, m_t^3, n_t^2 and n_t^3.

Substituting these expressions in the model, we can derive two sets of conditions which the constant in the solutions of the variables has to satisfy. The first set of equations is obtained by combining all terms with powers of ω, the second set by combining all constants. Each set consists of 10 equations and 10 unknowns. We give here only the first five equations of each set :

${}^1N_{01}^2 = {}^1\nu^2\,v_{01}^1$	${}^1N_{00}^2 = {}^1\nu^2\,v_{00}^1$
${}^2N_{01}^2 = {}^2\nu^2\,v_{01}^2$	${}^2N_{00}^2 = {}^2\nu^2\,v_{00}^2$
$N_{01}^2 = {}^1N_{01}^2 + {}^2N_{01}^2$	$N_{00}^2 = {}^1N_{00}^2 + {}^2N_{00}^2$
$N_{01}^2\,\omega = (1 - \lambda^2)\,N_{01}^2 + m_{01}^2\,\omega$	$N_{00}^2 = (1 - \lambda^2)\,N_{00}^2 + m_{00}^2$
$m_{01}^2\,\omega = n_{01}^2 - n_{01}^3\,\omega$ etc.	$m_{00}^2 = n_{00}^2 - n_{00}^3$ etc.

From these equations all constants, and consequently the time paths of all variables, can be determined.

DROP OUT

MODIFICATIONS OF THE MODEL

The basic model assumes that all students enrolled at secondary schools will one period later either join the labour force with sc ·ondary education or be enrolled in third-level education. This is a simplified picture of the reality :

not all students enrolled will graduate; some do not complete their studies, others fail their examinations (the criterion for completion of studies must be defined e.g. degree or diploma, specific number of years' enrolment etc.);

not all students will be able to complete their studies within the fixed time units, i.e. the assumed training period for each level of education;

not all second- and third-level graduates will join the labour force.

We shall ignore the second factor, the best way of taking it into account being to reduce the length of the time unit in which the flow variables of the model are measured (see below), but which makes the time structure of the model more complex.

We can modify the basic model to take into account the two other factors by leaving equations (1), (2), (5) and (6) unchanged, and reformulating equation (3) as follows :

(4.3) $m_t^2 = \mu^{21}\,n_{t-1}^2 + \mu^{22}\,n_{t-1}^3 - n_t^3$.

According to this equation, the number of persons joining the labour force with secondary education will equal a fraction μ^{21} of the number of students enrolled at secondary schools one time-period earlier. To this figure should be added those third-level students who do not complete their studies, assumed to be numerically proportional, with a coefficient μ^{22}, to the number of third-level students one period earlier, minus those who have completed their secondary education and continue their studies at the third level.

Equation (4) can be modified as follows :

(4.4) $m_t^3 = \mu^3 n_{t-1}^3$

According to this equation, not necessarily all students at the third level will graduate and join the third-level labour force.

As can be seen, the basic model assumes $\mu^{21} = \mu^3 = 1$ and $\mu^{22} = 0$. Further we have $\mu^{22} \leqslant 1 - \mu^3$, since $1 - \mu^3$ includes both the proportion of third-level students who do not enter the labour force at all and of those who do not complete their studies, while μ^{22} includes only the latter proportion.

An example

Assuming the following values for the μ-coefficients : $\mu^{21} = 0.7$; $\mu^{22} = 0.1$ and $\mu^3 = 0.8$ and the same values as taken in the Kyklos article for the other coefficients, we can calculate the balanced structure of the education system corresponding to a growth rate of total production of, say 30 per cent per time period and with $v_0 = 100$. The results (Case I) are given in Table 3 which includes for comparison the values of the variables when $\mu^{21} = \mu^3 = 1$ and $\mu^{22} = 0$ already calculated in the article (Case II).

TABLE 3. BALANCED GROWTH OF THE EDUCATIONAL SYSTEM WITH A GROWTH RATE FOR PRODUCTION OF 30 PER CENT PER TIME UNIT AND WITH TWO DIFFERENT SETS OF μ-COEFFICIENTS (Cases I and II)

Symbols	Case	I $\mu^{21} = 0.7$; $\mu^{22} = 0.1$; $\mu^3 = 0.8$			II $\mu^{21} = \mu^3 = 1$ $\mu^{22} = 0$		
	Time period	0	1	2	0	1	2
v	Volume of production	100	130	169	100	130	169
N^2	Manpower with secondary education	20.0	26.0	33.7	20.0	26.0	33.7
N^3	Manpower with third-level education	2.66	3.45	4.49	2.45	3.19	4.14
n^2	Students in secondary schools	13.7	17.8	23.2	9.3	12.1	15.8
n^3	Students in third-level education	1.33	1.73	2.24	0.98	1.27	1.65
m^2	Manpower with secondary education and under 6 years' employment	6.2	8.1	10.5	6.2	8.1	10.5
m^3	Manpower with third-level education and under 6 years' employment	0.82	1.06	1.38	0.75	0.98	1.27

A comparison between these cases shows that :

many more students have to be enrolled in Case I than in Case II : over 45 per cent more at secondary level and 35 per cent at third level;

the stock of manpower with third-level education has to be nearly 9 per cent larger to cover the increased need of teachers for the additional students. Secondary level, used only in production, is not affected.

ALTERNATIVE TREATMENT OF RETIREMENT

A further refinement of the model may consist in a more precise treatment of retirement. As the number of people leaving the labour force depends on the age composition of the stock of manpower, the assumption made in equations (2) and (5) of the basic model is too simple, and may be harmful when, as a consequence of rapid development, the age composition of the stock of qualified manpower is biased in favour of the younger generations. An acceleration in development will stress this bias even more. A more precise treatment requires a distinction to be made between the various age classes of the manpower stock and the application of retirement rates characteristic of each age class. This introduces far more time units than were in our basic model, making it considerably more complicated. We hope such attempts will be made in the future, but have refrained from doing so ourselves A simpler approach would be to adapt the numerical values of the coefficients λ^2 and λ^3 to the prevailing conditions, implying their possible (and probable) change over time in view of the type of problem considered.

To illustrate the impact of the envisaged rate of economic development on the values of λ, we made estimates of the latter based on the more exact treatment suggested above but restricting ourselves to the assumption that only retirement due to old-age takes place. This means that equation (2) is replaced by :

$$N_t^2 = N_{t-1}^2 - m_{t-T}^2 + m_t^2$$

where m_{t-T}^2 represents the number of those who entered the (second level) labour force T time units earlier; T standing for the productive life of an individual. It can be shown that this is equivalent to choosing :

$$\lambda^2 = \frac{\omega - 1}{\omega^T - 1}$$

where, as before, ω is the ratio of production in year t to that in year $t - 1$. For some different values of T and ω we then find the following values for λ^2 :

TABLE 4. VALUES FOR λ^2 FOR CERTAIN VALUES OF T AND ω

	$\omega = 1.2$	$\omega = 1.3$	$\omega = 1.4$
$T = 7$. .	0.077	0.057	0.042
$T = 8$. .	0.061	0.042	0.029
$T = 9$. .	0.048	0.031	0.020

These figures illustrate the impact of the rate of development on the age composition and the retirement ratio.

INCREASING THE NUMBER OF EDUCATION PROCESSES

For various purposes it will be useful to distinguish between a larger number of education processes, whether or not with the introduction of a larger number of production sectors. In the basic model the first three equations refer to secondary and the next three to third-level education, so that more stages of education will mean adding further triplets of equations—the manner of linking them together differing according to the type of educational subdivision chosen. A higher or lower level of education might be added, or one of the levels split into parallel components. A very useful refinement would be to split up third-level education into the humanities and technical and scientific subjects. The number of students entering these two branches would then be linked with the number of secondary school leavers by an equation replacing equation (3) and showing the following features :

$$m_t^2 = n_{t-1}^2 - n_t^3 - n_t^4$$

where n_t^3 and n_t^4 now stand for the number of students in the humanities and science departments respectively.

USING SMALLER TIME UNITS

The very simple time structure of our basic model, where only one lag appears in equations (3) and (4), was due to the assumption that secondary and third-level education each take the same time span (assumed to be six years) and to the choice of a time unit equal to that span; such a time structure considerably simplifies the calculations to be made.

A more refined time structure may be required, and would be desirable (though not absolutely necessary) if the duration of the two stages of education differs. The successive drop-out ratios for each consecutive year may also have to be considered.

All this is possible without changing the mathematical nature of the problems treated; the number of time units to be considered become more complicated. The equations still remain difference equations, and the possibility of defining and calculating "smooth paths" or of the numerical treatment of transition problems does not disappear. This type of generalization is less urgent than some of the others discussed.

THE ELIMINATION OF SURPLUSES OF A GIVEN EDUCATIONAL ATTAINMENT FROM THE LABOUR FORCE

THE ELIMINATION OF A SURPLUS IN ONE TIME UNIT

In some countries the available labour force with a given level of education may surpass the needs of the economy for this particular type of manpower. This may be expressed by unemployment, the inefficient use of labour and/or relatively low salaries, and may affect all graduates of a given level, (e.g. university) or only those in specific fields (law, humanities).

We shall now see how the basic model can be used for planning the elimination of such a surplus by considering a situation where there is an excess of third-level labour force and where all other variables, including manpower with second-level education, are already adapted to their balanced-growth values.

To reduce the surplus in period 1, fewer, or even no new students should be admitted during this period to third-level education. However, since the number of students graduating from secondary schools in period 1 is given (equation 4) and is adapted to the equilibrium growth, a reduction in the number of third-level students below the equilibrium value is possible only by creating a surplus in the labour force with secondary education. If we are to avoid this, the surplus third-level labour force can never be eliminated in one time unit. The following example shows whether it would be possible without this restriction.

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We use the bas'c model with the values of the coefficients as indicated. The existence of a surplus of third-level manpower implies that ν^3 can no longer be measured by the relation between N_t^3 (after deduction of the number of teachers) and v_t. A normative value for ν^3 must be assumed. The difference between the actual N^3 and the N^3 needed for production and teaching and estimated on the basis of normative values of ν^3 and, if necessary, the π's, measures the surplus manpower. (For $t = 0$ $\tilde{N}_0^3$ measures the third-level labour force needed and not the actual stock.) Let S_0^3 be the surplus third-level labour force and let all other variables in the initial situation ($t = 0$) be adapted to a growth in production by 30 per cent per time unit (see Table 5). If the elimination of S_0^3 is to take place in one time unit, the following sets of equations have to be satisfied :

For $t = 1$

(1) $\underline{N_1^2} = 0.2\,\underline{v_1} + S_1^2$
(2) $N_1^2 = 0.9\,\underline{N_0^2} + m_1^2$
(3) $m_1^2 = \underline{n_0^2} - n_1^3$
(4) $m_1^3 = \underline{n_0^3}$
(5) $N_1^3 = 0.9\,(\underline{\tilde{N}_0^3} + \underline{S_0^3}) + m_1^3$
(6) $N_1^3 = 0.02\,\underline{v_1} + 0.04\,n_1^2 + 0.08\,n_1^3 + S_1^3$

For $t = 2$

(7) $\underline{N_2^2} = 0.2\,\underline{v_2}$
(8) $\underline{N_2^2} = 0.9\,N_1^2 + m_2^2$
(9) $m_2^2 = n_1^2 - \underline{n_2^3}$
(10) $m_2^3 = n_1^3$
(11) $\underline{N_2^3} = 0.9\,N_1^3 + m_2^3$
(12) $\underline{N_2^3} = 0.02\,\underline{v_2} + 0.04\,\underline{n_2^2} + 0.08\,\underline{n_2^3}$

All underlined symbols may be considered as given : those in the first set of equations because they refer to the initial situation of the production volume; those in the second because they concern the production volume or variables which also appear in the equations for $t = 3$ which must lead to equilibrium values for all variables.

The equations (7) and (12) need not be considered since they contain only known variables. The remaining 10 equations are mathematically just sufficient to determine the 10 unknowns. Whether this will lead to an economically acceptable solution depends on the bounds to be set to the variables. Naturally all variables must be at least non-negative, but one might wish to set some additional constraints, for example $n_1^2 \geqslant n_0^2$ and $n_1^3 \geqslant n_0^3$, expressing the desire to diminish the number of students at both levels, not in absolute terms. Another additional constraint might be $S_1^2 \leqslant \bar{S}^2$, if the surplus labour force with secondary education should not exceed a certain maximum $\bar{S}^2$.

Whether a feasible solution with values of the variables satisfying the constraints exists depends on the size of the initial surplus S_0^3. Using the numerical values derived from Table 1 Case A for the underlined variables in the equations (1) to (6) and (8) to (11) above, we can determine the values of the unknown variables in terms of S_0^3. The results are given in Table 5 below.

TABLE 5. VALUES OF VARIABLES

Symbols	Time period 0	1	2
v	100	130	169
N^2. . . .	20	$26.1+0.81\ S_0^3$	33.7
S^2. . . .	—	$0.1+0.81\ S_0^3$	—
N^3. . . .	$2.45+S_0^3$	$3.19+0.9\ S_0^3$	4.14
S^3. . . .	S_0^3	$0.02+0.99\ S_0^3$	—
n^2. . . .	9.3	$11.8-0.73\ S_0^3$	15.8
n^3. . . .	0.98	$1.27-0.81\ S_0^3$	1.65
m^2. . . .	6.2	$8.0\ +0.81\ S_0^3$	$10.1+0.73\ S_0^3$
m^3. . . .	0.75	0.98	$1.27-0.81\ S_0^3$

The column of values for $t = 1$ shows that an adaptation in one time unit is not possible if $S_0^3 > 1.57$, because n^3 cannot be negative. We also find that unless $S_0^3 > 2.0$, the surplus of third-level labour force will increase from period 0 to 1. Since S_0^3 must be smaller than 1.57, this will always be the case. This increase in S^3 must be explained by the fact that the decrease in the number of students at both levels diminishes the demand for teachers in period 1, but their supply adapted to expected higher numbers cannot be decreased.

If the values of the solutions are not within the bounds imposed on them, the conclusion must be that there is no acceptable way of eliminating the surplus labour in one time unit and a solution must be found assuming an adaptation period of two time units.

THE ELIMINATION OF A SURPLUS IN TWO TIME UNITS

To find the solution in this case, we write down the equations for $t = 1$, $t = 2$ and $t = 3$. Again the underlined symbols can be considered as given for the reasons mentioned above. Their numerical values will be taken from Table 1 Case A.

For $t = 1$

(1) $N_1^2 = 0.2\ \underline{v_1} + S_1^2$

(2) $N_1^2 = 0.9\ \underline{N_0^2} + m_1^2$

(3) $m_1^2 = \underline{n_0^2} - n_1^3$

(4) $m_1^3 = \underline{n_0^3}$

(5) $N_1^3 = 0.9\ (\underline{\tilde{N}_0^3} + \underline{S_0^3}) + m_1^3$

(6) $N_1^3 = 0.02\ \underline{v_1} + 0.04\ n_1^2 + 0.08\ n_1^3 + S_1^3$

For $t = 2$

(7) $N_2^2 = 0.2\ v_2 + S_2^2$

(8) $N_2^2 = 0.9\ N_1^2 + m_2^2$

(9) $m_2^2 = n_1^2 - n_2^3$

(10) $m_2^3 = n_1^3$

(11) $N_2^3 = 0.9\ N_1^3 + m_2^3$

(12) $N_2^3 = 0.02\ v_2 + 0.04\ n_2^2 + 0.08\ n_2^3 + S_2^3$

For $t = 3$

(13) $\underline{N_3^2} = 0.9\, N_2^2 + m_3^2$

(14) $\underline{m_3^2} = n_2^2 — \underline{n_3^3}$

(15) $m_3^3 = n_2^3$

(16) $\underline{N_3^3} = 0.9\, N_2^3 + m_3^3$

Together we have 16 equations with 18 variables and, therefore, two degrees of freedom. We shall consider n_1^2 and n_1^3 as the free variables. They will be chosen in such a way as to minimize the costs connected with the existence of surpluses in the labour force of both education levels during the periods 1 and 2;

$$C = S_1^2 + S_2^2 + \gamma\,(S_1^3 + S_2^3) = \text{Min}$$

where γ measures the costs of a person with a third-level education in proportion to those of a person with a second-level.

Taking numerical values for the known variables, we can derive from the equations (1) to (16) the following solutions :

(5.1) $S_1^2\, 1.40 — n_1^3$

(5.2) $S_1^3 = 0.59 — 0.9\, S_0^3 — 0.04\, n_1^2 — 0.08\, n_1^3$

(5.3) $S_2^2 = 11.91 + 0.73\, S_0^3 + n_1^2$

(5.4) $S_2^3 = — 1.78 + 0.9\, S_0^3 + 0.036\, n_1^2 + 1.07\, n_1^3$

(5.5) $n_2^2 = 26.2 — 0.66\, S_0^3 — 0.9\, n_1^2$

(5.6) $n_2^3 = 2.77 — 0.73\, S_0^3 — 0.9\, n_1^3$

Assuming that $\gamma = 2$, we deduce from equations (5.1) to (5.4) and the cost function

$$C = — 12.89 + 4.33\, S_0^3 + 0.99\, n_1^2 + 0.98\, n_1^3$$

This function will be minimized with values for n_1^2 and n_1^3 which are the lowest possible. We must however, add a number of bounds to the values of the variables, at least those which impose that all surpluses and the n's have to be non-negative. These bounds can be derived from equations (5.1) to (5.6) :

(5.11) $n_1^3 \leqslant 1.40$

(5.21) $0.04\, n_1^2 + 0.08\, n_1^3 \leqslant 0.59 + 0.9\, S_0^3$

(5.31) $n_1^2 \geqslant 11.91 — 0.73\, S_0^3$

(5.41) $0.036\, n_1^2 + 1.07\, n_1^3 \geqslant 1.78 — 0.9\, S_0^3$

(5.51) $0.9\, n_1^2 \leqslant 26.2 — 0.66\, S_0^3$

(5.61) $0.9\, n_1^3 \leqslant 2.77 — 0.73\, S_0^3$

(5.71) $n_1^2 \geqslant 0$

(5.81) $n_1^3 \geqslant 0$

From (5.11) and (5.61) we deduce that $S_0^3 \leqslant 2.07$. With a larger surplus stock no absorption of this surplus is possible in two time units. For a numerical example we assume $S_0^3 = 1.80$.

The simple structure of the model makes it possible to derive from inspection of the inequalities (or their graphical representation) that $n_1^2 = 10.6$ [the minimum bound according to (5.31)], and $n_1^3 = 0$ will minimize the cost function. The values of the surpluses and the other n's are given in Table 6, Case A.

TABLE 6. VALUES OF CERTAIN VARIABLES IN TWO CASES OF ABSORPTION OF THIRD-LEVEL SURPLUS MANPOWER IN TWO TIME UNITS

Symbols	Case A				Case B			
	$t = 0$	1	2	3	$t = 0$	1	2	3
n^2	9.3	10.6	15.5	20.5	9.3	10.6	15.5	20.5
n^3	0.98	—	1.46	2.15	0.98	0.98	0.58	2.15
S^2	—	1.4	—	—	—	0.4	—	—
S^3	1.80	1.79	0.22	—	1.80	1.71	1.27	—

If we do not want to accept an absolute decrease in n_1^3 below n_0^3, we have to add a new constraint :

$$n_1^3 \geqslant 0.98$$

In this case we find the solutions for the variables as given in Case B of Table 6. The results show that now n_2^3 has fallen below the minimum bound set for n_1^3. It is not possible to exclude this by adding another new constraint $n_2^3 \geqslant 0.98$ since this would be inconsistent with (5.6) and (5.91). No absorption is possible in two time units without an absolute decrease in third-level students.

A comparison of cases A and B shows that the development of the number of secondary school students has to be the same in each case. A surplus of the labour force with a secondary education has to be created in period 1 only, and has to be much higher in case A than case B. The surplus stock of third-level manpower does not fall very much in either case in the first period, but does so in Case A in the second.

Part II

PLANNING MODELS FOR THE CALCULATION OF EDUCATIONAL REQUIREMENTS FOR ECONOMIC DEVELOPMENT

SPAIN

by
L. J. Emmerij

INTRODUCTION

This paper presents a number of variations of the Model and their applications to the actual situation in Spain. Thus, the interrelationship between the Spanish educational system and the economic development this country wishes to achieve has been expressed and studied in a simple mathematical form. This model approach, among the many techniques for studying this question, has its own set of advantages and pitfalls. It will be demonstrated, however, that the model's simplicity in no way impedes its tremendous capacity to be adapted to different situations.

The educational system in Spain is briefly described below. The section entitled "The Model and its Application to Spain" deals successively with: the original model, the disaggregation of the economy by sector; the correction for drop-outs and effective length of study; changing co-efficients; the introduction of more types of manpower and education and, finally, an attempt to shift the model to a marginal approach.

The economic development of Spain is expected to make considerable progress in the coming years. The first economic plan will start in 1964 and will cover the period until 1967. A heavy strain will be put on the educational system of the country, and the needs that this creates are recognised by all those who are concerned with these questions in Spain. A considerable amount of detailed work has been done by the Spanish M.R.P. team to establish a long-term educational plan with relation to the economic development perspectives.

It is hoped that this paper will be considered as an additional tool for their work.

THE SPANISH EDUCATIONAL SYSTEM

Primary Education

Compulsory education in Spain extends over a period of six years and is divided into two parts: a period of elementary education for children six years old to nine inclusive, and a so-called " perfection period " for children of 10 to 11 years inclusive. Children can obtain primary-school certificate at the age of 12 after having attended classes for six years, and after passing a final examination.

SECONDARY EDUCATION

Because of a certain amount of heterogeneity in the different types of education at second level it may be convenient to make a distinction—more or less arbitrarily—between three kinds of secondary education: general, technical and professional.

General Secondary Education. Three parts can be distinguished here. The elementary part which takes a minimum of four years to accomplish; the so-called *bachillerato superior* taking a minimum of two additional years; and a one year pre-university class, compulsory for those who want to pursue their studies at the university level. To enter secondary school, a child must be at least 10 years of age i.e. having completed the first cycle of primary education, and pass an entrance examination. It follows—if favourable conditions prevail—that he can obtain the *bachillerato elemental* diploma at 14; the *bachillerato superior* at 16, and has the possibility of entering an institute of higher learning at the age of 17.

Professional Secondary Education. Under this heading will be classified those types of secondary education that are predominantly concerned with the training of young men and women for a specific occupation of a non-technical character. In Spain these will include the primary teacher-training colleges (*enseñanza normal*) and the commercial schools (*enseñanza mercantil*).

The *bachillerato elemental* diploma is required for entry to the primary teacher-training colleges. After a three-year programme and a final examination the title of *Maestro* is awarded.

The commercial schools have the same entrance requirements as the primary teacher-training colleges, except that they are also open to those with a *bachillerato elemental laboral* diploma (see below). A first certificate can be obtained after a minimum of three years (*perito mercantil*); it is also possible to continue for a further three years to acquire the diploma of *professor mercantil*, giving access to the Department of Economics at University level.

Technical Education at Secondary Level. The types of education coming under this heading are also concerned with training for a specific occupation, but this time of a primarily technical character. The three types to be distinguished are easier to describe than to translate, they are: *formación profesional industrial*; *enseñanza media laboral*; and *enseñanza tecnica de grado medio.*

Formación Profesional Industrial. Its aim is to train qualified workers while at the same time giving them a minimum of general education. Through a system of intermediate steps its graduates may enter the general secondary schools as well as the *enseñanza media laboral.* This type of training is actually between primary and secondary education and might best be described as elementary vocational training. There are three parts: (*a*) professional initiation: two years after completion of the full six years of primary education; (*b*) apprenticeship: three years of training, minimum entrance age 14; (*c*) mastership (*Maestria*): a further two years.

Enseñanza Media Laboral. This is a combination of general secondary and technical training[1]. As in general secondary education there are two certificates: *bachillerato laboral elemental* (five years instead of four) and *bachillerato laboral superior* (two additional years). Entrance age: 10.

Enseñanza Tecnica de Grado Medio. The entrance into these schools is preceded by one preparatory year and one selective year for those with only the *bachillerato elemental* diploma; those with *bachillerato superior, perito mercantil* or a primary teacher-training college diploma have access directly to the selective year. Once these hurdles are passed, three years of training follow plus a practical period of three months. After this the final examination takes place.

This latter type of technical education borders on higher education and its graduates can best be described as sub-engineers.

Higher Education

This is given at the Universities and at the Institutes of Technology. For entrance into the University, the *bachillerato superior* is required and one pre-university year has to be completed. In some cases, as was mentioned above, it is possible to enter with other secondary certificates. The legal study period to obtain the *licenciado* is five years except in Medicine and Pharmacy where it is six. A doctorate can be acquired after a further year and presentation of a thesis. Most students, however, stop after the *licenciatura*.

Table 7. TOTAL ENROLMENTS AND GRADUATES IN 1960 FOR THE DIFFERENT LEVELS OF THE SPANISH EDUCATIONAL SYSTEM

	TOTAL ENROLLED	GRADUATES[1]
Primary education	3,751,469	
Secondary education:		
General secondary education	448,311	23,189
Professional secondary education:		
Primary teacher-training colleges	41,573	8,995
Commercial schools	25,745	6,037
Technical secondary education:		
Formación profesional industrial	79,099	2,946[2]
Enseñanza media laboral	16,555	1,454
Enseñanza tecnica de grado medio	38,891	3,636
Total secondary education:		
incl. Formación profesional industrial	650,174	46,257
excl. Formación profesional industrial	571,075	43,311
Higher education		
Science departments	14,142	468
Other departments	51,170	4,431
Institutes of technology	11,795	722
Total higher education	77,107	5,621

1. For a definition of " graduates " at secondary level see above.
2. High drop-out rate.

1. This is not true for all cases, for example, one specialisation is " administration and secretarial activities, " but, compared to the number of technical subjects, these are very few.

Entrance to the Institutes of Technology is again subject to the successful completion of a selective year and an initiation year. Thereafter five years are required to obtain the title of Engineer.

After this brief description of the Spanish educational system some quantitative information will be presented concerning total enrolments and graduates at the different levels in 1960[1]. From then on we shall consider secondary education as including all the different types mentioned above except *Formación profesional industrial.* We shall consider as graduates at secondary level those who obtain: the *bachillerato superior* (general or *laboral*); a diploma of a primary teacher-training college; either one of the two diplomas of the Commercial Schools, and also those who finish the *enseñanza tecnica de grado medio.*

Enrolment Ratios. Of the age group 10-19 inclusive, 12.7 per cent are enrolled in secondary schools (incl. *Formación profesional industrial*). If this latter group is excluded, 11.2 per cent are enrolled.

The ratio for higher education as a percentage of the 20-24 (incl.) age group is 3.5.

THE MODEL AND ITS APPLICATION TO SPAIN

THE ORIGINAL MODEL

The original model has been described in Part I so that a statement of the six equations of the system will suffice here. The equations considered to represent the most important aspects of the inter-relationship between the economy and the educational system are the following:

(1) $N_t^2 = \nu^2 v_t$

(2) $N_t^2 = (1 - \lambda^2) N_{t-1}^2 + m_t^2$

(3) $m_t^2 = n_{t-1}^2 - n_t^3$

(4) $m_t^3 = n_{t-1}^3$

(5) $N_t^3 = (1 - \lambda^3) N_{t-1}^3 + m_t^3$

(6) $N_t^3 = \nu^3 v_t + \pi^2 n_t^2 + \pi^3 n_t^3$

Coefficients. In order to apply the original model to the Spanish situation the coefficients had to be calculated. The values found were as follows:

$\nu^2 = 0.86$
$\nu^3 = 0.306$ G.N.P. in 1960 was expressed in billions of 1960 pesetas.

Because of the unstable position of the peseta prior to 1959, total volume of production for the initial and subsequent periods will be expressed in constant 1960 prices. The number of workers in 1960 with a higher or secondary education is expressed in thousands. For the calculation of ν^3, secondary-school teachers and professors were deducted from the third-level manpower. To facilitate international comparisons the two technical coefficients have also been calculated by expressing v in millions of U.S. dollars at the 1960 exchange rate. (Manpower is expressed in thousands.) Values found:

$\nu^2 = 0.052$
$\nu^3 = 0.0184$

1. See Table 7.

$\pi^2 = 0.05$

$\pi^3 = 0.08$

In 1960 there was one secondary-school teacher for 20 pupils and two professors for 25 students.

The values of the two coefficients λ^2 and λ^3, measuring deaths and retirements of the second- and third-level manpower stock respectively, are not very easy to establish with absolute accuracy. These values are quite obviously related to the age composition of the stocks under consideration. Difficulty arises from the fact that this age composition is likely to change over time depending upon the growth rates—past and future—of the manpower stocks. In the case of a country where this growth rate has been relatively low in the past and where the number of middle- and high-level personnel has to increase rapidly in the future, the values of λ^2 and λ^3 will fall rapidly over time, i.e. relatively fewer people will die or go into retirement because of the heavier weight of younger persons in the total stock.

Mr. A. K. Sen in his comments on the Model[1] proposed the following treatment of this " depreciation problem ": for any age group a certain proportion goes out due to death or retirement; this proportion will vary according to age group and will be 100 per cent for the period at which retirement comes.

Equation (2) will then read:

$$N_t^2 = m_t^2(1 - \lambda_t^2) + m_{t-1}^2(1 - \lambda_t^2)(1 - \lambda_{t-1}^2) + m_{t-2}^2(1 - \lambda_t^2)(1 - \lambda_{t-1}^2)(1 - \lambda_{t-2}^2) \dots\dots$$ [2]

A final period will be introduced by putting:

$$\lambda_{t-r}^2 = 1,$$

where r represents the number of periods at which retirement comes.

The main objection to the above method—besides some computational complications in the solution of the model—is that one would have to know the exact number of people who have entered the labour force every six years over the past 50 years or so. This kind of information—for different types of manpower—will be very hard to find in most countries. This, however, is no reason to throw the idea overboard. Mr. Sen's suggestion provides a clue towards what the *average* value of λ^2 and λ^3 will be if the manpower stocks grow at a certain rate over a long period of time, by assuming that the m's in the above equation have been growing at that same rate in the past.

For Spain it is most important that λ be as realistic as possible to reflect the slow growth of the N^2 and N^3 stocks in the past (it is only in recent years that a certain acceleration can be noticed) and the important speeding up of the growth of these stocks in the future with the resulting change in the age-composition. The Sen method has been applied to Spain using the latter possibility of eliminating the lagged variables as pointed out above. The implicit average values found are:

$\lambda^2 = 0.056$

$\lambda^3 = 0.077$

These values have been calculated by assuming an average active life for the N^2 and the N^3 stock of 48 years (average entrance age 17; average retirement age 65), and 42 years (23-65) respectively, and by applying the death rates

1. See: *The Residual Factor and Economic Growth,* O.E.C.D. 1964.

2. It will be noted that this time a λ has also been attached to m_t although this was not done either in the original model or in the Sen paper; this treatment seems to be a little more precise since the m's are clearly flow variables.

for the age groups corresponding to each of the lagged m's. Thus for m^2, death rates have been taken for the age groups 17-22; 23-28; 29-34; etc., and for m^3 for the age groups 23-28; 29-34; 35-40, etc. The above results then are the values for λ^2 and λ^3 assuming a 6 per cent annual growth of the stocks in past and future time periods. They have been retained for the following exercises with the model as being approximately the right averages for long-term projections starting from the situation in $t = -1$. It can indeed readily be seen that, given the slower growth of the stocks in the past, the heavy inflow of young graduates will bring the values of λ^2 and λ^3 down very rapidly, and that they will soon drop *below* the above calculated values. It will be only towards $t = 7$ that their values will actually be reached, assuming, of course, that the growth rate continues at the same rythm. One could be more precise by making the values of λ^2 and λ^3 change each time period. This possibility will be discussed later under the section " Changing Coefficients ". It is also true that the values of these coefficients should not be the same if for any reason the stocks grow at a different rate. All this will show that a certain degree of arbitrariness is almost unavoidable. Thus, while the values retained here are not absolutely precise for one period, they are likely to be a fairly good average over the long term.

The coefficients thus being established, the system of linear difference equations can be solved by assuming an exponential development over time. The rate of growth which has been retained by the Spanish officials responsible for the First Plan is 6 per cent per year or 42 per cent for a six-year period. Consequently the assumption introduced is that all variables will grow according to the general formula:

$Z_t = Z_0(1.42)^t$

This assumption will ensure a balanced growth of all variables over time.

The solution of the model gives the following initial values for the six unknowns expressed in terms of v_0:

$N_0^2 = 0.86v_0$; $N_0^3 = 0.355v_0$; $n_0^2 = 0.66v_0$; $n_0^3 = 0.1765v_0$;

$m_0^2 = 0.2885v_0$; $m_0^3 = 0.124v_0$

These are the values which determine the structure of the system and which have to be fulfilled during the initial period (t_0) if a balanced growth over time is to be obtained. The results in absolute figures, for the initial and three subsequent time periods, are presented in Table 8.

TABLE 8. BALANCED GROWTH OF THE EDUCATIONAL SYSTEM FOR A GROWTH RATE OF 6 PER CENT PER YEAR (42 PER CENT PER TIME PERIOD). BASIC MODEL

Production in billions of 1960 pesetas ; population in thousands.

ACTUAL	t_0 (1960)	t_1 (1966)	t_2 (1972)	t_3 (1978)
N^2	464.4	659.4	936.3	1,329.5
N^3	191.7	272.2	386.5	548.8
n^2	356.4	506.1	718.7	1,020.5
n^3	95.3	135.3	192.1	272.8
m^2	155.8	221.2	314.1	446.0
m^3	67.0	95.3	135.3	192.1
v	540.0	766.8	1,088.9	1,546.6

The actual values of the first four variables in 1960 were (in thousands): $N^2 = 464.4$; $N^3 = 203.0$; $n^2 = 571.1$; $n^3 = 77.1$.

According to the model, Spain had a sufficient number of high-level manpower in the initial period for the realisation of a 6 per cent annual growth rate. The actual number of pupils in secondary schools also surpasses the number required by the model. It is only for the number of third-level students that the opposite is true.

BREAKDOWN OF THE ECONOMY INTO SECTORS

One possibility of refining the model consists in the disaggregation of total volume of production into several sectors of the economy. The only limit that exists to the number of sectors into which the model can be broken down is the extent to which data are available for the calculation of the sector coefficients. For illustration purposes, the model will now be applied to Spain distinguishing three sectors 1, 2 and 3, (agriculture, industry and services respectively) using the method outlined in Part 1, " Sectoral Disaggregation of Production and Manpower Requirements. " Consequently equations (1) and (6) of the original model have each been broken down into three equations relating second- and third-level manpower in each of the sectors to the sector volumes of production. When adding the two definition equations:

(7) $N_t^2 = N_t^{21} + N_t^{22} + N_t^{23}$

(8) $N_t^3 = N_t^{31} + N_t^{32} + N_t^{33}$

and making no change in equations (2) through (5), a set of 12 equations is obtained with 12 unknowns:

$(N_t^{21}, N_t^{22}, N_t^{23}, N_t^2, N_t^{31}, N_t^{32}, N_t^{33}, N_t^3, n_t^2, n_t^3, m_t^2,$ et $m_t^3)$.

The second step consists in determining the constants:

$\bar{v}^i$ and $\bar{\bar{v}}^i (i = 1, 2, 3)$

It is assumed that the growth rates for each sector during the first time period will be 19 per cent (2.9 per cent per annum) for agriculture; 62 per cent (8.4 per cent per annum) for industry; and 37 per cent (5.4 per cent per annum) for services[1]. The additional condition that the overall growth rate of the economy is to be the same for each time period, required that:

$\bar{\bar{v}}^1 + \bar{\bar{v}}^2 + \bar{\bar{v}}^3 = 0$

The values found for these constants are the following:

$$\bar{v}^1 = 0.471\, v_0^1 \qquad \bar{\bar{v}}^1 = 0.529\, v_0^1$$
$$\bar{v}^2 = 1.538\, v_0^2 \qquad \bar{\bar{v}}^2 = -0.538\, v_0^2$$
$$\bar{v}^3 = 0.918\, v_0^3 \qquad \bar{\bar{v}}^3 = 0.082\, v_0^3$$

where:

v_0^1, v_0^2, v_0^3

are the sector volumes of production in 1960[2]. Having thus expressed the constants in terms of the initial values of the sector volumes of production, the solution to the system can now be ascertained by assuming that all variables grow according to the general formula:

$z_t = \bar{z}\omega^t + \bar{\bar{z}}$

1. As for the overall growth-rate, the figures advanced here for the three sectors are the targets as calculated by the Spanish M.R.P. team.

2. Pesetas 143.0; P 174.0; and P 223 billion (1960 value) respectively.

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Here again the barred and double barred symbols are constants. These have to be determined in such a way that they correspond to the growth path of the sector volumes of production. This can be done by substituting:

$\bar{z}\omega^t + \bar{\bar{z}}$ for z_t

in the model along the lines shown above (Part I), " Sectoral Disaggregation of Production. " Collecting all terms with ω^t and all constants, two sets of 12 equations each are obtained which determine the 12 barred and the 12 double-barred constants. Once the coefficients are known the system can consequently be solved.

Values found for the technical sector coefficients[1]:

$v^{21} = 0.082 \qquad v^{31} = 0.039$

$v^{22} = 0.702 \qquad v^{32} = 0.283$

$v^{23} = 1.482 \qquad v^{33} = 0.496$

(Volumes of production in billions of 1960 pesetas; manpower in thousands.)

The coefficients λ^2, λ^3, and π^2, π^3 keep their original values.

If we now solve the two sets of 12 equations, the following values are obtained:

$\bar{N}^{21} = 5.5$	$\bar{\bar{N}}^{21} = 6.2$		$N^{21}_0 = 11.7$
$\bar{N}^{22} = 187.9$	$\bar{\bar{N}}^{22} = -65.7$		$N^{22}_0 = 122.2$
$\bar{N}^{23} = 303.4$	$\bar{\bar{N}}^{23} = 27.1$		$N^{23}_0 = 330.5$
$\bar{N}^{2} = 496.8$	$\bar{\bar{N}}^{2} = -32.4$		$N^{2} = 464.4$
$\bar{N}^{31} = 2.6$	$\bar{\bar{N}}^{31} = 2.9$		$N^{31}_0 = 5.5$
$\bar{N}^{32} = 75.7$	$\bar{\bar{N}}^{32} = -26.5$		$N^{32}_0 = 49.2$
$\bar{N}^{33} = 127.7$	$\bar{\bar{N}}^{33} = 8.9$		$N^{33}_0 = 136.6$
$\bar{N}^{3} = 206.0$	$\bar{\bar{N}}^{3} = -14.7$	which gives	$N^{3}_0 = 191.3$
$\bar{n}^{2} = 366.8$	$\bar{\bar{n}}^{2} = -2.9$		$n^{2}_0 = 363.9$
$\bar{n}^{3} = 98.9$	$\bar{\bar{n}}^{3} = -1.1$		$n^{3}_0 = 97.8$
$\bar{m}^{2} = 162.5$	$\bar{\bar{m}}^{2} = -1.8$		$m^{2}_0 = 169.7$
$\bar{m}^{3} = 70.5$	$\bar{\bar{m}}^{3} = -1.1$		$m^{3}_0 = 69.4$
$\bar{v}^{1} = 67.4$	$\bar{\bar{v}}^{1} = 75.6$		$v^{1}_0 = 143.0$
$\bar{v}^{2} = 267.6$	$\bar{\bar{v}}^{2} = -93.6$		$v^{2}_0 = 174.0$
$\bar{v}^{3} = 204.7$	$\bar{\bar{v}}^{3} = 18.3$		$v^{3}_0 = 223.0$

These results are the same, with small differences, as those obtained by applying the one-sector model (see table 8). This was to be expected since the sector growth rates are such as to ensure an overall 6 per cent growth rate of the economy. Only if the sector growth rates were out of touch with the overall plan would substantially different results be obtained. This conclusion might prompt one to question what the advantages of this sector approach might be. They are quite clear: first, the more sectors distinguished, the more

1. For international comparisons:

$v^{21} = 0.0049 \qquad v^{31} = 0.0024$

$v^{22} = 0.0421 \qquad v^{32} = 0.0169$

$v^{23} = 0.089 \qquad v^{33} = 0.0289$

Volumes of production in millions of U.S. dollars at 1960 exchange rate; manpower in thousands.

homogeneous the type of manpower employed in them will become; secondly, the results may become more precise by using this approach simultaneously with that of changing technical coefficients (see below).

INTRODUCTION OF DROP-OUT AND EFFECTIVE PERIOD OF STUDY[1]

After the foregoing exercises it is now time to give attention to a few of the rigidities of the basic model. In the model it was assumed first that secondary and higher education are each effectively accomplished in a six-year period and secondly that all pupils or students who are in the educational system at a given time-period will graduate during the next one and will enter the labour force. These assumptions are concentrated in equations (3) and (4):

(3) $m_t^2 = n_{t-1}^2 - n_t^3$

(4) $m_t^3 = n_{t-1}^3$

Even if the legal period of study at the second and third levels were six years, the effective average length of study of those who graduate would be longer since many of the students repeat classes or are subject to other kinds of delays. As has been shown, the legal study period for the different types of secondary education in Spain varies from three to seven years, whereas for higher education it is between five and seven years. The second assumption neglects drop-out and those studying for cultural reasons, i.e. persons who do not enter the labour force upon completion of their studies. These factors: effective length of study, drop-out and " cultural " students, can be introduced into the system by writing the above two equations as follows:

(3) $m_t^2 = \alpha^2(\delta n_{t-1}^2 - \gamma n_t^3 + \sigma n_t^3)$

(4) $m_t^3 = \alpha^3 \delta^3 n_{t-1}^3$

α^2 and α^3 are the " participation rates " of secondary and third-level graduates respectively, i.e. the proportions who enter the labour force;

δn_{t-1}^2 and δn_{t-1}^3

are the numbers of students in the respective levels who actually graduated during the previous six years expressed as a proportion of those who were enrolled one time period before; γn_t^3 are those who actually enrolled in institutes of higher education during the previous six years, expressed as a proportion of total enrolment; and finally σn_t^3 are those who left the third level during the previous six years, before obtaining a degree, expressed again as a proportion of total enrolment.

The values found for Spain are as follows:

$\alpha^2 = \alpha^3 = 0.85;\ \delta^2 = 0.553;\ \delta^3 = 0.557;\ \gamma = 1.0;\ \sigma = 0.45$

It will be noticed that the labour-force participation rate is the same for both secondary-school and university graduates. In the calculation of these ratios an attempt has been made to eliminate short-term movements by taking a large age group. (This would remove the question of the young women who enter the labour force immediately after school but leave professional work shortly afterwards for marriage.) It may be noted that those leaving the University before obtaining a degree (drop-out) are assumed to have the same participation rate as the secondary-school graduates. No evidence to the contrary could be gathered. The two coefficients δ^2 and δ^3 can by no means be consid-

1. For a general explanation of the problem see Part I, " Drop Out " page 24, and for a fuller explanation of the particular technique see Part III, J. Blum's study on Turkey.

ered as the opposites of the drop-out ratios at the second and third levels; they tell us what proportions of the total enrolment in 1954 graduated between 1955 and 1960 inclusive. Only if the effective study period were exactly six years could it be said that the drop-out ratio is 0.445 and the success ratio 0.555. In the case under consideration, however, these coefficients are affected by the drop-out ratio and the study period of the average graduate. If they are held constant in future time periods the implicit assumption is that both these factors remain unchanged or that possible changes cancel out. The value for γ shows that those who entered third-level[1] education between 1955 and 1960 were 100 per cent of total enrolments for this level in 1960. Again the value of this coefficient is very much influenced by the length of study: it would be high (i.e. >1) when the effective study period is short and *vice versa*.

Finally, the value for σ indicates that 45 per cent of those enrolled in Institutes of Higher Learning during the previous six years (1955-60) left without obtaining a degree.

With the introduction of these new coefficients, and the establishment of their value, the system of equations can be solved along the same lines as for the original model. No new variables have been added and the structure of the model has not changed.

The values found for the variables in the initial period expressed in terms of v_0 are as follows:

$N_0^2 = 0.86v_0$; $N_0^3 = 0.415v_0$; $n_0^2 = 1.486v_0$; $n_0^3 = 0.436v_0$;

$m_0^2 = 0.2885v_0$; $m_0^3 = 0.145v_0$

The absolute numbers for the variables in the initial and subsequent time periods are shown in Table 9.

TABLE 9. BALANCED GROWTH OF THE EDUCATIONAL SYSTEM FOR A GROWTH RATE OF 6 PER CENT PER ANNUM TAKING INTO ACCOUNT DROP-OUT AND EFFECTIVE LENGTH OF STUDY

Production in billions of pesetas ; population in thousands.

	t_0 (1960)	t_1 (1966)	t_2 (1972)	t_3 (1978)
N^2	464.4	659.4	936.3	1,329.5
N^3	224.1	318.2	451.8	641.6
n^2	802.4	1,139.4	1,617.9	2,297.4
n^3	235.4	334.3	474.7	674.1
m^2	155.8	221.2	314.1	446.0
m^3	78.3	112.2	157.9	224.2
v	540.0	766.8	1,088.9	1,546.6

For the first four variables, actual values and values required according to the original model were:

	ACTUAL 1960	TABLE 8 t_0
N^2	464.4	464.4
N^3	203.0	191.7
n^2	571.1	356.4
n^3	77.1	95.3

The values for N_2 and m_2 have remained the same in Tables 8 and 9 since the number of second-level manpower is directly related only to the volume of production and since the growth rate remains the same in both cases. On the contrary, the values for N_2 and m^3 have gone up because more students at both secondary and third levels have to be trained. It may be noticed in passing that in the original model only those teachers and professors are included in N^3 who teach future graduates entering the labour force on completion of their studies. This limitation has now been removed since the figures for secondary students (n^2) and third-level students (n^3), as presented in Table 9 refer to the total student body for those levels including those that will leave before obtaining a degree. The values of the two variables are consequently much higher than those obtained through the original model.

By comparing the results of Table 9 for the initial situation with the actual numbers in 1960, we see that the Spanish educational system falls short of the figures as required by the model. If the difference for the secondary level amounts to 40 per cent, the discrepancy for the third level takes on gigantic proportions: three times the number of students is " required " than were actually enrolled in 1960.

This exercise with explicit drop-out and study-time coefficients could be repeated for the sector approach. The method would be the same as the one outlined above (page 39). The results would not differ very much from those shown in Table 9.

Changing Coefficients

As was noted in the foregoing example, the Spanish educational system might have difficulties in meeting the requirements resulting from the model; this is particularly true for university training. It may be argued that one of the reasons for this state of affairs, i.e. the high number of students required, is the assumption of constant technical coefficients (v^2 and v^3).

The value of these coefficients reflects the situation only as it happens to exist at a given moment in time and does not tell us very much about requirements or needs, not to speak of an optimum relationship between second- and third-level manpower and volume of production: it is really a mixture of supply and demand relationships. It may be possible to obtain more appropriate values for these coefficients through an examination of the values found for them in other countries through a cross-section analysis or through sectoral manpower forecasts.

In Spain, a long-term educational programme has been established by the Spanish team working in the framework of the Mediterranean Regional Project. This educational programme is mainly based upon long-term manpower forecasts. Those forecasts have been arrived at by setting up output and productivity targets by economic sector, thus giving global manpower requirements which in turn have been broken down by professional categories. In order to obtain educational requirements these professional categories have been translated into types and levels of education. The methods used are of the type outlined by H. S. Parnes[1].

Consequently the number of persons in the labour force with a second- and third-level education around 1975, as planned for in Spain, is known and

1. See H. C. Parnes, *Forecasting Educational Needs for Economic and Social Development*, O.E.C.D., Paris, 1962.

the two technical coefficients can then be calculated for that period ($t = 3$ in the model). Given a 6 per cent annual growth of volume of production, the values found are the following[1]:

$v_3^2 = 0.793$

$v_3^3 = 0.194$

v in billions of 1960 pesetas; manpower in thousands.

Thus it is noted that the above values for v^2 and v^3 are both less than their 1960 values. This is partly due to productivity increases of these two types of manpower and also (particularly for third-level manpower) to a more important increase in this type in industry compared to the service sector.

There seem to be several possibilities of incorporating this change in the value of the coefficients into the system. One method would be to introduce the new value for v^2 and v^3 in $t = 3$, and to solve the system for that time period assuming the other coefficients remain constant. One then has two sets of values for the variables: one for $t = 0$ and another for $t = 3$. It now remains to be seen what the values of the variables have to be in $t = 1$ and in $t = 2$. This problem can be solved using the method for the transition period as outlined in Part I " Transition Problems—with and without Foreign Aid "; this possibility has been tried out in a provisional draft of this paper.

As in most exercises with this method however, the evolution over time of the variables risks becoming somewhat erratic, showing sudden spurts upward in one time period to remain almost stagnant or, even worse, declining in the next one. This is due to the long time lags involved and to the fact that within each time period of six years the values of the coefficients are supposed to remain constant. In the case of changing technical coefficients, moreover, an additional objection arises. Solving the system for the base period with normal growth rate (6 per cent per year in this case) is equivalent to assuming that no change in these coefficients will occur between $t = 0$ and $t = 1$. The values found in $t = 0$ for n^2 and n^3 (enrolments) are such as to ensure an increase in N^2 and N^3 in the next time period based upon the original values of the technical coefficients. These remarks also apply to the values found at the end of the transition period ($t = 3$ in this case). Although the magnitude of the changes in the coefficients may differ from one time period to another, there seems to be no particular reason to consider that no change, or very little, occurs in one period and is heavily concentrated in others.

For all the above reasons a more refined method would be to assume that the coefficients change by a given percentage each time period, resulting in a continuous change over time. The method to incorporate this in the model would be as follows:

Suppose v_t^2 changes each time period by a percentage $(1 + \kappa_2)$, the time path of v_t^2 would then be:

(a) $v_t^2 = v_0^2(1 + \kappa_2)^t$

If $(1 + \kappa_2) = \phi_2$, this may be expressed as: $v_t^2 = v_0^2(\phi_2)^t$

The growth path of v is given by:

(b) $v_t = v_0\omega^t$

1. For international comparisons:
$v_3^2 = 0.048$ Manpower in thousands; v in millions of U.S. dollars.
$v_3^3 = 0.012$ 1960 Spanish prices and 1960 exchange rate.

Substituting (a) and (b) in equation (1) of the model one gets:

(1) $N_t^2 = v_t^2 v_t = v_0^2(\phi_2)^t v_0(\omega)^t$, or

(1a) $N_t^2 = \tilde{v}_0^2 \psi_2^t$ where

$\tilde{v}_0^2 = v_0^2 v_0$

$\psi_2 = (\phi_2 \omega)$

In the same manner one obtains:

(6a) $N_t^3 = \tilde{v}_0^3 \psi_3^t + \pi^2 n_t^2 + \pi^3 n_t^3$ where $\tilde{v}_0^3 = v_0^3 v_0$

$\psi_3 = (\phi_3 \omega)$

The values for v_3^2 and v_3^3 given at the beginning of this section, and which were obtained by means of detailed manpower forecasts, show an overall decrease of 8 per cent for v^2 and of 36.5 per cent for v^3 compared to their 1960 values. This is the equivalent of a 3 per cent and a 14 per cent decrease per time period for v^2 and v^3 respectively. The model will then take the following shape:

(1a) $N_t^2 = \tilde{v}_0^2 \psi_2^t$

(2) $N_t^2 = (1 - \lambda^2) N_{t-1}^2 + m_t^2$

(3) $m_t^2 = \alpha^2\{\delta^2 n_{t-1}^2 - (\gamma - \sigma) n_t^3\}$

(4) $m_t^3 = \alpha^3 \delta^3 n_{t-1}^3$

(5) $N_t^3 = (1 - \lambda^3) N_{t-1}^3 + m_t^3$

(6a) $N_t^3 = \tilde{v}_0^3 \psi_3^t + \pi^2 n_t^2 + \pi^3 n_t^3$.

It will readily be seen that the values for ψ_2 and ψ_3 are not the same, ψ_2 being 1.38 and ψ_3 1.22. The solution to the above system, with a different growth rate for each of the stocks to be solved simultaneously, can be obtained along the same lines as those outlined in Part I, " Generalisation of Demand Functions. "

The time path of each of the variables can be ascertained by equations of the following type:

$z_t = \bar{z}\psi_2^t + \bar{\bar{z}}\psi_3^t$ where $\bar{z}$ and $\bar{\bar{z}}$ are constants.

By substituting the above expression in the model and by collecting the terms, two sets of six equations each are obtained: one with powers of ψ_2 only and another with powers of ψ_3 only. The two sets determine the barred and double-barred constants. Values found for the constants are as follows:

$\bar{N}^2 = 464.4$	$\bar{\bar{N}}^2 = 0$	which gives	$N_0^2 = 464.4$
$\bar{N}^3 = 24.8$	$\bar{\bar{N}}^3 = 182.0$		$N_0^3 = 206.8$
$\bar{n}^2 = 463.3$	$\bar{\bar{n}}^2 = 137.7$		$n_0^2 = 601.0$
$\bar{n}^3 = 23.8$	$\bar{\bar{n}}^3 = 113.4$		$n_0^3 = 137.2$
$\bar{m}^2 = 147.0$	$\bar{\bar{m}}^2 = 0$		$m_0^2 = 147.0$
$\bar{m}^3 = 8.2$	$\bar{\bar{m}}^3 = 44.3$		$m_0^3 = 52.5$

The values of the variables for the next few time periods are shown in Table 10, they have been obtained from the growth formula:

$z_t = \bar{z}\psi_2^t + \bar{\bar{z}}\psi_3^t$

TABLE 10. BALANCED GROWTH OF THE EDUCATIONAL SYSTEM FOR A GROWTH RATE OF 6 PER CENT PER ANNUM (42 PER CENT PER TIME PERIOD) AND WITH A CHANGE IN THE VALUE OF ν^2 AND ν^3 OF —3 PER CENT AND —14 PER CENT PER TIME PERIOD RESPECTIVELY

Production in billions of 1960 pesetas ; population in thousands.

	ACTUAL VALUES 1960	WITH CONSTANT COEFFICIENTS TABLE 9 $t = 0$	t_0	t_1	t_2	t_3
N^2	464.4	464.4	464.4	640.9	884.4	1,220.5
N^3	203.0	224.1	206.8	256.2	318.0	395.5
n^2	571.1	802.4	601.0	807.4	1,087.4	1,467.7
n^3	77.1	235.4	137.2	171.1	214.0	268.3
m^2	n. a.	155.8	147.0	202.9	280.0	386.4
m^3	n. a.	78.3	52.5	65.3	81.5	101.9
v	540.0	540.0	540.0	766.8	1,088.9	1,546.6

The differences in the above results as compared to those shown in Table 9 are remarkable right from the base period. Compared to those " required " by the constant-coefficient model, enrolments may be far fewer for both levels, but particularly of course for the third-level. This gain is amplified by the phenomenon of drop-out and " cultural " students. Compared to the actual 1960 situation the results obtained are also much less discouraging, although enrolments as required by the model still exceed the actual figures for both levels (5.2 per cent for secondary education and 78 per cent for third-level education). This compares to 40 per cent and 200 per cent respectively, when constant technical coefficients were applied (see Table 9).

It does not appear that secondary education as a whole constitutes a bottle-neck, as far as manpower requirements are concerned. This bottle-neck is evident, however, in higher education. This finding has also been made by the Spanish M.R.P. team, independently of the model. We shall return to this problem in the concluding remarks.

One might also be inclined to vary the other coefficients over time, i.e. λ and π, although there are narrow limits here. The above method cannot be applied in these cases, however, because the change in the value of these coefficients does not affect the growth rate of all variables (N^2 remains unchanged). A more cumbersome method would be necessary consisting in solving the system of equations anew for each time period. The method would be to express the values of each of the variables in terms of the coefficients and v_t and then let the values of λ_t and π_t change per time period. This is perfectly feasible but it has not been undertaken in the framework of this paper.

It may justifiably be asked what exactly has been gained by the approach outlined in this section. After all, in order to derive the change in the value of the technical coefficients for Spain one had first to have the manpower forecasts. Such a situation will not be too frequently met in other countries. First of all, since even in the case of a country where manpower forecasts have been made the changes in the future values of the coefficients will never be rigorously correct, it is sufficient for this method to have only an approximate idea of the future changes. These can be ascertained by looking at past changes in the same country or by examining critically the values found for these coefficients in other countries. Secondly, the method outlined above shows a smooth development for all variables over time right from the base period, taking into account changes that will occur in the next period. It thus avoids the awkward movement of the variables from one time period to another resulting from the "transition-period method." Thirdly, it may very well be that one has more precise ideas about the evolution of the technical coefficients in certain sectors of the economy. If simultaneous use is made of the method described on page 39 and that described above of a continuous change in the coefficients over time, better results may be obtained.

More Types of Education

The changes in the technical and other coefficients as presented in the preceding section may be more significant if further differentiation is introduced into each of the two types of education. This, of course, gives rise to an increase in the number of coefficients. A knowledge of the future evolution of the total number of secondary and third-level students is very useful indeed, but for various reasons it seems preferable to make a distinction at least between scientific and technical education on the one hand and general education on the other[1]. First, depending on the situation in the initial period, the number of scientific and technical workers may have to increase more than that of workers with a non-technical background, or vice versa; this may be true *a fortiori* if we take each of the levels separately. Secondly, this distinction is also very useful for cost considerations; it is well known that the unit costs for scientific and technical students are higher, sometimes considerably so, than those for other students. Although the model is not directly concerned with this aspect of educational planning, there is no reason for not taking it into account if possible. With this in mind, it seems reasonable to suppose that more precise results are arrived at by examining the possible variations of each of the four technical coefficients thus obtained.

The model will now be presented with the differentiation in the educational system outlined above; it will be noted that the original model is used again for clarity's sake. The introduction of additional coefficients is always possible when the data are available for each of the levels now distinguished.

(1.1.) $N_t^1 = v^1 v_t$

(1.2.) $N_t^2 = v^2 v_t$

(6.1.) $N_t^3 = v^3 v_t + \pi^1 n_t^1 + \pi^3 n_t^3$

(6.2.) $N_t^4 = v^4 v_t + \pi^2 n_t^2 + \pi^4 n_t^4$

(2.1.) $N_t^1 = (1 - \lambda^1) N_{t-1}^1 + m_t^1$

(2.2.) $N_t^2 = (1 - \lambda^2) N_{t-1}^2 + m_t^2$

(5.1.) $N_t^3 = (1 - \lambda^3) N_{t-1}^3 + m_t^3$

(5.2.) $N_t^4 = (1 - \lambda^4) N_{t-1}^4 + m_t^4$

(3.1.) $m_t^1 = n_{t-1}^1 - n_t^3 - n_t^4$

(3.2.) $m_t^2 = n_{t-1}^2 - n_t^3 - n_t^4$

The 12 variables are:

N^1 persons in the labour force with a general secondary education;

N^2 ... with a professional and technical secondary education;

N^3 ... with a non-technical higher education;

N^4 ... with a scientific or technical higher education;

n^1, n^2, n^3, n^4 are the students at these respective levels and types of education;

1. It becomes more and more difficult to speak about "non scientific" students, with science progressing so rapidly in all fields. A student in economics might consider as an insult his being included under the heading of "non-scientific". It should be clear, however, that this manner of referring to a certain type of education does not imply a value judgment on our part.

(4.1.) $m_t^3 = n_{t-1}^3$

(4.2.) $m_t^4 = n_{t-1}^4$

m^1, m^2, m^3, m^4 are those that entered the labour force during the past 6 years after completing their respective studies.

The values of the coefficients for Spain in 1960 were found to be as follows:

$\left.\begin{array}{l} v^1 = 0.386 \\ v^2 = 0.474 \end{array}\right\}$ they add up, as will be understood, to 0.86 value of our former v^2

$\left.\begin{array}{l} v^3 = 0.157 \\ v^4 = 0.149 \end{array}\right\} 0.306$ Medical doctors, dentists and agronomists are included under scientific and technical personnel

$\pi^1 = 0.45$ $\quad \pi^3 = 0.075$

$\pi^2 = 0.60$ $\quad \pi^4 = 0.09$

$\lambda^1 = \lambda^2 = 0.056$ $\quad \lambda^3 = \lambda^4 = 0.077$

Thus with 12 equations, 12 unknowns and the coefficients calculated, the system can be solved.

If account is also taken of drop-out and effective length of study, the following coefficients should be added (see page 41):

$\delta^1 = 0.43$	$\gamma^3 = 1.25$	$\alpha^1 = 0.80$
$\delta^2 = 1.24$	$\gamma^4 = 0.88$	$\alpha^2 = 0.90$
$\delta^3 = 0.67$	$\sigma^3 = 0.60$	$\alpha^3 = 0.75$
$\delta^4 = 0.50$	$\sigma^4 = 0.35$	$\alpha^4 = 0.95$

The values attached to the above coefficients must be considered as approximations.

If it is now known how each of the technical coefficients, as distinguished in the extended model, will change *grosso modo* in future time periods, the first four equations will read as follows:

(1.1.) $N_t^1 = \tilde{v}_0^1 \psi_1^t$ where $\tilde{v}_0^1 = v_0^1 v_0$, $\psi_1 = \phi_1 \omega$

(1.2.) $N_t^2 = \tilde{v}_0^2 \psi_2^t$ where $\tilde{v}_0^2 = v_0^2 v_0$, $\psi_2 = \phi_2 \omega$

(6.1.) $N_t^3 = \tilde{v}_0^3 \psi_3^t + \pi^1 n_t^1 + \pi^3 n_t^3$ where $\tilde{v}_0^3 = v_0^3 v_0$, $\psi_3 = \phi_3 \omega$

(6.2.) $N_t^4 = \tilde{v}_0^4 \psi_4^t + \pi^2 n_t^2 + \pi^4 n_t^4$ where $\tilde{v}_0^4 = v_0^4 v_0$, $\psi_4 = \phi_4 \omega$

It is highly improbable that the same change over time will occur in the value of each of the coefficients so that in all likelihood we shall here be confronted with a system of variables that involves simultaneously four different rates. The solution is derived along the same lines as those described above on page 43; the evolution over time of each of the variables can be written in general form as:

$z_t = z_{01}\psi_1^t + z_{02}\psi_2^t + z_{03}\psi_3^t + z_{04}\psi_4^t$ where z_{01}, z_{02}, z_{03} and z_{04} are constants. By substituting this expression in the system, we can derive from it the conditions the constants have to satisfy by collecting all terms with ψ_1, with ψ_2, with ψ_3 and with ψ_4. We then find four sets of twelve equations each, determining z_{01}, z_{02}, z_{03} and z_{04}.

This exercise will not be performed in the context of this paper; the aim of this section has merely been to show that the model lends itself easily to disaggregation and that the only limits consist in the availability of empirical data for the calculations of the coefficients.

A Marginal Approach

It has been argued that the technical coefficients, v^2 and v^3, of the model reflect the relationship between the particular manpower stocks and the volume of production as it happens to exist at a given point in time. It is to be noted that the coefficients are affected by trends that go back as far as thirty or forty years (persons 50 or 60 years of age entered the labour force that long ago). It seems justified, and even preferable, to calculate the coefficients on a marginal basis by examining the changes in:

v, N^2 and N^3.

Thus, instead of calculating the coefficients by taking the actual stock and actual volume of production in a given year, we may now compute " marginal coefficients " on the basis of the actual changes, during the past six years, in the stock and the volume of production. The marginal model will then assume the following shape:

(1) $N_t^2 - N_{t-1}^2 = v^2(v_t - v_{t-1})$

(2) $N_t^2 - N_{t-1}^2 = (1 - \lambda^2)(N_{t-1}^2 - N_{t-2}^2) + (m_t^2 - m_{t-1}^2)$

(3) $m_t^2 - m_{t-1}^2 = (n_{t-1}^2 - n_{t-2}^2) - (n_t^3 - n_{t-1}^3)$

(4) $m_t^3 - m_{t-1}^3 = n_{t-1}^3 - n_{t-2}^3$

(5) $N_t^3 - N_{t-1}^3 = (1 - \lambda^3)(N_{t-1}^3 - N_{t-2}^3) + (m_t^3 - m_{t-1}^3)$

(6) $N_t^3 - N_{t-1}^3 = v^3(v_t - v_{t-1}) + \pi^2(n_t^2 - n_{t-1}^2) + \pi^3(n_t^3 - n_{t-1}^3)$

The difference in values of any variable over one time period, $z_t - z_{t-1}$ could be expressed as Δz_t.

Then the above model would appear as:

(1) $\Delta N_t^2 = v^2 \Delta v_t$

(2) $\Delta N_t^2 = (1 - \lambda^2)\Delta N_{t-1}^2 + \Delta m_t^2$

(3) $\Delta m_t^2 = \Delta n_{t-1}^2 - \Delta n_t^3$

(4) $\Delta m_t^3 = \Delta n_{t-1}^3$

(5) $\Delta N_t^3 = (1 - \lambda^3)\Delta N_{t-1}^3 + \Delta m_t^3$

(6) $\Delta N_t^3 = v^3 \Delta v_t + \pi^2 \Delta n_t^2 + \pi^3 \Delta n_t^3$

The coefficients have been calculated on the basis of actual changes observed in Spain during recent years.

$v^2 = 0.95$	Change in v in billions of 1960 pesetas.
$v^3 = 0.20$	Change in manpower in thousands.
$\pi^2 = 0.033$	During the previous six years (1955-1960) one teacher was
$\pi^3 = 0.10$	added for every 33 additional secondary students; at the third level one professor for every 10 additional students.
λ^2 and λ^3	will, in this exercise, retain their original values.

In order to solve the system of marginal equations, we have to make an assumption about the rate of change of the variables For this exercise we may assume that:

$z_t = \omega(\Delta z_{t-1}),\ z_t - z_{t-1} = \omega(z_{t-1} - z_{t-2})$

or that the differences between successive values of the variables will increase at an exponential rate. In general terms:

$$z_t - z_{t-1} = \omega^{t-1}(z_1 - z_0).$$

Our earlier assumption that each of the variables increases at an exponential rate, $z_t = z_0\omega^t$, would, of course, imply the new assumption. But the stronger assumption is not necessary. In fact, the marginal approach produces different results from the original approach only when the condition concerning the uniform growth rate of each of the variables is not imposed.

The results provided by the solution to the system of marginal equations appear in Table 11 where appropriate account has also been taken of drop-out and effective length of study (as already described, page 41). This method then gives the change that has to occur in the values of the variables in each time period in order to obtain a given increase in the volume of production. It says nothing about current levels—in terms of surpluses or deficits of certain stocks, for example. This, however, does not seem to be a major objection since, in exercises of this kind, one is usually looking for the additional needs of the different types of manpower necessitated by economic development. An analysis to see whether the best use is being made of existing stock should always precede the projections.

To give an idea of the values of the average variables in a given time period ($t = 3$ for example), the increases as shown in Table 11 have been added to the actual values in 1960. The result can be seen in column 4 of the same table. This can be usefully compared to the results arrived at by using the other methods presented in this paper[1].

It is clear that this marginal approach is really another way of dealing with the problem of changing (average) coefficients. The marginal values of these coefficients as observed in the past will not normally be the same as the average values at a given point in time, for this method will make the average values change from one period to the next until they coincide with the marginal ones. In this respect it is interesting to note that the marginal value of $\nu^3(0.20)$ is close to the value of ν_3^3 as presented in the section " Changing Coefficients " above.

Admittedly, this approach is far from being perfect; here again the problems of an optimum relationship between second- and third-level manpower on the one hand, and a given output of production on the other, does not yield a clear answer. The advantage of this method, however, is that it avoids the use of data that are very heterogeneous as far as the time periods are concerned. Comparisons in time and space of the marginal values of the coefficients would be very useful as long as knowledge about optimum relationships remains small[2].

1. For a summary of the results obtained with the different variations of the Model, see Table 12.

2. In a study prepared by the Netherlands Economic Institute " The Financing of Higher Education in Africa, " published in " The Development of Higher Education in Africa, " Unesco 1963, a cross-section analysis was presented establishing a relationship between change in the stock of third-level manpower and change in G.N.P. The change in the stock referred, however, to the gross increase in third-level manpower, i.e. number of graduates leaving the universities, and neglected deaths and retirements of those already in the labour force – precisely because figures concerning this stock were hard to find.

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TABLE 11. MARGINAL MODEL
VARIABLES—INCREASE PER TIME PERIOD
FOR AN ANNUAL GROWTH RATE OF 6 PER CENT; ACCOUNT TAKEN
OF MARGINAL COEFFICIENTS, DROP-OUTS AND
EFFECTIVE LENGTH OF STUDY

v in billions of 1960 pesetas ; population in thousands.

	t_1-t_0	t_2-t_1	t_3-t_2	t^3
ΔN^2	215.5	306.0	434.5	1,420.4
ΔN^3	62.1	88.2	125.2	478.5
Δn^2	310.0	440.2	625.1	1,946.4
Δn^3	65.3	92.7	131.6	366.7
Δm^2	72.1	102.4	145.4	n. a.
Δm^3	21.8	31.0	44.0	n. a.
Δv	226.8	322.1	457.4	1,546.6

As observed above, the results shown in Table 11 are planning figures and do not provide a check on the existing (average) figures for $t = 0$.

TABLE 12. SUMMARY OF RESULTS FOR TIME PERIODS t_0 AND t_3 OBTAINED WITH THE DIFFERENT VARIATIONS OF THE MODEL

v in billions of 1960 pesetas ; population variables in thousands.

	ACTUAL VALUES 1960	t_0			t_3			
		ORIGINAL MODEL TABLE 8	WITH DROP-OUT AND EFFEC-TIVE LENGTH OF STUDY TABLE 9	CHANG-ING COEFFI-CIENTS TABLE 10	ORIGINAL MODEL TABLE 8	WITH DROP-OUT AND EFFEC-TIVE LENGTH OF STUDY TABLE 9	CHANG-ING COEFFI-CIENTS TABLE 10	MAR-GINAL AP-PROACH TABLE 11
N^2	464.4	464.4	464.4	464.4	1,329.5	1,329.5	1,220.5	1,420.4
N^3	203.0	191.7	224.1	206.8	548.8	641.6	395.5	478.5
n^2	571.1	356.4	802.4	601.0	1,020.5	2,297.4	1,467.7	1,946.4
n^3	77.1	95.3	245.4	137.2	272.8	674.1	268.3	366.7
m^2	n. a.	155.8	155.8	147.0	446.0	446.0	386.4	n. a.
m^3	n. a.	67.0	78.3	52.5	192.1	224.2	101.9	n. a.
v	540.0	540.0	540.0	540.0	1,546.6	1,546.6	1,546.6	1,546.6

SUMMARY AND CONCLUSIONS

In the foregoing pages several variations of the model have been presented. For some of them the solutions were calculated, whereas for others only the extended model was shown and the method of solution was indicated, without any further elaboration. Table 12 gives a summary view of the results obtained in this paper for time periods $t = 0$ and $t = 3$.

The values for $t = 3$ show that Spain will need, according to the various results obtained by means of the model, between 1.5 and 2.3 million persons enrolled in secondary schools, and from 268,000 to 674,000 students at the third level. This corresponds, in 1975, for secondary education to an enrolment ratio of between 26 and 39 per cent of the 10 to 19 age group, and for

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third level education to 10 to 25 per cent of the 20 to 24 age group. Clearly the model using constant technical coefficients as applied to Spain results in exorbitant enrolment needs, particularly for the third level. This result might be expected because of the potential productivity increases in the different types of manpower. When changing coefficients are introduced into the model the required numbers enrolled for both levels become easier to provide, as can be seen from Table 12 and the enrolment ratio quoted above. These results correspond *grosso modo* to those obtained independently by the Spanish M.R.P. team.

The debate concerning technical coefficients is certainly not exhausted by introducing the method of a continuous change over time. We have mentioned several times that these coefficients are ambiguous in character: they not only indicate the labour input of a certain quality necessary for a given volume of production, but also reflect to some extent preferences and possibilities of individuals to pursue certain types of studies. These in turn are conditioned by income levels and the educational facilities available. It is in this context that we can speak of a " mixture of supply and demand relationship. " This, however, is one of the difficult problems of manpower forecasts in general, and we should like to state once and for all that the model under consideration is not a magic piece of machinery which solves all these questions by just putting a penny in the slot. The generalized demand function as put forward in Part I, " Generalization of Demand Functions, " is an interesting step towards a better understanding of how manpower stocks are related, dynamically, to the volume of production. It takes account of productivity changes by introducing per capita income. The method of the continuous change of the coefficients, as presented in this paper, is really a substitute for the generalized demand function. We have not applied the latter method here because we thought it premature to do so. In principle, the idea behind this method is the same as that behind the section " Changing Coefficients " above, i.e. the negative influence of productivity increase slows down the growth rate of manpower stocks. The provisional results of the regression made by the Netherlands Economic Institute on the basis of a cross section analysis with 19 observations showed this negative influence of productivity increase. The coefficient of (v/p) was —0.28. When this formula was applied to Spain the value for N^3 was found to be 201,400. This points to an existing deficit relative to the actual figure for 1960, if teachers and professors are excluded[1], although it may not be entirely certain whether they have been excluded in all the 19 observations used for the calculation of the regression coefficients. However, if the growth path of N^3 is calculated along the lines described in Part I, p. 21 *et seq.*, this increase would be 40 per cent per time period, or only slightly less than the 42 per cent using the original assumptions. This is because the opposite effects of the two coefficients attached to v and (v/p) more or less cancel each other out, and it is only the low population growth in Spain (emigration) which causes a slightly lower growth rate to be obtained. Once more, since these first results seemed to be provisional in character, they have not been further elaborated here.

The results obtained by means of the changing-coefficients method clearly show that, in view of the low enrolment figures for the base period (77,100 in 1960), third-level education will be the main bottleneck in Spain in the years to come, due to the increasing requirements of the economy and particularly of teachers. Here the possible qualitative implications of the quantitative exer-

1. $N^3 = \nu^3 v$ alone would give 166,000 for 1960.

cises presented in this paper can be seen. In view of the relatively important requirements of third-level manpower, a closer examination of the efficiency of the corresponding type and level of education becomes necessary. This implies an investigation into such problems as student-teacher ratios, legal length of study, drop-out, delays, etc., and is the reason, for example, why the M.R.P. team in Spain will suggest a shortening of the legal study period from 5 to 4 years for the *licenciado*.

The problem of " length of study " leads back to possible refinements of the model: the number of variations seems almost infinite. One exercise might be the introduction of a different length of study for each educational level. A secondary education of 6 years and a university education of 4 years as suggested above would imply working with time periods of two years, but would also require more precise data concerning drop-out.

We hope that the few examples given above will indicate some of the possibilities of what is apparently a very simple model.

The breakdown of the economy into sectors, together with the introduction of more types and levels of manpower and education, will provide any detail one may desire. It should be kept in mind, however, that it may be difficult to find reliable data for calculating the coefficients. The change-over-time of these coefficients, as presented in this paper, has yielded very promising results.

Research into the demand function in general, and the technical coefficients in particular, should provide us with a good idea of the direction and magnitude of the changes in values involved.

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Part III

PLANNING MODELS FOR THE CALCULATION OF EDUCATIONAL REQUIREMENTS FOR ECONOMIC DEVELOPMENT

TURKEY

by

J. BLUM[1]

INTRODUCTION

SCOPE OF THE PAPER

This paper presents various applications of the macro-models for educational planning described in Part I.

These models were developed in order to study some quantitative problems in adapting education to the economic development of a country. They are theoretical in nature and were not developed with any specific country in mind. This paper describes an experiment in which Turkey was chosen as a typical example of a country to which these models could be applied in order to test their general efficacy as practical instruments for educational planning. It is hoped that the results of such experiments will determine the practical usefulness of macro-models and show the best method of reformulating them for successful application to other countries.

BASIC DATA ON THE TURKISH ECONOMY

According to the preliminary results of the recent census, the total population of Turkey in 1960 was 27.8 million. The population has been growing and, as a result of a high birth rate and declining death rate, is expected to continue to increase at an average annual rate of 3 per cent. Emigration and immigration are insignificant. Of the total population aged 15 years and over in 1960, 80 per cent, or 13.0 million were economically active.

Turkey's total gross national product in 1961 was 49.2 billion Turkish lires (in current prices) or $197 per capita (using the official exchange rate of T.L. 9 = $1). Per capita income in Turkey is among the highest in the Eastern Mediter-

1. I wish to express my appreciation to the members of the Social Planning Department of the State Planning Organisation of the Turkish Government for all the assistance they have given me in gathering the data for this paper. I should also like to thank the State Institute of Statistics for making available to me the preliminary results of the 1960 Census and Professor Tinbergen for his useful comments and suggestions during the preparation of the paper.

ranean area, although it is substantially lower than in Western Europe, or in Greece, its western Mediterranean neighbour ($370 in 1961). Agriculture plays a dominant role in the Turkish economy, accounting for 41 per cent of the 1961 gross domestic product at factor cost (current prices) and employing 79 per cent of the labour force. Agricultural products constitute Turkey's major exports. Industry is still largely in its formative stages and produces primarily for internal consumption behind the protection of high tariff barriers. With about half the total industrial capacity supplied by government-owned State Economic Enterprises, industry accounted for 23 per cent of the 1961 gross domestic product and employed only 10 per cent of the labour force. The tertiary, or service sector, accounted for the remaining 36 per cent of G.D.P. and employed 11 per cent of the labour force.

The Five-Year Economic Plan for Turkey, prepared by the State Planning Organisation, calls for a 7 per cent target rate of annual growth, with a target rate of investment of 18 per cent of G.N.P., including substantial foreign assistance. If the planned rate of growth is achieved, per capita G.N.P. will increase at an annual rate of roughly 4 per cent.

Description of the Turkish Educational System

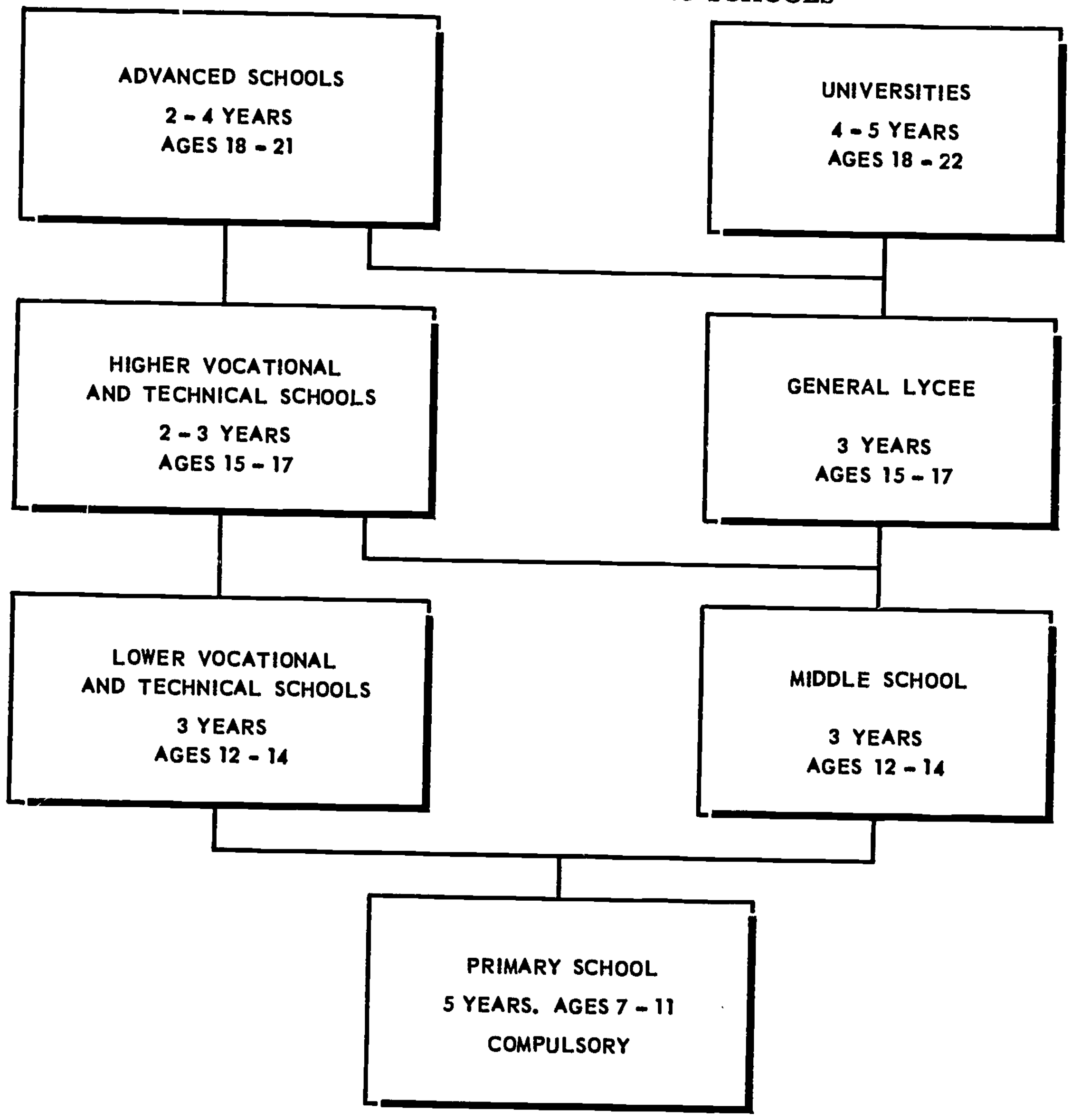
Education is free at all levels of schooling in Turkey. When the Turkish Republic was set up early in the century the schools were placed under the control of the Ministry of Education. Five years' primary education is compulsory in Turkey for children between the ages of 7 and 14, the normal primary school age being from 7 to 11 years.

Secondary education in Turkey is divided into two types—general education and vocational and technical education. Each type, in turn, is divided into two successive steps—a lower school and a higher school. A student passing his primary school leaving examination has the choice of continuing his studies at age 12 in either a " middle " general secondary school or one of the lower secondary vocational or technical schools. This first step in secondary education consists of three years of school for either type. Students graduating from a middle general secondary school may go on to the general " lycée, " which lasts three years, or enter one of the higher vocational or technical schools, for courses lasting two to three years, depending on the school. Graduates from a lower vocational or technical secondary school may go on to a higher vocational or technical secondary school, but not into a general lycée.

Higher education in Turkey also consists of two general types of schools —the universities and other advanced schools beyond the secondary level which are essentially technical and vocational in character and cover a wide range of subjects, including teacher training. Graduates from the general lycée may go on to a university or one of the advanced schools; graduates from the higher or lycée-level vocational and technical schools are limited to entry into one of the advanced vocational or technical schools. Higher education in Turkey lasts from two to five years; the length of study in one of the six universities is from four to five years.

To sum up, education in Turkey below the university level lasts 11 years, five at the primary schools and six at the secondary school levels. This compares with 13 years in Italy and 12 years in Greece. In the academic years 1960-61, 2.6 million children were enrolled in primary school, 292.6 thousand in middle general secondary schools, 76.6 in general lycées, 109.1 thousand in

DIAGRAM OF TURKISH PUBLIC SCHOOLS



vocational and technical secondary schools (both levels) and 62.2 thousand in institutions of higher education. As a proportion of school age population groups, total secondary school enrolments were 14.7 per cent and higher education enrolments 2.5 per cent.

APPLICATION OF THE BASIC MODEL

THE BASIC MODEL

The model first applied to Turkey was the basic model described in the introduction to Part I. It is made up of six equations, seven variables and six coefficients, as explained in Part I and shown below:

(1) $N_t^2 = v^2 v_t$

(2) $N_t^2 = (1 - \lambda^2) N_{t-1}^2 + m_t^2$

(3) $m_t^2 = n_{t-1}^2 - n_t^3$

(4) $m_t^3 = n_{t-1}^3$

(5) $N_t^3 = (1 - \lambda^3)N_{t-1}^3 + m_t^3$

(6) $N_t^3 = v^3 v_t + \pi^2 n_t^2 + \pi^3 n_t^3$

In applying this model and its several alternatives to Turkey, two basic assumptions were made. First, that the stock variables, N^2 and N^3 should refer only to *graduates* of secondary and third-level schools. For the secondary education manpower stock, N^2, this means that graduates from only the lycée-level schools, both general and technical or vocational are included. It does not include graduates of the middle-level secondary schools who did not complete their secondary education. For the third-level education manpower stock, N^3, this means that graduates of any of the higher-level or post-secondary schools are included. Secondly, it was assumed that the secondary school enrolment variable, n^2, should include enrolment for both the middle-level and the lycée-level schools.

Other assumptions were made also with respect to the measurement of the variables. All the variables in the models refer to a specific point in time. Five variables—v, N^2, N^3, n^2 and n^3—all refer to a particular year within a six-year period. The other two variables—m^2 and m^3—refer to a specific six-year period. 1961 was chosen as the initial year for the volume of production (v). The most recent data giving stocks of graduates in the Turkish labour force are the 1960 Census which was taken in October. Thus, in the models, the initial point in time for the manpower stocks N^2 and N^3 is October, 1960. For school enrolments, n_2 and n_3, the school year 1960-61 was taken as the initial point in time. The initial point in time for the new, educated entrants into the labour force was assumed to be the period 1955-1960. All these points in time are consistent with the choice of a time unit of six years for the models. For the period $t = 1$, for example, v would refer to 1967, the N's to October, 1966, the n's to the school year 1966-67, and the m's to the period 1961-1966.

To apply this basic model to Turkey, values had to be selected for the six coefficients, represented by Greek letters in the equations. A description of each coefficient and its selected value follows:

Technical coefficients. These coefficients assume that there exists a fixed linear relationship between the stocks of educated manpower in the labour force and the annual volume of production. The computation of these coefficients is indicated in general terms in Part I. The value of $v^2(0.039)$ in Turkey was the ratio of N_{1960}^2 manpower (in thousands) with a secondary education, to v_{1961}, total volume of production, expressed in millions of 1961 United States dollars. The value of $v^3(0.016)$ was the ratio of N_{1960}^3 manpower (in thousands) with third-level education, less teachers in secondary and third-level schools, to v_{1961}, total volume of production, expressed in millions of 1961 United States dollars.

Teacher-student ratios. These coefficients show the number of teachers with a third-level education required for any given number of secondary and third-level students. The values selected are more or less those planned for in the Turkish educational system in the next decade.

$\pi_2 = 0.03$, which implies a student-teacher ratio of 33.3 for secondary education and

$\pi_2 = 0.07$, which implies a student-teacher ratio of 14.3 for third-level education.

Manpower stock attrition rates. These coefficients express the percentage of persons in the stocks of educated manpower who will leave the stocks during a six-year period for reasons of death or retirement. The values were selected on the basis of the past overall death rate in Turkey and an assumed general retirement age of 65. It should be noted, however, that these coefficients are constant and, unfortunately, do not take into account a change in the age composition of the stocks. As the stocks increase in size, the withdrawals will correspondingly increase in size. Accordingly, the values selected for Turkey were the following:

$\lambda^2 = 0.15 \qquad \lambda^3 = 0.165$

The shorter working life for third-level manpower requires a higher value for the third-level attrition coefficient.

Determination of Balanced Growth Rates

As outlined in Part I "Some Applications of the Basic Model," it is assumed that the variables in the basic model will increase exponentially with time, or in symbols, $z_t = z_0\omega^t$. This assumption then provides the solution for the initial values for the education variables as well as for the growth path which these variables will follow, which would be consistent with the assumed growth in production. It is assumed that the volume of production (G.N.P.) in Turkey will increase at the rate of 7 per cent per annum or 50 per cent for every six-year period. Then $\omega = 1.5$, and $z_t = z_0 1.5^t$. The variables are put into this form and the values for the coefficients listed above are inserted in the system of equations. The system of equations is solved simultaneously with each variable being expressed as a proportion of v, i.e. the volume of production. The following relations are then found:

$N_0^2 = 0.03851\, v_0 \qquad N_0^3 = 0.01814\, v_0$
$n_0^2 = 0.04312\, v_0 \qquad n_0^3 = 0.01206\, v_0$
$m_0^2 = 0.01669\, v_0 \qquad m_0^3 = 0.00804\, v_0$

By substituting the value of v_0 into the above relationships, the initial values of the other six variables that put the model on a balanced growth path can be found. The value of v_0 for Turkey was taken equal to 5,468.1, the 1961 G.N.P. in Turkey expressed in millions of United States dollars. To find the future development of all the variables on the equilibrium path, these initial values are expanded by 1.5^t. Table 13 compares the actual values of the variables in the initial year in Turkey with those for the equilibrium path for the model and also shows the development of the variables on the equilibrium path over the first few time periods.

The model assumes that the relationship which exists in the base year between the educated manpower stocks and the volume of production is defined by the technical coefficients, and will continue to be the same in subsequent years. As the volume of production increases, the educated manpower stocks must correspondingly expand at the same rate. To allow these manpower stocks to expand at this rate, and to replace withdrawals from the stocks due to death and retirement, the schools must turn out a certain number of graduates every year. In effect, then, the model is based upon the existing educated manpower stocks and not the existing school enrolments. In the model, school enrolments become a function of the educated manpower stocks and their rate of expansion. In the development of the model over time, a certain balance between the stocks and the enrolments is accordingly required, despite the fact that in the base year, this balance may not, in reality, exist. It may not exist if, for example, there has recently been a very rapid increase in school enrolments and a smaller increase in stocks, or if the stocks are made up of a sizable number of persons who obtained their education abroad, so that the stocks are out of proportion with the country's school enrolments.

Given this, it is not at all surprising that the actual number of students enrolled in secondary schools in Turkey in the base year is inconsistent with the " equilibrium " number as determined by the model. As shown in Table 13, the actual enrolment in secondary schools in 1960-61 in Turkey was slightly more than double the number required by the model for the system to move along the equilibrium or balanced path. For third-level enrolment in the base year, the discrepancy between the actual and the equilibrium figures is much less. The actual enrolment in third-level schools in 1960-61 was only 5.6 per cent below the equilibrium number. Using the basic model, the result is that actual school enrolments in Turkey are out of balance with the educated manpower stocks and the requirements for expansion of the stocks to support the economic growth objective of the country.

Adaptation to a Balanced Growth Path

Given that the actual school enrolments are not in equilibrium according to the basic model, it is appropriate to consider what steps might be taken to put the educational system into a balanced or equilibrium state. This would be a transition problem similar to those described in Part I. Either the educational system can be brought into balance through internal measures, implying a temporary change in the coefficients used in the model for a number of periods, or an external solution can be found, implying imports or exports of foreign educated manpower. The second solution, in this case, would mean an export

Table 13. BALANCED GROWTH OF THE EDUCATIONAL SYSTEM FOR A GROWTH RATE OF 7 PER CENT PER ANNUM
Production in Millions of 1961 U.S. Dollars; Population in Thousands

The basic model.

Symbols	Variable	Actual value in $t = 0$	Equilibrium growth path 0	1	2	3
N^2	Manpower with secondary education	210.6	210.6	316.1	474.4	712.1
N^3	Manpower with third-level education	104.7	99.2	148.9	223.5	335.4
n^2	Students in secondary schools	477.3	235.8	353.9	531.2	797.4
n^3	Students in third-level schools	62.2	65.9	99.0	148.6	223.0
m^2	Manpower with secondary education and less than 6 years in labour force	n.a.	91.3	137.0	205.5	308.2
m^3	Manpower with third-level education and less than 6 years in labour force	n.a.	44.0	65.9	99.0	148.6
v	Volume of production	5,468.1	5,468.1	8,207.6	12,319.6	18,491.7
p^2	Secondary school age population	3,240	3,240	4,391	5,252	5,973
p^3	Third-level school age population	2,531	2,531	3,098	4,210	4,969
n^2/p^2	Secondary students as a proportion of secondary school age population (per cent)	14.7	7.3	8.1	10.1	13.4
n^3/p^3	Third-level students as a proportion of third-level school age population (per cent)	2.5	2.6	3.2	3.5	4.5

of secondary manpower and an import of third-level manpower, either temporarily or permanently. This section, however, will deal only with the first type of solution, that of self-help measures, such as described in Part I, " Some Applications of the Basic Model. "

Using exactly the same method for solving the transition problem without foreign aid, we find that the simplest possible solution (keeping the manpower stock attrition rates equal to their normal values), calls for a transition period of two time periods or 12 years, and either (*a*) changing the use of educated manpower in production or (*b*) changing the teacher-pupil ratios. The problem is complicated because, according to the simple model, far too many students were enrolled in secondary schools in the base year (1960-61). This means that at the end of the first six years the stock of secondary manpower will be 77 per cent greater than that required for an equilibrium development of the system, and the secondary school enrolment in that year need be large enough only to supply students for third-level schools. No new additions to the stock of secondary manpower need be made during the second six years of the transition period. This is shown in Table 14 below, which gives the values of the six variables in the base year and at the end of the first two time periods.

TABLE 14. VALUES OF VARIABLES FOR MAKING A TRANSITION WITHOUT FOREIGN ASSISTANCE

SYMBOLS	$t = 0$ BASE YEAR (ACTUAL)	$t = 1$ END OF PERIOD 1 (TRANSITION)	$t = 2$ END OF PERIOD 2 (GROWTH PATH)
N^2	210.6	557.7	473.8
N^3	104.7	149.6	223.5
n^2	477.3	148.4	531.2
n^3	62.2	98.6	148.6
m^2	—	378.7	(—) 0.2
m^3	—	62.2	98.6
v	5,468.1	8,207.6	12,319.6

The problem is what value to choose for the volume of production in $t = 1$. If the technical coefficient v^2 is left unchanged, then the stock of secondary manpower in $t = 1$ implies a volume of production of 14,300 million U.S. dollars, which is 162 per cent greater than the volume of production in the base year and represents an annual average growth during the first six years of almost three times the rate that is planned. Since economic growth does not depend solely upon the size of the secondary manpower stock, it would appear more reasonable to assume that the volume of production will grow as planned, i.e. increase by 50 per cent during a six-year period, and to work out the implications for the technical coefficients and the teacher-pupil ratios. Using this approach, the problem of transition can be solved through internal measures by increasing the use of secondary manpower in production by 77 per cent and either:

1. increasing the use of third-level manpower in production by 5.3 per cent,
2. increasing the teacher-pupil ratio in secondary education by 155 per cent,

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3. increasing the teacher-pupil ratio in third-level education by 100 per cent,
4. increasing both teacher-pupil ratios and the use of third-level manpower in production each by 4.8 per cent, or
5. some weighted combination of these possibilities.

A word of caution needs to be added at this point. In solving this transition problem, the number of unknowns exceeds the number of equations used, leaving a number of degrees of freedom. These degrees of freedom were used somewhat arbitrarily in this example. It would also be possible to use the degrees of freedom to find an optimum solution according to some criterion resulting in a linear programming problem. This type of treatment of a transition problem is found in Part I.

It should be pointed out also that, in the example given here, if the coefficients were left unchanged we should have a problem of surpluses of educated manpower in the labour force, and this problem could be treated using similar methods to those suggested in Part I, " The Elimination of Surpluses of a Given Educational Attainment from the Labour Force. "

BASIC MODEL REFORMULATED TO TAKE ACCOUNT OF SCHOOL DROP-OUTS, EFFECTIVE PERIOD OF STUDY AND AN ALTERNATIVE TREATMENT OF RETIREMENT

Reformulation of the Model

The basic model used in the preceding section is open to a number of criticisms on the grounds that it is too simplified and does not adequately represent the real world. In this section a number of changes will be introduced to make the model more realistic. The basic structure of the model will be left untouched; no new equations or variables will be added, but new coefficients will be added and the method for calculating the values of others will be altered.

It was noted in the preceding section that the manpower stock attrition rates used in the basic model disregarded the age composition of the educated manpower stocks and implied a constant proportion of the stocks withdrawing every six years due to death or retirement. This would be adequate only if the size of the stocks remained constant and the age composition of the stocks had an even distribution. But the model calls for a constant expansion in the size of the stocks with ever-increasing numbers of school graduates entering the labour force. In this case, the use of attrition rates that call for a constant proportion of the total stock to withdraw every six years implies that either the average retirement age is continually declining or that the average death rate is continually increasing, or both. This is obviously not very realistic. The alternative is to take account of the age composition of the stocks as they expand over time. A method for doing this is suggested in Part I " Alternative Treatment of Retirement " which will be used here. It will be assumed that the average productive life of secondary school graduates in the labour force is 8 time periods or 48 years, and for third-level graduates 7 time periods or 42 years. Using the formula given in Part I, the following retirement rates are found for a 7 per cent annual rate of growth:

$\lambda^2 = 0.020$ $\quad\quad$ $\lambda^3 = 0.031$

The difference in the values reflects the shorter working life assumed for third-level manpower. While these values may not be precise for any one time period, they do more adequately reflect the real situation over the long-term. It should be noted that these new rates do not take into account withdrawals from the stock due to death.

The basic model also includes other highly simplified assumptions with regard to the educational system and new entrants into the labour force. The period of study for both secondary and third-level education is assumed to be six years. In Turkey, the length of study in general secondary education is six years but some types of secondary vocational education take only five years; moreover, third-level education in Turkey lasts from two to five years or more. If all types of secondary and third-level education are to be included in the model, the actual effective period of study should be taken into account when the model is applied to a specific country.

In addition, the basic model assumes that all students enrolled in secondary schools in a given year will graduate within six years and either enter the labour force or go on to higher education. Similarly, the basic model assumes that all students enrolled in third-level schools in a given year will graduate and enter the labour force. These assumptions are also unrealistic when applying the model to a specific country. First, not all students enrolled in secondary or third-level schools will graduate. Some will fail to complete their studies, and this should be taken into account if we define, as we did, the manpower stocks to be made up of graduates only. Secondly, not all graduates will enter the labour force. This is particularly true of female graduates. It can be said that these graduates studied primarily for cultural reasons. This phenomenon also should be taken into account.

In Part I, " Drop Out, " a method is suggested to take account of these complicating, real-life factors. This involves reformulating equations (3) and (4) by adding new coefficients. But these reformulations ignore the problem of the effective length of study. According to Part I, the best way to handle this problem is to reduce the length of the time unit used in the model. However, this would needlessly complicate the model. The problem can be met by adding another coefficient and calculating the values for the new coefficient in an appropriate way, which is explained in the technical appendix, and which permits the six-year time unit to be retained. Using these methods, equations (3) and (4) would be reformulated as follows:

(3.1.) $m_t^2 = \alpha^2(\delta^2 n_{t-1} + \sigma n_{t-1}^3 - \gamma n_t^3)$

(4.1.) $m_t^3 = \alpha^3 \delta^3 n_{t-1}^3$

In equation (3.1) above, the term $\delta^2 n_{t-1}^2$ represents the number of secondary school graduates during a six-year period, γn_t^3 represents the number of these secondary school graduates who enrol in third-level schools, and σn_{t-1}^3 represents the number of students who have dropped out of third-level schools during the six-year period. The combination of these three terms is multiplied by a labour force participation rate, α^2, to arrive at the number of new entrants, m_t^2, into the stock of secondary level manpower stock, N^2, during a six-year period. In equation (4.1) above, the term $\delta^3 n_{t-1}^3$ represents the number of graduates from third-level schools during a six-year period, which is multiplied by α^3 a labour force participation rate, to determine m_t^3, the number of new entrants into the third-level manpower stock, N^3. A description of each of the new coefficients and their selected values for Turkey follows:

Graduation Rates

These coefficients show the percentage of total secondary and third-level enrolments in any one year that will graduate over a six-year period. The value for the third-level coefficient also takes into account that third-level education in Turkey takes less than six years to complete. The values selected for these coefficients are a weighted average of the six past six-year periods in Turkey.

$\delta^2 = 0.564 \qquad \delta^3 = 0.778$

Third-level Education Entrance Rate

This coefficient expresses the number of new third-level enrolments during a six-year period as a percentage of total third-level enrolment in any one year. Since third-level education in Turkey takes less than six years to complete, the value for this coefficient can be expected to be greater than 1, the more so the greater the concentration of enrolments in shorter term courses and the higher the attrition rate from third-level schools. The value selected represents the weighted average of the six past six-year periods in Turkey.

$\gamma = 1.455$

Third-level Education Attrition Rate

This coefficient is designed to represent the number of students who drop out of third-level schools during a six-year period expressed as a percentage of total third-level enrolment at the beginning of the period. Its value depends partly upon the values selected for δ^3 and γ, as explained in the technical appendix.

$\sigma = 1.071$

Labour Force Participation Rates

These coefficients show the percentages of secondary and third-level school graduates who enter the labour force on a permanent basis, ignoring short-term entrants such as women who leave the labour force a few years after entrance because of marriage. The values selected reflect the smaller proportion of female graduates from third-level schools than from secondary schools as well as the relatively high rate of labour force participation of the population 15 years of age and over in Turkey.

$\alpha^2 = 0.8 \qquad \alpha^3 = 0.9$

In comparing these coefficients with those suggested in Part I, the following relationships exist:

$$\mu^{21} = \alpha^2\delta^2$$
$$\mu^{22} = \alpha^2\sigma$$
$$\mu^3 = \alpha^3\delta^3$$

Thus, three coefficients have been used to represent relationships for which we have used five coefficients. In other words, the coefficients used are made up of two separate elements, which we have chosen to represent separately, since a change in either of the two separate elements will affect the combination. The new coefficient added here is γ, a third-level education entrance rate, to take account of the effective period of study for this level in Turkey, which enables a time unit of six years to be retained in the model.

ERIC

Determination of Balanced Growth Path

The method for finding the balanced growth path for this reformulated model is exactly the same as for the basic model as described on page 59. The result is shown in Table 15. This reformulated model will be referred to hereafter as the " complicated basic model. "

TABLE 15. BALANCED GROWTH OF THE EDUCATIONAL SYSTEM FOR A GROWTH RATE OF 7 PER CENT PER ANNUM

PRODUCTION IN MILLIONS OF 1961 U.S. DOLLARS, POPULATION IN THOUSANDS

The complicated basic model.

SYMBOLS	VARIABLE	ACTUAL VALUE IN $t = 0$	EQUILIBRIUM GROWTH PATH			
			0	1	2	3
N^2	Manpower with secondary education	210.6	210.6	316.1	474.4	712.1
N^3	Manpower with third-level education	104.7	105.0	157.7	236.7	355.2
n^2	Students in secondary schools	477.3	399.7	600.0	900.6	1,351.7
n^3	Students in third-level schools	62.2	79.7	119.6	179.5	269.4
m^2	Manpower with secondary education and less than 6 years in labour force	n.a.	73.0	109.6	164.5	246.9
m^3	Manpower with third-level education and less than 6 years in labour force	n.a.	37.2	55.8	83.8	125.7
v	Volume of production	5,468.1	5,468.1	8,207.6	12,319.6	18,491.7
n^2/p^2	Secondary students as a proportion of secondary school-age population (per cent)	14.7	12.3	13.7	17.1	22.6
n^3/p^3	Third-level students as a proportion of third-level school-age population (per cent)	2.5	3.1	3.9	4.3	5.4

The introduction of the alternative retirement rates, the values of which are considerably less than those used in the basic model, has the effect of reducing school enrolment requirements for both levels of education. In effect, the manpower stock replacement needs are assumed to be smaller, thus requiring smaller numbers of students for replacement purposes. However, the retirement rates used here are average long-term rates, which understate replacement needs during the first few time periods.

The new coefficients in the educational flow equations, on the other hand, have the effect of increasing school enrolment requirements. Additional enrolments are necessary to allow for school drop-outs and for graduates who will not enter the labour force.

The result of these two opposite effects is that for the base year (1960-61) the " balanced " secondary school enrolment requirement is 70 per cent higher than under the basic model, and the third-level school enrolment requirement is raised by 21 per cent. By comparing these new school enrolment requirements in the base year with the actual school enrolments in that year, we find that the actual secondary school enrolment is still too high for a balanced development, and the actual third-level school enrolment is further below the equilibrium enrolment. The actual secondary school enrolment is 19 per cent above the equilibrium level (compared with 101 per cent too many students under the basic model), and the actual third-level school enrolment is only 78 per cent of the equilibrium level (compared with 94 per cent of the quilibrium level found by using the basic model). The educational system, under this new model, is still out of balance with growth requirements; secondary education is less out of balance, but third-level education is more out of balance.

ERIC

ADAPTATION TO A BALANCED GROWTH PATH

To determine what internal steps might be taken to put the educational system into balance with growth requirements, the approach described in the preceding section for the basic model could be used, enabling a transition to the balanced growth path to be made within two time periods. However, the addition of six new coefficients to the model means that even more possibilities exist for making the transition without external measures (such as importing or exporting manpower). For example, under this complicated basic model, the transition could be made in just one time period by concentrating on temporary changes in labour force participation rates, school graduation rates and third-level school entrance and drop-out rates. No changes would be required in the technical coefficients, the teacher-pupil rates or the retirement rates. This is shown in the example given in the following paragraphs.

Since, in this example, the transition will be limited to just one time period, all variables for $t = 2$ must have the values shown on the equilibrium path in Table 15. But this means that the values of those variables in $t = 1$, which appear in the equations for $t = 2$, must be equal to the equilibrium values for $t = 1$ or otherwise the system would not remain on the equilibrium path in later time periods. These variables are N^2, N^3, n^2 and n^3. The system of equations for $t = 1$ would then be the following, assuming, for the moment, that all coefficients are also unknowns.

$$(1') \quad 316.1 = \nu_1^2 v_1$$

$$(2') \quad 316.1 = (1 - \lambda_1^2)\, 210.6 + m_1^2$$

$$(3.1') \quad m_1^2 = \alpha_1^2 (477.3\, \delta_1^2 + 62.2\, \sigma_1 - 119.6\, \gamma_1)$$

$$(4.1') \quad m_1^3 = 62.2\, \alpha_1^3 \delta_1^3$$

$$(5') \quad 157.7 = (1 - \lambda_1^3)\, 104.7 + m_1^3$$

$$(6') \quad 157.7 = \nu_1^3 v_1 + 600.0\, \pi_1^2 + 119.6\, \pi_1^3$$

Without changing the retirement rates, equations (2′) and (5′) determine the value of 109.7 thousand for m^2, and 56.2 thousand for m^3, the required new additions to the manpower stocks, for the stocks to reach their equilibrium level. If, however, the third level graduation and labour force participation rates were left unchanged, then equation (4.1′) would result in m_1^3 being equal to 43.6 thousand, 12.6 thousand *less* than required. To obtain the required number, the product of $\alpha^3\delta^3$ in equation (4.1′) would have to rise from 0.7002 to 0.9035, an increase of 29 per cent. This increase could be obtained by changing both coefficients or limiting the increase to the third-level graduation rate. For example, it might be assumed that it would be difficult to change the labour force participation rate of third-level graduates within a brief period of six years, so that the third-level graduation rate would have to rise from 0.778 to 1.004 in order to obtain the required 12.6 thousand additional entrants into the third-level manpower stock. Since third-level education in Turkey takes less than six years to complete, this increase in the value for δ^3 is not impossible to achieve. It implies a greater concentration of students in shorter third-level courses, and a decrease in the third-level drop-out rate. Or, it might be assumed that the labour force participant rate could be increased somewhat, say from 0.90 to 0.92, which would mean that the graduation rate would have to be increased to 0.982, an increase of 26 per cent. Many other combinations of changes in both coefficients would be possible.

A similar analysis can be made with equation (3.1.′). If the coefficients in the equation were left unchanged, the number of new entrants into the stock of manpower with a secondary education during the first period would be 129.4 thousand, 19.7 thousand *more* than required. Here, the problem is of a different nature. For third-level manpower, the problem was how to avoid a *shortage* of manpower, but for manpower with a secondary education, the problem is how to avoid having a *surplus* after the first six years.

As explained in the technical appendix to this paper, the values of σ and γ depend partly upon the value of δ^3, and a new value for δ^3 for the transition period implies new values for σ or γ or both. For example, if δ^3 is to be raised to 0.9E2 in equation (4.1′), then either σ must be decreased to 0.893 or γ must be increased to 1.548, or both coefficients have to be changed in some combination of an increase in γ and a decrease in σ. One such combination might be $\sigma = 1.015$, a decrease of 5.2 per cent, and $\gamma = 1.528$, an increase of 5 per cent. Using this combination in equation (3.1.′) with no changes in the other coefficients, we find that the number of new entrants into the stock of manpower with secondary education would be 119.7 thousand, 10.0 thousand more than required. Part of the problem of surplus has thus been met. The remaining surplus can be eliminated by either reducing the secondary school graduation rate by 4.6 per cent to 0.538, or by reducing the labour force participation rate of secondary school graduates by 8.8 per cent to 0.73, or by some combination of reductions in both.

With such a combination of changes in the coefficients of equations (3.1′) and (4.1′) as described above, the system of equations on page 66 becomes consistent and the transition becomes possible in only one time period without any alteration in the production objective, the use of educated manpower in production, or the teacher-pupil ratios as required in the simple model. The addition of the six new coefficients increases the number of measures available for bringing the existing educational system into balance with long-term growth requirements, and also makes possible a shorter transition period. If, however, the requirements for a one-period transition would put too severe a strain on the system, a longer transition could be considered with temporary changes in additional coefficients. It would also be possible to use linear programming techniques, such as those described in Part I, to work out a transition solution.

SECTORAL DISAGGREGATION OF PRODUCTION

Reformulation of the Model

It is usual in an expanding economy for the individual sectors of production to increase at different rates of growth over time. Moreover, the educated manpower requirements typically vary among the different sectors; for example, the requirements for third-level manpower in agriculture are less than in the industrial sector. Therefore, as is pointed out in Part I, an important elaboration of the model is to disaggregate the total national product into the various component sectors of production, each with its own rate of development over time and its own educated manpower requirements.

In this section, a disaggregated version of the complicated basic model presented in the previous section will be applied to Turkey. The disaggregation is limited to three major sections of production—agriculture, industry and services—together comprising the total gross domestic product of Turkey, as opposed to gross national product used in previous sections.

As described in Part I " Sectoral Disaggregation of Production and its Manpower Requirements, " the only changes needed in the model are limited to equations (1) and (6) to distinguish the different sectors of production plus the addition of two definitional equations. The new equations are as follows:

(1.1 a) $^{1}N_t^2 = {}^{1}v^2 v_t^1$

(1.1 b) $^{2}N_t^2 = {}^{2}v^2 v_t^2$

(1.1 c) $^{3}N_t^2 = {}^{3}v^2 v_t^3$

(1.1 d) $N_t^2 = {}^{1}N_t^2 + {}^{2}N_t^2 + {}^{3}N_t^2$

(6.1 a) $^{1}N_t^3 = {}^{1}v^3 v_t^1$

(6.1 b) $^{2}N_t^3 = {}^{2}v^3 v_t^2$

(6.1 c) $^{3}N_t^3 = {}^{3}v^3 v_t^3 + \pi^2 n_t^2 + \pi^3 n_t^3$

(6.1 d) $N_t^3 = {}^{1}N_t^3 + {}^{2}N_t^3 + {}^{3}N_t^3$

The symbols v^1, v^2 and v^3 represent the net output of agriculture, industry and services respectively, and $^{1}N^2$ measures the labour force with a secondary education in agriculture, $^{2}N^2$ the labour force with a secondary education in the industrial sector, etc. According to equation (6.1 c), teachers of both levels of education are included in the services sector, and are assumed to have a third-level education.

The following values for the sector technical coefficients for Turkey were found by using 1961 gross domestic income by major sectors expressed in millions of 1961 U.S. dollars. The data for trained manpower (in thousands) by sectors were derived from the 1960 census.

	v^2	v^3
Agriculture	0.007	0.002
Industry	0.040	0.013
Services	0.092	0.040
Total G.D.P.	0.045	0.018

The value of the third-level manpower technical coefficient for the services sector (teachers are excluded) in Turkey is twenty times greater than for the agriculture sector and three times greater than for the industrial sector. A similar disparity exists among the secondary-level manpower requirements of the three sectors. The table below gives the major reason for these disparities by showing the percentage distribution of secondary and third-level graduates in the labour force in 1960 in Turkey for the three sectors (teachers are included in the services sector):

TABLE 16. SECTOR DISTRIBUTION OF TRAINED MANPOWER IN 1960

In percentage.

SECTOR	TOTAL LABOUR FORCE	TRAINED MANPOWER	
		SECONDARY	THIRD-LEVEL
Agriculture	79.0 %	6.0 %	3.3 %
Industry	10.3	20.0	13.6
Services	10.7	74.0	83.1
Total	100.0	100.0	100.0

This distribution of educated manpower among the three sectors of production is not necessarily an optimum one, but is nevertheless used as though it were in applying the model.

Determination of the Balanced Growth Path

The method for determining the balanced or equilibrium growth path for this model is described in Part I, chapter 3. It is assumed that the total domestic product will grow at a constant rate—50 per cent each time period—and that production in each of the component sectors will increase according to:

$$v_t^i = v_{01}^i \omega^t + v_{00}^i \quad (i = 1, 2, 3)$$

During the first six years it is assumed that agricultural production will increase 26.5 per cent (4.0 per cent per annum), industrial production 87.0 per cent (11 per cent per annum) and output in the services sector 57.0 per cent (7.8 per cent per annum). These rates are in agreement with an annual average increase of 7 per cent in gross domestic income, or 50 per cent for the first six years, and result in the following sector constants:

$$v_{01}^1 = 1{,}023.8 \qquad v_{00}^1 = 911.9$$
$$v_{01}^2 = 1{,}849.4 \qquad v_{00}^2 = -784.4$$
$$v_{01}^3 = 1{,}929.8 \qquad v_{00}^3 = -233.6$$

It is assumed also that all other variables in the model will increase in the same manner. Thus, for each variable, the relationship:

$$z_t = z_{01}\omega^t + z_{00}$$

where z_{01} and z_{00} are constants and ω is equal to 1.5, is substituted in the 12 equations. From this, two sets of 12 equations each are derived by combining all terms with powers of ω and all terms consisting only of constants. Each set of equations is solved simultaneously in terms of the production constants, and the balanced growth path is determined by adding the results.

Table 17. BALANCED GROWTH OF THE EDUCATIONAL SYSTEM FOR A 7 PER CENT ANNUAL GROWTH RATE

The complicated basic model disaggregated by sectors of production.

Symbols	Variable	Actual value in $t = 0$	Equilibrium growth path 0	1	2	3
v	Volume of production (G.D.I.)	4,696.9	4,696.9	7,050.0	10,582.0	15,883.6
N^2	Manpower with secondary education	210.6	210.6	339.1	532.0	821.3
N^3	Manpower with third-level education	104.7	107.1	169.9	264.1	405.3
n^2	Students in secondary schools	477.3	481.4	723.4	1,086.5	1,631.0
n^3	Students in third-level schools	62.2	94.4	142.0	213.5	320.6
m^2	Manpower with secondary education and less than 6 years in labour force	n.a.	88.2	132.8	199.6	299.9
m^3	Manpower with third-level education and less than 6 years in labour force	n.a.	43.9	66.1	99.5	149.5
n^2/p^2	Secondary students as a proportion of secondary school age population (per cent)	14.7	14.9	16.5	20.7	27.3
n^3/p^3	Third-level students as a proportion of third-level school age population (per cent)	2.5	3.7	4.6	5.1	6.5

Thus, for example, the balanced growth path of the stock of manpower with a third-level education is determined by the following:

$$N_t^3 = 0.00214\, v_{01}^1 \omega^t + 0.01640\, v_{01}^2 \omega^t + 0.04822\, v_{01}^3 \omega^t + 0.00181\, v_{00}^1 + 0.01355\, v_{00}^2 + 0.04066\, v_{00}^3$$

The results of using this method in applying the sector model to Turkey for the first few time periods are shown in Table 17, which omits the individual sector manpower requirements.

The effect of disaggregating the complicated basic model by the major sectors of production is that for the base year (1960-61) the equilibrium secondary school enrolment is increased by 20 per cent, and the third-level school enrolment requirement is raised by 18 per cent. The reason for these increases is that the sectors which use most of the educated manpower (industry and services) will expand at rates greater than the rate for total production, at least during the first few time periods. In comparing these new school enrolment requirements with the actual school enrolment figures in the base year, we find that the actual secondary school enrolment in 1960-61 is now slightly less than the enrolment required for a balanced development of the system, and that the actual third level enrolment is still further below the equilibrium level. The actual secondary school enrolment is less than one per cent below the equilibrium level (compared with 19 per cent too many students under the complicated basic model), and the actual third-level enrolment is now down to 66 per cent of the equilibrium level (compared with 78 per cent of the equilibrium level using the complicated basic model). Under this new model secondary education in Turkey in 1960-61 was more or less in balance with growth requirements, but third-level education is even more out of balance than under the first two models used. Here again, transition problems could be worked out as for the previous models to determine what measures are necessary to bring the educational system into balance with manpower requirements.

A consequence of using the method described for determining the balanced growth paths for the variables in this sector model is that the rates of growth for the component sectors of production move asymptotically toward the rate of increase for total production. Eventually, the rate of increase in output in each of the sectors would be identical with that for total production. This seems somewhat unsatisfying, particularly if it is planned or expected that one or more sectors will expand at constant or fluctuating rates.

Another technique that could be used in order to find a growth path for a sector model would be to apply the complicated basic model to each of the component sectors, assume that sector growth rates will remain constant over time, and add together the results for the individual sectors. This technique, however, has the disadvantage of having total production increasing at an inconstant rate.

INCREASING THE NUMBER OF EDUCATIONAL PROCESSES

Requirements for educated manpower can be expected to differ not only among the various sectors of production but also among different types of educated manpower. In the models discussed in the previous sections it is assumed that secondary and third-level educated manpower are homogeneous in composition—that the types of skills embodied in the two levels differ only by level of education. It would be more realistic to take account of the hetero-

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geneous composition of skills embodied in these two levels of manpower, for example, by distinguishing many kinds of manpower by level of education, perhaps by broad occupational groups.

It is also assumed in the previous models that future requirements for both levels of educated manpower will increase at identical rates. Here again, it would be more realistic to allow for different rates of increase among the different kinds of educated manpower. For example, it seems reasonable to expect that the demand for scientific and technical third-level manpower will increase in the future at a greater rate than that for other types of third-level manpower.

Both objectives could be accomplished by increasing the number of educational processes and introducing the generalised demand functions described in Part I, " Generalization of Demand Function. " Increasing the number of educational processes will allow more distinction among the types of skill embodied in the manpower stocks and the use of the generalised demand functions will permit the requirements for the different types of skills to expand at different rates.

An example of a model incorporating these refinements is one that distinguishes two types of third-level education—scientific and technical education and all other third-level education. As is pointed out in Part I, " Increasing the Number of Education Processes, " the addition of each new educational process requires the addition of three equations to the model and a new linkage between the various types of education. Thus, in this example, the model will consist of nine equations of the character shown below:

$$(1.2)\quad N_t^2 = v^{20} v_t^{\,v^{21}} \left(\frac{v_t}{a_t}\right)^{v^{22}}$$

$$(2)\quad N_t^2 = (1 - \lambda^2) N_{t-1}^2 + m_t^2$$

$$(3.2)\quad m_t^2 = \alpha^2(\delta^2 n_{t-1}^2 + \sigma^3 n_{t-1}^3 + \sigma^4 n_{t-1}^4 - \gamma^3 n_t^3 - \gamma^4 n_t^4)$$

$$(4.1)\quad m_t^3 = \alpha^3 \delta^3 n_{t-1}^3$$

$$(5)\quad N_t^3 = (1 - \lambda^3) N_{t-1}^3 + m_t^3$$

$$(6.2)\quad N_t^3 = v^{30} v_t^{\,v^{31}} \left(\frac{v_t}{a_t}\right)^{v_{32}} + \eta \pi^2 n_t^2 + \pi^3 n_t^3$$

$$(7)\quad N_t^4 = v^{40} v_t^{\,v^{41}} \left(\frac{v_t}{a_t}\right)^{v_{42}} + (1 - \eta) \pi^2 n_t^2 + \pi^4 n_t^4$$

$$(8)\quad N_t^4 = (1 - \lambda^4) N_{t-1}^4 + m_t^4$$

$$(9)\quad m_t^4 = \alpha^4 \delta^4 n_{t-1}^4$$

In this model, N^4, n^4 and m^4 represent scientific and technical third-level manpower and N^3, n^3 and m^3 represent other types of third-level manpower. The linkage between the two types of third-level education and secondary education is provided in equation (3.2), which recognises that both types of third-level students will come from one type of secondary education assumed to be homogeneous in nature. It is assumed also that both types of third-level manpower serve as teachers in secondary schools, and this is reflected in equations (6.2) and (7). A certain proportion, η, of secondary school teachers is assumed to be graduates of non-scientific, non-technical third-level schools, and the remaining proportion, $1 - \eta$, is assumed to be graduates of scientific and technical third-level schools.

The generalised demand functions used in equations (1.2), (6.2) and (7) replace the simple proportionality previously assumed to exist between educated manpower stocks and the level of production with a new non-linear relationship. In these equations the educated manpower stocks are assumed to be a function of both the level of production v and per capita income v/a where a is equal to total population. When applying this model to a specific country the values for the v coefficients used in these equations can be derived by using regression analysis techniques on a series of observations for a number of different years. However, few countries, including Turkey, have time-series data giving the size of educated manpower stocks for a sufficient number of years to permit this kind of calculation. An alternative method for deriving the values of these coefficients is through cross-sectional regression analysis of observations taken from a number of different countries. This type of analysis is currently being carried out at the Netherlands Economic Institute.

The method for determining the balanced growth path for this model is the same as that described in Part I, " Generalization of Demand Functions " with appropriate changes to take account of the additional educational process.

Another useful refinement in the model would be to disaggregate the model just discussed by sectors of production. This could be done by altering equations (1.2), (6.2) and (7) in the same manner as in the previous section.

Because sufficient data were not available for calculating the values of many of the coefficients, none of the refinements suggested in this section could be applied to Turkey.

SUMMARY AND CONCLUDING REMARKS

Summary of Results

Three planning models were actually applied to Turkey, the results of which are summarized in Table 18 which shows the actual values of the variables in the year corresponding to $t = 0$, and the equilibrium values for $t = 0$ and $t = 3$. The first model applied was the simplest, the second somewhat more complicated because of the introduction of new coefficients, and the third the most complicated because it split the economy into three parts. At the same time, each model applied was increasingly more realistic as additional features of the educational system and manpower requirements were taken into account. But even the third model is very simple and the preceding section described how additional refinements could be made.

The comparison of actual school enrolments in the base years with the enrolments required by the model for a balanced development of the system is of particular interest in applying models of this type to a specific country. As was pointed out on page 58, the models are based upon the size of the existing educated manpower stocks and not upon the existing school enrolments. This makes it possible for the actual school enrolments not to coincide with the enrolments required by the models and such discrepancies did, in fact, occur with each of the models applied to Turkey. Under the simple basic model, actual and required third-level enrolments were almost identical, but the required secondary school enrolment was less than half of the actual enrolment in the base year. However, under the three-sector model, the results were reversed; actual and required secondary enrolments were almost identical but the required third-level enrolment was about 50 per cent greater than the actual enrolment.

It is interesting to note that, with each model applied, the level of the balanced growth paths for school enrolments at both levels rose successively

TABLE 18. SUMMARY OF RESULTS OBTAINED WITH DIFFERENT MODELS FOR $t = 0$ AND $t = 3$

Population in thousands.

SYMBOLS	ACTUAL VALUE IN $t = 0$	EQUILIBRIUM GROWTH PATHS					
		$t = 0$			$t = 3$		
		SIMPLE BASIC MODEL	COMPLICATED BASIC MODEL	SECTOR MODEL (COMPLICATED)	SIMPLE BASIC MODEL	COMPLICATED BASIC MODEL	SECTOR MODEL (COMPLICATED)
N^2	210.6	210.6	210.6	210.6	712.1	712.1	821.3
N^3	104.7	99.2	105.0	107.1	335.4	355.2	405.3
n^2	477.3	235.8	399.7	481.4	797.4	1,351.7	1,631.0
n^3	62.2	65.9	79.7	94.4	223.0	269.4	320.6
m^2	n. a.	91.3	73.0	88.2	308.2	246.9	299.9
m^3	n. a.	44.0	37.2	43.9	148.6	125.7	149.5
n^2/p^2 (%) ..	14.7	7.3	12.3	14.9	13.4	22.6	27.3
n^3/p^3 (%) ..	2.5	2.6	3.1	3.7	4.5	5.4	6.5

higher. Additional refinements of the models would presumably result in still different growth paths, which raises the question of which version of the model should be used for practical planning purposes.

It is obvious that, for Turkey, the simple basic model would not be a practical planning instrument. The assumptions with respect to educational flows are too simplified, and the results would not be politically acceptable. For example, the proportion of the secondary school age population which, as indicated by the model, should be enrolled in secondary schools in 1978-79, is less than the proportion actually enrolled in 1960-61. The results given by the two more complicated versions of the model are more acceptable and, in fact, are not too far out of line with what is actually planned in Turkey using micro-planning techniques. On this basis, these two models could be said to give a useful first approximation of educational needs in Turkey in the light of its economic development objectives.

This supposes, however, that the assumptions on which the models are based are realistic, particularly the assumptions as to how requirements for educated manpower will develop over time, at least in the short-run. The demand functions used in the models applied to Turkey assume a simple proportional relationship between educated manpower stocks and volume of production, and also that the size of the existing stocks is optional. Both assumptions are questionable. In the long run, the simple proportional relationship would result in the entire labour force requiring a third-level education. Moreover, in comparison with other countries, the present size of the manpower stock with a secondary education in Turkey would appear to be too small in relation to its level of production. The conclusion is that a great deal more experimentation should be made with the models before they are proposed as practical planning instruments.

AREAS FOR ADDITIONAL RESEARCH

For all their shortcomings, the models do represent a useful way of looking at the problems of educational planning in the light of the expected economic and social development in a country. The definitional equations, in particular, are very useful in looking at flows within the educational system and between the educational system and the labour force. All the major elements of these flows are contained in four equations, representing a very useful shorthand device for educational planners.

In order to improve the application of the models to specific countries, additional research and experimentation is needed along several lines. One type of work needed concerns demand functions. It has been proposed that application of the generalised demand function to specific countries would be useful. Experiments could also be made with other types of demand functions. For example, the use of constant technical coefficients in the two demand equations of the basic model would require the entire labour force to have at least a secondary-level education. To take this into account, Messrs. Emmerij and Williams have proposed the use of decreasing technical coefficients, based on past or expected future trends in the use of educated manpower. Another solution might be to vary the technical coefficients inversely to the proportion which these educated stocks represent of the total labour force. As the stocks of secondary and third-level manpower increase as a proportion of the total labour force, the requirements for additional educated manpower, limited by the size of the total labour force, would therefore grow at a decreasing rate.

Another useful type of research would be to study the development over time of the coefficients used in the definitional equations, particularly equations (3.1) and (4.1), and to determine how these definitional equations could be used as a means of planning educational flows. For example, by replacing the two demand function equations with two new equations determining how school enrolments at both levels might develop over time independently of educated manpower requirements, the model could be used as a supply rather than a demand model.

Attempts to explore further possible modifications in the models would also be useful. One such additional modification is suggested in the previous section. In addition, the models could be disaggregated by component sectors of production and different types of education in much greater detail. However, with each new refinement attempted, additional data are needed—data that become increasingly more difficult to secure.

TECHNICAL APPENDIX: CALCULATION OF EDUCATIONAL FLOW COEFFICIENTS

The values selected for the educational flow coefficients—graduation rates and third-level education entrance and attrition rates—used in equations (3.1) and (4.1) are based upon annual educational statistics for Turkey over a series of years. The methods described below were used in calculating their values.

Graduation Rates

The value for δ^2 for a given six-year period was found by dividing the number of lycée-level secondary school graduates for the period by the total secondary school enrolment (both lycée and lower levels) at the beginning of the period. The value for δ^3 was found in the same way, using university-level graduates for a six-year period and total third-level enrolment at the beginning of the period. So, for example, if 1954-55 to 1959-60 is the six-year period under consideration, the number of graduates during these school years is divided by total enrolment in 1954-55. These graduation rates, then, are not the usual rates based on cohorts or individual classes, but are rates calculated to fit the structure of the models which use a time unit of six years. In a country such as Turkey, where it usually takes six years to complete a full secondary school education, δ^2 will be less than one. But for third-level education, which usually takes less than six years to complete, the value of δ^3 can be greater than one depending upon drop-out rates and the degree of concentration of students in shorter term courses. The values for δ^2 and δ^3 for Turkey for six different six-year periods are given below in Table 19.

Third-level Education Entrance Rate

The value of γ for a six-year period was found by dividing the number of new third-level school enrolments during the period by the total third-level school enrolment at the end of the period. So, for example, if 1955-56 to 1960-61 is the six-year period under consideration, the number of new third-level enrolments during these six school years is divided by total third-level enrolments in 1960-61. When third-level education takes less than six years to complete, the value of γ can be expected to be greater than one, its value increasing the higher the drop-out rate from third-level schools and the greater the concentration of students in the shorter term courses. It is assumed that students who go on to third-level education enter third-level schools at the beginning of the school year immediately following that in which they graduate from secondary school. Therefore, the six-year period used for calculating the value of γ which corresponds to the six-year period used for the graduation rates is advanced by one year, e.g. for the period 1955-56 to 1960-61 γ corresponds to the period 1954-55 to 1959-60 for the δ's in the model. The values for γ for Turkey for the six different six-year periods corresponding to the δ's shown in Table 19 are also shown in the table.

Third-level Education Attrition Rate

The coefficient σ is designed to represent the number of students who drop out of third-level schools during a six-year period, expressed as a percentage of total third-level enrolment at the beginning of the period. In order to make σ consistent with the model, it must be defined in such a way as to take account of all students who were enrolled in third-level schools at the beginning (or the end) of a six-year period, e.g. 1954-55; those who graduated from secondary schools and the next year enrolled in third-level schools over a six-year period, e.g., 1955-56 to 1960-61; those who graduated from third-level schools over a six-year period, e.g. 1954-55 to 1959-60; and those who are still enrolled in third-level schools at the beginning (or the end) of the next six-year period, e.g. 1960-61. In this paper, this was done by defining σ as follows:

$$(10) \quad \sigma = \frac{n^3_{t-1} + \gamma n^3_t - n^3_t - \delta^3 n^3_{t-1}}{n^3_{t-1}}$$

or more conveniently as:

$$(10\,a) \quad \sigma = 1 - \delta^3 + (\gamma - 1)\frac{n^3_t}{n^3_{t-1}}$$

In this manner, the value of σ depends upon the values selected for δ^3, γ, and third-level enrolments at the beginning (or the end) of two consecutive

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six-year periods. Table 19 also gives the corresponding values for σ calculated by this method for six different six-year periods in Turkey.

The number of drop-outs from third-level schools over a six-year period can also be expressed as a percentage of total third-level enrolment at the beginning of the next period as Mr. Emmerij does in his paper. In this case, the following relationship would be used for calculating the value of σ:

$$(11) \quad \sigma = \gamma - 1 + (1 - \delta^3)\frac{n^3_{t-1}}{n^3_t}$$

TABLE 19. VALUES FOR EDUCATIONAL FLOW COEFFICIENTS FOR TURKEY FOR SIX DIFFERENT SIX-YEAR TIME PERIODS

TIME PERIOD	δ^2	δ^3	γ	σ
1	0.563	0.724	1.271	0.678
2	0.572	0.735	1.339	0.810
3	0.562	0.765	1.447	1.020
4	0.567	0.778	1.573	1.300
5	0.569	0.850	1.566	1.415
6	0.553	0.816	1.467	1.218
Weighted average	0.564	0.778	1.455	1.071

Part IV

PLANNING MODELS FOR THE CALCULATION OF EDUCATIONAL REQUIREMENTS FOR ECONOMIC DEVELOPMENT

GREECE

by
Gareth WILLIAMS

GENERAL REMARKS

The Planning Model of Educational Requirements consists of economic postulates, equations (1) and (6) in the basic model, and a set of definitions describing flows through the educational system and into the labour force. The usefulness of the model, therefore, depends upon the validity of the economic postulates and the accuracy with which the definitional equations describe any particular educational system. Two types of initial comment are necessary. The first is a critical examination of the theoretical and empirical basis of the economic relationships and the second is a consideration of technical refinements to enable the equations to correspond more closely to the actual educational flows in the country or region investigated.

In the simplest version the assumed economic relationships are a constant ratio between the number of secondary school graduates and the level of G.N.P., and a similar fixed relationship for university graduates, except insofar as these graduates are required to be teachers in secondary schools. Thus if G.N.P. grows at a constant exponential rate, the stocks of second- and third-level manpower will grow at the same exponential rate. Provided their numbers in the initial period are in equilibrium, the number of students in secondary schools and universities will also grow at the same rate. If they do not start in equilibrium, either the values of some of the coefficients must be changed temporarily to allow equilibrium to be reached, or the system becomes unstable and, as is usual with systems of difference equations, liable to violent fluctuations. The essential computing task therefore, in applying some version of the model to a particular set of data, is to establish the required initial values of the variables for whatever rate of growth is desired and, if necessary, to estimate changes in the values of the coefficients that will transform the disequilibrium initial situation to an equilibrium growth pattern.

The model is, in fact, very similar to that type of growth model which uses a constant average capital/output ratio and in which for any desired rate of growth of national income the necessary flow of savings/investment can be arithmetically determined. The "transition period" can be likened to the special savings effort which is temporarily necessary to promote "take-off" or to break " the vicious circle of poverty " in many of these more well-known models of economic growth.

The economic validity of the model depends upon the correctness of the assumption that there is a systematic relationship between certain types of manpower stock and national income.

Internationally comparable data on manpower stocks is still very scarce, but present indications are that this systematic relationship is a rather complicated one. For example, in Greece between 1951 and 1961 the value of v^2 declined by nearly 30 per cent and that of v^3 by over 30 per cent. Preliminary results of empirical investigations carried out by the Netherlands Economic Institute have shown by cross-sectional analysis of several countries that a reasonably good regression equation is[1]:

$$N^3 = 0.01062\, v^{1.22} \left(\frac{v}{a}\right)^{0.28}$$

There are, however, certain difficulties in using cross-sectional evidence of this type especially for small countries and regions. It indicates that in Greece in 1961 there was a surplus of 32,000 people with university education, or 50 per cent of the total. Data are not available which would allow a similar calculation for the second-level labour force.

The international comparability of the data that are available is open to considerable doubt. For example, in Greece the two-year training given to primary school teachers is usually considered as higher education; in many countries these teachers require only secondary education. Conversely, in Greece, the four-year post-gymnasium training received by sub-engineers is not usually considered as higher education.

Purely theoretical considerations suggest other difficulties. In the case of physical capital there is a basis in widely-held micro-economic theory for assuming that there are forces, at least in a competitive economy, driving the capital/output ratio to some degree of constancy. With a given technology and propensity to save there can be shown to be a tendency towards a stable capital coefficient. There is no such micro-economic infrastructure for assumptions about the relationship between stocks of educated manpower and the level of G.N.P.

It cannot be said that, in the past, in most parts of the world the decisions of those responsible for providing educational facilities have been determined purely or even mainly by economic considerations. The effect of economic needs on the size and structure of educational systems has been extremely indirect.

There are therefore both statistical and theoretical doubts to be expressed about the model. However, the distinction made at the beginning of the present paper between the economic relationships and the definitions has more than expositional significance. The former can be attached to any variable or set of variables whose long-term growth will be reasonably smooth and predictable. Some types of manpower stock, for example, can be given demographic significance. However, the advantage of the G.N.P. concept, as in many other branches of economics, is that it is a convenient summary of many of these factors. It is certainly convenient to use this as a preliminary hypothesis to test some of the other implications of the model and of aggregative educational planning in general.

1. The meanings of the symbols are given in Part I and also on p. 83 below. This equation is a specific example of the " Generalised Demand Function, " (see Part I).

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If there is not a systematic relationship of any sort between the educated labour force and certain indices of economic activity, or if there is not smooth long-run growth in these indices, the model and many other sorts of aggregate educational and long-term manpower planning become invalid. In the second case all the equations would become no more than a set of *ex post* definitions and of little interest.

Clearly education is not a homogeneous substance and some kinds of education have a more direct effect on economic growth than others. It is possible to distinguish very many different kinds of manpower in the economy each of which has a somewhat different educational background. The extent to which it is necessary or desirable to distinguish between these different types of manpower in working out the model is a matter for debate. The greater the degree of aggregation the greater the amount of substitutability of different types of manpower assumed. On the other hand, the greater the amount of disaggregation the more rigidity enters into the forecasts. One practical point can be made, namely that each new type of manpower with a different growth rate adds several equations to the system. In the present set of exercises only a very limited amount of disaggregation has been attempted. In some cases a breakdown by economic sectors may be useful, though in the absence of detailed long-period forecasts, it means making some special assumptions about sectoral rates of growth.

There are also several minor technical difficulties worth specifying[1]. In practice, the length of study is not the same for all levels of education. A simple method of eliminating this deficiency is the use of the γ coefficient[2], in equation (3). Another possibility would be a shorter time period. This is particularly useful in solving transition problems.

No contribution to output or stock is considered for courses started but not completed. An alternative hypothesis is possible, namely, that it is not the number of graduates in the labour force that affects output, but the number of years or even days or hours of education received by the whole labour force. For educational planning purposes, the distinction between graduates and years of study is not very important as long as the variables move in proportion to one another. It is only when the variables have different growth rates, or when δ (the correction factor for drop-outs) must be varied, that the numerical results will be affected. The content of the curriculum is also important. Clearly there would be strong grounds for expecting some of the coefficients to change if curricula become more adapted to the needs of the economy.

The non-entry into the labour force of large numbers of school leavers, especially girls, from secondary schools must be taken into account. To deal with this the coefficient α has been introduced. It is also possible to work throughout with total population figures, in which case the rationale of the v coefficients becomes a little complicated. This has been done on page 85 where it is considered more fully.

In some countries, particularly underdeveloped ones, a considerable number of third-level students attend foreign universities. Twenty-five per cent of all Greek university students are studying outside Greece. There are some slight complications in incorporating these into the model, the structure of

1. Some of these have been indicated in the more refined versions in Part I. However, since somewhat different symbols have been used here, and since some different solutions are suggested, slight repetition has been considered acceptable.
2. All additional coefficients are defined on page 83.

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which is such that the number of teachers and the number of students is simultaneously determined through the teacher/pupil ratios.

Conversely a considerable number of Greeks emigrate. The rate of emigration of educated manpower from Greece is so high that no realistic educational planning exercise can ignore it. The forces affecting the rate of emigration of skilled manpower are complex but they certainly are not the same as those affecting internal requirements. In fact, there is likely to be a negative relationship between them. A rise in G.N.P. increases home requirements of skilled manpower and reduces emigration. Conversely, a rise in N^3 for example, of itself will tend to increase emigration. In practice it is desirable to incorporate the emigrants with a special growth rate of their own, perhaps exogenous, perhaps related to the other variables of the system.

THE EDUCATIONAL SYSTEM IN GREECE

The educational system in Greece fits fairly conveniently into the conceptual framework of the model. The gymnasium course lasts six years, and graduates from gymnasia are able to proceed to institutions of higher education. Teachers in gymnasia are university graduates. The most important variation from the basic definitions of the model is the existence outside the main educational pyramid of a system of technical and vocational education at the secondary level which is new and growing rapidly in importance.

All children in Greece are required by law to attend school between the ages of six and twelve. The published statistical evidence indicates that nearly all do—although there is some falling off in attendance in the last year or two. After primary school several alternatives are possible. The child can leave school and enter the labour force (officially at fourteen, unofficially earlier). Or he can enter a gymnasium, or he can continue his education in one of the vocational, technical or professional schools which will fit him for a specific occupation. Only if he enters the gymnasium does he have any real chance of entering an establishment of higher education at the age of eighteen or nineteen—though arrangements have been made for a very few to enter universities from some forms of technical education. Pupils who fail to complete gymnasia have the opportunity to enter technical schools. Special provision is made for this by the division of the course into two three-year cycles, but as yet there is little evidence that a very high proportion of pupils leave gymnasia at the end of the third year. The vocational and professional schools are very heterogeneous; course lengths vary from one year in some agricultural and other vocational schools to seven years in the case of some professional schools. The average appears to be about three years. Attendance is usually part-time and in the evenings. Thus, in terms of both teachers and buildings, they use few resources. Technical education is also divided into two cycles, lower technical and secondary technical. Those in lower technical education come straight from the primary schools, while of those in secondary technical schools most are graduates of lower technical schools and some are people who have dropped out from the gymnasia.

The duration of higher education courses depends upon the subject of study and varies from two years in the training colleges for primary school teachers to six years in the faculty of medicine. The mode is four years.

Wastage appears to be very high in the gymnasia, with only 41 per cent of entrants eventually receiving the diploma. In the technical schools it is also high, but in universities it is low—though in most subjects a very large number of pupils repeat at least one year.

THE STRUCTURE OF GREEK EDUCATION

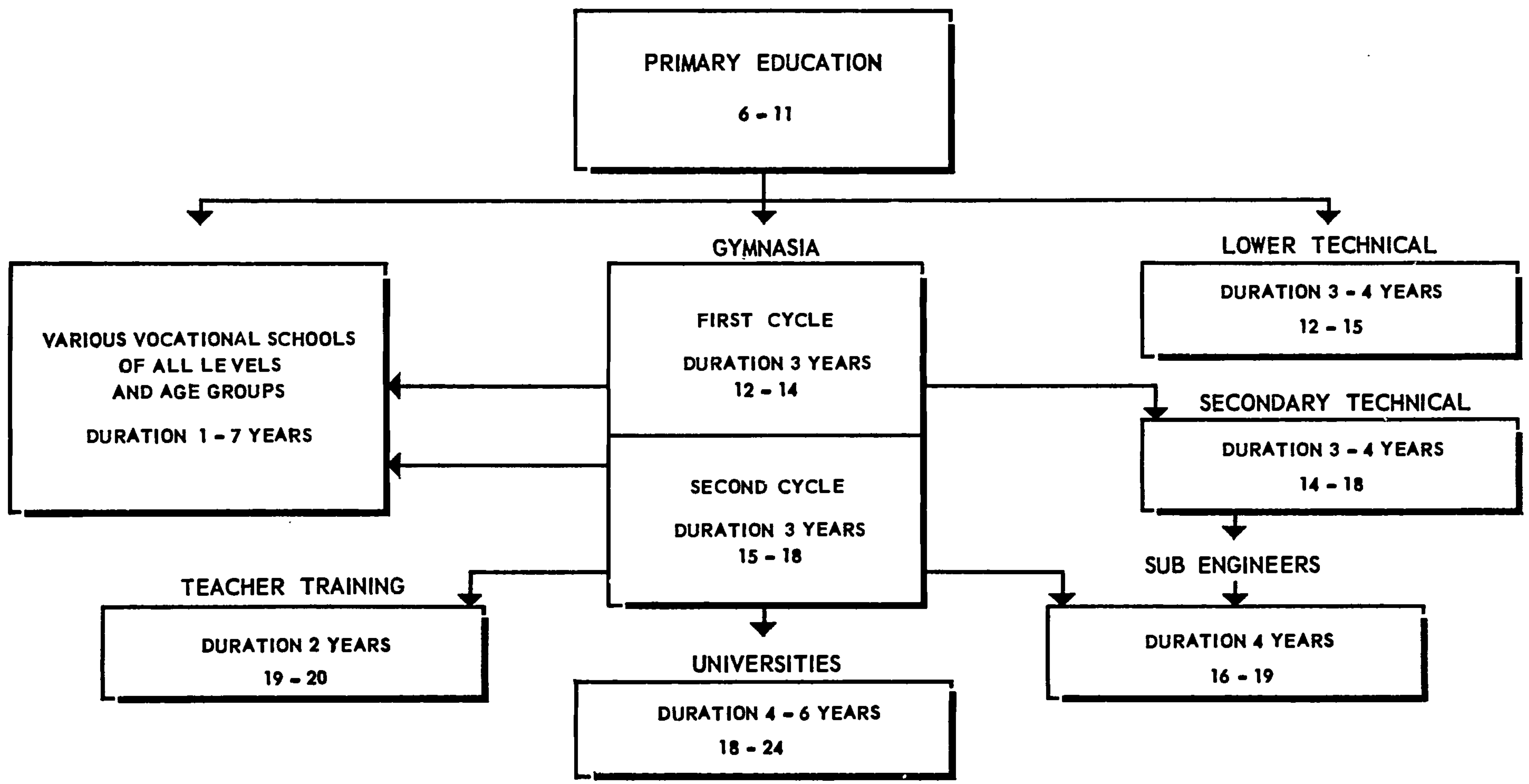

TABLE 20. TOTAL NUMBER OF STUDENTS ENROLLED IN THE VARIOUS LEVELS OF EDUCATION IN 1961

In thousands.

Primary schools	896.0
Secondary general	273.4
Technical schools (including School of Sub-Engineers)	32.4
Vocational schools	11.3
Teacher-training colleges	2.6
Universities (and other higher schools)	25.6
Foreign universities	8.8

THE DATA

In general, recent Greek data are adequate for detailed work on educational planning. There are very few historical series but information for the period 1951-1961 is plentiful.

The 1951 Census of Population included a short section which enables the general educational level of the population to be determined at that date. The 1961 Census of Population, of which the results of a 2 per cent sample elaboration became available at the end of 1962, deals with educational qualifications much more fully. The population, economically active and total, is classified by level of education; by educational specialisation, by economic sector and by occupation. Unfortunately the processing of the data does not make possible the cross classification of all these groupings.

In 1961, for the first time, figures are available about those who have received vocational, technical or professional training below university level.

The other major source of information about educational stock is the report of the Greek team participating in the Mediterranean Regional Project. These data were collected from special sample surveys in the various branches of economic activity. The approach was very different from the census, where the information is obtained from households. In this work the analysis of manpower stock uses occupational function rather than strict educational qualifications, but for many occupations the correlation between the two is high, especially since there are rigid legal educational requirements for many occupations in Greece. It provides a useful check on the census data, where there was some misinterpretation of the questionnaire.

Information about educational flows has been available in considerable detail since 1954, when the Ministry of Education began publishing annual statistical reports on general education. There is little information about numbers of students, course lengths or hours of study for the technical, professional and vocational schools. The figures used in this analysis are based on a study by P. Papadakis in connection with the Mediterranean Regional Project.

There is a serious deficiency in the data concerning the number of Greek students abroad. The sole figure available refers to the total number of Greek students in receipt of official foreign exchange allowances in foreign universities in 1961. As can be seen from Table 20 these students abroad comprise a substantial proportion of the total. Many of them do not return to Greece at the end of their studies.

The number of emigrants by occupation is published annually. For the original conversion of these into educational categories, I am indebted to J. Blum. The figures used here, however, have been modified in accordance with the very much increased rate of emigration in 1961 and 1962. In view of Greece's association with the European Economic Community this increased rate of outflow is expected to continue for some years.

The symbols used in the following exercises are identical with those of Part I except for the following modifications:

i) The addition of a prime (e.g. $N^{3\prime}$) to a stock figure means that it includes people with that level of education in the total population and not just the active labour force;

ii) The following symbols not previously defined have been used in some of the exercises in this paper:

N^1 — Number with a diploma of secondary technical or vocational education;

N^p — Number of primary school teachers (the superscripts 'p' and 'l' have been attached to all other relevant variables);

e — Number of emigrants in a six-year period;

λ' — Proportion of total population who die in a six-year period;

α — Labour force participation rate of recent graduates;

γ — Number of secondary-school graduates who have entered higher education considered as a proportion of the number of students in higher education at the end of the period (year t).

δ — Number of graduates of a particular level of education who graduate in a six-year period, expressed as a proportion of students of the same level at the end of the previous period ($t-1$).

σ — Number of university students who drop out in a six-year period expressed as a proportion of the university population at the end of the previous period. There is a simple arithmetical relationship between δ^3, γ^3 and σ^3 which has been worked out by J. Blum in his paper on Turkey.

Note. There is some strangeness and certainly some arbitrariness in defining σ and δ as proportions of student numbers in a previous period, but if expansion takes place smoothly it is not of great consequence. Strictly speaking both should be proportions of students in both the (t) and ($t-1$) periods but this would unnecessarily complicate their computation.

Some Remarks on the Calculation of the Coefficients

For most of the coefficients three values are possible. They can be calculated from the situation existing in a specific country in the base period. In this case, apart from possible temporary changes in the course of the transition to the balanced growth path, they will have the same value throughout the period.

Secondly, they can be given average or normal values which would be obtained from cross-sectional or time-series analysis from a number of countries. Thus abnormal values in a particular country in a particular period would not confuse the results. These values can in fact reveal if there are abnormalities in the country studied. Clearly this method has most application in the case of the strategic ν coefficient.

Thirdly, for many of the variables, target or forecast values can be used. Thus, for example π may be very low in the base period but government policy may be explicitly aimed at improving the pupil/teacher ratio. Thus there would be a strong case for using the target values rather than the existing values. It is

TABLE 21. VALUES OF THE VARIABLES AND COEFFICIENTS USED IN THE FOLLOWING EXERCISES

A. VARIABLES: VALUES IN THE BASE PERIOD (1961)

All values in thousands except where otherwise stated.

SYMBOL	VALUE (ACTIVE LABOUR FORCE)	VALUE OF PRIMED SYMBOL$^{3\prime}$ (N etc.) ENTIRE POPULATION
N^3	74.3	87.3
N^p	25.3	32.0
N^2	289.3	487.9
N^1	59.2	62.0
n^3	37.1 (28.3 in Greece, 8.8 abroad)	37.1
n^p	2.6	2.6
n^2	273.4	273.4
n^1	42.6	42.6

SYMBOL	VALUE IN PERIOD 1955-1961	FORECAST VALUE BASED ON 1961-62 ONLY
e^3	2.9	4.9
e^p	0.6	0.7
e^2	11.9	23.8
e^1	1.3	2.6

G.D.P. = ν = 3,101 (Millions in U.S. dollars at 1961 prices).
Population = π = 8,388.6 (thousands).

B. COEFFICIENTS: VALUES IN BASE PERIOD

$\nu^p = 0.025$ $\nu^{p\prime} = 0.021$
$\nu^3 = 0.0103$ $\nu^{3\prime} = 0.008$
$\nu^2 = 0.157$ $\nu^{2\prime} = 0.093$
$\nu^1 = 0.020$ $\nu^{1\prime} = 0.019$

	ACTUAL VALUE	TARGET OR FORECAST VALUE		ACTUAL VALUE	TARGET OR FORECAST VALUE
π^3	0.028	0.055	α^2	0.59	0.59
π^p	0.077*	0.050	α^1	0.95	0.95
π^2	0.030	0.035	γ^p	2.7	2.7
π^1	0.023	0.040	δ^p	2.5	2.5
λ^3	0.05	0.05	σ^p	0.4	0.4
λ^p	0.05	0.05	γ^3	1.3	1.3
λ^2	0.04	0.06 and (0.05)**	δ^3	1.4	1.4
λ^1	0.03	0.04	σ^3	0.3	0.3
α^3	0.85	0.85	δ^1	0.8***	0.8
α^p	0.79	0.79	δ^1	0.572	0.572

* This very high figure includes a considerable number of part-time teaching staff.
** 0.05 in exercises (1) and (2). Explanation in text.
*** Despite very high drop-out this figure is rather high because of the inclusion of a large number of vocational schools, with courses sometimes as short as one year. The figure for δ^1, if technical schools only are included, would be extremely low at around 0.15.

possible even to have forecast ν values if detailed manpower forecasting has been undertaken. The application of the model would then become an exercise in deciding how the required manpower targets can conveniently be reached.

To these a fourth technique may be added, namely the systematic change in the values of some of the coefficients through time. This may be necessary, for example, if changes in the rate of outflow of educated manpower are envisaged, thus changing the age structure of this section of the population.

In the table below the values of the coefficients actually existing in Greece are shown, along with target values of some of the coefficients estimated in the light of current social trends and government aims. It will be seen that very low values of λ have been observed and forecast. The low observed values are due to the very low average age of the educated population in 1961 and, with the rapid growth projected, this will continue. Only λ^2 will rise somewhat in the third exercise because of the slower rate of growth of this type of manpower. λ^1 has also been allowed to rise slightly because of its extraordinarily low value in 1961. As shown in the explanation of the symbols, a distinction should be made between λ and λ'. However, the actual values are so close to each other that this seemed an unnecessary complication at this stage.

SOME PRELIMINARY EXERCISES

Three very simple exercises with the model have been carried out in this paper to test some of its implications. These are: (1) a very slightly adapted version of the simplest model; (2) an exercise using the form of the demand function that takes some account of productivity growth, but allows N^2 and N^3 to grow at the same rate; (3) the introduction of more than two types of manpower with different rates of growth.

Exercise One: The Simple Model

In working out even a very simple exercise it seemed necessary to take account of the factors represented by the coefficients α, δ, σ and γ. This allows the definitional equations to represent more closely the actual situation in Greece. Thus the simplest version can be written:

(1) $N_t^2 = \nu^2 v_t$

(2) $N_t^2 = (1 - \lambda^2)N_{t-1}^2 + m_t^2$

(3) $m_t^2 = \alpha^2(\delta^2 n_{t-1}^2 - \gamma^3 n_t^3 + \sigma^3 n_{t-1}^3)$

(4) $m_t^3 = \alpha^3 \delta^3 n_{t-1}^3$

(5) $N_t^3 = (1 - \lambda^3)N_{t-1}^3 + m_t^3$

(6) $N_t^3 = \nu^3 v_t + \pi^2 n_t^2 + \pi^3 n_t^3$

The equations can be solved on the basis of existing ν^2 and ν^3 values to establish the initial values of the variables necessary for the 6 per cent per annum rate of growth of G.D.P.[1] at which the Greek government is aiming. The resulting initial conditions and the values for the first few time periods are shown in Table 22. For convenience the observed values for 1961 have also been written down.

Clearly these results must cause some misgivings, both because of the great discrepancy between the actual and desired number of secondary school students in the base period and because the maintenance of the 6 per cent per

1. Gross Domestic Product has been used in these exercises.

TABLE 22. ACTUAL VALUES OF THE MANPOWER AND EDUCATIONAL VARIABLES IN THE BASE PERIOD, REQUIRED VALUES FOR BALANCED EQUILIBRIUM GROWTH OF 6 PER CENT PER ANNUM (BASIC MODEL) AND FORECASTS OF SIZE OF POPULATION AGED 12-17

VARIABLES	ACTUAL VALUES $t = 0$	EQUILIBRIUM VALUES $t = 0$	$t = 1$	$t = 2$	$t = 3$
N^2	289.3	289.3	410.8	583.3	828.3
N^3	74.3	84.1	119.4	169.5	240.7
n^2	273.4	509.9	724.1	1,028.2	1,460.0
n^3	37.1	35.7	50.7	72.0	102.2
m^2	n. a.	95.8	136.0	193.1	274.2
m^3	n. a.	27.7	39.3	55.8	79.2
$P_{12\text{-}17}$	763.6	763.6	840.6	895.3	914.0

annum growth rate would require more second-level students by $t = 2$ than population of the appropriate age group. In fact a progressive raising of α^2 and δ^2 would tend to modify this result but not sufficiently to postpone for very long the physical population limit.

A possible conclusion would be that the 6 per cent rate of growth cannot be maintained. It is an interesting possibility because it might offer a clue as to why rich countries grow more slowly than poor countries. There is however something extremely pessimistic in assuming that once G.D.P. in Greece has doubled its present low level it can grow further only at a rate permitted by the number of people of secondary-school age. This exercise places considerable reliance on an unproven constancy of the values of v in an individual country at a specific point in time.

EXERCISE TWO: A MORE GENERAL AGGREGATE DEMAND FUNCTION

In this exercise the form of equations (1) and (6) were modified as suggested in Part I, " Generalized Demand Function. " Thus they now read:

$$(1)\quad N_t^2 = v^{20} v_t^{v^{21}} \left(\frac{v_t}{a_t}\right)^{v^{22}}$$

$$(6)\quad N_t^3 = v^{30} v_t^{v^{31}} \left(\frac{v_t}{a_t}\right)^{v^{32}} + \pi^2 n_t^2 + \pi^3 n_t^3$$

A cross-sectional analysis of several countries carried out by the Netherlands Economic Institute suggests that equation (6) should be:

$$(6)\quad N_t^3 = 0.01062\, v^{1.22} \left(\frac{v_t}{a_t}\right)^{0.28} + \pi^2 n_t^2 + \pi^3 n_t^3$$

There is insufficient statistical information available to compute values of the coefficients for equation (1) but since in general it appears that countries have about five times as many second-level people in the labour force as third-level, as a first approximation it may be deduced that:

$$(1)\quad N_t^2 = 0.0531\, v_t^{1.22} \left(\frac{v_t}{a_t}\right)^{-0.28}$$

The theoretical implications of the negative exponent attached to per capita income are discussed in Part I. In practice it means that a rapid rate of productivity growth reduces the need for educated manpower (as well as other types).

Thus if v is growing rapidly and the population slowly, as in Greece, the demands placed on the educational system might be expected to be modified. In Greece it is planned that:

$$v_t = v_0(1.42^t)$$

and forecast that:

$$a_t = a_0(1.043^t)$$

The result of these two factors when substituted into equations (1) and (6) is, however, that the required rate of growth of N^2 and N^3 is only slightly reduced. For example:

$$N_t^2 = N_0^2(1{,}4095^t)$$

One conclusion must be that unless the disparity between growth of v and growth of population is very large indeed, the values of the exponents as found in this regression equation will not appreciably modify in a downward direction the required growth rates of the educational variables.

However, another feature of this regression equation is that it allows " normal " values of N^3 (and more tentatively N^2) to be calculated. It suggests that at the present level of G.D.P. and productivity in Greece there is a considerable surplus of N^2 and N^3 manpower compared with what might be expected from the experience of many other countries. The comparison is as follows:

	ACTUAL	REQUIRED
N^3	64.2[1]	32.3[1]
N^2	289.3	161.6

This might suggest either that manpower is not being used efficiently, or that education in Greece is not so appropriate for economic growth as in other countries. In either case, educated manpower is not in quantitative terms a barrier to economic growth. It may however indicate that the structure of education needs changing.

On the assumption, therefore, that Greece should aim at " normal " values of the strategic coefficients an exercise was worked using the values of v and the rates of growth of v and a indicated above. The other equations were unchanged. The results of this calculation and the values of the variables for the first few time periods are given in Table 23.

TABLE 23. EQUILIBRIUM GROWTH PATH ASSUMING 6 PER CENT PER ANNUM OF GROWTH OF G.D.P. AND 0.7 PER CENT PER ANNUM RISE IN POPULATION. USING NORMAL VALUES FOR v^2 AND v^3 FOUND FROM OTHER COUNTRIES

MODEL BASED ON GENERALISED DEMAND FUNCTION

	ACTUAL VALUE	EQUILIBRIUM GROWTH PATH			
	$t = 0$	$t = 0$	$t = 1$	$t = 2$	$t = 3$
N^3	74.3	42.9	60.4	85.2	120.1
N^2	289.3	161.6	227.8	321.1	452.6
n^3	37.1	17.8	25.1	35.4	49.9
n^2	273.4	272.8	384.5	542.0	763.9
m^3	n. a.	52.7	74.3	104.7	147.6
m^2	n. a.	14.0	19.7	27.8	39.2

1. Excluding all teachers.

Clearly, if the " normal " values of the coefficients may be validly applied to Greece, there seems to be a very large surplus of manpower with general education. However, the above result is not really v ry much more encouraging than those found from the formulae based exclusively on Greek experience. On the one hand this technique has merely postponed until $t = 4$ the time when n^2 hits the absolute population barrier; on the other hand, for any realistic planning purposes, the very low values which were obtained by the model for n^3, third level enrolment, in the early time periods, present many problems. It is unreasonable to expect n^3 to fall from one period to a subsequent one. The major policy conclusion might be that in quantitative terms N^3 manpower will not provide a bottleneck in the foreseeable future. But if the productivity of the group with higher education is to increase, it is necessary that these students receive an education more suited to the needs of the economy. In this, the approach would agree in its conclusions with other more detailed studies of the Greek educational system.

Exercise Three: Several Types of Manpower Stock

In this exercise the model was elaborated to correspond slightly more closely to the educational system actually existing in Greece. Four new factors were introduced. First, primary school teachers who were ignored in the first two exercises have been included. Although it is usual in Greek statistics to call the education they receive " higher education " it seemed preferable to distinguish them from the rest of N^3 because in most countries they are not considered third-level manpower, and even in Greece they receive only two years' post-secondary education. Furthermore in more elaborate versions of the model it would be desirable to relate their growth in some way to the population of primary-school age. The second new feature is the explicit recognition of the existence of secondary-technical education which, as explained on page 80, is new but growing rapidly in importance. Thirdly, the value of ν^2 has been assumed to decline with time. This is partly a device to prevent n^2 hitting the population barrier but it also accords with empirical evidence in Greece between 1951 and 1961. Fourthly, emigration has been recognised as a drain on the educational system. The number of emigrants is assumed to be constant over time.

An initial comment needs to be made on the use in the exercise of figures of total population (in each educational category) rather than economically active population. In previous exercises it has been assumed that the ν coefficients reflect the relationship between economically active populations and G.D.P. This necessitates the use of the α coefficients (labour force participation ratios) since not all students will enter the active population. In this exercise different ν values have been calculated (see Table 22 relating total population to G.D.P.). There are two reasons for this:

1. Any usefulness of the present paper is that of exposition, and the principle of the use of the α coefficient has been established in other chapters. It has been used in the previous two exercises of this paper. In this exercise there were grounds for simplifying as much as possible an already elaborate exercise.
2. If " α " remains constant the numerical results will be the same. The absence of " α " when working with total population figures is a weakness only if changes in labour force participation rates are required as a possible solution to transition problems.

The main feature of this exercise, as stated above, is the attempt to incorporate several different varieties of manpower and to permit a different rate of growth for manpower with secondary-general education from that of the rest of the system. It is hoped that this can be considered a first step towards considerably greater proliferation of types of manpower stock, each growing at a rate appropriate to its function in the economy. In this exercise four types of manpower have been distinguished, those with higher education, those with secondary-general education, the primary-school teachers and those with secondary-technical education. This is an educational rather than an economic classification and was considered necessary because the education experience of the four types is so vastly different. In addition, emigration has been included as a drain on the educational system.

The system of equations describing this exercise is as follows:

(1) $N_t^1 = \nu^1 v_t$

(2) $N_t^2 = \nu_t^2 v_t$

(3) $N_t^p = \nu^p v_t$

(4) $N_t^3 = \nu^3 v_t^t + \pi^1 n_t^1 + \pi^2 n_t^2 + \pi^p n_t^p + \pi^3 n_t^3$

(5) $N_t^1 = (1 - \lambda^1) N_{t-1}^1 + m_t^1$

(6) $N_t^2 = (1 - \lambda^2) N_{t-1}^2 + m_t^2$

(7) $N_t^p = (1 - \lambda^p) N_{t-1}^p + m_t^p$

(8) $N_t^3 = (1 - \lambda^3) N_{t-1}^3 + m_t^3$

(9) $e_t^l \ + m_t^1 = \delta^1 n_{t-1}^1$

(10) $e_t^2 \ + m_t^2 = \delta^2 n_{t-1}^2 - \gamma^p n_t^p - \gamma^3 n_t^3 + \sigma^p n_{t-1}^p + \sigma^3 n_{t-1}^3$

(11) $e_t^p \ + m_t^p = \delta^p n_{t-1}^p$

(12) $e_t^3 \ + m_t^3 = \delta^3 n_{t-1}^3$

In this system v is required to grow at 6 per cent per annum or 42 per cent per time period. At the same time ν^2 is assumed to decline by 20 per cent per time period which accords with the decline in its observed value between 1951 and 1961 and also with likely developments if current manpower plans based on detailed occupational analyses are realised. Thus the system is subject to two rates of growth, those variables which are directly dependent on the value of v and those which depend on ν^2 and v. In addition e^1, e^2, e^p and e^3, are assumed to be constant, implying that emigration is determined by unknown exogenous forces. This will mean a constant drain on the educational system with the same values of the variables in each time period.

The method of solution used in this exercise is to solve the system for the two rates of growth indicated above in a manner exactly similar to that outlined in Part I under the heading " Generalisation of Demand Functions. " Another detailed account of the technique is given by L. Emmerij in " Changing Coefficients " and " More Types of Education " in his paper on Spain. The constant drain caused by emigration was estimated separately and could then be added to each variable in each time period. The way in which the calculations were made is indicated briefly in the following paragraphs.

Although the method adopted here is exactly similar to that of the two preceding papers a slightly different notation has been used. Neglecting for a moment emigration, the system of equations above was set down but the constant " e " in equations (9), (10), (11), (12) was omitted. Thus each of the variables in the system can be split into two components corresponding to the two different rates of growth. The first is that part of each variable which is directly related to v (G.D.P.). This is denoted by the extra superscript A. The second component is that whose rate of growth is modified by the declining value of the coefficient v^2. This is denoted by the superscript B. Thus each variable of the system can be written in the form:

$$z_t = {}^{A}z_t + {}^{B}z_t$$

The rate of growth of the first component can be expressed thus:

Since: $v_t = v_0\omega^t$

therefore: ${}^{A}z_t = {}^{A}z_0\omega^t$

The second is slightly more complicated in that it is the resultant of two growth rates:

(a) $v_t = v_0\omega^t$

(b) $v_t^2 = v_0^2\phi^t$

Since from equation (1) of the general system:

$$N_t^2 = v_t^2 v_t$$

it follows that for this component:

$${}^{B}z_t = v_0^2 v_0(\phi\omega)^t$$

The values of ω and ϕ which are implicit in current Greek economic plans are:

$\omega = 1.42$

$\phi = 0.8$

Thus the value of each variable in any time period can be found from the expression:

$$z_t = {}^{A}z_0(1.42^t) + {}^{B}z_0(1.136^t)$$

This expression can now be substituted into the model on page 89 and two sets of 12 equations precisely similar in form to the original model can be derived. In one of these sets all variables have the superscript A; in the other all have the superscript B. These two sets of equations can be solved independently in the usual way to arrive at the equilibrium base period values of the variables. These are shown below:

"A" COMPONENT		"B" COMPONENT		TOTAL VALUE OF VARIABLES	
${}^{A}N^1$	62.0	${}^{B}N^1$	0	N_0^1	62.0
${}^{A}N^2$	0	${}^{B}N^2$	487.9	N_0^2	487.9
${}^{A}N^p$	32.0	${}^{B}N^p$	0	N_0^p	32.0
${}^{A}N^3$	85.3	${}^{B}N^3$	6.0	N_0^3	91.3
${}^{A}n^1$	35.6	${}^{B}n^1$	0	n_0^1	35.6
${}^{A}n^2$	125.9	${}^{B}n^2$	169.4	n_0^2	295.3
${}^{A}n^p$	6.0	${}^{B}n^p$	0	n_0	6.0
${}^{A}n^3$	30.8	${}^{B}n^3$	0.9	n_0^3	31.7
${}^{A}m^1$	20.1	${}^{B}m^1$	0	m_0^1	20.1
${}^{A}m^2$	0	${}^{B}m^2$	84.4	m_0^2	84.4
${}^{A}m^p$	10.6	${}^{B}m^p$	0	m_0^p	10.6
${}^{A}m^3$	28.3	${}^{B}m^3$	1.0	m_0^3	29.3

The values of the variables for any subsequent time period can be calculated from the formula in the previous paragraph, i.e.:

$z_t = {}^{A}z_0\omega^t + {}^{B}z_0(\phi\omega)^t$

In order to estimate the drain on the educational system caused by emigration, the system of equations on page 89 was solved in a precisely similar way only this time " e " was included in equations (9), (10), (11), (12). Since the value of " e " is known (see Table 21) no more unknowns have been introduced and the system is still determinate. It is thus possible to compute a base period value for all the variables which would allow for the desired rate of emigration during the first time period. The equilibrium values of the variables without emigration were subtracted from these re-calculated values to find the drain on the educational system caused by emigration. This is shown below.

	VALUES OF VARIABLES WITH EMIGRATION	VALUES OF VARIABLES WITHOUT EMIGRATION (COL. 3 OF PREVIOUS TABLE)	DRAIN ON EDUCATIONAL SYSTEM CAUSED BY EMIGRATION
N_e^1	62.0	62.0	0
N_e^2	487.9	487.9	0
N_e^p	32.0	32.0	0
N_e^3	92.3	91.3	1.0
n_e^2	37.2	35.6	1.6
n_e^1	320.3	295.3	25.2
n^p	6.2	6.0	0.2
n_e^3	34.0	31.7	2.3
m_e^1	20.1	20.1	0
m_e^2	84.4	84.4	0
m_e^p	10.6	10.6	0
m_e^3	29.4	29.3	0.1

It is convenient to call the increase in each variable made necessary by emigration:

(i.e. column 3 above) ${}^{C}z_0$.

Thus the value of each of the variables in any time period can be defined by the expression:

$z_t = {}^{A}z_0\omega^t + {}^{B}z_0(\phi\omega)^t + {}^{C}z_0$

The results of this calculation are shown in Table 24 below. The actual observed values of the variables in the base period are also included for convenience.

SOME COMMENTS ON TRANSITION PROBLEMS

It is a straightforward arithmetical exercise to calculate values of the variables and coefficients for the period $t = 1$ which would transform the existing situation in $t = 0$ to the equilibrium growth path by $t = 2$. The technique has been explained in Part I. The result is that in the period $t = 1$ either the level of national income will be different from that planned or the values of some or all of the coefficients will be temporarily modified. A sharp demarcation should be made between those coefficients which realistically can be temporarily changed and those which cannot. For example, it seems reasonable to suppose a temporary deterioration in some pupil/teacher ratios but not reasonable to assume at the same time an improvement in wastage rates

TABLE 24. THE EQUILIBRIUM GROWTH PATH OF THE EDUCATIONAL SYSTEM WITH G.D.P. GROWING AT 6 PER CENT PER ANNUM AND ν^2 DECLINING BY 20 PER CENT IN 6 YEARS AND WITH A CONSTANT RATE OF EMIGRATION

	$t = 0$ ACTUAL	$t = 0$ REQD.	$t = 1$	$t = 2$	$t = 3$
N^1	62.0	62.0	88.0	125.0	177.5
N^2	487.9	487.9	554.3	629.7	715.3
N^p	32.0	32.0	45.44	64.5	91.6
N^3	87.3	92.3	128.9	180.7	254.0
n^1	42.6	37.2	52.3	73.5	103.7
n^2	273.4	320.3	396.1	497.2	633.3
n^p	2.6	6.2	8.8	12.3	17.4
n^3	37.1	34.0	47.1	65.7	91.9
m^1	21.2	20.1	28.5	40.5	57.5
m^2	83.6	84.4	95.9	108.9	123.7
m^p	7.8	10.6	15.0	21.4	30.3
m^3	21.1	29.4	41.3	58.3	82.3

(approximately the inverse of the δ coefficients) when in the subsequent equilibrium period pupil/teacher ratios will improve and wastage rates deteriorate. Similarly any change proposed in λ, perhaps because retirement ages are raised as a temporary expedient, might have to be associated with an appropriate change in α (labour force participation rate). It is certainly mathematically true that there are a very large number of ways in which equilibrium growth can be reached between $t = 0$ and $t = 2$, but in practice, many of these will have to be discarded.

A further constraint is provided by the values of the variables themselves. The values of many variables in $t = 1$ are predetermined by the existing values of other variables in $t = 0$. This may well mean that a violent change in their value is necessary to achieve the equilibrium situation in $t = 2$. It seems reasonable to introduce the constraint that in no period should the value of a variable fall below its value in a previous time period. Often this will mean lengthening the period of transition to three or more time periods.

Finally the use of six-year time periods may unduly lengthen the apparent necessary period of transition in educational specialisations in which course lengths are very much less than six years. In the case of n^p for example with its two year period of study, very large changes could be made in six years.

CONCLUDING REMARKS

The third exercise of this paper, despite its greater elaboration, does not sufficiently approach an actual situation to be immediately applicable for detailed planning purposes. But it does indicate a technique by which real plans might be approached. One way of making it more realistic would be to relate the rate of growth of primary-school teachers to the growth of primary-school population as well as G.N.P. In the present exercise a reduction of about 70 per cent in class sizes by $t = 3$ is implied. This is unlikely to occur. Again, despite the severe limitations on the rate of growth of N^2, secondary enrolment n^2, will have reached the physical population limit by $t = 4$ and any practical limit very much earlier. Whether this limitation can be overcome or whether it will instead provide a brake on the rate of growth needs to be explored.

Another useful elaboration is to distinguish between technical and non-technical higher education. There is a very large amount of non-technical higher education in Greece not related to the needs of the economy, resulting in an over-expanded civil service and over-qualification in many clerical ranks. The surpluses indicated in the second exercise certainly exist.

If a desirable educational profile of each occupation or each economic sector were known it would be possible to treat in a manner similar to that of *Exercise Three* a very large number of different manpower stocks each growing at a different rate. These could then be amalgamated and the total effects on the educational system estimated. If these requirements then exceeded the population limit, or were not otherwise feasible, the manpower forecast could be revised in the light of the knowledge that educated manpower of all or some types was going to be more expensive to train and to employ. The operation could be repeated until a suitable solution was found.

It may relevantly be asked in what way this would differ from the *ad hoc* methods of manpower and educational planning now in fairly general use. The answer is that in principle it does not. The advantage of the techniques used here is that they are reasonably concise and they are systematic, they collect all the variables under consideration into one set of equations. But this advantage is a sizeable one. The only caution note to be sounded is that no existing system of equations takes into account all the factors that may need consideration. But this is an argument for refining the system, not abandoning it.

A significant feature brought out by all the exercises is the important pivotal position of secondary-general education. Even when in themselves these manpower stocks are not required to grow rapidly, the expansion of all other parts of the system depend to some extent on growth in this branch. It is therefore quite explicit that an expansion in any branch of education needs to be accompanied by an appropriate expansion at the secondary level. This can all be deduced by non-mathematical logic. The equations give it quantitative expression.

The working out of the model makes explicit one general question which is implicit in much long term macro-educational planning which is simply, "Can it be done? Is it realistic?" It is usually said that global educational planning must be very long term. It is difficult, however, to forecast the independent variables and coefficients for very long periods ahead and the dependent variables are very sensitive to values of these coefficients. For example, an exercise was performed very similar to *Exercise Three* in which the rate of growth of v was assumed to be 5 per cent per annum rather than 6 per cent. If this were taken as a growth target it would imply that n^2 could be 25 per cent smaller in the base period and about 40 per cent smaller by 1980. In numbers of school-children this represents 80,000 and 250,000 respectively. It may well be that the ambitious growth target of 6 per cent per annum which the Greek Government has set itself will not be attained because of the impossibility of expanding the educational system rapidly enough.

It can be asserted, however, that all methods of long-term educational planning are subject to similar weaknesses. Clearly, models of this type must be refined considerably before they can be used as more than broad checks on other more piecemeal approaches. However, it does seem that this type of mathematical model, albeit considerably refined, will provide clues to the understanding of the mechanics of educational systems, and hence eventually to more informed policy making in this field.

Part V

AN APPRAISAL OF THE MODEL AND THE RESULTS OF ITS APPLICATION

by

Jan TINBERGEN and H. C. BOS

SCOPE OF THE MODEL

SOME MISUNDERSTANDINGS CONCERNING THE SCOPE OF THE MODEL

Let us first consider some of the misunderstandings about the scope of the model which became apparent during the discussions on its defects and merits. No model, however realistic or refined, will ever fully cover all the circumstances, be able to answer all the questions, or solve all the problems which might arise. The model and variants discussed in this report make no pretence of being able to do more than help solve some quantitative, long-term education-planning problems. It does not cover the qualitative aspects of education except insofar as these influence some of the coefficients used (teacher/pupil ratio, drop-out ratios and so on). Neither does it deal with the short-term policies needed to adapt the manpower stock in the last phase of training to the precise requirements of a large number of specified occupations. Its main objective is to guide major decisions concerning secondary and third-level education, i.e. the numbers of pupils, students, teachers and professors, and the schools required. These are long-term decisions, not in the sense that the action to which they refer will take place in the distant future, but that the educational processes to which they give rise will last a long time. Action will, in fact, be taken in the immediate future, any of the " paths " calculated being followed for only the first few years, after which a periodical revision—characteristic of any type of planning—will have to take place. As this will probably change the remainder of the path, it does not matter if the later years of the paths are unrealistic. Such a result, on the contrary, would be a warning that further study is needed. Even so, decisions of this type, which are meant to help shape the future stock of manpower, must be based on as true a picture of the future as is obtainable from existing information.

The models are essentially planning models, representing the needs for manpower and education. They are not an attempt at forecasting what might, but what should happen. Up to now they have made no pretence of dealing with the possible feed-back of education requirements at the general level of investment or with more far-reaching questions such as the need for social reform and change in élites, i.e. ruling groups and their particular type of training needed.

APPRAISAL OF THE MODEL AS AN INSTRUMENT OF RESEARCH

As we pointed out in Part I, when considering the relationship between education and economic growth, we must bear in mind that the education process:

i) provides new supplies of graduates to increase the existing stocks;
ii) consists of a series of inter-related steps;
iii) consumes part of the educated people it produces;
iv) may be accelerated by the importing of educated manpower.

All the versions presented are essentially based on these characteristics of the education process.

The model permits a variety of alternative versions, some of which have already been discussed. This flexibility is well illustrated by the contributions of Messrs. Emmerij, Blum and Williams who have added new variations which, to some extent, they have adapted to the conditions in the countries concerned. A more perfected method of dealing with fall-out was presented for each country: an example in which the coefficients change over time was presented for Spain; emigration, students abroad and surpluses of certain types of manpower were introduced for Greece, and a " marginal coefficients " version for planning future flows was added for Spain.

Macro-models may often present a useful first approximation, and the "exercises" carried out with the model, as well as some of its generalizations, allowed us in certain cases to observe the change in the results introduced by some of the refinements.

The introduction of what we have called the generalized demand function for manpower is more far-reaching than is implied if considered as an alternative only. Our cross-section formula for the relationship between manpower stock (of 3rd level) and two economic variables may be regarded as a " norm " in which random deviations for any single country have been eliminated. This norm enables us to appraise the existing stock of manpower and to estimate the degree of adequacy or of inadequacy, rather than taking it for granted, as the simplest version of the model does. For a country such as Greece, with a probable surplus in some types of manpower, this means an additional element of information.

We consider that up to now—and we are still very much in the experimental stage—such features as are common to several results may be regarded as evidence concerning the educational situation in the country concerned. In the next section we shall give some examples of this interpretation of our findings.

While refinements are possible, as has been shown in the preceding chapters, they do pose some additional problems too. Thus we have seen how the introduction of more economic sectors or more types of education raises the number of equations and of unknowns and can easily lead to very complicated systems. To be sure this stresses the necessity of using mathematics, because " common sense " calculations would then become altogether impossible. As long as the equations can be kept linear, there is not great difficulty from the mathematical side in handling them. If a considerable number of the equations are non-linear, even the mathematical treatment may encounter difficulties, although the complications they imply can still be dealt with more easily mathematically than otherwise.

Refinements also pose another problem, as we have seen: they require greater knowledge in the form of more coefficients, and it will not always be easy to measure their values. All the types of coefficients must be known either sectorwise or for each type of education considered, and there is a greater possibility that numerous substitution phenomena appear of which we know even less.

As in so many walks of life, there is an optimum solution in the form of a compromise between the degree of precision and the manageability of a model or method. This same compromise is characteristic of some of the best-known propositions of Keynesian economics. Manageability in this context implies not only how the model should be used but also how it should be explained to the policy-makers. This problem of communication is very important in planning.

APPRAISAL OF SOME CONCRETE FINDINGS

The exercises carried out with the model and its alternatives have already provided a range of possibilities far too diversified for us to consider them all here. Let us rather select a few examples from those intended for other explorations by persons interested in particular features of the educational structure.

The first refers simply to the values of the coefficients ν^2, ν^3, π^2 and π^3 and the question of their variation over time. Some values are given in Table 25.

TABLE 25. SOME VALUES FOR THE COEFFICIENTS ν AND π

	ν^2	ν^3	π^2	π^3
Greece	0.157	0.025	0.03	0.05
Spain	0.052**	0.018***	0.05	0.08
Turkey	0.039	0.016	0.03*	0.07*
United States	0.200	0.020	0.04	0.08

* Planned for next 10 years.
** Assumed to fall by 3 per cent per 6 years.
*** Assumed to fall by 14 per cent per 6 years.

Coefficients ν are measures in 1,000 man per million dollars (of about 1960 value, for U.S. 1950). Coefficients π are numbers of teachers per student.

While there is not too much difference among countries between the coefficients ν^3 and π^2, those between ν^2 and π^3 are large. The high coefficients ν^2 for the United States and Greece may indicate the considerable number of secondary-level manpower attending school for cultural rather than economic reasons. They may also, however, partly reflect understaffing for second-level manpower in the Spanish and Turkish economies. The relatively low figure for π^3 in Greece applies to a large number of European countries (and also to India) and represents the phenomenon of over-crowded or under-staffed universities characteristic of these countries. The fact that this phenomenon is not apparent in Spain and Turkey may be due to the relatively low number of students, whereas in the United States it is indicative rather of the ample resources available to universities.

If, as a consequence of a rising standard of living, the educational level of the lower-income countries approaches that of the wealthier countries—whether for economic or cultural reasons—a rise in coefficients must be expected. Saturation must eventually occur, however, as stated by various authors, when all the members of the required standard of a certain age group are included. Here we should keep in mind, however, what was said above, namely that our findings will be used only for the first few years and will then have to be reviewed.

Our second example refers to the numbers of those actually attending institutions of secondary and higher education and to those who, according to the models, should attend if the desired growth rate of the economy is to be attained. Table 26 shows some of the results obtained for these three countries.

TABLE 26. NUMBERS n_0^2 OF STUDENTS AT SECONDARY AND n_0^3 AT THIRD-LEVEL INSTITUTIONS ACTUALLY ATTENDING AND REQUIRED FOR THE ASSUMED RATE OF GROWTH g IN GREECE, SPAIN AND TURKEY

In thousands.

COUNTRY AND LEVEL	g	ACTUAL NUMBERS	REQUIRED NUMBERS: TYPE OF MODEL USED					
			ONE-SECTOR		THREE-SECTOR		GENERAL DEMAND NORM	FALLING COEFFICIENT ν
			" SIMPLE "	" COMPL. "	" SIMPLE "	" COMPL. "		" COMPL. "
Greece n_0^2	6	273	—	510	—	—	273	320
n_0^3	p.a	37*	—	36	—	—	18	34
Spain n_0^2	6 %	571	356	802	364	—	—	601
n_0^3	p.a.	77	95	235	98	—	—	137
Turkey n_0^2	7 %	477	236	400	—	481	—	—
n_0^3	p.a.	62	66	80	—	94	—	—

Note. The " simple " model is the one given in Part I.
The " complicated " model contains the coefficients α, γ, δ and σ as introduced by Mr. Blum.
* Includes students abroad.
The dashes indicate that the relative figures were not calculated in the country reports.

The following conclusions appear to be justified:

a) there are too few third-level students in Spain and Turkey, and possibly too many in Greece;

b) the introduction into the models of coefficients α, γ, δ and σ as proposed by Mr. Blum is very significant; the use of three sectors instead of one considerably alters the results for Turkey; for Greece, the introduction of the international cross-section norm for manpower demand considerably reduces the requirements for second- and third-level students, whereas Mr. Emmerij reduces them by the introduction of falling coefficients.

Other examples, such as the total number of people with second- or third-level education, the number of teachers or of imported manpower, etc. will be left for the reader to analyse.

GENERAL CONCLUSION; FUTURE WORK

Our general conclusion concerning the exercises carried out is that they have led to a considerable amount of concrete and orderly research and to the disentangling of a number of important questions referring to the main problem: the planning of the volume of education for the next few years. They have also led to the formulation of more complicated but manageable models for this planning, and to a large variety of alternatives to the coefficients to be used. While the time available to the three Fellows was not sufficient for them to tackle the many conceivable variants, the selection made has shown the desirability of a larger-scale effort for the more systematic calculation of practical alternatives. For these, all the coefficients and variables found to be relevant should be used, and a sufficient number of sectors and types of education as well as alternative value sets of the coefficients. They should represent alternative improvement programmes for the existing situation, including changes in the length of some of the education processes. Perhaps they could be combined with the results of studies concerning qualitative changes, i.e. in curricula and methods, in some of the processes.

We should repeat, however, the remarks made in Part I concerning the limitations of the models, which are not intended for the short-term planning of the final phases of education. These are characterized by a very large number of education processes directed at obtaining practical skills of which an enormous variety exists in a modern society. For these processes there is less need for a comprehensive treatment such as that aimed at by the models.

A general word of caution should be added. The economics of education is a very new subject the study of which has only just begun. Our empirical knowledge, although increasing rapidly, is still limited. Unlike other, more "typically economic" subjects, the economics of education represents only one of its many aspects. All this means that very broad research programmes must be planned and carried out before we can attain the degree of reliability characterizing older branches of research. Each single project of research, such as the present, must be seen as a modest contribution of an experimental nature.

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