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Three studies attempted to assess the relative effects of two methods of instruction: (1) teaching by a carefully structured sequence of questions (intermediate guidance) and (2) teaching by stating each concept with illustrations and then having the students practice the behavior embodied in these concepts (maximal guidance). One of the studies used fourth grade subjects and a programed presentation of addition and multiplication of integers; the second used sixth grade subjects and a programed presentation of modulus seven arithmetic; the third used eighth grade subjects and a programed presentation of selected topics from vector arithmetic. Within each of three ability subgroups, the subjects were randomly assigned to one of the two treatments. The results of the studies indicated that (1) the high ability subgroup had mean scores significantly greater than those for the middle ability subgroups which in turn had mean scores significantly higher than those of the low ability subgroups, and (2) the significant differences pertaining to treatment effects favored the maximal guidance treatment. It was concluded, therefore, that when mathematics instruction is provided by means of a linear program, a relatively high level of guidance should be provided. (RP)



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Comparison of Two Teaching Techniques in Elementary School Mathematics

Otto C. Bassler

George Peabody College for Teachers

Nashville, Tennessee

July 1968

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Summary

Three studies were carried out to assess the relative effects of two methods of instruction when achievement and transfer were measured by a posttest and a retention test. The methods of instruction were (1) teaching the students by a carefully structured sequence of questions (intermediate guidance) and (2) teaching the students by stating each concept with illustrations and then having the students practice the behavior embodied in these concepts (maximal guidance). One of these studies used fourth grade subjects and a programmed presentation of addition and multiplication of integers, the second used sixth grade subjects and a programmed presentation of modulus seven arithmetic, and the third used eighth grade subjects and a programmed presentation of selected topics from vector arithmetic.

In each study, the subjects consisted of all students in the specified grade at Mitchell-Neilson Elementary School in Murfreesboro, Tennessee. The subjects in each study were categorized into high, intermediate, and low ability subgroups on the basis of scores on standardized mathematics tests. Within each ability subgroup, the subjects were randomly assigned to one of the two treatments.

The programmed instructional units were presented to the subjects in classroom size groups in the form of programmed textbooks. In all of the studies there was a programmed booklet to be completed by each subject on each of eight instructional days. The instructional time varied from day to day and study to study. An average time per day was approximately 30 minutes.

On the class day(s) immediately following instruction, a posttest consisting of three subtests was administered. These subtests were designated achievement, horizontal transfer, that is, the application of the instructional content to novel situations, and vertical transfer, that is, the application of the instructional content to the learning of new and higher level mathematical content. Approximately four weeks after the end of instruction, a retention test was administered. This test was a shortened form of the posttest in the fourth and sixth grade studies and a parallel test in the eighth grade study. The items on each retention test were changed in context from those on the posttest to prevent direct recall.

This testing procedure yielded nine sets of data for each study, that is, three posttest measures, three retention measures and three ratios defined to be a retention measure divided by the corresponding posttest measure. Treatment by ability level analyses of variance using orthogonal comparisons were performed to test the following five comparisons for each set of data:



- 1. The mean of the intermediate guidance treatment group is equal to the mean of the maximal guidance treatment group.
- 2. The mean of the high ability subgroup is equal to the average of the means for the lower ability subgroups.
- 3. The mean of the intermediate ability subgroup is equal to the mean of the low ability subgroup.
- 4. In the high ability subgroup the difference between the means of the intermediate and maximal guidance treatments is equal to the corresponding difference in the lower ability subgroups, that is, the interaction of hypothesis 1 by hypothesis 2.
- 5. In the intermediate ability subgroup the difference between the means of the intermediate and maximal guidance treatments is equal to the corresponding difference in the low ability subgroup, that is, the interaction of hypothesis 1 by hypothesis 3.

The results of these analyses were that on each of the posttest measures a distinct ordinal relation existed between the ability subgroups, that is, the high ability subgroup had mean scores significantly larger than those for the middle ability subgroup which in turn had mean scores significantly higher than the low ability subgroup. This relation also occurred on the three retention test measures with one exception in each study. These exceptions occurred when comparing mean scores of the intermediate ability subgroup with those of the low ability subgroup on retention horizontal transfer in the fourth and eighth grade study and retention vertical transfer in the sixth grade study. These three comparisons yielded no significant differnce in means.

The significant differences pertaining to treatment effects when they existed all favored the maximal guidance treatment. This occurred in the fourth grade study when vertical transfer was measured by the retention test; in the sixth grade study when vertical transfer retention ratios were compared, and in the eighth grade study for the high ability subgroup when posttest vertical transfer and retention test achievement were measured.

On the basis of these results and observing the necessary precautions, it was concluded that when mathematics instruction is provided by means of a linear program, a relatively high level of guidance should be provided.

Introduction

One of the pressing needs of our time is to ascertain teaching methods which will maximize student learning. This problem is made

difficult by the multitude of variables affecting the teaching process, the variety of contents to be taught, and the many interactions of variables which almost certainly exist.

Statement of Problem and Definition of Terms

The present study was designed to test the relative effects of two levels of one of the variables involved in the teaching process, that is, the amount of guidance provided for the learner. The relative effects of intermediate guidance and maximal guidance were assessed by a posttest and a retention test, both of which measured achievement and transfer. A second purpose of this study was the identification of any differential effectiveness the methods may have for children of different ability levels.

The term discovery has been used and misused in so many different ways related to teaching and learning that it is practically meaningless unless it is given a precise operational definition for a particular study. For this reason, the teaching methods to be used in this study are defined in terms of the amount of guidance provided for the student in the learning situation. Guidance, as used here, is the teacher-directed activity provided in the instructional process. Intermediate guidance includes the statement of the problem, student recall of relevant concepts through a series of carefully structured questions, and questions by the teacher designed to lead the student to the desired behavior. An alternative name for this type of teaching is "guided discovery." Maximal guidance includes the statement of the problem, the statement of each intermediate concept with illustrations, and finally exercises which require the students to practice the behaviors embodied in these previously exhibited concepts. This is sometimes called tell-and-practice teaching or expository teaching.

Two types of transfer, designated as horizontal transfer and vertical transfer, were assessed on the posttest and the retention test. Horizontal transfer is the application of the instructional content to either novel situations from the physical world or parallel mathematical content. For example, if the instructional content is designed to teach addition of integers (positive, negative, and zero), these concepts could then be applied to temperature problems, money problems, or score in a two person game. Alternatively, if modulus 7 arithmetic is the instructional content, then horizontal transfer could be modulus 5 arithmetic which has identical properties as the instructional content but different computational facts. Vertical transfer is the application of the instructional content to the learning of new and higher level mathematical concepts. Using either of the contents previously mentioned, one type of vertical transfer would be solution of open sentences of the type a + x = b where a and b are numbers in the system. To assess this type of transfer a certain amount of new



This investigation dealt with students learning selected mathematics topics at the fourth, sixth and eighth grade levels. The mathematics topics were addition and multiplication of integers for the fourth grade study, modulus 7 arithmetic for the sixth grade study, and selected topics from vector arithmetic for the eighth grade study. Each of these three separate studies used the same design to test relative differences between intermediate and maximal guidance teaching methods at three designated ability levels of subjects. They were not exact replications, however, since student population and instructional content varied.

Related Literature

The guided discovery (intermediate guidance) teaching method has a long history dating from the Socratic method of instruction and it is possible to find a great many references to this teaching technique. The big surge toward discovery teaching in mathematics learning occurred with the advent of the changing mathematics curricula classified as the "new math." Thus, curriculum projects such as the University of Illinois Committee on School Mathematics and the Madison Project place great emphasis on instruction which fosters student discovery. One proponent of this view (Taba, 1963) states that one does not truly understand that which is not discovered and one cannot learn that which is not understood.

Kersh (1964), reporting on some general claims made for discovery teaching, states the conjectures that students who learn by discovery (a) develop interest in the task; (b) understand what is learned and are better able to transfer and retain the learned concepts; and (c) learn a strategy for discovering new generalizations. He continues by stating that there is some research evidence in favor of claim (a) but little if any research to substantiate claims (b) and (c). Ausubel (1961), on the other hand, favors reception learning where the entire content of what is to be learned is presented to the learner in final form and the learner only needs to internalize the material for future recall and use. His convictions pertaining to reception learning seem to be based on the assumption that discovery learning is too time consuming. He does admit, however, that student discovery may be an effective method of teaching elementary school students.

Most of the research pertaining to the guided discovery method of teaching has been conducted using subjects at the sixth grade level or above. In a prelude to this study, however, Bassler (1968) found that second grade children can learn using either method but no comparisons of the treatment effects were made. Eruner (1966) also reports a study where four eight-year-old children learned some sophisticated

mathematical concepts by a discovery method. Unfortunately, there have been no comparisons of the two instructional techniques uncovered by this investigator with subjects below the fifth grade level.

Kittrell (1957) is one investigator who looked at the effect of external direction on subjects in the sixth grade. He found that some external direction during learning aids discovery, transfer, and retention of principles for his group of subjects. Another study (Gagne, et al., 1962) investigating the effects of varied amounts of guidance and repetition used seventh grade subjects. The findings of this study indicate that high guidance paired with high repetition produced superior performance when compared with low guidance, low repetition. When the guidance and repetition variables were tested singly, however, no significant differences were found. Worthen (1968), using fifth and sixth grade subjects, found that expository teaching was preferable for immediate recall while guided discovery teaching yielded better results when retention and transfer of heuristics were measured. The general trend of these studies for sixth and seventh grade subjects leads to the conclusion that some guidance in learning produces better performance, but the amount of guidance to be provided is open to debate.

For ninth and tenth grade subjects it appears that guided discovery is an effective method of teaching. This method seems to facilitate transfer and retention of the learned concepts better than a more direct method of teaching (Kersh, 1962; Ray, 1961; Gagne & Brown, 1961). At the college level of instruction, the effects of guided discovery and direct teaching appear to be mixed. The direct teaching appears to be a preferable method of instruction on post-learning tasks, but retention achievement and retention transfer seem to be better when guided discovery teaching techniques are used (Haslerud & Meyers, 1958; Kersh, 1958; Craig, 1956). In fact, Craig's (1956) conclusion seems to apply in general to the research in the field, "the evidence indicates that teachers and experimenters should be liberal with information designed to assist learners in the discovery of principles" (p. 234). The meaning of Craig's phrase "liberal with information" remains a research problem open to investigation.

Objectives

The purpose of this study was to compare the relative effects of two teaching methods, intermediate guidance, and maximal guidance, when achievement, horizontal transfer and vertical transfer were assessed by a posttest and a retention test. The treatment effects were studied across three mathematics ability levels (high, intermediate, and low)



as measured by standardized tests of achievement and aptitude. Based on the assumption that the "guided discovery" teaching method is a means to enhance transfer and retention and the results of the studies reviewed in the related literature section, it is hypothesized that:

- 1. On the posttest, subjects who receive instruction under intermediate guidance will perform better on achievement, horizontal transfer, and vertical transfer tasks than will subjects who receive instruction under maximal guidance.
- 2. On the retention test, subjects who receive instruction under intermediate guidance will perform better on achievement, horizontal transfer, and vertical transfer tasks than will subjects who receive instruction under maximal guidance.
- 3. On the posttest and the retention test, high ability subjects will perform better than middle ability subjects who in turn will perform better than low ability subjects regardless of the instructional treatment.

Each of the above hypotheses was tested for the experimental subjects participating in the fourth, sixth, and eighth grade studies.

Procedures

The general design of the three studies was the same. There were, however, modifications which were necessitated by time considerations or by administrative procedures. The present section will describe the general procedures for the three studies as well as deviations from these procedures for a specific study.

Instructional Materials

Programmed instructional units representative of the two treatments were prepared and presented to the subjects in the form of programmed textbooks. The treatments that were being considered were intermediate guidance and maximal guidance. The programmed unit for the intermediate guidance treatment consisted of a carefully constructed sequence of questions or commands which required the students to perform physical manipulation of objects or "discover" various subconcepts leading to the final task. The only type of information which was given to the subjects in this instructional unit was notation that is commonly used or names of the concepts with which the subjects have been working. A parallel program was constructed for the maximal guidance treatment. The subjects in this program, however, had each concept explained and illustrated for them prior to any exercises or practice work on these



concepts to be completed by the subjects. Each of the programs was designed to consume eight 30 to 40 minute instructional periods.

The two programs, representative of the two treatments, consisted of (1) frames informing the student of the task; (2) instructional frames requiring a response by the student; and (3) exercises requiring a response from the student. All of the instructional frames provided immediate confirmation of response. This was accomplished in the fourth grade study by printing the correct response on the back of the page and in the sixth and eighth grade study the correct response was printed at the top of the next page. Most of the exercises also permitted immediate confirmation of response, but there were several exercises each day without this feature. It was felt that these frames which did not permit response verification would provide a check on the effectiveness of the program as well as providing a means to deter the students from looking at the correct response prior to making their response.

At the fourth grade level the instructional content was the meaning of integers, addition of integers, and multiplication of integers. The development proceeded on an intuitive basis with manipulation of physical objects playing an integral part. The topics for each day of instruction were as follows:

- Day 1 Introduction to integers, writing integers, and interpreting the meaning of integers.
- Day 2 Addition of integers of like sign.
- Day 3 Addition of integers of opposite sign or zero.
- Day 4 Number line and addition of integers using the number line.
- Day 5 Multiplication of a positive integer times another integer.
- Day 6 Multiplication of a negative integer times another integer.
- Day 7 Properties of multiplication of integers.
- Day 8 The distributive property of multiplication over addition.

In the sixth grade study the instructional content was a development of a finite mathematical system in terms of clock arithmetic. The content for each day of instruction was as follows:

- Day 1 Introduction to a clock with seven numbers and clock addition.
- Day 2 Continuation of clock addition and properties of clock addition.
- Day 3 Properties of clock addition and introduction to clock subtraction.



Day 4 - Properties of clock subtraction.

Day 5 - Clock multiplication.

Day 6 - Properties of clock multiplication.

Day 7 - Distributive property of multiplication over addition.

Day 8 - Clock division.

The eighth grade instruction dealt with selected topics from vector arithmetic which was presented in an intuitive but relatively abstract manner. Specifically, the eight instructional days presented the following content:

Day 1 - A review of integers and computations with integers.

Day 2 - Coordinate system and definition of vector.

Day 3 - Length of vector and multiplication of a scalar times a vector.

Day 4 - Vector addition.

Day 5 - Properties of vector addition.

Day 6 - Vector multiplication (dot product).

Day 7 - Properties of vector multiplication.

Day 8 - Applications of vectors, vector addition, and vector multiplication.

The selection of these instructional contents for the three studies was influenced by two considerations. In the first place, it was felt that the content should be novel for the students. This does not imply that the topics to be taught were more difficult than the mathematics curriculum at the specified level, rather, these topics reflected meaningful mathematical ideas that could be presented at a level which is comparable in difficulty to the usual curriculum. The second consideration was to retain a thread of commonality throughout the three studies. That is, each of the three instructional contents was a development of a mathematical system which included a set of elements and one or more defined operations on these elements.

The Posttest and Retention Test

A posttest consisting of three separate measures designated as achievement, horizontal transfer, and vertical transfer was administered to all subjects on the class day(s) immediately following instruction. In the fourth and sixth grade studies there were two days of testing. The achievement test and half of the horizontal transfer test were administered on Day 9 and on Day 10 the second half of the horizontal transfer test and the vertical transfer test were administered. In the eighth grade study there was only one day of testing. On this day

achievement, horizontal transfer, and vertical transfer were assessed. The retention test was a shortened version of the posttest in the fourth and sixth grade studies and parallel to the posttest in the eighth grade scudy. All items on the retention test were changed in context from similar items on the posttest to prevent direct recall, but the content of the items remained the same. The retention test was administered approximately four weeks following the end of instruction.

As has been mentioned previously, the achievement portion of the criterion measure is a direct test of the effects of instruction. In this portion of the test, the arithmetic processes and properties which were taught during instruction were tested. The horizontal transfer subtest consisted of applications of the instructional materials to novel physical situations or to mathematical situations with a slight change in context. The vertical transfer subtest consisted of new and higher level mathematical tasks. In some cases a minimal amount of additional instruction was provided. This occurred when the students were told the meaning of an open sentence and its solution in the fourth grade. In other cases it was a generalization of the instructional content such as the extension of the concepts of two dimensional vectors to three dimensional vectors in the eighth grade.

The tests were prepared in booklet form just as were the instructional materials. The students were not given confirmation of responses on the criterion measures so no answers were printed on the back of the page or the top of the next page.

Pilot Studies

In order to ascertain the effectiveness of the prepared programmed presentations of the instructional content, pilot studies were conducted at the Demonstration School of George Peabody College for Teachers. In the fourth and sixth grade studies one intact class of children constituted the tryout sample. Due to a lack of time, the eighth grade pilot study was not administered in a class situation; rather, it was conducted with two high ability eighth grade students on a one-to-one pupil-observer basis.

The purposes of the pilot studies were to detect any existing flaws in the programmed material, to gain some insight into the feasibility of the content being taught at the designated grade level, to obtain an estimate of the length of time necessary to complete each day's learning materials and to determine, if possible, the adequacy of the criterion measures. In general, these goals were accomplished and modifications based on the results of the pilot studies were made both in the programmed instruction and the criterion measures.



In the fourth and sixth grade pilot studies revisions were made in portions of the program which caused excessive student questions during administration of the program or on any frame which one-third or more of the students in a particular treatment had errors. A further indication for necessary revisions was obtained from the no-answer frames. When checking these frames, it became evident when a subject was not playing the game fairly, that is, there was no indication of mistakes on a concept until a check frame was inserted. This could either be a fault of the program or of the individual student and so instances which exhibited this pattern were carefully scrutinized for possible modification.

In the eighth grade pilot study, any hesitation or problems which the two students had were noted and made the basis for possible modification. A second source of information, useful to the investigator in revision, was obtained in an individual interview immediately following instruction. At that time, the students were asked for their reactions and criticisms. In general, after working with individual children, it was felt that this method was at least as helpful as pilot studies presented to intact classes.

The time necessary for completion was carefully observed in the pilot studies to determine if the instructional times were approximately equal for the two treatments. In the fourth grade, completion times were written on the front of each booklet. This practice, however, tended to indicate to some of the students that a premium was placed on time and they tended to hurry through the booklets. When this was observed the practice of putting times on booklets was stopped and subjective judgments were made pertaining to time necessary for completion for each of the treatments. It was estimated in each study that these times were approximately equal. In the fourth grade the usual time range for completion was approximately 10 minutes to 40 minutes; in the sixth grade, approximately 15 minutes to 45 minutes; and in the eighth grade, about 35 minutes to 45 minutes. These ranges seemed to indicate that the daily programs were about the proper length.

Subjects

The subjects for these studies consisted of all students in the specified grades at Mitchell-Neilson Elementary School in Murfreesboro, Tennessee. This procedure yielded four classes or approximately 135 students for the fourth grade study; four classes or approximately 120 students for the sixth grade study; and three classes or approximately 105 students for the eighth grade study. Approximate numbers of students are reported here since the number of students varied throughout each study and some subjects were eliminated due to illness or for failure to complete the instructional program.



The subjects in the fourth grade study were categorized into high, intermediate, and low ability subgroups on the basis of their scores on the Arithmetic Computation and Arithmetic Problem Solving and Concepts of the Metropolitan Achievement Test which was administered at grade placement 3.7. The average of the two grade level achievements for each subject was obtained and used as a single number for this classification. Based on this procedure, the low ability subgroup had grade placement which ranged from 2.2 to 3.7; the intermediate subgroup had grade placement ranging from 3.8 to 4.2; and the high ability subgroup had grade placement ranging from 4.3 to 6.0. The particular cut off values were used in an attempt to have approximately equal numbers of subjects in each subgroup.

The subjects in the sixth and eighth grade studies were categorized into ability levels on the basis of the Mathematics subscore of the Sequential Tests of Educational Progress (STEP) and the Quantitative subscore of the School and College Ability Test (SCAT). Since these measures are not comparable, the scores could not meaningfully be averaged. It was determined that high ability subjects have scores in the high ability range on both measures, middle ability subjects have scores in the middle ability range on both measures, and low ability subjects have scores in the low ability range on both measures. As a result, some of the ranges for ability groups have a slight degree of overlap on the two measures. This method of grouping permitted an ability classification for almost all of the subjects and allowed approximately an equal number of subjects for each ability classification. Table 1 gives the raw score bands for each ability group on both subscores for the two grade levels.

Table 1

Raw Score Bands for the Three Ability Groups on the STEP and SCAT Subscores for the Sixth and Eighth Grade Study

| | Sixth G | Eighth Grade | | |
|---------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Ability | STEP | SCAT | STEP | SCAT |
| High Int. Low | 258-274 250-261 230-253 | 266-287 258-269 243-261 | 272-296 253-275 230-263 | 283-315 272-287 255-272 |

Within each ability subgroup, subjects were randomly assigned to one of the two treatment groups. As a result of this method of assigning subjects to ability level and treatment, there were approximately an equal number of subjects per cell.

Administration of Instructional Materials

On the first day, detailed instructions were given to all subjects in the use of the booklets which were in the form of programmed text-In the fourth grade study, common directions were read to all students by their teachers. The directions in this case also informed the students of the major goal of instruction, that is, to learn more about some new numbers, and also how to use the learning aids which were provided. After the instructions were read, the children were permitted to ask questions about the use of the booklets. These questions were answered and then the children were instructed to begin work. On each subsequent day of the study, brief instructions were given after the student had received a booklet representative of the treatment to which the student had been assigned. Each booklet was considered as one day's work. There were, however, some subjects who completed more than one booklet during one class period to make up for an absence. When the students had begun work on the programmed instructional units, questions were discouraged. In cases where the student persistently demanded an answer, the investigator's response was a reflection of the treatment to which the student was assigned.

In the sixth and eighth grade studies, the subjects were instructed in the use of the booklet and the purpose of the booklets in the first several pages of the first booklet. This instruction included one trial page for the student to complete, followed by a description of the procedures to follow if he responded correctly or if he responded incorrectly. Again questions were freely answered pertaining to the use of the booklets but were discouraged after instruction had begun.

On Day 9, the first day of the posttest, all students were informed that today there would be no correct answers with which they could compare their responses. They were not told, however, that this was a test situation.

If a student was absent on a given day of instruction or if he was unable to finish a booklet on a given day, he would begin the next day at the point where he stopped on the preceding day. In this way each student could proceed with as much of the eight days of instruction as possible without missing any of the programmed presentation. It was administratively decided that data for a subject would be usable if he had completed all eight booklets even though, for example, booklet six was not completed on Day 6 but rather on Day 7. One reason for this decision was that it seemed to more closely

parallel the classroom situation which also has to cope with absences and individual differences.

The Experimental Design

As has previously been reported the subjects were assigned to levels by means of an ability measure and then, within levels, randomly assigned to treatment groups. This resulted in a treatment by levels design as indicated below.

Table 2

Experimental Design for Each of the Three Studies

| Ability | Intermediate Guidance | Maximal Guidance |
|---------|--------------------------|---------------------|
| Low | n > 10 | n > 10 |
| Int. | n > 10 | n ≥ 10 |
| High | n > 10 | n > 10 |

Analysis of variance by orthogonal comparisons was used to test null hypotheses pertaining to the effects of ability, treatment, and interaction for achievement, horizontal transfer, and vertical transfer. In order to facilitate these analyses subjects were randomly eliminated to obtain equal n in each cell.

Specifically, the hypotheses tested for each analysis and the corresponding λ weights for each comparison are reported in Table 3.

Table 3

Hypotheses and λ Weights for Testing the Significance of Ability, Treatment, and Interaction Effects on Each Measure

| | Interme | diate G | Maximal Guidanc | | | |
|---|---------|-------------|-----------------|------|------|-----|
| Hypothesis | High | Int. | Low | High | Int. | Low |
| $^{\mu}$ IG = $^{\mu}$ MG | 1 | 1 | 1 | -1 | -1 | -1 |
| $\mu_{\text{HA}} = \frac{\mu_{\text{IA}} + \mu_{\text{LA}}}{2}$ | 2 | -1 . | -1 | 2 | -1 | -1 |
| $\mu_{\mathbf{I}\mathbf{A}} = \mu_{\mathbf{L}\mathbf{A}}$ | 0 | 1 | -1 | 0 | 1 | -1 |
| $\mu_{\text{IG}}^{-\mu_{\text{MG}}}$ $\mu_{\text{HA}}^{-\mu_{\text{MG}}} = \left(\mu_{\text{IG}}^{-\mu_{\text{MG}}}\right) \frac{\text{IA} + \text{IA}}{2}$ | 2 | -1 | -1 | -2 | 1 | 1 |
| $\left(\mu_{\mathrm{IG}} - \mu_{\mathrm{MG}}\right)_{\mathrm{IA}} = \left(\mu_{\mathrm{IG}} - \mu_{\mathrm{MG}}\right)_{\mathrm{LA}}$ | 0 | 1 | - 1 | 0 | -1 | 1 |

Findings and Analyses

In order to use the design stated in the last section, it was decided to randomly eliminate subjects to have an equal number of observations for each cell. This method of operation permitted analyses on 14 subjects per cell in the fourth grade study, 13 subjects per cell in the sixth grade study, and 11 subjects per cell in the eighth grade study. These analyses were all made using the .05 level of significance.

For each of the three studies, the criterion measures were constructed by the investigators for the express purpose of testing concepts or extensions of concepts taught during the instructional phase of the study. As a result, it was assumed that these measures possessed adequate content validity. Also of concern was the question of reliability; that is, the consistency of the measure if used repeatedly in similar situations. An indication of the reliability of the criterion measures is presented in the ensuing sections which deal with the results of the three studies independently.

Results of the Fourth Grade Study

One method of gaining an indication of the reliability of a measure

is to compute the correlation coefficient of the scores obtained on parallel forms of the same test given over a time interval. In the present study, the retention test for each measure was a shortened form of the posttest and, as such, it cannot be strictly regarded as a parallel form. The correlation, however, between scores on the posttest and the retention test does yield an indication of the reliability of the criterion measure. These correlation coefficients between posttest and retention test scores for each treatment on each criterion measure are given in Table 4. These values all show a significant positive relation between posttest and retention test measures. Hence, it was assumed that each criterion measure possessed adequate reliability for subsequent comparisons of cell means.

Table 4

Correlation Coefficients Between Posttest and Retention
Test Scores for Each Treatment and the Total
Group for Each Criterion Measure,
Fourth Grade Study

| | Treatm | | |
|---------------|--------------|---------|-------|
| Measure | Intermediate | Maximal | Total |
| Achievement | .59 | .82 | .71 |
| Hor. Transfer | .66 | .75 | .73 |
| Ver. Transfer | .61 | .61 | .62 |

The statistic
$$\frac{Z_{r_1} - Z_{r_2}}{\sqrt{\frac{2}{N-3}}}$$
 where $Z_r = \frac{1}{2} \ln \frac{1+r}{1-r}$ and $N = 42$ was

used to test the equality of the obtained correlation coefficients for treatments on each criterion measure. This statistic has approximately a standard normal distribution. The results of these tests indicated that for the achievement measure the correlation coefficient for the maximal guidance treatment was significantly larger than that for the intermediate guidance treatment. No other significant differences were found.

The means and variances of scores on the postlearning measures are reported in Table 5, while the analogous scores on the retention test measures are given in Table 6.



Means and Variances of Scores on Three Postlearning
Measures for the Experimental Treatments Within
Ability Groups, Fourth Grade Study

| Treatment Within | Achie | vement | Horz. | Transfer | Vert. | Transfer |
|------------------|-------|--------|-------|----------|-------|----------|
| Ability Level | Mean | Var. | Mean | Var. | Mean | Var. |
| Int. Guidance | | | | | | |
| High Abil. | 10.5 | 11.65 | 10.6 | 5.17 | 6.3 | 9.14 |
| Int. Abil. | 10.6 | 8.57 | 9.1 | 12.84 | 4.3 | 9.45 |
| Low Abil. | 7.0 | 8.92 | 5.8 | 7.26 | 3.9 | 5.76 |
| Max. Guidance | | | | | | |
| High Abil. | 11.4 | 5.34 | 12.4 | 36.26 | 8.1 | 13.21 |
| Int. Abil. | 9.9 | 10.23 | 10.0 | 14.62 | 5.9 | 4.84 |
| Low Abil. | 6.0 | 7.08 | 5.1 | 8.23 | 3.2 | 6.49 |
| | | | | | | |

Table 6

Means and Variances of Scores on Three Retention
Measures for the Experimental Treatments Within
Ability Groups, Fourth Grade Study

| Treatment Within | Achie | vement | Horz. | Transfer | Vert. T | ransfer |
|------------------|-------|--------|-------|----------|---------|---------|
| Ability Level | Mean | Var. | Mean | Var. | Mean | Var. |
| Int. Guidance | | | | | | |
| High Abil. | 8.1 | 6.90 | 5.9 | 5.59 | 2.9 | 6.38 |
| Int. Abil. | 7.1 | 5.30 | 4.7 | 4.84 | 1.6 | 2.57 |
| Low Abil. | 5.6 | 8.40 | 3.4 | 5.79 | .6 | .86 |
| Max. Guidance | | | | | | , |
| High Abil. | 9.0 | 6.15 | 7.6 | 14.40 | 4.1 | 6.29 |
| Int. Abil. | 8.2 | 8.02 | 4.4 | 9.48 | 3.3 | 4.84 |
| Low Abil. | 4.7 | 5.60 | 2.9 | 4.53 | 1.3 | 2.84 |

It should be noted that the decrease in means from posttest measures to retention test measures is accounted for in part by the total possible scores that could be obtained. To obtain comparable scores from the



posttest and retention test the retention achievement scores should be multiplied by a factor of 24/19; the retention horizontal transfer scores should be multiplied by 27/18; and the retention vertical transfer scores should be multiplied by 25/14.

Graphs of the mean scores obtained by the subjects are given in Figure 1. The means for each measure on the posttest and retention test for each treatment within ability groups are shown. These graphs give a good indication of the ordinal relation existing for the ability groups on each measure.

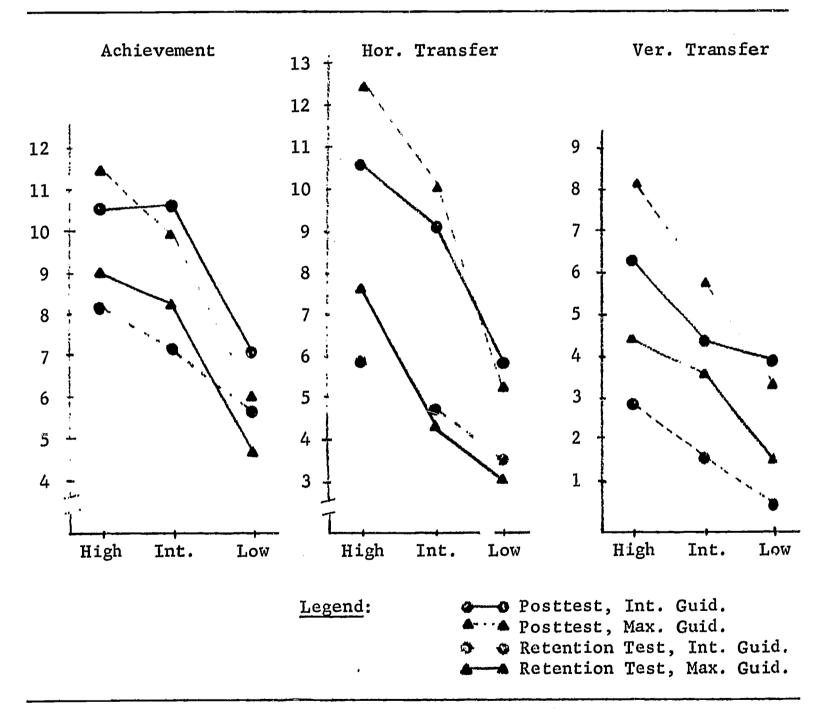


Figure 1. Means of Scores on Three Measures for the Experimental Treatments Within Ability Groups, Fourth Grade Study

To determine the significance of treatment effects, ability effects, or interaction effects, six analyses of variance by orthogonal comparisons were performed on the six sets of data summarized in Table 5 and Table 6. The summary analysis of variance tables are reported in Tables A-1 through A-6 in the Appendix.

Homogeniety of variance was tested using Cochran's Test at the .05 level of significance and found to be tenable in all cases except for the posttest horizontal transfer analysis. Based upon the study of Boneau (1960) and the problems encountered in interpreting any analyses with transformed data, it was decided to conduct the analysis of variance even though the assumption of homogeneity of variance was violated. This followed from Boneau's conclusion that with equal numbers per cell, violation of the assumption of homogenous variances produces a minimal effect on the distribution of F.

The results of the six analyses of variance may be summarized as follows:

- 1. On each posttest and retention test measure the high ability subgroup scored significantly higher than did the lower ability subgroups.
- 2. On each measure except the retention test horizontal transfer measure, the intermediate ability subgroup scored significantly higher than the low ability subgroup.
- 3. On the retention test vertical transfer measure, the maximal guidance treatment group scored significantly higher than did the intermediate guidance treatment group.
- 4. There were no other significant treatment effects nor were there any significant interactions between ability and treatment.

The data from the posttest were also compared with the data from the retention test by means of ratios. To obtain a meaningful retention ratio the raw scores were first transformed to per cent correct scores. The retention ratio was then defined as the per cent correct on the retention test divided by the per cent correct on the posttest. If the per cent correct on the posttest was zero, that is, a zero denominator, the retention ratio was defined as one. These ratios, the means and variances of which are reported in Table 7, were then analyzed for each of the three measures: achievement, horizontal transfer, and vertical transfer.

Even though there was lack of homogeneity of variance as indicated by Cochran's Test on two of the measures, analyses of variance were conducted to test the five previously stated comparisons. The results



Table 7

Means and Variances of Retention Ratios for the Experimental Treatments Within Ability Groups, Fourth Grade Study

| Treatment Within | Achie | vement | Horz. T | ransfer | Vert. | Transfer |
|------------------|-------|--------|---------|---------|-------|----------|
| Ability Level | Mean | Var. | Mean | Var. | Mean | Var. |
| Int. Guidance | | | | | | |
| High Abil. | 1.01 | .09 | .83 | .08 | .71 | • 35 |
| Int. Abil. | .87 | .06 | .80 | .07 | .79 | .94 |
| Low Abil. | 1.06 | .36 | 1.04 | 1.04 | .30 | . 15 |
| Max. Guidance | | | | | | |
| High Abil. | .99 | .03 | .98 | .18 | 1.05 | .63 |
| Int. Abil. | 1.06 | .08 | .61 | .09 | 1.06 | .63 |
| Low Abil. | 1.11 | .35 | .97 | .46 | .66 | .78 |

of these analyses are reported in the Appendix and indicate only one significant difference; that is, the mean retention ratio on the vertical transfer test for the intermediate ability subgroup was significantly larger than the comparable ratio for the low ability subgroup.

To gain some further insight into the degree of relationship between the three criterion measures, several correlation coefficients were computed. These are reported in Table 8.

Table 8

Correlation Coefficients Between Postlearning Measures and Between Retention Measures for Each Treatment and the Total Group, Fourth Grade Study*

| | P | osttest | Retention Test | | | |
|------------|-----------------|---------|-----------------|-----------------|------|-----------------|
| Treatment | r _{AH} | rAV | r _{HV} | r _{AH} | rAV | r _{HV} |
| Int. Guid. | 60 | .53 | .54 | .38 | .51 | .52 |
| Max. Guid. | .70 | .77 | .61 | .52 | .67 | .65 |
| Total | .65 | .65 | .58 | .47 | . 59 | .59 |

^{*(}A = Achievement, H = Horizontal Transfer, and V = Vertical Transfer) -

All of these correlation coefficients show a significant positive relation between the measures. There were, however, no significant differences between the values obtained for the correlation coefficients for the two treatments.

Results of the Sixth Grade Study

As in the fourth grade study an indication of the reliability of the instruments used to measure achievement, horizontal transfer, and vertical transfer was obtained from correlation coefficients computed for scores on the posttest and the retention test. These correlation coefficients are reported in Table 9.

Table 9

Correlation Coefficients Between Posttest and Retention Test Scores for Each Treatment and the Total Group for Each Criterion Measure, Sixth Grade Study

| Measure | Treatm | | |
|---------------|--------------|---------|-------|
| | Intermediate | Maximal | Total |
| Achievement | .73 | .66 | .69 |
| Hor. Transfer | .74 | .65 | .68 |
| Ver. Transfer | .60 | .59 | .58 |

These correlation coefficients are not quite as large as those reported in the fourth grade study, but they all show a significant positive relation between the posttest and retention test measures. This in turn led to the assumption that the measuring instruments had acceptable reliability. There were no significant differences when correlation coefficients for intermediate guidance were compared with those for maximal guidance.

The means and variances of scores on the postlearning measures are reported in Table 10 and these values on retention test measures are given in Table 11. As in the fourth grade study, retention test scores and posttest scores are not directly comparable. If, however, retention achievement scores were multiplied by 30/21, retention horizontal transfer scores were multiplied by 25/14, and retention vertical transfer scores were multiplied by 33/17, then these values would be comparable to the corresponding posttest scores.



Table 10

Means and Variances of Scores on Three Postlearning
Measures for the Experimental Treatments Within
Ability Groups, Sixth Grade Study

| Treatment Within | Achie | vement | Horz. | Transfer | Vert. | Transfer |
|------------------|-------|--------|-------|----------|-------|----------|
| Ability Level | Mean | Var. | Mean | Var. | Mean | Var. |
| Int. Guidance | | | | | | |
| High Abil. | 18.5 | 8.10 | 12.8 | 10.18 | 14.8 | 19.10 |
| Int. Abil. | 16.8 | 15.82 | 12.4 | 8.85 | 12.8 | 9.05 |
| Low Abil. | 13.2 | 12.79 | 8.5 | 3.79 | 8.8 | 14.79 |
| Max. Guidance | | | | | | |
| High Abil. | 19.6 | 20.55 | 13.6 | 14.09 | 13.2 | 36.79 |
| Int. Abil. | 15.4 | 17.62 | 11.7 | 12.06 | 11.3 | 16.06 |
| Low Abil. | 13.8 | 9.10 | 9.4 | 7.01 | 8.8 | 8.44 |

Table 11

Means and Variances of Scores on Three Retention Measures for the Experimental Treatments Within Ability Groups,
Sixth Grade Study

| Treatment Within | Achie | vement | Horz. | Transfer | Vert. | Transfer |
|------------------|-------|--------|-------|----------|-------|----------|
| Ability Level | Mean | Var. | Mean | Var. | Mean | Var. |
| Int. Guidance | | | | | | |
| High Abil. | 13.9 | 5.61 | 8.4 | 4.08 | 4.7 | 4.98 |
| Int. Abil. | 12.8 | 4.90 | 8.1 | 2.53 | 4.0 | 4.92 |
| Low Abil. | 10.3 | 5.75 | 6.1 | 3.92 | 2.9 | 3.76 |
| Max. Guidance | | | | | | |
| High Abil. | 12.8 | 7.26 | 8.2 | 4.64 | 5.9 | 13.76 |
| Int. Abil. | 11.8 | 4.64 | 6.7 | 5.14 | 3.5 | 3.94 |
| Low abil. | 10.5 | 6.40 | 5.8 | 3.52 | 3.5 | 3.17 |

Graphs of the mean scores which were reported in Table 10 and Table 11 are pictured in Figure 2. As in the fourth grade study, the graphs show a definite and expected ordering of the means with regard to ability. It can also be seen that there are minimal treatment effects.

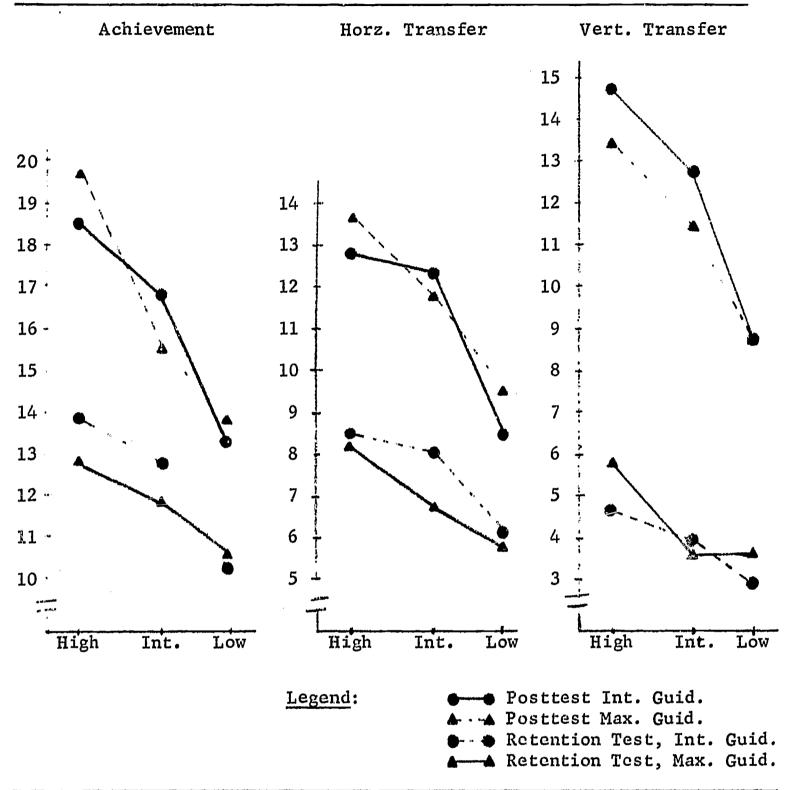


Figure 2. Mean Scores on Three Measures for the Experimental Treatments Within Ability Groups, Sixth Grade Study

To determine the significance of treatment effects, ability effects or interaction effects, six analyses of variance by orthogonal comparisons were performed. The summary tables for these analyses are reported in Tables A-10 through A-15 in the Appendix. As before, even though the assumption of homogeneity of variance was violated on both vertical transfer measures, the raw data were still subjected to analyses of variance.

The results of these analyses of 'variance were as follows:

- 1. On each posttest and retention test measure the high ability subgroup scored significantly higher than did the lower ability subgroups.
- 2. On each measure except the retent on test vertical transfer measure, the intermediate ability subgroup scored significantly higher than the low ability subgroup.
- 3. There were no significant treatment effects nor were there any significant interactions between ability and treatment.

The data from the posttest were also compared with the data from the retention test by means of retention ratios. The retention ratio was again defined as the per cent correct on the retention test divided by the per cent correct on the posttest. This ratio was defined to be one if the raw score on the posttest was zero. The means and variances for the retention ratios on each of the three measures are reported in Table 12.

Table 12

Means and Variances of Retention Ratios for the Experimental
Treatments Within Ability Groups, Sixth Grade Study

| Treatment Within | Achie | vement | Horz. T | ransfer | Vert. T | ransfer |
|------------------|-------|--------|---------|---------|---------|---------|
| Ability Level | Mean | Var. | Mean | Var. | Mean | Var. |
| Int. Guidance | | | | | | |
| High Abil. | 1.09 | .04 | 1.21 | .09 | .60 | .07 |
| Int. Abil. | 1.12 | .03 | 1.21 | .06 | .61 | .09 |
| Low Abil. | 1.17 | .08 | 1.26 | .11 | .67 | .11 |
| Max. Guidance | | | | | | |
| High Abil. | .95 | .03 | 1.14 | .17 | .92 | .24 |
| Int. Abil. | 1.14 | .06 | 1.03 | .06 | .67 | .17 |
| Low Abil. | 1.12 | .07 | .90 | .38 | 1.15 | .13 |

It is interesting to note that these ratios are quite large for the achievement and horizontal transfer measures which indicates on the average there was a gain in performance on these measures after approximately a four week lapse in time.

The three analyses of variance by orthogonal comparisons yielded the following results:

- 1. The high ability subgroup had a mean achievement retention ratio that was significantly smaller than the comparable mean retention ratio for the lower ability subgroups.
- 2. The maximal guidance treatment had a significantly larger mean vertical transfer retention ratio than did the intermediate guidance treatment.
- 3. There were no other significant ability or treatment effects.

For the summary analysis of variance tables, the reader is referred to Tables A-16 through A-18 in the Appendix.

Correlation coefficients, indicating the degree of relationship between postlearning measures and between retention measures, are reported in Table 13. Each of these correlation coefficients indicates a significant positive relation between the measures. No significant differences between the values obtained for the correlation coefficients for the two treatments were found.

Table 13

Correlation Coefficients Between Postlearning Measures and Between Retention Measures for Each Treatment and the Total Group, Sixth Grade Study*

| Treatment | Po | sttest | Retention Test | | | |
|------------|-----------------|--------|-----------------|-----------------|-----|-----------------|
| | r _{AH} | rAV | r _{HV} | r _{AH} | rAV | r _{HV} |
| Int. Guid. | .85 | .58 | .61 | .66 | .38 | .47 |
| Max. Guid. | .69 | .70 | .60 | .64 | .51 | .41 |
| Total | .76 | .64 | .60 | .65 | .43 | .42 |

^{*(}A = Achievement, H = Horizontal Transfer, and V = Vertical Transfer)

Results of the Eighth Grade Study

As in the previous studies an indication of the reliability of the criterion measures was obtained from correlation coefficients computed for scores on the posttest and the retention test. These correlation coefficients are reported in Table 14. In this study each retention test measure was a parallel form of the corresponding posttest measure. Hence these values may be considered as giving a better indication of



Table 14

Correlation Coefficients Between Posttest and Retention
Test Scores for Each Treatment and the Total
Group for Each Criterion Measure,
Eighth Grade Study

| | Treatme | | |
|---------------|--------------|---------|-------|
| Measure | Intermediate | Maximal | Total |
| Achievement | .70 | .82 | .77 |
| Hor. Transfer | .76 | .81 | .78 |
| Ver. Transfer | .82 | .85 | .82 |

reliability than was obtained in the previous studies. Since these correlation coefficients are quite large, it was assumed that the criterion instruments were sufficiently reliable to permit subsequent comparisons of group means. There were no significant differences when the correlation coefficients for the two treatments were compared.

The means and variances of scores on the postlearning measures are reported in Table 15 and the analogous scores on the retention measures are given in Table 16. It should be noted that the possible number of items on each criterion measure was 18. Thus, the means in Table 16 when compared with the comparable means in Table 15 gives an indication the amount of retention over a time period of about four weeks.

Means and Variances of Scores on Three Postlearning Measures for the Experimental Treatments Within Ability
Groups, Eighth Grade Study

| Treatment i th | in Ach: | evement | Horz. | Transfer | Vert. | Transfer |
|----------------|---------|---------|-------|----------|-------|----------|
| Ability Lagal | Mear | ı Var. | Mean | Var. | Mean | Var. |
| Int. Guicance | | | | | | |
| High Abil | 9.0 | 4.80 | 7.2 | 4.96 | 7.1 | 6.69 |
| Int. Abil | | 14.26 | 3.4 | 15.66 | 5.0 | 5,80 |
| Low Abil. | 3.1 | 2.49 | 1.4 | 1.46 | 2.4 | 5.46 |
| Max. Guidance | | | | | | |
| High Abil | 10.1 | 23.35 | 6.7 | 11.42 | 9.3 | 14.01 |
| Int. Abil | 5.8 | 6.16 | 4.0 | 6.20 | 4.3 | 5.62 |
| Low Abil. | 3.2 | 2 4.16 | 1.4 | 3.06 | 1.4 | 1.66 |

Table 16

Means and Variances of Scores on Three Retention Measures for the Experimental Treatments Within Ability
Groups, Eighth Grade Study

| Treatment Within | Achie | vement | Horz. | Transfer | Vert. | Transfer |
|------------------|-------|--------|-------|----------|-------|----------|
| Ability Level | Mean | Var. | Mean | Var. | Mean | Var. |
| Int. Guidance | | | | | | |
| High Abil. | 6.7 | 4.82 | 5.6 | 10.45 | 6.4 | 7.85 |
| Int. Abil. | 4.3 | 3.22 | 2.0 | 5.40 | 3.4 | 2.27 |
| Low Abil. | 2.6 | 2.67 | 1.7 | 3.22 | 1.9 | 1.29 |
| Max. Guidance | | | | | | |
| High Abil. | 9.7 | 22.32 | 5.9 | 17.69 | 8.8 | 22.56 |
| Int. Abil. | 4.6 | 5.87 | 3.3 | 5.62 | 4.3 | 6.02 |
| Low Abil. | 2.7 | 2.62 | .9 | 1.49 | 2.4 | 2.87 |

Graphs of the mean scores obtained by the subjects are pictured in Figure 3. As in both of the previous studies, the ordinal relationship for the ability groups is quite obvious. There are, however, relatively few other differences suggested by observing the graphs.

To determine if there were significant differences between treatments, ability levels, or if there were any interaction effects, an analysis of variance by orthogonal comparisons was conducted on each of the six sets of data. In each case Cochran's Test indicated a lack of homogeneity of variance but for reasons previously mentioned, each analysis of variance was performed using the raw data.

The results of these analyses of variance may be summarized as follows:

- 1. On each posttest and retention test measure, the high ability subgroup scored significantly higher than did the lower ability subgroups.
- 2. On each measure except the retention test horizontal transfer measure, the intermediate ability subgroup scored significantly higher than the low ability subgroups.
- 3. There were no significant main effects due to treatment.
- 4. Two significant interactions were found. On the posttest vertical transfer measure and the retention test achievement measure the high ability maximal



guidance group had a significantly greater mean score than the high ability intermediate guidance group while for lower ability subgroups the two treatments produced similar mean scores.

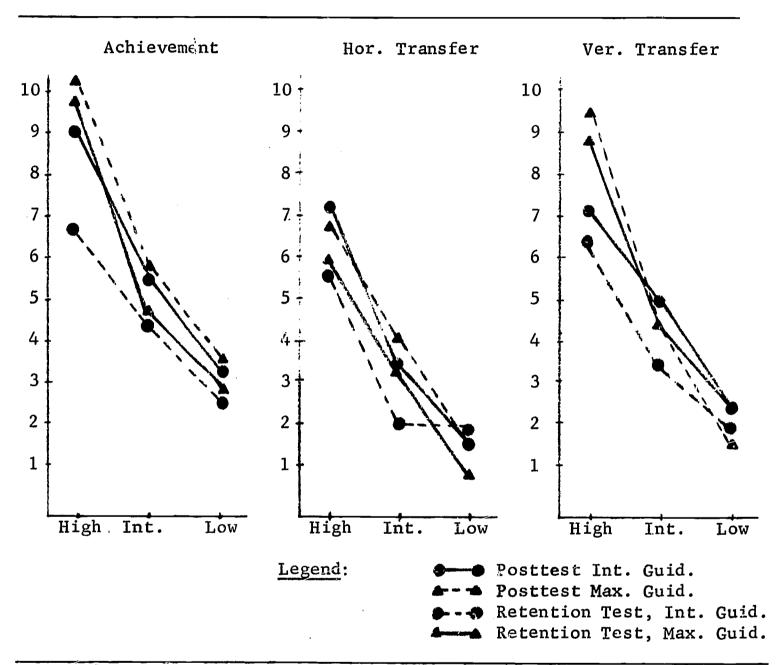


Figure 3. Mean Scores on Three Measures for the Experimental Treatments Within Ability Groups, Eighth Grade Study

Summary analysis of variance tables for these six analyses are reported in Tables A-19 through A-24 in the Appendix.

When the data from the posttest were compared with the data from the retention test by means of a retention ratio, the means and variances of ratios, as reported in Table 17, were obtained. The retention ratio in this case was obtained by dividing the retention test score for an individual by his posttest score. As in the previous

studies if a posttest score was zero, the retention ratio was defined to be one. Analyses of variance were conducted on these sets of data and summary tables appear in the Appendix. These analyses yielded no significant differences with regard to the main effects of ability and treatment or interactions between ability and treatment

Table 17

Means and Variances of Retention Ratios for the Experimental
Treatments Within Ability Groups, Eighth Grade Study

| Treatment Within | Achie | vement | Horz. T | ransfer | Vert. | Transfer |
|------------------|-------|--------|---------|---------|-------|----------|
| Ability Level | Mean | Var. | Mean | Var. | Mean | Var. |
| Int. Guidance | | | | | | |
| High Abil. | .76 | .06 | .78 | .19 | .95 | .12 |
| Int. Abil. | .93 | .71 | .69 | .17 | .90 | .58 |
| Low Abil. | 1.05 | .76 | 1.14 | 1.06 | 1.01 | .42 |
| Max. Guidance | | | | | | |
| High Abil. | 1.05 | .33 | .91 | .19 | .92 | .07 |
| Int. Abil. | .77 | .11 | .88 | .34 | 1.20 | .72 |
| Low Abil. | .98 | .77 | .76 | .18 | 1.23 | .37 |

Correlation coefficients were computed between postlearning measures and between retention measures and are reported in Table 18. All of these correlation coefficients show a significant positive relation between the measures. The only significant difference occurred when the scores on the posttest achievement measure were correlated with the scores on the posttest horizontal transfer measure. In this case, the correlation coefficient for the maximal guidance treatment was significantly larger than that for the intermediate guidance treatment.

Table 18

Correlation Coefficients Between Postlearning Measures and Between Retention Measures for Each Treatment and the Total Group, Eighth Grade Study*

| | Po | sttest | | Reten | tion Te | st |
|---------------------------|-----------------|----------|---------------------------------------|----------|----------|-----|
| Treatment r _{AH} | r _{AV} | r_{HV} | $\overline{\mathbf{r}_{\mathrm{AH}}}$ | r_{AV} | r_{HV} | |
| Int. Guid. | .48 | .73 | .55 | .57 | .89 | .84 |
| Max. Guid. | .78 | .86 | .78 | .70 | .83 | .76 |
| Total | .64 | .81 | .67 | .64 | .85 | .76 |

^{*(}A = Achievement, H = Horizontal Transfer, V = Vertical Transfer.)

Summary of Results

In each of the three studies the criterion measures distinguished clearly between ability levels. There was only one exception to this generalization in each study. This exception occurred when the intermediate ability subgroup was compared with the low ability subgroup on a retention test transfer measure. When significant treatment effects or interaction effects occurred, they favored the maximal guidance treatment. In the fourth grade study the maximal guidance group exceeded the intermediate guidance group on the retention test vertical transfer measure; in the sixth grade study the maximal guidance group had a significantly larger mean vertical transfer retention ratio than did the intermediate guidance group; and in the eighth grade study intermediate guidance was a preferable method of instruction for the high ability subgroup when performance was measured by posttest vertical transfer and retention achievement.

Correlation coefficients were computed between various posttest measures and retention test measures. In every case, these correlation coefficients indicated a significant positive relation between the measures being compared.

Discussion

One of the fundamental problems encountered in these studies was the mode of instruction. It appears that programmed instruction introduces a variable into experimental studies just as perplexing as the teacher variable which it eliminates.

In the subjective opinion of the proctors, there appeared to be some type of a novelty effect which lasted from one to three instructional periods. During this time the subject appeared to be interested in the tasks set forth in the programs, made honest attempts to answer the questions, and was concerned if his answer differed in the slightest detail from the answer stated in the program. On the average, the length of time spent on the program during the first few days of administration was longer than on later days. This also seemed to be an indication that many students were making an honest effort to learn.

When the instructional content became somewhat more difficult or when the novelty effect waned, many of the subjects ceased learning. This became evident from an analysis of the scores on several check frames which did not provide immediate confirmation of results and from the lower average time used in completion of an instructional booklet. In these studies it seemed that many of the subjects would read the frame and, before answering the question on the page, would look at the correct answer. This effect was by no means restricted to



subjects from any one ability level although it did seem to be more prevalent at the lower ability levels. An interesting study would be to investigate if there are any personality traits which are linked to this lack of motivation to learn.

One result of this failure to use the programmed booklets correctly was a contamination of the treatments under consideration. This occurred if the subjects in the intermediate guidance treatment looked at a correct response prior to making their own responses. Under these conditions the intermediate guidance subjects would be provided with a type of maximal guidance. Thus it is possible that comparisons were being made for two different amounts and types of maximal guidance.

A second consequence of the failure of our instructional program to motifate learning, was the relatively low level of performance on the criterion measures. Prior to conducting these studies it was anticipated that average posttest achievement scores would be about 70 per cent. This was not accomplished in any of the three studies in that the average posttest achievement per cent for the fourth grade study was 39, for the sixth grade study was 54, and for the eighth grade was 34. This low level of achievement would certainly restrict the amount of transfer, either horizontal or vertical, that could be assessed and would account in part for the very low level of performance on these measures.

This lack of learning as witnessed by scores on the criterion measures may quite possibly account for the relatively few significant differences that were obtained. It would also give some credence to all of the significant differences favoring the maximal guidance treatment since the intermediate guidance treatment demanded more original responses from the subjects. This, in turn, may have caused subjects having the intermediate guidance treatment to stop learning earlier in the programs of instruction.

The low level of scores on the performance measures may also account for the number of analyses in which the assumption of homogeniety of variance was violated. This inequality of variances seems to be directly related to a restriction of the range since generally the maximum variance occurred in a high ability cell and the minimum variance in low ability cell. It is interesting to observe that in all but one of the analyses of variance on the criterion measures (ratios excluded) where there was a lack of homogeneity of variance, the maximum variance occurred in the maximal guidance, high ability cell. This would tend to imply that, at least for high ability students, the maximal guidance treatment seems to produce a greater variability in performance than the intermediate guidance treatment.



Conclusions

Any conclusions that are drawn from these three studies must be interpreted in light of the previous discussion and the fact that selected mathematical topics were being taught. With this in mind it was concluded that when students learn using linear programmed instruction, the program should provide a relatively high level of guidance. This conclusion seems to be valid for all ability levels at each grade level studied. It seems that motivation to learn is a limiting factor to the intermediate guidance treatment and when more information is provided the learner in the programmed instruction there is a tendency to continue learning. This conclusion in no way infers that in a normal classroom situation the maximal guidance teaching technique produces superior performance.

One implication inherent in this conclusion is that when students are learning without the benefit of a teacher and human interaction, the better instruction is that which provides a liberal amount of guidance. This tends to agree with B. F. Skinner's view that the error rate in a program should be as low as possible and this cannot occur if the students are expected to make minor generalizations and independent judgments (intermediate guidance) within the program of study.

Recommendations for Further Study

There are a number of avenues relevant to the present studies that seem to merit further research. One of these studies would be to repeat these studies but make a greater attempt to motivate the subjects to learn. This may be done by administering the programs in small groups of two or three students and one proctor, provide some human interaction between the subjects and the proctor or administer the programs with a teaching machine which would not permit response verification until a response has been made. In any case some revisions in the programs of study would be necessary.

Another interesting study would be to vary the instructional treatments even further along the amount of guidance continuum. This would eventually necessitate definite differential results between two amounts of guidance in the instructional process and help to specify limits on the guidance continuum within which instruction should remain.

It would also be of interest to further investigate the effect of maximal guidance on variability. In the present study it was observed that the high ability maximal guidance subgroup had a larger variance than other subgroups on a number of performance measures. This seemed to occur across grade levels implying that it may be a general phenomenon relating to the maximal guidance treatment. It may be that this treatment produces a bimodal distribution



which may be a function of variables not directly related to ability. In any case, further exploration pertaining to variability seems warranted.

Finally, it may be worthwhile to test the effects of intermediate guidance versus maximal guidance over levels of creative ability rather than mathematical ability. It seems reasonable that an intermediate guidance treatment may be more appealing to a creative type of individual while maximal guidance would be preferable for a more conforming individual.



Appendix

Summary Analyses of Variance Tables for Each Performance Measure in Each Study



Table A-1
Posttest Achievement, Fourth Grade Study

| Source of Variation | d.f, | Sum of Squares | Mean Square | F | F .95 |
|----------------------------------|------|-------------------|----------------|-------|-------|
| (1) Between treatments | 1 | 1.19 | 1.19 | .14 | 3.96 |
| (2) High abil. vs. low abil. | 1 | 125.15 | 125.15 | 14.50 | |
| (3) Int. abil. vs. low abil. | 1 | 196.88 | 196.88 | 22.81 | |
| (4) Treatment by abil.(1) X (2) | 1 | 14.29 | 14.29 | 1.66 | |
| (5) Treatment by abil. (1) X (3) | 1 | .45 | .45 | .05 | |
| Within Cells | 78 | 673.29 | 8.63 | | |
| To tal | 83 | 1011.24 | | | |

Table A-2
Posttest Horizontal Transfer, Fourth Grade Study

| | • | Sum of | Mean | F | F |
|----------------------------------|------|---------|--------|-------|------|
| Source of Variation | d.f. | Squares | Square | | .95 |
| (1) Between treatments | 1 | 9.33 | 9.33 | .66 | 3.96 |
| (2) High abil. vs. low abil. | 1 | 306.72 | 306.72 | 21.82 | |
| (3) Int. abil. vs. low abil. | 1 | 236.16 | 236.16 | 16.80 | |
| (4) Treatment by abil.(1) X (2) | 1 | 13.15 | 13.15 | .94 | |
| (5) Treatment by abil. (1) X (3) | 1 | 9.45 | 9.45 | .67 | |
| Within Cells | 78 | 1096.86 | 14.06 | | |
| Total | 83 | 1671.67 | | | |

Table A-3
Posttest Vertical Transfer, Fourth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F .95 |
|----------------------------------|------|-------------------|----------------|-------|-------|
| (1) Between treatments | 1 | 18.11 | 18.11 | 2.22 | 3.96 |
| (2) High abil. vs. low abil. | 1 | 154.29 | 154.29 | 18.93 | |
| (3) Int. abil. vs. low abil. | 1 | 33.02 | 33.02 | 4.05 | |
| (4) Treatment by abil.(1) X (2) | 1 | 9.05 | 9.05 | 1.11 | |
| (5) Treatment by abil. (1) X (3) | 1 | 19.45 | 19.45 | 2.39 | |
| Within Cells | 78 | 635.64 | 8.15 | | |
| Total | 83 | 869.56 | | | |

Table A-4
Retention Test Achievement, Fourth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F.95 |
|---------------------------------|------|-------------------|----------------|-------|------|
| | u.r. | | | | |
| (1) Between treatments | 1 | 2.68 | 2.68 | .40 | 3.96 |
| (2) High abil. vs. low abil. | 1 | 87.15 | 87.15 | 12.95 | |
| (3) Int. abil. vs. low abil. | 1 | 85.02 | 85.02 | 12.63 | |
| (4) Treatment by abil.(1) X (2) | 1 | 2.62 | 2.62 | .39 | |
| (5) Treatment by abil.(1) X (3) | 1 | 15.02 | 15.02 | 2.23 | |
| Within Cells | 78 | 525.07 | 6.73 | | |
| Total | 83 | 717.56 | | | |

Table A-5
Retention Test Horizontal Transfer, Fourth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F.95 |
|---------------------------------|------|-------------------|----------------|-------|------|
| (1) Between treatments | 1 | 2.33 | 2.33 | .31 | 3.96 |
| (2) High abil. vs. low abil. | 1 | 158.15 | 158.15 | 21.26 | |
| (3) Int. abil. vs. low abil. | 1 | 27.16 | 27.16 | 3.65 | |
| (4) Treatment by abil.(1) X (2) | 1 | 22.15 | 22.15 | 2.98 | |
| (5) Treatment by abil.(1) X (3) | 1 | .02 | .02 | .003 | |
| Within Cells | 78 | 580.14 | 7.44 | | |
| Total | 83 | 789.95 | | | |

Table A-6
Retention Test Vertical Transfer, Fourth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F •95 |
|---------------------------------|------|-------------------|----------------|-------|----------|
| (1) Between treatments | 1 | 29.76 | 29.76 | 7.52 | 3.96 |
| (2) High abil. vs. low abil. | 1 | 63.15 | 63.15 | 15.95 | |
| (3) Int. abil. vs. low abil. | 1 | 30.02 | 30.02 | 7.58 | |
| (4) Treatment by abil.(1) X (2) | 1 | .001 | .001 | .001 | |
| (5) Treatment by abil.(1) X (3) | 1 | 4.02 | 4.02 | 1.02 | |
| Within Cells | 78 | 309.00 | 3.96 | | |
| Total | 83 | 435.95 | | | |

Table A-7
Achievement Retention Ratio, Fourth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F.95 |
|---------------------------------|------|-------------------|----------------|------|------|
| (1) Between treatments | 1 | .107 | .107 | .66 | 3.96 |
| (2) High abil. vs. low abil. | 1 | .012 | .012 | .07 | |
| (3) Int. abil. vs. low abil. | 1 | .208 | .208 | 1.28 | |
| (4) Treatment by abil.(1) X (2) | 1 | .106 | .106 | .65 | |
| (5) Treatment by abil.(1) X (3) | 1 | .074 | .074 | .46 | |
| Within Cells | 78 | 12.63 | .162 | | |
| Total | 83 | 13.14 | | | |

Table A-8
Horizontal Transfer Retention Ratio, Fourth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F .95 |
|----------------------------------|------|-------------------|----------------|------|----------|
| (1) Between treatments | 1 | .024 | .024 | .08 | 3.96 |
| (2) High abil. vs. low abil. | 1 | .050 | .050 | .16 | |
| (3) Int. abil. vs. low abil. | 1 | 1.230 | 1.230 | 3.86 | |
| (4) Treatment by abil.(1) X (2) | 1 | .370 | .370 | 1.16 | |
| (5) Treatment by abil. (1) X (3) | 1 | .043 | .043 | .13 | |
| Within Cells | 78 | 24.92 | .319 | | |
| Total | 83 | 26.63 | | | |

Table A-9
Vertical Transfer Retention Ratio, Fourth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F.95 |
|---------------------------------|------|-------------------|----------------|------|------|
| (1) Between treatments | 1 | 2.166 | 2.166 | 3.93 | 3.96 |
| (2) High abil. vs. low abil. | 1 | .572 | .572 | 1.04 | |
| (3) Int. abil. vs. low abil. | 1 | 2.684 | 2.684 | 4.87 | |
| (4) Treatment by abil.(1) X (2) | 1 | .002 | .002 | .004 | |
| (5) Treatment by abil.(1) X (3) | 1 | .027 | .027 | .05 | |
| Within Cells | 78 | 43.01 | .551 | | |
| Total | 83 | 48.47 | | | |

Table A-10
Posttest Achievement, Sixth Grade Study

| Course of Mariation | d.f. | Sum of Squares | Mean | F | F.95 |
|---------------------------------|-------|-------------------|--------|-------------|--------------|
| Source of Variation | и. д. | | | | |
| (1) Between treatments | 1 | . 12 | . 12 | .01 | 3.98 |
| (2) High shil. vs. low abil. | 1 | 310.26 | 310.26 | 20.47 | |
| (3) Int. abil. vs. low abil. | 1 | 88.92 | 88.92 | 5.87 | |
| (4) Treatment by abil.(1) X (2) | 1 | 11.31 | 11.31 | . 75 | |
| (5) Treatment by abil.(1) X (3) | 1 | 13.00 | 13.00 | .57 | |
| Within Cells | 72 | 1091.69 | 15.16 | | |
| Total | 71 | 1515.29 | | | |

Table A-11
Posttest Horizontal Transfer, Sixth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F.95 |
|---------------------------------|------|-------------------|----------------|-------|------|
| (1) Between treatments | 1 | 2.17 | 2.17 | .21 | 3.98 |
| (2) High abil. vs. low abil. | 1 | 125.64 | 125.64 | 12.44 | |
| (3) Int. abil. vs. low abil. | 1 | 123.08 | 123.08 | 12.19 | |
| (4) Treatment by abil.(1) X (2) | 1 | 2.56 | 2.56 | .25 | |
| (5) Treatment by abil.(1) X (3) | 1 | 7.69 | 7.69 | .76 | |
| Within Cells | 72 | 727.54 | 10.10 | | |
| Total | 77 | 988.68 | | | |

Table A-12
Posttest Vertical Transfer, Sixth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F •95 |
|----------------------------------|------|-------------------|----------------|-------|----------|
| (1) Between treatments | 1 | 19.51 | 19.51 | 1.04 | 3.98 |
| (2) High abil. vs. low abil. | 1 | 219.39 | 219.39 | 11.66 | |
| (3) Int. abil. vs. low abil. | 1 | 138.94 | 138.94 | 7.38 | |
| (4) Treatment by abil.(1) X (2) | 1 | 2.83 | 2.83 | .15 | |
| (5) Treatment by abil. (1) X (3) | 1 | 8.48 | 8.48 | .45 | |
| Within Cells | 72 | 1355.08 | 18.82 | | |
| Total | 77 | 1744.22 | | | |

Table A-13
Retention Test Achievement, Sixth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Squa r e | F | F.95 |
|---------------------------------|------|-------------------|-------------------------|-------|------|
| (1) Between treatments | 1 | 8.67 | 8.67 | 1.39 | 3.98 |
| (2) High abil. vs. low abil. | 1 | 68.01 | 68.01 | 10.90 | |
| (3) Int. abil. vs. low abil. | 1 | 46.17 | 46.17 | 7,40 | |
| (4) Treatment by abil.(1) X (2) | 1 | 2.31 | 2.31 | .37 | |
| (5) Treatment by abil.(1) X (3) | 1 | 5.56 | 5.56 | .89 | |
| Within Cells | 72 | 449.23 | 6.24 | | |
| Total | 77 | 579.95 | | | |

Table A-14
Retention Test Horizontal Transfer, Sixth Grade Study

| | | Sum of | Mean | F | F |
|----------------------------------|------|---------|--------|-------|------|
| Source of Variation | d.f. | Squares | Square | | .95 |
| (1) Between treatments | 1 | 6.78 | 6.78 | 1.58 | 3.98 |
| (2) High abil. vs. low abil. | 1 | 46.31 | 46.31 | 10.77 | |
| (3) Int. abil. vs. low abil. | 1 | 26.33 | 26.33 | 6.12 | |
| (4) Treatment by abil.(1) X (2) | 1 | 1.85 | 1.85 | .43 | |
| (5) Treatment by abil. (1) X (3) | 1 | 4.33 | 4.33 | 1.01 | |
| Within Cells | 72 | 309.69 | 4.30 | | |
| Total | 77 | 395.29 | | | |

Table A-15
Retention Test Vertical Transfer, Sixth Grade Study

| | | Sum of | Mean | F | F |
|---------------------------------|------|---------|--------|------|------|
| Source of Variation | d.f. | Squares | Square | | .95 |
| (1) Between treatments | 1 | 3.71 | 3.71 | .59 | 3.98 |
| (2) High abil. vs. low abil. | 1 | 57.85 | 57.85 | 9.27 | |
| (3) Int. abil. vs. low abil. | 1 | 3.25 | 3.25 | .52 | |
| (4) Treatment by abil.(1) X (2) | 1 | 6.16 | 6.16 | .99 | |
| (5) Treatment by abil.(1) X (3) | 1 | 4.33 | 4.33 | .69 | |
| Within Cells | 72 | 449.08 | 6.24 | | |
| Total | 77 | 524.37 | | | |
| | | | | | |

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Table A-16
Achievement Retention Ratio, Sixth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F.95 |
|----------------------------------|------|-------------------|----------------|------|------|
| | 1 | .06 | .06 | 1.00 | 3.98 |
| (1) Between treatments | _ | | | | 3.70 |
| (2) High abil. vs. low abil. | 1 | . 24 | .24 | 4.00 | |
| (3) Int. abil. vs. low abil. | 1 | .02 | .02 | .33 | |
| (4) Treatment by abil.(1) X (2) | 1 | .07 | .07 | 1.17 | |
| (5) Treatment by abil. (1) X (3) | 1 | .02 | .02 | .33 | |
| Within Cells | 72 | 4.07 | .06 | | |
| Total | 77 | 4.45 | | | |

Table A-17 Horizontal Transfer Retention Ratio, Sixth Grade Study

| | | Sum of | Mean | F | F.95 |
|---------------------------------|------|---------|--------|------|------|
| Source of Variation | d.f. | Squares | Square | | .95 |
| (1) Between treatments | 1 | .28 | .28 | 2.55 | 3.98 |
| (2) High abil. vs. low abil. | 1 | .01 | .01 | .09 | |
| (3) Int. abil. vs. low abil. | 1 | .09 | .09 | .81 | |
| (4) Treatment by abil.(1) X (2) | 1 | .02 | .02 | .18 | |
| (5) Treatment by abil.(1) X (3) | 1 | .02 | .02 | .18 | |
| Within Cells | 72 | 8.20 | .11 | | |
| Total | 77 | 8.63 | | | |

Table A-18
Vertical Transfer Retention Ratio, Sixth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | ¥ | F.95 |
|----------------------------------|------|-------------------|----------------|------|------|
| (1) Between treatments | 1 | .80 | .80 | 4.21 | 3.98 |
| (2) High abil. vs. low abil. | 1 | .04 | .04 | .21 | |
| (3) Int. abil. vs. low abil. | 1 | .27 | .27 | 1.42 | |
| (4) Treatment by abil.(1) X (2) | 1 | .13 | .13 | .68 | |
| (5) Treatment by abil. (1) X (3) | 1 | .11 | .11 | .58 | |
| Within Cells | 72 | 13.66 | .19 | | |
| Total | 77 | 15.02 | | | |

Table A-19
Posttest Achievement, Eighth Grade Study

| | | Sum of | Mean | F | F |
|---------------------------------|------|---------|--------|-------|------|
| Source of Variation | d.f. | Squares | Square | | •95 |
| (1) Between treatments | 1 | 3.41 | 3.41 | 0.36 | 4.00 |
| (2) High abil. vs. low abil. | 1 | 383.52 | 382.52 | 39.99 | |
| (3) Int. abil. vs. low abil. | 1 | 73.84 | 73.84 | 7.70 | |
| (4) Treatment by abil.(1) X (2) | 1 | 3.34 | 3.34 | 0.35 | |
| (5) Treatment by abil.(1) X (3) | 1 | 0.02 | 0.02 | 0.002 | |
| Within Cells | 60 | 575.64 | 9.59 | | |
| Total | 65 | 1039.77 | | | |

Table A-20 Posttest Horizontal Transfer, Eighth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F .95 |
|---------------------------------|------|-------------------|----------------|-------|----------|
| (1) Between treatments | 1 | .06 | .06 | .008 | 4.00 |
| (2) High abil. vs. low abil. | 1 | 288.07 | 288.07 | 40.46 | |
| (3) Int. abil. vs. low abil. | 1 | 59.11 | 59.11 | 8.30 | |
| (4) Treatment by abi1.(1) X (2) | 1 | 2.19 | 2.19 | .31 | |
| (5) Treatment by abil.(1) X (3) | 1 | 1.11 | 1.11 | .16 | |
| Within Cells | 60 | 427.45 | 7.12 | | |
| Total | 65 | 7 7 8.00 | | | |

Table A-21
Posttest Vertical Transfer, Eighth Grade Study

| Source of Variation | аe | Sum of | Mean | F | F.95 |
|---------------------------------|------|---------|--------|-------|------|
| | d.f. | Squares | Square | | |
| (1) Between treatments | 1 | 0.38 | 0.38 | 0.06 | 4.00 |
| (2) High abil. vs. low abil. | 1 | 356.73 | 356.73 | 52.69 | |
| (3) Int. abil. vs. low abil. | 1 | 84.57 | 84.57 | 12.49 | |
| (4) Treatment by abil.(1) X (2) | 1 | 34.01 | 34.01 | 5.02 | |
| (5) Treatment by abil.(1) X (3) | 1 | 0.20 | 0.20 | 0.03 | |
| Within Cells | 60 | 406.36 | | | • |
| Total | 65 | 882.25 | | | |

Table A-22
Retention Test Achievement, Eighth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F.95 |
|---------------------------------|------|-------------------|----------------|-------|------|
| (1) Between treatments | 1 | 21.88 | 21.88 | 3.16 | 4.00 |
| (2) High abil. vs. low abil. | 1 | 324.61 | 324.61 | 46.91 | |
| (3) Int. abil. vs. low abil. | 1 | 34.57 | 34.57 | 5.00 | |
| (4) Treatment by abil.(1) X (2) | 1 | 28.18 | 28.18 | 4.07 | |
| (5) Treatment by abil.(1) X (3) | 1 | .02 | .02 | .003 | |
| Within Cells | 60 | 415.18 | 6.92 | | |
| Total | 65 | 824.45 | | | |

Table A-23
Retention Test Horizontal Transfer, Eighth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F .95 |
|---------------------------------|------|-------------------|----------------|-------|----------|
| (1) Between treatments | 1 | .97 | .97 | .13 | 4.00 |
| (2) High abil. vs. low abil. | 1 | 211.28 | 211.28 | 28.90 | |
| (3) Int. abil. vs. low abil. | 1 | 19.11 | 19.11 | 2.61 | • |
| (4) Treatment by abil.(1) X (2) | 1 | .01 | .01 | .001 | |
| (5) Treatment by abi1.(1) X (3) | 1 | 12.02 | 12.02 | 1.64 | |
| Within Cells | 60 | 438.73 | 7.31 | | |
| Total | 65 | 682.12 | | | |

Table A-24
Retention Test Vertical Transfer, Eighth Grade Study

| Source of Variation | d.f. | Sum of Squares | Mean Square | F | F.95 |
|---------------------------------|------|-------------------|----------------|-------|------|
| (1) Between treatments | 1 | 26.73 | 26.73 | 3.76 | 4.00 |
| (2) High abil. vs. low abil. | 1 | 306.07 | 306.07 | 43.05 | |
| (3) Int. abil. vs. low abil. | 1 | 31.11 | 31.11 | 4.38 | |
| (4) Treatment by abil.(1) X (2) | 1 | 11.52 | 11.52 | 1.62 | |
| (5) Treatment by abil.(1) X (3) | 1 | .20 | .20 | .03 | |
| Within Cells | 60 | 426.73 | 7.11 | | |
| Total | 65 | 802.36 | | | |

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Table A-25
Achievement Retention Ratio, Eighth Grade Study

| | Sum of | Mean | F | F.95 |
|------|---------|---|--|---|
| d.f. | Squares | Square | | |
| 1 | .0079 | .0079 | .02 | 4.00 |
| 1 | .0102 | .0102 | .02 | |
| 1 | .2912 | .2912 | . 64 | |
| 1 | .6136 | .6136 | 1.34 | |
| 1 | .0218 | .0218 | .05 | |
| 60 | 27.420 | .457 | | |
| 65 | 28.370 | | | |
| | | d.f. Squares 1 .0079 1 .0102 1 .2912 1 .6136 1 .0218 60 27.420 | d.f.SquaresSquare1.0079.00791.0102.01021.2912.29121.6136.61361.0218.02186027.420.457 | d.f. Squares Square 1 .0079 .0079 .02 1 .0102 .0102 .02 1 .2912 .2912 .64 1 .6136 .6136 1.34 1 .0218 .0218 .05 60 27.420 .457 |

Table A-26
Horizontal Transfer Retention Ratio, Eighth Grade Study

| | | Sum of | Mean | F | F | |
|----------------------------------|------|---------|--------|------|------|--|
| Source of Variation | d.f. | Squares | Square | | .95 | |
| (1) Between treatments | 1 | .007 | .007 | .02 | 4.00 | |
| (2) High abil. vs. low abil. | 1 | .006 | .006 | .02 | | |
| (3) Int. abil. vs. low abil. | 1. | .290 | .290 | .82 | | |
| (4) Treatment by abil.(1) X (2) | 1 | .185 | .185 | .52 | | |
| (5) Treatment by abil. (1) X (3) | 1 | .888 | .888 | 2.51 | | |
| Within Cells | 60 | 21.240 | .354 | | | |
| _. Total | 65 | 22.250 | | | | |

Table A-27
Vertical Transfer Retention Ratio, Eighth Grade Study

| | | Sum of | Mean | F | F |
|---------------------------------|------|---------|--------|------|------|
| Source of Variation | d.f. | Squares | Square | | 95 |
| (1) Between treatments | 1 | .421 | .421 | 1.11 | 4.00 |
| (2) High abil. vs. low abil. | 1 | .340 | .340 | .89 | |
| (3) Int. abil. vs. low abil. | 1 | .050 | .050 | .13 | |
| (4) Treatment by abil.(1) X (2) | 1 | .308 | .308 | .81 | |
| (5) Treatment by abil.(1) X (3) | 1 | .021 | .021 | .06 | |
| Within Cells | 60 | 22.82 | .380 | | |
| Total | 65 | 23.96 | | | |

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| ABSTRACT | | | who molective offects | of two me | thods |
| Three st | udies were c | arried out t | to assess the relative effects | sttest ar | nd a |
| of instructi | on when achi | evement and | transfer were measured by a portruction were (1) teaching the | student b | у а |
| retention te | st. The met | nods of that | estions (intermediate guidance) | and (2) | teach |
| carefully st | ructured seq | dence or due | ncept with illustrations and the | en having | g the |
| ing the stud | ents by stat | ting each con- | lied in these concepts (maximal | guidance | 2), |
| students pra | ictice the be | Havror emboo | e subjects and a programmed pre | sentation | nof |
| One of the s | tuales used | tourth grade | gers, the second used sixth gra | de subjec | cts |
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The results indicated that in almost all cases there was a distinct ordinal relation for the three ability subgroups. Where this failed to be significant, the mean scores were so low that comparisons were not meaningful. cant differences pertaining to treatment effects all favored the maximal guidance treatment. This occurred in the fourth grade study when retention vertical transfer was measured, in the sixth grade study when vertical transfer retention ratios were compared, and in the eighth grade study for the high ability subgroup when posttest vertical transfer and retention achievement were measured. It was concluded that, when teaching mathematical concepts by programmed instruction, a relatively high level of guidance should be provided.



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