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This guide for teachers of mathematics in the Career Guidance Program has been designed to help to develop a new interest in education for pupils who have at some point in their career lost interest in academic learning. The program was organized for the ninth grade, since this was the terminal year of the junior high school and since at this time young people must decide whether to continue their education. In this guide, mathematical concepts are presented functionally in realistic social situations for the purpose of helping indifferent, reluctant learners become active, interested participants. Included are topics related to maps, measuring, statistics, time, positive and negative numbers, using of money, and decimals. (RP)

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CAREER
GUIDANCE
SERIES

Mathematics

BOARD OF EDUCATION • CITY OF NEW YORK

Foreword

The aim of the Career Guidance Program is to develop a new interest in education for pupils who have at some point in their career lost interest in academic learning, and in their future in general. It is hoped that by providing these pupils with strong and wholesome incentives they not only will be redirected but they will also be motivated to prepare themselves for a productive life as self-sustaining adults.

This guide for teachers of mathematics in the Career Guidance Program has been designed with this objective in mind. Mathematical concepts are presented functionally in realistic social situations for the purpose of so motivating these youngsters that they will change from indifferent, reluctant learners to active, interested participants. This bulletin has been developed after three years of experimentation and evaluation in Career Guidance classes.

I wish to thank the many devoted teachers and supervisors who so generously contributed to this uniquely valuable guide.

Deputy Superintendent
JOSEPH O. LORETAN*

July, 1966

**Deceased*

pupil, individually, at least once a week. An industrial arts teacher was also assigned full-time to instruct the pupils in prevocational and avocational skills.

Three years of experimentation and a study of similar programs throughout the nation showed that a new teaching approach was essential in every subject area, if these youngsters were to be rehabilitated and redirected. Adaptations or "watered-down" versions of the traditional curriculum without a modified approach presented learning situations which were only too familiar and were filled with the failures and frustrations of the past. It was also evident that once these pupils had spent some time in a Career Guidance class they began indicating that they no longer wanted to leave school to go to work; they now wanted to prepare themselves for high school.

Thus, in February 1963, a team of specialists in each of the curriculum areas began to work on specially-designated teaching guides in Guidance and Job Placement, Language Arts, Speech, Social Studies, Science, Mathematics and Industrial Arts. To prepare these guides the curriculum specialists visited each of the schools that had been in the Career Guidance Program from two to five years and studied the teacher-prepared materials in use, observed and conferred with the pupils in the classes, and interviewed the teachers and supervisors to become oriented with the pupils' backgrounds, aspirations, cultures, interests, and needs. Workshop committees composed of teachers, advisors, and assistant principals were organized to work with each curriculum specialist. As the teaching material was developed it was tried out experimentally in selected schools and evaluated.

By September 1963, teaching guides in seven subject areas were made available in mimeograph form to all the schools in the program. The subject matter developed departed largely from the job-centered themes and concentrated on the skills and subject matter necessary for further study in high school; less on theory and more on the functional and manipulative aspects of each subject area so as to present the pupils with true-to-life problems and situations. Beginning September 1963, the area of Office Practice was included to prepare the pupils with immediate saleable skills for obtaining part-time jobs and to instill a desire for further vocational work in high school.

Through a continuous program of evaluation by teachers, supervisors, and curriculum consultants, the teaching guides were revised and extended and the present series evolved: Guidance and Job Placement, Language Arts, Mathematics, Social Studies 1, Social Studies 2, Social Studies 3, Speech, Science, Industrial Arts, and Office Practice (Pupil Material, Teacher's Manual).

In September 1965, with the reorganization of the schools in New York City, the 8th grade became the terminal grade in some junior high schools. Thus, the Career Guidance Program was placed in the 8th grade of seventeen of these schools. At present there is a total of fifty-two schools in the Career Guidance Program.

Acknowledgments

This resource guide was prepared under the general direction of Max Rubinstein*, Assistant Superintendent, Junior High School Division, with the cooperation of Martha R. Finkler†, Acting Associate Superintendent, Junior High School Division. Irving Anker, appointed Staff Superintendent in February 1966, Office of Junior High Schools, has encouraged this project with his deep interest and cooperation.

Gida Cavicchia, Coordinator of the Career Guidance, served as project director with the cooperation of Willia Peace, Coordinator of Pupil Personnel of the Career Guidance Program.

Ada Sheridan, Junior High School Mathematics Coordinator, was the principal writer of the original material during the school year 1962-63, and of the revised material after a year of tryout and evaluation. She worked with the supervisors and teachers of mathematics in the Career Guidance Program of the following schools: J13M, J118M, J139M, J55X, J60X, J118X, J136X, J136K, J210K, J263K, J142Q, J204Q.

Marguerite Ferrerio, Sonia Koslow, Margaret Lalor, Estelle Gillman, and Martin Weber, Junior High School Reading Coordinators, edited the material.

Acknowledged with gratitude and appreciation is the assitance given by the many supervisors and teachers who used the material in classroom tryout and played a part in the evaluation and revision:

Manhattan—J3, J13, J22, J43, J44, J88, J99, J115, J118, J120, J139, J164

Bronx—J22, J55, J98, J118, J133, J136, J142

Brooklyn—J33, J49, J50, J64, J126, J136, J166, J210, J263

Queens—J16, J142

George Grossman, Acting Director of Mathematics, and Frank Wohlfort, Acting Assistant Director of Mathematics, served as consultants.

Special thanks are extended to William H. Bristow, Assistant Superintendent, Bureau of Curriculum Development, and Leonard Simon, Acting Assistant Director, Bureau of Curriculum Development.

Elena Lucchini, assistant to Editor Aaron N. Slotkin, Bureau of Curriculum Development, designed the cover and was responsible for overall production.

*Now Dist. Supt., Dist. # 29, Q
†Retired

Contents

Introduction

This resource guide for teachers of mathematics in the Career Guidance Program provides material for the development of concepts and skills necessary for recognizing and handling quantitative aspects in practical problem situations. The mathematics of travel, earnings, consumer purchases, taxes, savings, games, and sports are treated at an appropriate level of difficulty through a mature approach. Thus, it is possible for the teacher to reteach mathematical concepts and skills in informal and socially meaningful situations. In addition, the program provides time (two periods a week) for the systematic testing and re-teaching of fundamental concepts and skills of arithmetic. In keeping with the overall plans for mathematics K-12, a modern approach in teaching and some contemporary principles, concepts, and topics have been included. If pupil ability and interest warrant it, and time permits, the teacher should present further topics as presented in the revised curriculum bulletin, "General Mathematics—9th Year", or in any of the latest mathematics textbooks.

The objectives of this guide are:

1. to help the pupils continue the development of concepts and processes previously taught

2. to help the pupils realize that mathematics is an indispensable tool of daily living

3. to help the pupils apply mathematical skills, according to their interest and ability, in the field of work and recreation and in their everyday experiences as consumers of goods and services

4. to help the pupils use mathematics as a tool in other areas of schoolwork as well as in future studies in mathematics

5. to develop an appreciation of mathematical thinking.

The material in this guide is presented in the form of daily lesson plans. Two lessons may be combined and taught in one for the more apt pupils, or one lesson may be divided into two or more lessons. It is suggested, however, that the units be taught consecutively for optimum results, since the sequence of presentation is graded. At the beginning of each unit, materials and, in some cases, directions have been included to assist the teacher. For effective teaching of each unit the materials needed should be acquired or prepared before the actual teaching of the unit begins.

Unit I

LOCATING SITES AND TRAVELING IN NEW YORK CITY

The specific objectives of this Unit are to have the pupils learn the basic geometric concepts, and the ability to use them.

1. Point, Line and Plane
2. Direction
3. Reading and Interpreting a Map
4. Horizontal and Vertical Axes

Teaching Aids

1. New York City Streets and Roads Map
   May be obtained at gasoline service stations.
2. Atlas of New York City — Five Boroughs — No. 145
   World Map Co. Inc., P. O. Box 336, Tarrytown, N. Y.
3. Official New York City Subway Map
   N. Y. City Transit Authority, 370 Jay Street,
   Brooklyn, N. Y. 11201
4. Wall Map of New York City
5. Board Graph
6. Worksheets for Lessons 1 and 3

Correlation
Guidance and Social Studies

Lesson 1

LOCATING OUR SCHOOL

Motivation

Discuss possibility of part time work as a C. G. pupil.

If the employer wants you to locate or tell exactly where your school is and how you would travel to work, what would you tell him?

Evaluate responses. Point out need for specific directions.

Lead to aim of the lesson.

Aim

How can we locate our school?

Development

What is the exact address of the school?

What is the best known public site north of the school? south, east and west of the school? (Park, River, etc.)

Starting at the school, in what *direction* is (name a site)?
(Continue for other directions.)

Starting at (name a site) in what direction is the school?

Have pupils realize that it is easier to locate a special place if we have a known starting place or *point of origin.* (Distribute diagrams — prepare board diagram.)

Write name, or number, of streets in front of school, back of school, to left and right of school. (Teacher labels board diagram.)

Write name, or number, of street one block above front of school; below school; left, right of school.

Relate *above* with *north; below* to *south,* etc. Establish fact that *facing north* — south is the opposite direction, east is to the right, west is the opposite direction, or to the left.

Name best known site north of school and write it above the diagram.

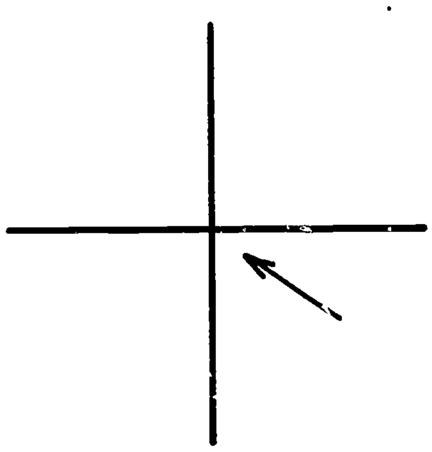
Name best known site south of school and write it below diagram. Continue for east and west of school.

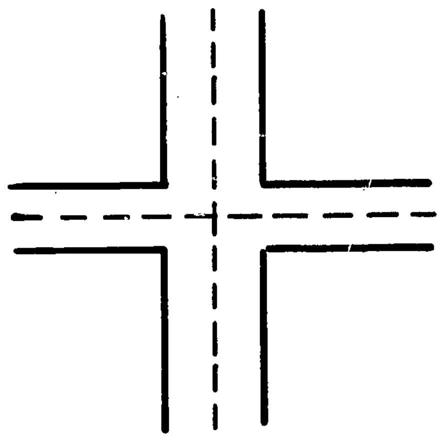
Using diagram, have pupil trace school street until he comes to the first crossing — or crossroads. Ask pupils if anyone knows another name for crossroad.

If no one responds, tell them a *crossroad* may be called an intersection.

Explain that when two lines meet or cross each other, we say the lines *intersect.*

The point where the lines cross is called the point of *intersection.* Relate to intersections of streets and roads.





Have pupils continue tracing school street to next intersection and write name, or number, of street.

Do you know what we call lines that run from left to right or right to left?

← → If possible, obtain response, *horizontal lines.*

(Continue with lines running up and down. ↑ ↓ *(vertical lines)*

Summary

We can locate a special place by telling in what direction and how far it is from a known place.

Practice

Write names, or numbers, of horizontal streets and roads on diagram.

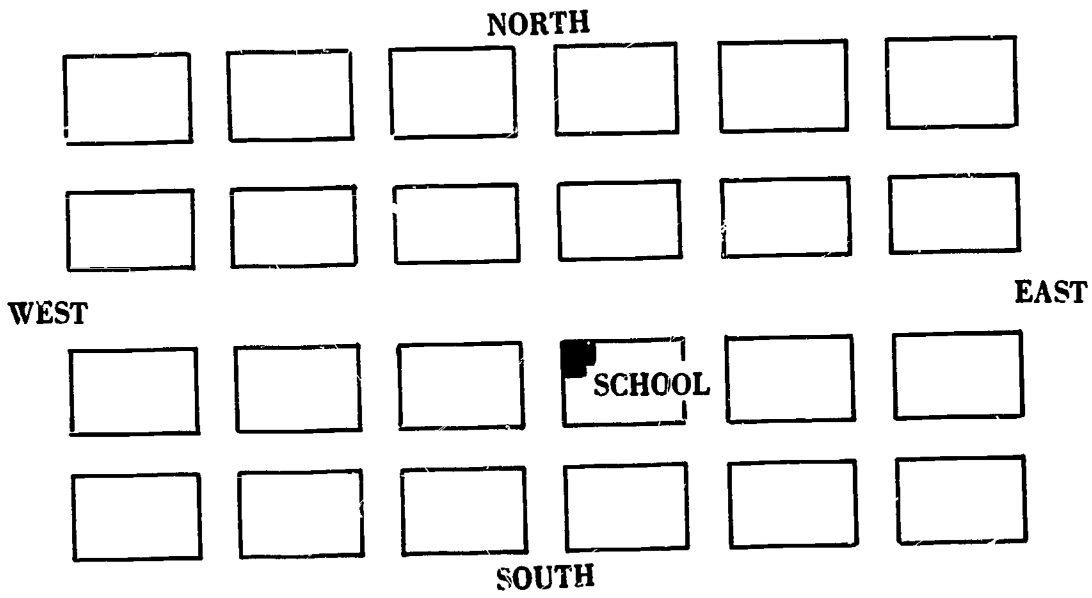
Write names or numbers of vertical streets and roads on diagram.

(Name of one street) intersects ....

Homework

Draw a diagram locating your home. (Clear, complete, attractive diagrams posted.)

Adapt diagram to individual school location. Duplicate for pupil use.



Lesson 2

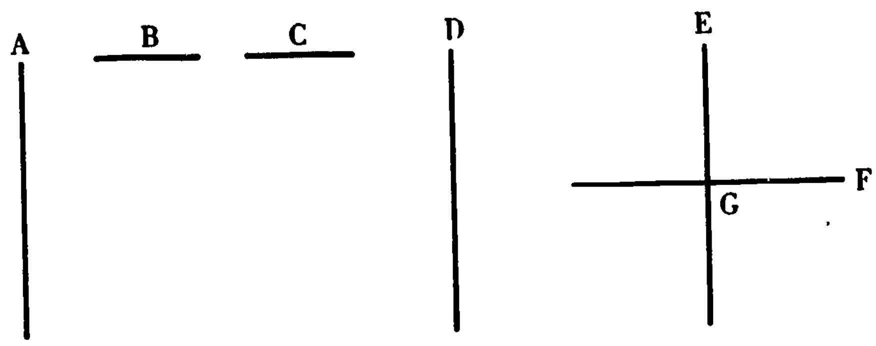
USING A MAP TO LOCATE SPECIAL PLACES

Warmup (all or part)

The opposite of:

above is below   west is .... is north
right is ....   below is ....   east is ....
north is ....   left is ....

Horizontal lines are lines that run from .... to ....
Vertical lines are lines that run from .... to ....



Lines .... , ... and .... are horizontal lines.
Lines .... , ... and .... are vertical lines.
Lines E and F intersect at point ....

Motivation

We can locate a special place if we relate it to a known or familiar site. How do we locate or find a special site if there is no one to tell us? Have pupils suggest ways. Lead to use of maps.

Aim

How can we use maps to locate sites?

Development

To locate our school on a map, what is the first thing we should know? (Answer is *borough*. Write appropriate borough on board.)

Is the school in the northern, southern, eastern or western part of the borough?

Think of the streets near and around our school. *East* 12th Street, *West* 190th Street, could help us approximate eastern or western part of the borough. (Write appropriate direction under borough.)

The size of the number of the street could help us approximate northern or southern part of the borough of Manhattan. 12 is a small number, therefore, 12th Street would probably be in the southern part; 190 is a large number, therefore, it would probably be in the northern part of the borough.

(Distribute New York City Streets and Roads Map. First, look at city map. Have pupils pick out borough, and locate it in relation to other boroughs. South of ...., etc.)

Do the numbered streets run vertically or horizontally?
Lines, not horizontal nor vertical are called *oblique* lines. *(slanting)*

In what part of the borough is the lowest numbered street? Highest?
Do the street numbers get larger, or *increase*, going N. S. E. or W.?
Do the street numbers get smaller, or *decrease*, going N. S. E. or W.?

Have pupils locate the school according to:
1. Nearest streets and/or avenues
2. In relation to a familiar site in the vicinity.

(Ask similiar questions to familiarize pupils with city map.)

Summary

Using a New York City map to locate a site:

1. Find the borough.
2. Find the part of the borough. *(north, northeast, etc.)*
3. Find the street or avenue the site is on.
4. Find the nearest avenue or road that intersects the address street or avenue.

Practice

Assign a number of familiar sites for pupils to locate. Give the name and address of the site. Have the pupils find borough, and the names of the four streets or avenues of block site is on.

Suggested Sites

Yankee Stadium, Fordham University, Couty Court (Bronx)

Board of Education, Coney Island, Brooklyn Museum, Prospect Park (Brooklyn)

Madison Square Garden, United Nations, Metropolitan Museum of Art, City Hall, Rockefeller Center (Manhattan)

Shea Stadium, Kennedy Airport, LaGuardia Airport, Forest Park (Queens)

Homework

Assign 3 or 4 sites for the pupils to locate. To vary the assignment give them the location and have pupils name the site.

Lesson 3

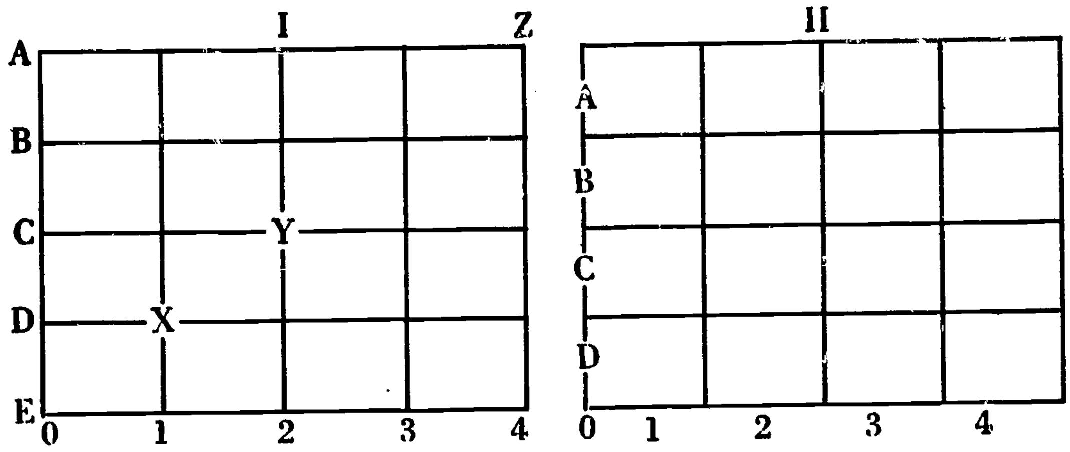
USING A MAP INDEX

Warmup

Complete the following, using Diagram I:

1. Lines ...., ..., ..., are horizontal lines.
2. Lines ...., ..., ..., are vertical lines.

3. Point X is the intersection of lines D and ....
4. Point Y is the intersection of lines .... and ....
5. Point Z is the .... of lines .... and ....



Motivation

We can locate places in New York City if we have a city map, and know the exact address and borough of the place to be located.

Have pupils locate Hayden Planetarium, 81st Street and Central Park West, in Manhattan. (After a short time, stop class. Ask: How many have located Hayden Planetarium? How many pupils need more time?)

Aim

How can we use maps to save time in locating places?

Development

Note differences between graphs I and II. In Graph I, distance from 0 to 1, 1 to 2, etc. called *intervals;* as are distances from A to B, etc. in Graph II, the space area of box in row D-1 is the interval.

Have pupil place index finger on right hand on number 4 of base, Graph II; index finger of left hand on letter D of vertical starting place.

Have pupils run fingers in appropriate direction until they meet or intersect. Continue with B-6, A-2, etc.

Explain: Horizontal base line is called the *Horizontal Axis.* First vertical line is called the *Vertical Axis.*

Ask pupils to explain term *index finger.* (*forefinger usually used in point-*

ing) Develop meaning of *index. (points out, indicates, guides)* Look at *street index* on map. (in Atlas at end of book)

What are the major divisions of index? *(Boroughs)*
How are numbered streets listed? *(Chronologically)*
How are street names listed? *(Alphabetically)*
What information does the *key* give us? *(letter and number of axes)*
How can the *key* help us find the street or avenue? *(Point of intersection of horizontal and vertical directions in key will indicate approximate location)*

Have pupils use Manhattan street index to locate Museum of Natural History.
Locate Central Park West on map; follow to 79th Street.
Using index, locate Grand Central Station, 42nd Street and Lexington Avenue in Manhattan.

Summary

To locate a special place quickly and correctly, we use a map of the area and the map index.

Practice

Have pupils individually locate specific sites using index. (Suggested: sites in other boroughs listed in lesson 2)

Homework

Distribute copies of Pupil Worksheet for home completion.

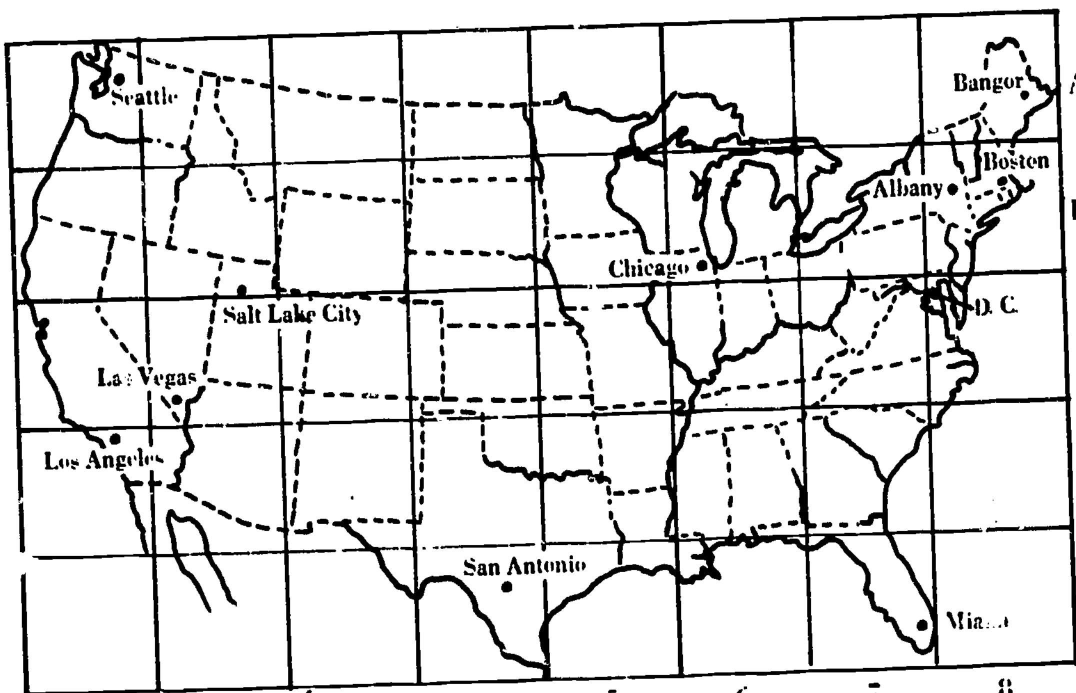
PUPIL WORKSHEET

Fill in the missing information:

City	*Key*
Albany, New York	B — 8
Bangor, Maine	A — ....
Boston, ...	.... — ....
Chicago, Illinois	.... — ....
Las Vegas, ...	.... — 2
Los Angeles, ...	D — 1
Miami, Florida	.... — ....
Salt Lake City, Utah	.... — ....

San Antonio, ... E—4
Seattle, ... A— ....
Washington, District of Columbia C—8

The United States



Lesson 4

TRAVELING BY SUBWAY

Warmup

The opposite of:

North is ....
East is ....
Downtown is ....
East side is ....
Horizontal lines run .... and ....
Vertical lines run .... and ....
Lines that are not horizontal or .... are called oblique.
Lines that meet, or ...., are intersecting lines.

Motivation

Before holidays, junior high school boys can get jobs delivering packages, flowers, candy, etc. In the job interview, questions asked are: "How would you travel from one part of the city to another? What subway would you take?"

Aim

How can we find the subway line that will take us closest to the place we are going?

Development

City maps help us locate streets and avenues. The New York City Transit Authority prints and gives away subway maps.

Have pupils open and examine maps. Note: Horizontal and vertical axes; *Map and Station Guide* instead of index.

Summary

To locate and travel to places in New York City, use a city street map and subway map.

1. Take west side subway lines to get to west side of boroughs.
2. Take east side subway lines to get to east side of boroughs.
3. Take express train for speed, but be sure to change to local train if site you want is near a local station.
4. If you have to change from one division to another, be sure you know the transfer station.

Practice

Starting from school, how would you get to:

1. Yankee Stadium — River Avenue and 161st Street, Bronx
2. Madison Square Garden — 8th Avenue and West 50th Street, Man.
3. Hayden Planetarium — 81st Street and Central Park West, Man.

   Answer these questions for each of the places above:

   a. At what stations would you take the train?
   b. At what stations would you get off the train?
   c. What changes would you make if any are needed?

Homework

To travel by subway to each of the following places:

The Empire State Building — Fifth Avenue and 34th St., Manhattan
Grant's Tomb, 125th Street and Riverside Drive, Manhattan
Public Library, 42nd Street and Fifth Avenue, Manhattan

1. Which subway station would you leave from if you started the trip from home?
2. At which station would you get out for each of the above place?
3. What transfers would you make to get to each of the above places, if you began your trip from home?

TEST

Correct spelling of mathematical terms

locate	origin	horizontal	index	
vertical	guide	intersect	axis	direction

ANSWER ALL THE QUESTIONS.

1. Lines that run left and right, or East and ...., are .... lines.
2. Lines that run up and down ...., or........ and South are................lines.
3. Lines that meet and cross are .... lines.
4. The starting place may be called the 'point of ....'
5. Facing North:
   West is to our ....
   East is to our ... .
   South is . ...
6. In the box:
   Write the 4 points of directions.
   Draw an arrow pointing South.
7. We can use a .... to locate a special place quickly.
8. What do we call: the horizontal base line? ....
   the first vertical line? ....
9. Using a map index the *key* is A - 5:
   we will find A on the .... axis
   5 on the .... axis

10. The street we are looking for will be about the place
A and 5 .... or meet.

11. What can we use to find the correct subway train to travel to work, or to the theatre? ....

12. What does *express train* mean? ....

13. What does *local train* mean? ....

Use maps for the following:

14. Starting from the subway station nearest your school, how would you travel to Times Square, 42nd Street and Broadway, Manh[illegible]an?
    a. What subway line would you use? ....
    b. Would you travel by express or local train? ....
    c. If necessary, where would you transfer? ....

15. Locate the school on the map.
    a. What is the key for the street address of the school? ....
    b. *Key* for the street directly in back of school is ....
    c. Name a well known place north of school ....

Unit II

MATHEMATICS IN DAILY LIVING

The specific objectives of this unit are to have the pupils realize:

1. The need for mathematical concepts and skills to handle quantitative aspects in the field of work, recreation, as a consumer of goods and services, and as groundwork in other areas of school work
2. There are facts, concepts and skills they need to learn.

Teaching Aids

Pupil inventory test

Correlation

Guidance

Lesson 1

MATHEMATICS IN WORK AND PLAY

Warmup

$9 \times 7 =$	$81 \div 9 =$	$20 \times 100 =$
$14 + 23 =$	$62 - 29 =$	$300 - 46 =$

Motivation

Relate the mathematical facts, skills, judgments, etc., to the pupil's use daily. Start with time they arise, size of clothes, amount of food they eat, distance to school, money for fare, lunch, etc. Have the pupils realize that the mathematics we learn in school is to help us find the answers to the *how much* and *how many* questions we meet daily.

Aim

How do we find the total number of things when we combine smaller groups or the number of things when we separate a group?

Development

Ask the pupils questions to elicit the following information:

1. Combining two sets. *(addition)*
2. Combining a number of equivalent sets. *(mulitiplaction)*
3. Separating one set from another to find the difference between the numbers of the two sets or to find the number of elements remaining. *(subtraction)*
4. Separating one set into smaller equivalent subsets. *(division)*
5. Review terms involved: *Sum, Total, Remainder, Difference, Factor, Product, Divisor, Quotient*

Conclude that for the four operations, it is important to understand when to use each, and how to compute quickly.

Summary

There are four fundamental operations: *Addition, Multiplication, Subtraction, Division*

Practice and Homework

Assign addition, subtraction, multiplication, and division examples.

Lesson 2

INVENTORY TIME

Motivation

Explain briefly that in order to gain the greatest benefit from the mathematics periods it is important for each pupil to be aware of the concepts and facts he knows and can use and those he may need to learn.

Development

Distribute mimeographed copies of inventory test.

Have the pupils work independently, and be sure they realize there is no penalty for errors.

Warn them not to spend too much time on any one example, but to answer all the items they know and are sure of first, then return to others.

Note

Teacher corrected papers should be returned to pupils after areas of common weaknesses and specific individual needs are noted.
No homework assignment.

INVENTORY TEST FOR SEPTEMBER

Name .... Class ....

School .... Date ....

1. The numeral 69 may mean .... tens and .... ones
2. The numeral 69 may mean 5 tens and .... ones
3. In the numeral 150 the 5 means 5 ....
4. In the numeral 105 the 5 means 5 ....
5. In the numeral 510 the 5 means 5 ....

Add:

6. 34	7. 57	8. 4.3	9. .26	10. 865
52	34	7.6	.91	98

Subtract:

11. 47	12. 72	13. 501	14. 3.7	15. 6.77
24	48	169	1.0	.02

What fundamental operation is performed when:

16. The Product is found .... 17. The Difference is found ....

18. The Remainder is found .... 19. The Quotient is found ....

Which is larger?

20. $\frac{1}{3}$ of a number or $\frac{2}{4}$ of a number? ....

21. 6.72 or .911? ....

22. 10% of 30 or 30% of 10? ....

23. $\frac{4}{6} + \frac{1}{3}$ or $\frac{5}{6}$? ....

Find:

24. $\frac{3}{4}$ of $\frac{5}{8}$ = ...

25. $4\frac{3}{4} \div 1\frac{1}{2}$ = ...

26. $\begin{array}{r} 54 \\ \underline{\times\ 6} \end{array}$

27. $\begin{array}{r} 19.8 \\ \underline{\times\ \ 6} \end{array}$

28. $6\overline{)\,.522}$

29. $.6\overline{)\,522}$

30. $\begin{array}{r} 8\frac{3}{4} \\ \underline{-3\frac{1}{8}} \end{array}$

31. $\begin{array}{r} 8.154 \\ \underline{-2.317} \end{array}$

32. $\begin{array}{r} 325 \\ \underline{\times\ 37} \end{array}$

33. $21\overline{)\,504}$

Write the missing words:

34. Factor × Factor = ...

35. Quotient = Dividend ÷ ....

36, 37, 38. 7841 = 7 .... + 8 .... + 4 tens + 1 ....

or

39. 7841 = ...

40. $\frac{3}{10}$ = .3 or ....%

Unit III

SALARIES–TAXES–TAKE HOME PAY

The main objectives of this Unit are to help the pupil learn:

1. To understand the computation involved in salaries and taxes
2. To report and record data
3. To prepare a data table
4. To read and interpret data tables.

Teaching Aids

1. Employer's Tax Guide
   Bureau of Internal Revenue.
2. Worksheet for lesson 3 and lesson 6.

Correlation

Social Studies (Topic II, "You and the Working World")

Lesson 1

SALARY AND WAGES

Warmup

75×2	75×4	75×6	75×8	75×10
90×1	90×3	90×5	90×7	90×9

Motivation

Part-time employment is usually paid by the hour. To know how much money will be received on pay day, we must know:

How many hours the *employee* worked
How much the *employer* paid for each hour of work

Aim

How can we find, and keep accurate records of earnings?

Development

If the pay is $1.25 for one hour, then how much would you earn in 2 hours? 3 hours? 5 hours?

Develop first table with class.

Let us make a list or table of earnings for $1.25 *per hour*.
To know what the table means, we will give it a *title*.

Wages at $1.25 per hour	
1 hour at $1.25 =	$ 1.25
2 hours × 1.25 =	
3 hours × 1.25 =	
Continue to	
10 hours × 1.25 =	$12.50

Have pupils develop a table for $1.50 per hour up to 10 hours. Be sure to have pupils write a title, "Wages at $1.50 Per Hour," above the table.

Using the charts, have pupils find:

How much money an employee would earn at $1.25 an hour if he worked 6 hours? 9 hours? 2 hours? 3½ hours?

Similarly for $1.50 per hour.

Have pupils realize we can find the answer quickly and accurately by using the tables.

When pupils work after school they sometimes work a day or two at one job, and a day or two at another job. If the pay is different, a table could be made for each rate of pay. An easier way would be to keep a diagram or *chart.*

Have pupils suggest a title. Select something such as *Wages Earned.*

Have pupils write the title of the chart.

Select rates: 75¢, 90¢, $1.25, $1.50. At left write, *Rate of Pay.* List selected rates, starting with the highest.

Under the last rate have pupils draw and extend a horizontal line.

Divide the line into 10 approximately equal parts. Then number parts from 1 to 10. Write, *Hours Worked* at the left.

Have pupils draw a vertical line cutting off rate of pay.

Have pupils draw vertical and horizontal lines completing grid.

Place diagram on board:

WAGES EARNED PER HOUR											
Rate of Pay	$1.50										
	$1.25										
	.90										
	.75										
Hours Worked		1	2	3	4	5	6	7	8	9	10

Have pupils compare chart with map divisions, e.g., Horizontal Axis, Vertical Axis, Key and Intersecting Points.

Summary

We can find, and keep accurate records of, earnings by constructing and using charts and tables.

Practice

Using tables developed, and warm up exercises, fill all squares in the diagram. Then answer the following:

How much would you earn if you worked:

1. 7 hours at 75¢ per hour?
2. 4 hours at 90¢ per hour?
3. 3 hours at $1.50, and 4 hours at $1.00 per hour? etc.

Homework

Make a wage chart showing wages at 50¢, 60¢, 70¢, 80¢, 90¢, and $1.00 for 1 to 10 hours.

Lesson 2

EARNINGS AND TAXES

Warmup

1/5 of $5.00	1/5 of $ 9.00	1/5 of $11.60
1/5 of 1.00	1/5 of 12.00	1/5 of 17.00
1/5 of 2.00	1/5 of 2.50	1/5 of 7.50

Background

Every able-bodied citizen earns money for:

Necessities (food, clothing, shelter)
Luxuries (automobiles, toasters, washing machines)
Entertainment (parties, theatre, vacations)

All citizens need protective services such as protection of person and property. The U. S. Government, or Federal Government, provides some of these services, such as armed forces for protection against invasion; judicial laws to protect personal rights and privileges; etc. To pay for these services, the government requires that the employer withhold, or deduct, a part of the employee's earnings, or income. The money deducted for government use is called *Income Tax*, or *Withholding Tax.*

A tax is also deducted to protect the earner when he is disabled, ill, or too old to work. This tax is known as the *Social Security Tax,* or *F.I.C.A.* All employees pay the F.I.C.A. Tax.

Each worker is allowed an exemption of $600, which is tax free, for his own support, and $600 exemption for every person he supports. Every person whom an employee supports is called an *exemption.* If the worker supports only himself, he claims one exemption. If he supports himself and his mother, he claims two exemptions, etc. Employees who earn less than $600 do not pay *Income Tax.* In that case, the amount withheld from his weekly wages is returned, or *refunded,* by the government at the end of the year.

The government prints an *Employer's Tax Guide* to aid with computing payroll deductions. (Distribute copies of Guide.)

Aim

How do tables and charts help in keeping salaries and taxes accurate?

Development

Have pupils use *Wages Earned* chart they developed to find earnings in each of the following:

Rate of pay	*Hours worked*
$ .75 per hour	7 hours
1.00 per hour	17 hours
1.15 per hour	22 hours
1.25 per hour	24 hours

After checking answers, have pupils examine tax charts to note and explain: *Wages are at least; but less than* and exemptions.

Have pupils prepare table, listing answers to above problems. Have them find tax for each by using charts. (Do not have them compute Take Home Pay.)

SALARY	INCOME TAX (1 exemption)	SOCIAL SECURITY TAX
$ 4.20		
10.70		
19.80		
24.00		

Summary

Tables and charts are used to find salaries and taxes quickly and correctly.

CLASS PRACTICE AND HOMEWORK

Use the tables to find the correct salary, income tax and Social Security tax and list them on the chart. He claimed only *one* exemption.
An employee worked:

1. 10 hours at $ .90 per hour
2. 15 hours at .80 per hour
3. 9 hours at 1.00 per hour
4. 19 hours at 1.00 per hour
5. 22 hours at 1.15 per hour
6. 32 hours at .90 per hour
7. 17 hours at 1.25 per hour
8. 40 hours at 1.50 per hour
9. 23 hours at 1.00 per hour
10. 48 hours at .75 per hour

SALARY	INCOME TAX	SOCIAL SECURITY TAX
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		

Lesson 3

TAKE HOME PAY

Warmup

$50.00	$43.40	$ 6.60	$50.00
—6.60	—1.56	+1.56	—8.16

$33.75	$30.05	$ 3.70	$33.75
—3.75	—1.05	+1.05	—4.75

Motivation

In order to help the employee pay taxes, the government has the employer withhold a small part of his earnings every pay day.

If the salary is $50.00, but there is $41.84 in the pay envelope, how does the employer explain the *deductions* to the employee?

Aim

How are salary deductions reported to the employee?

Development

Explain: Either on the pay check or with the cash pay envelope there is a statement that looks something like this sample: (On board)

Total Wages	F.I.C.A. or S.S. Tax	Federal Income Tax	Other	Amount Received
$50.00	$1.56	$6.60		$41.84

The statement may be in this form:

Salary		$50.00
Deductions		
Federal Income Tax	$6.60	
F.I.C.A.	1.56	
Other		
Amount Enclosed		41.84

Ask pupils: Why is $6.60 deducted? Why is $1.56 deducted?

What does *other* mean? (Hospitalization, State Tax, Social Security, etc.)
How much money does the employee actually take home?
The money the earner takes home is called *Take Home Pay.*
How did the employer know that *$41.84* was the correct amount of money?

Lead pupils to the generalization:

Salary — Deductions = Take Home Pay
(Salary minus Deductions equals Take Home Pay)

Have pupils prepare a pay statement.

Salary $33.75. Deductions, etc. as sample on page 23.

Have pupils find "Take Home Pay," using Tax Tables, and claiming one exemption.

Working with the same salary, $33.75, have pupils find *Take Home Pay*, if 2 exemptions are claimed.

Summary

Rate of Pay × Hours worked = Salary
Salary — Deductions = Take Home Pay
Tax tables are used for accuracy and speed.
Duplicate for distribution and have pupils complete:

PRACTICE AND HOMEWORK

Salary	Exemptions	Income Tax	F.I.C.A.	Take Home Pay
1. $28	1			
2. 28	2			
3. 36	1			
4. 44	0			
5. 44	2			
6. 36	1			
7. 50	1			
8. 21	2			
9. 21	1			
10. 40	2			

For extra credit, find the *Take Home Pay* of the above salaries if 3 exemptions were claimed.

TEST

Name______________________________ Class______________

School______________________________ Date______________

1. A person's salary is the .... he earns for work he does.

   To know what his salary will be, a worker must know

2. How many .... he has worked, and

3. The rate of .... for each hour.

To save time we say:

4. Hours Worked × Rate of Pay = ...

What is the salary if an employee has worked:

5. 17 hours at 75¢ per hour? Salary is $.....................

6. 28 hours at $1.25 per hour ?.....................

7. Is the salary earned and the take home pay usually the same amount of money?
   Check either: Yes .... or No ....

8. What does the employer withhold or deduct from the worker's salary?

   ....

9. Every one who works must pay the Social Security ....

10. Every one who earns $600 or more pays a Federal ....

11. Which is *less* money, the:
    salary earned or *take home pay?* ....

12. To save time we say:
    .... — Deductions = Take Home Pay.

13. Employers use Tax .... to deduct the amount of money for taxes.

14. If an employee worked 48 hours, was paid $1.25 per hour, and claimed only 1 exemption: (use tables)

    a. What was his salary? $................................
    b. How much was deducted for Income Tax? $................................
    c. How much was deducted for S.S. Tax? $................................
    d. What was his Take Home Pay? $................................
       If he claimed 2 exemptions
    e. What would his Income Tax be? $................................
    f. What would his S.S. Tax be? $................................

How To Use the Wage-Bracket Table Method of Income Tax Withholding

Income tax withholding tables for single and married taxpayers have been provided for the following payroll periods in the order named: Weekly, biweekly, semimonthly, monthly, and daily or miscellaneous. The payroll period and marital status of the employee determines the table to be used.

If the wages exceed the highest wage bracket in the applicable table, in determining the amount to be deducted and withheld, the wages may, at the election of the employer, be rounded to the nearest dollar.

WEEKLY — SINGLE
PAYROLL PERIOD — PERSONS

And the wages are—		And the number of withholding exemptions claimed is—										
At least	But less than	0	1	2	3	4	5	6	7	8	9	10 or more
		The amount of income tax to be withheld shall be—										
$0	$4	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
4	5	.10	0	0	0	0	0	0	0	0	0	0
5	6	.20	0	0	0	0	0	0	0	0	0	0
6	7	.40	0	0	0	0	0	0	0	0	0	0
7	8	.50	0	0	0	0	0	0	0	0	0	0
8	9	.70	0	0	0	0	0	0	0	0	0	0
9	10	.80	0	0	0	0	0	0	0	0	0	0
10	11	.90	0	0	0	0	0	0	0	0	0	0
11	12	1.10	0	0	0	0	0	0	0	0	0	0
12	13	1.20	0	0	0	0	0	0	0	0	0	0
13	14	1.40	0	0	0	0	0	0	0	0	0	0
14	15	1.50	0	0	0	0	0	0	0	0	0	0
15	16	1.70	0	0	0	0	0	0	0	0	0	0
16	17	1.80	0	0	0	0	0	0	0	0	0	0
17	18	2.00	0	0	0	0	0	0	0	0	0	0
18	19	2.10	.20	0	0	0	0	0	0	0	0	0
19	20	2.30	.30	0	0	0	0	0	0	0	0	0
20	21	2.40	.40	0	0	0	0	0	0	0	0	0
21	22	2.60	.60	0	0	0	0	0	0	0	0	0
22	23	2.70	.70	0	0	0	0	0	0	0	0	0
23	24	2.90	.90	0	0	0	0	0	0	0	0	0
24	25	3.00	1.00	0	0	0	0	0	0	0	0	0
25	26	3.20	1.10	0	0	0	0	0	0	0	0	0
26	27	3.40	1.30	0	0	0	0	0	0	0	0	0
27	28	3.50	1.40	0	0	0	0	0	0	0	0	0
28	29	3.70	1.60	0	0	0	0	0	0	0	0	0
29	30	3.90	1.70	0	0	0	0	0	0	0	0	0
30	31	4.10	1.90	0	0	0	0	0	0	0	0	0
31	32	4.20	2.00	.10	0	0	0	0	0	0	0	0
32	33	4.40	2.20	.20	0	0	0	0	0	0	0	0
33	34	4.60	2.30	.40	0	0	0	0	0	0	0	0
34	35	4.70	2.50	.50	0	0	0	0	0	0	0	0
35	36	4.90	2.60	.70	0	0	0	0	0	0	0	0
36	37	5.10	2.80	.80	0	0	0	0	0	0	0	0
37	38	5.20	3.00	.90	0	0	0	0	0	0	0	0
38	39	5.40	3.10	1.10	0	0	0	0	0	0	0	0
39	40	5.60	3.30	1.20	0	0	0	0	0	0	0	0
40	41	5.80	3.50	1.40	0	0	0	0	0	0	0	0
41	42	5.90	3.60	1.50	0	0	0	0	0	0	0	0
42	43	6.10	3.80	1.70	0	0	0	0	0	0	0	0
43	44	6.30	4 00	1.80	0	0	0	0	0	0	0	0
44	45	6.40	4.10	2.00	0	0	0	0	0	0	0	0
45	46	6.60	4.30	2.10	.20	0	0	0	0	0	0	0
46	47	6.80	4.50	2.30	.30	0	0	0	0	0	0	0
47	48	6.90	4.70	2.40	.50	0	0	0	0	0	0	0
48	49	7.10	4.80	2.60	.60	0	0	0	0	0	0	0
49	50	7.30	5.00	2.70	.70	0	0	0	0	0	0	0
50	51	7.50	5.20	2.90	.90	0	0	0	0	0	0	0
51	52	7.60	5.30	3.00	1.00	0	0	0	0	0	0	0
52	53	7.80	5.50	3.20	1.20	0	0	0	0	0	0	0
53	54	8.00	5.70	3.40	1.30	0	0	0	0	0	0	0
54	55	8.10	5.80	3.60	1.40	0	0	0	0	0	0	0
55	56	8.30	6.00	3.70	1.60	0	0	0	0	0	0	0
56	57	8.50	6.20	3.90	1.70	0	0	0	0	0	0	0
57	58	8.60	6.40	4.10	1.90	0	0	0	0	0	0	0

SINGLE PERSONS — WEEKLY PAYROLL PERIOD

And the wages are—		And the number of withholding exemptions claimed is—										
At least	But less than	0	1	2	3	4	5	6	7	8	9	10 or more
		The amount of income tax to be withheld shall be—										
$58	$59	$8.80	$6.50	$4.20	$2.00	$0.10	$0	$0	$0	$0	$0	$0
59	60	9.00	6.70	4.40	2.20	.30	0	0	0	0	0	0
60	62	9.20	6.90	4.70	2.40	.50	0	0	0	0	0	0
62	64	9.60	7.30	5.00	2.70	.70	0	0	0	0	0	0
64	66	9.90	7.60	5.30	3.10	1.00	0	0	0	0	0	0
66	68	10.30	8.00	5.70	3.40	1.30	0	0	0	0	0	0
68	70	10.60	8.30	6.00	3.70	1.60	0	0	0	0	0	0
70	72	10.90	8.60	6.40	4.10	1.90	0	0	0	0	0	0
72	74	11.30	9.00	6.70	4.40	2.20	.30	0	0	0	0	0
74	76	11.60	9.30	7.00	4.80	2.50	.50	0	0	0	0	0
76	78	12.00	9.70	7.40	5.10	2.80	.80	0	0	0	0	0
78	80	12.30	10.00	7.70	5.40	3.10	1.10	0	0	0	0	0
80	82	12.60	10.30	8.10	5.80	3.50	1.40	0	0	0	0	0
82	84	13.00	10.70	8.40	6.10	3.80	1.70	0	0	0	0	0
84	86	13.30	11.00	8.70	6.50	4.20	2.00	.10	0	0	0	0
86	88	13.70	11.40	9.10	6.80	4.50	2.30	.30	0	0	0	0
88	90	14.10	11.70	9.40	7.10	4.80	2.60	.60	0	0	0	0
90	92	14.50	12.00	9.80	7.50	5.20	2.90	.90	0	0	0	0
92	94	14.90	12.40	10.10	7.80	5.50	3.20	1.20	0	0	0	0
94	96	15.30	12.70	10.40	8.20	5.90	3.60	1.50	0	0	0	0
96	98	15.70	13.10	10.80	8.50	6.20	3.90	1.80	0	0	0	0
98	100	16.10	13.40	11.10	8.80	6.50	4.30	2.10	.10	0	0	0
100	105	16.80	14.10	11.70	9.40	7.10	4.80	2.60	.60	0	0	0
105	110	17.80	15.10	12.60	10.30	8.00	5.70	3.40	1.30	0	0	0
110	115	18.80	16.10	13.40	11.10	8.80	6.50	4.30	2.10	.10	0	0
115	120	19.80	17.10	14.40	12.00	9.70	7.40	5.10	2.80	.80	0	0
120	125	20.80	18.10	15.40	12.80	10.50	8.20	6.00	3.70	1.50	0	0
125	130	21.80	19.10	16.40	13.80	11.40	9.10	6.80	4.50	2.30	.40	0
130	135	22.80	20.10	17.40	14.80	12.20	9.90	7.70	5.40	3.10	1.10	0
135	140	23.80	21.10	18.40	15.80	13.10	10.80	8.50	6.20	3.90	1.80	0
140	145	24.80	22.10	19.40	16.80	14.10	11.60	9.40	7.10	4.80	2.50	.60
145	150	25.80	23.10	20.40	17.80	15.10	12.50	10.20	7.90	5.60	3.30	1.30
150	160	27.30	24.60	21.90	19.30	16.60	13.90	11.50	9.20	6.90	4.60	2.40
160	170	29.30	26.60	23.90	21.30	18.60	15.90	13.20	10.90	8.60	6.30	4.00
170	180	31.60	28.60	25.90	23.30	20.60	17.90	15.20	12.60	10.30	8.00	5.70
180	190	34.10	30.80	27.90	25.30	22.60	19.90	17.20	14.50	12.00	9.70	7.40
190	200	36.60	33.30	29.90	27.30	24.60	21.90	19.20	16.50	13.80	11.40	9.10
200	210	39.10	35.80	32.40	29.30	26.60	23.90	21.20	18.50	15.80	13.10	10.80
210	220	41.80	38.30	34.90	31.50	28.60	25.90	23.20	20.50	17.80	15.10	12.50
220	230	44.80	40.80	37.40	34.00	30.70	27.90	25.20	22.50	19.80	17.10	14.40
230	240	47.80	43.80	39.90	36.50	34.20	29.90	27.20	24.50	21.80	19.10	16.40
240	250	50.80	46.80	42.70	39.00	35.70	32.30	29.20	26.50	23.80	21.10	18.40
250	260	53.80	49.80	45.70	41.70	38.20	34.80	31.40	28.50	25.80	23.10	20.40
260	270	56.80	52.80	48.70	44.70	40.70	37.30	33.90	30.60	27.80	25.10	22.40
270	280	59.80	55.80	51.70	47.70	43.60	39.80	36.40	33.10	29.80	27.10	24.40
280	290	62.80	58.80	54.70	50.70	46.60	42.60	38.90	35.60	32.20	29.10	26.40
290	300	65.80	61.80	57.70	53.70	49.60	45.60	41.60	38.10	34.70	31.30	28.40
300	310	68.80	64.80	60.70	56.70	52.60	48.60	44.60	40.60	37.20	33.80	30.50
310	320	71.80	67.80	63.70	59.70	55.60	51.60	47.60	43.50	39.70	36.30	33.00
320	330	74.80	70.80	66.70	62.70	58.60	54.60	50.60	46.50	42.50	38.80	35.50
330	340	77.80	73.80	69.70	65.70	61.60	57.60	53.60	49.50	45.50	41.40	38.00
340	350	80.80	76.80	72.70	68.70	64.60	60.60	56.60	52.50	48.50	44.40	40.50
350	360	83.80	79.80	75.70	71.70	67.60	63.60	59.60	55.50	51.50	47.40	43.40
		30 percent of the excess over $360 plus—										
$360 and over		85.30	81.30	77.20	73.20	69.10	65.10	61.10	57.00	53.00	48.90	44.90

SOCIAL SECURITY EMPLOYEE TAX TABLE

4.20 percent employee tax deductions (3.85% for old age, survivors, and disability insurance plus 0.35% for hospital insurance benefits)

Wages		Tax to be withheld	Wages		Tax to be withheld	Wages		Tax to be withheld	Wages		Tax to be withheld
At least	But less than		At least	But less than		At least	But less than		At least	But less than	
$0.00	$0.12	$0.00	$15.36	$15.60	$0.65	$30.84	$31.08	$1.30	$46.31	$46.55	$1.95
.12	.36	.01	15.60	15.84	.66	31.08	31.31	1.31	46.55	46.79	1.96
.36	.60	.02	15.84	16.08	.67	31.31	31.55	1.32	46.79	47.03	1.97
.60	.84	.03	16.08	16.31	.68	31.55	31.79	1.33	47.03	47.27	1.98
.84	1.08	.04	16.31	16.55	.69	31.79	32.03	1.34	47.27	47.50	1.99
1.08	1.31	.05	16.55	16.79	.70	32.03	32.27	1.35	47.50	47.74	2.00
1.31	1.55	.06	16.79	17.03	.71	32.27	32.50	1.36	47.74	47.98	2.01
1.55	1.79	.07	17.03	17.27	.72	32.50	32.74	1.37	47.98	48.22	2.02
1.79	2.03	.08	17.27	17.50	.73	32.74	32.98	1.38	48.22	48.46	2.03
2.03	2.27	.09	17.50	17.74	.74	32.98	33.22	1.39	48.46	48.70	2.04
2.27	2.50	.10	17.74	17.98	.75	33.22	33.46	1.40	48.70	48.93	2.05
2.50	2.74	.11	17.98	18.22	.76	33.46	33.70	1.41	48.93	49.17	2.06
2.74	2.98	.12	18.22	18.46	.77	33.70	33.93	1.42	49.17	49.41	2.07
2.98	3.22	.13	18.46	18.70	.78	33.93	34.17	1.43	49.41	49.65	2.08
3.22	3.46	.14	18.70	18.93	.79	34.17	34.41	1.44	49.65	49.89	2.09
3.46	3.70	.15	18.93	19.17	.80	34.41	34.65	1.45	49.89	50.12	2.10
3.70	3.93	.16	19.17	19.41	.81	34.65	34.89	1.46	50.12	50.36	2.11
3.93	4.17	.17	19.41	19.65	.82	34.89	35.12	1.47	50.36	50.60	2.12
4.17	4.41	.18	19.65	19.89	.83	35.12	35.36	1.48	50.60	50.84	2.13
4.41	4.65	.19	19.89	20.12	.84	35.36	35.60	1.49	50.84	51.08	2.14
4.65	4.89	.20	20.12	20.36	.85	35.60	35.84	1.50	51.08	51.31	2.15
4.89	5.12	.21	20.36	20.60	.86	35.84	36.08	1.51	51.31	51.55	2.16
5.12	5.36	.22	20.60	20.84	.87	36.08	36.31	1.52	51.55	51.79	2.17
5.36	5.60	.23	20.84	21.08	.88	36.31	36.55	1.53	51.79	52.03	2.18
5.60	5.84	.24	21.08	21.31	.89	36.55	36.79	1.54	52.03	52.27	2.19
5.84	6.08	.25	21.31	21.55	.90	36.79	37.03	1.55	52.27	52.50	2.20
6.08	6.31	.26	21.55	21.79	.91	37.03	37.27	1.56	52.50	52.74	2.21
6.31	6.55	.27	21.79	22.03	.92	37.27	37.50	1.57	52.74	52.98	2.22
6.55	6.79	.28	22.03	22.27	.93	37.50	37.74	1.58	52.98	53.22	2.23
6.79	7.03	.29	22.27	22.50	.94	37.74	37.98	1.59	53.22	53.46	2.24
7.03	7.27	.30	22.50	22.74	.95	37.98	38.22	1.60	53.46	53.70	2.25
7.27	7.50	.31	22.74	22.98	.96	38.22	38.46	1.61	53.70	53.93	2.26
7.50	7.74	.32	22.98	23.22	.97	38.46	38.70	1.62	53.93	54.17	2.27
7.74	7.98	.33	23.22	23.46	.98	38.70	38.93	1.63	54.17	54.41	2.28
7.98	8.22	.34	23.46	23.70	.99	38.93	39.17	1.64	54.41	54.65	2.29
8.22	8.46	.35	23.70	23.93	1.00	39.17	39.41	1.65	54.65	54.89	2.30
8.46	8.70	.36	23.93	24.17	1.01	39.41	39.65	1.66	54.89	55.12	2.31
8.70	8.93	.37	24.17	24.41	1.02	39.65	39.89	1.67	55.12	55.36	2.32
8.93	9.17	.38	24.41	24.65	1.03	39.89	40.12	1.68	55.36	55.60	2.33
9.17	9.41	.39	24.65	24.89	1.04	40.12	40.36	1.69	55.60	55.84	2.34
9.41	9.65	.40	24.89	25.12	1.05	40.36	40.60	1.70	55.84	56.08	2.35
9.65	9.89	.41	25.12	25.36	1.06	40.60	40.84	1.71	56.08	56.31	2.36
9.89	10.12	.42	25.36	25.60	1.07	40.84	41.08	1.72	56.31	56.55	2.37
10.12	10.36	.43	25.60	25.84	1.08	41.08	41.31	1.73	56.55	56.79	2.38
10.36	10.60	.44	25.84	26.08	1.09	41.31	41.55	1.74	56.79	57.03	2.39
10.60	10.84	.45	26.08	26.31	1.10	41.55	41.79	1.75	57.03	57.27	2.40
10.84	11.08	.46	26.31	26.55	1.11	41.79	42.03	1.76	57.27	57.50	2.41
11.08	11.31	.47	26.55	26.79	1.12	42.03	42.27	1.77	57.50	57.74	2.42
11.31	11.55	.48	26.79	27.03	1.13	42.27	42.50	1.78	57.74	57.98	2.43
11.55	11.79	.49	27.03	27.27	1.14	42.50	42.74	1.79	57.98	58.22	2.44
11.79	12.03	.50	27.27	27.50	1.15	42.74	42.98	1.80	58.22	58.46	2.45
12.03	12.27	.51	27.50	27.74	1.16	42.98	43.22	1.81	58.46	58.70	2.46
12.27	12.50	.52	27.74	27.98	1.17	43.22	43.46	1.82	58.70	58.93	2.47
12.50	12.74	.53	27.98	28.22	1.18	43.46	43.70	1.83	58.93	59.17	2.48
12.74	12.98	.54	28.22	28.46	1.19	43.70	43.93	1.84	59.17	59.41	2.49
12.98	13.22	.55	28.46	28.70	1.20	43.93	44.17	1.85	59.41	59.65	2.50
13.22	13.46	.56	28.70	28.93	1.21	44.17	44.41	1.86	59.65	59.89	2.51
13.46	13.70	.57	28.93	29.17	1.22	44.41	44.65	1.87	59.89	60.12	2.52
13.70	13.93	.58	29.17	29.41	1.23	44.65	44.89	1.88	60.12	60.36	2.53
13.93	14.17	.59	29.41	29.65	1.24	44.89	45.12	1.89	60.36	60.60	2.54
14.17	14.41	.60	29.65	29.89	1.25	45.12	45.36	1.90	60.60	60.84	2.55
14.41	14.65	.61	29.89	30.12	1.26	45.36	45.60	1.91	60.84	61.08	2.56
14.65	14.89	.62	30.12	30.36	1.27	45.60	45.84	1.92	61.08	61.31	2.57
14.89	15.12	.63	30.36	30.60	1.28	45.84	46.08	1.93	61.31	61.55	2.58
15.12	15.36	.64	30.60	30.84	1.29	46.08	46.31	1.94	61.55	61.79	2.59

SOCIAL SECURITY EMPLOYEE TAX TABLE—Continued

4.20 percent employee tax deductions (3.85% for old age, survivors, and disability insurance plus 0.35% for hospital insurance benefits)

Wages		Tax to be withheld	Wages		Tax to be withheld	Wages		Tax to be withheld	Wages		Tax to be withheld
At least	But less than		At least	But less than		At least	But less than		At least	But less than	
$61.79	$62.03	$2.60	$77.27	$77.50	$3.25	$92.74	$92.98	$3.90	$108.22	$108.46	$4.55
62.03	62.27	2.61	77.50	77.74	3.26	92.98	93.22	3.91	108.46	108.70	4.56
62.27	62.50	2.62	77.74	77.98	3.27	93.22	93.46	3.92	108.70	108.93	4.57
62.50	62.74	2.63	77.98	78.22	3.28	93.46	93.70	3.93	108.93	109.17	4.58
62.74	62.98	2.64	78.22	78.46	3.29	93.70	93.93	3.94	109.17	109.41	4.59
62.98	63.22	2.65	78.46	78.70	3.30	93.93	94.17	3.95	109.41	109.65	4.60
63.22	63.46	2.66	78.70	78.93	3.31	94.17	94.41	3.96	109.65	109.89	4.61
63.46	63.70	2.67	78.93	79.17	3.32	94.41	94.65	3.97	109.89	110.12	4.62
63.70	63.93	2.68	79.17	79.41	3.33	94.65	94.89	3.98	110.12	110.36	4.63
63.93	64.17	2.69	79.41	79.65	3.34	94.89	95.12	3.99	110.36	110.60	4.64
64.17	64.41	2.70	79.65	79.89	3.35	95.12	95.36	4.00	110.60	110.84	4.65
64.41	64.65	2.71	79.89	80.12	3.36	95.36	95.60	4.01	110.84	111.08	4.66
64.65	64.89	2.72	80.12	80.36	3.37	95.60	95.84	4.02	111.08	111.31	4.67
64.89	65.12	2.73	80.36	80.60	3.38	95.84	96.08	4.03	111.31	111.55	4.68
65.12	65.36	2.74	80.60	80.84	3.39	96.08	96.31	4.04	111.55	111.79	4.69
65.36	65.60	2.75	80.84	81.08	3.40	96.31	96.55	4.05	111.79	112.03	4.70
65.60	65.84	2.76	81.08	81.31	3.41	96.55	96.79	4.06	112.03	112.27	4.71
65.84	66.08	2.77	81.31	81.55	3.42	96.79	97.03	4.07	112.27	112.50	4.72
66.08	66.31	2.78	81.55	81.79	3.43	97.03	97.27	4.08	112.50	112.74	4.73
66.31	66.55	2.79	81.79	82.03	3.44	97.27	97.50	4.09	112.74	112.98	4.74
66.55	66.79	2.80	82.03	82.27	3.45	97.50	97.74	4.10	112.98	113.22	4.75
66.79	67.03	2.81	82.27	82.50	3.46	97.74	97.98	4.11	113.22	113.46	4.76
67.03	67.27	2.82	82.50	82.74	3.47	97.98	98.22	4.12	113.46	113.70	4.77
67.27	67.50	2.83	82.74	82.98	3.48	98.22	98.46	4.13	113.70	113.93	4.78
67.50	67.74	2.84	82.98	83.22	3.49	98.46	98.70	4.14	113.93	114.17	4.79
67.74	67.98	2.85	83.22	83.46	3.50	98.70	98.93	4.15	114.17	114.41	4.80
67.98	68.22	2.86	83.46	83.70	3.51	98.93	99.17	4.16	114.41	114.65	4.81
68.22	68.46	2.87	83.70	83.93	3.52	99.17	99.41	4.17	114.65	114.89	4.82
68.46	68.70	2.88	83.93	84.17	3.53	99.41	99.65	4.18	114.89	115.12	4.83
68.70	68.93	2.89	84.17	84.41	3.54	99.65	99.89	4.19	115.12	115.36	4.84
68.93	69.17	2.90	84.41	84.65	3.55	99.89	100.12	4.20	115.36	115.60	4.85
69.17	69.41	2.91	84.65	84.89	3.56	100.12	100.36	4.21	115.60	115.84	4.86
69.41	69.65	2.92	84.89	85.12	3.57	100.36	100.60	4.22	115.84	116.08	4.87
69.65	69.89	2.93	85.12	85.36	3.58	100.60	100.84	4.23	116.08	116.31	4.88
69.89	70.12	2.94	85.36	85.60	3.59	100.84	101.08	4.24	116.31	116.55	4.89
70.12	70.36	2.95	85.60	85.84	3.60	101.08	101.31	4.25	116.55	116.79	4.90
70.36	70.60	2.96	85.84	86.08	3.61	101.31	101.55	4.26	116.79	117.03	4.91
70.60	70.84	2.97	86.08	86.31	3.62	101.55	101.79	4.27	117.03	117.27	4.92
70.84	71.08	2.98	86.31	86.55	3.63	101.79	102.03	4.28	117.27	117.50	4.93
71.08	71.31	2.99	86.55	86.79	3.64	102.03	102.27	4.29	117.50	117.74	4.94
71.31	71.55	3.00	86.79	87.03	3.65	102.27	102.50	4.30	117.74	117.98	4.95
71.55	71.79	3.01	87.03	87.27	3.66	102.50	102.74	4.31	117.98	118.22	4.96
71.79	72.03	3.02	87.27	87.50	3.67	102.74	102.98	4.32	118.22	118.46	4.97
72.03	72.27	3.03	87.50	87.74	3.68	102.98	103.22	4.33	118.46	118.70	4.98
72.27	72.50	3.04	87.74	87.98	3.69	103.22	103.46	4.34	118.70	118.93	4.99
72.50	72.74	3.05	87.98	88.22	3.70	103.46	103.70	4.35	118.93	119.17	5.00
72.74	72.98	3.06	88.22	88.46	3.71	103.70	103.93	4.36	119.17	119.41	5.01
72.98	73.22	3.07	88.46	88.70	3.72	103.93	104.17	4.37	119.41	119.65	5.02
73.22	73.46	3.08	88.70	88.93	3.73	104.17	104.41	4.38	119.65	119.89	5.03
73.46	73.70	3.09	88.93	89.17	3.74	104.41	104.65	4.39	119.89	120.12	5.04
73.70	73.93	3.10	89.17	89.41	3.75	104.65	104.89	4.40	120.12	120.36	5.05
73.93	74.17	3.11	89.41	89.65	3.76	104.89	105.12	4.41	120.36	120.60	5.06
74.17	74.41	3.12	89.65	89.89	3.77	105.12	105.36	4.42	120.60	120.84	5.07
74.41	74.65	3.13	89.89	90.12	3.78	105.36	105.60	4.43	120.84	121.08	5.08
74.65	74.89	3.14	90.12	90.36	3.79	105.60	105.84	4.44	121.08	121.31	5.09
74.89	75.12	3.15	90.36	90.60	3.80	105.84	106.08	4.45	121.31	121.55	5.10
75.12	75.36	3.16	90.60	90.84	3.81	106.08	106.31	4.46	121.55	121.79	5.11
75.36	75.60	3.17	90.84	91.08	3.82	106.31	106.55	4.47	121.79	122.03	5.12
75.60	75.84	3.18	91.08	91.31	3.83	106.55	106.79	4.48	122.03	122.27	5.13
75.84	76.08	3.19	91.31	91.55	3.84	106.79	107.03	4.49	122.27	122.50	5.14
76.08	76.31	3.20	91.55	91.79	3.85	107.03	107.27	4.50	122.50	122.74	5.15
76.31	76.55	3.21	91.79	92.03	3.86	107.27	107.50	4.51	122.74	122.98	5.16
76.55	76.79	3.22	92.03	92.27	3.87	107.50	107.74	4.52	122.98	123.22	5.17
76.79	77.03	3.23	92.27	92.50	3.88	107.74	107.98	4.53	123.22	123.46	5.18
77.03	77.27	3.24	92.50	92.74	3.89	107.98	108.22	4.54	123.46	123.70	5.19

Unit IV

MEASUREMENT

The specific objectives of this unit are to review and extend the meaning and use of linear measurement and to learn the following:

1. A unit of measure
2. Finding the perimeter of a square, rectangle
3. A formula
4. The formula for perimeter
5. Constructing scale drawings

Teaching Aids

1. Map of New York City
2. Measurement strips as indicated
3. Worksheets for Lesson 2

Correlation

Science, Industrial Arts

Lesson 1

SCALE DRAWINGS

Warmup

1. We use a .... to locate places. *(map)*
2. When we use ...., index, or guide, we save time. *(key)*
3. *Scale unit=1 mile* means each space represents .... miles. *(1)*
4. How many miles would 2 spaces represent? *(2)*

Motivation

This lesson is based on *The Atlas of New York City,* recommended for Unit I.
Have pupils look at maps and find the *scale.*

Aim

What does the *scale* on a map mean, and how is it used?

Development

Lead the pupils to discover the meaning of *Unit of Measure* by asking the following or similar questions:
Have pupils turn to page 1. (map of lower Manhattan)

1. What does *Scale in Feet* mean? What do the *arrows* and *One Mile* mean?
2. Take the strip of paper and mark off the distance between 0 (zero) and the end of one mile.
3. Fold the paper at the one mile mark, tear apart, and write *one mile* on the measured part.
4. Is the strip really 1 mile long? *(represents)*
5. What part of a mile does the strip represent? *(1 whole mile)*
6. How many miles would using the strip twice represent?
7. One half of the strip would represent what part of a mile?
8. What is the smallest *measure* the strip represents?
10. The strip is the smallest measure or the *Unit of Measure* we will use to measure parts of the map.

11. Does any one know how many city blocks are in 1 mile? *(about 20 blocks)*
12. What does *about* mean? (not exact, more or less than, approximate)
13. *About 20 blocks* could mean any number between 15 and *? (25)*
14. Place the Unit of Measure along Tenth Avenue, with lower edge on 14th Street. Count the number of blocks from 14th Street to end of Unit. What street is about 1 mile north of 14th Street?
15. Without using Unit, start at tip of Manhattan, Brooklyn Battery Tunnel and count twenty blocks north.
16. What street is about 1 mile north of tip of Manhattan? (Use different answers to point out that block counting is not a reliable measure. Blocks are not the same size, due to rivers, highways, parks, etc.)
17. Now, use Unit of Measure. What street is about 1 mile north of the tip of Manhattan? *(Chambers Street)*
18. Have pupils measure the width of Manhattan at 34th Street and the approximate length, using maps on pages 1, 2 and 4.
19. How did the map makers show the Island of Manhattan, which is about 9 miles long and 2 miles wide, on a little more than 2 pages? (Ask leading questions to have pupils discover that blocks and streets are drawn small enough to fit space. Mention models, miniatures, photographs, blue prints, etc. Compare size of people on T.V. screen, regular movie screen and very wide screen. Images on screen are different only in size.)
20. The map shows the streets, blocks, parks, etc. in New York City exactly as they are in shape and location, but drawn much smaller in size. This is called *drawing to scale* or *scale drawing*.
21. Why isn't the whole island of Manhattan on one page? (Have pupils realize that details would have to be so small they wouldn't be clear.)

Summary

1. A scale drawing is a drawing of a large thing reduced in size to fit the paper.
   Examples: blue prints, maps, diagrams, etc.
2. A scale drawing may be a drawing of a small thing enlarged in size to show each part clearly.
   Samples: cells, insects, seeds in science, etc.

3. The scale used must be small enough for the drawing to fit the paper, and large enough for all parts of the drawing to be clear.
4. A *Unit of Measure* is the smallest *whole measure* being used.

Practice

Have pupils use prepared *Unit* for 1 mile to find the approximate length and width of other four boroughs.

Homework

What is the *Unit of Measure* in each of the following measurements:

1. 4 inches
2. 9 feet
3. 7 yards
4. 2 miles
5. 3 lbs.

Lesson 2

MEASURES OF LENGTH

Warmup

Correct your homework:

Measurements	*Unit of Measure*
4 inches	1 inch
9 feet	1 foot
7 yards	1 yard
2 miles	1 mile
3 lbs.	1 lb.

Motivation

How can we measure the length of the classroom, or the distance from the front to the back of the room? *(ruler, yardstick, pacing)*

What would the Unit of Measure be if we used the yardstick? *(1 yard* or *1 inch)*

What would the Unit be if we used a ruler? *(1 foot* or *1 inch)* Pacing? *(1 pace)*

Have one tall pupil and one short pupil pace the distance. Why are the number of paces different? *(length of pace)*

Aim

How do we measure length?

Development

Have 2 pupils use a yardstick. Compare number of yards. (There should be a slight difference.)
Why are measurements different if distance was measured with the same unit? Have pupils give and explain reasons.

Have all pupils measure the length and width of their desks and write measurements. Compare results. Explain that all measurement is approximate, due to size and thickness of marker, measuring tool, care taken, etc.

Note that using yardstick and ruler, discrepancies are very small. Ask pupils for reasons. Lead to and explain *Standard Units,* set by the United States Bureau of Weights and Measures.

What do we use to measure line segments? *(ruler)*
What do we use to measure the length of materials? *(yardstick, tape measure)*
What unit do we use to measure distance between cities? *(Miles)*

What is the measure of a standard ruler? *(1 foot* or *12 inches)*
Then: 1 foot = 12 inches (on board)

What is the inch measure of 1 yard? *(36 inches)*
What is the foot-measure of 1 yard? *(3 feet)*
Then: 1 yard = 3 feet or 36 inches (on board)

How many feet in 1 mile? *(5280)*
If we know how many feet are in a mile, how do we find the number of yards in a mile? Then: 1 mile = 1760 yards, or 5280 feet (on board)

Have pupils draw a line segment freehand. A little below, place a dot at each end of line segment. Above, write *length of line segment;* between dots write *distance between points* as follows:

length of line segment

• *distance between points* •

Have pupils measure the line segment, and emphasize that the measurement is not exact.

Through questioning, have pupils understand that measuring length and distance is measuring in a line and is called *Linear Measure.* In linear measurement, what is the smallest unit we use? *(1 inch)*

Have pupils study markings and dimensions on ruler; then have them put rulers away.

After they copy the summary in their notebooks tell them that they will have an opportunity to check their powers of observation.

Summary

To measure length or distance, we use the standard units:

1 inch
1 foot = 12 inches
1 yard = 3 feet = 36 inches
1 mile = 1760 yards = 5280 feet

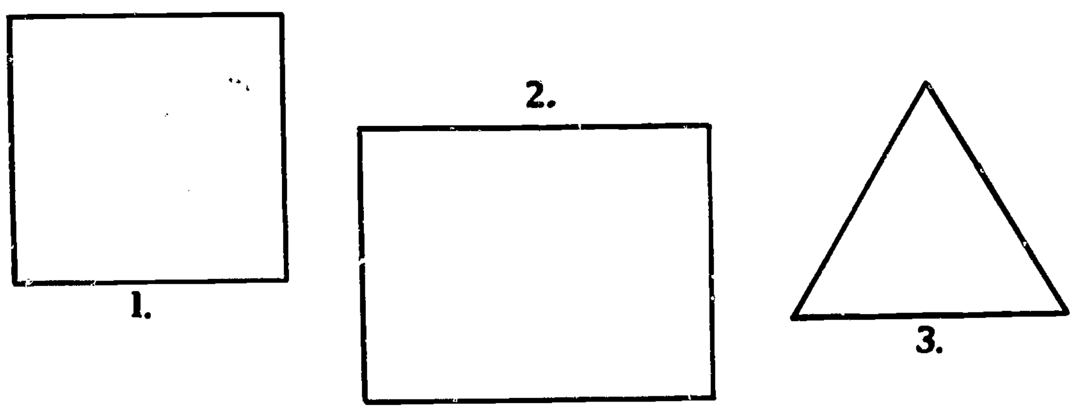
Practice

On the 6-inch strip, place all the markings that are on your ruler. (to be approximated, not copied.)
Discuss smaller units: ½ inch, ¼ inch, ⅛ inch, $\frac{1}{16}$ inch
When completed, compare with ruler.

Duplicate worksheet that follows:

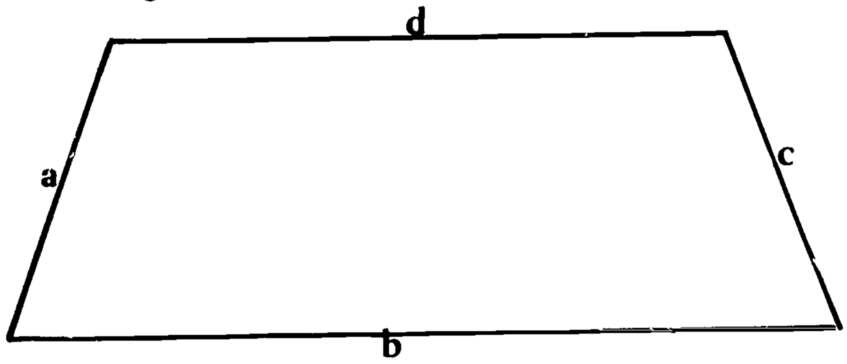
HOMEWORK

Measure each side. Write the measurement for each side.



4. Scale Drawing

Scale: 1 inch $=$ 5 miles



How many miles are represented by line segment a? ....

How many miles are represented by line segment b? ....

How many miles are represented by line segment c? ....

How many miles are represented by line segment d? ....

Be sure to bring this sheet to class for the next lesson.

Lesson 3

MEASURING DISTANCE AROUND OBJECTS

Warmup

? $=$ 3 feet	1 foot plus $\frac{1}{2}$ foot $=$? feet, $=$? inches
1760 yards $=$?	8 inches plus 3 inches $=$ ....inches
1 mile $=$? feet	2 yards plus ? yards $= 2\frac{3}{4}$ yards

Motivation

In general, pupils believe *scale drawings* and *measures of length* are used only by architects, draftsmen, engineers, and expert craftsmen who are far removed fron. us. Let us stop and look closely. The clothes we wear were made to special measurements; our shoes have length and width; and our height is measured in feet and inches. What is the distance you travel from home to school? Everything about us and everything we toucb or hold involves measurement.

Aim

Knowing and using measures of length.

Development

Look at figure 1 on your homework paper.
What is the length of each side? *(2 inches)*
What is another way of saying that the sides are the same size? *(the four sides are equal in length)*

About the middle of the square draw a vertical line segment from the top horizontal line to the bottom.
Measure the line segment you just drew.
Between the line segment you just drew and the edge draw another vertical line segment and measure it. What do the line segments measure? *(2 inches)*
Would you say the two horizontal line segments are equally distant from each other?
Have the pupils extend the horizontal line segments. Would these lines ever meet or intersect?

Lines that have the same direction and never meet because they are always the same distance apart are called *parallel lines.*

Measure distance across box. Are the vertical lines parallel? Why?
Do you know what we call a 4-sided figure with right angles whose sides are equal? *(square)*
How would you describe a square? *(a quadrilateral with equal sides, equal angles)*

Measure figure 2. Are the *opposite* sides parallel? Are all the angles and sides equal? Is it a square?
A quadrilateral with four right angles is called a *rectangle.* Notice that its opposite sides are parallel and equal.

Measure the distance around the 2-inch square. What is your answer? *(8 inches)*
Do you know what we call the distance around a thing or place? *(perimeter)*
How can we find the *perimeter* of a rectangle? a triangle?
What is the perimeter of figure 4?

To save time in writing measurements, we may abbreviate them, or we may use symbols to represent them. (Review: *in., ft., yd., mi.*)

Summary

Parallel lines are lines that have the same direction and are always the same distant apart.

A square is a quadrilateral with all 4 sides equal; opposite sides parallel, and 4 right angles.

A rectangle is a quadrilateral with both pairs of opposite sides equal and parallel and 4 right angles.

Practice

Draw 2 horizontal parallel lines 2½ inches apart. Drew 2 vertical lines parallel and 4 inches apart to close the rectangle.
How can we find the perimeter? What is the perimeter?

Homework

Draw a square, a rectangle, and a triangle of different sizes.
Find the perimeter of each.

Lesson 4

USING MEASUREMENT

Warmup

1. 365 means 300 plus 60 plus 5
2. 219 means 200 plus ? plus ?
3. $6.22 means 6 dollars plus 2 dimes plus 2 cents, or $6.00 plus .20 plus ?
4. $2.63 means $2.00 plus ? plus .03
5. Add $6.22 and $2.63

Motivation

When we add whole numbers, we add ones to ones, tens to tens, etc. Continue with adding and subtracting dollars and cents.

Use warmup exercises to demonstrate. When we add 47¢ and 35¢ what do we do? Have the pupils explain *exchange*. (10¢ for 1 dime, or 10 ones for one ten)

Aim

How can we add feet and inches?

Development

Ask pupils to think of a square with sides 8 inches long.
Have them find the perimeter. *(32″)*

Send several pupils to the board to draw lines approximately 32 inches, 1 foot, and 8 inches.
Have other pupils check with a yardstick.

Review linear facts: 1 foot is the same length as 12 inches.
On the measured line segment 32″, have pupils mark off feet intervals. (use ruler)
Stress 32″ and 2′8″ have the same meaning in length, mean the same distance.

Have pupils generalize that the *length*, or distance, may be expressed in any unit or combination of units of linear measure. However, to make it easier to visualize, understand, and use, we often think in terms of the largest unit possible.

Have pupils work several exercises of the type:

$$\begin{array}{rr} & \text{2 feet 5 inches} \\ \text{plus} & \underline{\text{1 foot 8 inches}} \end{array} \qquad \begin{array}{r} \text{5 feet 7 inches} \\ -\underline{\text{2 feet 8 inches}} \end{array} = \begin{array}{r} \text{4 feet 19 inches} \\ -\underline{\text{2 feet 8 inches}} \end{array}$$

Review and stress working with like or similar things and exchanging "units of measure."

Summary

In measuring length, we may exchange *units of measure* without actually changing the length.

Practice

Assign a number of exercises in addition and subtraction of linear measures, with and without exchanging units.

Homework

Have pupils construct 2 squares and 2 rectangles to given size (Keep numbers small.)

Have them find the perimeter of 1 square and 1 rectangle.

Lesson 5

USING A FORMULA

Warmup

36 inches may be written as .... ft.

5 feet may be written as .... in.

3 yards may be written as .... inches, *or* .... feet.

Write in terms of larger units:

28 inches 28 feet 5 feet 15 inches

Motivation

Have pupils look at homework papers.

How did you find the perimeter of the square? rectangle?
(Answers will probably be, "Add actual measures of sides.")

How would you explain finding the perimeter, if you did not know the length of the sides?
(Answers will probably be, "Add the length of all the sides.")

Do we need to measure all four sides?

Aim

How can we find the perimeter of any square or rectangle quickly and accurately?

Procedure

When we add the lengths of the sides, we are adding: side plus side plus side plus side.

To save time, we can write: *s plus s plus s plus s,* or $s+s+s+s$.

Examine the squares and the rectangles.

Have pupils realize that the squares and rectangles have *length* and *width.*

Have pupils write *length* and *width* in appropriate places on all four figures.

Have pupils repeat method of finding the perimeter.

Have them write: $P=s+s+s+s$, or $4\ s$

Ask pupils if they can write the same thing using the words *length* and *width.*

Continue asking questions to simplify the explanation of perimeter in order to develop the formula as follows:

P = side + side + side + side *or* P = s + s + s + s (square)

(rectangle)

P = length + width + length + width *or* P = l + w + l + w

P = length + length + width + width *or* P = l + l + w + w

P = 2 lengths + 2 widths or P = 2 l + 2 w

P = 2 times the length plus 2 times the width

To save time we write it: $P = 2\,l + 2\,w$

Have pupils measure the four figures substituting the measures of the length and the width. Check results. Explain that a short way of finding the perimeter of a rectangle is to use the *formula* $P = 2\,l + 2\,w$.

Summary

A formula is a rule that tells us how to find an answer quickly and accurately.

The formula for perimeter of a square is $P=4s$ and for a rectangle it is $P=2\,l+2\,w$.

Practice and Homework

Assign the construction of several squares and rectangles.
Have pupils find the perimeter with the formula.

Lesson 6

FINDING PERIMETER

Spend this lesson constructing a number of varied figures.

Have pupils decide on a *formula,* if they can.

Note: If all sides of a figure are equal (as in squares, regular pentagon, regular hexagon, etc., formulas would be 4s, 5s, 6s, respectively.

Review meaning of $P = 2\,l + 2\,w$.

Practice using formula to find P.

TEST

Prepare questions covering all aspects of linear measure, using formulas for finding perimeter, and constructing and measuring figures.

Unit V

MEASURES AT WORK

The specific objectives of this unit are to help the pupils understand:

1. Area is a surface measurement
2. Units of area
3. How to find the area of simple figures
4. The formulas for area of square and rectangular regions

Teaching Aids

1. Rulers, graph paper
2. Squared materials (pupil constructed)
3. Square yard chart
   Winston Company, 1010 Arch Street, Philadelphia 7, Pa.
4. Work sheets Lesson 1 and 4

Correlation

Industrial Arts

Lesson 1

MEASURING SURFACES

Warmup

1. $6 \times 10 =$	3. $9 \times 9 =$	5. $7 \times 7 =$
2. $8 \times 7 =$	4. $6 \times 18 =$	6. $8 \times 7 =$

Motivation

What are some units of length? *(inch, foot, yard, mile)*
What unit would you usually use to measure a line segment on paper? *(inch)*
What unit would you use to measure the length of the teacher's desk? *(foot, inch)*; the width of a room? *(yard, foot)*
Unit to measure distance from Times Square in Manhattan to our Junior High School? *(mile)*

Have pupils summarize: For short lengths — use small units
For longer lengths — use large units

Distribute copies of worksheet for this lesson.

What do we call the distance around a square?
What is the perimeter of the square?
If the square is a block of wood and you want to put a gilt frame around it, how would you know how much gilt frame was needed?
(Find the perimeter of the block.)
If you want to cover the wood space or surface with plastic material, how much plastic would you need?

Aim

How do we measure surface?

Development

Have pupils try to measure the surface enclosed by a square with rulers and realize that units of length cannot measure surface. Have the pupils conclude that a unit of measure that will cover the space is needed.

Have pupils measure and mark the midpoint of each side of the square, draw horizontal and vertical line segments dividing the large square into 4 smaller squares and shade upper left and lower right square so that each unit stands out clearly.

Measure the length and width of each small square.
Find the perimeter of each small square.
If the area of one of the small squares is our unit of measure, could we use it to buy the correct amount of plastic to cover the *surface,* or *area,* of the block of wood?
Have the pupils realize that they could buy as much plastic as the 4 small squares would cover.
Have the pupils conclude that the small square region can be a unit of surface measurement. Since it is 1 inch in length, 1 inch in width, and square, we call it *1 square inch.*

Have the pupils draw a three-inch line (on worksheet). Then, have pupils construct a rectangle 3 inches long and 1 inch wide and mark off the square inches in the rectangular region. What is the perimeter?

What is the area of the rectangular region? (3 square inches)

Have pupils discuss the difference between units of length and units of area, and between a rectangle and a rectangular region.

Summary

Area or surface is measured by square units.

The smallest unit of area we will use is the square inch.

A square inch is the area of a one-inch square.

Practice

Construct 2 rectangles.

1. 3 inches long and 2 inches wide
2. 3 inches long and 3 inches wide

Find the perimeter and the area of each.

Homework

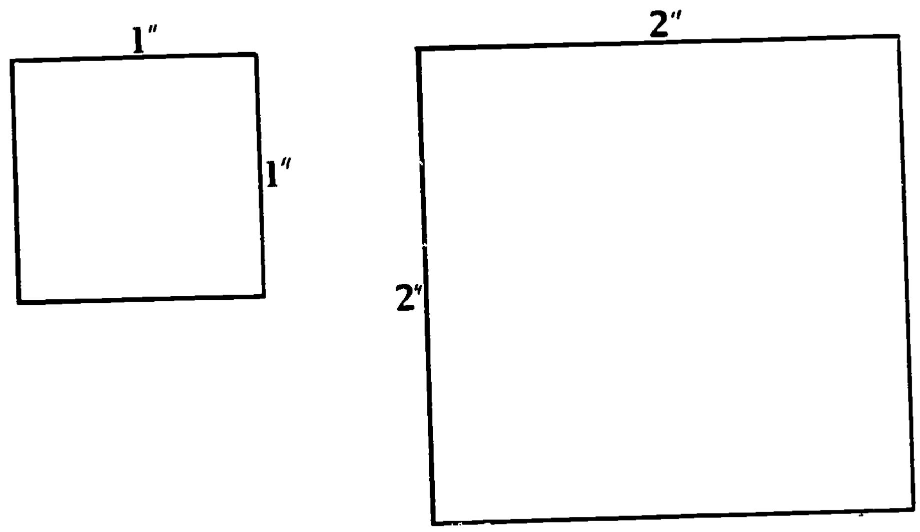
Construct 4 rectangles 6 inches long and 1 inch wide. Draw lines to show the 1-inch units.

Construct 12 one-inch squares.

Cut out the 4 rectangles and 12 squares.

Pupil Worksheet

(Prepare worksheet for pupils to use in development of lesson.)



Other figures will be drawn by pupils.

Lesson 2

SQUARE MEASURE

Warmup

The smallest unit of length is ....

1 foot, 1 yard, 1 mile are units of ....

1 foot = ...inches, 1 yard = ..., 1 yard = ...inches

We use square units to measure surface, or ....

Motivation

To paint a wall, we must find the area of the wall to know how much paint to buy.

To carpet a floor, what must we know before we can buy the carpeting?

If the area is small, we can use small units, such as, square inches. What can we use to measure larger surfaces?

Aim

What are some of the larger units of area?

Development

Have pupils draw a square on graph paper. (12 boxes by 12 boxes)
If the scale is *the length of 1 box = 1 inch,* what is the perimeter of the 12 by 12 square?
If the scale is *1 box = 1 square inch,* how many square inches are there in the figure? (count)

Write *144 square inches* on the board. Remind pupils that they are using a square unit and that they are measuring area.
To show that you are measuring area, you write *square* inch.
According to the scale, the length is 12 inches, or 1 foot. What is the width?
If the square is 1 foot long and 1 foot wide, what unit of square measure would you call it? *(1 square foot)*

Show pupils models of 1 square inch and 1 square foot.
How many square inches are there in a square foot?

Follow the same procedure for 1 square yard. Start with graph drawing $3' \times 3'$, scale *1 box = 1 square foot* and end with *1 square yard = 9 square feet.* Show pupils model of a square yard.

Area of land is measured in acres. 1 acre = 4840 square yards
1 square mile = 640 acres

Summary

Units of Area: 1 square inch is the smallest unit in square measure
1 square foot = 144 square inches
1 square yard = 9 square feet
1 acre = 4840 square yards
1 square mile = 640 acres

Practice

1 ft. = ... inches   1 sq. ft. = ... square inches

How many feet in 1 yard?

How many square feet in 1 square yard?

If 1 square yard of carpeting cost \$3.25, how much will 2 square yards cost?

How much will 3 square yards cost? How much will 7 square yards cost?

How much will a $9' \times 12'$ rug cost?

Homework

1. Draw a rectangle 4 inches long and 3 inches wide.

   Use the 1-inch squares prepared in last lesson to cover the rectangular region.

   How many square inches are needed to cover the region?

   What is the area enclosed by the rectangle? *(area of the rectangle)*

2. Draw a rectangle 6 inches long and 2 inches wide.

   Use the 6 square inch strips to cover the area of the rectangle.

   How many rows of 6 square inches are there in the rectangle?

   How many square inches in the rectangle?

   What is the area of the rectangular region?

Lesson 3

FINDING THE AREA OF A RECTANGLE

Warmup

1 square foot = ... square inches
1 square yard = ... square feet
1 acre = ... square yards
1 square mile = ... acres

$5 \times 6 =$ .... $9 \times 12 =$ .... $3 \times 18 =$ ....

Motivation

If you were being paid 10¢ for each square foot you painted, and the wall to be painted was 5 feet long and 7 feet high, how much would you earn if you painted the whole wall?

To find out how much money you would earn, what must you know? *(how many square feet are to be painted)*

When we measure the number of square units in a surface, what are we measuring? *(area)*

Aim

How do we measure a surface? How do we find an area?

Development

What do we know about 1 square foot? *(It is 1 foot long, 1 foot wide, and equal to 144 square inches.)*

Have pupils suggest ways of representing the area of the $5' \times 7'$ wall. If not suggested by them, you suggest a scale *diagram*, or drawing.

Have pupils construct a $5' \times 7'$ rectangle, scale: *1 square = 1 square foot.* If each square represents 1 square foot, count the number of square feet in the wall. *(35 square feet)*

Since the wall is not very large, it is easy to make a scale drawing and count the squares, but think how long it would take if the wall were very large.

Let us look at the diagram. Can you think of an easier way of finding the area? Allow pupils to suggest methods and guide them to the following procedures:

> The base of the diagram represents 5 square feet.
> Have them observe that there are 7 rows of 5 square feet.
> Elicit that 7×5 rows of square feet = 35 square feet.

Have them compare this area with the counted area.

Is there another way we could find the area? (5 columns of 7 square feet or 5×7 square feet = 35 square feet.)

Let us summarize: Length (5 feet) $\times$ width (7 feet) = Area (35 sq. feet). To save time, we say Length $\times$ Width = Area, or Area = Length $\times$ Width. Can you write a formula for finding the area of a rectangle from the above summary? *($A = LW$)*

If we called the length (the base) b and the width (the height) h, what would the formula be? *($A = bh$)* How would you read this formula? *(Area = base × height)*

If the area of the wall is 35 square feet and you charge 10¢ per square foot to paint it, how much money would you earn?

Summary

To measure the area of a rectangular region, multiply the number of units in the length by the number of rows in the width.

The formula for finding the area of a rectangular region is:

$$A = L \times W \textit{ or } W \times L \text{ or } A = lw \textit{ or } wl$$

Sometimes the length is called the *base* and width is called the *height*.

Then, the formula for the area would be written: $A = bh$

Practice

1. On graph paper, make a scale drawing of a room 8 feet by 12 feet. Scale: *1 unit = 1 foot*
2. If a rectangle is 7 yards long and 4 yards wide, what is the area it encloses?

Homework

1. Find the area of a farm 18 miles long and 22 miles wide.

   What formula would you use? ....

   Estimate the area. .... *(about 20 × 20)*

   Find the computed area. ....

   Make a scale drawing of the farm on graph paper.

   Count the number of squares in the diagram.

   Check your answer.

2. Measure the area of one room in your home or a classroom.

   Make a scale drawing of the room.

Lesson 4

MEASURING AREA

Warmup

Estimate, then compute: 1. 17×23 2. $96 \div 8$ 3. 74×62

Motivation

Use formula $A = L \times W$ to find area enclosed by rectanble or square.

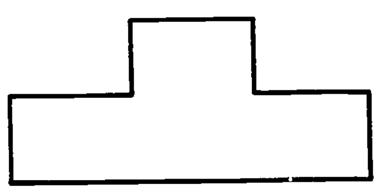
Sometimes the rooms, building lots, or farm we measure are neither perfect squares nor rectangles.

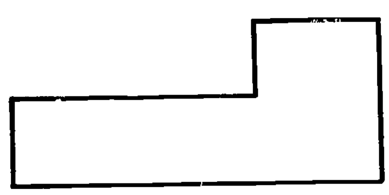
Aim

How do we find the area of rooms not in the shape of a square, or a rectangle?

Development

Prepare worksheet with these, or similar, figures plus measurements:





Have the pupils separate the figures into squares or rectangles.
Find the area of each part, then combine.

Assign Practice and Homework exercises.

Note

If pupil interest and ability warrant, teach any or all of the following:

1. The area enclosed by a parallelogram (leading to the area of a triangular region)
2. Finding the length of a square or rectangle when the width and the area are known $\frac{a}{w} = 1$
3. Finding the width of a square or rectangle when the length and area are known, e.g., 5 l = 30
4. Teach preparation lessons at end of Unit V.

Optional: For Industrial Arts Projects

FLOOR TILING: Lesson 1

NUMBER OF TILES NEEDED TO TILE A FLOOR

Materials (May be obtained from Industrial Arts teacher)

1. 2 feet $\times$ 3 feet hardboards with floor plans outlined with $\frac{1}{4}$ inch moulding (1 per group of 3 or 4 pupils)
2. Floor tiles ($3'' \times 3''$, $9'' \times 9''$, $12'' \times 12''$)
3. Graph paper

Motivation

Display layout board. Discuss how it can be used in the shop and in the math class. Show tiles.

Aim

How do we find the number of tiles needed to cover the surface of a floor?

Development

Review meaning of, and finding perimeter; units of measure used to find P; formula for finding perimeter.

Have pupils find the perimeter of layout board.

Follow same procedures reviewing area and units of square measure.

1. Have pupils find area of layout board.
   Have pupils find area of $3'' \times 3''$ tile, $9'' \times 9''$ tile, $12'' \times 12''$ tile.
   Write on the board:
   area of layout board = 6 square feet
   area of $12'' \times 12''$ tile = 144 square inches or 1 square foot.
   Have the pupils place 1 ft. sq. tiles on layout board. Ask how many tiles were needed? *(6)* Then 6 foot square tiles will cover 6 square foot area.
   Lead the pupils to see that 6 sq. ft. $\div$ 1 sq. ft. = 6
   Remind them that we can divide only like units.

2. Area of layout board — 6 sq. ft.
   area of $9'' \times 9''$ tile — 81 sq. inches

   Have pupils cover area of board with actual tile $9'' \times 9''$. Tiles needed: about 11.

   Can we divide to find the approximate number of tiles? (Have pupils realize they must change measurements to like units.)
   To change 6 sq. ft. to square inches, what must we do?
   Elicit from pupils 1 sq. ft. $= 144$ sq. inches, then, multiply $6 \times 144 = 864$ sq. inches.

   Find area of $9'' \times 9''$ tile. *(81 sq. in.)*
   Now divide: $31\overline{)865} = 10\frac{2}{3}$
   Therefore, 11 tiles will be needed.

3. Have pupils find the number of $3'' \times 3''$ tiles needed to cover layout board.

Summary

To find the number of tiles needed to cover a surface

1. Find the area of the surface to be covered
2. Find the area of one tile
3. Divide: Area of surface $\div$ area of one tile $=$ approximate number of tiles

Note: This gives a number less than or equal to the number of tiles needed.

Lesson 2

THE COST OF TILING A FLOOR

Procedure

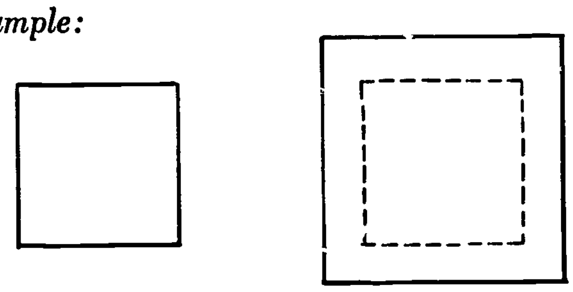
1. Find the approximate number of tiles needed
2. Find cost of one tile
3. Multiply the number of tiles by the cost of one tile

UPHOLSTERING Lesson 1

MATERIAL NEEDED TO COVER A FOOTSTOOL

Procedure

1. Find the area of the surface to be covered.
2. Through diagrams show the extra material needed for tacking and finishing: *Example:*



3. Elicit from pupils: All materials are bought by length measurement. Width has been established by cloth manufacturers. Some common widths are: 24″, 36″, 48″, 54″. The upholsterer buys the width which is best suited for his needs, and which will cost the least.

Lesson 2

COST OF MATERIAL NEEDED IN UPHOLSTERING

Procedure

1. Fid the area of the footstool top.
2. Find the amount of material needed.
3. Find the cost of the material "by the yard" as it is sold. Remind the pupils that a "yard of material" is not only the yard length but also the manufacturer's width.
4. To find the cost of the material needed, multiply number of yards, or part of a yard, needed by the cost for one yard. If material costs \$3.75 per yard:
   (a) 3 yards will cost $3 \times \$3.75$
   (b) $2\frac{1}{2}$ yards will cost $2\frac{1}{2} \times \$3.75$
   (c) $\frac{3}{5}$ of a yard will cost $\frac{3}{5} \times \$3.75$

GENERAL REVIEW

Three class periods should be spent on corrective measures.

Select a number of items from each unit test.

Include questions to test concepts and understandings.

Select a number of mathematical terms from the pupil's vocabulary lists.

Prepare a diagnostic test.

Tabulate errors to locate common areas of weakness.

Review or reteach weak areas.

Unit VI

A NEW LOOK AT NUMBERS AND NUMERALS

The specific objectives of this unit are to:

1. Review and practice basic facts and fundamental operations.
2. Introduce the terms, Commutative, Associative, Distributive properties, and Inverse operation
3. Relate the name of each property with its meaning

Teaching Aids

1. Rulers
2. Board graph
3. Worksheets for lesson 1.

Lesson 1

CONSTRUCTING AND USING AN ADDITION AND SUBSTRACTION FACTS CHART

Warmup

$$\begin{array}{cccccccc} 8 & 9 & 17 & 17 & 9 & 5 & 14 & 14 \\ \underline{+9} & \underline{+8} & \underline{-9} & \underline{-8} & \underline{+5} & \underline{+9} & \underline{-9} & \underline{-5} \end{array}$$

$17 = 9 + \square$	$17 = 8 + \square$	$14 - 9 = \square$	$74 - 5 = \square$
$17 - 9 = \square$	$17 - 8 = \square$	$14 = 9 + \square$	$14 = 5 + \square$

Motivation

Have pupils examine warmup exercises.

Have them note and discuss similarities. Guide them to conclude:

Subtracting a number is the opposite or *inverse of adding that number.*

Therefore, if we can add 9, we can subtract 9 and return to the same number. e.g. $(14 - 9 = 5$ and $5 + 9 = (14)$

Refer to previous lessons where we used "tables" for accuracy and speed.

Suggest the possibility of using a table to check addition and subtraction, *sums* and *differences.*

Aim

Can a chart be used to check sums and differences?

Development

Have pupils prepare a grid as illustrated. *(See top of next page.)*

Have pupils label chart: "Addition and Subtraction Facts"

Work with pupils to complete chart, putting sums in appropriate squares.

Have pupils recognize the pattern.

After chart is completed, have pupils find sums for a number of simple addition facts, e.g. 3 on vertical axis $+ 4$ on horizontal axis $= 7$ at point of intersection. Using chart, check sums.

Have them discover how to use the chart to check subtraction facts, e.g., 7 at point of intersection of 3 and 4. 7 minus $3 = 4$; 7 minus $4 = 3$.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	
1	1	2	3	4	5	6	7	8	9		
2	2	3	4	5	6	7	8	9			
3	3	4	5	6	7	8	9				
4	4	5	6	7	8	9					
5	5	6	7	8	9						
6	6	7	8	9							
7	7	8	9								
8	8	9									
9	9										
10											

ADDITION AND SUBTRACTION FACTS

Have pupils find answers as quickly as they can to the following exercises:

$7+6=$ $\quad 8+7=$ $\qquad 15-9=$ $\quad 14-6=$

$5+9=$ $\qquad 17-8=$

Check answers using chart. Rewrite any incorrect combinations. Correct answers and study. Start with a new list.

Suggest method of increasing accuracy and speed in addition and subtraction.

Summary

An addition and subtraction chart can be used to find and correct answers.

With a little practice, we can increase speed and accuracy in addition and subtraction.

Practice

Write the answers as quickly as you can. (Duplicate for pupils)
Use chart to check answers. (Start in class and complete at home)

$9+5=$	$7+5=$	$15-6=$	$14-8=$
$7+8=$	$3+9=$	$17-9=$	$11-5=$
$6+7=$	$9+2=$	$12-4=$	$11-2=$
$7+9=$	$8+5=$	$11-8=$	$17-8=$
$8+6=$	$4+9=$	$16-7=$	$14-6=$

Homework

Prepare a grid for use in our next lesson — 10 squares on each axis.

Lesson 2

CONSTRUCTING AND USING A MULTIPLICATION AND DIVISION CHART

Warmup

7	10	$9 \times 5 = ?$
7	10	
$+7$	10	$5 \times 9 = ?$
	$+10$	
		then:
$21 = 3 \times ?$	$40 = 4 \times ?$	$9 \times 5 = 5 \times ?$
$21 \div 3 = ?$	$40 \div 4 = ?$	and
$21 = 7 \times ?$	$40 = 10 \times ?$	$45 \div 9 = ?$
$21 \div 7 = ?$	$40 \div 10 = ?$	$45 \div 5 = ?$

Follow same procedure indicated in lesson 1 of this unit to construct and use a multiplication and division table. Allow as much practice as possible. To save class time, use the grid which the pupils prepared as previous day's homework.

Have pupils explain the meaning and add to the vocabulary list:

Factor, Product, Division, Dividend, Quotient

Summary

Dividing by a number is the opposite or inverse of multiplying by that number. With a little practice, we can multiply and divide accurately and quickly.

Assign practice and homework similar to Lesson 1.

Lesson 3

THE COMMUTATIVE PROPERTY OF ADDITION

Warmup

If $69 + 12 = \square$; then $81 - 69 = \square$ and $81 - 12 = \square$
Why is the above true?

$79 = 32 + \square$; $79 - 32 = \square$; $79 - \square = 32$

$86 = 16 + 70$ then $86 - \square = 70$; $86 - 16 = \square$

Motivation

Does knowing that subtracting is the inverse of adding help us find answers quickly and accurately? Why? *(if we know one fact we know the related facts.)* Have several pupils give examples of addition and subtraction related facts. Elicit from pupils that knowing a multiplication fact helps us find the quotients. e.g., $63 = 7 \times \square$ then $7\overline{)\,63} = \square$ and $9\overline{)\,63} = \square$

Have pupils give examples of multiplication and division facts.
Tell the pupils there are other properties to help us in mathematics.
One property is the *commutative* property.

Aim

What is the commutative property?

Development

What are the related facts of $12 + 13$?
What are the related facts of $13 + 12$?
Have the pupils explain why the related facts of the two combinations are similar. Have the pupils realize that: $12 + 13 = 25$ and $13 + 12 = 25$

Therefore $12+13=13+12=25$

Summarize that changing the *order* of the numbers added does not change the sum. This fact is known as the *commutative* property, of addition.

Practice using the commutative property.

$3 + 9 = \square$ $\quad$ $9 + 3 = \square$ then $3 + 9 = 9 + 3$

$15 + 9 = \square$ $\quad$ $9 + 15 = \square$ then .... etc.

Have the pupils experiment with subtraction facts.
Does the commutative property work with subtraction? Why? (We cannot change the order of the numbers subtracted without changing the remainder.)

Summary

The commutative property of addition tells us we may change the order of the *addends* (numbers added) without changing the sum.

Practice and Homework

Assign a number of exercises in addition and have the pupils change the order of the addends and add again. Include vertical examples with more than two addends.

Lesson 4

THE COMMUTATIVE PROPERTY OF MULTIPLICATION

Follow the same procedures as in lesson 3 of this unit.
Have pupils experiment to see if the property works for division.

Summary

The commutative property of multiplication tells us we may change the order of the factors without changing the product.

Addition and Multiplication are commutative.

Subtraction and Division are not commutative.

Practice and Homework

Assign as many basic facts as possible for the pupils to find the products and commute the factors.

Lesson 5

THE ASSOCIATIVE PROPERTIES OF MATHEMATICS

Warmup

What is the commutative property for addition; multiplication?

Show that 37 and 53 are commutative

$$\begin{array}{r} 37 \\ 26 \\ \underline{54} \end{array} \qquad \text{and} \qquad \begin{array}{r} 53 \\ \underline{\times 12} \end{array}$$

Motivation

What are the inverse operations?
What operations are commutative? What operations are not commutative?
Does knowing the inverse operations and the commutative property help us in mathematics? How?

Today we will learn about a new property and how it can help us.
What does $3+2+8$ mean? $(3+2)+8$ or $3+(2+8)$?

Aim

How does knowing the associative property help us?

Development

Have the pupils find the sums of the following as quickly as possible:

1.	$3+2+8=\square$	$5+8=\square$	$3+10=\square$
2.	$17+4+3+\square$	$21+3=\square$	$20+4=\square$

Have them realize that the three parts of item 1 have the same sum. Ask them which addition they found easiest to perform; which sum they found quickest. Proceed similarly with item 2.
Remind them that we add only two numbers at a time.
When we add 3+2+8 we are adding: $3+2=5$, then $5+8=13$

To show how we are grouping to add we may use brackets:
$(3+2)+8=13$, or $5+8=13$
We may group the addends $3+(2+8)$ or $3+10=13$

Have the pupils group the addends of: $17+4+3$
Have them realize that adding $(17+3)+4$ is an easier and quicker way of finding the sum.

Assign several practice exercises and have the pupils judge the groupings easiest to add. Do the same for multiplication.

Conclude: Changing the way we group the addends does not change the sum. The same property holds also for multiplication.

Have the pupils try changing the grouping of the factors in:

$3 \times 2 \times 4$, $6 \times 2 \times 4$, $2 + 5 + 8$

Have them realize that grouping to tens, and grouping to multiplication facts they are more familiar with, will help them work quickly and more accurately.

Conclude: Changing the way we group the factors does not change the product.
Changing the way we group the addends in addition and the factors in multiplication is known as *associative* property of mathematics.

Summary

The *associative* properties tell us:

1. Changing the way we group the addends in addition does not change the sum.
2. Changing the way we group the factors in multiplication does not change the product.
3. Using the associative properties helps us find sums and products more quickly and accurately, and in different ways.

Practice

Assign practice exercises to show that the pupils have used an associative property.

Homework

1. What does the commutative property of addition tell us; of multiplication?
2. What does the associative property tell us in addition; in multiplication?
3. Show by an example how addition is commutative.
4. Show by an example how multiplication is commutative.
5. Is division associative? Is subtraction associative?
6. Give examples to show your answer to 5 is correct.

Lesson 6

SPECIAL NUMBERS: ZERO AND ONE

Warmup

$$
\begin{array}{rrrrrr}
9 & 29 & 324 & 0 & 0 & \\
\underline{+0} & \underline{+0} & \underline{+0} & \underline{+9} & \underline{+29} & 0\times 6 \\
 & & & & & \\
6 & 36 & 469 & 1 & & \\
\underline{+1} & \underline{\times 1} & \underline{\times 1} & \underline{\times 6} & \underline{\times 36} & 0\times \frac{2}{3}
\end{array}
$$

Motivation

Review the meaning of the fundamental operations of addition and multiplication.

Have the pupils look at sums and products in warm up and tell any special facts they note. Lead to aim.

Aim

What are the special properties of zero and one?

Development

With a number of exercises such as:

$5+0=5;\ 0+5=5;$ have the pupils realize that when zero is added to any number, the sum is always that number.

Have the pupils try adding any number, other than zero, to a given number to see for themselves that the sum is always different from the given number.

Have the pupils generalize.

Zero is the only number which when added to another number preserves the other number's *identity*. Hence, *zero* is called the *identity* element for addition.

Proceed similarly with multiplying by *one.*

Have the pupils generalize.

Any number multiplied by *one* is the number itself. Hence *one is the identity* element for multiplication.

Summary

Zero is the identity element for addition.

One is the identity element for multiplication.

Practice and Homework

Assign a number of exercises.

Lesson 7

DISTRIBUTIVE PROPERTIES OF MULTIPLICATION OVER ADDITION

Warmup

Use the laws of mathematics you know to help you work quickly and accurately.

$$\begin{array}{r} 73 \\ \times 3 \\ \hline \end{array} \qquad \begin{array}{r} 11 \\ \times 68 \\ \hline \end{array} \qquad \begin{array}{r} 135 \\ \times 17 \\ \hline \end{array} \qquad 22\overline{)792}$$

Motivation

Review briefly: inverse operations, commutative and associative properties, identity for addition, and identity for multiplication.

Tell the pupils there is another important property of multiplication and addition known as the *distributive property.*

Aim

What is the distributive property?

Development

Have the pupils find the products: $(3 \times 4) + (3 \times 5) = 3 \times (4 + 5)$

$$12 \quad + \quad 15 \quad = 3 \times 9 = 27$$

Have the pupils find the products:

$$4 \times 32 = \qquad (4 \times 30) + (4 \times 2) =$$

$$\begin{array}{r} 63 \\ \times\ 3 \\ \hline \end{array} = \begin{array}{r} 60 + 3 \\ \times 3 \\ \hline \end{array} = \begin{array}{r} 60 \\ \times\ 3 \\ \hline \end{array} + \begin{array}{r} 3 \\ \times 3 \\ \hline \end{array}$$

Remind the pupils that the Hindu Arabic system of numeration is an additive system:

$13 = 10 + 3$ therefore $2 \times 13 = (2 \times 10) + (2 \times 3)$

$128 = 100 + 20 + 8$ $\qquad 3 \times 128 = (3 \times 100) + (3 \times 20) + (3 \times 8)$

When we multiply 3×128, we are distributing the multiplier, using it once for each addend.

$$\begin{array}{r} 10 \\ \underline{\times 13} \end{array} = 13 \times (10 + 2) = (13 \times 10) + (13 \times 2)$$

This property is known as the *distributive* property of multiplication over addition.

Summary

The distributive property tells us that when we must find a product of a number and a sum, e.g., (3×28), we may distribute the multiplier, using it once for each addend: $(3 \times 20) + ((3 \times 8)$

then, $(3 \times 20) + (3 \times 8) = 3 \times (20 + 8) = 3 \times 28$

or $3 \times 28 = 3 \times (20 + 8) = (3 \times 20) + (3 \times 8)$

Practice and Homework

Assign a number of multiplication examples. Have the pupils demonstrate the use of the distributive law as follows:

$$\begin{array}{r} 29 \\ \hline \times\ 6 \end{array} = \begin{array}{r} 20 + 9 \\ \hline \times 6 \end{array} = \begin{array}{r} 20 \\ \hline \times 6 \end{array} + \begin{array}{r} 9 \\ \hline \times 6 \end{array}$$

TEST

1. Which answer is the largest number?

   a. 9 and 5?　　b. 15×2?

   c. $20 + 4$?　　d. 19 from 30?　　1. ....

2. $8 \times 9 =$

   a. 3×12?　　b. 3×10?

   c. 3×24?　　d. 3×20?　　2. ....

3. $14 \times 15 =$

   a. $(10 \times 4) + 51$?　　b. $14 + (50 \times 1)$?

   c. $(10 + 4) + (50 + 1)$?　　d. $14 \times (50 + 1)$?

   3. ....

4. Give an example for each of the following:

   a. The commutative property for addition   4a. ....

   b. The commutative property for multiplication   4b. ....

   c. The associative property for addition   4c. ....

   d. The associative property for multiplication   4d. ....

   e. The distributive property of multiplication   4e. ....

   f. The identity element for addition   4f. ....

   g. The identity element for multiplication   4g. ....

5. Write the missing numeral:

   a. $21 + 49 = 49 +$ ....   5a. ....

   b. $(4 \times 2) \times 8 =$ .... $\times (2 \times 8)$ 5b. ....

   c. $(10 +$ .... $) + 5 = 10 + (6 + 5)$ 5c. ....

   d. $5 \times 4 + 5 \times 3 =$ .... $\times (4 + 3)$ 5d. ....

   e. $12 \times 15 = 15 \times$ ....   5e. ....

6. State which property is used in each of the following:

   a. $(5 + 3) + 1 = 5 + (3 + 1)$ 6a. ....

   b. $7 \times (4 + 5) = (7 \times 4) + (7 \times 5)$ 6b. ....

   c. $8 \times 7 = 7 \times 8$ 6c. ....

   d. $3 \times (6 \times 4) = (3 \times 6) \times 4$ 6d. ....

   e. $24 + 15 = 15 + 24$ 6e. ....

Unit VII

STATISTICS

The specific objective of this unit is to have the pupil learn:

1. To read, interpret and construct statistical tables and graphs.
2. To determine the Mean, Median, Mode and Range from statistical tables and graphs.

Teaching Aids

1. Statistical data.
   (May be obtained from newspapers, almanacs, experiences or any other source available.)
2. Work sheets lesson 1, 2, 3.

Correlation

Social Studies, Science.

Lesson 1

READING AND INTERPRETING STATISTICAL TABLES

Warmup

Write the missing numerals:

5, 10, —, 20, —, —, —, 40

15, 30, —, —, 75, —.

450, 400, ——, 300, ——, ——, ——, 100, 50.

Motivation

In previous lessons we have collected number facts and listed them.

We called the number facts "the data," and the listed data "a table."

A better name in some cases is a *statistical table.*

Aim

How do we use statistical tables?

Development

Have the pupils list the following facts as given:

Five members were to be selected for a Student Court in Thomas Paine Junior High School. The voting showed the following results:

Al 268, Anna 119, Ben 201, Bob 278, Fred 312, Grace 134, Harry 165, Inez 302, John 281, Lydia 268, Marie 178, Sam 149.

VOTING RESULTS FOR STUDENT COURT

Names	*No. of Votes*	*Names*	*No. of Votes*
Al	268	Harry	165
Anna	119	Inez	302
Ben	201	John	281
Bob	278	Lydia	268
Fred	312	Marie	178
Grace	134	Sam	149

Have the pupils answer the following questions:

1. Who was elected?
2. How many votes did each winner receive?
3. How many votes were cast?
4. Did Fred receive a majority of the votes?

Ask the pupils to suggest other ways of organizing the data and to select the organization they think is best. (Listed according to number of votes received) Have the pupils prepare a table starting with pupil receiving largest number of votes, e.g., Fred 312, Inez 302, etc.

Have the pupils realize that the order in which the facts are organized can help getting the information sought quickly and easily.

Summary

We get exact number facts and information from statistical tables.

Practice

Distribute duplicated copies of following table.

HOME RUN DISTANCE IN SOME AMERICAN LEAGUE BALL PARKS

City	*Park*	*Feet from Plate to Fence*		
		Lf	*Cf*	*Rf*
New York	Yankee Stadium	301	461	296
Boston	Fenway Park	315	420	302
Cleveland	Municipal Stadium	320	410	320
Detroit	Briggs Stadium	340	440	325
Chicago	Comiskey Park	352	415	352
Washington	Griffith Stadium	350	401	320
Baltimore	Memorial Stadium	309	425	309
Kansas City	Kansas City Stadium	330	421	353

Have pupils answer questions such as:

1. In which parks is the distance from home plate to fence in left field greater than in right field?
2. Is there a park in which the distance from home plate to the fence in right field is greater than the distance to the fence in left field? Which?

3. In which park is the distance from home plate to center fiield the greatest?
4. How much greater is that distance than the next longest distance? (see #3)
5. Babe Ruth hit the longest home run on record in 1919. The ball traveled 587 feet. In which park could this be a home run?

Homework

Have pupils select a statistical table from a newspaper or the Almanac, and note the information they can obtain from the table.

Lesson 2

PREPARING A STATISTICAL TABLE

Warmup

6×375 $\quad$ 23×214 $\quad$ $21\overline{)903}$

9×918 $\quad$ $9\overline{)1908}$

Motivation

Discuss the need for organizing information in the form of a table for quicker interpretation.

Aim

How do we construct a statistical table?

Present a problem, such as: The salesman who rents the caps and gowns for graduation wants to know the height of each boy and girl in our class so that he can order the correct sizes from his manufacturer. In what form shall we send the information?

Have pupils discuss ways of presenting the information on height. (possible answers: by rows, an alphabetical list of the class, lists of boys and lists of girls, size place list, grouping by sizes)

Help pupils develop tables based on their suggestions.

1. Random order

Heights of Boys and Girls in Class 9–10 (as reported)			
Boys	*Girls*	*Boys*	*Girls*
5′ 6″	5′ 6″	5′ 11″	5′ 0″
5′ 6″	58″	5′ 6″	5′ 2″
5′ 1″	61″	5′ 5″	68″
5′ 4″	4′ 7″	5′ 6″	5′ 4″
5′ 9″	5′ 8″	5′ 6″	5′ 2″

2. Organized order (size place, size place by sex)
Suggest an arrangement which would be more helpful to the salesman.

Heights of Pupils in Class 9–10 (in order of size)	
Height in Feet and Inches	*Inches*
4′ 7″	55
4′ 10″	58
5′ 0″	60

Have pupils realize that repetitions occur and the salesman might find the information more useful if he could see at a glance the number required for each height.

3. Frequency table
Have pupil draw up a table which shows the frequency of each height.

Heights of Pupils in Class 9–10

Height	*Tallies*	*Number*			
55	/	1	65	/	1
58	/	1	66	~~////~~	6
60	/	1	68	/	2
61	//	2	69	/	1
62	//	2	71	//	2
64	//	2			

Have pupils answer the following questions:

How many pupils are 55″ tall? 71″ tall? 59″ tall?

How many are more than 61″ tall? less than 58″ tall? at least 66″ tall?

Why is this arrangement more helpful to the salesman than the other types of arrangements?

Summary

To prepare a statistical table:

1. Collect all the number facts.
2. Organize the facts so that they give the information quickly and easily:
   a. Listing numbers in order, from largest to smallest, or smallest to largest.
   b. Preparing a frequency table.

Practice and Homework

Duplicate for pupil use.

1. Underline words that describe the word in the first column:
   a. Increase: more, larger, less, higher, smaller
   b. Decrease: more, lower, less, larger, smaller
   c. Represent: present, stand for, numeral, in place of
   d. Numeral: sign, symbol, mark, digit
   e. Number: size, quantity, amount, count

2. COMPARISON OF SALES IN DEPARTMENT STORES 1964 AND 1965
(week period ending May 14, 1965)

Federal Reserve Districts	*Difference in Percent*
Boston	+1
New York	+9
Philadelphia	+3
Cleveland	+4
Richmond	—1
Atlanta	+3
Chicago	+1
St. Louis	0
Minneapolis	+4
Kansas City	—4
Dallas	—7
San Francisco	0

a. Would you say that there was a general increase or general decrease in sales? Explain.

b. Which cities did better this year than last year?

c. Which city made the poorest showing?

d. What happened in the San Francisco and St. Louis areas?

e. What was the increase in New York over last year?

3. Prepare another statistical table using the same data, but change the order so that the city showing the greatest *increase* is first, the city showing the greatest *decrease* is last.

Lesson 3

STATISTICAL GRAPHS

Warmup

1. Round to multiples of 10: 39, 63, 75, 81
2. Round to multiples of 100: 125, 363, 415, 450
3. Round to multiples of 1000: 1298, 1723, 2050, 2509

Motivation

Compare statistical tables with graphs. Both are used to compare number fact. Both must have a title. Table has headings for clarification. Axes of graphs are labeled for clarification.

Review importance of "scale" used. (Value of each space)

Organization of table may be in ascending or descending order.
Graph is usually in ascending order. Must be in chronological order.
We must prepare a table of the facts before we can construct a graph.

Aim

To interpret a statistical graph.

Development

Have the pupils prepare a statistical table from the facts listed below. (A frequency table)

Remind them to include a title, and to organize the facts.

There are 30 pupils on the basketball squad.

The heights are:

60″	62″	64″	66″	63″
59″	71″	67″	72″	68″
61″	65″	63″	63″	
67″	68″	64″	69″	
70″	66″	69″	61″	
68″	65″	72″	64″	

Before the pupils start ask:

Which is the shortest height? *(59″)*

Which is the tallest height? *(72″)*

Explain that the lowest to the highest number values is called the *Range.*

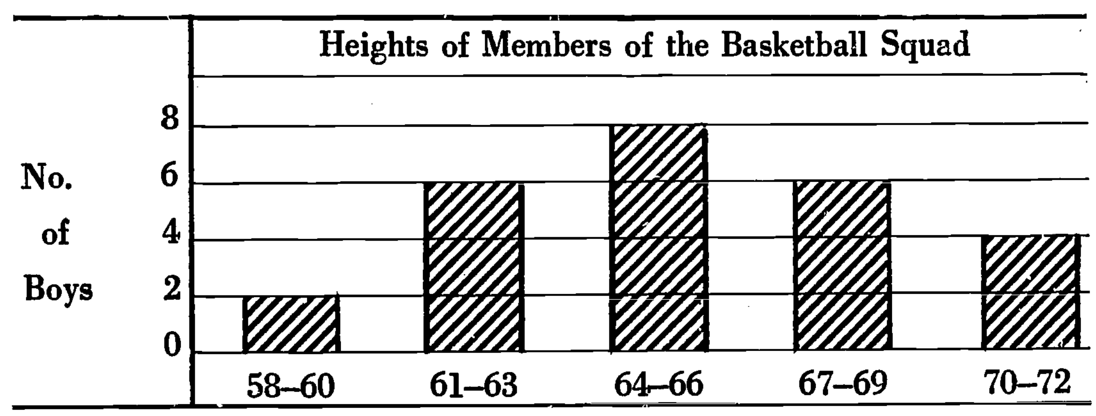
Have the pupils realize that all the heights fall between 58″ and 73″.

When table is completed have the pupils answer the following and similar questions:

How many members are 72″ tall? 67″ tall?

What height measurement has the greatest number of members? etc.

Distribute duplicate of bar graph.



Have the pupils answer the following and similar questions:

What scale was used? Range?

What is the title of the graph?

Which height measurement contains the most members? The least members?

How many members are on the squad?

Have the pupils realize the graph contains the same facts as the table they prepared.

Summary

Graphs picture statistical data.

We can get the information we seek quickly and easily from graphs.

Practice and Homework

Assign statistical data and questions.

Have pupils prepare a table and a graph from which they will answer the questions assigned.

Lesson 4

FINDING AN AVERAGE VALUE

Warmup

1) $\begin{array}{r} 70 \\ 80 \\ \underline{66} \end{array}$	2) $\begin{array}{r} 31 \\ 32 \\ 44 \\ \underline{33} \end{array}$	3) $\begin{array}{r} 50 \\ 60 \\ 55 \\ \underline{95} \end{array}$
4) $3\overline{)216}$	5) $4\overline{)140}$	6) $4\overline{)260}$

Motivation

Review through discussion the meaning of *average* attendance, *average* weight, *average* number of pupils present for the week, etc.

Have the pupils realize that sometimes it is necessary to obtain a picture of a group of scores for better understanding.

One way of obtaining such a picture is through the *average* score.

Aim

How do we find an average?

Development

Use warmup exercise, assuming that numbers represent test grades:

a. The average is the mark that would give the same number of points if all three or four tests had *identical* grades, e.g.

$$\begin{array}{r} 70 \\ 80 \\ \underline{66} \\ 216 \end{array} \qquad 3\overline{)216}^{\,72} \qquad \begin{array}{r} 72 \\ \underline{\times\ 3} \\ 216 \end{array}$$

Have the pupils realize that if they were to take 8 points from the highest score and distribute them 6 to 66 and 2 to 70 they would then have three identical scores.

b. Have the pupils find the average of the following by taking points from higher scores and adding them to lower scores:

a) 30, 40, 50 b) 55, 65, 60 c) 64, 68, 60

Have the pupils conclude it is easier and quicker to add the scores, than to divide by the number of addends.

2. Review computation of arithmetic averages with such problems as:

a. In last year's Red Cross Drive the girls of class 9-3 made the following contributions: 20¢, 10¢, 15¢, 15¢, 10¢,10¢, 15¢, 20¢, 15¢, 20¢. The boys of class 9-13 made the following contributions: 7¢, 12¢, 15¢, 20¢, 16¢, 14¢, 8¢, 13¢, 15¢, 8¢, 14¢, 12¢, 15¢, 16¢, 15¢. Find the average contribution made by each boy and by each girl.

Solution.

Pupil thinks: Add all the contributions made by each group (girls, boys); divide each total by the number of contributors.

He fiinds that the average contribution for the girls was 15¢, for the boys 13¢.

b. Pose problems with extreme values and have pupil discover how they alter the average. Use data in 2a, but substitute a 5-dollar contribution for one of the 10¢ contributions.

Solution.

Pupil finds the average. He discovers that the average contribution of the girls has changed from 15¢ to 60¢, thus giving a false impression of what is commonly thought of as the "average" contribution. After many such situations, guide the pupil to sum up his observations.

Summary

1. To find the average of a group of scores we add the scores then divide the sum by the number of scores added.
2. The value of each score is important in finding an average.
3. Extreme values give a false impression of the "average".

Practice and Homework

Assign a number of problems. Have pupils find the average score, number, height, weight, etc.

Lesson 5

MEANING OF THE MEAN AS A MEASURE OF CENTRAL TENDENCY

Warmup

1. What is the average length of:
   a. 368 ft., 369 ft., 367 ft.
   b. .8 ft., .5 ft., .4 ft., .3 ft.
2. What is the average weekly salary of a person who earns $52.00 the first week, $68.00 the second week, and $48.00 the third week?

Motivation

Through discussion help the pupil realize that as the number of cases and values increase, an easier method for finding the average is needed.
Have pupils suggest methods. Elicit, if possible, or suggest using a frequency table.

Aim

What is *average?*

Development

Use data in problem 2a in lesson 4, to develop the following table:

Red Cross Drive							
Contributions by Girls				Contributions by Boys			
Amount	Tally	No. of Contributors	Total Contribbutions	Amount	Tally	No. of Contriutors	Total Contributions
10¢	///	3	30¢	7¢	/	1	7¢
15¢	~~////~~	5	75¢	8¢	//	2	16¢
20¢	///	3	60¢	12¢	//	2	24¢
		11	165¢	13¢	/	1	13¢
			$1.65	14¢	//	2	28¢
				15¢	////	4	60¢
				16¢	//	2	32¢
				20¢	/	1	20¢
						15	200¢
							$2.00
Average for boys: 13¢				Average for girls: 15¢			

After other problems similar to the one above, have the pupils learn that this type of "average" is called the *mean.*

Summary

The "Mean" or "Average" is *one* value that represents an entire group.

Practice and Homework

Assign problems, such as the following:

1. Find the mean of the daily mileage covered by a salesman whose daily trips were:
   125, 75, 140, 220 and 90 miles.
2. Mrs. Smith's gas bills from January to December were:
   $3.60, $8.25, $4.20, $3.25, $2.85, $2.45, $2.25, $2.25, $2.20, $2.45, $2.85, and $3.25. What was the mean cost per month for gas services?
3. What does the expression "Mean Temperature" mean?

Lesson 6

MEANING OF THE MEDIAN

Warmup

Have the pupils find: $\frac{1}{2}$ of 4, 10, 40, 5, 11, 41

Motivation

Review method of finding the average. Have pupils realize the average is a measure of central tendency. Tell them another measure of central tendency is the median or middle score.

Aim

How do we find the median?

Development

Choose seven boys (girls) in the class. Let them arrange themselves in size places, and have each write his height on the board.

1. With reference to their heights, which boy is in the middle? Why?
2. How many boys are below the middle one? How many above?
3. With reference to the numbers on the board, which is the middle number? How do you know?

This middle position, or value, is called *the median.* (The median is the middle point so chosen in a series that there are as many numbers above it as there are below it, when they are arranged in order of size.) Divide the number of cases by 2 (or take ½): 7=3R1, then the median is the fourth number, e.g.,

3 — 1 — 3
↑
Median
↓
67, 66, 65, 64, 63, 62, 61

If we have 8 cases, the midpoint is halfway between 4 and 5.

$$\overset{4}{\overbrace{67,\ 66,\ 65,\ 64,}}\ \overset{\text{Median}}{\big|}\ \overset{4}{\overbrace{63,\ 62,\ 61,\ 60}}$$

$$\downarrow$$

$$63\frac{1}{2}$$

Summarize

To find the median of an odd number of cases:

1. Divide the number of cases into two equal parts.
2. The odd number in the middle position is the median.

To find the median of an even number of cases:

1. Divide the number of cases into two equal parts.
2. Then find the value halfway between the 2 middle values.

Practice and Homework

Have pupils practice finding the median in problems with an even number of cases and with an odd number of cases.

Have pupils find the mean of the same problems.

Have the pupils compare the mean and median for each problem.

Lesson 7

MEANING OF MODE

Warmup

Assign several multiplication and division exercises.

Motivation

Discuss measures of *central tendency*.

Review meaning of Mean and Median.

Review preparation of a frequency table.

Lead to another measure of central tendency, called the *Mode*.

Aim

What statistical information does the Mode give us?

Development

Present the following problem:

The owner of a dress store always keeps a record of the sizes of dresses sold during each week.

Size	*Number Sold*	*Size*	*Number Sold*
10	4	16	15
12	6	18	4
14	9	20	2

Which dress was sold in greatest quantity?

Following regular lesson format, have the pupil learn that the number which occurs most frequently in a list of numbers is called the *mode*. It is found by *inspection* and is used when the score, or number, that occurs most frequently is needed.

The mode is a measure of central tendency and it can be found by inspection. (no computation is necessary)

Help the pupils realize that:

The mean is found by computation.

The median is found by counting.

The mode is found by inspection.

The three measures may differ in any set of data.

Each measure has a special purpose or advantage.

Suggested Practice

1. Have the pupil find the mean, median, and mode from data he gathered in the Lesson 3.
2. Have pupils indicate their choices of occupation. Have them make up a frequency table from these data and find the mode.
3. On Friday, April 14, 1960 the New York Times reported that the median income for American families was $5,300 for the previous year. The average *(mean)* was $6,520. The mode was $4,600.
   a. What income was received most frequently by an American family?
   b. How much less is the mode than the mean?
   c. What fractional part of the number of American families received more than $5,300? What part received less than $5,300?
   d. The average (mean) was $250 higher than that of last year. What was the mean last year?
   e. The median income of American families rose to $5,417 as reported

June 20, 1960. How much of an increase is this over the median reported April 14, 1960.

TEST

1. Why is the mode the easiest of the measures of central tendency to find?

   How do you find the median when you have an odd number of scores?
   How do you find the median when you have an even number of scores?
   Why is the mean the most difficult to find?

2. The pupils in Class 7-2 and 8-3 estimated the number of beans in a jar (secret ballot). The number was 100. They tabulated their responses in a frequency table.

Number	*7-2*	*8-3*
0 - 24	//	
25 - 34	~~////~~	/
35 - 44	////	//
45 - 54	~~////~~	~~////~~ /
55 - 64	////	///
65 - 74	/	//
75 - 84	///	//
85 - 94	//	/
95 - 104		//
125 - 134		
145 - 154		
105 - 114		
115 - 124		
125 - 134	/	
135 - 144		
145 - 154	/	

In what respect is this grouping different from other tables you have made?

How many pupils are there in 7-2? In 8-3?

What was the highest estimate in 7-2? The lowest? The range?

What was the highest estimate in 8-3? The lowest? The range?

What is the *mode* in this distribution for the Grade 7 class? For the Grade 8 class?

What is the *median* for the Grade 7 class? For the Grade 8 class?

What class made better estimates? Explain. (8-3 had smaller range.)

Unit VIII

ROUND AND ROUND IN TIME

The specific objectives of this unit are to:

1. Review the meaning and use of a unit of measure
2. Review units of time measurement
3. Learn the properties of a circle
4. Learn the properties and classification of angles
5. Measure angles
6. Learn the relationship of the circumference to the diameter of a circle
7. Find the circumference of a circle by using the formula $C = 2\pi$

Teaching Aids

1. World globe
2. Board compass, pupil compass
3. Seven inch paper disks

Optional Aids and Books

1. Primary Time Telling Kits
   Watchmakers of Switzerland, Information Center
   750 Fifth Avenue, New York 19, New York
2. The Earth Is Your Space Ship — McGraw Hill
3. The First Book of Time — Franklin Watts

Lesson 1

MEASURING TIME

Warmup

1. \$59.06	2. 5 ft. 6 in.	3. 25 square inches	4. 38 hours
—35.77	—3 ft. 7 in.	—17 square inches	—19 hours

5. 1 square yard = ... square feet

6. 1 square foot = ... square inches

Motivation

What is length? (Distance between points)
What is area? (Surface measure)
What is time? Can we see it? Hear time? Touch time?

Have pupils realize that the passing of time includes passing from lightness to darkness or day to night, etc.
Where does our light come from? *(Sun)*

What causes day and night? (Briefly recall and discuss rotation of earth on its axis and its orbit around the sun. Use globe to demonstrate.) The rotation of the earth towards the sun and away from the sun gives us our days. What does one complete orbit around the sun give us? *(1 year)*

Can we stop time? Can we start time?

Aim

How do we measure time?

Development

How do we measure time?
How do we measure area? *(square units)*
To measure length and area we use measuring tools such as: rulers, yardsticks, square inches, etc.

What measuring tools do we use to measure time? *(clocks, watches, calendars)*
Did man always have the clocks we use today?

What were some ways man measured time long ago? *(sun dial, sand glass, position of moon and stars, etc.)*

Look at the calendar we use today. Has it always been eaxctly the same?

(Point out that changes were made sometimes to please powerful rulers, and sometimes to improve or unify recordkeeping.)

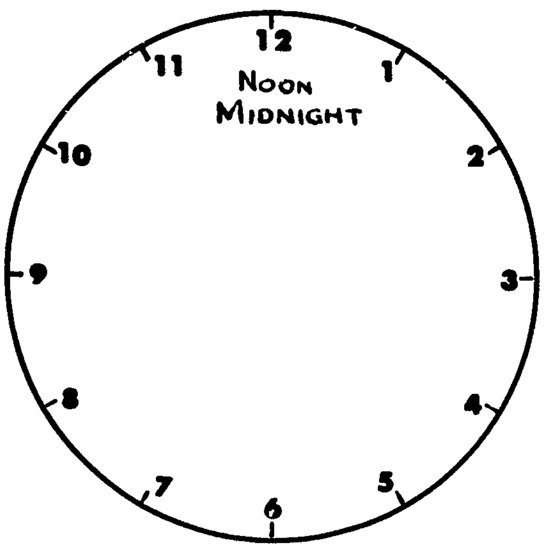
How much time does the commonly used calendar represent? *(1 year)*
Since time is measured by the movement of the earth, what does *1 year* really mean? *(The earth has completed one orbit around the sun, passing through the four seasons.)*

How has man divided the year or what are the units of time used on the calendar? *(year, month, week, day)*
How long is a year?

How much time does the commonly used watch or clock represent? *(one half of earth's rotation)*
What does *12 o'clock noon* mean? *(The sun is directly overhead at a selected spot on the earth's surface.)*
What does *12 o'clock midnight* mean? *(Selected spot directly opposite noon position after having rotated one half way round.)*
Distribute copies of worksheet.
Have pupils attach strips to clock face. Using clock and calendar develop summary for day's lesson.

Summary

UNITS OF TIME	Approx. 4½ weeks or 30 days — 1 month
1 day = 24 hours	12 months = 1 year
1 hour = 60 minutes	365 days = 1 year (approximately)
1 minute = 60 seconds	10 years = 1 decade
7 days = 1 week	100 years = 1 century



Practice

1. If you spend 6 hours in school, what part of a full day are you in school?
2. How many minutes pass when the minute hand moves between the numerals *2* and *7?*
3. What does *2:20 A.M.* mean? *2:20 P.M.* mean?
4. Write without words: 20 minutes past 3 in the afternoon
   20 minutes before 4 in the afternoon.

Homework

Assign a number of specific exercises to review units of time, and test the meaning and use of time measurements.

Lesson 2

EARTH, TIME, AND CIRCLES

Warmup

$\frac{1}{2}$ of $24 =$	$\frac{1}{2}$ of $12 =$	$2 \times 12 =$ ....
$\frac{1}{4}$ of $24 =$	$\frac{1}{4}$ of $12 =$	$4 \times 6 =$ ....
$\frac{1}{6}$ of $24 =$	$\frac{1}{6}$ of $12 =$	$6 \times 4 =$ ....

Motivation

We know that time is measured by the movement of the earth on its axis and in its orbit.

We know that one complete rotation is called one day, and one day equals 24 hours. Where does the day start? Where does it end?

Aim

How were time units decided upon?

Development

For convenience, and so that man all over the earth measured time the same way, mathematicians and map makers divided the distance around the earth into 24 equal parts. Then they drew *imaginary* lines around the earth which they called *meridians*. The first meridian is known as the *Zero Meridian*. It passes through Greenwich, England.

(Use the globe to show that each meridian around the earth divides it into 2 equal parts.)

When the sun is directly over the meridian, we say it is *noon*. The same meridian, on the opposite side just starting toward the sun, is the midnight hour.

Distribute copies of worksheet.

This is a copy of the 24 hour clock. We know that the hours from midnight to noon are the morning hours, or Ante Meridian Time.

Ante Meridian means before the sun is directly over the meridian, or *before meridian*.

What does *2:20 A.M.* really mean?

The hours from noon to midnight are the evening, or *Post Meridian* hours, meaning *Passed Meridian* time.

What does *2:20 P.M.* really mean?

In the armed forces, the 24 hour clock is sometimes used. Look at the diagram. 4 figures are used for each hour.

What time does 0100 represent? 0600? etc.

Write the missing numerals on the 24 hour worksheet.

Have the pupils compare the 12 hour clock with the 24 hour clock and discuss the direction *clockwise*.

Summary

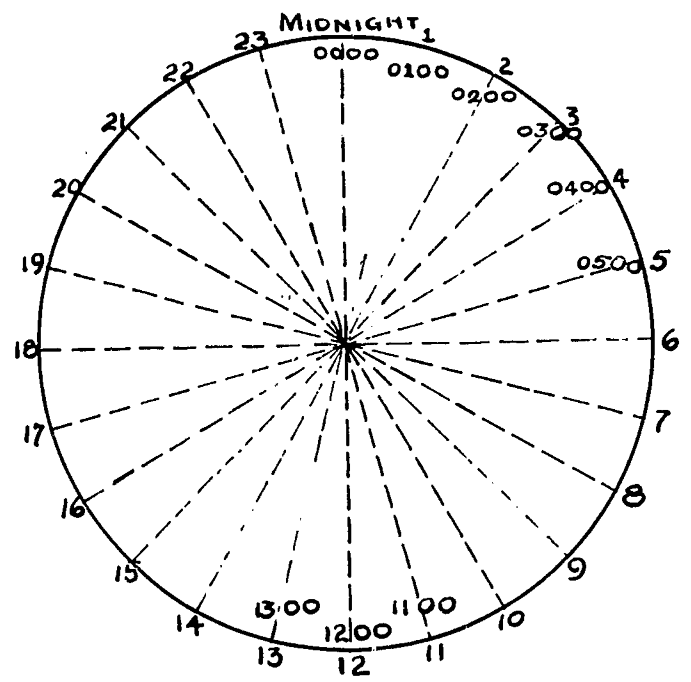
Time is measured by the movement of the earth in relation to the sun.

Practice and Homework

Complete timetable.

Midnight		Noon	
12 midnight	— 0000	12 noon	— 1200
1 a.m.	— 0100	1 p.m.	— ....
2 a.m.	—	2 p.m.	— ....
....	— 0300	3 p.m.	— ....

4 a.m. —	4 p.m. — ....
5 .... —	5 p.m. — ....
6 a.m. —	6 p.m. — ....
7 a.m. —	7 p.m. — ....
8 a.m. —	8 p.m. — ....
9 a.m. —	9 p.m. — ....
10 a.m. —	10 p.m. — ....
11 a.m. —	11 p.m. — ....



Lesson 3

CIRCLE MEASUREMENT

Warmup

$\frac{1}{2}$ of 360 = ... $\frac{1}{2}$ of 180 = ...

$\frac{1}{4}$ of 360 = ... $\frac{1}{2}$ of 90 = ...

$\frac{1}{6}$ of 360 = ... $90 + 45 =$ ....

Motivation

We know that time units were established by dividing a circle, representing the earth's equator, into 24 equal parts.

Aim

How do we divide round or circular areas evenly?

Development

Distribute 7 inch discs.

Have pupils measure the distance across the disc at its widest part, draw a line horizontally across, and mark the center. (3½ inches from edge) Have pupils label the line segment drawn *the diameter*.
What does the *diameter* do to the circle? *(Divides it into 2 equal parts)*

Have pupils draw a vertical diameter (perpendicular to horizontal diameter) dividing the disc into 4 equal parts.

Have them note that the distance from the center to the edge is the same all around. Have them draw one line segment and label it *Radius*. Explain that the length of the radius determines the size of the circle.

Demonstrate by using board *protractor* and/or lengths of string.

Have pupils learn the vocabulary of the circle: *circumference, center, radius, diameter,* and label all parts on the disc.

A long time ago, it was decided that one complete rotation of the radius equals 360 *degrees*, or 360°.

Distribute diagrams. Use diagrams to have pupils realize that the size of the circle does not change the number of degrees in the circle, since the rotation at the center is always the same.

Summary

A complete rotation *(a circle)* equals 360°

Circumference = Distance around the circle, or length of circle

Center = Point within the circle equally distant from every point on the circle.

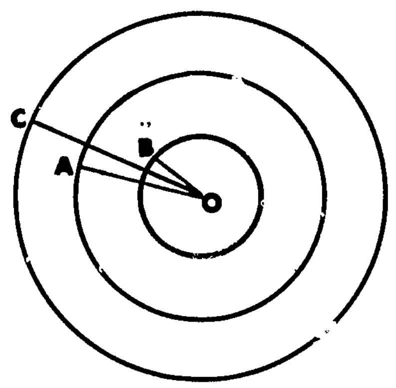
Radius = Length from the center to the circle

Diameter = A straight line through the center connecting 2 points on the circle.

Practice and Homework

1. How many degrees are there in the circle with radious AO?
2. How many degrees are there in the circle with radious CO?
3. How many degrees are there in the upper ½ of the larger circle?
   in the lower ½ of the larger circle?
4. How many degre are there in the upper ½ of the smaller circle?
   in the lower ½ of the smaller circle?
5. How many degrees are there in ½ of any circle?
6. How many degrees are there in ¼ of any circle?

Measure the diameter and circumference of 3 circular objects in your home, such as glasses, plates, records, baskets, etc. Use a tapemeasure. if you have one. If not, use a string, then measure the length of the string If you use a string, measure the length of the string with a ruler.



Lesson 4

MEASURING THE CIRCUMFERENCE OF A CIRCLE

Warmup

1. Change $3\frac{1}{7}$ to an improper fraction. ....
2. Change $3\frac{1}{7}$ to a decimal (2 decimal places)
3. $4\overline{)13}$ 4. $6\overline{)38}$ 5. $8\overline{)42}$
6. $\frac{22}{7} \times 14 =$ 7. $3.14 \times 14 =$

Motivation

To measure the circumference, or distance, around small objects is easy. We can't use a ruler or a yardstick, but what can we use? (tape measure made of steel cloth, or string) We can also roll circular object along a yard stick. (Have a pupil roll a plate or record along the yardstick.) How would we measure the circumference of a large circle?

Aim

How do we measure the circumference of a circle?

Development

Have the pupils prepare a table, using the following headings:

Article, Diameter, Circumference, Circumference $\div$ Diameter
(In last column use 3 + rather than exact quotients)

In appropriate columns write the measurements of article measured in motivation.

Have pupils measure several other round articles and write measurements in table.

Have pupils write in table, articles and measurements from homework assignment.

Have pupils examine the table and note relationship of diameter and circumference. Conclude that in every case the circumference is a little more than 3 times greater than the diameter.

After measuring a very large number of circumferences, the mathematicians observed that every circumference is approximately $3\frac{1}{7}$ times the diameter of the circle.

Have pupils take the diameter of articles listed in the table, multiply by $3\frac{1}{7}$, $\frac{22}{7}$, or 3.14 and see how close they get to the actually measured circumference.

Have pupils generalize: Circumference equals approximately $3\frac{1}{7}$, $\frac{22}{7}$, or 3.14 times the diameter.

Have pupils abbreviate: $C = 3\frac{1}{7} \times D$ (or $C = 3\frac{1}{7} \times D$)

Summary

Circumference is the distance around the circle.

The measurement of the circumference is always a little more than 3 times the length of the diameter.

To find the circumference, we multiply the diameter by $3\frac{1}{7}$, $\frac{22}{7}$ or 3.14.

Practice and Homework

Assign a number of exercises, reviewing properties of a circle, and finding the circumference.

Lesson 5

FORMULA FOR THE CIRCUMFERENCE OF A CIRCLE

Warmup

$\frac{22}{7} \times 154$ $\quad$ $(2 \times 77) \times \frac{22}{7}$ $\quad$ 3.14×154 $\quad$ $3\,1 \times (2 \times 77)$

What is the circumference of a circle whose diameter is 14 inches?

Motivation

Continue with previous lesson.

Development

If we know the diameter of a circle, how do we find the circumference?

Have pupils draw a circle. (trace a coin or draw freehand)

Have them recognize that a diameter is 2 radii or $2 \times$ the radius.

Then: $C = 2 \times \text{radius} \times 3\frac{1}{7}$ is the same as $C = D \times 3\frac{1}{7}$

Mathematicians use the Greek letter π (pi) to express the number which approximately is equal to $3\frac{1}{7}$, $\frac{22}{7}$ or 3.14.

Summary

To write the formula for the circumference: $C = \pi D$ or $C = 2\pi R$

Practice and Homework

Prepare a number of exercises using this formula.

Lesson 6

ANGLES

Warmup

$\frac{1}{2}$ of $360° =$	$\frac{1}{2}$ of $180° =$ ....
$\frac{1}{4}$ of $360° =$	$\frac{1}{4}$ of $180° =$ ....
$\frac{1}{10}$ of $360° =$	$\frac{1}{3}$ of $180° =$ ....

Motivation

Have pupils take out clock face with time hands attached. Compare clock hands to radius of a circle. By rotating hands, have pupils realize that *angles* are formed.

Have pupils rotate 2 pencils, 2 rulers, their hands, the pages of a book, etc. The point of rotation of one line about another is called the *vertex.*

Aim

How do we measure angles?

Development

We know a complete rotation equals 360° and $\frac{1}{2}$ rotation equals 180°

Have pupils use clock face to see that hands seem to form part of a straight line when $\frac{1}{2}$ rotation is made.

Distribute worksheet. Note that only angles are drawn. Look at the first angle. It appears to be a straight line. It is an angle that measures 180 degrees. It is called a straight angle. Then a straight angle $= 180°$; $\frac{1}{4}$ of a rotation $= 90°$; then an angle with $\frac{1}{4}$ rotation is a 90° angle and is called a *right angle.*

Stress that a right angle forms a perfect corner. Illustrate with squares and rectangles. Proceed similarly with *acute angle* (less than 90°) and *obtuse angle* (more than 90°, but less than 180°).

Have pupils learn to label angles.

Distribute half circles. Show pupils board protractor. By folding semicircle in half, then quarters, etc. have them mark and number of degrees, starting with zero at the right to 180° at left. Have pupils turn semicircle over and number degrees starting with zero at the left.

Show pupils how to measure angles using the protractor

Summary

An angle has 2 sides and a vertex.

The size of an angle depends upon the amount of rotation.

Angles are measured in degrees.

Kinds of angles: acute, right, obtuse and straight angles.

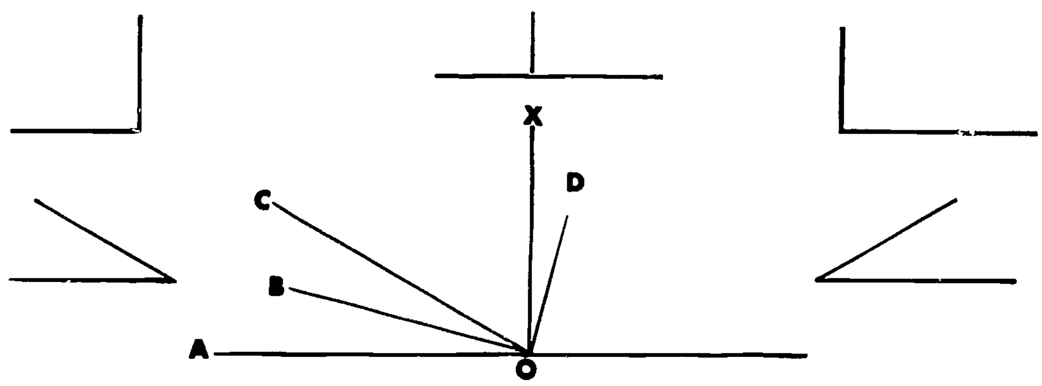
Practice and Homework

A..................................●..................................B

O

1. ∠ AOB is a .... angle. 2. ∠ AOB = ... degrees.

Measure and label the following angles.



In the above figure, name:

a right angle: .... a straight angle: ....

an acute angle: .... an obtuse angle: ....

Prepare a unit test including:

Units of time measurement
Properties of a circle
Relationship between diameter and the circumference
Formula for the circumference of a circle
Finding the circumference of a circle
A value of pi (approximate)
Parts of an angle
Kinds of angles
Reading protractor measurements of angles

Unit IX

THE LANGUAGE OF MATHEMATICS

The specific objectives of this unit are to have the pupils learn to:

1. Recognize mathematical statements of equality and inequality
2. Determine the truth or falsity of mathematical statements
3. Understand the meaning of an open sentence
4. Understand meaning and use of replacements for pronouns, symbols, letters, and frames in open sentences
5. Understand and use the terms: range domain, variable, replacement set, solution set, empty set, place holder
6. Solve open sentences (find solution sets)

Lesson 1

MATHEMATICAL SENTENCES

Warmup

Write 1 or 2 simple words to explain the meaning of the following words:

Sample: decrease — lessen

1. compare	6. difference
2. increase	7. raise
3. remainder	8. factor
4 double	9. sum
5. clue	10. perimeter

Motivation

Write the following on the board:

1. The United States Government is experimenting with rockets.
2. Two plus eight equals ten.
3. $2+8=10$
4. One half of 22 is 2.

Ask the pupils if the first statement is a sentence. Why? *(It expresses a complete thought.)*

Ask the pupils to consider statements 2 and 3.
Do they express complete thoughts? Are they sentences? How does sentence 3 differ from the other sentences? *(Mathematical numerals and symbols are used to express the thought.)*

Aim

What are mathematical sentences?

Development

Use the sentence: "John built a model rocket," to elicit from the pupils what they have learned about sentences in the language arts class:

1. Expresses a complete thought.
2. Has a subject, verb, and object.
3. Nouns and pronouns are used to indicate persons and/or things.
4. A verb is used to express the action.
5. The object expresses the result of the action.

Use $9 \times 4 = 36$ to elicit from the pupils that mathematical sentences express a complete thought.
Have the pupils conclude that a *complete* mathematical expression is a *mathematical sentence* or statement.

Have the pupils judge which of the following expressions (and similar expressions) are mathematical sentences.

$8 + 7 = 15$	$1 + 7 = 7 + 1$	$2 + 2 = 5$
$\frac{18}{9}$	$18 = 30 - 12$	9:15 a.m.
$6 + 2 = 3 + 5$	a right angle	
$9 >$	two right angles	

Stress the fact that a mathematical sentence must express a complete thought. It may be true or false.

Summary

A mathematical sentence is a statement that expresses a complete thought. It may be true or false.

Practice and Homework

Assign exercises of the type: $34 \times 2 = \triangle$

$3.01 + 4.1 = 7.11$

$189 \bigcirc 3 = 63$

Lesson 2

MATHEMATICAL STATEMENTS

Warmup

$2\frac{1}{2} + 2\frac{1}{2} =$ ....

The sum of 6 and 3 equals the product of 3 and ....

Reduce to lowest terms: $\frac{15}{25} =$ .... $189 \div 3 =$ ....

Motivation

On the board write:

George Washington's birthday is celebrated on July 4th.

Ask the pupils: Is the sentence a true statement? Why not?
Then the sentence is a *false* statement.

Write: Abraham Lincoln's birthday is celebrated on February 12th.
Ask the same questions and conclude that *the sentence is a true statement.*

Aim

Develop: A mathematical sentence is a true or false statement:

Development

Have the pupils judge the truth or falsity of the following sentences:

1 yard $= 3$ ft.	$3 \times 6 = 9 + 2$
$75¢ + 50¢ = \$1.00$	$\frac{1}{2} = 75\%$
19 is greater than 18	$.3 + .5 = .35$

Have the pupils realize that *mathematical sentences that can be judged definitely true or definitely false are mathematical statements.*

Note: Review, or teach, if necessary, the meaning and use of the symbols:

$=$ is equal to, or names the same number as

$\neq$ is not equal to or does not name the same number as

$>$ is greater than or names a number larger than

$<$ is less than or names a number less than

Have the pupils label the following T for true or F for false:

$2\frac{1}{2} + 2\frac{1}{2} = 5$	One half or $4.2 = .3 \times 7$
$\frac{6}{10} = \frac{30}{50}$	$6 < 3$
$\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$	A right angle $= 90°$

Have the pupils explain how they determined each sentence as either true or false.

Summary

A sentence that can be judged definitely true or definitely false is a mathematical statement.

Practice and Homework

Assign incomplete statements of the following type:

1. Write the symbol that will make each sentence a true statement:
   a $4+9 \; \square \; 13$ (=)
   b. $29 \; \square \; 7 \times 3$ (or ≠)
   c. $(8 \times 9 \div 9 \; \square \; 8$
   d. $42 \; \square \; 29$

2. Judge the following sentences as T or F:
   a. $\frac{16}{100} \neq .016$
   b. $\frac{2}{5} = 60\%$
   c. $\frac{4}{3} < 1$
   d. $1 = .9999$

Lesson 3

STATEMENTS OF EQUALITY AND INEQUALITY

Warmup

Label *true* or *false*, each of the following:

1. $3 \times 8 = 4 + 9$
2. $8 + 7 = 10 + 5$
3. $19 \neq 2 + 17$
4. $\frac{1}{2}$ of $96 = 69$
5. $36 = 72 \div 2$
6. $3 \times \frac{2}{3} = 2$

Motivation

Is the sentence $8 + 7 = 15$ true or false? Why? *($8 + 7$ and 15 name the same number)*

Is the sentence $(3 \times 8) = (4 + 9)$ a true statement? Why? *($3 \times 8 = 24$; $4 + 9 = 13$; 24 is not equal to 13.)*

Does $(3 \times 8) = (4 + 9)$ name the same number?

Aim

What is meant by *statement of equality* and *inequality?*

Development

Have the pupils select the statements that express equality in the warmup. (2, 3, 5, 6)

Have the pupils realize that a statement may be a true statement or a false statement.

Have the pupils judge the truth or falsity, by labeling each of the following T or F:

A. $3 \times 2 = 2 \times 3$	B. $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$
$1 + (9 + 6) = (1 + 9) + 6$	$40\% = \frac{1}{4}$
$5 \times 9 = 54$	$39 = 19 + 19$

Group A are true *statements* of *equality.*

Group B are false *statements* of *equality.*

Judge *True* or *False*:

C. $6 \times 5 > 9 + 8$	D. $\frac{2}{5} + \frac{2}{3} > \frac{5}{3} + \frac{3}{2}$
$13 + 20 > 17 + 13$	
$4 \times 13 < 9 \times 8$	$50\% < \frac{1}{2}$
	$54 < 6 \times 9$

Have the pupils realize that every statement above is a statement about the inequality of two numbers.

Group C are true statements of *inequality.*

Group D are false statements of *inequality.*

Summary

A statement of equality

1. expresses an equality
2. may be a true statement or may be a false statement

A statement of inequality

1. expresses an inequality
2. may be a true statement or may be a false statement

Practice and Homework

Have the pupils practice determining statements of equality and inequality.

Lesson 4

OPEN SENTENCES

Warmup

Practice in recognizing statements of equality and inequality.

Have the pupils judge statements true or false.

Motivation

Ask the pupils to judge the following sentences as true or false.

1. She is president of the class.
2. $4 \times \text{..................} = 0$

Have the pupils realize that they cannot judge these sentences true or false.

In #1 they must *replace* the word *she* with the name of a person.

In #2 they must replace the blank with a number.

Aim

What is an incomplete sentence in mathematics?

Development

In the sentence $4 \times \text{..................} = 0$, we cannot judge the sentence *true* or *false* because some information is missing

If we replace the blank with *0*, we have $4 \times 0 = 0$. True or False?

If we replace the blank with *2* we have $4 \times 2 = 0$. True or False?

Have the pupils judge the following:

	True	*False*	*Cannot Tell*
1. $4 \times 9 = 36$			
2. $7 + 13 \neq 20$			
3. $9 \times 0 = n$			
4. $3.2 \times 4 = 12.8$			
5. $32 \times n = 16$			
6. $n + 2 = 8$			
7. $\boxed{} + 2 = 8$			
8. Twenty One $\boxed{}$ 21			
9. $3n = 21$			
10. $96 + 6 = n$			

Have the pupils explain why 3, 5, 6, 7, 8, 9, 10 cannot be judged true or false.

These mathematical sentences are called *open sentences.*

Have the pupils replace the letter, or box in each open sentence with a numeral, or relation, that will make it a true statement.

Summary

A mathematical sentence which cannot be judged either true or false is called an *Open Sentence.*

Practice and Homework

Have the pupils practice recognizing open sentences.

Lesson 5

SOLUTION SETS OF OPEN SENTENCES: REPLACEMENT SETS

Warm Up

Practice in recognizing true and false statements, statements of equality and inequality.

Motivation

Review the meaning of an *open sentence;* a true statement of equality; a true statement of inequality. Lead to the aim of the lesson.

Aim

How do we change an open sentence to a *true* statement of equality or a *true* statement of inequality?

Development

Have the pupil consider the sentence:
It is the borough of New York which has the largest area.

Ask the pupils to determine what kind of a sentence it is; equality, inequality, true, or false Conclude: It is an open sentence.

To change it to a true sentence, what information is needed? *(number, names, and sizes of boroughs)* Then, to make the sentence a true sentence we must *replace* the word *it* with the name of a borough.

Have the pupils list the set of 5 boroughs *(Manhattan, Bronx, Queens, Brooklyn, Richmond)*

Have them realize that they may *replace* the pronoun *it* with the name of a borough, or *any member of the set* of boroughs.
If they use *Manhattan,* the sentence becomes, "Manhattan is the borough of New York which has the largest area."
Have them judge the truth or falsity of the statement.
Have them replace the pronoun *it* with each member of the set of boroughs, until they arrive at a true sentence.

Note: Explain briefly that a set is a collection of things. Each thing in the set is a *member* or *element* of that set.
Have the pupils realize that the possible replacements for the pronoun come from the set of 5 boroughs.

The set from which we get the replacement to make the open sentence a true or false sentence is called the *replacement set.*

Have the pupils consider the sentence: $x + 2 = 8$

Limit the values of x to the replacement set $x = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$.

Have the pupils experiment using elements of the set that would result in a false sentence before they *solve* the open sentence:

$x + 2 = 8 \qquad 6 + 2 = 8 \qquad x = 6$

Have pupils find which value or values for n from the replacement set: $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ will result in a true statment. $n - 9 = 1$.

Summary

To change an open sentence to a true statement:

1. Use the replacement set of values for the unknown word, symbol, or letter in the open sentence.
2. Replace the unknown word, symbol, or letter by the members, or elements, of the replacement set.
3 An element of the replacement set which makes the open sentence a

true statement of equality or inequality is called a solution of the open sentence. The set of such elements is called the *solution set* of the open sentence.

4. A solution must be an element of the replacement set.
5. The solution set is the set of all these solutions.

Practice

Solve, using the given replacement set: $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

1. $n + 6 = 9$
2. $3 - n = 0$
3. $5n = 10$
4. $\frac{n-1}{2} = 3$
5. $n > 3$
6. $n < 6$
7. $7n + 1 = 22$
8. $5n - 8 = 27$
9. $n - 3 > 4$

Homework

Assign exercises similar to practice exercises.

Lessons 6 and 7

THE MEANING AND USE OF: VARIABLE; DOMAIN; SOLUTION SETS; EMPTY SET

Warmup

Solve, the replacement set: $\{1, 2, 3, 4, 5, 6, 7\}$

A 1. $x + 2 = 6$
2. $3x = 21$
3. $36 + 6 = x$

B. 1. $x + 7 = 7 + x$
2. $x + 2 < 6$
3. $2n = 11$

(Do not check answers. They will be used in developing the lesson.)

Motivation

Check answers to exercises in group A.
In # 1 of Group B the pupils should have different answers. (all seven elements of the set)

Aim

How do we show the solution for an open sentence?

Development

Consider the sentence $x + 7 = 7 + x$

Have the pupils replace x with *each* member of the replacement set.
Have them realize that each replacement results in a true statement.
Have them discuss any objections to multiple solutions.

Remind them that in life we may solve a problem in one of many ways. e.g. To pay for a 45¢ lunch, we may give the cashier: 4 dimes and 1 nickel, a quarter and 2 dimes, 9 nickels, or a half dollar and get a nickel in change, etc

Have them realize that each amount named would solve the problem of paying for the lunch.

Because the letter x *represents a quantity* that may be replaced by a number of values we say x is a VARIABLE. Because the *elements of the replacement set* are the *only values* we can use to solve the open sentence, we say the *replacement set is the* DOMAIN *of the variable.*

Solve, using replacement set: $\{\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\}$

Open Sentence	Solution
1. $n + 2 = 2\frac{1}{2}$	1. $n = \frac{1}{2}$
2. $2n = 5$	2. $n = 2\frac{1}{2}$
3. $4n = 10$	3. $n = 2\frac{1}{2}$
4. $n < 2\frac{1}{2}$	4. $n = \frac{1}{2}, 1, 1\frac{1}{2}, 2$

What is the variable? (n)
What is the domain of the variable? $\{\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\}$

Have the pupils realize that the solution to an open sentence may be one value or many values taken from the replacement set.
(Explain that a set may have 1 element, many elements, or no elements.)

The set of solutions to an open sentence is called the *solution set.*
What is the solution for $2n = 11$ if the domain is $\{1, 2, 3, 4, 5\}$?

Remind the pupils we can use *only the values in the replacement* set to find the solution.
Have the pupils replace n with each of the 5 elements, concluding that

there is no solution. (Domain contains only whole numbers and $2\frac{1}{2}$ is not a whole number.)

To show that there is no element in the solution set we write:

Solution set is $\{\ \}$, or $\emptyset$ and say $\{\ \}$, or $\emptyset$ is the empty set, or the null set.

Summary

1. The solution to an open sentence is the solution set.
2. The replacement set is the domain of the variable.
3. The solution set must be elements of the domain.
4. To show an empty set (a set with no elements) we write $\{\ \}$, or $\emptyset$.
5. To check the solution we replace the variable by each element of the solution set. If the open sentence becomes a true sentence, the solution is correct.

Suggested for Practice and Homework Assignments

Find the solution set for each of the following equalities and inequalities. The domain in each case is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

A. 1. $4\,a = 36$ ($\{9\}$)

2. $a - 7 = 16$ ($\{23\}$)

3. $\frac{3n}{4} = 16$ ($\{\ \}$, or $\emptyset$)

4. $n < 4$ ($\{1, 2, 3,\}$)

5. $n - 3 > 3$ ($\{7, 8, 9, 10\}$)

6. $\frac{x}{2} + 6 > 8$ ($\{6, 8, 10\}$)

B. The domain in each of the following cases is:

$$\{\tfrac{1}{4}, \tfrac{1}{2}, \tfrac{3}{4}, 1, 1\tfrac{1}{4}, 1\tfrac{1}{2}, 1\tfrac{3}{4}, 2\}$$

1. $4n = 6$ ($\{\ \}$ or $\emptyset$)

2. $n + 1\frac{1}{4} = 2\frac{1}{2}$ ($\{1\frac{1}{4}\}$)

3. $2a - \frac{3}{4} = \frac{3}{4}$ ($\{\frac{3}{4}\}$)

4. $n > 1$ ($\{1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2\}$)

5. $2a < 1\frac{1}{4}$ ($\{\frac{1}{4}, \frac{1}{2}, 1\}$)

6. $2n - 1 = 1\frac{1}{2}$ ($\{\frac{1}{2}\}$)

C. The domain in each case is $\{\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3\}$

1. $4n = 6$ ($\{\ \}$ or $\emptyset$)

2. $n + 1\frac{1}{4} = 2\frac{1}{2}$ ($\{1\frac{1}{4}\}$)

3. $2a - \frac{3}{4} = \frac{3}{4}$ ($\{\frac{3}{4}\}$)

4. $n > 1$ ($\{1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2\}$)

5. $2a < 1\frac{1}{4}$ ($\{\frac{1}{4}, \frac{1}{2}, 1\}$)

6. $2n - 1 = 1\frac{1}{2}$ ($\{\frac{1}{2}\}$)

TEST

1. Judge these sentences as true or false: Use label T or F.

   a. $6+4=8+2$

   b. $4+0=0$

   c. $3\times 2=2\times 3$

   d. $6\times 0=6$

   e. $\frac{6}{8} = \frac{12}{16}$

   f. $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

   g. $8\times 9\neq 12\times 6$

   h. $.57 > .6$

2. Write a ✔ in the proper column

   a. $8\times 12\frac{1}{2}=100$

   b. \$25 is \$3 more than Jack has

   c. $8\div n=4$

   d. $3.2\times 6=n$

Sentence	True	False	Open
a.			
b.			
c.			
d.			

3. Solve the open sentences

   The replacement set in each case is $\{1, 2, 3, 4, 5, 6, 7, 8\}$

   a. $n+6=9$

   b. $\frac{3x}{4}=6$

   c. $n>1$

   d. $y\neq 9$

4. In item 3 above

   a. What is the variable in a?............ b?............ d?............

   b. What is the domain of each variable?

   c. What do we call the solution set of *c?*

   d. Which is an equality *b* or *c?*

Unit X

READING IN MATHEMATICS

A major objective of every mathematics program is to develop the ability to solve mathematical problems.

The specific objectives of this unit are to have the pupil realize that:

1. A mathematical problem is a quantitative question or situation that must be answered, resolved, or solved.
2. A mathematical problem may be in a verbal or non-verbal form.
3. Ability in fundamental skills and processes is needed.
4. The ability to read is of the utmost importance in understanding and visualizing verbal problems.
5. Experiences, judgments, and skills are necessary to find the solution to problems.
6. Visualizing the quantitative situation diagrams and graphic representations are valuable aids.

Lesson 1

MATHEMATICAL PROBLEMS

Warmup

1. $5. + $5. = △
2. 398 + 614 = △
3. 10 + △ = 18
4. 852 + △ = 948
5. John earned $5 on Monday and $5 on Tuesday. How much did he earn altogether.
6. Mr. Smith paid $852 for rent in 1961. In 1962 his rent for the year was $948. How much more did he pay in 1962 tha in 1961?

Motivation

Look at the warmup. Which examples, would you say, were easier to answer?

Pupils will probably answer "examples #1, 2, 3, and 4 were easier than problems #5 and 6."

Ask pupils why they think problems #5 and 6 are more difficult to answer. (Stress need for ability to read.)

Aim

How can knowing how to read help us solve problems in mathematics?

Development

In problem #1: How did you arrive at the answer '10'?

After several pupils have given reasons, summarize:
Most of us can read and understand number symbols, and we learned a long time ago that 5 addtd to 5 equals 10.

Look at problem #5: What is the difference between problem #5 and #1? What is alike in problem #1 and problem #5?

Encourage pupils to express their opinions. Challenge their answers in thinking and reasoning.

Conclude: Reading and understanding the words is a deciding factor in solving problem #5.

Emphasize: If we can read, we can quickly "see" that $5 must be added to $5. Therefore problem #5 is the same as problem #1. However, if we cannot read, we may not see that problem #5 is the same as #1.

Follow the same procedure to compare problem #4 and problem #6.

Summary

In order to *see* and understand a problem so that we can solve it, we must be able to read and to know the meaning of the words.

Practice and Homework

Solve the following: *Answers*

1. $10 \times 367 = n$ — 1.
2. It cost 30¢ to have 1 shirt laundered. How much does it cost for 4 shirts? — 2.
3. A mirror is 2 feet wide and 5 feet long. How many feet of wood are needed for a frame? — 3.
4. $1.2 + 2.34 + 1.78 = x$ — 4.
5. $\begin{array}{r} 98.15 \\ -18.37 \\ \hline \end{array}$ — 5.
6. Since there are 3 feet in one yard, how many feet are there in 100 yards? — 6.
7. Round to the nearest *Ten:* 164 1,288 — 7.
8. Round to the nearest *Hundred:* 99 3,669 — 8.

Lesson 2

READING VERBAL PROBLEMS

Warmup

Will the answer be *larger* or *smaller* than 34 to the following?

1. 34×17 ....
2. 34—17 ....
3. 34+17 ....
4. 34—17 ....

Motivation

We know that in order to solve verbal problems, we must be able to read and understand the words.

We can understand almost all the words we hear, but — — — —.

Aim

How can we understand written words we do not recognize?

Development

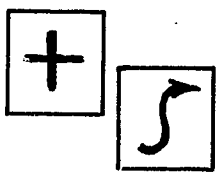
Before we talk about understanding written words, let us think about just *words*.

What is a word?
Have pupils try to explain *a word*. Conclude that a spoken word is a sound (or group of sounds) that has meaning in a particular language.

Talk to English-speaking people means *saying words;* to others it may mean nothing. *Hablo,* to Spanish speaking people, means ....

Why do we talk?
Have pupils discuss reasons for speaking. (describing things and places, expressing ideas, thoughts, opinions, making needs known, acquiring information and knowledge.)

Have pupils realize that knowing many words helps us to express our ideas and makes for better understanding of the ideas and thoughts of others.



What does a red traffic light mean? A green light?
What do these signs on the side of roads mean?
What do *U.S.A., F.B.I., J.H.S.* mean?
Have the pupils realize that signs and letters are *symbols* for directions, places, and things.

Letters are the symbols we use to write words. Each letter stands for a sound. We combine the sounds to say the word. If we know the sound of the written word, we usually know the meaning. When we recognize written words and understand the idea or thought they express, we say we are reading.

To read words, we must know the sound each letter, or group of letters, stands for. Think of the sounds and letters we learned in the lower grades. The sound for *f* is sometimes written as *ph* or *ough* as in *fine, photograph,* and *rough.*

Have the pupils say and give the meaning of the following words: problem, account, number, expenses, approximate, estimate, operation, process.

Have the pupils say the syllables in the words

Where can we find the meaning of words?
(Review dictionary skills. Suggest pocket dictionary.)

Summary

We can read written words by combining the sounds of the letters.

We can learn the meaning of words we don't know by using the dictionary.

Words we know and use in speaking make up our *speaking* vocabulary.

Words we know, can spell, and use in writing make up our *writing* vocabulary.

Words we can read and understand make up our *reading* vocabulary.

Practice and Homework

Directions: Read the paragraph below. Write a list of the words you *do not know*. Use the dictionary to find the meaning of these words. Be prepared to tell, in your own words, what the paragraph means.

"The good reader not only sees words accurately, secures an understanding of their meanings, and reacts thoughtfully to what he reads; in addition, he combines the ideas gained through reading with previous experiences, so that new or clearer insights, broader interests, and improved ways of thinking and behaving result. This step is the heart of the learning act in reading." William S. Gray, 1959

Lesson 3

UNDERSTANDING VERBAL PROBLEMS

Note: The aim of this lesson is to have the pupils realize that to understand the meaning of words, phrases, and sentences, previous associations, experiences, environment and background must be considered.

Sometimes first associations are erroneous. Therefore, careful rereading may be necessary.

Suggested: Informal discussion lesson. Present one word and have pupils describe reactions, etc.

Motivation

How do you react, or what do you do, when you see a sign with the word, *stop?*
How do you react, or what do you do, when you see the word, *danger?*
What do you do when you see a red traffic light?

Have the pupils realize that their reactions are almost automatic, because safety education and past experiences have made reactions almost habits.

Point out that *one word* conveys a wide field of understanding because it brings to mind past teachings and experiences.

Aim

How can we use previous learnings and past experiences to help us solve problems in mathematics?

Development

What are problems in mathematics?
Have the pupils realize that quantitative situations, or situations, involving shape, size, amount, comparisons, etc., that require regrouping, uniting or separation, etc., that occur in daily living are the *problems* that we solve or resolve.

Mathematics gives us a logical, organized, and sometimes quick way of finding the answers to the quantitative situations.
Have the pupils suggest aids that help in solving problems.
List them as suggested, then list them in logical sequence, such as the following:

1. Recognize written words or look in the dictionary to find the meaning.
2. Remember all you know about the subject of the problem.
3. Write down the information that the problem gives.
4. Write an open sentence using the number facts given.
5 Estimate the answer to see if it makes sense.
6. Draw a simple diagram to *see* the problem.

7. Find the exact answer.
8. Check.

Summary

The above eight points.

Practice and Homework

Distribute copies of work sheet.

Work problems #2 and #9, with the whole class, eliciting background that can be used, experiences that would help, variety of diagrams to clarify meaning of problem, etc.

1. If you know the *product* and one factor, how do you find the other factor?
2. How can you find the perimeter of a square if you know the length of only one side?
3. If you want to paper a wall in your room what must you measure before buying the paper, the perimeter or the area?
4. What must you know to find the circumference of a circle?
5. To find how much tax was taken from Mr. X's pay, you must know his earnings and his ....
6. A car was bought for a certain price and sold for a higher price How do you find the amount of money gained?
7. The sum of a number and 12 is 76. What is the number?
8. John and Saul start from the same place. John travels at the rate of 24 miles an hour. Saul travels at the rate of 30 miles an hour in the opposite direction. After 3 hours how far apart are they?
9. Draw a diagram to explain problem #8.

Lesson 4

ESTIMATING THE ANSWER

Warmup

Round to the nearest ten: 39, 42, 65, 78, 86

Round to the nearest hundred: 286, 325, 176, 281, 450

3

7. Find the exact answer.
8. Check.

Summary

The above eight points.

Practice and Homework

Distribute copies of work sheet.

Work problems #2 and #9, with the whole class, eliciting background that can be used, experiences that would help, variety of diagrams to clarify meaning of problem, etc.

1. If you know the *product* and one factor, how do you find the other factor?
2. How can you find the perimeter of a square if you know the length of only one side?
3. If you want to paper a wall in your room what must you measure before buying the paper, the perimeter or the area?
4. What must you know to find the circumference of a circle?
5. To find how much tax was taken from Mr. X's pay, you must know his earnings and his ....
6. A car was bought for a certain price and sold for a higher price How do you find the amount of money gained?
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8. John and Saul start from the same place. John travels at the rate of 24 miles an hour. Saul travels at the rate of 30 miles an hour in the opposite direction. After 3 hours how far apart are they?
9. Draw a diagram to explain problem #8.

Lesson 4

ESTIMATING THE ANSWER

Warmup

Round to the nearest ten: 39, 42, 65, 78, 86

Round to the nearest hundred: 286, 325, 176, 281, 450

Add $10 + 40 + 20 + 10 =$ Add: $400 + 100 + 100 + 300 =$

Subtract: 30 from $80 =$ Subtract: 200 from $700 =$

Motivation

Review and check homework. Locate, clarify, and correct, if possible, specific weaknesses and difficulties. (In every lesson stress importance of reading and review, when indicated.)

Ask if anyone knows a quick and easy way to check answers. (Cue then by having them look at day's warm up.) Have the pupils discover that by rounding the numbers and computing they can arrive at an answer that is 'about right.'

Ask if anyone knows what the rounding of numbers is called. Write on board: *Estimate* *Estimated* *Estimation*

Aim

How can estimating answers help us?

Development

When we round numbers, the answer is called an *estimate*. What does that mean?

After several pupils have responded, suggest the following experiments:

1. Without counting, write down the number of children in the class.
2. Without checking, write down the number of panes in the windows. Compare pupil answers.

Now, have the pupils count the number of pupils in the class. Compare answers. Have pupils count the number of panes. Compare answers.

Have the pupils note that when they actually counted, they all had the same answer because it was the exact answer.
The answers they guessed, were approximate or estimated, a little more or a little less than the exact answer.
Conclude: An estimate may be a little more, the same, or a little less, than the exact or counted answer.

How do we estimate answers?
Have pupils look at warmup. Guide them to realize that to get an estimate, we round the numbers and perform the indicated process.

Which is easier to find, the estimate or the exact answer? Why?
(Most pupils can add and subtract numbers ending in zero without trouble. They may need some help with multiplying and dividing numbers ending in zero. Do not have pupils say they "add zeros" or "cross out zeros." Demonstrate.)

$$10 \times 2 = 20 \qquad\qquad 10 \times 20 = 200$$

Write zero in one's place to show that there are no ones.

$$20 \div 10 = 2 \qquad\qquad 200 \div 10 = 20$$
$$40 \div 10 = 4 \qquad\qquad 400 \div 10 = 40$$
$$40 \div 20 = 2 \qquad\qquad 400 \div 200 = 2$$

Have pupils practice: (First compute, then use short cuts.)

Multiply:	$10 \times 30 =$	$20 \times 40 =$
	$20 \times 30 =$	$20 \times 400 =$
Divide:	$60 \div 10 =$	$600 \div 10 =$
	$90 \div 30 =$	$600 \div 300 =$ etc

Estimate the answer; then, find the exact answer.

$$\begin{array}{r} 24 \\ +59 \\ \hline \end{array} \qquad \begin{array}{r} 20 \\ +60 \\ \hline 80 \end{array} \text{ Estimate} \qquad \begin{array}{r} 24 \\ +59 \\ \hline 83 \end{array} \text{ Exact sum}$$

Compare the estimate and the exact sum.

$$\begin{array}{r} 424 \\ -276 \\ \hline \end{array} \qquad \begin{array}{r} 36 \\ \times 23 \\ \hline \end{array} \qquad 27\overline{)639}$$

Summary

To estimate an answer:

1. Round the numbers and compute.
2. Compare the estimate with the exact answer.
3. If the exact answer is a little more or a little less than the exact answer, the answer is probably correct.

Practice and Homework

Assign exercises. Have pupils estimate, then find the exact answer.

TEST

Estimate Only:

1. Thomas A. Edison was born in 1847 and lived until 1931. How old was he when he died?	1. Estimate ....
2. A train travels 63 miles per hour. How far will it travel in 9 hours?	2. Estimate ....
3. What is the perimeter of a room that is 12 feet long and 9 feet wide?	3. Estimate ....
4. Mr. Wilson's take-home pay is $260 per month. How much does he take home in one year?	4. Estimate ....
5. In New York City, army records for the month of February showed the following enlistments:	5. Estimate ....

89 men were seventeen years old
116 men were eighteen years old
62 men were nineteen years old
45 men were twenty years old

How many men under twenty joined the army in February?

GENERAL REVIEW

It is suggested that a review of concepts and skills presented to date be given at this time.

Use no more than three periods.

Unit XI

POSITIVE AND NEGATIVE NUMBERS

The specific objectives of this unit are to:

1. Help the pupil see the need for an extension of the number system to include negative numbers:
2. Interpret negative and positive numbers in terms of opposite directions.
3. Have the pupil associate numbers with points on the number line.

Lesson 1

NEGATIVE NUMBERS

Warmup

$$\begin{array}{r} 15 \\ +12 \\ \hline \end{array} \qquad \begin{array}{r} 145 \\ +107 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ -3 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ -5 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ -6 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ -7 \\ \hline \end{array}$$

Motivation

Discuss addition. Have the pupils realize that addition with whole numbers is always possible; however, subtraction is not possible with the numbers of arithmetic we have been using.

Aim

Is there a different kind of number that will make subtraction always possible?

Development

Have the pupils study the following examples:

$$\begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array} \qquad \begin{array}{r} 4 \\ -3 \\ \hline 1 \end{array} \qquad \begin{array}{r} 3 \\ -3 \\ \hline 0 \end{array} \qquad \begin{array}{r} 2 \\ -3 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ -3 \\ \hline \end{array}$$

Have the pupils discover that as the minuend decreases by one, the difference decreases by one.

Then the difference of $\begin{array}{r} 2 \\ -3 \\ \hline 1 \end{array}$ could be 1 less than zero and $\begin{array}{r} 1 \\ -3 \\ \hline 2 \end{array}$ 2 less than zero.

We could indicate *one less than zero* as *—1* and read it as *negative one.* How would you indicate *2 less than zero? (—2)*. How would you read *—2? (negative 2)*

Then — 1, — 2, — 3 . . .
are *negative numbers* and mean:

— 1 = 1 less than zero
— 2 = 2 less than zero
— 15 = ? etc.

Summary

Subtraction is always possible if we use negative numbers that are numbers that are smaller than zero, e.g., — 10 means 10 less than zero.

Practice

$$\begin{array}{r} 5 \\ -4 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ -4 \\ \hline \end{array} \qquad \begin{array}{r} 3 \\ -4 \\ \hline \end{array} \qquad \begin{array}{r} 18 \\ -14 \\ \hline \end{array} \qquad \begin{array}{r} 14 \\ -18 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ -83 \\ \hline \end{array} \qquad \begin{array}{r} 125 \\ -135 \\ \hline \end{array} \qquad \begin{array}{r} 98 \\ -100 \\ \hline \end{array}$$

Homework

More practice with negative numbers as differences.

Lesson 2

POSITIVE AND NEGATIVE NUMBERS

Warmup

Find the solution set. In each case consider as the domain:

$$\{-9, -2, -1, 0, 1, 2, 3\}$$

$$x + 3 = 5 \qquad x + 3 = 3 \qquad x + 7 = 4$$

Motivation

Have pupils recall situations where they have seen a scale with values above and below zero.

Aim

What is the difference between 3 and -3?

Development

Have pupils prepare a thermometer scale:

20° above 0
15° above 0
10° above 0
0°
5° below 0
10° below 0
15° below 0
20° below 0

Have pupils consider ways of expressing 5° above zero, 5° below 0 (+ 5°, — 5°) etc.
Have pupils reasilze that + 10° and — 10° can be thought of as being on opposite sides of, or in *opposite directions* from zero.

What is the opposite of negative? *(positive)*
If — 5 is negative 5 what is + 5? *(positive 5,* or *5)*
Have pupils give the opposite, or negative, situation of the following:

1. 10 lbs. overweight or + 10
2. Earning $15 or + 15
3. 8° rise in temperature + 8°
4. aboxe zero +
5. 20 yard gain + 20

Summary

Negative numbers are in the opposite direction of positive numbers.
For every positive number there is a negative number.

Practice

Write *above zero* or *below zero* for each of the following:

1. — 5°	4. + 98.6°
2. — 3°	5. — 55°
3. + 32°	6. + 10°

Homework

More practice with negative and positive numbers.

Lesson 3

SIGNED NUMBERS AND THE NUMBER LINE

Warm Up

Write the opposite of

1. + 7 .... 1. 36 ....

2. — 18 ....   2. 74 ....

3. —125 ....   3. —32 ....

4. 10 ....   4. 84 ....

Motivation

Discuss the meaning of negative and positive numbers, opposites and opposite direction.

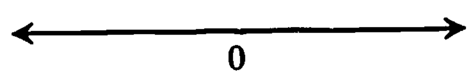
Have pupils realize that a numeral without a + or — sign is considered a positive number.

Aim

How do we show negative numbers on a number line?

Development

Have the pupils draw a number line.

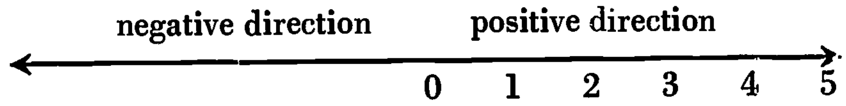


Have the pupil pick a point on the line and label it *0*.

Have the pupils mark off 5 points at even intervals to the right of zero.

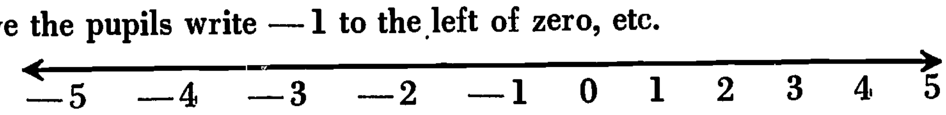
Then mark off at even intervals 5 points to the left of zero.

Starting at zero and moving to the right, to show increase, label each point with a positive number.



To show the negative numbers, have the pupils move in the opposite or negative *direction.* (For this reason they are also called "directed numbers")

Have the pupils write —1 to the left of zero, etc.



Summary

On the number line, the positive numbers are usually shown to the right of zero, the negative numbers are to the left of zero. Each number to the right is greater than any number to its left.

3 is greater than 1.   —3 is greater than —5.

Practice and Homework

Provide much practice in using positive and negative numbers and in associating these numbers with points in the number line.

Lessons 4 and 5

ADDITION OF POSITIVE AND NEGATIVE NUMBERS

Warmup

Practice using directed numbers and reading directed numbers on a number line.

Motivation

Discuss the use of "+" and "—" as symbols to denote opposite directions for positive and negative numbers and "+" and "—" to mean *add* and *subtract* as signs of operation.

Aim

How do we add directed numbers?

Motivation

Have the pupils construct a number line as shown:

West, or Negative Direction — East, or Positive Direction

W ←——————————————————————→ E

—7 —6 —5 —4 —3 —2 —1 0 +1 +2 +3 +4 +5 +6 +7

Maple Avenue

We will assume that walking East is walking in the positive direction and walking West is walking in the negative direction.

Have the pupil refer to the number line to answer such questions as:

1. If a boy walks from point $+1$ to point $+3$, in what direction is he walking? (East, or positive)

2. If he walks from point 0 to $+2$, in what direction is he walking? (East, or positive)

Continue asking similar questions. Include points -3 to $+4$; 0 to -4; -1 to -3; etc.

If we consider the distance between points to be street blocks then:

1. How many blocks does a boy walk, if he started at Maple Avenue and walked 5 blocks East?
2. 5 blocks West? etc.

Have the pupils represent the number of blocks walked as a directed number.

Continue with questions:

1. If he walked five blocks East and then an additional 6 blocks East, how many blocks did he walk?

   Have the pupils realize that we can represent this as $(+5) + (+6) = 11$. (Note the two meanings of the $+$ sign.)

2. If a boy started at Maple Avenue and walked 2 blocks west, then an additional 5 blocks west, how many blocks did he walk?

   $(-2) + (-5) = -7$, or he walked 7 blocks in the negative direction.

Then: $\begin{array}{r} (+5) \\ +\,(+6) \\ \hline +11 \end{array}$ or $\begin{array}{r} 5 \\ +6 \\ \hline 11 \end{array}$ (Positive sign is not necessary)

and $\begin{array}{r} (-2) \\ +\,(-5) \\ \hline -7 \end{array}$ or the sum of $\begin{array}{r} -2 \\ -5 \\ \hline \end{array}$ is -7

Find the sums:

$\begin{array}{r} 4 \\ 9 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} -5 \\ -10 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 6 \\ +\,(-4) \\ \hline \end{array}$ or $\begin{array}{r} 6 \\ -4 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 6 \\ 0 \\ \hline \end{array}$

$\begin{array}{r} -8 \\ -6 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} -8 \\ -6 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 8\ - \\ 10 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 0 \\ 4 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 0 \\ -6 \\ \hline \end{array}$

Have the pupil use the number line to see at what point on the line they would be, if they have trouble adding signed numbers, which represent opposite directions.

Summary

The sum of two positive numbers is a positive number.

The sum of a positive number and zero is a positive number.

The sum of two negative numbers is a negative number.

The sum of a negative number and zero is a negative number.

Practice and Homework

Find the sum:

$$\begin{array}{r} 9 \\ 7 \\ \underline{3} \end{array} \qquad \begin{array}{r} -7 \\ -6 \\ \underline{-4} \end{array} \qquad \begin{array}{r} +25 \\ +10 \\ \underline{-\ 6} \end{array} \qquad \begin{array}{r} +26 \\ -\ 6 \\ \underline{+\ 5} \end{array}$$

$$\begin{array}{r} -\ 5 \\ -16 \\ 20 \\ \underline{-25} \end{array} \qquad \begin{array}{r} 15 \\ -12 \\ 8 \\ \underline{-20} \end{array} \qquad \begin{array}{r} +2.5 \\ -3.7 \\ -5.0 \\ \underline{} \end{array}$$

TEST

1. What is the opposite situation of each of the following:
   a. A loss of $5?
   b. A profit of $25?
   c. An increase of 10 lbs.?
   d. 18° rise in temperature?
   e. 6 steps forward?

2. Express each of the following as a directed number:
   a. 5 games won
   b. 3 points below average
   c. a deposit of $10
   d. 300 feet above sea level
   e. a withdrawal of $10.

3. On the number line, locate the points listed below.

—6 —5 —4 —3 —2 —1 0 1 2 3 4 5 6

a. negative 3

b. negative $4\frac{1}{2}$

c. positive 5

d. five less than 0

e. two more than zero

4. a. A negative number is in the .... of a positive number.

   b. For every positive number there is a ....

   c. $\begin{array}{r} +26 \\ -10 \\ +32 \\ \hline \end{array}$ d. $\begin{array}{r} -14 \\ -21 \\ +13 \\ \hline \end{array}$ e. $\begin{array}{r} +7 \\ 0 \\ -7 \\ \hline \end{array}$

Note: Refer to any *Mathematics 8* textbooks and curriculum bulletins for more work on these topics.

Unit XII

EQUATIONS

Note: Before presenting this unit be sure the pupils have completed and reviewed Unit IX of this Manual.

The specific objectives of this unit are to

1. Introduce the pupils to use variables in equations.
2. Learn the meaning of an equation.
3. Understand equivalent equations.
4. Learn and use the principles of addition, subtraction, multiplication, and division.
5. Solve equations.

Lesson 1

MEANING OF EQUATION

Warmup

Judge the following as true or false:

1. $7 \times 28.60 = 200.20$
2. The sum = the total of the addends
3. Product $\div$ by one factor = the other factor
4. $\frac{369}{9} = n$
5. $6 \times 9 = 7 \times 8$

Motivation

Have the pupils select the statements of equality in warm up exercises.

Elicit from the pupils the meaning of a statement of equality. (The two quantities are equal, or the two sides name the same number.)

Explain that a statement of equality is usually called an *equation.*

Aim

What are the parts of an equation?

Development

Have the pupils study the following equations:

$8 + 5 = 13$	$3x - 10 = (2 \times 5) + 15$
$4 + 3 = 3 + 4$	$12 = 3 + n$

Have the pupils realize that an equation consists of two quantities equal to each other. $8 + 5 = 13$ or $13 = 13$

$4 + 3 = 3 + 4$ or $7 = 7$ etc.

Have the pupils read the quantity on the left side of the equal sign for each equation, e.g., $8 + 5$; $4 + 7$, etc.

Then, read the quantity on the right side of the equal sign of each equation.

The quantity on the left side is referred to as *the left member of the equation;* the quantity on the right side is referred to as the *right member of the equation.*

Have the pupils identify the left and right member of each equation. Compare the *true* sentence with the open sentences:

$3+5=8$	$n=4+2$
$4+4=3+5$	$16=8x$
$16=8\times 2$	$\frac{n}{2}=5$
$4\times 4=2\times 8$	$6-4=n$

Have the pupils rewrite each equation in its simplest form, e.g., $3+5=8$, $8=8$, $n=4+2$, $n=6$, etc

Summary

An equation is a statement of equality.

It has two members: a left member and a right member. The value of one member is equal to the value of the other member.

Practice and Homework

Practice in recognizing the members of an equation and rewriting the equation in its simplest form. Discuss *solution set* of an open sentence.

Lesson 2

EQUIVALENT EQUATIONS

Warmup

Exercises involving directed numbers.

Motivation

Have the pupils consider the fractions

$$\frac{8}{16},\quad \frac{2}{4},\quad \frac{1}{2}$$

Elicit from the pupils that the numerals are different but that the numbers each numeral represents is the same. (If necessary, demonstrate with a diagram.) Remind them that they are called *equivalent* fractions.

Review the fact that a quantity (number) may be expressed in many ways. (numerals)

Have the pupils consider: $x + 2 = 10$ and $x = 8$

Aim

What are equivalent equations? (equations that have same solution sets)

Development

Consider the equations $3 + 2 = \,' + 4$, $2 + 2 + 1 = 6 - 1$, $5 + 0 = 5$, $7 - 2 = 5$.

Have the pupils rewrite each equation in its simplest form:

A	B
$3 + 2 = 1 + 4$	$\rightarrow 5 = 5$
$2 + 2 + 1 = 6 - 1$	$\rightarrow 5 = 5$
$5 + 0 = 5$	$\rightarrow 5 = 5$
$7 - 2 = 5$	$\rightarrow 5 = 5$

Is each equation a true statement of equality?

Could you write $(3 + 2 = 1 + 4) = (7 - 2 = 5)$?

Have the pupil realize that each equation is a different way of expressing $5 = 5$.

The simplest form (group B) is the easiest way of expressing the quantities because the truth of the equation is *obvious.*

Return to the equations presented in motivation.

Are the equations $x + 2 = 10$ and $x = 8$ equivalent equations?

Elicit from the pupils: Both are open sentences. The variable is the same.

To judge the truth of the equations we must find the solution sets for each.

Have the pupils find the solution of $x + 2 = 10$. $\{8\}$

What is the value of "x"? $(x = 8)$

How do we know that $x = 8$? (When x is replaced by 8 the equation is a true statement of equality.)

Do we have to find the value of "x" in $x = 8$? (The value of x is obvious.) Hence, the solution set here too is $\{8\}$.

Solve the following pairs of equations:

Equations	*Value of the variable*	*Solution Set*
a. $4x = 8$, $2x = 4$	a. $x =$	$\{2\}$

b. $\frac{n}{3} = 2$, $2n = 12$	b. $n =$
c. $20 = 4a$, $10 = 2a$	c. $a =$
d. $3b + 2 = 11$, $3b = 9$	d. $e =$
e. $22 = 2y + 6$, $16 = 2y$	e. $y =$

Have the pupils note:

1. In each pair of equivalent equations, the variable is the same.
2. The solution sets are the same.
3. The solutions of each equation of a pair are the same.

Have the pupils replace the variable by its value.

$$\text{e.g., } 4x = 8,\ 2x = 4,\ x = 2,$$
$$8 = 8,\ 4 = 4,\ 2 = 2, \text{ etc.}$$

Have the pupils conclude that each equation is a true statement of equality.

Summary

Equations in the same variable, whose solution sets are the same are called *equivalent equations.*

Practice and Homework

The domain in each of the following cases is the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Solve the following collections of equations:

1. $2x - 1 = 5$, $2x = 6$, $x = 3$
2. $x + 3 = 5$, $x = 2$
3. $2x + 3 = 5$, $2x = 2$
4. $\frac{x}{2} - 3 = 4$, $\frac{x}{2} = 1$, $x = 2$
5. $3x + 2 = 11$, $3x = 9$, $x = 3$
6. If the variable is the same in a collection of equations and the solutions sets of the equations in the collection are the same, what kind of equations are they?
7. Fill in the missing steps in solving equations
   a. Find the solutions of the equation.
   b. Replace .... (the variable by its value) to see if the equation is a .... statement of ....
   c. If the statement is true you have .... the equation.

Lessons 3 and 4

REVIEW

It is suggested that at this time a review of concepts, skills, and terms of the contemporary mathematics presented be reviewed before proceeding to some of these properties in finding solution sets of equations.

A vocabulary test, to be used diagnostically, is suggested, e.g.,

1. Commutative
3. Solution Set
2. Open Sentence
4. Variable
5. Equation, and other words taken from the daily summaries.

Tabulation of errors will quickly pinpoint areas of common weaknesses which then can be retaught.

Lesson 5

USING SUBTRACTION TO WRITE EQUIVALENT EQUATIONS

Warmup

Judge the truth or falsity (use T and F) and give reason:

1. $17 = 15$ .... because ....
2. $6 + 3 = 8$ .... because ....
3. $\frac{1}{2} = \frac{2}{4}$ .... because ....
4. $A = lw$ .... because ....
5 $3x = 9$ .... because ....
6. $\frac{n}{4} = 8$ .... because ....

Motivation

Review steps in solving an equation. Have pupils solve the equations: (1) $n + 3 = 12$, (2) $n - 7 = 2$, (3) $n = 9$
(Establish that the domain for all equations used in this lesson is to be the set of the numbers they know.)

Elicit that the solution is "9" for each equation; that they are equivalent equations; that they have the same solution set: $\{9\}$
Ask the pupils the differences they observe among the equations.
Have them analyze the solution.

Aim

How do we use subtraction to solve equations?

Development

Have the pupils rewrite the three equations replacing the variable by 9.

(1) $9 = 3 = 12$ Note that: 3 has been *added to* 9

(2) $9 - 7 = 2$ 7 has been subtracted from 9

(3) $9 = 9$

Have the pupils practice solving similar equations subtracting the quantity added to the variable, from both members of the equation.

$x+7=11$, $a+15=45$, etc., using the form $(x+7)-7=(11-7)$, etc.

Summary

Have the pupils generalize:

If the same number is subtracted from both members of an equation, the result is an equivalent equation.

Practice and Homework

Assign equations for solution of the type $y + 11 = 23$. (Domain the set of all the numbers they know.)

Lesson 6

USING ADDITION TO WRITE EQUIVALENT EQUATIONS

Warmup

Solution of equations involving subtraction.

Motivation

Have the pupils solve: $n - 7 = 2$.
Ask them to analyze their solution.

Aim

How we use addition to solve equations.

Development

Following the same procedures in lesson 3 of this unit, conclude

$$(n - 7) + 7 = (2 + 7) = 9$$

Have the pupils generalize that if the same number is added to both members of an equation, the result is an equivalent equation.
Tell the pupils that the generalizations they arrived at in lesson 3 and 4 are principles of addition and subtraction.

Summary

The principle of addition:
If the same number is added to:....................
The principle of subtraction:
If the same number is subtracted from ....
(The Golden Rule: Do unto one side, etc.)

Practice and Homework

Have the pupils use the addition and subtraction principles to solve many simple equations.

Lessons 7 and 8

THE PRINCIPLES OF MULTIPLICATION AND DIVISION

Development

Following the same procedure suggested in lessons 3 and 4, develop the principles of multiplication and division in lessons 7 and 8, respectively.

Sequence suggested:

1. Solve $2w = 6$, $3n = 21$, $\frac{a}{8} = 5$
2. Analyze $(2 \times 3) = 6$, $(3 \times 7) = 21$, $\frac{(40)}{8} = 5$

   Therefore $w = 3$, $n = 7$, $a = 40$
3. Have them experiment using multiplication or division to arrive at the solution.

   Recall that to maintain equality, whatever is done to one member must also be done to the other member of the equation.
4. Then $2x = 6$

   $$\frac{2x}{2} = \frac{6}{2}$$

   $$x = 3$$
5. Similarly $3n = 21$, $\underline{\frac{3n}{3} = \frac{21,}{3}}$ $\underline{n = 7}$

   Have pupils note equivalency of the underlined equations.
6. $\frac{a}{8} = 5$ $\quad (8x)\ \frac{a}{8} = (8x)\ 5,$ $\quad a = 40$

   Check the solutions to see if we have a true statement of equality.
7. Have the pupils solve:

   $\frac{n}{7} = 52;$ $\quad 9n = 108;$ $\quad 3n = 12;$

   $\frac{x}{3} = 7;$ $\quad 3n = 1.2;$ $\quad \frac{y}{30} = 3$
8. Lead to the generalization and lesson summary.

Summary

Multiplying and Dividing as Inverse Operations:

1. If a number is multiplied and then divided by the same non-zero number, the result is the original number.

   $(2a = 6)$ $(2 \text{ x } a = 6)$ $\frac{(2) \text{ x } a}{2} = \frac{(6)}{2}$ $(a = 3)$
2. If a number is divided and then multiplied by the same non-zero number, the result is the original number: $\frac{n}{3} = 7$, $(3 \text{ x } n)$

   $$\frac{3n}{3} = 3 \times 7, \quad n = 21$$

3. The multiplication and division principles:
   If both sides of an equation are multiplied or divided by the same non zero number, the result is an equivalent equation.

Practice and Homework

Solution of simple equations using addition, subtraction, multiplication.

Lesson 9

SOLVING EQUATIONS

Review

1. The symbolic language of mathematics
2. Open sentences
3. Replacement Sets, Domain, Solution Sets
4. Statements of Equality and Inequality
5. Truth or falsity of statements
6. The meaning of an equation
7. The solution of an an equation
8. *Golden Rule* for finding equivalent equations
9. How the *Golden Rule* helps in solving equations
10. Steps in solving an equation
    a. Using the principles of addition, subtraction, multiplication and/or division to obtain a series of simpler equivalent equations.
    b. The simplest equation gives the value for the variable.
    c. Replace the variable by its value.
    d. If the resulting equation is a true statement, the replacement value is the correct solution.
11. When do we use addition, subtraction, etc.?
12. Have the pupils realize that they have been using letters to represent numbers. (variables)
13. Have the pupils explain the "Golden Rule."
14. Allow much practice in solving equations.

TEST

(suggested credits encircled)

1. (6) An equation has .... members; the value of one member is .... in value to the other member.

2. (3) An equation is a statement ....

3. (12) The equations $4x = 8$, $2x = 4$, $x = 2$, are .... equations because the .... is the same and the .... are the same.

4. (15) What principle was used to solve each of the following equations?

   a. $n + 24 = 10$, $n + 4 - 4 = 10 - 4$, $n = 6$

   b. $y - 7 = 21$, $y - 7 + 7 = 21 + 7$, $y = 28$

   c. $3x = 213$, $\frac{3x}{3} = \frac{213}{3}$ $x = 71$

5. (4) What is the principle of multiplication as applied to solving equations?

6. (15) In the equation $\frac{a}{4} = 12$

   a. What is the variable?

   b. Write an equivalent equation.

   c. What is the solution?

   d. What principle did you use?

7. (40) Solve the following equations: (Domain the set of all known numbers) (Show the equivalent equations)

   a. $3x = 18$ e. $\frac{n}{3} = 8$

   b. $12y = 6$ f. $84 = 7y$

   c. $24b - 16 = 82$ g. $x + 29 = 57$

   d. $\frac{a}{30} = 10$ h. $\frac{m}{4} = 15$

8. (5) How do you check the solution?

Optional (for extra credit) 9. $\frac{3}{2x} = 8$ 10. $15 = \frac{3y}{4}$

Unit XIII

SOLVING PROBLEMS

The specific objectives of this unit are to help the pupils:

1. Improve reading skills in order to understand verbal problems.
2. Use equations to solve verbal problems.
3. Solve equations.

Note: Before this unit the pupils should have completed Unit IX and Unit XII.

Lesson 1

ALGEBRAIC EXPRESSIONS

Warmup

Common Fraction	Decimal	Per Cent
$\frac{1}{2}$	....	50%.
....	.75	....
....	....	20%
$\frac{3}{10}$	....	....

Motivation

Have the pupils express in words:

$3n = 9$ (3 times a number $= 9$)

$\frac{y}{2} = 8$ (a number $\div 2 = 8$) etc.

Aim

Using letters to represent numbers.

Development

Have the pupils express:

Two *more than ten*	*as*	$10 + 2$
Two *more than twelve*		....
Two more than one hundred		....
Ten *increased* by two		....
Forty increased by two		....
A number increased by two		$(n + 2)$

In a similar manner develop *decreased by less than, twice a number, one half a number,* 20% of a number, etc.

Express algebraically: (different letter should be used)

1. Seven times a number $(7 \times n)$
2. A number divided by three $\left(\frac{y}{3}\right)$

3. Sam is 14 years old. How old was he 5 years ago? How old will he be 3 years from now?

   Sam's age; let a = the *number* of years in Sam's age now.

   Sam's age 3 years from now $= a + 3$

   Sam's age 5 years ago $= a - 5$

Summary

In algebraic expressions letters are used to represent unknown numbers.

Practice and Homework

(Work several with pupils before they work independently.)
Express algebraically each of the following:

1. 4 more than d
2. At $5 each, express the cost of n books.
3. If d represents the number of dollars in a month's rent, what does 12d represent?
4. If x represents the number of pupils in a class that has 15 girls, what does x — 15 represent?
5. If t represents the number of hours in the 2400 mile flight of a jet plane, what does $\frac{2400}{t}$ represent?
6. The length of a room is 15 ft. The width is 5 ft. less than the length. What is the width?
7. The length of a room is x feet. Express the width of a room, if it is 5 ft. less than the length.
8. You have 4 nickels in your pocket. Jack has 2 nickels more than you. How many nickels has Jack?
9. You have a certain number of nickels in your pocket. Jack has 2 more than you. Express the number of nickels Jack has.
10. How many feet are there in 1 yard? 3 yards? 5 yards? y yards?
11. How many inches are there in 1 foot? in 2 feet? in 4 feet? in c feet?
12. How many cents in one dollar? in 3 dollars? in 4 dollars? in g dollars?
13. How many cents in one nickel? in 2 nickels? in 10 nickels? in n nickels?

Lesson 2

STATING A PROBLEM IN OPEN SENTENCE FORM

Warmup

Let n represent the number in each sentence. Express algebraically, then solve:

1. 3 less than the number is 6
2. 4 less than the number is 3.20
3. 5 more than the number is $6\frac{1}{2}$
4. 6 multiplied by some number is 24

Motivation

Refer to warm up.

How did you solve the open sentences? (used principles of addition, subtraction, etc.)

How do we know the solution is correct? (replaced variable with value makes sentence a true statement of equality)

Have the pupils write an open sentence for the *problem*:
John spent $4 for a pair of sneakers. He had $3.20 left. how much did he have before he bought the sneakers?

Aim

How do we express a problem as an open sentence?

Development

Have the pupils consider the problem posed in motivation and answer questions similar to:

1. What does the problem tell us? (spent $4, left $3.20)
2. If we let d represent the number of dollars John had, how would you express algebraically the spending of $4? (d — 4)
3. How would you show the $3.20 he had left? (d — 4 = 3.20)
4. What kind of sentence did we write? (open sentence)

Have the pupils recognize the open sentence as the same as problem #2 in warmup.

Note: This would be a good place to stress the importance of reading and understanding what is read.

Have the pupils realize that a verbal problem is easy to solve if we can translate it as an open sentence.

Have the pupils practice (help when necessary). Express each of the following problems in the form of an open sentence:

(Do not solve the problems at this time.)

1. A boy can ride a bicycle at the rate of 6 miles per hour. How long would it take to ride 30 miles?
2. Sam earned \$5.24 more than his brother. Sam earned \$68.75. How much did his brother earn?
3. A pilot covered a distance of 1440 miles in 8 hours. What was his average rate per hour?
4. Mae's father owed \$50 for a record player. This is to be paid in 12 equal monthly payments. How much does he pay each month?
5. The sum of the three angles of a triangle is 180°. If one angle is 20°, a second is 35°, how many degrees does the third angle contain?

Summary

To express a problem as an open sentence (help pupils summarize):

1. Read the problem carefully.
2. Use a letter to represent the unknown quantity.
3. Find the expressions that are equal.
4. Write an open sentence.

Practice and Homework

Identifying the Correct Open Sentences

Have pupil select the sentences which express the following:

1. A number increased by 5 is 8. Let n represent the number

   $n - 5 = 8$ $\quad$ $n + 5 = 8$ $\quad$ $5n = 8$ $\quad$ $\frac{1}{5}n = 8$

2. A boy is 10 years old. He is ¼ as old as his father. Let a represent his father's age

   $a - \frac{1}{4} = 10$ $\quad$ $a + \frac{1}{4} = 10$ $\quad$ $4a = 10$ $\quad$ $\frac{1}{4}a = 10$

3. A certain number of pencils cost 56¢. The price of 1 pencil is 7¢. Let $n =$ the number of pencils

$n - 7 = 56$ $\quad n + 7 = 56$ $\quad 7n = 56$ $\quad \frac{n}{7} = 56$

4. The perimeter of a square is 60 inches. Let $s =$ the side

$s - 4 = 60$ $\quad \frac{s}{4} = 60$ $\quad 4s = 60$ $\quad s + 4 = 60$

5. The area of the floor of a room whose width is 12 feet is 180 square feet. Let $b =$ the length

$b + 12 = 180$ $\quad 12b = 180$ $\quad b + 12 = 180$ $\quad \frac{b}{12} = 180$

Lesson 3

USING EQUATIONS TO SOLVE PROBLEMS

Warmup

Express algebraically the following problems. (Do NOT solve them.)

1. One number is 9 times another. What is the number?
2. One number is 17 more than another number. What is the number?
3. I am thinking of a number. If I add 72 to it, I shall have 91. What is the number?

Motivation

Review method of expressing problems algebraically. Have the pupils realize that the open sentence (equation) resulting may be solved by using the principles of addition, etc.

Aim

To use equations in the solution of problems.

Development

Pose the *problem*:

Joe has 7 times as much money as Tom.
If Joe has 98¢, how much does Tom have?

Help the pupils:

1. Select a letter to represent the amount of money Tom has (T = Tom's money)
2. Express the problem algebraically. ($7T = 98¢$)
3. Decide which principle to use in solving the equation. (Division)
4. Write the equivalent equations until the value of the variable is obvious.
5. Replace the variable by its value to check the truth or falsity of the equality. (Tell the pupils they have verified the solution.)

Have the pupils follow the same procedures to solve problems assigned. (Choose appropriate problems, or similar problems, from list at end of this unit.)

Summary

To solve problems:

1. Read the problem carefully to understand:
   a. what you are to find
   b. what information the problem gives
2. Use a letter to represent the quantity you are to find.
3. Form an equation. (open sentence)
4. Solve the equation.
5. Check or verify the solution.

Practice and Homework

Assign problems from list at end of this unit.

Lessons 4 and 5

REVIEW: SOLVING PROBLEMS

Have pupils solve problems selected from: the group which follows, textbook, or other sources, by using algebraic equations.

TEST

Select a number of problems and exercises from those listed below.

Suggested problems for practice, homework assignments, and unit test.

1. If a number is multiplied by 5, the result obtained is 90. What is the number?
2. If 6 times a number is 96, what is the number?
3. Joe has 7 times as much money as Tom. If Joe has 98¢, how much does Tom have?
4. Mr. Jones is 14 times as old as his son. How old is the son, if Mr. Jones is 42 years of age?
5. I am thinking of a number. If you add 16 to the number, the result will be 54. What is the number?
6. A number increased by 27 equals 78. What is the number?
7. Mr. Henry who is 76 years of age is 49 years older than his grandson. How old is his grandson?
8. Last year at J.H.S. 57 there were 192 more pupils in the 8th grade than in the 7th grade. How many 7th grade pupils were there, if 736 pupils were in the 8th grade?
9. Marion wishes to buy a new car priced at $2150 by trading in her old car and paying an additional $1575. How much money is the dealer allowing her for trading in her old car?
10. After Jack made a deposit of $3.75 in his bank account, his total savings were $22.15. What was his balance before the last deposit?
11. To solve these equations, what should be done to each member of the equation?
    $6x = 42$; $5a = 145$; $2b = 9$; $39 = 3b$; $8t = 72$; $.15x = 4.50$
12. Match the equations in A with those in B. Tell what has been done to both sides of the equation to get the equation in B.

A	B
$5x = 45$	$x = 14$
$12x = 144$	$x = 30$
$x + 8 = 29$	$x = 9$
$15 + x = 45$	$x = 12$
$9x = 126$	$x = 21$

Solve these problems by equations and check the results:

13. A junior high school bought 9 baseball suits for its team for $83.70. What was the cost of each suit?

14. The area of one triangle is 3 times the area of another triangle. If the area of the larger one is 120 sq. ft., what is the area of the smaller?

15. In a school cafeteria four times as many containers of milk were sold as bottles of soda in one week. If 648 containers of milk were sold, how many bottles of soda were sold?

16. A baseball team won 3 times as many games as it lost. It won 87 games. How many games did it lose?

17. I am thinking of a number. If I multiply it by 12, the product is 168. What number am I thinking of?

18. Henry has a number of fish in his aquarium. Since he wished to have 25 altogether, he bought 8 more. How many did he have before his purchase?

19. Jack was told that he is underweight and should gain 11 more pounds in order to weigh 135 pounds. How much does he weigh now?

20. I am thinking of a number. If I add 103 to it, I shall have 240. What is the number?

21. Mary wishes to buy a typewriter for $79.50. She needs to save $22 more. How much has she now?

22. a. A boy cut a board into two pieces. One piece is 10 inches long; the other is four times as long. How long is the larger piece? What is the total length of the two boards?
    b. If the number of inches in the shorter piece is represented by x, how would you represent the length of the longer piece? Represent the total length.

23. A board which is 63 inches long is to be cut into two pieces so that one will be 8 times as long as the other. How long should the pieces be?

24. a. One number is 5 times another. The smaller number is 8. What is the larger? What is the difference between the two numbers?
    b. If the smaller number is represented by n, how would you represent the larger number? How would you represent their difference?

25. One number is 6 times another. Their difference is 35. Find the numbers.

26. a. The length of a rectangular table is twice its width. If the table is 12 inches wide, how long is it? What is the perimeter?
    b. If the table is x inches wide, how would you express the length? Represent the perimeter.

27. Four times a number is increased by 6. The result is 66. What is the number?

28. The larger of two numbers is 19 more than the smaller number. The sum of the numbers is 75. What are the numbers?

29. In a class there are twice as many girls as there are boys. There are 39 pupils in the class. How many boys are there? How many girls?

30. Catherine is paid 75¢ an hour to baby sit, plus 30¢ for carfare. One evening she received $4.05. How many hours did she baby sit?

31. A bottle and cork cost $1.10. If the bottle costs $1 more than the cork, what is the cost of each? (5¢; $1.05)

32. Peter is twice as old as his baby sister. Their father is 16 times as old as the baby. The sum of all three ages is 38. How old is the baby? How old is Peter? How old is Peter's father?

33. The sum of two consecutive numbers is 89. What are the numbers?

34. The sum of two consecutive numbers is 49. What are the numbers?

35. If there are 3 consecutive numbers, such that the sum of the first and third is 28, what are the numbers?

36. Harry's piggy bank contains only nickels and pennies. There are 3 times as many nickels as pennies. How many pennies has he, if the total amount of money is $3.36? (Use p for pennies)

37. Peter has twice as many dimes as pennies in his pocket. The total he has is $1.47. How many pennies and how many dimes has he?

38. Leonard has a number of nickels in his coin jar. He has 3 more pennies than he has nickels. He has 99¢ altogether. How many nickels has he? How many pennies?

39. What are 4 principles that help you solve equations? Express them as a single rule.

40. Four times a number is decreased by 6. The result is 26. What is the number?

Unit XIV

SPENDING MONEY WISELY

The specific objectives of this unit are to help the pupil:

1. gain an insight into the mathematics associated with daily living
2. gain a sense of value in spending money
3. be aware of the advantages and disadvantages of cash and credit payments
4. improve ability in arithmetic computation

Teaching Aids

1. Sales slips — may be obtained from G-1 Supply list, Board of Education of the City of New York
2. See "Resource Materials"

Lesson 1

PLANNED SPENDING

Warmup

Practice in dollar and cents computation.

Motivation

Discuss pupils' daily expenses: carfare, lunch, entertainment, etc.

Have pupils write the amount of money each had at the beginning of the previous day.

Have them write the amount of money left at the end of the previous day.

Have them list the items the money was spent for. (If necessary, use an imaginary situation.)

Have pupils review individual lists, and indicate total unnecessary expenditures. Conclude that planned spending, or a *budget* would help them save money.

Aim

To prepare a personal budget.

Development

Have the pupils answer questions of the type:

How much money do you spend for carfare weekly? The average weekly amount spent for lunch? Price of haircut? How often is hair cut? Average cost per week (which should be put aside) for haircut? Stress the importance of taking care of necessities first.

Have pupils prepare a table showing the best weekly allotment of earnings or allowance, including regular savings. Explain that planned spending is living on a budget.

Discuss advantages of a budget. Suggest they follow the budget planned for one week and report success or failure.

Summary

Living on a budget insures living within income and prepares for expenses that may come.

Practice and Homework

Start preparation of a family budget with the class, allowing approximately 30% for food, 25% for shelter, 15% miscellaneous, 10% for insurance and recreation, and 5% for savings.

Have the pupils suggest a monthly income that is real and reasonable for them.

Have pupils complete budget at home.

Lesson 2

THE WISE CONSUMER

Warmup

Practice in meaning of percent and using percent.
Include: 100% = the whole amount; a percent of a number is a part of that number (if the percent is less than 100).

Motivation

Discuss buying an article of interest to most pupils such as, a recording, sports equipment, camera, etc. Point out that the article may be bought in a number of places. Have the pupils consider the advantages of having many shops to compare prices, quality, service, etc.

Aim

Saving money by shopping wisely.

Development

Elicit from pupils ways of saving money, such as, waiting for sales, buying at the end of the season, taking advantage of discounts, buying in quantity, etc. Point out quality, maintenance, standard brands, etc.
Have them judge which is preferable: good quality at a slightly greater cost, or less cost and poor quality.
Use newspaper advertisements for comparison.

Use a discount problem to review finding a percent of a number, to find the difference between the original price and sale price, profit, loss, etc.

Summary

The wise consumer saves money by taking advantage of sales and discounts.

Practice and Homework

Prepare worksheet, or assign in text, a number of consumer problems.

In solving the problems pupils should show:

1. Open sentence
2. Equivalent equations
3. Solution
4. Verification — checking.

Lesson 3

SALES SLIPS AND RECEIPTS

Warmup

Practice in taking 4% of amounts of money.

Motivation

Discuss various kinds of markets, shops, and stores.

Elicit from the pupils that in most cases a sales slip is given to the *buyer*.

When bills are paid, receipts are given to the *payer*.

Aim

Are sales slips and receipts necessary?

Development

What would you do if you bought a pair of shoes and when you opened the packages at home you found two left shoes?

When you return a package, what is the first thing the sales clerk asks for? Why? (sales slip establishes purchase was made at that store, indicates the cost, amount of sales tax paid, sales person involved, etc.)

We can say that the sales slip is proof of the purchase.

Ask the pupils to suggest other values of the sales slips.
(buyer-record of expenditure, seller-record of sale and money received)

Have the pupil answer questions such as:

What is a receipt? (proof of amount paid and purpose for which it was paid)

What kind of organizations usually give receipts? (gas, electric and telephone companies; loan companies; renting agents; insurance companies; etc.)

Distribute copies of sales slips. Discuss items to be filled in.

Discuss city sales tax, percent taken of total amount, etc.

Have pupils write out sales slips from an assigned list of purchases.

Have them find total cost, amount of sales tax, and total paid.

Summary

A sales slip is proof of purchase and should be kept until item bought is completely satisfactory.

A receipt is a proof of payment.

Practice and Homework

Have the pupils find total cost of a number of items, sales tax of various amounts, and other similar problems.

Have them write a sales slip for each problem.

Lesson 4

CASH OR CREDIT?

Warmup

Practice in finding a percent of a number.

Motivation

Discuss the importance of living within one's income, and having enough money for necessities. Occasionally, people want a costly thing for which they have not saved enough money. Ask pupils to suggest ways of obtaining these costly articles, such as, television sets, hi-fi sets, cars, etc.

Aim

Is it wise to buy goods on the installment plan?

Development

Discuss the meaning of "*installments.*" Pose a problem similar to: A T.V. set is sold for $300 cash, or 10% down, and $19.20 a month for 15 months.

Explain and discuss: cost if cash is paid, *installment price,* and *carrying charge.*

Have pupils consider the advantages and disadvantages of buying on the installment plan. Caution pupils about excessive carrying charges.

Work a number of problems with pupils using diagrams for clarity.

Summary

Advantages of installment buying:

The buyer has the use of the article immediately.

The buyer must save his money to meet the payments.

Making payments on time establishes credit.

Disadvantages of installment buying:

The installment price is more than the cash price.

The buyer may buy more than he can *afford.*

If payments are not met, article and money paid is lost.

Practice and Homework

A number of problems in which the pupil must find:

- number of installments
- amount of each installment
- carrying charge
- total amount paid.

Lesson 5

EXPENSES FOR RECREATIONAL ACTIVITIES

Warmup

Estimation and computation of multiplication and division exercises.

Motivation

Have pupils discuss meaning of, "All work and no play, makes Jack a dull boy."

Include the benefits to health and disposition, of relaxation, enjoyment, and change of routines and environment.

Have pupils list recreational activities. Keep within pupils' experiences and income status, e.g., movies, house parties, picnics in parks, no cost or low cost theatres, etc.)

Vacation activities; city parks, pools, camps and travel to other states, etc.

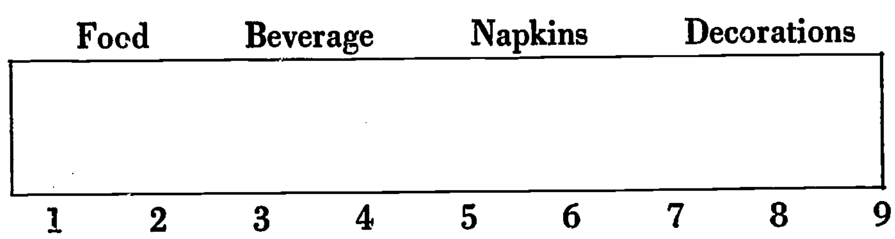
Aim

To plan recreational activities to *insure* enjoyment at low cost.

Development

Plan an imaginary house party with the class. State a specific amount of money available. Have pupils suggest: number of guests, refreshments desired, etc. Have them compute cost for each item. Total costs and compare with amount available. Have pupils realize that planning and organization is needed.

To simplify planning, suggest a table and diagram. (Rectangle Graph)



Amount available — $9.

Refreshments:	$3.50	Food
	3.00	Beverages
	.50	Napkins
	2.00	Decorations

Using a diagram helps to see if expenses exceed the money available.

Have the pupils plan theatre parties, picnics, etc. Point out that if the expenses are too high, less expensive refreshments can be used (Kool-aid, or frozen lemonade instead of sodas), decorations may be omitted, etc.

Have the pupils discuss means and cost of transportation to beaches, amusement parks, etc. Least expensive would be public conveyances.

Summary

Activities should be planned and organized to insure enjoyment and not waste money.

Using tables and diagrams makes planning easier.

Practice and Homework

Have pupils draw a rectangle graph in planning each of two activities.

Lesson 6

TRAVELING FOR PLEASURE

Warmup

Practice in solving equations.

Motivation

Everyone, young and old, looks forward to vacation time. Have pupils give reasons for vacations and why people look forward to them.

Discuss students' vacation activities.

Discuss adult workers' vacation activities.

Have pupils express their opinions of an enjoyable vacation trip.

Development

Review planning and organizing activities, consideration of costs, money available, etc.

Indicate that there are additional considerations in traveling, such as, length of vacation, distances to travel, reservations, etc.

Very often people who own cars prefer to drive to their vacation spots. How can the time needed for traveling be computed?

At an average rate of 40 miles per hour, how long will it take to drive 100 miles? 200 miles, 150 miles? Have pupils arrange a table:

RATE IN MPH	TIME IN HOURS	DISTANCE IN MILES
40	3	?
40	$4\frac{1}{2}$	?
40	6	?
40	$6\frac{1}{4}$	?

Guide the pupils to discover: Rate x Time = Distance

After practice in finding distance traveled, time taken, rate traveled, develop the formula D = RT. Have the pupils define *transportation.*

Summary

Distance = Rate x Time Formula. D = RT

Practice and Homework

Problems using distance formula. Suggest diagrams to "see" the problem.

Have pupils find the distance traveled, when rate and time are known; the time taken when rate and distance are known; the rate when the time and distance are known, for various means of transportation.

Compare costs, time, speeds of air, train and car travel.

Lesson 7

COST OF CAR TRANSPORTATION

Elicit facts needed to compute costs: cost of gasoline, oil, etc., miles per gallon of gasoline, etc.

Have the pupils compute: number of gallons of *fuel* to travel given distances, amount of fuel used, total cost is known, etc.

TEST

1. What does the formula $D = RT$ mean?
2. A train travels at a speed of 90 miles per hour. How long will it take to travel 405 miles?
3. A T.V. set cost $225. Mr. X bought the set on the installment plan. He paid $21 a month for 12 months.
   a. How much did he pay for the T.V. set?
   b. How much more did he pay than he would have paid for it in cash?
   c. To find the sales tax, did the salesman take 4% of $225 or 4% of $252?
4. How would you save more money on a coat that cost $37.50
   a. If the price were reduced $5?
   b. If it were sold at a discount of 10%?
5. A supermarket sold canned tomatoes at the sale price of 6 cans for $1.74, or 3 cans for 90¢. How much did mother save by buying 6 cans?
6. Mr. W. bought a house for $18,500. The real estate salesman's commission was 5% of the cost.
   a. How much did the salesman receive?
   b. Mr. B. paid 30% of the cost as a down payment. How much did he pay?

Unit XV

SAFE AND SECURE

The specific objectives of this unit are to lead the pupils to:

1. appreciate the benefits of regular savings
2. understand the purposes and advantages of various kinds of insurance
3. realize the importance of evaluating advantages and disadvantages of a situation before making a decision

Materials Needed

1. bank deposit and withdrawal slips
2. blank checks and stubs
3. insurance tables (optional)
4. see "Resource Materials"

Lesson 1

SAVE MONEY TO EARN MONEY!

Warmup

Practice changing percents to decimals, percents to common fractions, and common fractions to decimals and percents.

Motivation

Have pupils consider:
If you saved 5¢ per day, how much money will you have saved at the end of a week? 10 weeks? ½ year? one year? Suppose you saved 10¢ a day? n¢ a day? etc.

Have the pupils realize that most of us cannot save large amounts daily, weekly, or monthly, but that everyone can save a little. Small amounts grow into larger amounts.

Aim

Why is best to keep savings in a savings bank?

Motivation

A. Have the pupils answer questions, such as:

1. What does the bank do with the money deposited? (invests it)
2. Why does the bank *invest* the *deposits?*
3. Have banks the right to use other people's money to make money for themselves?
4. Explain that *depositor* are paid for the use of their money by receiving *interest* on their savings.
5. What is *interest*? How are records kept?
6. How do we know our money is safe?

B. Develop, in sequence suggested, the following facts:

1. The depositor's record of money deposited is the individual "*account book.*"
2. Banks record of money received is the *deposit slip*. Information is recorded on individual's account card. (Distribute deposit slips.)
3. *Withdrawal slips* are bank's receipts for money paid back to

depositor *on demand.* This information is also recorded on individual's account card. Distribute withdrawal slips.

4. The U. S. Federal Government safeguards depositor's money.
5. *Interest* is the money paid the depositor by the bank for the use of his money.

Summary

Savings deposited in a bank are safe and earn more savings.

Practice and Homework

Have the pupils practice using signed numbers, (do not include interest at this time) by preparing and completing the tables below:

Date	*Deposits*	*Withdrawals*	*Balance*
	$2.00	$ .75	
	3.00	1.50	
	1.00		
	5.00		

Have the pupils write the deposits as positive numbers, the withdrawals as negative numbers.

Lesson 2

THE SIMPLE INTEREST FORMULA

Warmup

Changing common fractions to equivalent decimal fractions and percents.

Motivation

Review: The bank pays interest on money deposited.

The bank gets the money to pay the interest by making money on the money deposited.

One way of making the money is by making loans to people, businesses, etc., and charging the borrowers for the use of the money. This charge is also called interest.

(If possible, discuss compound interest briefly.)

Aim

How does the bank know how much interest to give a depositor and how much interest to charge on a loan?

Development

A. Establish:

1. The amount of money deposited is called the *principal.*
2. The percent of interest paid is called the *rate.*
3. The length of *time* the money is in the bank, or how long the money is borrowed.

B. 1. What does principal mean? the rate? the time?

2. Explain that, for interest purposes, we usually use 360 days to mean one year.
3. What part of a year is 90 days? 180 days? 120 days?

C. 1. If you had \$75 in the bank for 1 year and the rate of interest was 3%, how much interest would you receive?
Have the pupils realize they must multiply:
($\$75 \times .03 \times 1 = \$2.25$)
   a. Find how much interest was paid for one-half a year. (1 year's interest is \$2.25, ½ of \$2.25)
   b. Find how much interest was paid for 2 years.
   ($1 \text{ yr.} = \$2.25$, $2 \text{ yrs.} = 2 \times \$2.25$)
   or (a) ($75 \times .03 \times \frac{1}{2}$); (b) ($75 \times .03 \times 2$)

2. Have the pupils realize that to find the interest we multiply the principal by the rate by the time.
3. Then: Interest = Principal × Rate × Time.
4. Abbreviating: I = PRT, which is known as the *interest formula.*

D. If a man borrowed \$350 from a bank and agreed to pay 4% per year, how much interest would he pay for (a) 1 year? (b) 2 years? (c) ½year? Use the interest formula: I = PRT.
a. $\$350 \times .04 \times 1 = I$ b. $\$350 \times .04 \times 2 = I$, etc.

Summary

To find the amount of interest, we use the interest formula: I = PRT.

Practice and Homework

Select problems from list below.

1. John wanted to buy a typewriter for $65. He borrowed this amount for one year at 3% interest. How much interest did John pay? What amount did he pay at the end of the year to cover principal and interest?

2. Paul borrowed $500 from the student loan fund to finish his training at a business college. He was charged interest at the rate of 4% per year. How much did he return to the fund at the end of the year?

3. Frak's father borrowed $350 from his friend to put a new roof on his house. The rate of interest charged for the loan was 2% per year. If Frank's father kept the money for ½ year, how much interest did he have to pay? What is the total amount he paid at the end of the half year?

4. Janet's father borrowed $240 for one year to pay a doctor's bill. He agreed to pay interest at the rate of 3% per year. Using the facts, match item in Column A with appropriate item in Colum B.

A	B
$240 borrowed	amount
one year	principal
3%	time
$7.20	interest rate
$247.20	interest

5. Complete the table:

Monthly Deposit	Number of years (T)	Total Deposit (P)	Rate of Interest (R)	Interest earned
$ 2	10		3½%	
$ 4	5		3%	
$ 8	7		4%	
$10	4		3%	
$20	10		2%	

Lesson 3

BANK SERVICES

Following usual lesson procedures develop the meaning and use of:

1. commercial banks
2. checking accounts (regular and special)
3. writing checks and stubs
4. the advantages of paying by check
5. traveler's checks
6. bank checks

Lesson 4

PROTECTION OF PERSON AND PROPERTY

Warmup

Practice in fundamental operations

Addition of directed numbers

Motivation

Have the pupils consider and explain the statement:
Insurance is a way of saving.

Aim

Why is insurance a safeguard against financial want or loss?

Development

A. 1. Have the pupils name emergencies in life that may occur when earnings and savings alone would not be enough to pay costs. (accident, fire loss, theft, serious illness, property damage or loss, untimely death)

2. Have them realize that insurance is a way of sharing the burden

of large losses. When many individual persons pay small amounts a large amount of money, or *reserve fund*, is created. From this fund large amounts can be paid to those insured persons who suffer loss.

3. There are many types of insurance, one for each different kind of risk.

B. Kinds of Insurance

1. Elicit from the pupils the types of insurance they are familiar with, including:
   a. automobile insurance
   b. fire insurance
   c. health and accident insurance
   d. unemployment insurance
   e. old age insurance
   f. life insurance

2. Insurance terms

Note: The following concepts and terms apply to all types of policies.

   a. The person taking out the insurance is called, the *insured*.
   b. The *policy* is the written agreement the insured makes with the insurance company.
   c. The *face value* is the amount of money the person is insured for.
   d. The *term* is the period of time during which the policy is in effect.
   e. The *premium* is the payment that the insured pays for the insurance.

   Premiums are paid *annually, semi-annually, quarterly, monthly* or *weekly*.

   The amount of the premium paid depends on the kind and amount of insurance carried and the rate that is being charged for each $100 of insurance.

C. Have the pupils consider fire insurance for buildings.

1. Would the rate be greater for a house in a country setting or in a city setting? Why?

2. Have pupils realize that the cost of the insurance depends on the kind of building, the use made of the building, the proximity and efficiency of the fire department, etc.

3. Would the rate be greater for a gasoline storage plant or for a tool plant? Why? Stress: the greater risk the greater rate.

D. Find the amount of the premium

1. Property owners usually insure their property for from 50% to 80% of its value.
2. If a house is insured for $8000 and the insurance rate is $.35 per $100 for a year, what is the yearly, or annual, premium?
3. Remind the pupils the rate is for each $100 of insurance. ($8000 \div 100 = 80$; $80 \times .35 = \$28$.)
4. Have the pupil realize that the owner pays $28 for $8000 protection. If his house should be damaged by fire, he could collect up to $8000.

Summarize

Insurance protects an individual and his property against financial loss.

By paying a small annual premium, a large amount of money may be received in case of loss or damage.

Practice and Homework

1. A man insured his house for $12,000 against fire for 1 year. If the rate is $.48 per $100 find the premium for the year.
2. Mr. Roberts insured his house against fire for $10,500 for 1 year. If the annual rate is $42 per $100, find the premium for the year.
3. Complete each of the following:

Face	*Rate per $100*	*Annual Premium*
$ 4,500	$.25	?
6,200	.32	?
7,000	.38	?
9,500	.41	?
12,500	.46	?
15,800	.49	?
25,000	.62	?
46,000	.75	?
115,000	.90	?

Lesson 5

JUDGMENTS AND DECISIONS

Warmup

Solve the following and verify each solution:

$13n - 27 = 103$ $\qquad$ $317 \times 7 = R$

$882 \div 21 = x$ $\qquad$ $\frac{315}{S} = 105$

Motivation

Have the pupils explain how they verify equations. (Replace the variable by its value, then *judge* the equation to be true or false.) What does "judge" mean? What does "judgment" mean?

Have the pupils suggest situations in which they must use judgment before making a decision.

Aim

Should we make quick or snap decisions?

Development

A. Which job pays the better salary? Job A pays \$60 per week. Job B pays \$240 per month. Before we can decide, let us consider the facts:

$$\$60 \times 52 = \$3120 \text{ per year}$$
$$\$240 \times 12 = \$2880 \text{ per year}$$

Now it is easy to see which job pays more.

B. Job A pays more money. Job B pays \$240 less than job A, but the employer insures all his employees and pays all premiums.
Each employee has accident and health insurance and life insurance.
Have the pupils realize they must decide which is more important, security or more money.
Have the pupils consider, judge and decide, a number of situations including graduating from high school or leaving school and getting any job that can be found.

Summary

Before decisions are made, all the facts should be considered and judged.

Practice and Homework

Have the pupils arrive at a decision, then explain reasoning leading to it. Have the pupils realize that the decision, in most cases, depends on individual needs, desires, ambition and outlook on life.

What is your decision in each of the situations listed below? Explain.

1. If you found a wallet containing $7 and the owner's name and address, would you
   a. return wallet and money to rightful owner?
   b. return the wallet, but keep the money?
   c. keep the money and throw the wallet away?
2. If you had to park your car, would you
   a. park in a "No Parking" zone to save time?
   b. take some time to find a legal place to park?
3. If your employer gave you a choice of insurance, would you choose
   a. an accident, health and life insurance?
   b. a twenty year endowment policy?
4. Would you open a savings account in a bank that paid
   a. a 3½% interest rate
   b. a 3% interest rate
5. If you lost a wallet containing your name and address, would you want the finder to
   a. return the wallet and money?
   b. return the wallet without the money?
   c. keep the money and throw the wallet away?

   (Compare this answer with answer to item #1.)

Unit XVI

$$25_{TEN} = 100_{FIVE} = 11001_{TWO} = 34_{SEVEN}$$

The specific objectives of this unit are to lead the pupils to:

1. a better understanding of the decimal system of numeration
2. an awareness of other systems of numeration
3. recognize and express quantities in other systems of numeration
4. appreciate the logical structure of a system of numeration

Teaching Aids

See *Resource Materials*

Lesson 1

BASE 10: DECIMAL SYSTEM OF NUMERATION

Warmup

What is the value of the:

1. 5 in 365 ....
2. 6 in 365 ....
3. 3 in 365 ....
4. 0 in 1,037
5. 1 in 21,480 (no hundreds)
6. 3 in 32,141

Motivation

How many symbols (digits) do we use to express a number?
What are the ten symbols?
Using the symbols "3" and "8", does $38 = 83$? Why not? (Place value)
What is our system of writing numbers? (Hindu-Arabic Decimal System)

Aim

To learn the *structure* of the Hindu — Arabic System

Development

A. We use 10 symbols as digits to express numbers.

1. What number does the numeral "1" name?
2. What number does "11" express? ($10 + 1$ or 1 ten $+ 1$)
3. Why does the digit "1" in the first place express only one object, while the symbol "1" in the second place expresses ten things? Establish that "1" in the second place expresses one group of ten things, or (1×10).
4. What does the numeral "111" express?
   Have the pupils realize $111 = (1 \times 100) + (1 \times 10) + (1)$
5. Have the pupils prepare a *Place Value* chart as follows:

PLACE VALUE CHART

Order of Place	5th Place	4th Place	3rd Place	2nd Place	1st Place
Name of Place	Ten Thousands	Thousands	Hundreds	Tens	Ones
Value of Place	10,000 1 x (10x10x10x10)	1000 1x(10x10x10)	100 1 x (10 x 10)	10 1 x 10	1 1 x 1

(Continue with added spaces.)

6. Have the pupils note that the place values increase by ten, and that there are 10 number symbols. We can say it is based on 10, or is a base 10 system.
7. Have the pupils expand each numeral as shown, e.g.,

   a. $3{,}000 = 3 \times 1000$ or $3 \times (10 \times 10 \times 10)$

b. $700 =$	d. $6{,}000 =$
c. $90 =$	e. $60{,}000 =$

   Have the pupils learn that "place" may be expressed as "position." We say the Hindu-Arabic system is a *positional* system.

Summarize

In the Hindu-Arabic system of *numeration.*

1. There are 10 digits or symbols to express number.
2. It is a positional system.
3. The value of each place is 10 times as great as the value of the place to its right.
4. The value of each place is 1/10 of the value of the place to its left.
5. It is a base ten system of numeration.

Practice and Homework

Have the pupils give the value of numerals (assigned) in terms of 10.

1. $80 =$ 2. $600 =$ 3. $4000 =$ etc.

(give all rounded numbers in preparation for next lesson.)

Lesson 2

EXPONENT: POWERS OF A NUMBER

Warmup

Write the value of each numeral as shown in "a":

a. $7{,}000 = 7 \times 1{,}000$ or $7 \times (10 \times 10 \times 10)$

1. 50	3. 5,000
2. 500	4. 50,000

Motivation

Study of the Hindu-Arabic System is continued.

Aim

To learn new ways of expressing number in the Hindu-Arabic System.

Development

A. 1. Refer to warm up exercises

2. Tave pupils note that the number of times 10 is taken as a factor determines the place, or position, and the value of the digit.

3. $50 = 5 \times 10$ (10 as factor 1 time)
$500 = 5 \times (10 \times 10)$ (10 as a factor 2 times), etc.

4. Have the pupils evaluate:
   a. $5 \times$ (10 as a factor once) = (50)
   b. $5 \times$ (10 as a factor 3 times) = (5000)
   c. $5 \times$ (10 as a factor 0 times) = (5)

   Remind the pupils that in "c" it is not 5×0, but since 10 is not a factor, 5 cannot be in any other "place" but the one's place.

5. Have the pupils express, or evaluate, as indicated in terms of factors of 10
   a. 210 =
   b. .... $= 7 \times$ (10 as a factor 1 time) + 9 + (10 as a factor 0 times)
   c. 460 =
   d. 1200 =
   e. 502 =

B. The pupils should realize that it is awkward to express numbers with factors of ten.

1. Tell them that there is a very simple way of showing how many times 10 is a factor.

2. To write 10 as a factor 3 times, we write 10^3
   a. What is the value of 10^3? (1000)
   b. What does the little three above and to the right of 10 mean? (10 is a factor 3 times)

c. What does 10^2 mean? The value of 10^2?

d. What is the value of 10^0? (1)

3. Have the pupils learn:

a. the small number notation at the right of and above the 10 is called an *exponent.*

b. The exponent tells us how many times 10 is to be taken as a factor.

C. Return to the place chart prepared in lesson 1. In the fifth row, have the pupils express the powers of 10 with exponents
e.g. 10^4, 10^3, 10^2, 10^1, 10^0 or 1

Summary

The Hindu-Arabic system of numeration is a base 10 system. To show the power of the base, exponents are used. ($10^3 = 10 \times 10 \times 10$, or 1000.)

Practice and Homework

Evaluate:

1. $10^4 + (3 \times 10^3) + (2 \times 10^2) + (4 \times 10^1) + 6 =$
2. $(9 \times 10^2) + 9 =$
3. $(7 \times 10^3) + (2 \times 10^3) + 8 =$

Express the following using exponents to show the power of the base 10:

1. $375 =$ 2. $600 =$ 3. $4252 =$

Lesson 3

BASE 5—A QUINARY SYSTEM

Warm Up

////, IV, 4, all express the same number

IV is a .... numeral 4 is a .... numeral

//// are marks, or tallies, that show .... things.

Motivation

Review base 10 system of numeration
Ten digits 0 — 9
Positional system — place value
Addition system

The value of each place is 10 times the value of the place to its right.
Could we have a system of numeration with only 5 digits?
What would the base be?

Aim

To express numbers in a base 5 system of numeration

A. 1. In a base 5 system, what could the symbols be? (0, 1, 2, 3, 4)

2. What are the names of the first four places in *base 10?*

   Write the value of each place using exponents:

   (thousands hundreds tens ones)
   (10^3 10^2 10 1)

   Have the pupils note that the value of the second place is the base.

3. Write the value of the first four places in base 5 using exponents:

   (5^3 5^2 5 1)

4. Have the pupils show how many times 5 is taken as a factor in each place: ($5 \times 5 \times 5$) 5×5 5 1)

5. What could we call each place?
   One hundred twenty-fives twenty-fives fives ones

6. What does 1 in tens place mean in base 10? *(1 ten)*

7. What would 1 in fives place mean in base 5? *(1 five)*

8. What would 1 in tweny-fives place mean in base 5? (1 twenty-five)

9. a. 2 in twenty fives place base 5 means ....

   b. 4 in one hundred twenty-fives place means ....

   c. what does the numeral 13 mean in base 5?
   (1 five + 3 ones) (We would write this as 13_{Five})

10. Have the pupils prepare a Place Value Chart for base 5 as they did for base 10 in Lesson 1 of this unit.

Summary

Base 5 is called a Quinary System of numeration.

1. There are 5 symbols to express number.
2. It is a positional system.
3. The value of each place is 5 times as great as the value of the place to its right.
4. The value of each place is $\frac{1}{5}$ of the value of the place to its left.
5. It is an additive system.

Practice and Homework

Write the numbers from 1 to 10 in base ten and base five.

Remember that in base 5 you can use only the symbols 0, 1, 2, 3, 4.

Lesson 4

EXPRESSING NUMBERS IN BASE 5

Warm Up

Prepare a chart showing numbers from 1 to 10 in base 10 and base 5. Have pupils compare homework with chart.

BASE 10		BASE 5		
Tens	*Ones*	*Fives*	*Ones*	
	1		1	
	2		2	
	3		3	
	4		4	
	5	1	0	
	6	1	1	(1 five + 1 one)
	7	1	2	
	8	1	3	
	9	1	4	
1	0	2	0	(2 fives)

Motivation

What is the largest number that we can write in ones place base 10? (9) In base 5? (4)

What is the largest number that can be written in tens place and one's place base 10? (99) Base 5? (44)

Aim

Expressing larger number in base 5.

Development

1. Have the pupils write the value of the first four places base 5 (written in base 10).
   (125 25 5 1)
2. To show that numerals are in base 5, we write a 5 to the right and below numeral. 11
   Base 5
3. What does 12_5 which is read "one, two, base 5," mean?
   (1 five + 2 ones)
4. What is the value of 12_5 in base 10? $(5 + 2 = 7)$
   What is the value of the base 5 numerals in base 10?

Place Value	125	25	5	1		Base 10 value
			3	1_5	3 fives + 1 one or	16
		1	0	4_5		
	1	1	0	0_5		
			1	3_5		

Summary

Any number may be expressed in base 5 by writing the symbols in places that have a large value. Each place is 5 times as great as the value of the place to the right of it. (Note: These are written in base 10.)
e.g.

5×125	5×25	5×5	5×1
625	125	25	5

Practice

Have the pupils write numbers in base 5, and give the value of base 5 numerals in base 10.

SUGGESTED TEACHING AIDS AND REFERENCES

Books for Class Library

Author	*Title*	*Item No.*	*Publisher*
Bendick, Jeanne	How Much and How Many	30250	
Bendick & Levin	Take a Number	30613.5	
Bendick, Jeanne	The First Book of Time		Franklin Watts, Inc.
Epstein, Sam & Beryl	The First Book of Measurement	30250.3	
Epstein, Sam & Beryl	First Book of Codes & Ciphers	31407.4	
Schwartz, Julius	The Earth is Your Space Ship		McGraw Hill
Simon & Bendick	The Day the Numbers Disappeared		McGraw Hill

Teacher References

		Publisher
Curriculum Bulletin	General Mathematics 9th Year	
Curriculum Bulletin	Mathematics Grade 7, 7X	Board of Education
Curriculum Bulletin	Mathematics Grade 8	Board of Education
Brueckner, Grossnickle and Reckzeh	Developing Mathematical Understandings	John C. Winston Co.

Textbooks

Salten, Frankson & Dever	Mathematics: A Basic Course	Cambridge, Bronxville
Brown, Simon, Snader	General Mathematics, Bk 2	Laidlow
Fehr, Schult	Mathematics in Life	D. C. Heath & Co.

Teaching Aids (no cost)

Author	*Company and Address*
Why Study Math? Planning Now for Your Career	General Electric Co., Public Relations Div. 1 River Road, Schenectady 5, N. Y.
The Story of Figures Fascinating Figure Puzzles	Burroughs Corporation 6071 Second Ave., Detroit, Michigan
History of Measurements Measurement—(posters)	Ford Motor Corp., Educational Relations Dept. 300 Schaefer Road, Dearborn, Mich.
Area & Perimeter Calculator	Libby-Owens-Ford Glass Co. Public Relations Dept., Nicholas Bldg., Toledo 3, Ohio
Square Yard Chart	Winton Company 1010 Arch Street, Phila. 7, Pa.
The Day of the Two Moons	Association of American R.R. Transportation Bldg., Washington 6, D. C.
The Story of Time & Timekeepers Telling Time How a Watch Works	The Hamilton Watch Company, Educ. Service Dept., Lancaster, Pa.
Money Management—Your Budget Guiding Family Spending	Good Housekeeping 57th Street & 8th Ave., N. Y. 19

Moneys of the World	Curtis Circulation Co. Independence Square, Phila. 5, Pa.
Sharing the Risk Money Management & Life Insurance	Institute of Life Insurance Educ. Div., 488 Madison Ave., N. Y. 22
The Age of Flight	United Aircraft Corp. 400 Main St., East Hartford, Conn.

SUGGESTED RESOURCE MATERIALS

(From *General Mathematicss 9th Year*)

Source	*Item*	*Suggested Applicable Areas*
1. G. 1 Supply List N.Y.C. Bd. of Educ.	1″ gummed squares Sales Slips Petty Cash Slips Deposit Slips Blank Checks and Stubs Compass; Pupil, Blackboard Graph Paper Blackboard Graph Charts Protractors and Rulers	Measurement, Area Consumer Mathematics " " Banking " Time Time
2. B.A.V.I. (Bur. Audio-Visual Instr.) 131 Livingston St. Brooklyn, N. Y.	Films & Filmstrips Lists published regularly	All areas
3. Bowery Savings Bank 110 E. 42 St., N. Y. C.	Computer Cost of Credit	Banking
4. Burroughts Corp. Detroit 32, Mich.	Fascinating Figure Puzzles	Geometry
5. Chrysler Corp. Dept. of Public Relations Detroit 31, Mich.	Problems from Industry Others	Equations Measurement
6. Duodecimal Society of America	An Excursion in Numbers Others	Contemporary Math
7. Federal Reserve Bank of N. Y. 33 Liberty St., N. Y. C.	Study of Checks Others	Consumer Math
8. Ford Motor Co. Educ. Relations Dept. Dearborn, Mich.	How Long is a Rod? History of Measurement Average Stopping Distances Others	Measurement
9. General Motors Corp. Educ. Relations Dept. Detroit 2, Mich.	A Measure of Progress Precision Others	Measurement

10. Hartford Ins. Co. 690 Asylum Ave. Hartford, Conn.	Glossary of Insurance Terms General Information Bulletin	Insurance Statistics
11. International Business Machines 590 Madison Ave. New York 22, N. Y.	World of Numbers Mathematics Chart Others	Contemporary Mathematics
12. Marchant Calculators Oakland 8, Calif.	From Oq to Googol	Measurement Contemporary Math
13. Scott, Foresman & Co. 19-00 Pollitt Dr. Fairlawn, N. J.	Using Ratio to Solve Problems Number Line Others	Fundamentals Equations Measurement
14. U. S. Dept. of Health, Educ. & Welfare Office of Educ. Washington 25, D. C.	Aids for Math Education Circs. #346 - 347, 348 - 431 - 584, 586 and others	Many areas
15. Webster Publish. Co. 1154 Reco Blvd. St. Louis, Mo.	Number Line Others	Signed Numbers
16. World Book Co. 750 Third Avenue New York 17, N. Y.	Notes for the Math Teacher Problem Solving Others	Equations

Note: Many of the above listed organizations issue catalogues of their educational publications.