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PREPARATION FOR GRADUATE STUDY IN MATHEMATICS.

Committee on the Undergraduate Program in Mathematics, Berkeley, Calif.

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The purpose of this booklet is to provide interim guidance for those colleges that cannot immediately effect the recommendations of the Committee on the Undergraduate Program in Mathematics (CUPM) 1963 report. The recommendations in the 1963 booklet were for the first four years of a seven-year program leading to the PhD. and a career in mathematical research. Considered in this report are the undergraduate curricula in Abstract Algebra and Real Analysis. (RP)

MATHEMATICAL ASSOCIATION OF AMERICA

PREPARATION FOR GRADUATE STUDY IN MATHEMATICS

A program in partial fulfillment of the goals set out in the Panel's earlier pamphlet, *Pregraduate Preparation of Research Mathematicians.*

NOVEMBER 1965

The Committee on the Undergraduate Program in Mathematics, supported by the National Science Foundation and appointed by the Mathematical Association of America, is charged with making recommendations for the improvement of college and university mathematics curricula at all levels and in all educational areas. Moreover, the Committee devotes its energies to reasonable efforts in realizing these recommendations. The present membership of the Committee is as follows:

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To reflect one of its major concerns, the Committee has established a Panel on Pregraduate Training, with current membership as follows:

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FOREWORD: THE GOAL

The task of the Panel on Pregraduate Training is to recommend programs for undergraduates who intend to go on to graduate work in mathematics. The first stage of the Panel's work, to consider an ideal undergraduate program for the future research mathematician, was completed in 1963, and the conclusions have been published in a booklet.*

The recommendations in the 1963 booklet are for the first four years of a seven-year program leading to a solid Ph.D. and a career in mathematical research. The booklet contains guidelines and several detailed sample course outlines. Note, however, that in forming these recommendations the Panel made no allowance for the student's possible deficiencies in preparation or tardiness in selecting a goal, for inadequacies of staff, for lack of suitable textbooks, etc. Furthermore, the program was designed for the very gifted undergraduate working under ideal circumstances. Some schools already offer programs compatible with the 1963 recommendations. At many others, however, the recommendations cannot be put into effect very soon. For them, the 1963 booklet provides a goal. The present booklet provides interim guidance.

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*Preliminary Recommendations for Pregraduate Preparation of Research Mathematicians, May 1963. Reprinted with minor modification under the title Pregraduate Preparation of Research Mathematicians, 1965. Distributed through CUPM, P. O. Box 1024, Berkeley, California 94701.

THE PRESENT RECOMMENDATIONS

With the goal clearly in mind, the Panel has turned to the urgent practical task of recommending specific underg aduate curricula at those schools where it is unlikely to be achieved quickly. The present recommendations are first steps toward the goal.

The lower division (first two years). Another subcommittee of CUPM, the Subcommittee on a General Curriculum in Mathematics for Colleges (GCMC), has recently drawn up a program in mathematics designed for almost all students who require training in mathematics. This program was developed with full realization of the great diversity of problems in the various schools. It includes the standard basic subjects, but the order of topics is sometimes unconventional. Furthermore, the GCMC committee recommends attitudes toward the subject matter that are more consistent with current developments in mathematics than those that many students acquire today. The Panel feels that the first part of the GCMC program (Mathematics 1, 2, 3, 4, 5) can also serve well for the pregraduate training of mathematicians, preferably with some increase in rigor and depth. The Panel makes the following observations:

1. The first two years of the GCMC program include a thoughtful study of calculus through partial derivatives and multiple integrals, and of linear algebra through the elementary theory of vector spaces and linear transformations.

2. If all mathematics students follow the first two years of the GCMC program, those who decide relatively late on graduate work in mathematics will not lose very much, even though early work in the spirit of the 1963 recommendations is very desirable for the future professional mathematician.

3. The first two years of the GCMC program are within the reach of all schools, even if they have to offer a pre-calculus course as preparation.

The lower division of the GCMC program and its relation to pregraduate training are discussed in more detail later on.

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*A General Curriculum in Mathematics for Colleges, 1965. Distributed through CUPM, P. O. Box 1024, Berkeley, California 94701.

The upper division (last two years). Because of the heterogeneity of the mathematical world, the Panel recognizes that no single curriculum will work for all schools. We therefore recommend the following priorities.

1. A minimal upper division program for mathematics majors who intend to continue the study of mathematics in graduate school appears in the GCMC report (page 19). Any college not already offering a comparable program should take immediate steps to do so.

2. If, however, the college can also supply courses designed especially for the pregraduate student, then it should provide one, or if possible both, of the one-year sequences (analysis and algebra) described below. The choice, if only one can be given, should of course be determined by the staff's capabilities. The one-semester analysis course (Mathematics 11) of the GCMC recommendations is itself aimed primarily at the pregraduate student; it could be replaced by the one-year course (Mathematics 11-12). On the other hand, the one-semester algebra course (Mathematics 6) of the GCMC serves many purposes; it should be retained for these purposes and the one-year algebra course described below should be added for the pregraduate student.

The Panel has consulted many outstanding graduate mathematics departments; it turns out that a solid grounding in algebra and analysis is what they most want from incoming students. If time and resources permit, it is of course desirable to introduce the pregraduate student to a broader range of material, but not at the expense of depth in algebra and analysis.

Introducing the program. Many departments of mathematics can adopt the present recommendations now. Many departments, indeed, are already offering even more substantial programs; they are referred to the 1963 recommendations and urged to proceed as far and as fast as they can in the directions suggested there. We have no universal advice for departments that feel unable to go as far as the present recommendations now; but we encourage such departments to seek individual advice from CUPM, for example through its Consultants Bureau.

Objectives of the program. Our concern is with all prospective graduate mathematics students regardless of their destination in today's diversified mathematical profession. The mathematical involvement of the professional mathematician, heavily influenced by the rapid developments in computer science, continues to broaden and deepen in education, in industry and in government.

The subject matter recommended for the pregraduate program is discussed briefly above and in more detail below. This subject matter is, of course, very important; but equally important are the spirit and tone of the teaching, not only because they are reflected in the ultimate quality of the student's performance, but also because they can influence the student to decide for or against a career in mathematics.

The student should be introduced to the language of mathematics, in both its rigorous and idiomatic forms. He should learn to give clear explanations of some fundamental concepts, statements, and notations. He should develop facility with selected mathematical techniques, know proofs of a collection of basic theorems, and acquire experience in constructing proofs. He should appreciate the power of abstraction and of the axiomatic method. He should be aware of the applicability of mathematics and of the constructive interplay between mathematics and other disciplines. He should begin to read mathematical literature with understanding and enjoyment. He should learn from illustration and experience to cultivate curiosity and the habit of experimentation, to look beyond immediate objectives, and to make and test conjectures. In short, the student must be helped to mature mathematically as well as to acquire mathematical information.

There are many ways in which the student can be helped to mature mathematically. He can be taught in special "honors" classes for superior students. In the earlier stages he can be given independent reading assignments in textbooks, and later he can be assigned the more difficult task of reading papers in journals. He can be taught through "reading courses." He can make reports in seminars and colloquia. He can prepare an undergraduate thesis containing work original for him although not necessarily original in the larger sense. He can be taught through the "developmental course" in which he is led to develop a body of mathematical material under the guidance of the professor. <u>In general, the Panel feels very strongly that every pregraduate curriculum should include work to develop mathematical self-reliance, initiative and confidence</u>.

<u>Identifying students</u>. Far too few college students successfully complete a graduate program in mathematics. The Panel recommends strongly that every effort should be made to identify pregraduate mathematicians as early as possible, preferably when they enter college (a task often complicated by the students' own incorrect notions about their mathematical capabilities).

Lower division courses. As we have already said, the Panel regards the basic sequence Mathematics 1,2,3,4,5 of the GCMC recommendations as essential for the pregraduate student. We comment briefly on this program.

For the first semester of college-level mathematics, the GCMC presents a course dealing with the integral and differential calculus of the elementary functions, together with the associated analytic geometry. To meet the needs of the future graduate student most effectively, this course should be designed with three specific objectives in mind. First, the course should build a strong intuitive concept of limits based on concrete examples. These examples can be drawn from geometry, physics, biology, etc. This may be followed by setting down a strong enough axiom system about limits to encompass their elementary properties obtained intuitively. The student should be told which of his current axioms will be future theorems. The formal definition of a limit is too difficult to be swallowed whole by the student at this point: our greatest service to the student would be to give him a firm intuitive grasp of the concept.

The second objective of this course should be to improve the student's ability to handle mathematical rigor. This can be done, for example, by using the axioms about limits in a rigorous development of the calculus.

The third and final purpose of this course should be to teach the student to calculate. For certainly, it is the ability to calculate with the calculus that makes the calculus the powerful tool that it is.

In the next calculus courses, Mathematics 2 and 4, the GCMC is concerned, in part, with the possible introduction of a fair amount of multivariable calculus much earlier than is customary. This is less important for pregraduate mathematics students than for some other students, since pregraduate students will take all the courses.

The attitude in presenting the material is more important. After courses 1 or 3 there should be a gradual but considerable increase of mathematical maturity. Course 11-12 treats continuity, differentiation, and integration at the level of sophistication required in the theory of "real variables." Consequently, the study of these concepts in courses 2 and 4 should bring the student to an insight which makes the transition easier. There will be little in Mathematics 5 to help in this direction. Thus the student must be led in Mathematics 2 and 4 to a considerable appreciation of rigor and to the effective personal use of mathematical language.

The GCMC recommends an introductory course in probability, Mathematics 2P, for all students in their first two years. The present Panel feels that the student preparing for graduate work in mathematics might better be making faster progress toward upper division courses, deferring his work in probability.

Mathematics 3 is a short course in linear algebra. The Pregraduate Panel concurs with the GCMC committee in recommending that this course should come no later than the beginning of the second year.

Upper division courses. Mathematics 5 is material frequently presented as the second half of a course called "advanced calculus." Certainly the pregraduate student, whatever his branch of mathematical study, needs to acquire skill in the techniques and understanding of the concepts of mappings between Euclidean spaces of dimension at least 2 (i.e., systems of several functions of several variables).

After Mathematics 1,2,3,4,5, the Panel recommends for pregraduate students, for reasons discussed earlier, a year course in abstract algebra instead of the GCMC Mathematics 6, and a year course in real analysis instead of the GCMC Mathematics 11. Possible outlines for these courses are presented below to indicate the flavor and scope that the Panel considers desirable for the pregraduate student; the analysis course, which is the same as the GCMC Mathematics 11-12, is reproduced here for the reader's convenience.

We repeat that any department that can add more than these two courses should turn to the 1963 recommendations for further suggestions.

ABSTRACT ALGEBRA (COURSE OUTLINES)

The purpose of this year course is to introduce the student to the basic structures of abstract algebra and also to deepen and strengthen his knowledge of linear algebra. It provides an introduction to the applications of these concepts to various branches of mathematics. [Prerequisite: Mathematics 3.]

OUTLINE A

1. Groups. (10 lessons) Definition. Examples: Vector spaces, linear groups, additive group of reals, symmetric groups, cyclic groups, etc. Subgroups. Order of an element. Theorem: Every subgroup of a cyclic group is cyclic. Coset decomposition. Lagrange theorem on the order of a subgroup. Normal subgroups. Homomorphism and isomorphism. Linear transformations as examples. Determinant as homomorphism of GL(n) to the nonzero reals. Quotient groups. The first two isomorphism theorems. Linear algebra provides examples throughout this unit.

2. Further group theory. (10 lessons) The third isomorphism theorem. Definition of simple groups and composition series for finite groups. The Jordan-Hölder theorem. Definition of solvable groups. Simplicity of the alternating group for $n > 4$. Elements of theory of p-groups. Theorems: A p-group has nontrivial center; a p-group is solvable. Sylow theory. Sylow theorem on the existence of p-Sylow subgroups. Theorems: Every p-subgroup is

contained in a p-Sylow subgroup; all p-Sylow subgroups are conjugate and their number is congruent to 1 modulo p.

3. Rings. (10 lessons) Definition. Examples: Integers, polynomials over the reals, the rationals, the Gaussian integers, all linear transformations of a vector space, continuous functions on spaces. Zero divisors and inverses. Division rings and fields. Domains and their quotient fields. Examples: Construction of field of four elements, embedding of complex numbers in 2 x 2 real matrices, quaternions. Homomorphism and isomorphism of rings. Ideals. Congruences in the ring of integers. Tests for divisibility by 3, 11, etc., leading up to Fermat's little theorem, $a^{p-1} \equiv 1 \pmod p$, and such problems as showing that $2^{32} + 1 \equiv 0 \pmod{641}$. Residue class rings. The homomorphism theorems for rings.

4. Further linear algebra. (continuing Mathematics 3) (12 lessons) Definition of vector space over an arbitrary field. (Point out that the first part of Mathematics 3 carries over verbatim and use the opportunity for some review of Mathematics 3.) Review of spectral theorem from Mathematics 3 stated in a more sophisticated form (e.g., as in reference [4]). Dual-space adjoint of a linear transformation, dual bases, transpose of a matrix. Theorem: Finite-dimensional vector spaces are reflexive. Equivalence of bilinear forms and homomorphism of a space into its dual. General theory of quadratic and skew-symmetric forms over fields of characteristic not two. The

canonical forms. (Emphasize the connections with corresponding material in Mathematics 3.) The exterior algebra defined in terms of a basis -- two-and three-dimensional cases first. The transformation of the p-vectors induced by a linear transformation of the vector space. Determinants redone this way.

5. Unique factorization domains. (12 lessons) Primes in a commutative ring. Examples where unique factorization fails, say, in $Z[\sqrt{-5}]$. Definition of Euclidean ring, regarded as a device to unify the discussion for Z and $F[x]$, F a field. Division algorithm and Euclidean algorithm in a Euclidean ring; greatest common divisor; Theorem: If a prime divides a product it divides at least one factor; unique factorization in a Euclidean ring. Theorem: A Euclidean ring is a principal ideal domain. Theorem: A principal ideal domain is a unique factorization domain. Gauss's lemma on the product of two primitive polynomials over a unique factorization domain; Theorem: If R is a unique factorization domain so is $R[x]$.

6. Modules over Euclidean rings. (14 lessons) Definition of module over an arbitrary ring viewed as a generalization of vector space. Examples: Vector space as a module over $F[x]$ with x acting like a linear transformation. Module homomorphism. Cyclic and free modules. Theorem: Any module is a homomorphic image of a free module. Theorem: If R is Euclidean, A an $n \times n$ matrix over R, then by elementary row and column transformations A can be

diagonalized so that diagonal elements divide properly. Theorem: Every finitely generated module over a Euclidean ring is the direct sum of cyclic modules. Uniqueness of this decomposition, decomposition into primary components, invariant factors and elementary divisors. Application to the module of a linear transformation, leading to the rational and Jordan canonical forms of the matrix. Several examples worked in detail. Similarity invariants of matrices. Characteristic and minimal polynomials. Hamilton-Cayley Theorem: A square matrix satisfies its characteristic equation. Application of module theorem to the integers to obtain the fundamental theorem of finitely generated abelian groups.

7. Fields. (10 lessons) Prime fields and characteristic. Extension fields. Algebraic extensions. Structure of $F(a)$, F a field, a an algebraic element of some extension field. Direct proof that if a has degree n, the set of polynomials of degree $n-1$ in a is a field, demonstration that $F(a) \cong F[x]/(f(x))$, where f is the minimum polynomial of a. Definition of $(K:F)$, where K is an extension field of F. If $F \subset K \subset L$ and $(L:F)$ is finite, then $(L:F) = (L:K)(K:F)$. Ruler-and-compass constructions. Impossibility of trisecting the angle, duplicating the cube, squaring the circle (assuming π transcendental). Existence and uniqueness of splitting fields for equations. Theory of finite fields.

OUTLINE B (Including Galois theory).

A course culminating in and climaxed by Galois theory can be constructed by compressing the topics in Outline A into somewhat less time and adding material at the end. Outline B suggests such a course. The appeal of Galois theory as a part of a year course in algebra is obvious. The material ties together practically all the algebraic concepts studied earlier and establishes a clear connection between modern abstraction and a very concrete classical problem. The cost is equally obvious; the depth of much of the earlier material must be reduced, or several topics eliminated. Whether the advantages justify the cost is debatable.

Recognizing that there is merit on both sides, the Panel offers Outline B as an alternative to Outline A with the following words of caution and explanation:

(a) For most pregraduate students today, Outline A probably represents the better balance between coverage and pace.

(b) Outline B contains all the material in Outline A plus Galois theory. Thus Outline B should be attempted only where more time is available or the students are clearly capable of an accelerated pace. Accordingly, each unit in this outline is assigned a range of suggested times, the extremes representing these two alternatives.

(c) In order to break up the rather substantial concentration on group theory at the beginning of Outline A, some of this material has been moved to a position near the end of Outline B, where it fits naturally with the Galois theory. This same shift may be made in Outline A by taking the units in the order 1, 3, 4, 5, 6, 7, 2.

1. Groups. (6-10 lessons) (Outline A, unit 1)

2. Rings. (7-10 lessons) (Outline A, unit 3)

3. Further linear algebra. (10-12 lessons) (Outline A, unit 4)

4. Unique factorization domains. (10-12 lessons) (Outline A, unit 5)

5. Modules over Euclidean rings. (12-14 lessons) (Outline A, unit 6)

6. Fields. (7-10 lessons) (Outline A, unit 7)

7. Galois theory. (8-10 lessons) Automorphisms of fields. Fixed fields. Definition of Galois group. Definition of Galois extension. Fundamental theorem of Galois theory. Separability. Equivalence of Galois extension and normal separable extension. Computation of Galois groups of equations. Existence of Galois extensions with the symmetric group as Galois group. Theorem on the primitive element: If $(L:F)$ is finite and there exist only a finite number of intermediate fields then $L = F(a)$ for some $a \in F$.

8. Further group theory. (9-10 lessons) (Outline A, unit 2)

9. Galois theory continued. (8-10 lessons) Hilbert's theorem 90: If L is finite and cyclic over F, g is a generator of the Galois group, and x is an element of L of norm 1 over F, then

$x = y\,(yg)^{-1}$ for some $y \in L$. Also the additive form of Hilbert's theorem 90. Galois groups of $x^n - a$. Definition of solvability by radicals. Theorem: An equation is solvable by radicals if and only if its Galois group is solvable. The unsolvability of the general equation of degree n, $n \geq 5$. Other examples. Roots of unity and cyclotomic fields.

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REAL ANALYSIS (COURSE OUTLINE)

The purpose of this course is to introduce the student to the concepts and techniques of "real variable theory," laying the foundations for his later study of complex analysis and Lebesgue measure and integration. The level of abstractness is increased, but not overwhelmingly so, by the systematic use of metric spaces, even though Euclidean spaces are still the principal objects of study. [Prerequisite: Mathematics 4.]

First semester - 39 lessons.

a. Real numbers. (6 lessons) The integers; induction. The rational numbers; order structure, Dedekind cuts. The reals defined as a Dedekind-complete field. Outline of the Dedekind construction. Least upper bound property. Nested interval property. Denseness of the rationals. Archimedean property. Inequalities ([7]is a good source of problems). The extended real number system.

b. Complex numbers. (3 lessons) The complex numbers introduced as ordered pairs of reals; their arithmetic and geometry. Statement of algebraic completeness. Schwarz inequality.

c. Set theory. (4 lessons) Basic notation and terminology: membership, inclusion, union and intersection, cartesian product, relation, function, sequence, equivalence relation, etc.; arbitrary unions and intersections. Countability of the rationals; uncountability of the reals.

d. Metric spaces. (6 lessons) Basic definitions: metric, ball, boundedness, neighborhood, open set, closed set, interior, boundary, accumulation point, etc. Unions and intersections of open or closed sets. Subspaces. Compactness. Connectedness. Convergent sequence, subsequence, uniqueness of limit. A point of accumulation of a set is a limit of a sequence of points of the set. Cauchy sequence. Completeness.

e. Euclidean spaces. (6 lessons) R^n as a normed vector space over R. Completeness. Countable base for the topology. Bolzano-Weierstrass and Heine-Borel-Lebesgue theorems. Topology of the line. The open sets; the connected sets. The Cantor set. Outline of the Cauchy construction of R. Infinite decimals.

f. Continuity. (8 lessons) (Functions into a metric space) Limit at a point, continuity at a point. Continuity; inverses of open sets, inverses of closed sets. Continuous images of compact sets are compact. Continuous images of connected sets are connected. Uniform continuity; a continuous function on a compact set is uniformly continuous. (Functions into R) Algebra of continuous functions. A continuous function on a compact set attains its maximum. Intermediate value theorem. Kinds of discontinuities.

g. Differentiation. (6 lessons) (Functions into R) The derivative. Algebra of differentiable functions. Chain rule. Sign of the derivative. Mean value theorems. The intermediate value theorem for derivatives.

L'Hospital's rule. Taylor's theorem with remainder. One-sided derivatives; infinite derivatives. (This material will be relatively familiar to the student from his calculus course, so it can be covered rather quickly.)

Second semester - 39 lessons

h. The Riemann-Stieltjes integral. (11 lessons) [Alternative: the Riemann integral.] Upper and lower Riemann integrals. [Existence of the Riemann integral: for f continuous; for f monotonic.] Monotonic functions and functions of bounded variation. Riemann-Stieltjes integrals. Existence of $\int_a^b f d\alpha$ for f continuous and α of bounded variation. Reduction to the Riemann integral in case α has a continuous derivative. Linearity of the integral. The integral as a limit of sums. Integration by parts. Change of variable. Mean value theorems. The integral as a function of its upper limit. The fundamental theorem of the calculus. Improper integrals. The gamma function ([11], 367-378; [10], 285-297).

i. Series of numbers. (11 lessons) (Complex) Convergent series. Tests for convergence (root, ratio, integral, Dirichlet, Abel). Absolute and conditional convergence. Multiplication of series. (Real) Monotone sequences; lim sup and lim inf of a sequence. Series of positive terms; the number e. Stirling's formula, Euler's constant ([11], 383-388). Again, see [7] for problems.

j. Series of functions. (7 lessons) (Complex) Uniform convergence; continuity of uniform limit of continuous functions. Equicontinuity; equicontinuity on compact sets. (Real) Integration term by term. Differentiation term by term. Weierstrass approximation theorem. Nowhere-differentiable continuous functions.

k. Series expansions. (10 lessons) Power series, interval of convergence, real analytic functions, Taylor's theorem. Taylor expansions for exponential, logarithmic, and trigonometric functions. Fourier series: orthonormal systems, mean square approximation, Bessel's inequality, Dirichlet kernel, Fejér kernel, localization theorem, Fejér's theorem, Parseval's theorem.

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