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By-Buckland, M.K.; Woodburn, I.

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A research project is being conducted to construct a mathematical model of the operations of an academic library to be used in making managerial decisions. As part of this project, this report examines Bradford's Law of Scattering and the fall-off of use of documents as they age. A series of mathematical analyses indicate how these two laws can be used together to indicate optimal decisions in the management of collections of journals. These decisions include the number of titles to be taken, the length of time retained, and the choice of binding policies. Imaginary petroleum libraries in various circumstances are used to illustrate the conclusions. (Author/JB)

University of Lancaster Library Occasional Papers, no. 1

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PREFACE

This research report is the first of a series of papers which the Library intends to publish at irregular intervals. It presents, in preliminary form, one aspect of a research project which has been supported since January 1967 by the Office for Scientific and Technical Information; this project is designed to construct a mathematical model of the operation of an academic library, in order that managerial decisions may be made on a rational rather than on an intuitional basis.

The research team comprises Mr. Ian Woodburn, M.A., and Mr. Michael K. Buckland, B.A., A.L.A.. Further reports will be issued in the coming months.

A. Graham Mackenzie

Librarian and Principal Investigator

May 1968

"The law deduced conforms to the mathematician's criterion of being no possible practical use whatever."

Bradford on his Law of scattering, 1946[1].

Summary: Bradford's Law of scattering and the fall-off of use of documents as they age are briefly described. A series of mathematical analyses follows which indicate how these two laws can be used together to indicate optimal decisions in the management of collections of journals. These decisions include the number of titles to be taken, the length of time retained and the choice of binding policies. Imaginary petroleum libraries in various circumstances are used to illustrate the conclusions. The assumptions are then reviewed.

1. INTRODUCTION

Library management is a subjective matter and decisions are taken in the light of the librarian's understanding of the facts and his experience - modified no doubt by the opinions and experiences of his staff and users. At the same time there are a number of statistical regularities in the way in which people use libraries. The purpose of this paper is to examine how the librarian can use two of the best known of these regularities to help his understanding of the facts when decisions about the management of his library need to be taken.

1.1 BRADFORD'S LAW OF SCATTERING

Bradford's Law of scattering, which was developed as long ago as 1934[2], is essentially a law of diminishing returns in the use made of scientific serials. Although work in the field has concentrated on pure and applied sciences, there appears no reason to suppose that the use of serials in the social sciences and humanities would not follow the same law. Various papers have been written on Bradford's Law of scattering and the interested reader should refer to them (e.g. refs. 1 - 7). The investigators have used a recently presented formula in order to demonstrate how a law of diminishing returns could be used. This formula, given by Leimkuhler[7] states, with slightly different notation, that of if R_N references on a given subject are derived from N journal titles, then the n most productive of these journals would yield R_n references, where

$$R_n = R_N \frac{\log(1 + \beta n/N)}{\log(1 + \beta)}$$

or $R_n = \alpha R_N \log(1 + \beta n/N)$

where $\alpha = 1/\log(1+\beta)$

and β is a constant characteristic of the subject field and the logarithm is to base e. This implies that the nth most productive title yields r_n references where

$$r_1 = R_1$$

$$r_n = R_n - R_{n-1} \qquad n > 1$$

The investigators would stress that the analyses which follow presuppose a law of diminishing returns of some kind, but any other such formula could have been used instead. Further, although the conclusions of the present report are only valid for collections of journals, the investigators believe that if more were known about the use of monographs, then collections of monographs could very likely be treated in a similar manner.

1.2 OBSOLESCENCE

The fall-off of use of documents as they age is even better known than Bradford's Law of scattering. Again the investigators referred to a convenient review article, by Cole[8], and took the first formula given in order to develop their ideas. Equally they would stress that any other formula could have been used.[9] The one used states that if $r_n(x)$ is the number of r_n references which are older than x years, then

$$r_n(x) = r_n\, e^{-\lambda x}$$

where λ is a constant characteristic of the subject concerned.

2. ANALYSES WHICH ASSUME THAT ALL TITLES ARE RETAINED FOR THE SAME LENGTH OF TIME

2.1 POTENTIALLY MOST USEFUL STOCK PATTERN

If we assume that a library can accommodate M volumes, then how many titles n, retained for x years, would give the most useful service? It is assumed that all titles are kept for the same length of time before being discarded (x years); and the definition of "most useful" (which we retain throughout the present report) is that "of maximal immediate availability" - i.e. the stock which meets the largest amount of the demand falling upon the library. The problems caused by lending and the effect of duplication are not considered in this report.

Demand will be characterised by R_N references to N journals. If all N titles are acquired and retained for ever then the total unsatisfied demand is zero. Otherwise the unsatisfied demand, U, is made up of two components:

(i) the journals which are not taken; $R_N - R_n$.

(ii) the parts of journals which are taken but which have been discarded at an age of x years: $R_n e^{-\lambda x}$.

Therefore $U = (R_N - R_n) + R_n e^{-\lambda x}$

Now if we assume that the n titles are the n most productive of the total of N, then

$$R_n = \alpha R_N \log(1 + \beta n/N)$$

where $\alpha = 1/\log(1 + \beta)$

so that $U = (R_N - R_n) + R_n e^{-\lambda x}$

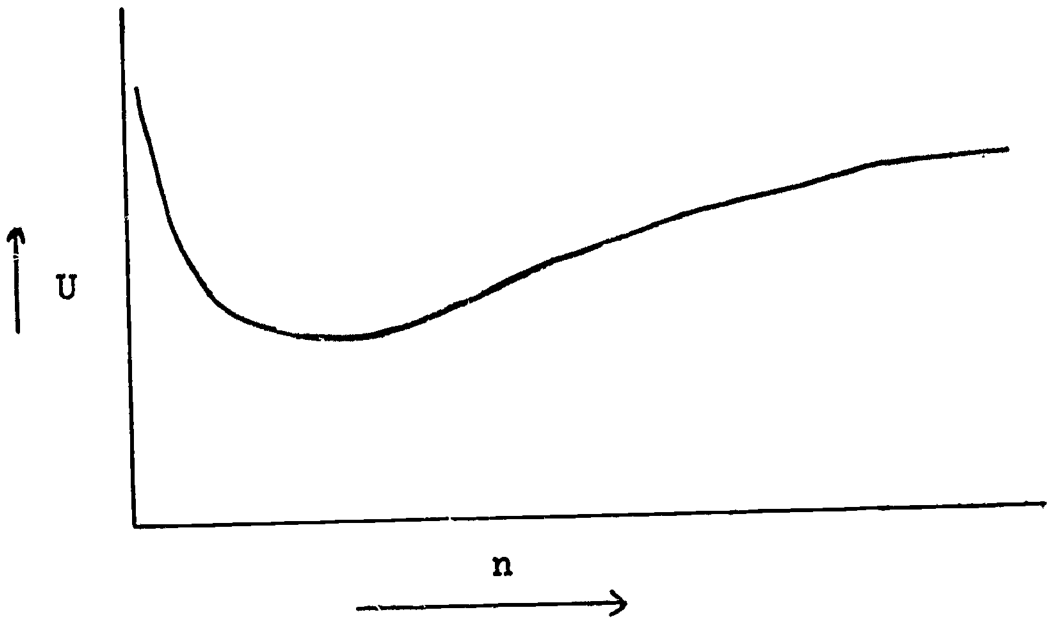
$$= R_N + R_n (e^{-\lambda x} - 1)$$

$$= R_N \{ 1 + \alpha(e^{-\lambda x} - 1) \log (1 + \beta n/N)\}$$

Now M = nx

so that $U = R_N \{ 1 + \alpha(e^{-\lambda M/n} - 1) \log (1 + \beta n/N)\}$

This function has the following shape:



When we select the value of n which corresponds to the minimal value of U, we have the most useful stock pattern.

This analysis uses fundamentally the same approach as that of Cole[8], who produces data on scattering and obsolescence in the field of petroleum. He examines the imaginary case of a petroleum library which can accommodate about 2,000 title/years and receives 2,000 requests a year. He concludes that 190 titles, all retained for 11 years would constitute the most useful stock pattern and that this would satisfy about 75% of the requests. It should be added that in this particular case the results are not very sensitive to variations in the number of titles taken. Sixty more, or sixty fewer titles, with a corresponding adjustment to the retention period, would make little difference to the number of requests satisfied.

2.2 BEST VALUE FOR MONEY

In the previous analysis, the aim was to establish the best use of limited space: the M most useful volumes. A more practical question is how to make the best use of a limited amount of money. This is a different problem and consequently has a rather different answer.

We assume a budget of £B per annum which must pay for:

(i) Acquisitions

(ii) Storage (in the form of rent - or rent equivalent as interest on capital investment-, light, heat and other overheads).

How many titles, n, retained for x years would give best value for money? What is the best allocation of the budget between acquisitions and storage?

Let c_1 be the average purchase cost per title per annum and c_3 be the average storage cost per volume per annum. For present purposes a volume is defined as one title/year. Since each of the n titles is to be retained for x years

$$B = n(c_1 + c_3 x)$$

$$\therefore n = \frac{B}{c_1 + c_3 x}$$

Of the total demand of R_N references, we know that the number of references in the n most productive titles is R_n where

$$R_n = \alpha R_N \log (1 + \beta n/N)$$

Since $R_n e^{-\lambda x}$ references occur after the n titles have been discarded at age x, the usefulness of the collection will be

$$R_n - R_n e^{-\lambda x} = R_n(1 - e^{-\lambda x})$$
$$= \alpha R_N(1 - e^{-\lambda x}) \log (1 + \beta n/N)$$

Substituting for n we seek to maximise

$$\alpha R_N(1 - e^{-\lambda x}) \log (1 + \beta B / (c_1 + c_3 x) N)$$

with respect to x. The resultant values of x and n give the best policies and the effect of variations in the size of budget B can readily be calculated.

We can conveniently illustrate this analysis by calculating optimal policies for imaginary petroleum libraries. The data used by Cole, who changed his formulae to base 10, imply that when we work with Leimkuhler's formula, the use made of a petroleum library is defined when $\beta = 256$ (when $R_N = 2{,}000$ and $N = 490$) and $\lambda = 0.2303$. We show below the results for two imaginary budgets for two imaginary petroleum libraries: one in central London, where storage costs are very high and one deep in the country where storage costs are very low. As in the previous analysis the results should not be regarded as more than indicative.

	City library	Rural library
Assumptions:		
Annual acquisitions costs	£5 per title	£5 per title
Annual storage costs	£0.125 per vol.	£0.033 per vol.
Requests received	2,000 per an.	2,000 per an.
Conclusions:		
Annual budget £1,000		
Titles taken	140	175
Retention period	18 years	22 years
Vols. in stock	2520	3850
Requests satisfied	76%	80%
Annual budget £1,500		
Titles taken	205	260
Retention period	18 years	23 years
Vols. in stock	3690	5980
Requests satisfied	83%	88%

2.3 OPTIMAL BINDING POLICIES

In this analysis we assume binding costs must be paid from the same budget as purchase and storage costs. The problem is to determine which combination of acquisition, binding and discarding policies will give the best value for any given budget. (This report is not concerned with loan policies or duplication and so the choice between part-binding and volume-binding is not considered; it is assumed that all titles are bound in the same manner.) Attention is concentrated on the two decisions: (i) When to bind; and (ii) How far it is worth paying extra for faster binding.

(i) At first sight there is a good case for delaying binding for a while until the drop in the rate of use with time makes the temporary absence of a volume from the shelves less inconvenient to the user. Indeed this would argue for indefinite postponement - or rather not binding at all! Until the penalty for not binding and more expecially the cost of delaying binding is better understood and measured, it does not seem possible to indicate mathematically whether material should or should not be bound - still less <u>when</u> it should be bound. In the following analysis it is assumed that, as a matter of policy, titles will be bound and that material will be sent for binding at an average of a years. The effects of choosing different values of a on the usefulness of the library can be calculated.

(ii) The time taken to bind material is defined as b years. This is the length of time that material is absent from the shelves. It is assumed that there is some choice in this matter and that although in general the cheapest binding rates will be chosen there is always the possibility of choosing to pay a little extra for a more rapid service.

How far would this be justifiable?

We proceed as in the previous analysis.

Let B be the annual budget

c_1 be the average purchase cost per title per annum,

c_2 be the average binding cost per title per annum,

(N.B. The value of c_2 will depend upon the choice of binding time b, although this need not be a continuous function), and

c_3 be the average storage cost per volume per annum.

Therefore since n titles are to spend b years at binding and be discarded after x years

$$B = n(c_1 + c_2(b) + c_3 . x)$$

$$\therefore \quad n = \frac{B}{c_1 + c_2(b) + c_3 . x}$$

Of the total demand of R_N references, we know that the number of references in the n most productive titles is R_n where

$$R_n = \alpha R_N \log (1 + \beta n/N)$$

The number of references satisfied before titles are sent to binding at age a is $R_n - R_n e^{-\lambda a}$. The number satisfied after a period of b years at binding will be $R_n e^{-\lambda(a+b)}$ less those lost by discarding at age x, which amount to $R_n e^{-\lambda x}$. The total usefulness of the collection will, therefore, be

$$R_n - R_n e^{-\lambda a} + R_n e^{-\lambda(a+b)} - R_n e^{-\lambda x} = R_n(1 - e^{-\lambda a} + e^{-\lambda(a+b)} - e^{-\lambda x})$$

Substituting for n and R_n, this becomes

$$\alpha R_N(1 - e^{-\lambda a} + e^{-\lambda(a+b)} - e^{-\lambda x}) \log (1+ \beta B/(c_1 + c_2(b)+c_3 x) N)$$

which when <u>maximised</u> with respect to b and x denotes the best combination of policies. The effect on maximised usefulness of variations in the size of the budget B and the time of binding a can be easily determined.

To illustrate this analysis we suppose that the librarians of our imaginary petroleum libraries have three options open to them.

Binder no. 1 charges on average 22/- per volume, but material is absent from the shelves for about three months.

Binder no. 2 charges on average 25/- per volume, but material is absent from the shelves for one month.

Binder no. 3 charges on average 30/- per volume but material is absent from the shelves for only about one week. (We assume one fiftieth of a year.)

We come to the same conclusions for both libraries for annual budgets of £1,000, £1,500 and £2,000. If the material is sent to binding at an average of two years or less then choice of binder no. 3 would, by a very narrow margin, result in the best library service even though the substantially higher cost of binding means that fewer titles can be bought. If, however, material is sent at an average age of five years then, by an even narrower margin, binder no. 2 becomes the best choice.

3 ANALYSES WHICH DO NOT ASSUME THAT ALL TITLES ARE RETAINED FOR THE SAME LENGTH OF TIME

Investigators in this area have generally tended to assume that all titles are to be retained for the same period of time (e.g. Cole[8], Hanson[10], Meadows[11], etc.). This assumption has the great virtue of simplicity but, unless we are to deny the existence of scattering and obsolescence, it must necessarily lead to less than optimal results. In the following analyses we determine an individual discarding age for each title: the more heavily used titles are kept longer than the less heavily used. It is assumed for simplicity that all titles in a given collection have the same obsolescence rate, but different obsolescence rates for different titles could be used if known.

3.1 POTENTIALLY MOST USEFUL STOCK PATTERN

If we assume that a library can accommodate only a limited number of volumes, what combination of acquisition and discarding policies would be most useful? How many titles should be purchased and for how long should each be retained?

We define r_n as the number of references to the nth title.

$$r_1 = R_1$$

$$r_n = R_n - R_{n-1} \qquad n > 1$$

We do not assume that all titles are retained for the same length of time and we define x_n as the age at which the nth title is discarded. The amount of the demand for the nth title which occurs after that title has been discarded is $r_n e^{-\lambda x_n}$ so that the total satisfied demand, S, is,

$$R_N - \sum_{n=1}^{n=N} r_n e^{-\lambda x_n}$$

Since we seek the most useful stock we seek to maximise S with respect to $x_1, x_2, x_3, \ldots x_N$. We define M as the number of volumes which the library can hold. The restriction is, therefore,

$$\sum_{n=1}^{n=N} x_n = M$$

(a) Retention for whole years only

If we consider the retention policy only in terms of whole years, then $x_1, x_2, x_3, \ldots x_N$ can only have integer values and a convenient approximation to the optimal solution can be derived as follows. Since the problem is to define the M most useful volumes, we would not wish to include a volume which satisfied, say, one request a year if it meant the exclusion of another volume which would have been used more than once a year. We should need, therefore, to observe the fall-off of use of each title and ensure that, at the discarding point of each, its usefulness was similar to that of the other titles. Otherwise the restriction on the number of volumes would mean that the over-prolonged retention of one title would cause the premature discarding of volumes which would have been more useful. In other words, the optimal solution is when the marginal utility of further retention is the same for all titles. We define V as the marginal rate of usefulness at which the titles are to be discarded, although since we are concerned with whole years, this can only be done approximately. The usefulness of the volume of the nth title which is x years old is

$$r_n e^{-\lambda(x-1)} - r_n e^{-\lambda x}$$

The optimal discarding ages $x_1, x_2, x_3, \ldots x_N$ will define the most useful volumes when for each title the age of discarding

x_n is the highest value for x for which

$$r_n\, e^{-\lambda(x-1)} - r_n\, e^{-\lambda x} > V$$

and the most useful M volumes are defined by the value of V which satisfies the condition that

$$\sum_{n=1}^{n=N} x_n = M$$

We can profitably compare analysis 2.1 with this analysis. Let us re-examine the case of a petroleum library with 2,000 volumes to satisfy 2,000 requests. We find that if we accept the restriction that all titles are to be accepted for the same length of time, then at best (with about 190 titles retained for about 11 years) we could expect to satisfy 75% of the requests. If, however, as in this analysis we can choose a different retention policy for each title, then by acquiring 420 titles with retention periods varying from 1 to 23 years we can satisfy no less than 80% of the requests with 2,000 volumes.

(b) <u>Unrestricted retention</u>

If we do not insist that only whole years are considered and $x_1, x_2, x_3, \ldots x_N$ are not integers but continuous variables then we can define more precisely the marginal utility of retaining each title.

$$S = R_N - \sum_{n=1}^{n=N} r_n\, e^{-\lambda x_n}$$

$$\therefore \frac{dS}{dx_n} = \lambda . r_n\, e^{-\lambda x_n}$$

Since the marginal utility is to be equalised for all titles then we may define a variable w such that

$$\frac{dS}{dx_n} = \lambda . r_n e^{-\lambda x_n} = w \qquad n = 1, 2, 3, \ldots N.$$

The optimal solution is achieved when the value of w is such that the restriction $\sum_{n=1}^{n=N} x_n = M$ is satisfied.

Alternatively we can define q as a Lagrangian multiplier and define the Lagrangian function

$$S' = S + q(\sum_{n=1}^{n=N} x_n - M)$$

then $\frac{dS'}{dx_n} = r_n e^{-\lambda x_n} . \lambda + q$

Therefore S is maximised when

$$r_n e^{-\lambda x_n} = - {}^{q}/_{\lambda} \qquad n= 1, 2, 3, \ldots N.$$

and $\sum_{n=1}^{n=N} x_n = M.$

Both of these methods imply that each title will be retained until an age at which the remaining demand for that title is the same as the remaining demand for any other title. This would not be true if the obsolescence rate had not been the same for each title.

3.2 THE BEST PURCHASING POLICY WHEN SETTING UP A NEW LIBRARY

How many titles and how long a back-set of each title should be purchased in order to establish a library which will satisfy a given percentage of demand at minimal cost? This analysis is very similar to the previous one, but introduces the concept of designing a collection to meet a specified percentage of demand instead of achieving a collection of a specified size. We assume in this analysis that the cost price per volume does not vary significantly between titles nor between years. It follows that the result would be both the most useful selection as well as the best value for money. If the costs did vary then a more complex analysis would be needed to indicate the selection which would give the best value for money. The problem is to determine the minimal number of volumes which will satisfy the requisite proportion of requests.

The total number of satisfied requests is S where

$$S = R_N - \sum_{n=1}^{n=N} r_n e^{-\lambda x_n}$$

and the collection is defined in terms of $x_1, x_2, x_3, \ldots x_N$. Again we seek to equalise the marginal usefulness of extending each back-set further in time. This time, however, the restriction is not that the collection must reach a specified size but that the collection must satisfy a specified proportion, μ, of the demand such that

$$S = \mu R_N.$$

Consequently we proceed as before seeking the value of V which will meet this restriction. Analysis 2.1 indicated that 2,000 volumes could at best satisfy 75% of demand if the restriction were accepted that all titles must be retained for the same length of time. This present analysis indicates that without this restriction, 75% of demand could be satisfied by 1400 volumes.

3.3 OPTIMAL LIBRARY SIZE AND MINIMAL COSTS

If requests for items not in stock are to be satisfied by interlibrary loans, then what combination of purchasing and discarding policies will minimise library costs? We do not assume that all titles must be retained for the same length of time.

There are two methods of satisfying requests:

(i) by the acquisition and storage of titles,

(ii) by interlibrary loan.

Let c_1 be the average purchase cost per title per annum

c_3 be the average storage cost per volume per annum

c_4 be the average cost per interlibrary loan.

F be the total overall cost

and m be the number of titles purchased.

The total overall cost F will comprise four parts: the sum of the purchase costs of the m titles purchased, the sum of the storage costs of the volumes purchased and not yet discarded at age x_n, the sum of the interlibrary loan costs for requests for discarded material and the sum of interlibrary loan costs for requests for titles not acquired at all.

$$F = \sum_{n=1}^{n=m} c_1 + \sum_{n=1}^{n=m} c_3 . x_n + \sum_{n=1}^{n=m} c_4 . r_n e^{-\lambda x_n} + \sum_{n=m+1}^{n=N} c_4 . r_n$$

The problem is to determine the values of x_1, x_2, x_3, ... x_N and m which minimise F.

If we consider the retention policy only in terms of whole years, then x_1, x_2, x_3, ... x_N can only have integer values and a convenient approximation can be achieved by examining each title and volume separately. We know the total number of requests for each title

$$r_1 = R_1$$

$$r_n = R_n - R_{n-1} \qquad n > 1$$

We can also estimate the number of requests likely to fall on each volume of each title. The volume of the nth title which is x_n years old is likely to be subject to

$$r_n e^{-\lambda(x_n-1)} - r_n e^{-\lambda x_n} \text{ requests.}$$

The cost of satisfying these requests by interlibrary loan would be

$$c_4(r_n e^{-\lambda(x_n-1)} - r_n e^{-\lambda x_n}).$$

It is clearly economical to retain any purchased title until the age at which its usefulness has dropped to the level at which the requests which still occur can be more cheaply satisfied by interlibrary loans than by continued storage. In other words the best x_n is the highest value of x_n for which

$$c_4(r_n e^{-\lambda(x_n-1)} - r_n e^{-\lambda x_n}) > c_3$$

However, in view of the cost of purchasing the title in the first place it might still not be worth having. The number of requests which it would satisfy before being discarded is

$$r_n - r_n e^{-\lambda x_n}$$

and it would only be worth having if the cost of satisfying these requests by interlibrary loan were more than the combined cost of purchase and storage whilst retained, i.e.

if $$c_4(r_n - r_n e^{-\lambda x_n}) > c_1 + c_3 . x_n .$$

The minimal cost occurs, then, when for each title we select the largest value of x_n which satisfies the condition

$$c_4(r_n e^{-\lambda(x_n-1)} - r_n e^{-\lambda x_n}) > c_3$$

and we require only such titles as satisfy the condition

$$c_4(r_n - r_n e^{-\lambda x_n}) \geq c_1 + c_3 x_n$$

The number of titles which satisfy this last condition is the optimal value of m. Having thus determined the optimal values of $x_1, x_2, x_3, \ldots x_N$ and m, we can calculate the costs, the size and other details of our imaginary libraries.

	City library	Rural library
Assumptions		
Acquisition cost	£5 per title	£5 per title
Storage cost	£0.125 per vol.	£0.03 per vol.
Interlibrary loan cost	£1 per loan	£1 per loan
Conclusions		
Titles taken	50	62
Retention range	11-24 yrs.	16-30 years
Total volumes	744	1,230
Overall cost (F)	£1160	£1095
Satisfaction from stock	58%	63%

It might well be decided as a matter of policy deliberately to choose a solution other than that indicated above. For example, to satisfy a larger proportion from stock. If we define V as the marginal rate of usefulness then we can substitute V for c_4 in the restrictions above and select a value for V which will result in the desired percentage of satisfaction from stock being achieved at minimal cost.

3.4 THE COST OF REDUCING DELAYS

In the previous analysis the aim was to minimise the cost of providing a library service to meet a specified level of demand and the choice between satisfaction from stock and satisfaction by interlibrary loan was solely on a basis of the cost to the library. No account was taken of the fact that there is a delay in satisfaction by interlibrary loan. On the other hand causing library users to wait is generally regarded as undesirable. The investigators are not aware of any method of objectively measuring the cost to be assigned to this delay; nevertheless the cost to the librarian of reducing the delay can be explored.

We define initially as constants

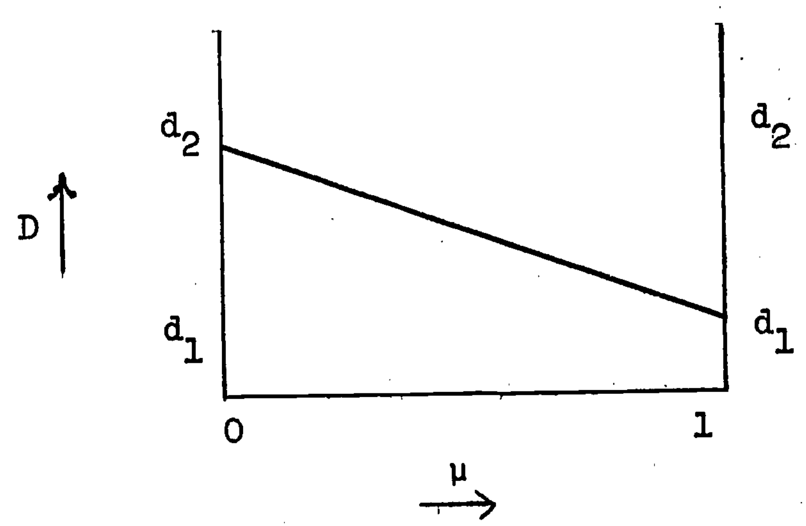
d_1 average delay in satisfying requests from stock

d_2 average delay in satisfying requests by interlibrary loan

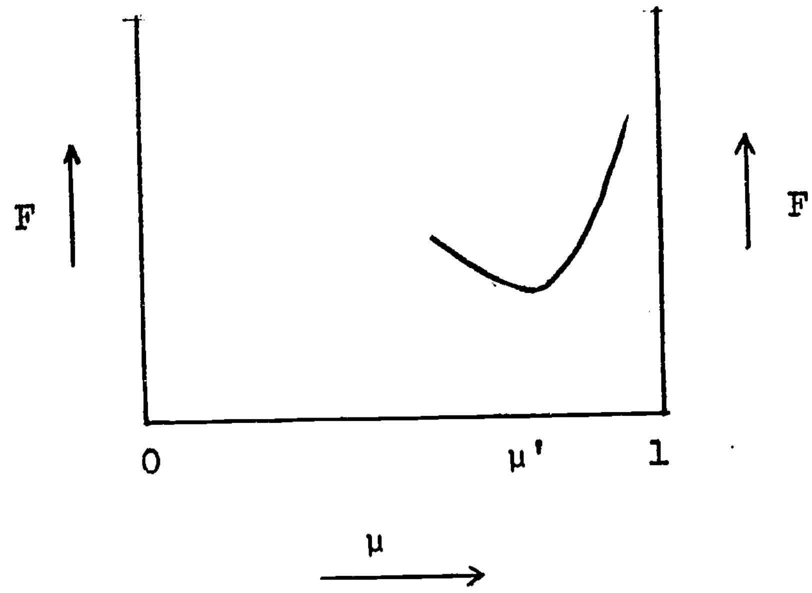
and as a variable D overall average delay. We assume that $d_1 < d_2$ and note that D depends upon the proportion μ of requests satisfied from stock

$$D = \mu . d_1 + (1 - \mu)\, d_2$$
$$= d_2 + \mu\,(d_1 - d_2)$$

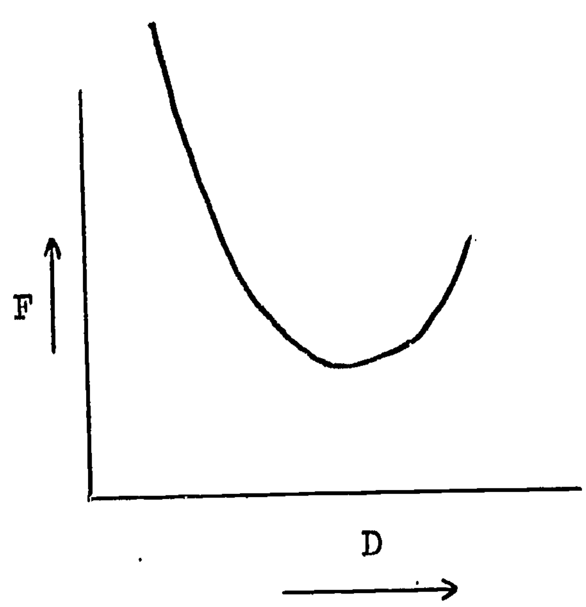
and since $d_1 < d_2$ and $0 \leq \mu \leq 1$ then $d_1 \leq D \leq d_2$ and $D \to d_1$ as $\mu \to 1$ we reduce D by increasing μ.



The previous analysis has shown that the cost, F, of supplying requests is related, under any given circumstances, to the proportion μ satisfied from stock. Assuming the optimal policies this can be calculated. As the next graph shows, F reaches a minimum when $\mu = \mu'$.



Hence if we increase the size of the collection so that the proportion of requests satisfied from stock μ increases from 0 to μ', both average delays and unit-costs are reduced. However if we continue to increase this proportion beyond μ' towards 1, then the continuing reduction in average delays is only achieved at the price of ever more rapidly increasing unit-costs. If objective data were available on the cost to be associated with various delays, then an optimal solution could be established. Until then we can only establish the effects on average delays and on unit costs of any choice of μ, either assuming optimal policies or for any given non-optimal combination of policies, and use the information derived to help a subjective choice.



It has hitherto been assumed that the average delays d_1 and d_2 are constants. Consequently only the effect of increasing library size was explored as a means of reducing the average delay. This is, of course, unrealistic. Even if interlibrary loans are being arranged as speedily as possible at any given unit cost, the delays can generally be reduced further at the cost of a rise in unit costs by use of telex, telephone or telefacsimile, by investing in better finding lists and union catalogues, by more or better staff, by investment in improved external lending facilities and so on; in the extreme case one could dispatch the enquirer by road, rail or air to another library holding the required material. It remains to be seen in any given situation how far these factors could reduce delays more economically than by increasing the size of the library.

4 REVIEWING THE ASSUMPTIONS

A large number of assumptions have been made during these analyses. Most of them were made for the sake of simplicity. The two fundamental assumptions are that there are two recognisable patterns in the demand for journal literature by workers in any field. More specifically it is assumed that the law of diminishing returns operates when the number of journals in a library is increased, and that there is an obsolescence effect in individual titles. These two assumptions appear to be universally accepted in the professional literature. The formulae used have been chosen somewhat arbitrarily, but any others could have been used in the same manner. (It may be noted that Cole[6] has examined the case when the titles concerned are not the most productive.)

In Section 2 the practice of other investigators has been followed in assuming that all titles are to be retained for the same length of time before discarding. This was not done because it is in itself a justifiable assumption; it does in fact lead to non-optimal results, but it does have the virtue of simplicity and of showing the <u>manner</u> in which calculations based on observable patterns of library use can be used to help make policy decisions. This assumption is relaxed in Section 3.

Apart from their productivity in terms of the law of scattering, it is assumed that titles do not differ significantly in various other respects: their obsolescence rates, their purchase price, their size and the cost of their binding. This has been done for the sake of simplicity and for ease of calculation. If data indicated that the journals concerned did differ significantly in one or more of these respects then these variations could easily be incorporated into the calculations; similarly the effects of having different binding policies for different titles could well be explored. As the assumptions are relaxed so the prove more convenient to use this type of analysis to compare various different policies rather than attempt to compute an optimal solution.

In Section 3.3 Optimal library size and minimal costs and section 3.4 The cost of reducing delays it was implicitly assumed that the amount of provision from stock would not affect the pattern of demand itself. Since physical accessibility is known to be a factor affecting the demand for library services, as Harris[12] and Rosenberg[13,14] have demonstrated, it is likely that even with excellent interlibrary loan facilities, items not in stock will seem less accessible to the user. Consequently users might tend not to request interlibrary loans even though they would have consulted the item had it been in stock and immediately available. Would this matter? Certainly it would tend to reduce costs. It is a pity that more is not known about the factors affecting the demand for library services.

It must be stressed that the present report is more in the nature of a working paper since there is a good deal of scope for refining and developing the analyses in various ways. (For example, the implications of having a library which serves users in more than one subject-field, of the budget being used for other purposes and of the cost of discarding not being neglible, are fruitful fields for further enquiry). The authors would welcome critical comments on the approach which has been adopted.

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