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Developed as part of a series of materials in the electrical engineering sequence developed under contract with the United States Office of Education, this laboratory manual provides nine laboratory projects suitable for a second course in electrical engineering. Dealing with resonant circuits, electrostatic fields, magnetic devices, and electronic devices, it involves the following projects: (1) electrostatic field plotting, (2) amplifier frequency response, (3) oscillator circuits, and (4) analog computer applications. There are also projects on the characteristics (1) triodes, (2) inductors, (3) transformers, and (4) transistors. (DH)

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EE 25 Laboratory Manual

Electrical Engineering Department
Syracuse University

Contract No. OE 4-10-102
U.S. Office of Education

ELECTRICAL LABORATORY II

EE 25 is a continuation of EE 24.

The student should refer to the appendix of EE 24 manual for such information as laboratory safety, characteristics of components and instruments, etc.

Work will be recorded in a bound notebook as in EE 24.

The projects will cover resonant circuits, electrostatic fields, magnetic devices, and electronic devices.

B. S.

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EE 25

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E.E. 25

Project III - 1

Electrostatic Field Plotting

I. Introduction

Frequently, problems in electrostatics involve conducting and dielectric bodies of complicated geometric configurations. Exact analytic solutions to these problems cannot be obtained always. Approximate graphical solutions to these problems can be obtained using the analogue device called field plotting.

By field plotting, usually, we obtain constant potential surfaces; i.e., we determine a family of surfaces in which each surface represents the set of all points in the region at the same (perhaps arbitrarily chosen) electrostatic potential.

It is known that the direction of the field intensity vector $\vec{E}$ at a point is normal to the equipotential surface containing that point (see page 3 - 16 of your notes, Introduction to Electrical Science).

Furthermore, because an electrostatic field is conservative, the potential ϕ and the field intensity $\vec{E}$ at a point are related (in rectangular coordinates) by the formula:

$$\vec{E} = -\left[\vec{u}_x \frac{\partial\phi}{\partial x} + \vec{u}_y \frac{\partial\phi}{\partial y} + \vec{u}_z \frac{\partial\phi}{\partial z} \right] = -\nabla'\phi \qquad (1)$$

where the vector operator $\nabla = \vec{u}_x \frac{\partial}{\partial x} + \vec{u}_y \frac{\partial}{\partial y} + \vec{u}_z \frac{\partial}{\partial z}$

and the minus sign appears because of the definition of $\vec{E}$ and ϕ.

Consequently, if a number of equi-potential surfaces in a region can be determined, along with the numerical value of the potential on the surfaces, the $\vec{E}$-field (i.e. the force on a unit positive charge) is also determined and the problem is solved.

See the attached appendix for an additional discussion of the relation between the potential, ϕ, and the electric field intensity, $\vec{E}$.

II. The Analogue Method

An experimental or analogue method for obtaining constant potential surfaces in an arbitrary two-dimensional electrostatic field will now be illustrated.

In fig. 1 are shown two conducting bodies in an isolated region at potentials ϕ_1 and ϕ_2, respectively. To determine the resulting field in this case, the apparatus of fig. 2 is used.

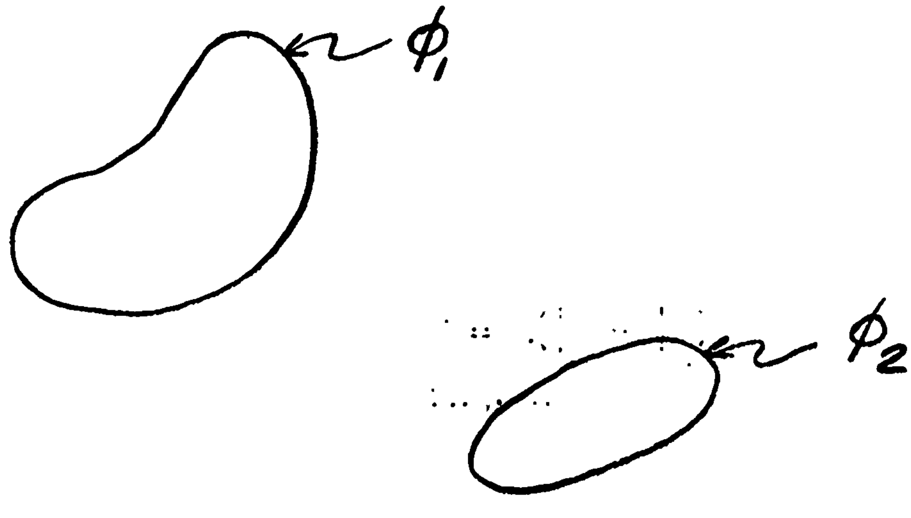
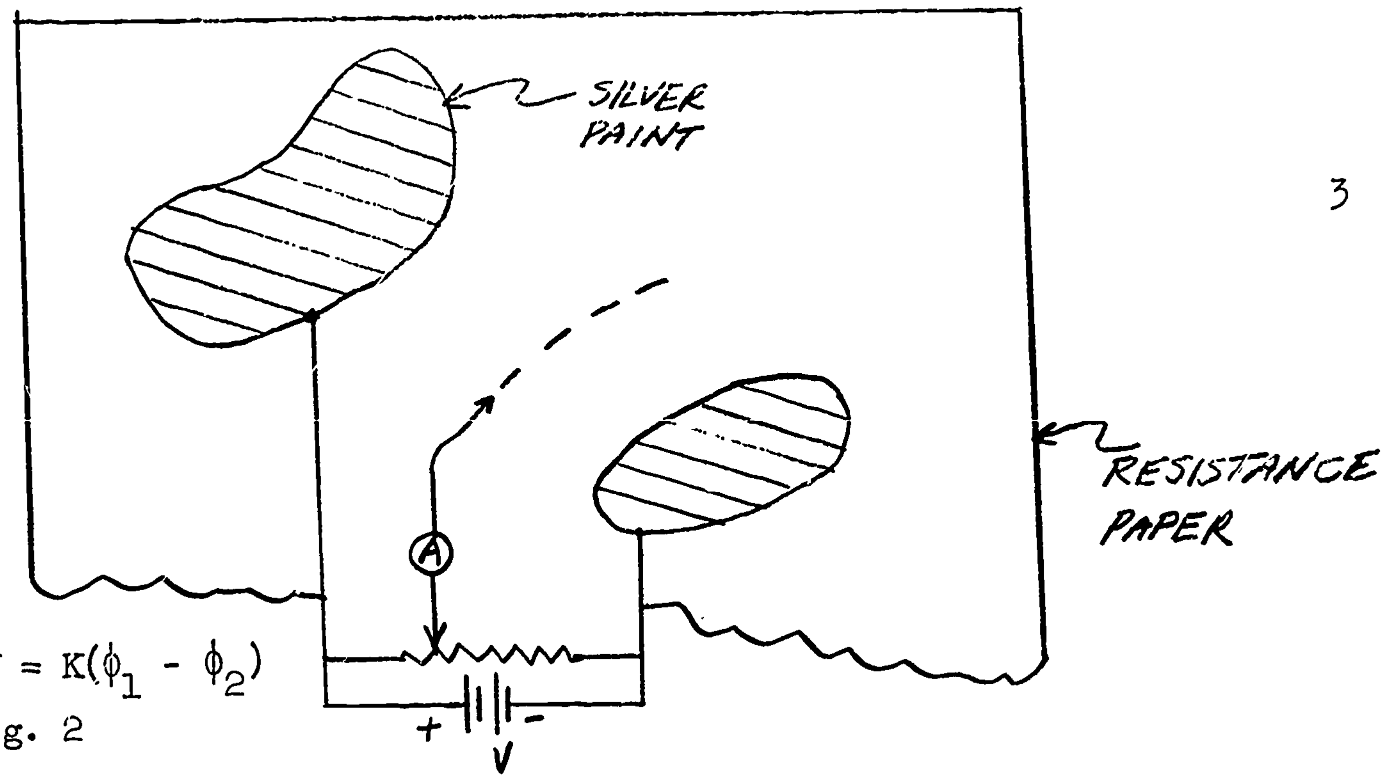


fig. 1



$V = K(\phi_1 - \phi_2)$

fig. 2

To simulate the conditions of fig. 1, a special resistance paper is used. On this, copies of the conducting bodies are painted using silver paint. A battery is connected between these two painted areas to represent the difference of potential $(\phi_1 - \phi_2)$. Across the battery is connected a calibrated potentiometer.

Connected to the variable arm of the potentiometer is an ammeter and a probe. If the probe is moved along a path on the resistance paper so that the ammeter reading is always zero, that path is the line of constant potential corresponding to the setting of the potentiometer.

Thus a field plot is obtained.

III Requirements

Several configurations of conducting bodies on resistance paper have been prepared. Obtain a plot of a family of equipotential surfaces for these configurations.

Two different configurations are shown in figures 3 and 4.

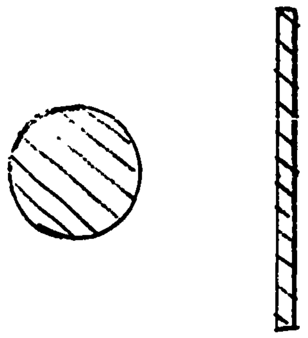


fig. 3

fig. 4

For the arrangement in fig. 3 obtain at least three equipotential lines between the two conducting bodies. For fig. 4, obtain seven.

Can an exact analytical expression be obtained for the equipotential surfaces. Give an intuitive explanation for their configuration. Can the plot obtained in fig. 3 be predicted from that obtained in fig. 4?

Obtain a field plot for the arrangement shown in fig. 5.

Can an exact analytical solution be found for these equipotential surfaces? Can you explain their configuration on an intuitive basis?

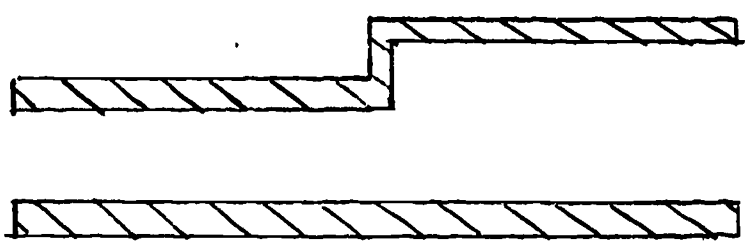


fig. 5

References: Electrical Engineering Science, Clement and Johnson, McGraw-Hill Co., Chap. I.

Introduction to Electromagnetic Engineering, Harrington, McGraw-Hill Co., Chap. 5.

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La Place's Equation in Rectangular Coordinates

The equation describing the behavior of the potential of an electrostatic field in a charge free region is called La Place's Equation. It can be derived by the application of Gauss' Law to an incremental volume.

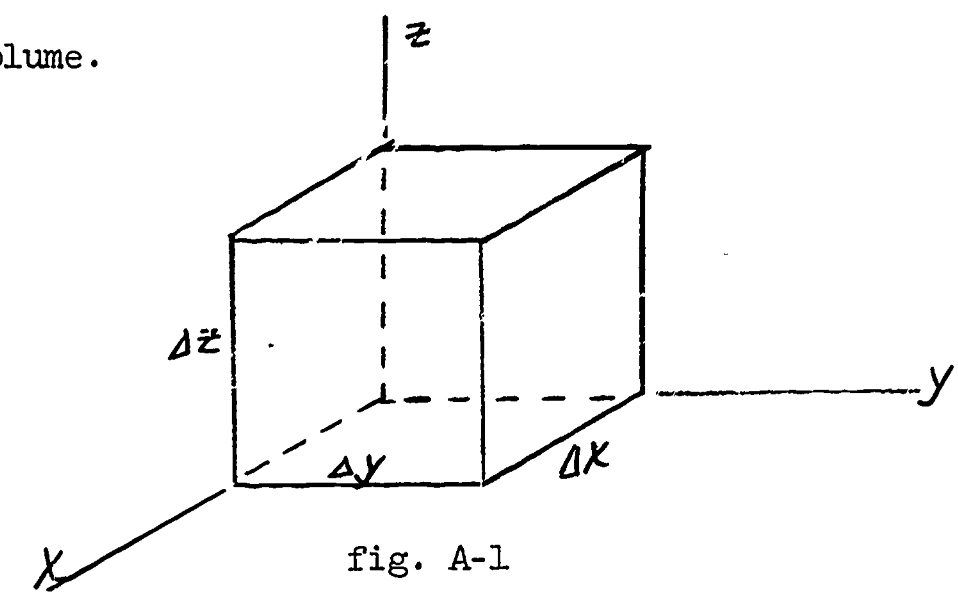

fig. A-1

Gauss' Law is given as equation (3-8) page (3-9) of your notes, Introduction to Electrical Science by LePage and Balabanian, as:

$$\oint\!\!\oint \vec{E} \cdot \vec{u}_n \, da = \frac{q}{\epsilon} \qquad (3\text{-}8)\ (1)$$

Let us compute the left-hand side of (3-8) for the incremental volume in rectangular coordinates shown in fig. 1.

$$\begin{aligned} &- E_x dydz + \left(E_x + \frac{\partial E_x}{\partial x} dx\right) dydz \\ &- E_y dxdz + \left(E_y + \frac{\partial E_y}{\partial y} dy\right) dxdz \\ &- E_z dxdy + \left(E_z + \frac{\partial E_z}{\partial z} dz\right) dxdy = \frac{\rho}{\epsilon_0} dxdydz \end{aligned} \qquad (2)$$

where: $q = \rho dxdydz$ and ρ is the charge density.

Equation (2) reduces to:

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\epsilon_o} \qquad (3)$$

But equation (3) may be written as

$$\left(\vec{u}_x \frac{\partial}{\partial x} + \vec{u}_y \frac{\partial}{\partial y} + \vec{u}_z \frac{\partial}{\partial z} \right) \cdot \left(\vec{u}_x E_x + \vec{u}_y E_y + \vec{u}_z E_z \right) = \frac{\rho}{\epsilon_o} \qquad (4)$$

if $\left(\vec{u}_x \frac{\partial}{\partial x} + \vec{u}_y \frac{\partial}{\partial y} + u_z \frac{\partial}{\partial z} \right)$ is treated as a vector operator. The symbol used for this vector operator is ∇, usually referred to as 'del'.

Equation (4) may be written as:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_o} \qquad (5)$$

But if the electrostatic potential at a point is ϕ_p, the electrostatic field at that point is given by:

$$\vec{E}_p = -\left(\vec{u}_x \frac{\partial \phi_p}{\partial x} + \vec{u}_y \frac{\partial \phi_p}{\partial y} + \vec{u}_z \frac{\partial \phi_p}{\partial z} \right) \qquad (6)$$

$$= - \nabla \phi$$

Note: $\vec{E}$ must be everywhere normal to equi-potential surfaces because no work is done when a test charge moved along such a surface.

Therefore, equ. (5) reduces to

$$\nabla \cdot \nabla \phi = \nabla^2 \phi$$

$$= \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{-\rho}{\epsilon_o} \qquad (7)$$

Equation (7) is called Poisson's Equation and when $\rho = 0$, it reduces to

$$\nabla^2\phi = 0 \tag{8}$$

which is called La Place's Equation.

Although the form of ∇^2 will be different in different coordinate systems, the fundamental relationships expressed by equs. (6), (7) and (8) are independent of the coordinate system. Furthermore, equations of this form describe any force field which is governed by an inverse square law (for example, the gravitational field: $F = G\frac{M_1M_2}{r^2}$.)

The simplest application of the above equation occurs probably in analysis of the field between two large parallel conducting plates connected by a battery. This is shown fig. 2. Fringing or edge effects are neglected. From symmetry E_x and $E_z = 0$. Therefore,

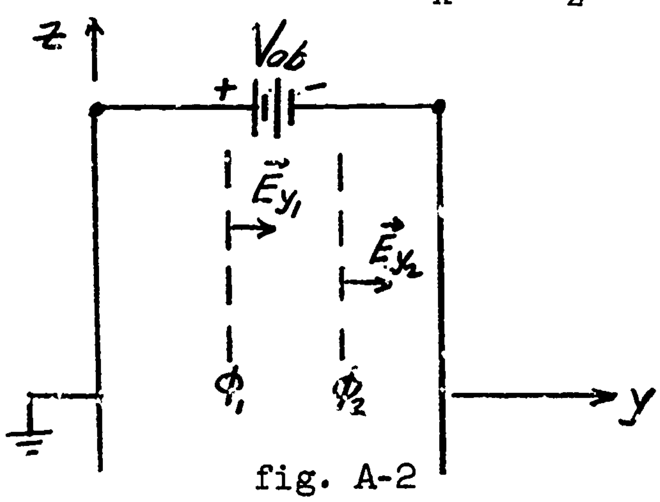

fig. A-2

equation (8) reduces to:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial y^2} = 0$$

Therefore:

$$\frac{d\phi}{dy} = K_1$$

and:

$$\phi = K_1 y + K_2.$$

To determine K_1 and K_2, apply the boundary conditions: At $y = 0 = a$, $\phi = 0$; therefore $K_2 = 0$. At $y = (b-a) = d$, $\phi = -V_{ab}$; therefore, $K_1 = \dfrac{-V_{ab}}{d}$.

Therefore:

$$\phi = \frac{-V_{ab}}{d} y. \qquad (a)$$

Note:

$$\vec{E} = -\nabla\phi = -\vec{u}_y \frac{\partial\phi}{\partial y} \qquad (b)$$

Therefore $\vec{E}$ is in the $\vec{u}_y$ direction; its magnitude is constant $= \dfrac{V_{ab}}{d}$. The equipotential surfaces are planes parallel to the conducting plates and $\phi_y = \dfrac{V_{ab}}{d} y$.

Thus equs. (a) and (b) completely describe the field.

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Characteristics of Inductors

I. Introduction

In electrical circuit design it is important to know how well available physical components approximate the models assumed in the original design.

For instance, in many control and computer applications the nonlinear properties of ferromagnetic inductors are used. On the other hand, linear inductors are needed in many filtering or frequency discriminating networks. However, often ferromagnetic materials are used in these latter cases to reduce the physical size of the components. This limits the range of the amplitude of applied signal for which a given component can be considered linear.

Furthermore, there is inherent distributed capacity among the coil windings of an inductor which (along with the inductance) gives the coil a self-resonant frequency. This restricts the useful frequency range for such an inductor (or coil).

In this project we wish to establish the useful amplitude and frequency ranges of an inductor having a powdered molybdenum permalloy core. At the same time, we will gain experience in additional measurement techniques.

II. Determination of Amplitude Range

One should recall that the following relations for a linear inductance (see sec. 4.7-4.8, pages 121-127, Electrical Engineering Science, Clements and Johnson)

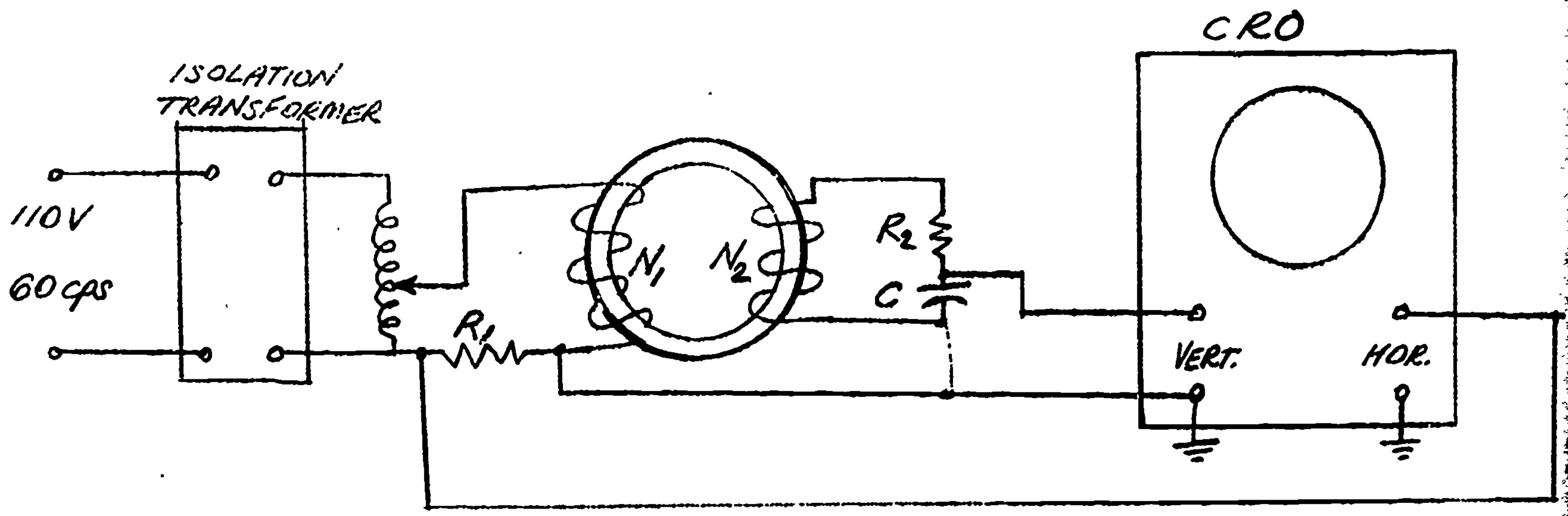
$$e = n\frac{d\phi}{dt} = L\frac{di}{dt}.$$

or:

$$L = N \frac{d\phi}{di} = N \frac{\Delta\phi}{\Delta i} = N \frac{\phi}{i}.$$

Therefore, the inductor will be linear (or constant) if the ratio of the magnetic flux threading its winding, ϕ, to the current flowing in that winding (which produces the flux), i, is constant (independent of the magnitude of the current and flux).

The ϕ vs. i curve (the hysterisis loop) can be observed utilizing the current shown below in Fig. 1.



R_1 = 10 ohms, 2 watts

R_2 = $(10)^6$ ohms

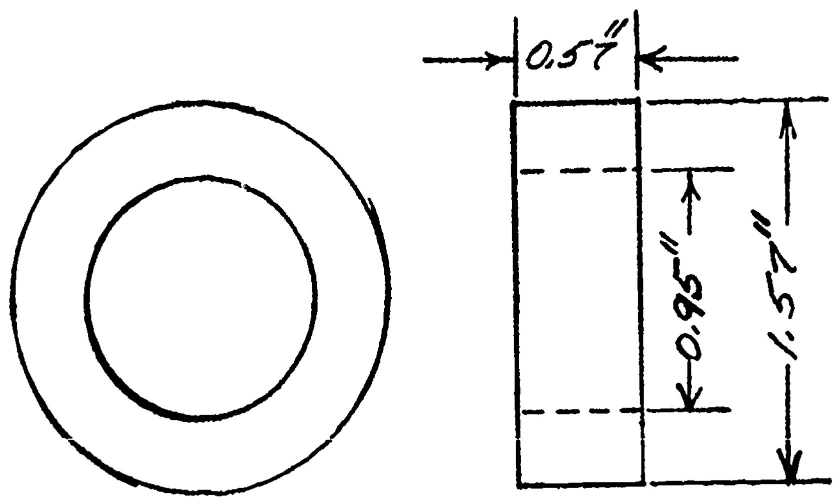
C = 0.1 microfarads

N_1 = 800 turns

N_2 = 200 turns

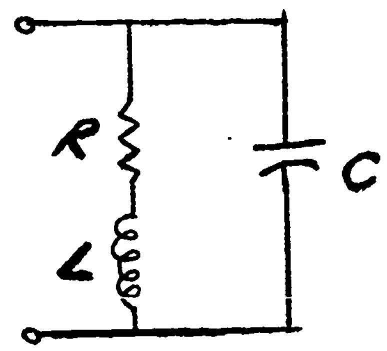
Figure 1.

The physical dimensions of the molybdenum permalloy core are shown in Figure 2.



Dimensions of Core

Fig. 2



Equivalent Circuit of Inductor

Fig. 3

Requirement: Determine the approximate range of linear operation of the inductor. State this range in terms of maximum allowable flux density (B, webers per square meter) and magnetic field intensity (H, amperes per meter) in the core.

III. Determination of Frequency Range

For our purposes the upper limit of the useful frequency range of the inductor will be somewhat less than its self-resonant frequency. This self-resonant frequency can be determined by measuring the impedance of the inductor versus frequency. (Note: The energy losses of the inductor will increase with an increase in frequency of applied signal and in some applications this will limit the useful frequency range of the inductor rather than its distributed capacitance.)

Assemble an impedance bridge using decade resistance, inductance, and capacitance boxes and determine the components in the equivalent circuit of the inductor shown in Fig. 3. If this assembly becomes unsatisfactory as the frequency of the applied signal increases, use a General Radio Co. bridge to complete your measurements.

If the values of components in the equivalent circuit vary with frequency, these values must be shown as graphs rather than as fixed quantities.

Note: The value of self-resonant frequency obtained by this method can be checked using Lissajous figures on the oscilloscope.

Requirement:

1. Determine the self-resonant frequency of the inductor.

2. Using the value of inductance found at a frequency 1000 cps, determine the permeability of the core material. Compare this with the value of $\mu = B/H$ obtained from the curve observed in the arrangement in Fig. 1. List possible causes of discrepancies.

IV. References

1. Introduction to Electrical Science, Clement and Johnson, paragraphs 3.8, 3.11, 4.7, 4.8.

2. Electronic Instrumentation, Prensky, Chapters 4 and 5.

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Oct. 15, 1963

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Project IV - 1

Transformer Characteristics

Transformers are used in many different applications in electrical engineering. In different applications, differ nt characteristics may be desirable.

Power Transformers

When transformers are used to change voltage and current levels in the transmission of large amounts of power, two characteristics are of interest:

1) Efficiency = $\frac{\text{Output Power (100)}}{\text{Input Power}} = \frac{\text{Output (100)}}{\text{Output + Losses}}$ (for rated full load conditions)

2) Regulation = $\frac{V_{out}\text{ (no load)} - V_{out}\text{ (full load)}}{V_{out}\text{ (full load)}}(100)$.

Frequently, facilities are not available in the laboratory to measure the performance of transformers under full-load conditions. Therefore, other tests are performed which allow the calculation of efficiency and regulation.

Efficiency Determination

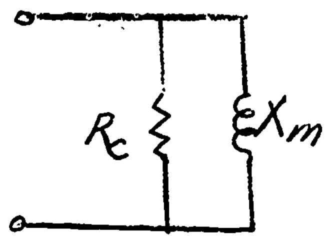
There are two major sources of power loss in an iron core transformer: 1) the core losses; and 2) winding losses.

The core losses do not vary appreciably with load. However, the winding losses equal the winding resistance times the current squared.

Therefore, by measuring the power supplied to the transformer when the secondary is an open-circuit (the primary current is small), the value of the core losses is obtained.

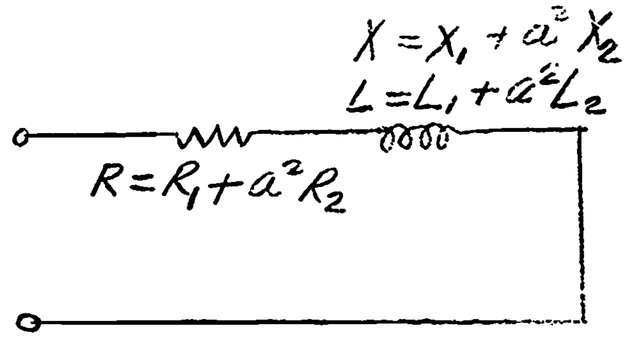
If we short-circuit the secondary and allow rated current to flow, the power dissipated is the winding loss, approximately. Note: The required input voltage for rated current under short-circuit conditions is small. Do not apply full line voltage.

The equivalent circuits representing the transformer under the two conditions of test are shown below:



Open circuit test

Fig. 1

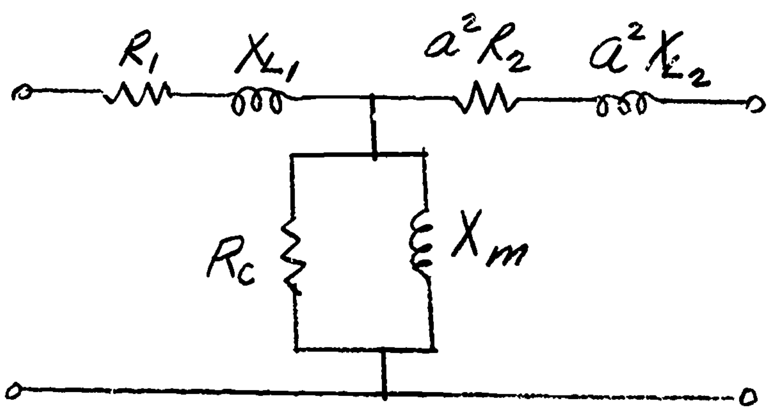


Short Circuit Test

Fig. 2

Equivalent circuit

An equivalent circuit for low frequencies and arbitrary load conditions is:



Equ. Circuit (Referred to Primary)

Fig. 3

R_1 = Resistance of primary winding

X_{L_1} = Leakage reactance of primary

a = Effective turns ratio

R_c = Equivalent core loss Resistor

X_m = Mutual Reactance

The elements of the equivalent circuit in Fig. 3 can be obtained from the open and short-circuit test. They may also be obtained by bridge measurements.

Regulation

The regulation may be calculated from the equivalent circuit of Fig. 3.

Problem

Determine the efficiency, regulation, and equivalent circuit of a laboratory filiament transformer. Determine the equivalent circuit from open and short circuit data (note: assume: $R_1 = a^2 R_2$, $X_{L_1} = a^2 X_{L_2}$) and bridge measurements.

References: Electrical Engineering Science, Clement and Johnson, Chapter 17.
Introduction to Electrical Science, LePage and Balabanian, Chapter 4.

Required Equipment

Filament Transformer

Variac

Isolation Transformer

Oscilloscope

Impedance Bridge

G.R. Type 1650

Graphical Determination of Average Power

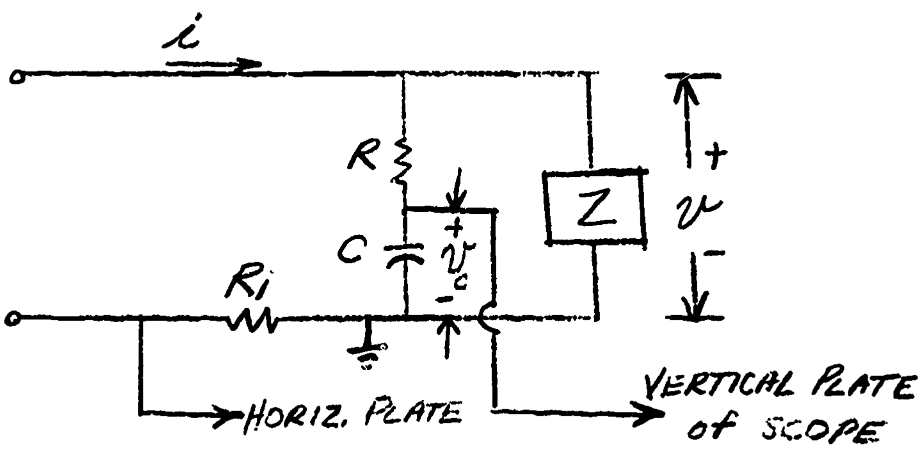


Fig. 1

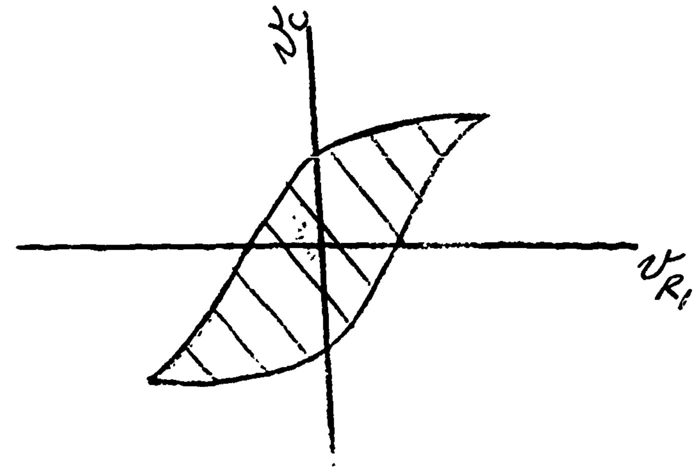


Fig. 2

(1) $P_{ave} = \frac{1}{T}\int_0^T v\, i\, dt$ where T is period of v and i.

It is required that: $RC \gg T$; $[R^2 + (\frac{1}{\omega C})^2]^{1/2} \gg |Z|$.

$$v_c = \frac{1}{c}\int \frac{v}{R}\, dt$$

(2) $v = R\, C \frac{d\, v_c}{d\, t}$

(3) $i = \frac{v_{R_1}}{R_1}$

$$P_{ave} = \frac{1}{T}\int_0^T v\, i\, dt = \frac{1}{T}\int_0^T RC \frac{d\, v_c}{dt} \frac{v_{R_1}}{R_1}\, dt$$

(4) $= \frac{1}{T} \frac{RC}{R_1} \int v_{R_1}\, d\, v_c$

The integral in eq. (4) is represented by the area in Fig. 2.

Thus, the average power dissipated is a two-terminal impedance Z can be determined by using the circuit shown in Fig. 1 and measuring the area in Fig. 2.

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Project I

VACUUM TUBE TRIODE CHARACTERISTICS

I. Introduction

In this project we wish to investigate some of the external characteristics of vacuum tube triodes. The study of the physical principles of operation and design will not be undertaken at this time.

The schematic representation of a triode is shown in Figure 1. The entire structure is contained in an evacuated envelope which is represented by the circle in Fig. 1. The heater maintains the cathode at a sufficiently high temperature so that electrons are emitted from the surface of the cathode under the influence of an electric field. The heater is not shown usually in circuit diagrams.

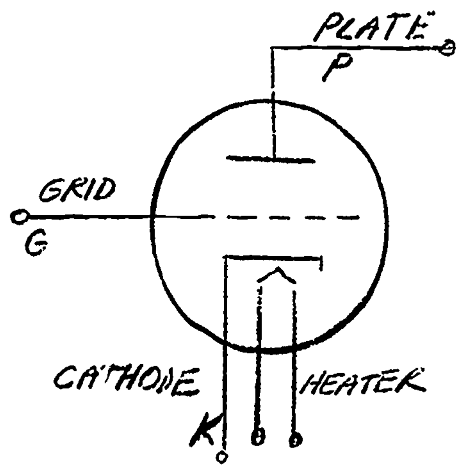


Fig. 1. Vacuum Tube Triode

The cathode, then, is the source of free electrons. Its surface is coated with a low work-function material (for receiving type tubes). In order not to damage the cathode surface, the heater must be functioning before the plate-to-cathode voltage is applied.

If the grid is ignored, the cathode and plate (or anode) form a vacuum tube diode. See Fig. 2. For instance, if the grid were connected directly to the plate, the triode would function as a diode. When the plate-to-cathode voltage is positive, some of the

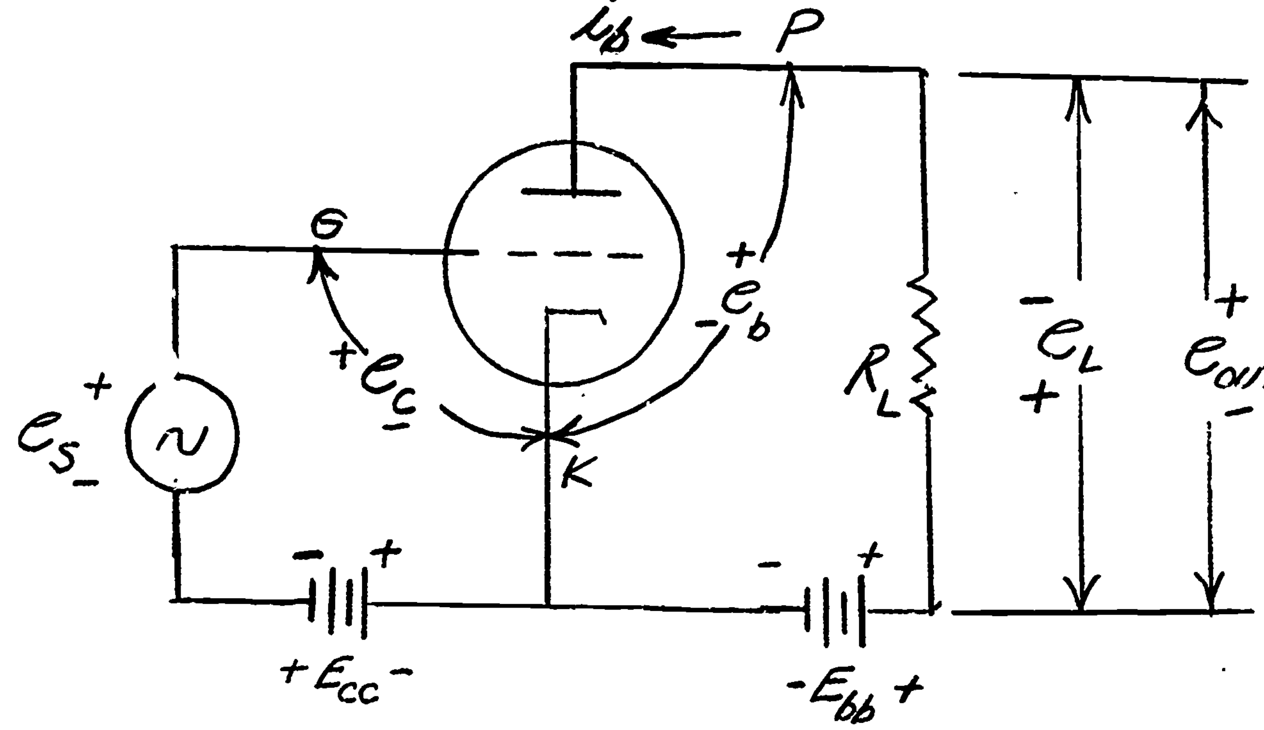


Figure 2. Elementary Triode Amplifier Circuit.

liberated cathode electrons readily traverse the tube causing a plate current. If the voltage polarity is reversed, current will not flow.

The grid is a fine-wire, wide-mesh screen structure located close physically to the cathode. Because of this closeness, a small difference of potential between grid and cathode (compared to plate-cathode potential) produces a relatively large electric field at the surface of the cathode. Thus, a small grid-to-cathode voltage results in a large change in plate current and load voltage (e_L). Consequently, the circuit of Fig. 2 exhibits the property of <u>power</u> gain; i.e., the signal power dissipated in load resistor is greater than the power supplied by the signal source, e_S. This additional power is supplied by the bias supply E_{bb}. The action of the triode converts this d. c. power to signal power.

In receiving tubes the grid can be damaged easily if it conducts appreciable current. In this work the grid will be maintained always at a lower potential than the cathode by the bias E_{cc} and the grid current will be zero.

It was pointed out that the bias supply E_{bb}, is a source of power. That portion of this power which is not dissipated in the load R_L is dissipated by the plate of the tube. This plate dissipation manifests itself in the form of heat and the maximum values of plate voltage and current specified by the tube manufacturer must be observed or the tube will be destroyed.

II. Graphical Representation; Plate characteristics

At this point, it is necessary to outline a method for quantitatively describing a triode. This will allow us to measure and evaluate its performance; to design amplifiers to meet stated specifications; and to analyze existing circuits.

The performance of a triode can be given by stating the relationship among its plate current, i_b, its plate-to-cathode voltage, e_b, and its grid-to-cathode voltage, e_c. It is assumed that the grid current is zero. Since the relationship is nonlinear over most of the allowed operating ranges, often this information is presented graphically. In order to present this data on a two-dimensional sheet, one variable is held constant and the other two are varied. The most frequently used data is the so-called plate characteristics, i_b vs. e_b, e_c = constant. (See Fig. 3.)

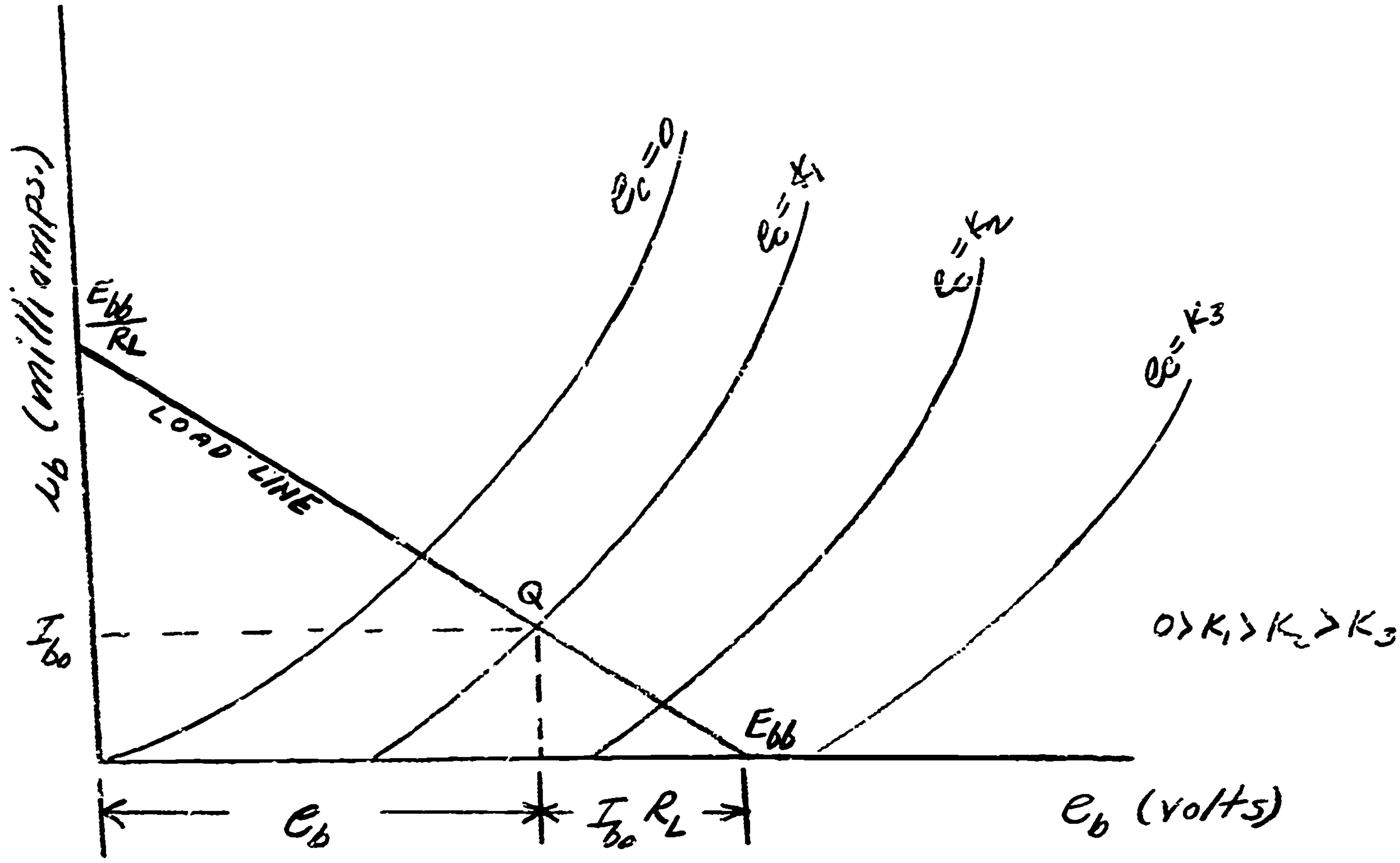


Fig. 3. Plate Characteristic - Typical Triode.

These are static characteristics; for instance, if in Fig. 2, $e_s = 0$ and $E_{cc} = k_1$, the state of the triode would be described by the quiescent operating point Q. If, now, e_s becomes a time-varying voltage, the operating point Q will move along the load-line in accordance with the new value of $e_c = (E_{cc} + e_s)$. Thus, the performance of the triode can be predicted for any given input signal e_s, load R_L, and bias voltages E_{cc} and E_{bb}.

It is convenient to represent the various circuit parameters as a sum of a d. c. term and a varying quantity. The present discussion will be limited to linear class A_1 (class A indicates that plate current is always present; class $\underline{A_1}$ indicates that there is no grid current) mode of operation. Then the d. c. term is the quiescent or bias value of the parameter and the varying term arises from the presence of a signal. Referring to Fig. 2:

$$e_c = E_c + e_g \quad \text{(in general)}$$

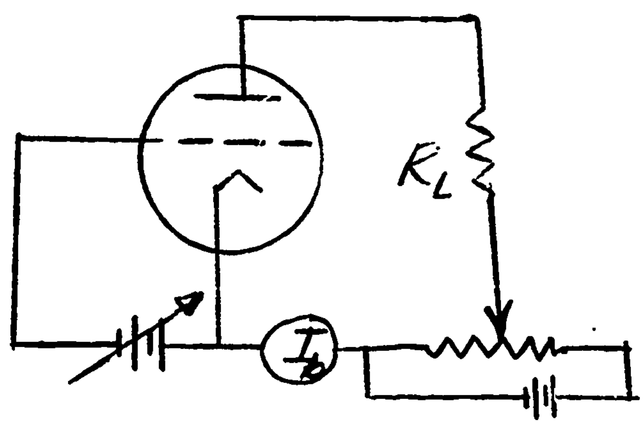
$$= E_{cc} + e_s \quad \text{(in this particular case)}$$

$$e_b = E_b + e_p$$

$$= E_{bb} - i_b R_L = (E_{bb} - I_b R_L) - i_p R_L$$

$$i_b = I_b + i_p = I_{bo} + i_p.$$

<u>Problem 1</u>. Obtain the static plate characteristics for the triode type 6J5. Refer to a manufacturer's tube manual for typical characteristics of a 6J5. A test circuit such as shown in Fig. 4 may be used.



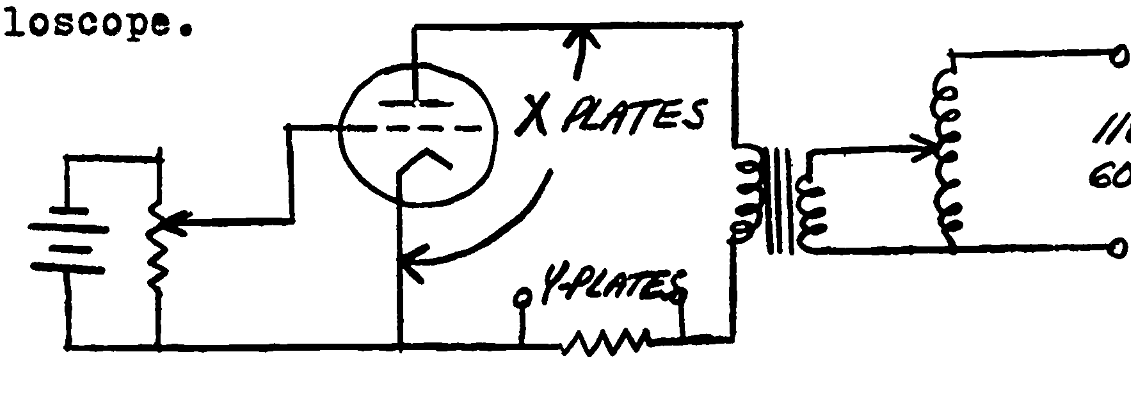
R_L = 18000 ohms

10K Potentiometer

Fig. 4. Test Circuits

Only obtain a few specific curves in the family of plate characteristics since this information is readily available.

Display some plate characteristics and a load line on the oscilloscope.



110V
60 cps

Fig. 4, b.

Problem 2. Voltage Gain. If a sinusoidal voltage $e_s = \sqrt{2}\ E_s \sin \omega t$ is applied to input, the rms value of the signal output voltage may be represented as E_p where $e_b = E_b + e_p$ and $e_p = \sqrt{2}\ E_p \sin (\omega t + \pi)$. The magnitude of the voltage gain is defined as:

$$A = \frac{E_p}{E_s}$$

When $E_{bb} = 300$ volts, $E_{cc} = -8$ volts, $\sqrt{2}\ E_s = 2$ volts and $R_L = 18000$ ohms, determine A from the plate characteristics.

III. Equivalent AC Circuit

Analytically, we proceed as follows:

$$i_b = f(e_b, e_c) \tag{1}$$

Furthermore, if the range of operation is restricted to the region in which the plate characteristics are approximately straight, parallel lines equidistant apart for equal changes in grid voltage, i_b can be represented as:

$$i_b = f(E_b E_c) + \frac{\partial i_b}{\partial e_b} \Delta e_b + \frac{\partial i_b}{\partial e_c} \Delta e_c \tag{2}$$

Define:

$$\frac{\partial i_b}{\partial e_b} = \lim_{\Delta e_b \to 0} \left. \frac{di_b}{de_b} \right|_{e_c = \text{const.}}$$

$$= \frac{1}{r_p} ; \tag{3}$$

$$\frac{\partial i_b}{\partial e_c} = g_m . \qquad (4)$$

The quantity r_p is called the dynamic plate resistance: g_m is called the transconductance.

Eqns. 2 reduces to:

$$I_{bo} + \Delta i_b = I_{bo} + \frac{1}{r_p} \Delta e_b + g_m \Delta e_c$$

or

$$i_p = \frac{1}{r_p} e_p + g_m e_g . \qquad (5)$$

If in eqn. 2 the current i_b is fixed (i.e., $i_b = I_{bo}$ and $\Delta i_b = 0$):

$$- \frac{de_b}{de_c} \Bigg|_{i_b=\text{const.}} = \frac{\dfrac{\partial i_b}{\partial e_c}}{\dfrac{\partial i_b}{\partial e_b}} = g_m r_p = \mu . \quad (6)$$

The quantity μ is defined as the amplification factor. Equation (5) may be written as:

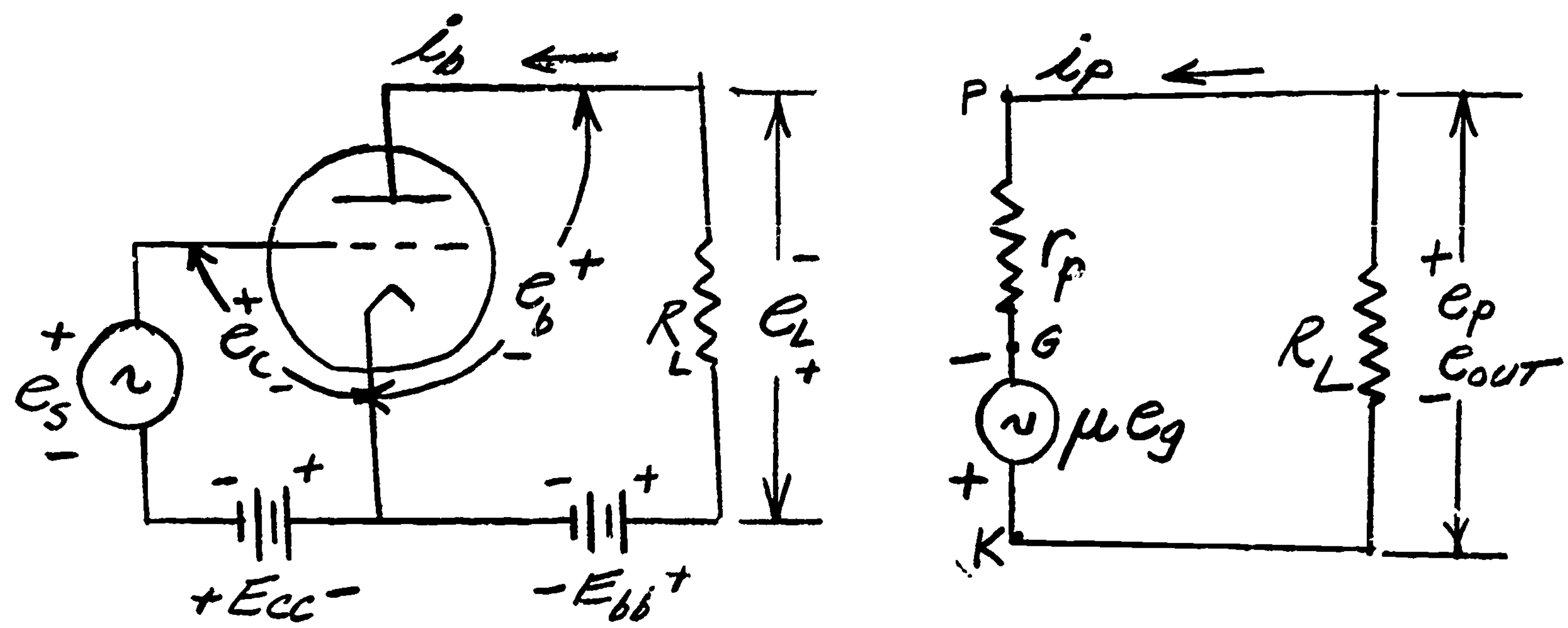
$$i_p = \frac{1}{r_p} (-i_p R_L) + g_m e_g$$

or

$$\mu e_g = i_p (r_p + R_L). \qquad (7)$$

Equation (7) describes the circuit shown in Fig. 5b.

This circuit is the equivalent a.c. circuit for the circuit shown in Fig. 5a. It describes the a. c. or signal behavior of



(a) (b)

$$e_c = E_c + e_g = E_{cc} + e_s$$

$$e_b = E_b + e_p = (E_{bb} - I_{bo}R_L) - i_p R_L$$

$$i_b = I_b + i_p = I_{bo} + i_p$$

Figure 5. Equivalent A. C. Circuit.

the circuit and does not include the information about the quiescent parameters. Note that the assumed polarities of μe_g and e_p and the reference direction of i_p in (b) <u>are not arbitrary</u> but are determined by those assumed in Fig. 5a.

<u>Problem 2</u>. Equivalent Circuit Calculations.

1. Determine μ, g_m, r_p from the static characteristics.

2. Compare the computed value of voltage gain A obtained using Fig. 5b with that obtained graphically using the plate characteristics.

3. Note that if the amplitude of e_s is small compared to E_{cc}, the average value of the plate current, I_b, does not change when the signal e_s is applied. Observe that as the amplitude of e_s is increased, I_b does increase. This indicates that the tube behavior is becoming nonlinear. Demonstrate this fact analytically.

4. In Figure 5b, if μe_g and r_p represent a source (or generator) and R_L represents a load, maximum power will be delivered to R_L when $R_L = r_p$ for a fixed μe_g and r_p. Verify this statement.

Use this fact to determine r_p experimentally (E_{cc} = -8 volts, E_{bb} = 300 volts).

Notice that we have been considering the terminal characteristics (expressed analytically as $i_b = f(e_b, e_c)$) of the triode without studying the physical bases which give rise to these characteristics. Frequently, engineers are required to use devices with only this type information, but most successful engineers use every opportunity to gain further knowledge about things with which they work.

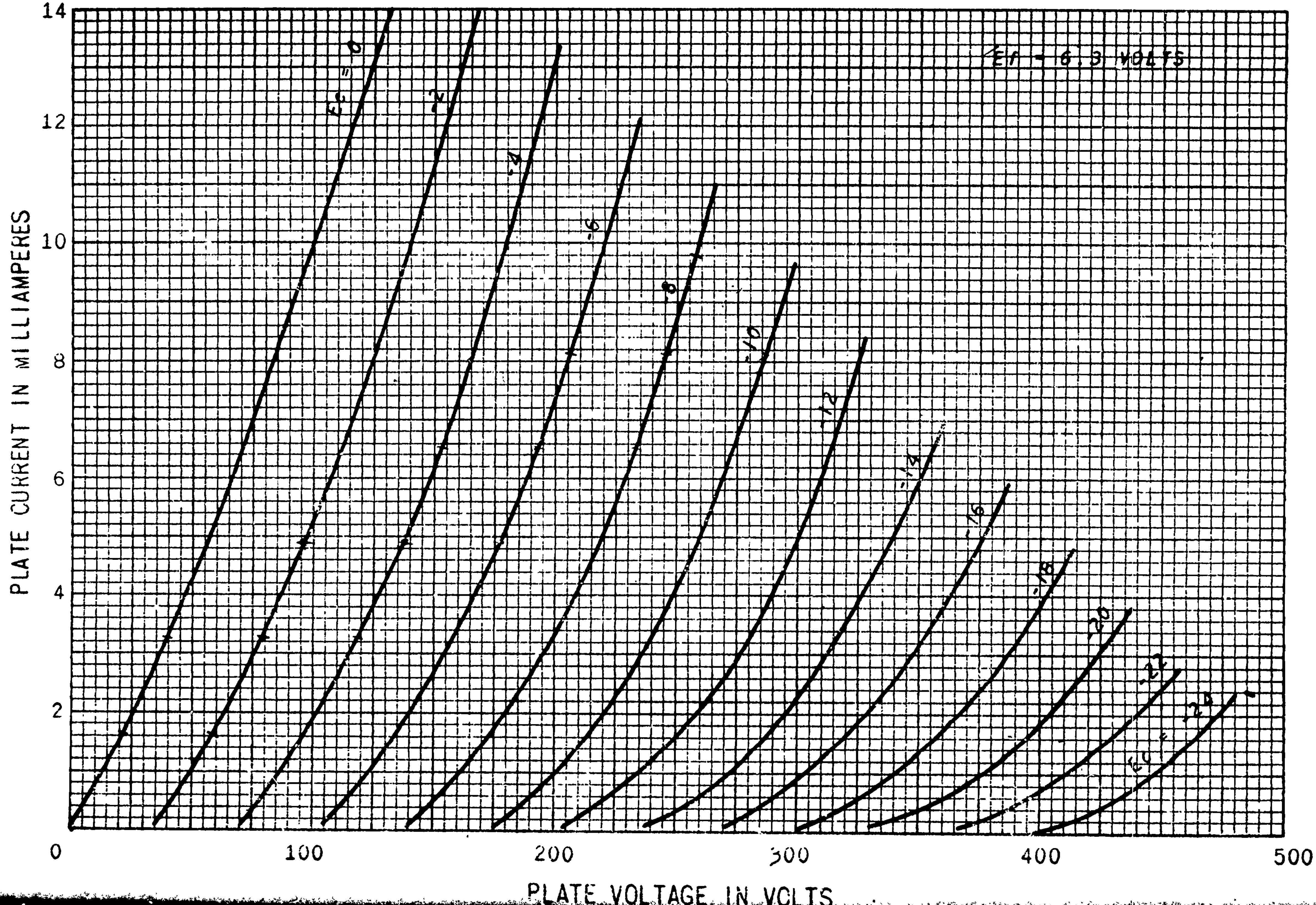
IV. References

1. RCA Receiving Tube Manual
2. APPLIED ELECTRONICS, T. S. Gray
3. Fundamentals of Vacuum Tubes, A. V. Eastman

B. S. 1/31/64

AVERAGE PLATE CHARACTERISTICS

TYPE 6J5



DEPARTMENT OF ELECTRICAL ENGINEERING

EE 25

PROJECT 2

I. INTRODUCTION

In the first project the characteristics of a linear class A_1 triode amplifier in its most elementary form was studied. It is rare that an amplifier is constructed this way.

II. THE SELF-BIASED TRIODE

For instance, usually the grid-to-cathode bias is obtained by a self-biasing resistor R_k as shown in figure 1.

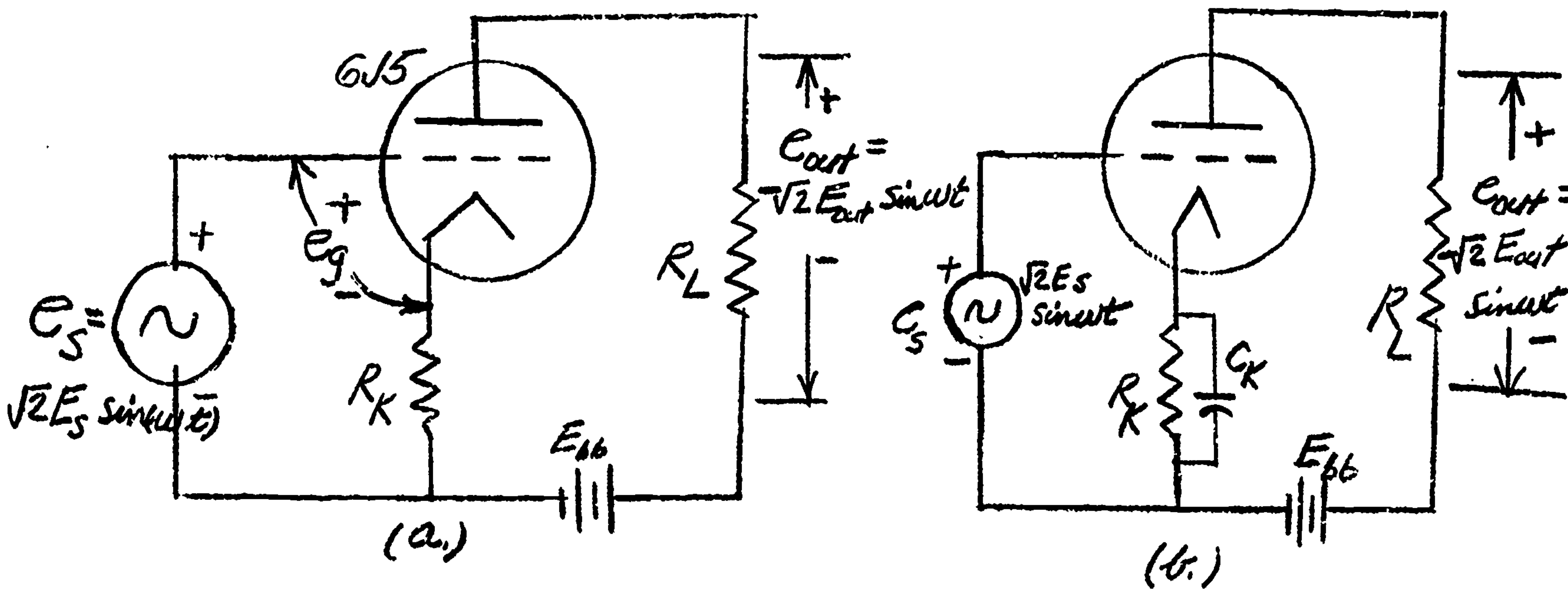


Fig. 1 Self-Biased Triode Amplifier

Problem:

In figure 1 if R_L = 18,000 ohms and E_{bb} is 300 volts, determine the value of R_k which will provide E_c = -6 volts.

Notice that in figure 1, a:

$$e_g = e_s - i_p R_k \quad (1)$$

In other words the grid-to-cathode terminals are not accessible to the input signal e_s and an additional signal voltage ($i_p R_k$) which is proportional to the output (current, in this case) is applied to the input. This is an example of feedback in an amplifier circuit. By placing a capacitor C_k of appropriate size in parallel with R_k, this feedback effect can be eliminated.

Problem:

Determine the value of C_k in figure (1, b), required so that the magnitude of the voltage gain $|A| = \frac{E_{out}}{E_s}$ will be constant for all sinusoidal signals e_s whose frequency is above 200 cps.

Problem:

Compare the magnitude of the voltage gain $|A|$ for the circuits of figure (1, a) and (1, b).

a) Check the gain as the frequency of e_s is changed from 50 to 5,000 cps.

b) When the frequency of e_s is 1,000 cps., check the gain when the peak value of $e_s = \sqrt{2}E_s$ varies from 0 to 6 volts.

c) Verify the experimental results of (a) and (b):

   1) Analytically, by developing an equivalent ac circuit for the circuits of figures (1, a) and (b) from which the gain may be calculated.

   2) Graphically, by constructing a family of e_s = constant - curves on the plate characteristics of a 6J5 triode. From these the gain for the circuit of figure (1, a) may be obtained graphically; the gain in figure (1, b) may be obtained from the conventional plate characteristics.

d) Determine the plate efficiency and the circuit efficiency of the amplifier in figure (1, b)

1) $$\text{Plate Eff.} = \frac{E_{out}^2}{R_L} \Big/ E_{bb} I_b \quad (100)$$

where E_{out} is the rms value of the a.c. component of the output voltage.

$$\text{Circuit Eff.} = \frac{E_{out}^2}{R_L} \Big/ (E_{bb} I_b + E_f I_f) \quad (100)$$

where E_f and I_f are the rms filament voltage and current, respectively.

III. THE CATHODE FOLLOWER

The use of triodes in the amplifier circuit configuration of figure 1 is only one of many applications. When used in a different configuration, the circuit displays different characteristics.

For example, figure 2 shows a cathode-follower circuit in which the output voltage is obtained from the cathode circuit instead of the plate circuit.

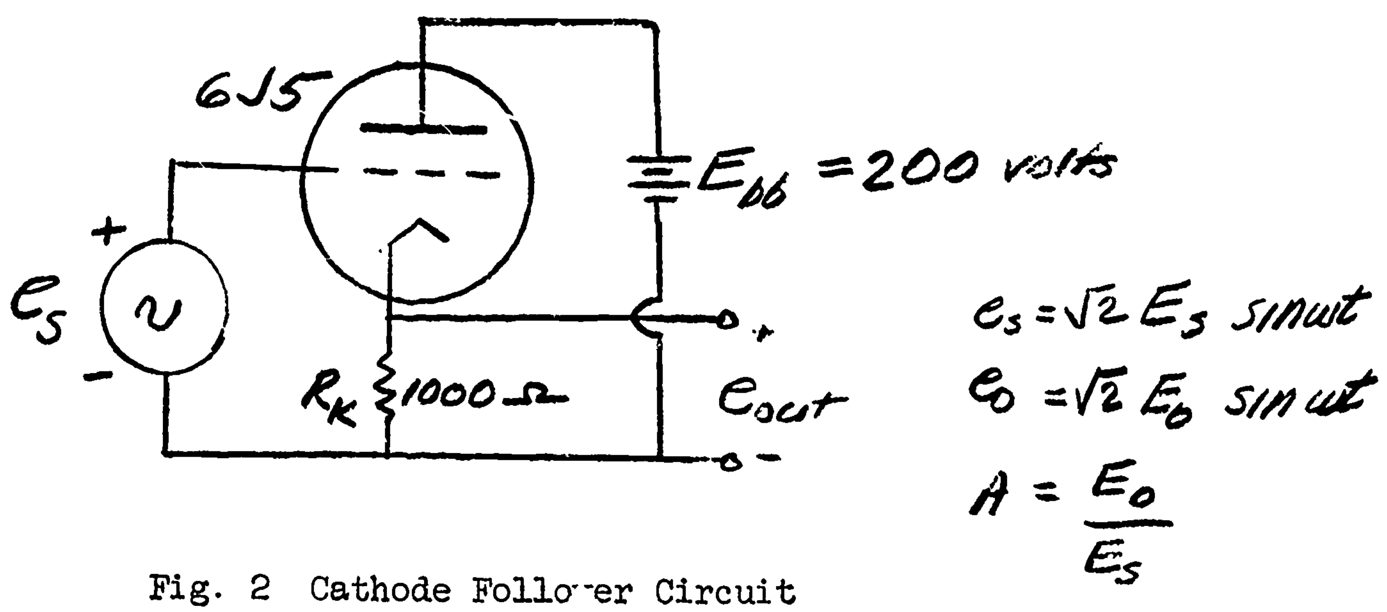


Fig. 2 Cathode Follo--er Circuit

Problem:

a) Determine an equivalent a.c. circuit for the amplifier shown in figure 2.

b) Determine the voltage gain A for this circuit.

c) Determine the output resistance (impedance) of this circuit. The output resistance R_o is defined as:

$$R_o = \frac{E_1}{I_1} \quad \text{in figure 3}$$

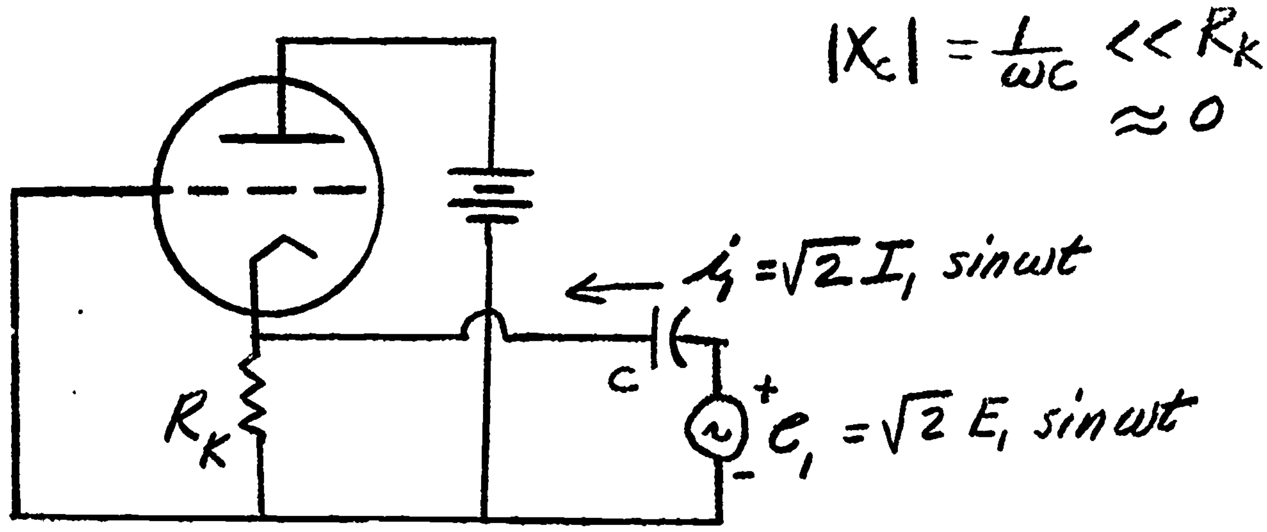


Fig. 3 Output Resistance Determination

If the equivalent a.c. circuit in part (a) is reduced to a Thevinin's equivalent circuit, the equivalent resistance in that circuit equals the output impedance. (See pages 362-366, Electrical Engineering Science, Clement and Johnson)

d) Check the values of gain and output resistance experimentally.

e) Compare A and R_o for the circuit of figure(1, b) with that obtained for figure 2.

IV REFERENCES (See Project 1)

Department of Electrical Engineering

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Project III

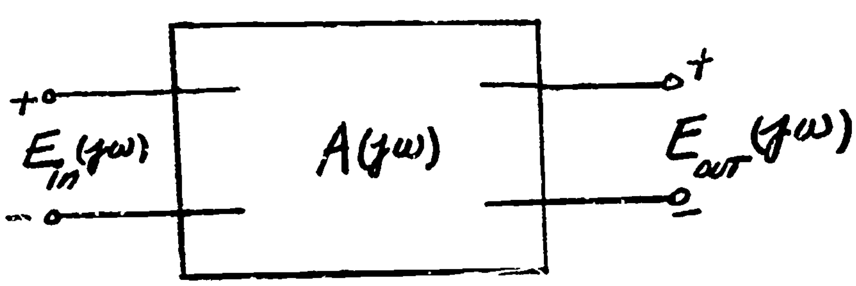
AMPLIFIER FREQUENCY RESPONSE

Introduction

It is useful to define various measures of performance for equipment. For instance the frequency response characteristic of a linear class A amplifier indicates over what frequency range (this may be interpreted as time characteristics of the signal, also) of input signals the circuit output will be an amplified faithful replica of the input. (See Fig. 1)

R-C Coupled Amplifier

In this project the frequency response of an R-C coupled (resistance-capacitance) triode amplifier will be examined. Frequently, a single stage (one vacuum tube or transistor) does not provide sufficient gain and two or more stages will be cascaded. It is important that bias voltages in one stage do not cause undesired effects in other stages.



$$A(j\omega) = \frac{E_{out}(j\omega)}{E_{in}(j\omega)}$$

Figure 1. Definition of Frequency Response of a Linear Network.

This can be assured by using coupling capacitors between stages to provide dc isolation. For instance, in Fig. 2, the coupling capacitors are represented by C_c.

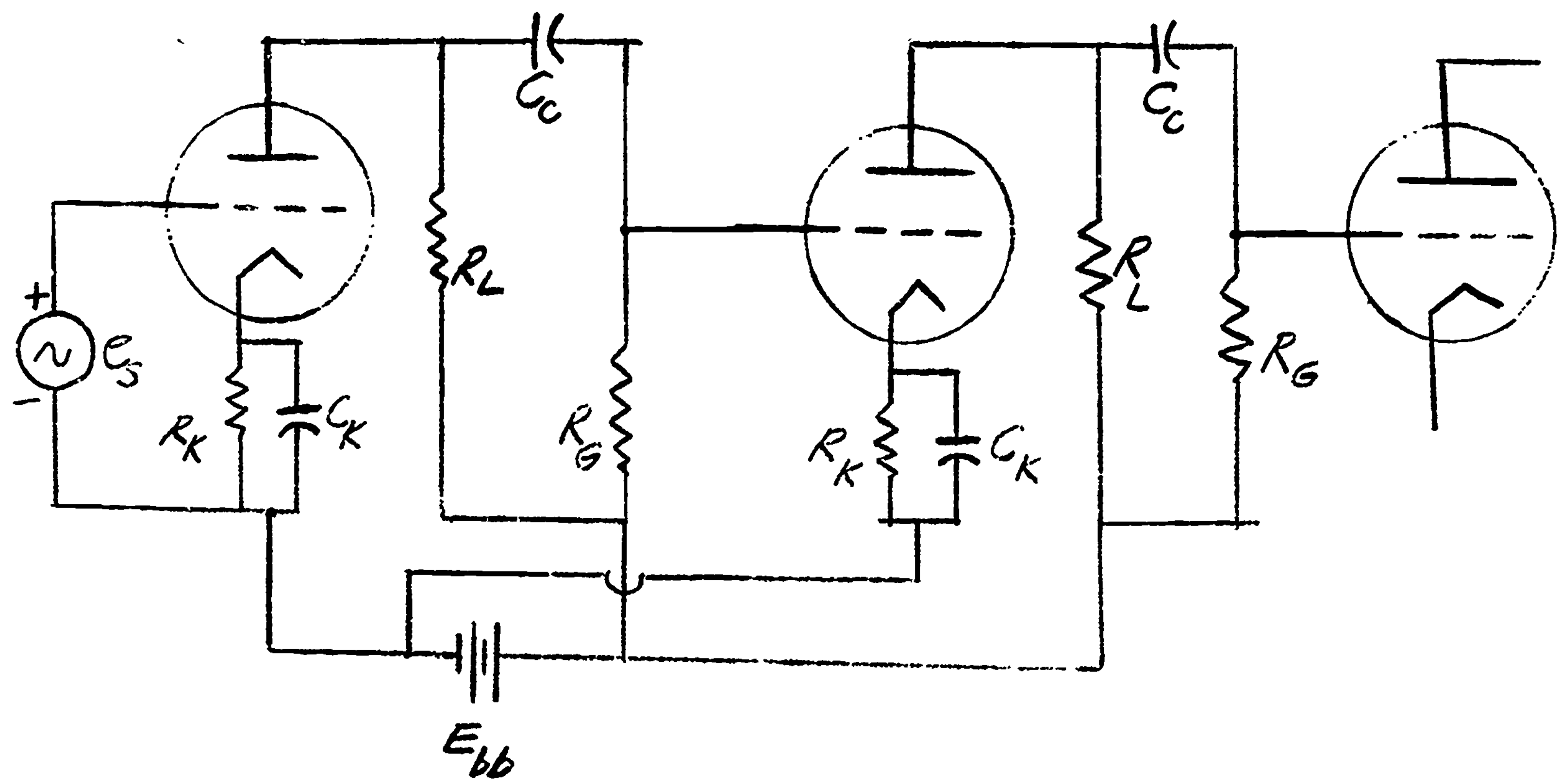
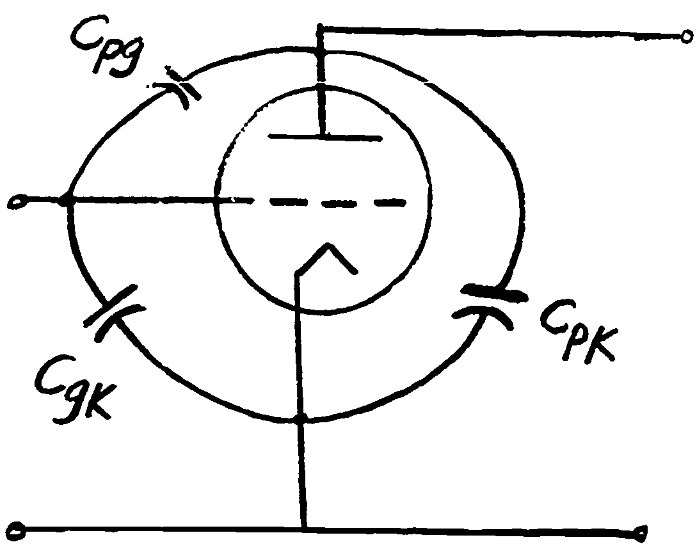


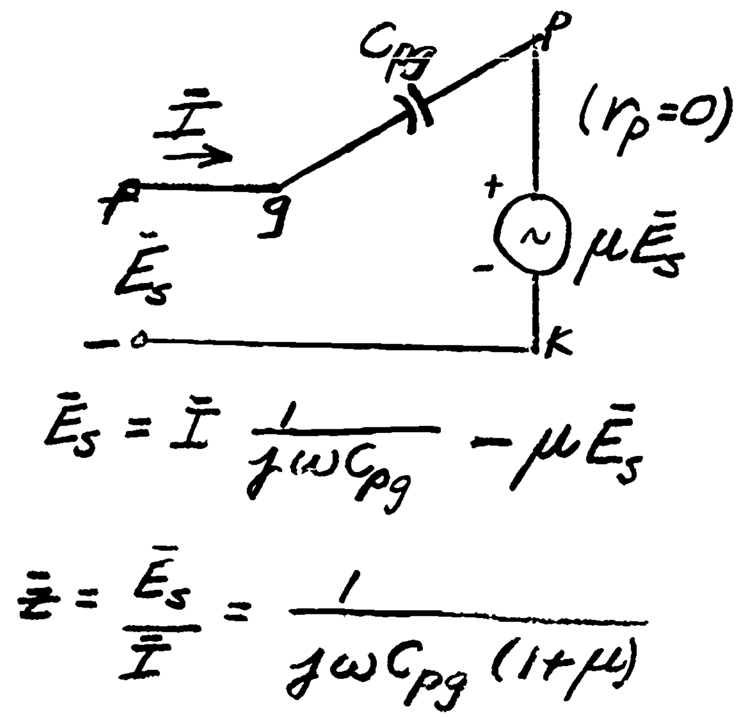
Fig. 2. Cascaded R-C Coupled Amplifier.

However, these capacitors limit the low frequency response of the amplifier. In fact, if $f = \omega/2\pi$ is the lowest frequency in a signal e_s which we wish to amplify without distortion, it is required that $1/\omega C_c < 0.1\ R_g$. Furthermore, any reactive elements in the circuit will affect the frequency response. In addition to the capacitors and inductors specified by the designer, there are present always the inherent interelectrode and wiring capacitances (also lead inductances) which place an upper limit on the frequency response of an amplifier. (See Fig. 3a).



Triode Interelectrode Capacitances

Fig. 3a.



Effect of C_{gp} in Ideal Voltage Amplifier

Fig. 3b.

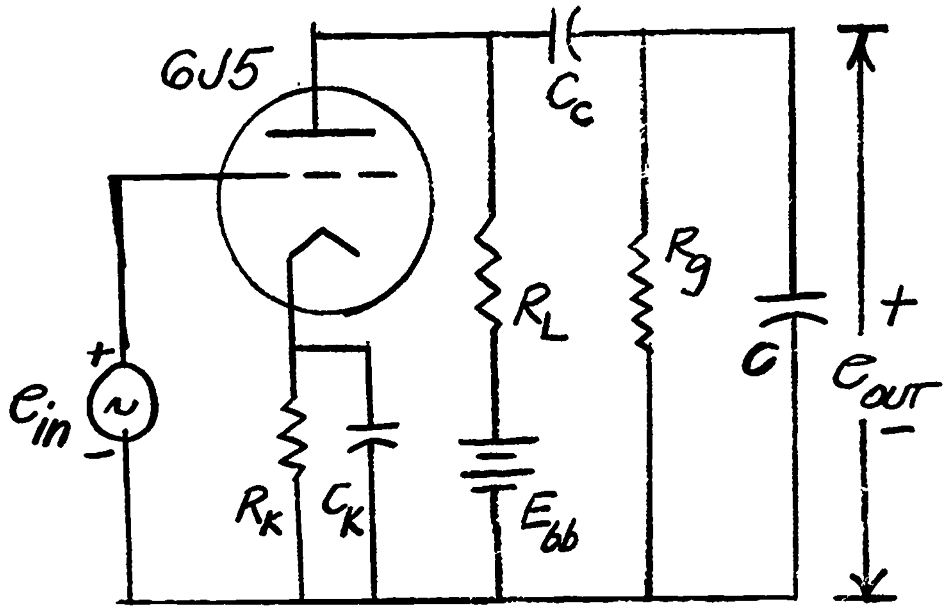
The interelectrode capacitances can be measured accurately and are included in manufacturers' data. Wiring capacitance is difficult to estimate accurately. Sometimes the values of interelectrode capacitances are doubled in an initial design to account for this added capacitance. In addition, the effect of C_{gp} at the input terminals is "amplified" as indicated in Fig. (3b). Note that in cascaded amplifiers (Fig. 1) the input to the second amplifier is the output of the first. At any rate the frequency response of an amplifier (or any network) is determined accurately usually by measurement.

Problem

Determine the freq'ency response,

$$A(j\omega) = \frac{E_{out}(j\omega)}{E_{in}(j\omega)},$$

of the circuit shown in Fig. 4. The effect of succeeding stages on the frequency response has been approximated arbitrary by the 100 μμf capacitor.



$E_{bb} \doteq 300V$

$|E_{in}| = 1V$

$R_k = 1000$ ohms

$R_L = 18{,}000$ ohms

$R_g = 270{,}000$ ohms

$C_c = 0.01\mu f$

$C_k = 0.2\ \mu f$

$C = 100\ \mu\mu f$

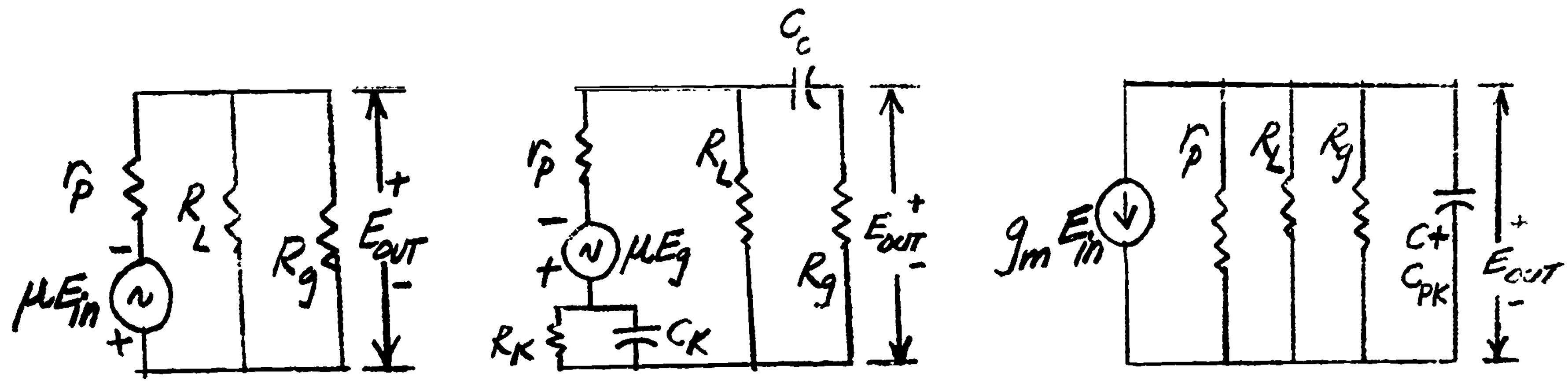
Fig. 4. R-C Coupled Amplifier.

Equivalent Circuits

To facilitate analysis, equivalent circuits for the amplifier are considered usually in three frequency ranges:

1) The midband range where the gain is constant and the reactive elements have negligible effects. 2) The low-frequency range where the effects of C and C_{pk} are negligible. 3) The high-frequency range where C_k and C_c may be neglected. See Fig. 5.

Determine the frequency range in which each of the above equivalent circuits is valid.



Midband   Low Frequency   High Frequency

Fig. 5. Equivalent Circuits

DB Gain vs. Frequency

It is convenient to discuss the gain in the units of Decibels. See Clement and Johnson, E. E. Science, pp. 417-420.

$$G_{db} = 20 \log_{10} A(j\omega)$$
$$= 20 \log_{10} \left[\frac{E_{out}(j\omega)}{E_{in}(j\omega)} \right]$$

Plot $|G_{db}|$ vs. frequency (on a logarithmic scale) (see pg. 419, Clement and Johnson).

The voltage and power output will be a maximum in the midband range. When the gain has decreased by 3 db from the midband gain, the power has decreased by one-half. This will occur at frequency below, f_1, and above, f_2, the midband. The bandwidth BW, is defined as:

$$BW = f_2 - f_1$$

Calculate and check experimentally the bandwidth of the amplifier.

References

Electrical Engineering Science, Clement and Johnson, Chapter 14.

Applied Electronics, T. S. Gray.

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Project IV

TRANSISTOR CHARACTERISTICS

Introduction

In this project, the terminal characteristics of transistors used as linear class A amplifiers will be examined. The vacuum tube triode was invented in 1907. Since then, there have been major modifications such as tetrodes and pentodes; and high-frequency generators such as klystrons and magnetrons have been invented. However, for the general purpose of linear amplification no alternative to the vacuum tube was available until the invention of the transistor in 1948.

Characteristics of Transistors

Transistors have some desirable characteristics (and some undesirable ones) which vacuum tubes do not possess.

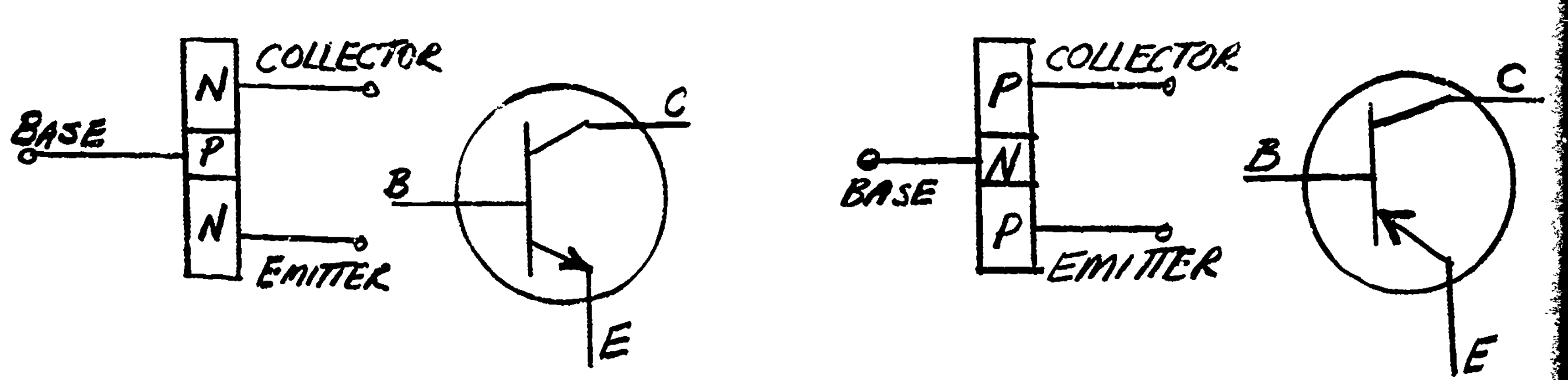
Vacuum tubes have a limited (though frequently long) life. If nothing else fails first, the cathode deteriorates and ultimately renders the tube useless. In principle, a transistor should have an indefinite life when used properly. In fact, manufacturers have not yet achieved this goal.

As linear amplifiers, vacuum tubes are relatively inefficient. Most tubes require a supply of continuous filament power and the relatively large plate voltages (and currents) result in high plate dissipation. On the other hand, transistors require no filament power and the low junction voltages produce small amounts of internal power dissipation.

However, since transistors are semiconductor devices their characteristics depend strongly on their operating temperatures. Because of their small physical size, they can dissipate only limited amounts of heat. Excessive currents and voltages can cause temperature increases which result in permanent damage.

Static Characteristics

A junction transistor consists of an encapsulated sandwich of 'p' and 'n' type semiconductor materials* as illustrated in Fig. 1



N-P-N Transistor P-N-P Transistor

Figure 1

For an explanation of the operating principles of a transistor, refer to the texts listed in the reference section.

In Figure 1, notice that the base and emitter form a p-n junction and the base and collector form another. To operate the transistor as a class A amplifier, the base-to-emitter junction will be biased in the forward direction and the base-to-collector junction will be biased in the reverse direction.

*Review Ch. 2, pg. 53-60, EE Science, Clement and Johnson.

Problem

A number of unidentified p-n-p transistors have been received as a gift. It is desired to use them in the common-emitter configuration as shown in Fig. 2. To predict the amplifier performance, static characteristics and an ac equivalent circuit are desirable.

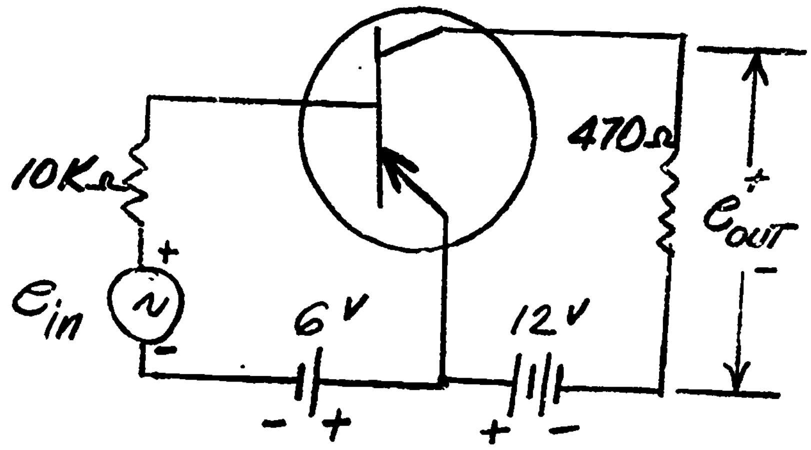


Fig. 2. Transistor Amplifier

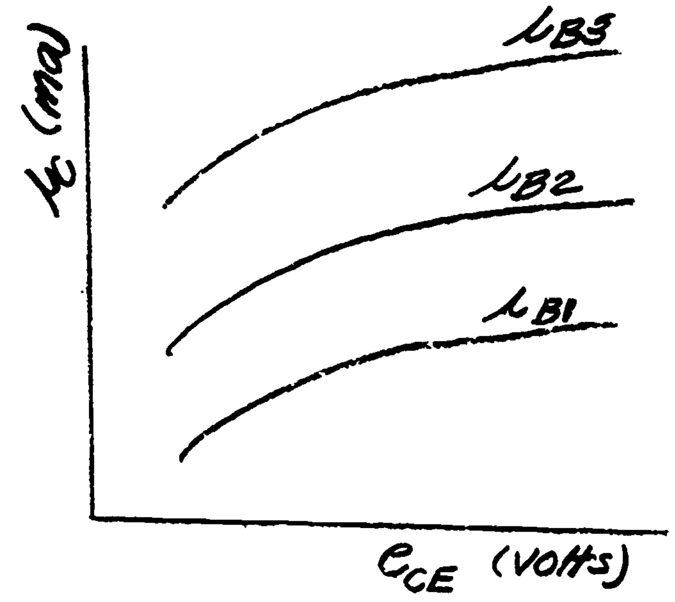


Fig. 3. Collector Characteristic

For the vacuum tube triode it is useful to describe the plate current as a function of plate voltage and grid voltage.

Here the static collector characteristic, the collector current as a function of collector-to-emitter voltage and base current ($i_c = f(e_{ce}, i_B)$), is useful.

Obtain the static collector characteristic as shown in Fig. 3. Let e_{ce} vary from 0 to 10 volts; let i_B vary from 0 to 1 ma. in 100 microampere steps. (see Fig. 4, pg. 5, project I)

Equivalent ac Circuit

If $i_c = f(e_{ce}, i_B)$,

$$\Delta i_c = \frac{\partial i_c}{\partial e_{ce}} \Delta e_{ce} + \frac{\partial i_c}{\partial i_B} \Delta i_B$$

$$= g_o \, \Delta e_{ce} + \alpha_{cB} \, \Delta i_B$$

Develop a collector ac equivalent circuit which includes the parameters g_o and α_{cB}.

Determine g_o and α_{cB} for this transistor from the static characteristic.

Set up the circuit shown in Fig. 2. Apply a sinusoidal signal, $e_{in} = E_{in} \sin \omega t$.

Measure: $|A_V| = \frac{E_{out}}{E_{in}}$; $A_I = \frac{I_c(ac)}{I_B(ac)}$;

$$G = \frac{P_{out}(ac)}{P_{in}\ (ac)} \ ; \qquad \text{eff.} = \frac{P_{out}(ac)\ (100)}{P_{in}\ (\text{Total})}$$

Check these measurements using the static characteristics and the equivalent ac circuit.

References

G. E. Transistor Manual

Electronic Circuits, E. J. Angelo, Ch. 8 and 9.

Electronics, (2nd Edition) Millman.

2N322, 2N323, 2N324

Outline Drawing No. 2

The 2N322, 2N323, 2N324 are alloy junction PNP transistors intended for driver service in audio amplifiers. They are miniaturized versions of the 2N190 series of G.E. transistors. By control of transistor characteristics during manufacture, a specific power gain is provided for each type. Special processing techniques and the use of hermetic seals provides stability of these characteristics throughout life.

SPECIFICATIONS

ABSOLUTE MAXIMUM RATINGS: (25°C)

Voltage			
Collector to Emitter (R_{EB} = 10K)	V_{CER}	−16	volts
Collector to Base	V_{CBO}	−16	volts
Current			
Collector	I_C	−100	ma
Power			
Collector Dissipation	P_{CM}	140	mw
Temperature			
Operating	T_A	−65 to 60	°C
Storage	T_{STG}	−65 to 85	°C

TYPICAL ELECTRICAL CHARACTERISTICS: (25°C)

		2N322	2N323	2N324	
D.C. Characteristics					
Base Current Gain (I_C = −20 ma; V_{CE} = −1v)	h_{FE}	50	75	95	
Collector to Emitter Voltage (R_{EB} = 10K; I_C = −.6 ma)	V_{CER}	−16	−16	−16	volts
Collector Cutoff Current (V_{CB} = −16v)	I_{CO}	−10	−10	−10	µa
Max. Collector Cutoff Current (V_{CB} = −16v)	I_{CO}	−16	−16	−16	µa
Small Signal Characteristics					
Frequency Cutoff (V_{CB} = −5v; I_E = 1 ma)	f_{ab}	2.0	2.5	3.0	mc
Collector Capacity (V_{CB} = −5v; I_E = 1 ma)	C_{ob}	25	25	25	µµf
Noise Figure (V_{CB} = −5v; I_E = 1 ma)	NF	6	6	6	db
Input Impedance (V_{CE} = −5v; I_E = 1 ma)	h_{ie}	2200	2600	3300	ohms
Current Gain (V_{CE} = −5v; I_E = 1 ma)	h_{fe}	45	68	85	
Thermal Characteristics					
Thermal Resistance Junction to Air		.25	.25	.25	°C/mw
Performance Data Common Emitter					
Power Gain Driver (V_{CC} = 9v)	G_e	42	43	44	db
Power Output	P_o	1	1	1	mw

2N1097, 2N1098

Outline Drawing No. 2

The General Electric Types 2N1097 and 2N1098 are alloy junction PNP transistors intended for low power output and audio driver service in entertainment equipment. These types are similar to the General Electric Types 2N322 and 2N323 except for h_{FE} limits.

SPECIFICATIONS

ABSOLUTE MAXIMUM RATINGS: (25°C)

Voltage			
Collector to Emitter (R_{EB} = 10K)	V_{CER}	−16	volts
Collector to Base	V_{CBO}	−16	volts

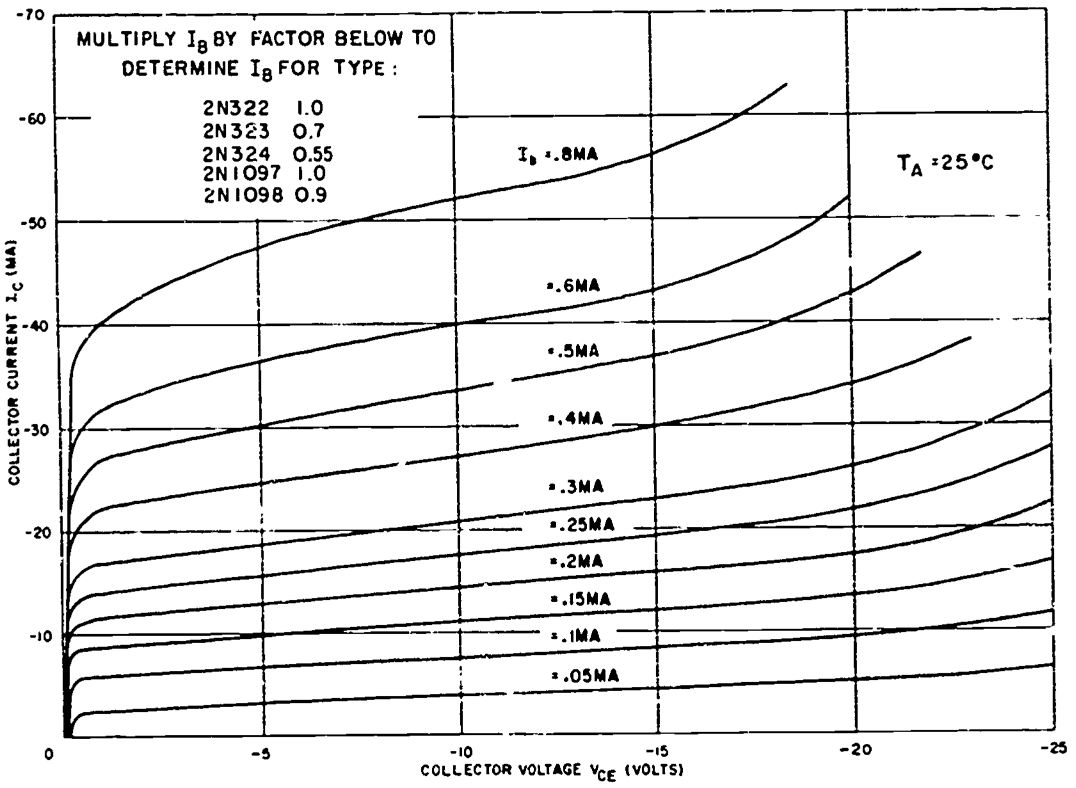
Current			
Collector	I_C	−100	ma
Temperature			
Storage	T_{STG}	−65 to 85	°C
Operating Junction	T_J	60	°C
Power			
Transistor Dissipation*	P_{AV}	140	mw

ELECTRICAL CHARACTERISTICS: (25°C)

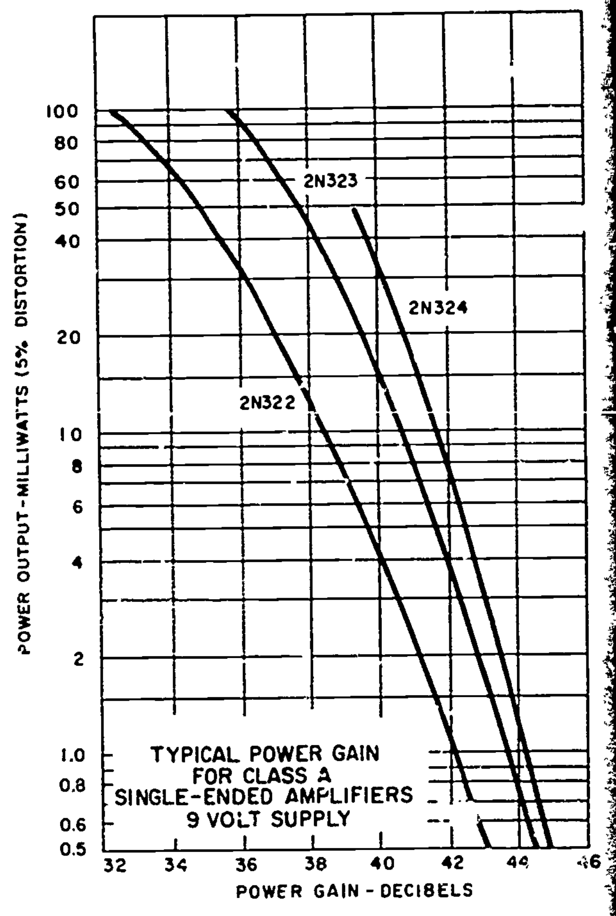
		2N1097	2N1098	
D-C Characteristics				
Collector Current (V_{CB} = −16v)	I_{CBO}	−16	−16	µa max.
Forward Current Transfer Ratio (I_C = 20 ma; V_{CE} = 1v)	h_{FE}	34-90	25-90	
Low Frequency Characteristics (V_C = −5v; I_E = −1 ma; f = 1 KC)				
Output Capacity (Typical)	C_{ob}	25	25	µµf
Forward Current Transfer Ratio (Typical)	h_{fe}	55	45	

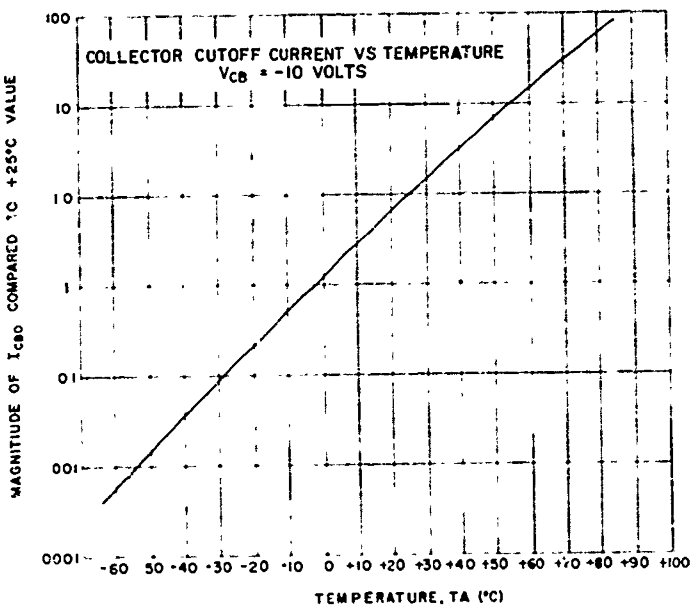
*Derate 4 mw/°C increase in ambient temperature over 25°C.

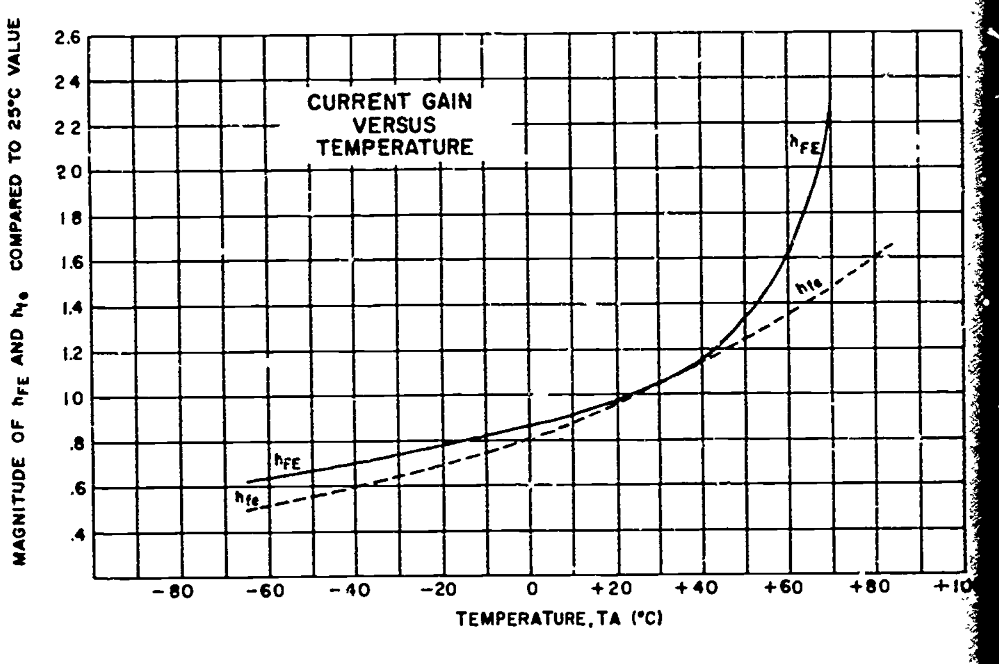
TYPICAL CHARACTERISTICS

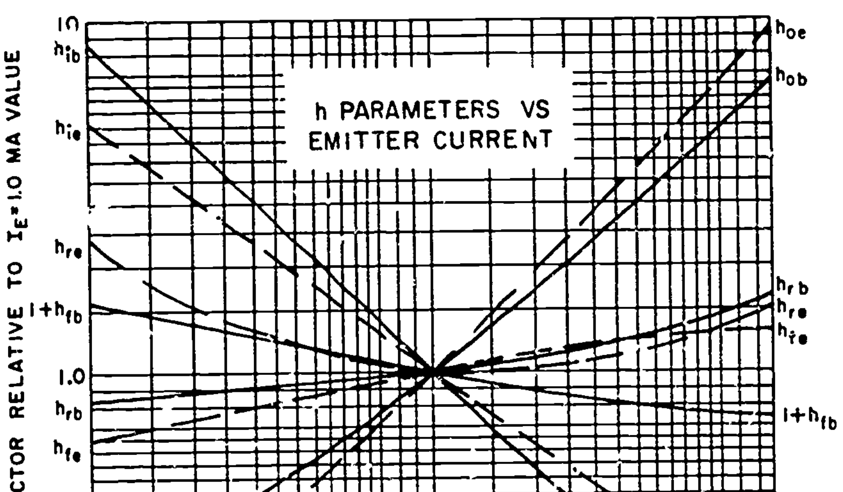


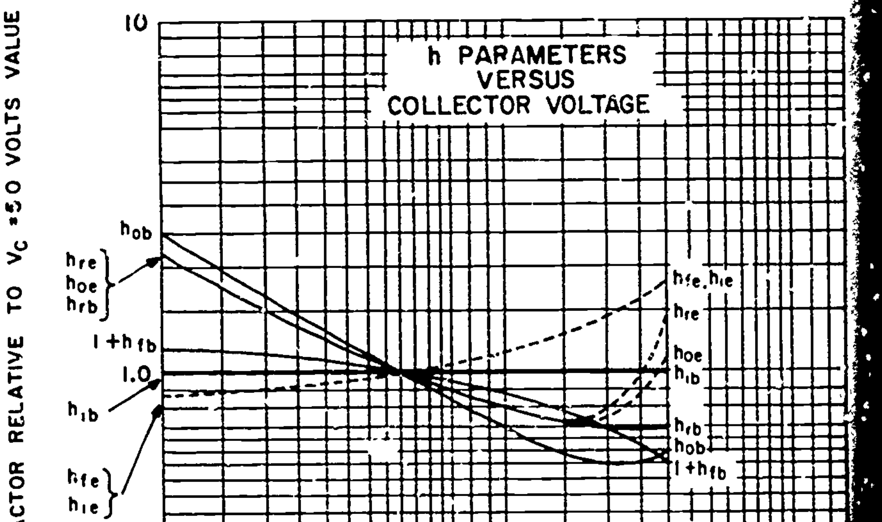
COMMON EMITTER COLLECTOR CHARACTERISTICS











EE 25

PROJECT V

Oscillator Circuits

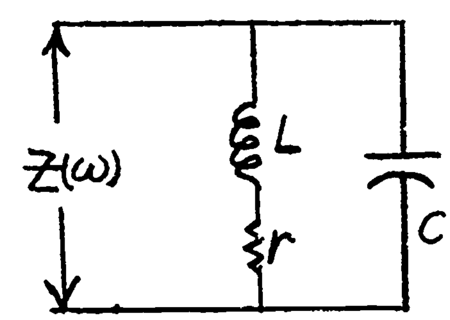
Introduction

In this proje e behavior of two types of oscillator circuits are examined. Prior to the examination of these circuits, the frequency response of a parallel-tuned circuit will be investigated.

The student should consult the following:

1) Class notes on Oscillator Circuits; 2) EE Science, Clement and Johnson, pg. 158-161, regarding "skin effect;" and 3) EE Science, Clement and Johnson, pg. 452-461, regarding "Q."

Frequency Response of Parallel-Tuned Circuit



L is one coil of a G. R. variable inductor.

C is Heathkit decade box.

Adjust C so that $\omega_{res.} = 2\pi f_{res.} \approx 5(10)^4$.

Fig. 1. Parallel-Tuned Circuit.

Using an impedance bridge, determine $Z(\omega) = (R + jX)$ for the circuit shown in Fig. 1. Plot R and X versus frequency.

Note that r is the inherent resistance of the inductor having an inductance L. Because of "skin effect" r varies with frequency and the exact variation cannot be given as a simple, accurate, analytical expression for all inductors. From the above data determine the values of r and Q of the inductor at the resonant frequency of the circuit.

What are the limiting errors in the values of r and Q. (See Appendix VI of EE 24 notes.)

Figure 2 represents the circuit of Fig. 1 which has been modified so that $Q'_{res.} = 0.1\ Q_{res.}$ of the inductor in Fig. 1. On the same sheet plot $|Z(\omega)|$ versus frequency for the circuits of Fig. 1 and Fig. 2.

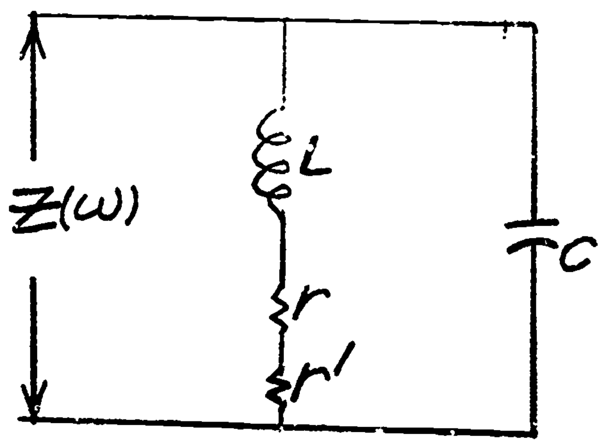


Fig. 2. Low-Q Circuit

<u>Tuned-Collector Oscillator Circuit</u>

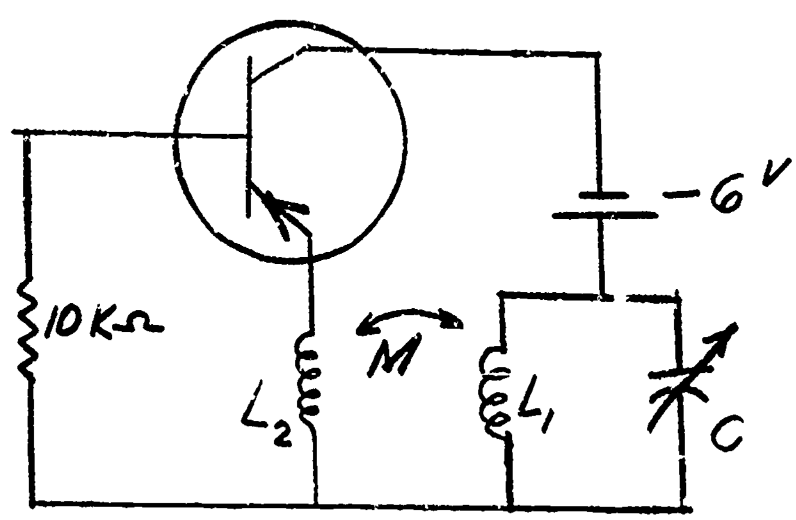


Fig. 3. Transistor Oscillator

In the circuit shown in Fig. 3, the same type transistor is used as that studied in project IV.

Determine the approximate value of α_{CB} required to initiate oscillation.
What is the approximate frequency of oscillation.

Investigate the performance of the oscillator shown in Fig. 3. Include such things as its frequency, voltage amplitude, and power output ranges.

Phase-Shift Oscillator

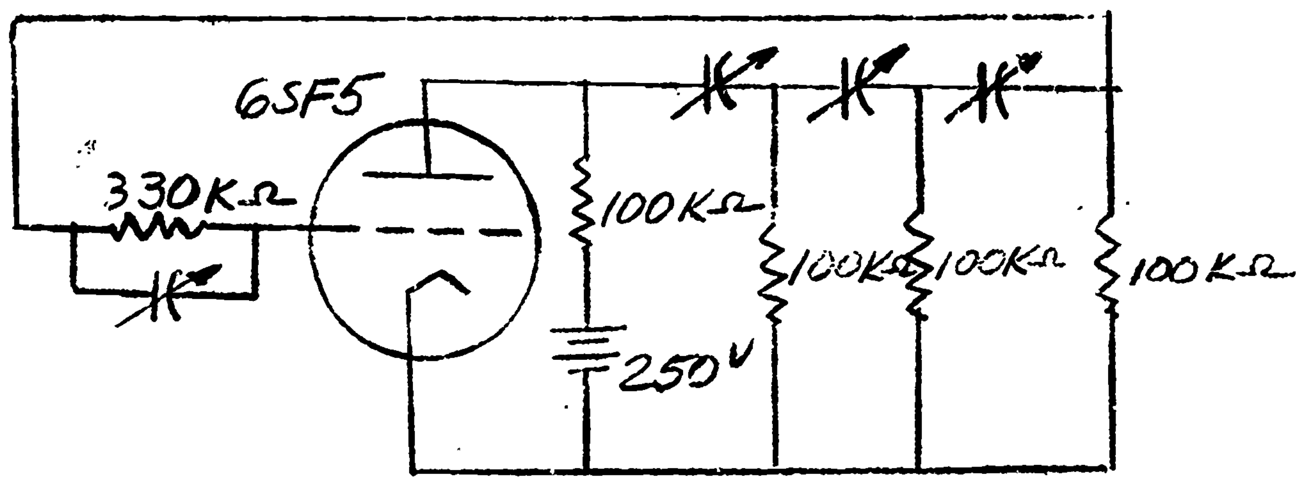


Fig. 4. Phase Shift Oscillator

Determine the approximate value of μ required to initiate oscillations in the circuit in Fig. 4. What will be the frequency of the oscillation?

Investigate experimentally the performance of this oscillator.

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EE 25 Project VII

APPLICATIONS OF ANALOG COMPUTERS

I. Introduction

In analyzing some physical problems, particularly those described by differential equations, it is more convenient sometimes to use an electronic analog computer than a digital computer. For instance, the solution can be displayed graphically while the system parameters (including initial and boundary conditions) are varied.

In the analog computer, the various parameters and mathematical operations of an equation are simulated by electronic circuits. The computer operator has access to the terminals of these circuits through a patch panel. The output of the computer is a voltage (which is a function of time). This voltage can be used to actuate a recording pen (or other equipment) or can be displayed on an oscilloscope. When this voltage and time are "scaled" properly (or calibrated), they represent the dependent and independent variables, respectively, of the original equation.

II. An Example

The operation of an analog computer will be illustrated further by considering a specific problem. Consider the equation:

$$a_1 \frac{d^2x}{dt^2} + a_2 \frac{dx}{dt} + a_3 x + a_4 x^3 = f(t); \qquad (1)$$

$$\text{at } t = 0,\ x = X_o,\ \frac{dx}{dt} = \dot{X}_o.$$

Equation (1) is rewritten as:

$$\frac{d^2x}{dt^2} = \frac{f(t)}{a_1} - \frac{a_2}{a_1}\frac{dx}{dt} - \frac{a_3}{a_1}x - \frac{a_4}{a_1}x^3 \qquad (2)$$

In general, one always solves for the highest-order derivative in the equation. Where this is done, only integrators (and no differentiators) are needed to simulate the problem. This is a practical consideration; it is much easier to build a workable stable system involving only integrators than one including differentiators.

The key to understanding the analog computer is in this next step. A block diagram representing Eq. (2) is drawn in Fig. 1.

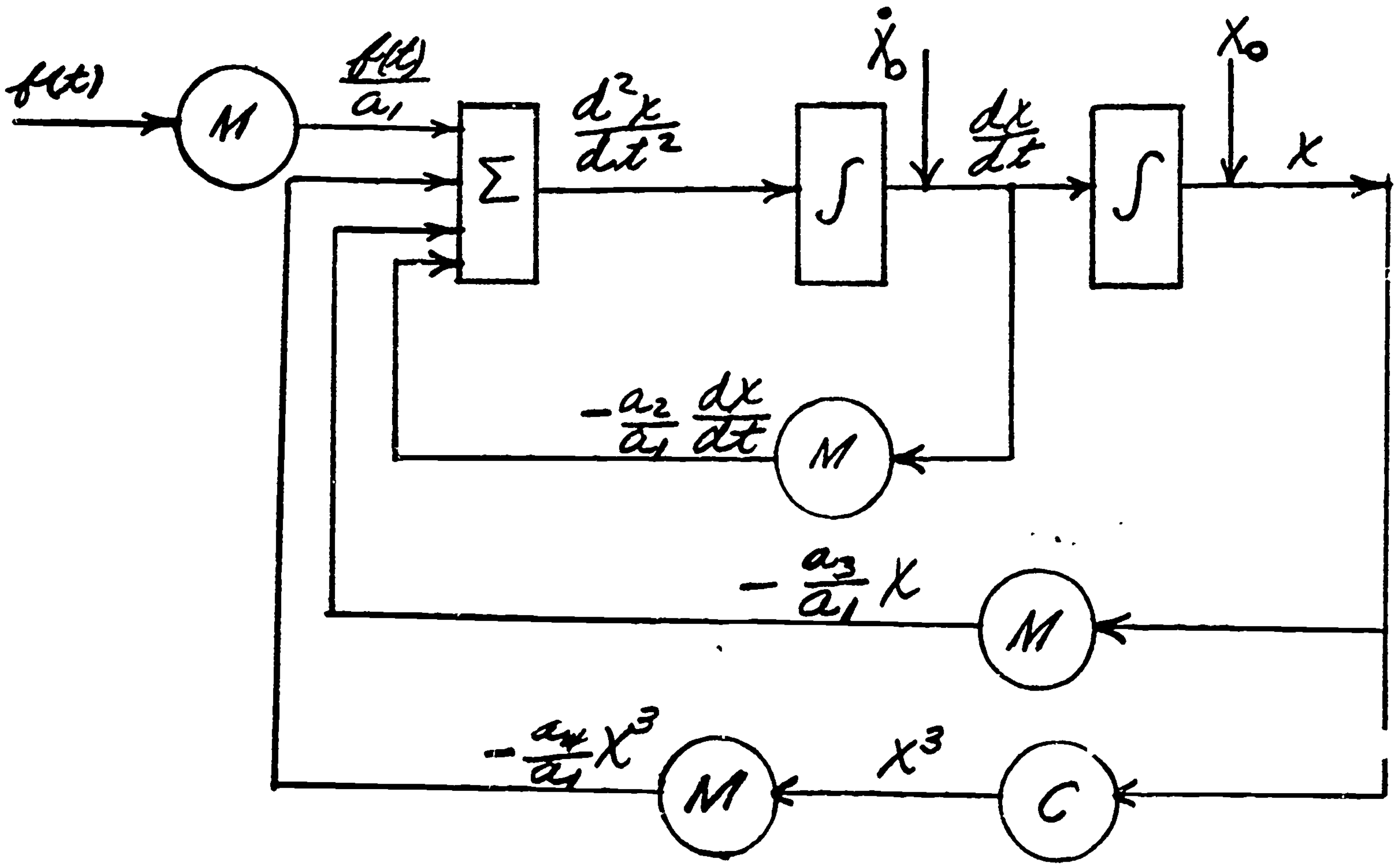


Fig. 1. Block Diagram: $\frac{d^2x}{dt^2} = \frac{f(t)}{a_1} - \frac{a_2}{a_1}\frac{dx}{dt} - \frac{a_3}{a_1}x - \frac{a_4}{a_1}x^3$

If the block diagram in Fig. 1 can be implemented and if x(t) can be observed, this yields a solution to Eq. (1).

To implement this block diagram, the following components are needed:*

1) an arbitrary function generator. This supplies the driving function f(t). Frequently, this is accomplished using diode circuits.

2) a constant multiplier, M . This is accomplished with potentiometers and amplifiers. When the constant multiplier is minus one (-1), it is called an inversion. This is accomplished by a unity gain amplifier.

3) a summer, Σ. It will be shown in the next section that summing occurs when several signals are applied to the input of a single amplifier.

4) an integrator. This will be discussed in the next section.

5) other special functions. For instance, in this problem, x^3 must be generated. In other problems, x^2, log x, xy, etc., may be needed. These may be obtained in a variety of ways. For instance, in the model TR-10 (available in the laboratory), x^2 is generated from x using a diode circuit. The product xy is obtained from the relation:

$$xy = \frac{1}{4}\left[(x + y)^2 - (x - y)^2\right].$$

The function, sin ωt, is obtained by solving the equation, $\frac{d^2x}{dt^2} + \omega^2 x = 0$, on the computer!

III. The Operational Amplifier (Integrator, Inverter, Summer, Constant Multiplier)

Modern electronic analog computers are attractive tools because (among other things) high-gain stable d.c. amplifiers can be built which provide excellent integrators. This will be demonstrated in what follows. First,

*Refer to material in the list of references for detailed information about these components.

consider the circuit shown in Fig. 2. It follows that:

$$i_1 + i_2 + \cdot\cdot + i_i = i_a + i_f, \tag{3}$$

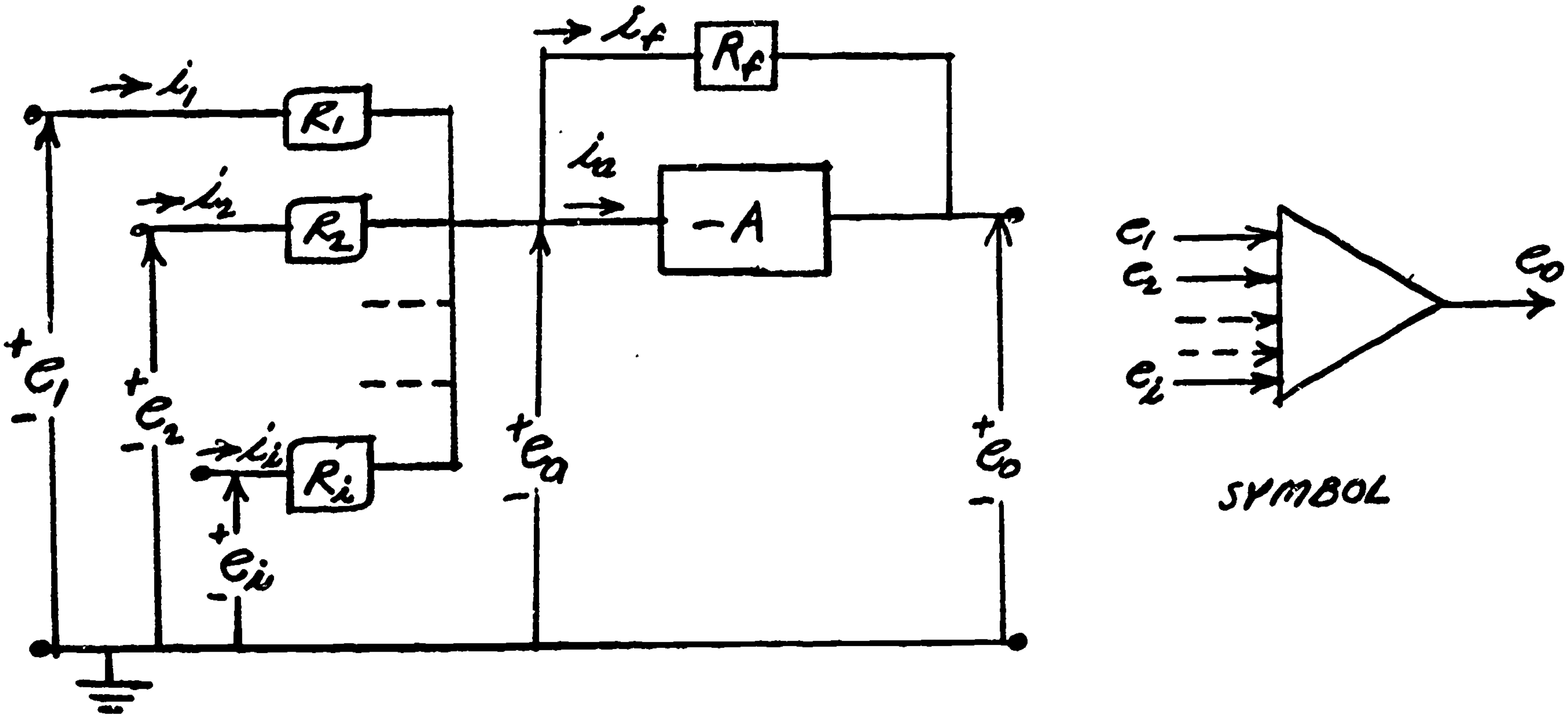


Fig. 2. OPERATIONAL AMPLIFIER

It may be assumed that $i_a = 0$. The input terminal to amplifier (-A) is the grid of a vacuum tube or base of a transistor. Eq(3) may be written as:

$$\frac{e_1 - e_a}{R_1} + \frac{e_2 - e_a}{R_2} + \dots \frac{e_i - e_a}{R_i} = \frac{e_a - e_o}{R_f} \tag{4}$$

Furthermore:

$$e_a = \frac{e_o}{-A} \tag{5}$$

Therefore:

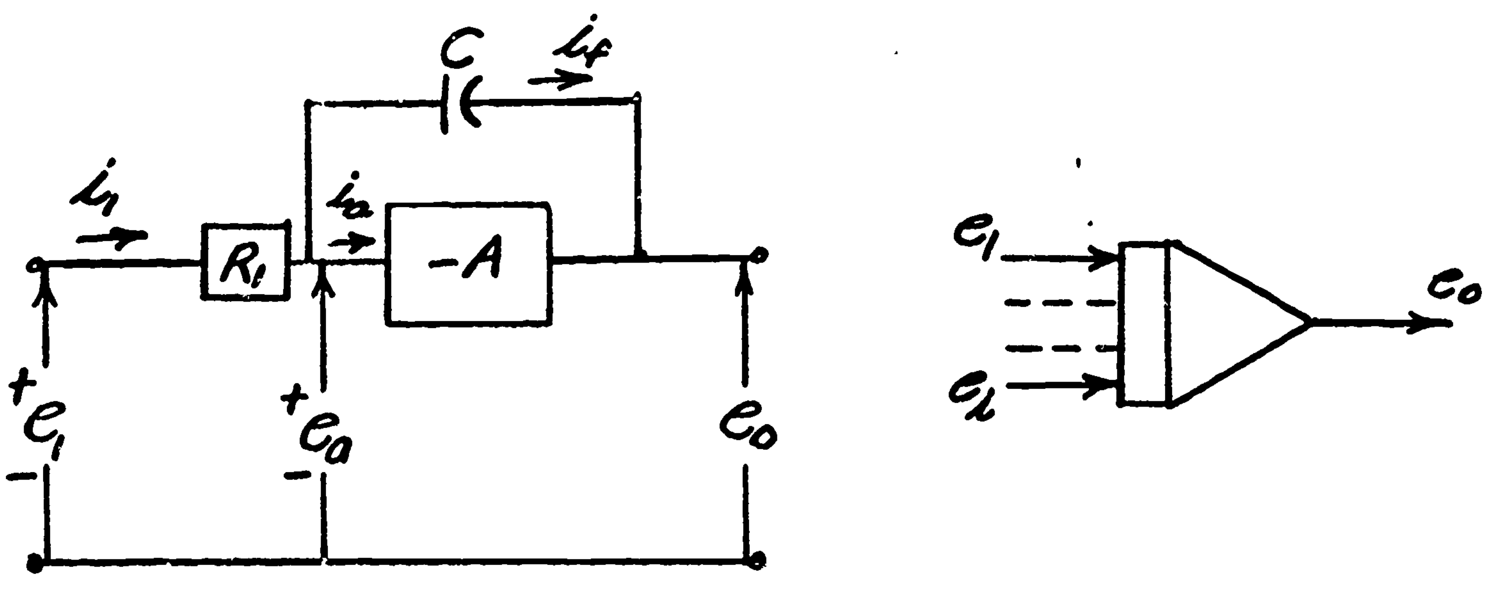
$$e_o \left[1 - \frac{R_f}{A}\left(\frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_i}\right)\right] = -\left[\frac{R_f}{R_1} e_1 + \frac{R_f}{R_2} e_2 + \dots \frac{R_f}{R_i} e_i\right] \tag{6}$$

If A is 100 or greater and the R's are approximately equal, (6) is approximated by:

$$e_o = -\left[\frac{R_f}{R_1} e_1 + \frac{R_f}{R_2} e_2 + \dots \frac{R_f}{R_i} e_i\right]. \tag{7.}$$

Eq.(7) indicates that the circuit in Fig. 2 performs the functions of summing, inverting, and constant multiplying. In the TR-10 computer, several input (and output) jacks are available for each amplifier. Resistor R_f is 100,000 ohms and wired permanently into the circuit; 10,000 ohms and 100,000 ohms plug-in resistors are available for the R_i's which give multiplying constants of 10 and 1, respectively. There always is inversion in one stage of amplification.

If R_f is replaced by a capacitor in Fig. 2, this circuit becomes an integrator. This is shown in Fig. 3.



a) circuit b) symbol

Fig. 3. INTEGRATOR

Now:

$$i_1 = i_f + i_a ; \qquad (8)$$

but i_a is negligible. Therefore:

$$i_1 = \frac{e_1 - e_a}{R_1} = i_f = C \frac{d}{dt} (e_a - e_o). \qquad (9)$$

Again $e_a = \frac{e_o}{-A}$ and A is 100 or greater.

Consequently:

$$e_o = - \frac{1}{R_1 C} \int_0^t e_1 \, dt + E_0 \qquad (10)$$

Eq.(10) shows that the circuit of Fig. 3 is indeed an integrator.

In the TR-10, C is wired permanently in the circuit and equals 10 microfarads. Again, R_1 may be 10,000 ohms or 100,000 ohms. The integrators include jacks (marked IC) to allow the insertion of initial conditions, E_0.

IV. Scaling

There is one additional requirement to be met before "answers" can be obtained from the computer. This involves the so-called "amplitude and time scaling"; i.e., converting the output voltage of the computer and the variable, time, to units of the independent and dependent variables of the original equation to be solved.

Before beginning the discussion of scaling, the block diagram of Fig. 1 is redrawn in Fig. 4 using conventional symbols.

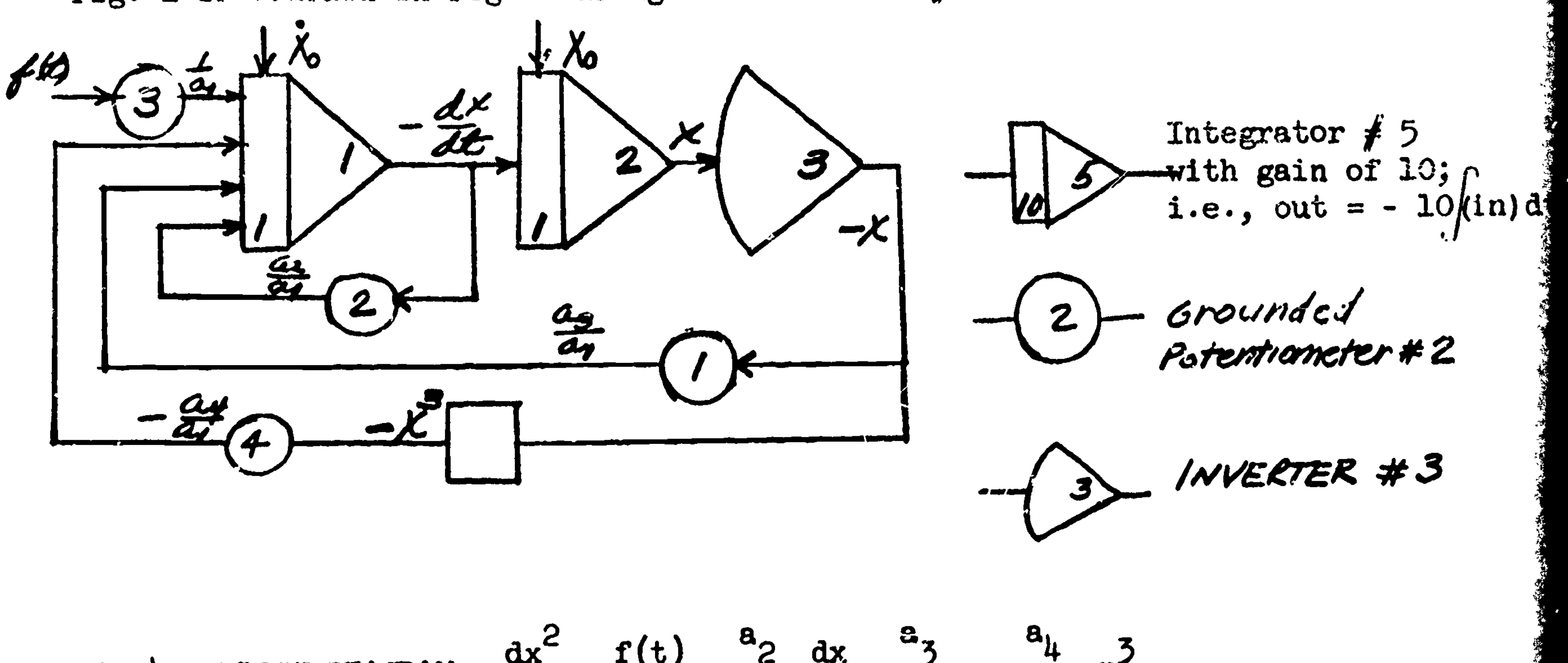


Fig.4. BLOCK DIAGRAM: $\frac{dx^2}{dt^2} = \frac{f(t)}{a_1} - \frac{a_2}{a_1}\frac{dx}{dt} - \frac{a_3}{a_1} x - \frac{a_4}{a_1} x^3$

A. Time Scaling

In any physical system there may be natural resonant frequencies and time constants which characterize its response. Also, the forcing function f(t) may be periodic or, in general, will vary with time. On the other hand, the computer may have a limited frequency response. For instance, when a mechanical recorder is used, this has a limited frequency response. Also, high frequency signals within the computer usually decreases its accuracy.

All components of the computer must be able to respond and perform in a manner which will describe the equation being solved. Also, the output solution should include the range of interest of the independent variable.

For the TR-10, the manufacturer specifies that the solution should occupy a time of 15 to 60 seconds.

In the problem of Fig. 4 and Eq.(1) the natural frequency can be approximated as $[a_3/a_1]^{1/2}$, the damping coefficient is $\frac{a_2}{2a_1}$, and f(t) is specified in a particular problem. With this information one decides how many cycles or time constants, etc. should be included in the 15 to 60 second time interval of the solution and picks a value of β in Eq.(11) below to meet these requirements. Let:

$$t = y/\beta, \qquad (11)$$

$$\frac{dx}{dt} = \frac{dx}{dy}\frac{dy}{dt} = \beta\frac{dx}{dt}, \qquad (12)$$

$$\frac{d^n x}{dt^n} = \beta^n \frac{d^n x}{dy^n}. \qquad (13)$$

Eq.(1) is transformed to:

$$a_1\beta^2 \frac{d^2x}{dy^2} + a_2 \frac{\beta dx}{dy} + a_3 x + a_4 x^3 = f(y/\beta) \tag{14}$$

Eq.(14) is solved on the computer (after amplitude scaling) and the solution to eq.(1) is obtained by

$$x(t) = x(y/\beta) \ . \tag{15}$$

This time scaling assures that the time variations of the signals are restricted to a range which can be accommodated by all components of the computer.

Of course, the independent variable t need not represent time, but might be some other variable such as a spacial coordinate.

As an example, consider

$$\frac{dx}{dt} + \frac{x}{100} = 0 \ ; \text{ at } t = 0, \quad x = 150 \, . \tag{16}$$

The solution to (16) is

$$x = 150\epsilon^{-\frac{t}{100}} \ . \tag{17}$$

Suppose we wish to display a solution corresponding to five time constants of this equation within 60 seconds. In Eq.(16), five time constants corresponds to 500 seconds. Therefore, let:

$$y = \beta t = \frac{t}{10} \quad \text{and (16) becomes} \tag{18}$$

$$\frac{1}{10} \frac{dx}{dy} + \frac{x}{100} = 0 = \frac{dx}{dy} + \frac{x}{10} \tag{19}$$

Now Eq.(19) would be solved (after amplitude scaling,

E. Amplitude Scaling

All analog computers are built to operate within a certain voltage range. For instance in the TR-10 the range is: $-10 \text{ volts} \leq V \leq +10 \text{ volts}$. Any signals greater than 10 volts which are applied to any amplifier in the circuit will overload this amplifier causing it to function improperly and give erroneous results. Consequently, the magnitude of all signals (or voltages) in the computer must be maintained less than 10 volts.

In the problem illustrated by Fig. 1 and Fig. 4, f(t) and initial conditions X_o and $\dot{X}_o$ will have magnitudes and dimensions (and units) specified by the problem to be solved. However, they will be entered as voltage inputs to the computer. They give rise, in turn, to $\frac{d^2x}{dt^2}$, $\frac{dx}{dt}$, x, x^3 whose values are related through the equation (problem) being solved. For instance, in Fig. 4 if $X_o > 2.15$ volts, $X_o^3 = X^3(\text{at } t = 0) > 10$ volts and our solution would not be correct. In estimating the peak values of all the amplifier signals the time-scaled equation (14) must be used. Suppose in (14) that x is equal approximately to $X_o \, \epsilon^{-Ky} \cos my = X_o \, \epsilon^{-\beta Kt} \cos \beta mt$. Then

$$\left| \frac{dx}{dy} \right|_{peak} < m \; |x|_{peak}$$

$$\left| \frac{d^2x}{dy^2} \right| < m^2 \; |x|_{peak}, \text{ etc.}$$

After making these estimates Eq(14) is transformed again as follows:

Let $\quad Z = \frac{x}{h}$, (20)

where h is chosen to insure that the peak values of all variables (or signals) in Fig. 4 are less than 10 volts. Eq(14) becomes

$$a_1 \beta^2 h \frac{d^2Z}{dy^2} + a_2 h\beta \frac{dZ}{dy} + a_3 hZ + a_4 h^3Z^3 = f(y/\beta); \qquad (21)$$

$$\text{at } t = 0,\ Z_o = \frac{X_o}{h}\ ;\ \dot{Z}_o = \frac{\dot{X}_o}{h}.$$

Equation (21) is the equation programmed on the computer to solve the original equation 1.

Finally:

$$x(t) = h\ Z(y/\beta). \qquad (22)$$

Consider the amplitude scaling of equation (19):

$$\frac{dx}{dy} + \frac{x}{10} = 0\ ;\ y = 0,\quad x = x_o = 150.$$

The peak value of x is x_o and $\frac{dx}{dy} = - \frac{x}{10}$.

$$\text{Let } Z = \frac{x}{20}\ ;\quad Z_o = \frac{150}{20} = 7.5 \text{ volts}. \qquad (23)$$

The equation solved on the computer is:

$$\frac{dz}{dy} + \frac{z}{10} = 0; \qquad (24)$$

and

$$x(t) = 20\ Z\ (y/\beta) = 20\ Z(10y). \qquad (25)$$

Note if all the signals were small, say less than 1, h in Eq.(20) would be less than 1. Peak signals should be of the order of 5 volts to give good signal-to-noise ratios.

V. Accuracy

Nothing has been said about the accuracy of the analog computer. An accuracy of at least three significant figures is maintained usually in commercial equipment. Errors are introduced because components are not ideal; i.e., the amplifiers draw some current as do other components such as potentiometers; also, the gain -A of the amplifiers is finite not infinite. Also, errors are introduced by the operator in initial settings.

After you become familiar with the elementary operation of the analog computer, you should concern yourself with its accuracy. See the references listed.

VI. Problems

Solve: 1) $\frac{dx}{dt} + \frac{x}{10} = 0$; at $t = 0$, $x = 1$.

2) $\frac{d^2x}{dt^2} + 25x + 10x^3 = 0$; at $t = 0$, $x = 5$, $\dot{x} = 0$.

3) $\frac{d^2x}{dt^2} + 0.01\frac{dx}{dt} + x = 0$; $t = 0$, $x = 10$, $\dot{x} = 0$.

4) $\frac{d^2x}{dt^2} - 0.1\frac{dx}{dt} + x = 0$; $t = 0$, $x = 0$, $\dot{x} = 5$.

5) $\frac{d^2x}{dt^2} - K(1 - x^2)\frac{dx}{dt} + x = 0$; $t = 0$, $x = 1$, $\dot{x} = 1$.

solve for: $K = 10$, $K = 1$, $K = 0.1$

Predict the form of solution you expect from the analog computer. Resolve any differences between predictions and results.

VII. References

1) Instruction Manual for Model TR-10 Analog Computer

2) Analog Computer Techniques, C.L. Johnson, McGraw-Hill Book Co.

NOTE: Both references have extensive bibliographies

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