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ACTION WITH FRACTIONS, ADDITION AND SUBTRACTION.

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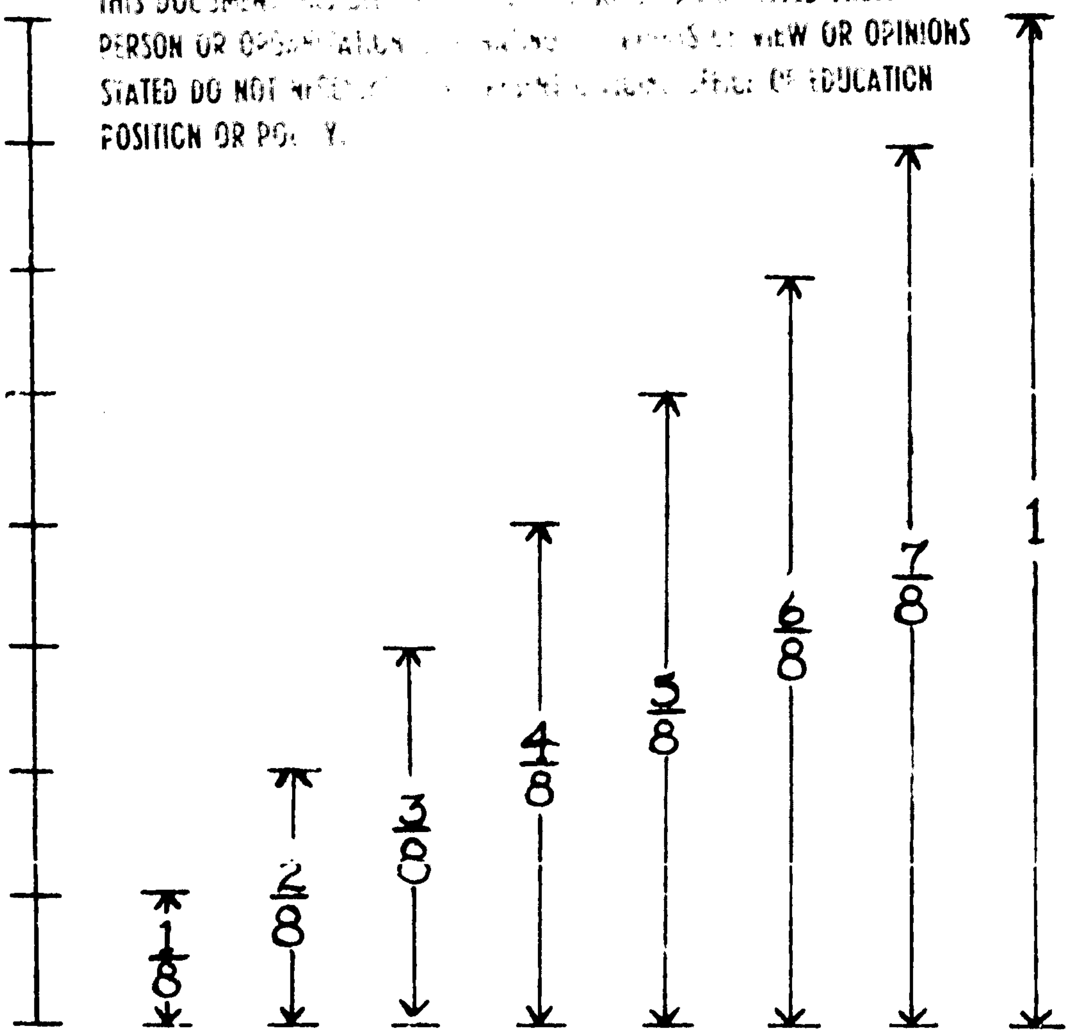
THIS BOOKLET, ONE OF A SERIES, HAS BEEN DEVELOPED FOR THE PROJECT, A PROGRAM FOR MATHEMATICALLY UNDERDEVELOPED PUPILS. A PROJECT TEAM, INCLUDING INSERVICE TEACHERS, IS BEING USED TO WRITE AND DEVELOP THE MATERIALS FOR THIS PROGRAM. THE MATERIALS DEVELOPED IN THIS BOOKLET INCLUDE (1) NUMBER RELATIONSHIPS, (2) EQUIVALENT FRACTIONS, (3) ADDITION AND SUBTRACTION OF RATIONAL NUMBERS, (4) LEAST COMMON DENOMINATORS, AND (5) SUPPLEMENTARY ACTIVITIES WITH RATIONAL NUMBERS. ACCOMPANYING THESE BOOKLETS WILL BE A "TEACHING STRATEGY BOOKLET" WHICH WILL INCLUDE A DESCRIPTION OF TEACHER TECHNIQUES, METHODS, SUGGESTED SEQUENCES, ACADEMIC GAMES, AND SUGGESTED VISUAL MATERIALS. (RP)

ACTION WITH FRACTIONS



ADDITION & SUBTRACTION

ESEA Title III

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MODELS AND NUMBERS

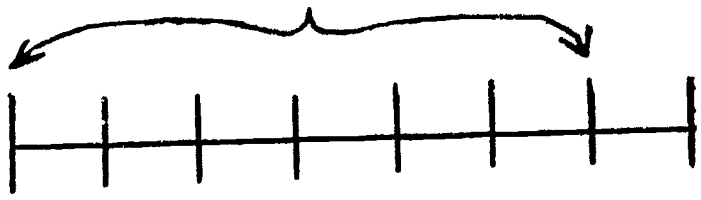
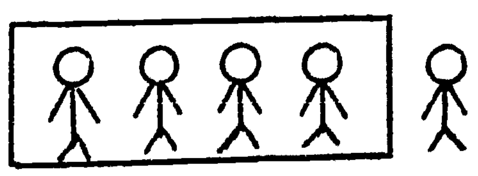
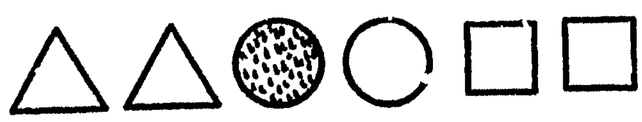
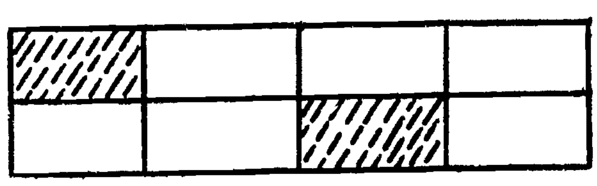
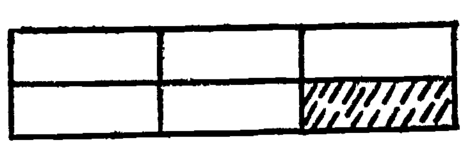
A pair of numbers can be used to describe certain drawings. The number pair is found by:

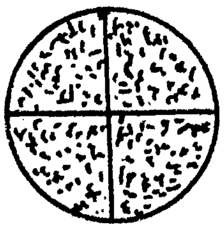
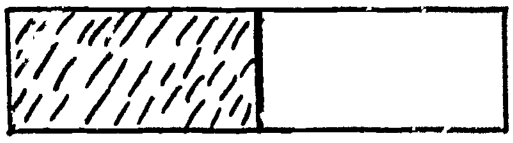
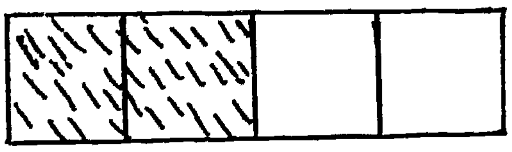
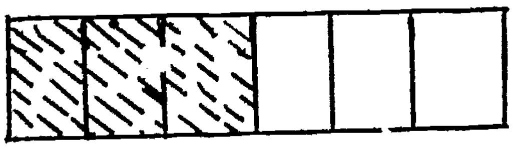
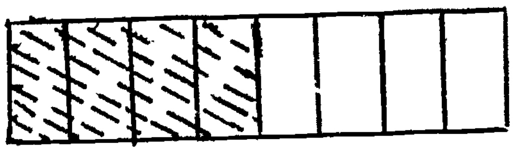
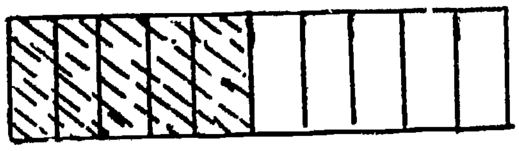
A. Counting the total number of objects, or counting the total number of equal parts of <u>an</u> object.

B. From the total, count the number of parts we have shown an "interest in." This can be done by shading, a line drawn around some part of the total, or in many other ways. You can usually tell from the drawing.

Activities

Apply these two ideas and complete the missing values. Compare the number we are "interested in" to the total as shown in the first example.

	Model	"Interested In"	Total	Comparison
1.	X X X X X X	2	6	$\frac{2}{6}$
2.		6	____	____
3.		____	5	____
4.		1	____	____
5.		____	8	____
6.		____	____	$\frac{1}{6}$

		"Interested In"	Total	Comparison
7.		4	____	____
8.		1	2	____
9.		____	4	$\frac{2}{4}$
10.		____	____	$\frac{3}{6}$
11.		4	____	$\frac{}{8}$
12.		____	____	____

Discuss the last 5 problems. Compare the number we are "interested in" (part) to the total.

Activities

Give the number comparison for each statement below. Use the first as an example.

Example: 1. Bob passed the football 8 times and completed 3.

Answer: $\frac{3}{8}$

______

2. A class has 30 students and 15 are girls.

Answer: ______

3. A marksman fired 4 shots and scored 3 perfect times.

Answer: ______

4. A team lost 2 of 10 games.

Answer: ______

5. In the last 3 weeks it has rained 4 days. (Be careful and think.)

Answer: ______

6. A group of 50 light bulbs contains 3 that will not burn.

Answer: ______

7. John has 14 hits in his last 28 times at bat.

Answer: ______

8. One-half of a class of 20 students made an "A."

Answer: ______

Comparing Rational Numbers--Number Relationships

The numbers we have been working with are sometimes called fractions. They are also called "positive rational numbers." They can be described as a number comparison (using whole numbers) where the bottom number is not zero. In writing our number comparisons, we placed one number above a horizontal line and one number below the line. The names for these two numbers are:

Denominator: The number below the horizontal line which tells the total number of objects or the total number of equal parts of an object.

Numerator: The number above the horizontal line which tells how many of the total that are of special interest, or how many of the total we are discussing.

Three important "relations" used in comparing whole numbers were:

Relation Symbol	Meaning
=	"is equal to"
>	"is greater than"
<	"is less than"

Activities

Here are some number sentences using these relations. See if you can tell which ones are true and which ones are false. Circle your choice.

1. $\frac{3}{4} > \frac{1}{2}$ True or False

2. $\frac{2}{3} < \frac{3}{4}$ True or False

3. $\frac{5}{6} = \frac{10}{12}$ True or False

4. $\frac{5}{6} > \frac{7}{12}$ True or False

5. $\frac{4}{6} < \frac{2}{3}$ True or False

6. $\frac{7}{8} = \frac{10}{16}$ True or False

7. $\frac{5}{6} > \frac{11}{13}$ True or False

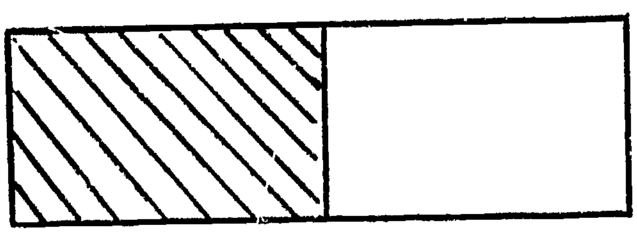
8. $\frac{12}{24} > \frac{9}{24}$ True or False

9. $\frac{7}{8} < \frac{5}{8}$ True or False

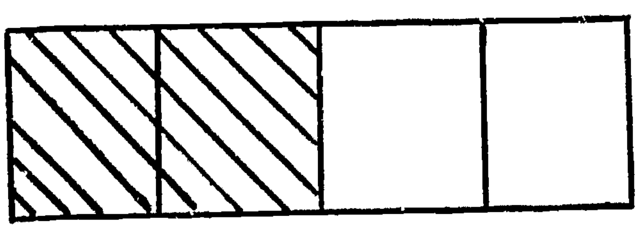
10. $\frac{2}{4} < \frac{3}{4}$ True or False

Did you find the last three problems easy? They are easy to answer because they have the same "denominator." Notice that when this is true you need only to compare numerators.

Notice that the drawings below show that there are "many names" for the same number. From this idea maybe we can "rename" rational numbers such that they have equal denominators.

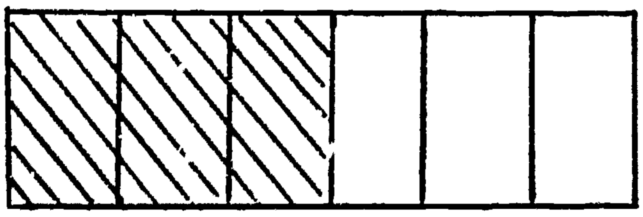


$\frac{1}{2}$

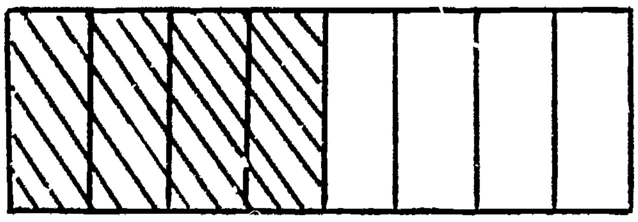


$\frac{2}{4}$

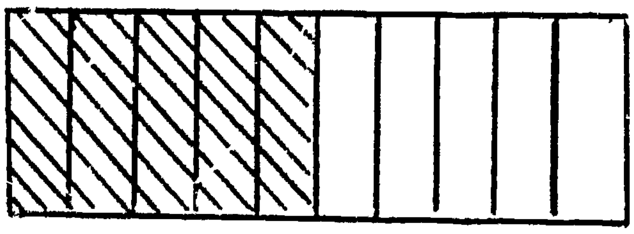
$$\frac{(1)}{(2)} \frac{X}{X} \frac{(2)}{(2)} = \frac{2}{4}$$



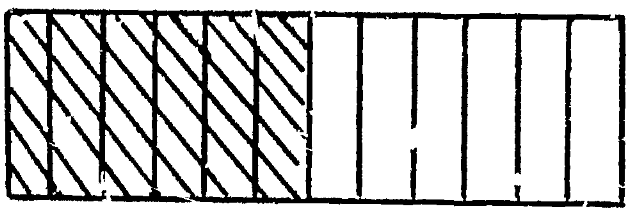
$\frac{3}{6}$ $$\frac{(1)}{(2)}\frac{X}{X}\frac{(3)}{(3)} = \frac{3}{6}$$



$\frac{4}{8}$ $$\frac{(1)}{(2)}\frac{X}{X}\frac{(4)}{(4)} = \frac{4}{8}$$



$\frac{5}{10}$ $$\frac{(1)}{(2)}\frac{X}{X}\frac{(5)}{(5)} = \frac{5}{10}$$



$\frac{6}{12}$ $$\frac{(1)}{(2)}\frac{X}{X}\frac{(6)}{(6)} = \frac{6}{12}$$

Could we contiaue on with this idea? See if you can supply 3 more.

$$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \_, \_, \_, \ldots\}$$

These are called "equivalent fractions," that is, different names for the same .umber.

Examine the way the equivalent fractions are written below:

$$\{\frac{2}{3}, \quad \frac{2}{3}\frac{X}{X}\frac{(2)}{(2)}, \quad \frac{2}{3}\frac{X}{X}\frac{(3)}{(3)}, \quad \frac{2}{3}\frac{X}{X}\frac{(4)}{(4)}, \quad \frac{2}{3}\frac{X}{X}\frac{(5)}{(5)}, \ldots\}$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$\{\frac{2}{3}, \quad \frac{4}{6}, \quad \frac{6}{9}, \quad \frac{8}{12}, \quad \frac{10}{15}, \ldots\}$$

Notice that $\frac{2}{3}$ was multiplied by: $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and $\frac{5}{5}$. These are different names for the same number. What is the most common name for this number? What happens when you multiply by <u>one</u>?

Activities

Use the example below to write equivalent fractions (rename fractions).

Example: $\frac{5}{6} = \frac{\square}{24}$

Solution: $\frac{5}{6} \frac{X}{X} \frac{(4)}{(4)} = \frac{\boxed{20}}{24}$ or $\frac{5}{6} \times \frac{4}{4} = \frac{\boxed{20}}{24}$

1. $\frac{1}{2} \frac{X}{X} \frac{(\ )}{(\ )} = \frac{12}{24}$ or $\frac{1}{2} \times \frac{\square}{\square} = \frac{12}{24}$

2. $\frac{1}{3} \times \frac{\square}{\square} = \frac{4}{12}$

3. $\frac{1}{5} \times \frac{\square}{\square} = \frac{3}{15}$

4. $\frac{5}{6} \times \frac{\square}{\square} = \frac{25}{30}$

5. $\frac{3}{7} \times \frac{\square}{\square} = \frac{21}{49}$

Fill in the missing values for these equivalent fractions

6. $\{\frac{3}{4}, \frac{\square}{8}, \frac{9}{\square}, \frac{\square}{\square}, \frac{15}{\square}, \frac{\square}{24}, \ldots\}$

7. $\{\frac{\square}{\square}, \frac{6}{20}, \frac{9}{\square}, \frac{\square}{40}, \ldots\}$

8. Draw a line between pairs of equivalent fractions. One example is given.

a. $\frac{2}{3}$	$\frac{4}{12}$
b. $\frac{3}{10}$	$\frac{2}{14}$
c. $\frac{3}{4}$	$\frac{4}{6}$
d. $\frac{5}{6}$	$\frac{10}{25}$
e. $\frac{3}{8}$	$\frac{5}{10}$
f. $\frac{1}{7}$	$\frac{6}{20}$
g. $\frac{2}{5}$	$\frac{12}{16}$
h. $\frac{1}{2}$	$\frac{10}{12}$
i. $\frac{1}{3}$	$\frac{9}{24}$

It was shown earlier that it is easy to compare two fractions if the denominators are equal. That is, it is easy to tell if the two fractions are equal or which is greater.

Activities

In the problems below, <u>first</u> rename the fractions so the denominators are equal then place the correct symbol (" >," is greater than, or "<," is less than) between them.

Example: $\frac{3}{5}, \frac{3}{4}$

Solution: $\frac{3\ (4)}{5\ (4)}, \frac{3\ (5)}{4\ (5)}$

$\frac{12}{20}, \frac{15}{20}$ Since 12 "is less than" 15, then:

$$\frac{12}{20} < \frac{15}{20}$$

and $\frac{3}{5} < \frac{3}{4}$

1. $\frac{1}{2}, \frac{1}{3}$

2. $\frac{3}{8}, \frac{2}{5}$

3. $\frac{1}{4}, \frac{1}{3}$

4. $\frac{3}{4}, \frac{1}{2}$

Make all three denominators equal, and arrange the fractions in order from "smallest" to "largest."

5. $\frac{2}{3}, \frac{3}{5}, \frac{1}{2}$

6. $\frac{5}{6}, \frac{6}{7}, \frac{3}{4}$

7. $\frac{1}{2}, \frac{2}{3}, \frac{9}{12}$

8. $\frac{5}{6}, \frac{4}{12}, \frac{9}{12}$

If you want to check quickly and see if two rational numbers are equivalent, try this way: multiply the numerator of one "times" the denominator of the other. If your two products are equal, then the fractions are equivalent (different names for the same number).

Activities

Below are two examples. Arrows are drawn to illustrate the pairs we are multiplying.

Example I

$$\frac{3}{4} \quad \frac{9}{12} \quad \begin{array}{l} \longrightarrow 4 \times 9 = 36 \\ \longrightarrow 3 \times 12 = 36 \end{array}$$

These are equivalent (36 = 36).

Example II

$$\frac{5}{6} \quad \frac{10}{18} \quad \begin{array}{l} \longrightarrow 6 \times 10 = 60 \\ \longrightarrow 5 \times 18 = 90 \end{array}$$

These are not equivalent (60 "is not equal" to 90).

Use the idea of "cross multiplying" to decide if the pair of fractions are equivalent.

1. $\frac{2}{3}$ and $\frac{9}{12}$

2. $\frac{3}{4}$ and $\frac{12}{16}$

3. $\frac{9}{10}$ and $\frac{27}{30}$

4. $\frac{11}{12}$ and $\frac{55}{72}$

5. $\frac{7}{8}$ and $\frac{14}{16}$

6. $\frac{1}{9}$ and $\frac{10}{90}$

Adding Rational Numbers

The drawing below shows how the desks are arranged in a classroom.

X X X X
X X X X
X X X X
X X X X
X X X X
X X X X

There are 6 rows with 4 in each row. What rational number would we use if we were "interested in" the first two rows as compared to the total?

X X X X
X X X X
X X X X
X X X X
X X X X
X X X X

It would be: $\frac{8}{24}$

Use the same idea to describe the third row. It would be: $\frac{4}{24}$

If we add, using these numbers,

$$\frac{8}{24} + \frac{4}{24} =$$

What should the answer be? The sum of the number of desks in the "first three rows" compared to the total.

Then:

X X X X
X X X X
X X X X
X X X X
X X X X
X X X X

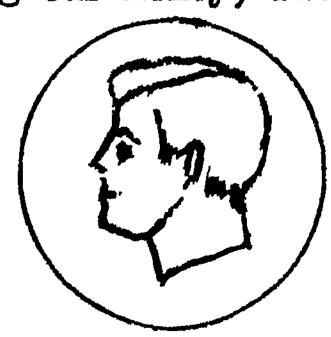
$$\frac{8}{24} + \frac{4}{24} = \frac{12}{24}$$

We can rename whole numbers and show we add "numerators" when denominators are equal. For example:

$$5 + 3 = 8$$

or $$\frac{5}{1} + \frac{3}{1} = \frac{8}{1}$$

In counting our money, rational numbers are often used. For example:



50¢ or $\frac{1}{2}$ dollar, or $\frac{50}{100}$.

This is called "quarter dollar," or 25¢, or $\frac{1}{4}$ of a dollar, or $\frac{\square}{100}$.

This is called "one dime," and we know it is $\frac{1}{10}$ of a dollar, or $\frac{\square}{100}$.

This is called "five cents" or a nickel, and it is $\frac{1}{20}$ of a dollar, or $\frac{\square}{100}$.

This is called "one cent." We call it a penny, and, since there are 100 of them in a dollar, it is also $\frac{\square}{100}$.

Many times we add these amounts. If we add one dime and one penny, we know we don't have 2 cents or 2 dimes. What we have to know is that a dime and a penny have other names. Use a name so the dime and penny have a common denominator.

$$\text{So: } \frac{10}{100} + \frac{1}{100} = \frac{11}{100} \text{ or 11 cents}$$

At times we must add two or more fractions where the denominators are not the same (equal). We have to rename and get equal denominators before adding. Using our examples with money, we may have:

One-half dollar + one-quarter dollar =

$$\frac{1}{2} + \frac{1}{4} =$$

Or, renaming $\frac{1}{2}$ we have: $\frac{1}{2}\frac{(2)}{(2)} = \frac{2}{4}$

Now: $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

Another way would be: $\frac{50}{100} + \frac{25}{100} = \frac{75}{100}$

Is $\frac{75}{100}$ "equivalent to" $\frac{3}{4}$?

<u>Lowest Common Denominators</u>

Suppose we wanted to add: $\frac{1}{2} + \frac{2}{3} = \boxed{\quad\quad}$. We would first get a "common denominator" (equal denominators). To do this, we rename both fractions. Just for fun, let's write quite a few different names for each.

$$\frac{1}{2} \longrightarrow \{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \frac{9}{18}, \ldots\}$$

$$\frac{2}{3} \longrightarrow \{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}, \ldots\}$$

Do you notice any denominators among the different names for $\frac{1}{2}$ that are also denominators of the different names for $\frac{1}{3}$? Choose the ones we could add (same denominators).

For: $\frac{1}{2} + \frac{2}{3} =$

We have: $\frac{3}{6} + \frac{4}{6} = \frac{7}{6}$

Or: $\frac{6}{12} + \frac{8}{12} = \frac{14}{12}$

Or: $\frac{9}{18} + \frac{12}{18} = \frac{21}{18}$

Are $\frac{7}{6}$, $\frac{14}{12}$, and $\frac{21}{18}$ equivalent fractions? If so, then all answers are correct as they name the same number.

In many cases, the "smallest" or "lowest" common denominator is used. In the example above it is 6. Notice that 6 "is the least" number that both 2 and 3 will divide into and leave a zero remainder.

Activities

In the problems below, circle which of the numbers in each set is the "lowest" common denominator. Rename the fraction and add.

1. $\frac{1}{8} + \frac{1}{6} = \square$ $\{12, 24, 36, 18\}$
2. $\frac{5}{6} + \frac{7}{12} = \square$ $\{12, 24, 36, 6\}$
3. $\frac{3}{8} + \frac{7}{8} = \square$ $\{16, 8, 24, 12\}$
4. $\frac{2}{3} + \frac{3}{4} + \frac{1}{12} = \square$ $\{8, 12, 24, 36\}$
5. $\frac{3}{5} + \frac{11}{15} + \frac{2}{5} = \square$ $\{15, 30, 45, 60\}$

Using addition ideas, solve each of the following problems.

6. Sally spent $\frac{3}{4}$ of an hour studying mathematics and $\frac{1}{3}$ of an hour studying history. How long did she study?

7. In a meatloaf, Ruth used $\frac{7}{8}$ lb. beef and $\frac{1}{4}$ lb. of pork. How much did the meat in the meatloaf weigh?

8. Tom ate $\frac{1}{5}$ of his mother's cake, and his younger brother ate $\frac{1}{10}$ of the cake. How much of the cake did they both eat?

9. Tom rode his bicycle $\frac{5}{8}$ of a mile and stopped for a coke. Then he rode $\frac{3}{4}$ of a mile farther to Bill's house. How far did Tom ride getting to Bill's house?

10. $\frac{2}{3} + \frac{5}{6} =$

11. $\frac{1}{6} + \frac{5}{9} =$

12. $\frac{1}{4} + \frac{1}{3} + \frac{1}{2} =$

13. $\frac{3}{10} + \frac{7}{20} =$

14. $\frac{5}{12} + \frac{7}{18} =$

15. $\frac{4}{25} + \frac{3}{100} =$

16. $\frac{7}{25} + \frac{3}{10} =$

17. $\frac{10}{3} + \frac{17}{6} =$

18. $\frac{9}{8} + \frac{5}{12} + \frac{17}{15} =$

Subtracting Rational Numbers

Suppose I have $\frac{1}{2}$ dollar and I owe John a quarter. How much will I have left after I pay John?

$$\frac{1}{2} - \frac{1}{4} =$$

This, of course, is subtraction. We already know subtraction is the inverse of addition. We can use our knowledge of changing rational numbers to equivalent numbers.

$$\frac{1}{2} \text{ dollar} = \frac{2}{4} \text{ dollars (2 quarters)}$$

$$\text{So: } \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$$

To do subtraction with rational numbers we must also express rational numbers with the same denominator. When we have written both the subtrahend and minuend with the same denominator, then we subtract the numerators.

Activities

Suppose we try the subt·· ction problems below.

1. $\frac{1}{3} - \frac{1}{5} =$

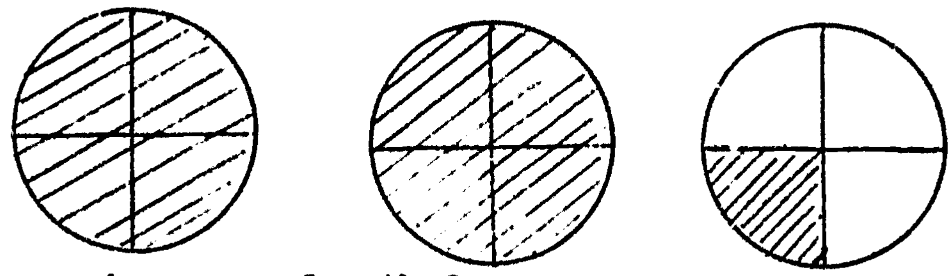
2. $\frac{4}{7} - \frac{1}{14} =$

3. $\frac{3}{4} - \frac{1}{10} =$

4. $\frac{5}{8} - \frac{3}{16} =$

So far, we have only considered rational numbers whose values were less than one. That is, the denominator was "greater than" the numerator. These are called common fractions. In the first arithmetic books they were called "vulgar" fractions because at that time the words "common" and "vulgar" had the same meaning.

What can we do if we wish to work with a rational number where the numerator is greater than the denominator? When would we have such a situation? Well, we might have to feed pizzas to a crowd of people. If there were a crowd, we'd need more than one pizza, but we couldn't afford to give each person a whole pizza. Let's give everyone a quarter of a pizza. We must feed 9 people.



We would use how many fourths?

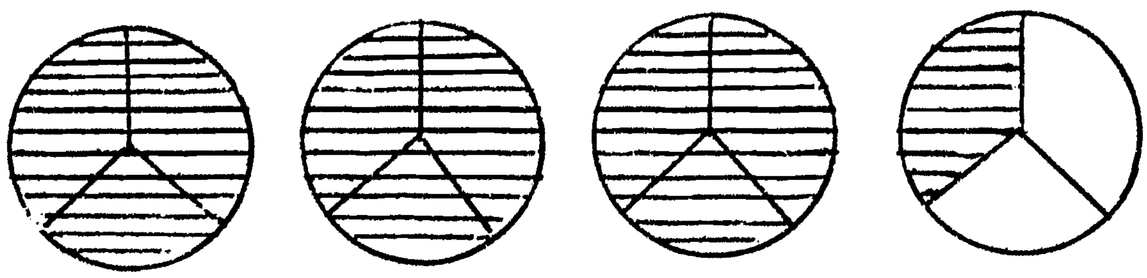
" $\frac{9}{4}$ " Correct!

Could we say 2 complete pizzas and $\frac{1}{4}$ more?

$$2 + \frac{1}{4} \quad \text{or} \quad 2\frac{1}{4}$$

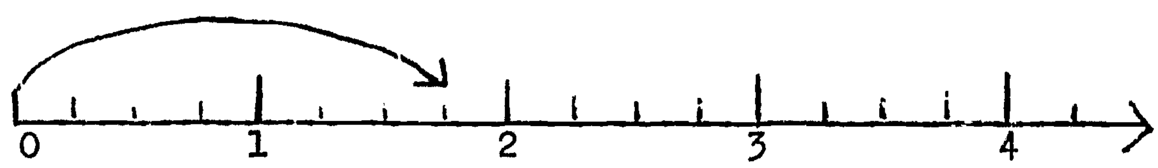
<u>Activities</u>

Example: How would you express these numbers, counting the shaded parts?

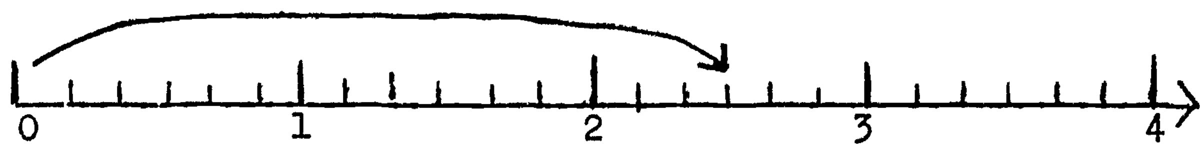


$= 3\frac{1}{3}$ or $\frac{\square}{3}$

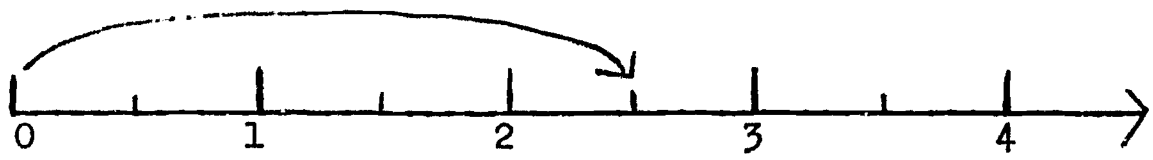
What fractions are shown below on each number line?

1. 

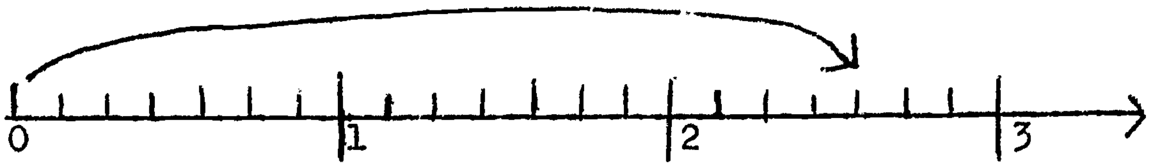
$= \frac{\square}{\square}$

2. 

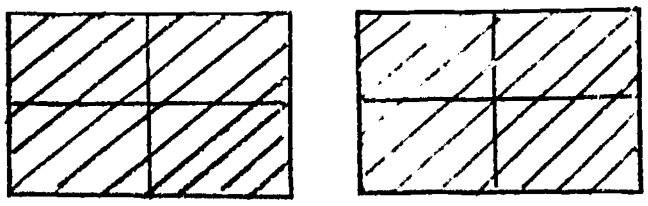
$= \frac{\square}{\square}$

3. 

$= \frac{\square}{\square}$

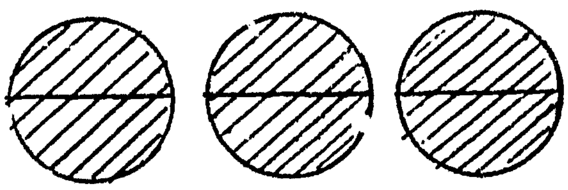
4.  $= \frac{\square}{\square}$

5. Count the number of fourths that are shaded.



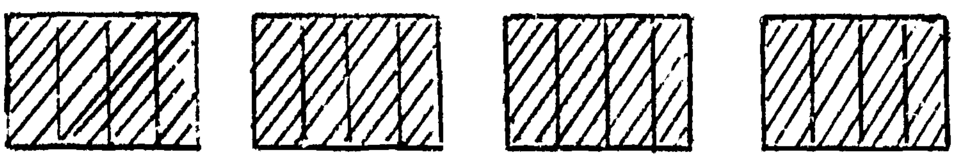
= ____ fourths $= \frac{8}{4} =$ ____ whole objects

6. Count the number of halves that are shaded.



= ____ halves $= \frac{\square}{\square} =$ ____ whole objects

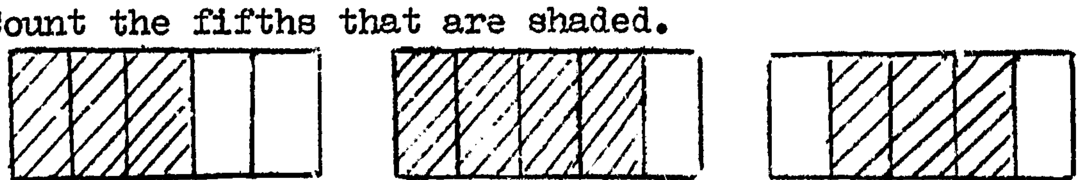
7. Count the number of fourths that are shaded.



=

______ fourths $= \frac{\square}{\square} =$ ______ whole objects

8. Count the fifths that are shaded.

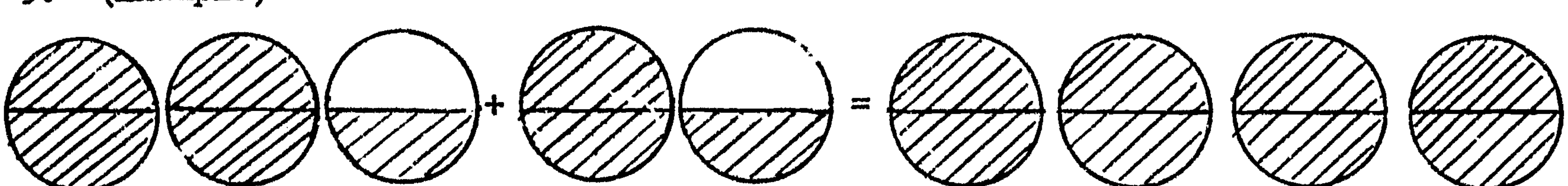


=

______ fifths $= \frac{\square}{5} =$ ______ whole objects

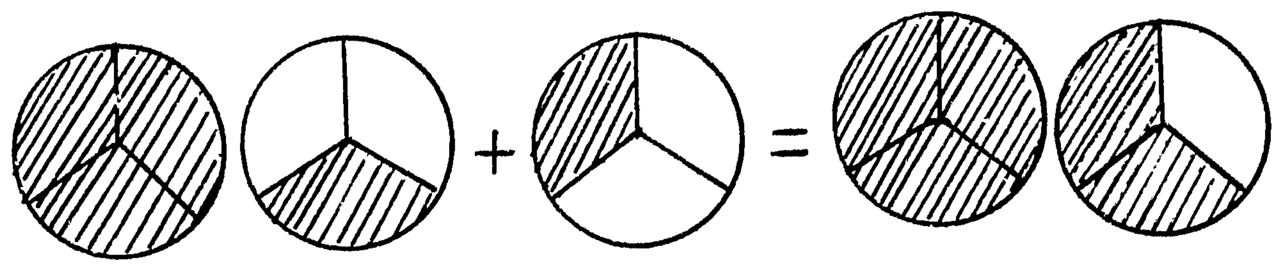
Each model below shows an addition problem. Use the first as an example.

9. (Example)



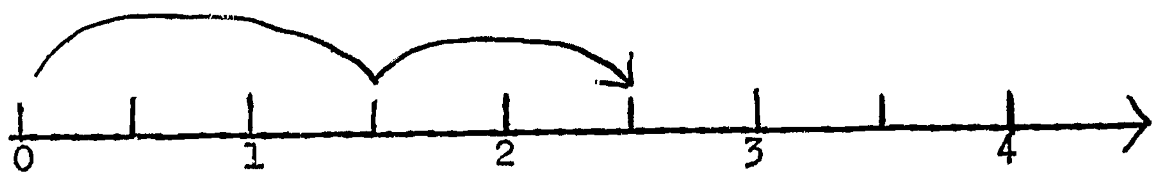
$2\frac{1}{2} + 1\frac{1}{2} = 4$

10.



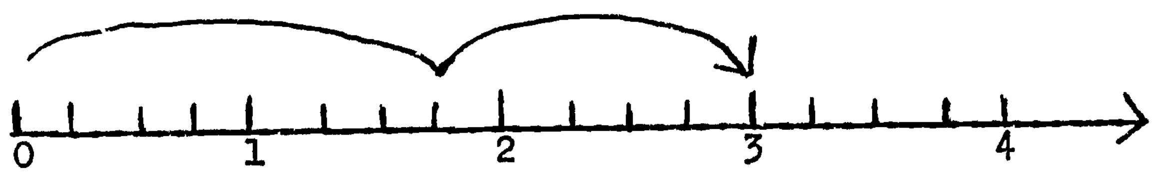
______ + ______ = ______

11.



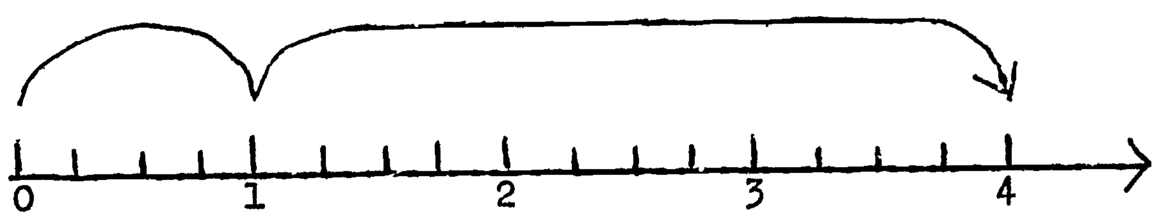
$\square + \square =$

12.



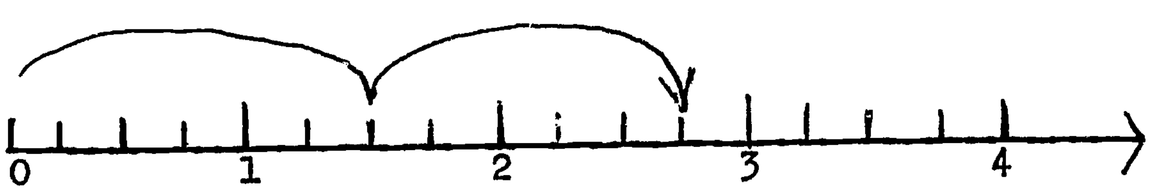
$\square + \square =$

13.



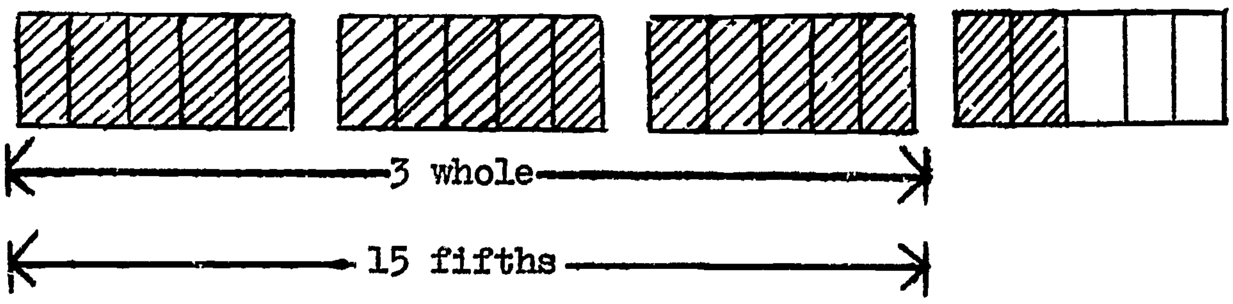
$\square + \square =$

14.



$\square + \square =$

Most people find an easy way to rewrite a number when there is a whole number and a common fraction. (We call these <u>mixed numbers</u>.) To change $3\frac{2}{5}$ to a rational number, multiply 3 X 5 to get 15, which is the number of 5ths in 3 wholes. Now add the 2 extra fifths so you have 17 fifths.



15 fifths + 2 fifths = 17 fifths

We write: $3\frac{2}{5} = \frac{17}{5}$

Supply the missing number in each of these.

$2\frac{1}{2} = \frac{(\ \ )}{2}$ $\qquad$ $\frac{9}{4} = 2\frac{(\ \ )}{(\ \ )}$

$3\frac{1}{7} = \frac{(\ \ )}{7}$ $\qquad$ $2\frac{45}{10} = \frac{(\ \ )}{10}$

$14\frac{1}{2} = \frac{(\ \ )}{2}$ $\qquad$ $\frac{27}{3} = (\ \ )$

$10\frac{2}{5} = \frac{52}{(\ \ )}$ $\qquad$ $11\frac{19}{12} = \frac{(\ \ )}{12}$

$\frac{68}{5} = (\ \ )\frac{3}{5}$ $\qquad$ $(\ \ )\frac{(\ \ )}{(\ \ )} = \frac{87}{12}$

Let's add some mixed numbers.

$2\frac{1}{4} + 3\frac{3}{4} =$

$2 + \frac{1}{4} + 3 + \frac{3}{4} =$

$2 + 3 + \frac{1}{4} + \frac{3}{4} =$

$5 + \frac{4}{4} =$

$5 + 1 = \underline{\underline{6}}$

Remember $2\frac{1}{4}$ can be written as $2 + \frac{1}{4}$.
Then we used the commutative property of addition to make our problem easier.

Let's try: $3\frac{1}{4} + 2\frac{1}{3} =$

Method I

$$3 + \frac{1}{4} + 2 + \frac{1}{3}$$

$$3 + 2 + \frac{1}{4} + \frac{1}{3}$$

$$5 + \frac{3}{12} + \frac{4}{12}$$

$$5 + \frac{7}{12}$$

$$5\frac{7}{12}$$

Method II (change to improper fractions)

$$\frac{13}{4} + \frac{7}{3}$$

$$\frac{39}{12} + \frac{28}{12}$$

$$\frac{67}{12}$$

$$5\frac{7}{12}$$

Try some of these:

$4\frac{3}{4} + 2\frac{3}{16} + 1\frac{1}{4} =$ $\qquad$ $10\frac{5}{8} + 12 + 7\frac{15}{16} =$

$6\frac{1}{4} + 4\frac{1}{2} + \frac{7}{16} =$ $\qquad$ $2\frac{7}{16} + 1\frac{5}{16} + 1\frac{5}{8} + 2\frac{3}{8} =$

Did you use the commutative law? It will make the problem easier if you will.

In the following problems, write an estimate, and check by adding.

1. $2\frac{2}{8}$	2. $7\frac{2}{3}$	3. $\frac{7}{8}$
$\frac{7}{8}$	$2\frac{1}{3}$	$5\frac{3}{8}$
$1\frac{3}{8}$	$8\frac{1}{4}$	$5\frac{1}{4}$
4. $7\frac{1}{3}$	5. $2\frac{1}{5}$	6. $9\frac{2}{7}$
$2\frac{2}{3}$	$4\frac{3}{10}$	$3\frac{1}{14}$
$7\frac{7}{9}$	$6\frac{1}{2}$	$1\frac{5}{7}$

7. $\begin{array}{r} 8\frac{2}{3} \\ 3\frac{3}{4} \\ 7\frac{1}{6} \\ \hline \end{array}$

8. $\begin{array}{r} 5\frac{1}{5} \\ 4\frac{3}{5} \\ 6\frac{1}{2} \\ \hline \end{array}$

9. $\begin{array}{r} 9\frac{1}{3} \\ 8\frac{2}{9} \\ 5\frac{2}{3} \\ \hline \end{array}$

10. $\begin{array}{r} 5\frac{1}{3} \\ 3\frac{2}{3} \\ \frac{1}{6} \\ \hline \end{array}$

11. $\begin{array}{r} 4\frac{3}{4} \\ 2\frac{5}{8} \\ 7\frac{1}{2} \\ \hline \end{array}$

12. $\begin{array}{r} 7\frac{1}{5} \\ 2\frac{9}{10} \\ 3\frac{1}{2} \\ \hline \end{array}$

13. $\begin{array}{r} 8\frac{5}{6} \\ 2\frac{1}{2} \\ 9\frac{2}{3} \\ \hline \end{array}$

14. $\begin{array}{r} 3\frac{7}{8} \\ 4\frac{5}{6} \\ 9\frac{5}{12} \\ \hline \end{array}$

15. $\begin{array}{r} 7\frac{3}{5} \\ 6\frac{1}{3} \\ 4\frac{4}{5} \\ \hline \end{array}$

16. $\begin{array}{r} 7\frac{1}{9} \\ 5\frac{1}{2} \\ 2\frac{1}{3} \\ \hline \end{array}$

17. $\begin{array}{r} 4\frac{1}{2} \\ 8\frac{3}{4} \\ 3\frac{1}{4} \\ 5\frac{1}{2} \\ \hline \end{array}$

18. $\begin{array}{r} 9\frac{2}{5} \\ 4\frac{1}{10} \\ 1\frac{3}{10} \\ 2\frac{1}{2} \\ \hline \end{array}$

19. $\begin{array}{r} 5\frac{1}{8} \\ 8\frac{3}{4} \\ 6\frac{3}{8} \\ 10\frac{1}{2} \\ \hline \end{array}$

Now solve these subtraction problems and see how well you estimate.

Example I: $2\frac{3}{4} - 1\frac{1}{2} =$

1st Method	2nd Method
$2\frac{3}{4} - 1\frac{2}{4}$	$\frac{11}{4} - \frac{3}{2}$ (We must have the same denominator for our fractions.)
$(2 - 1) + (\frac{3}{4} - \frac{2}{4})$	$\frac{11}{4} - \frac{6}{4}$
$1 + \frac{1}{4}$	$\frac{5}{4}$
$1\frac{1}{4}$	$1\frac{1}{4}$

Example II: $2\frac{1}{5} - 1\frac{3}{10} =$

$2\frac{2}{10} - 1\frac{3}{10}$	$\frac{11}{5} - \frac{13}{10}$
$1\frac{12}{10} - 1\frac{3}{10}$	$\frac{22}{10} - \frac{13}{10}$
$\frac{9}{10}$	$\frac{9}{10}$

Which method do you prefer? Both are correct, and it would be a good idea to be able to do both of them.

How would you do this problem?

$$\begin{array}{r} 16 \\ -\ 9\frac{3}{5} \\ \hline \end{array}$$

16 could be rewritten as $15\frac{5}{5}$ or as $15 + \frac{5}{5}$.

Now: $\begin{array}{r} 16 \\ -\ 9\frac{3}{5} \\ \hline \end{array}$ or $\begin{array}{r} 15\frac{5}{5} \\ -\ 9\frac{3}{5} \\ \hline \end{array}$

Try these:

$$\begin{array}{r} 20 \\ -\ 11\frac{6}{7} \\ \hline \end{array} \qquad \begin{array}{r} 17 \\ -\ 12\frac{1}{3} \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ -\ 1\frac{2}{5} \\ \hline \end{array} \qquad \begin{array}{r} 18 \\ -\ 11\frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 16\frac{1}{5} \\ -\ 9\frac{3}{5} \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 15\frac{6}{5} \\ -\ 9\frac{3}{5} \\ \hline \end{array} \qquad \begin{array}{r} 17\frac{1}{10} \\ -\ 8\frac{3}{10} \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 16\frac{11}{10} \\ -\ 8\frac{3}{10} \\ \hline \end{array}$$

$$\begin{array}{r} 19\frac{1}{7} \\ -\ 12\frac{2}{7} \\ \hline \end{array} \qquad \begin{array}{r} 18\frac{1}{3} \\ -\ 13\frac{2}{3} \\ \hline \end{array} \qquad \begin{array}{r} 15\frac{3}{16} \\ -\ 11\frac{1}{8} \\ \hline \end{array} \qquad \begin{array}{r} 20\frac{6}{7} \\ -\ 10\frac{2}{7} \\ \hline \end{array} \qquad \begin{array}{r} 20\frac{1}{3} \\ -\ 12\frac{2}{3} \\ \hline \end{array}$$

Try these mentally:

1. $\frac{4}{5} - \frac{2}{5} =$
2. $\frac{7}{8} - \frac{3}{8} =$
3. $\frac{5}{6} - \frac{2}{6} =$
4. $\frac{3}{4} - \frac{1}{2} =$
5. $\frac{9}{10} - \frac{3}{10} =$
6. $\frac{5}{6} - \frac{1}{6} =$
7. $5\frac{3}{4} - 2\frac{1}{4} =$
8. $4\frac{5}{8} - 2\frac{1}{8} =$
9. $6\frac{3}{4} - 2 =$
10. $9\frac{9}{10} - 5 =$
11. $7\frac{3}{8} - 2\frac{3}{8} =$
12. $7\frac{3}{4} - 2\frac{1}{2} =$
13. $10 - 4\frac{1}{2} =$
14. $15 - 3\frac{1}{2} =$
15. $18 - 7\frac{1}{2} =$
16. $32 - 6\frac{1}{2} =$

17. $50 - 20\frac{1}{2} =$

18. $8 - 2\frac{1}{4} =$

19. $8 - 2\frac{3}{4} =$

20. $8 - 2\frac{3}{8} =$

21. $8 - 2\frac{1}{16} =$

22. $12 - 6\frac{3}{5} =$

23. $10 - \frac{5}{6} =$

24. $7 - \frac{5}{8} =$

25. $16\frac{1}{2} - 12\frac{1}{2} =$

26. $8\frac{3}{4} - 2\frac{5}{8} =$

27. $\frac{7}{8} - \frac{1}{2} =$

28. $\frac{5}{6} - \frac{1}{2} =$

29. $\frac{13}{16} - \frac{1}{6} =$

30. $\frac{7}{12} - \frac{1}{2} =$

31. $\frac{7}{8} - \frac{1}{4} =$

32. $\frac{13}{16} - \frac{1}{4} =$

33. $\frac{7}{12} - \frac{1}{4} =$

34. $8\frac{7}{12} - 2\frac{1}{4} =$

35. $20 - 5\frac{1}{4} =$

36. $25 - 10\frac{3}{8} =$

37. $3\frac{5}{6} - 2\frac{1}{3} =$

38. $5\frac{7}{8} -$ [illegible] $\frac{1}{4} =$

39. $5\frac{3}{4} - 2\frac{1}{2} =$

40. $2 - \frac{1}{2} =$

Below there are lots of problems in addition and subtraction using rational numbers. The operations are all mixed up, so be sure you read the problem carefully. There are some tricky problems.

1. $20\frac{6}{7} - 10\frac{2}{7} =$

2. $14\frac{3}{4} + 8\frac{15}{16} + 1\frac{1}{4} =$

3. $5\frac{3}{4} + 4\frac{1}{8} + \frac{5}{16} =$

4. $33\frac{3}{8} - 16\frac{7}{12} =$

5. $\frac{49}{63} - \frac{7}{9} =$

6. $10\frac{1}{2} + 10\frac{2}{4} =$

7. $26\frac{3}{8} - 10\frac{2}{3} =$

8. $10\frac{1}{2} + 10\frac{2}{3} + 15\frac{1}{6} + 5 =$

9. $\frac{3}{4} + \frac{1}{2} + \frac{1}{4} =$

10. $3\frac{3}{4} + 2\frac{1}{4} =$

11. $25\frac{1}{4} - 19\frac{3}{4} =$

12. $15\frac{3}{8} - 1\frac{3}{10} =$

13. $14\frac{7}{6} - 15\frac{1}{6} =$

14. $17\frac{1}{5} + 17\frac{1}{5} =$

15. $\frac{1}{6} + \frac{2}{3} =$

16. $\frac{7}{8} - \frac{3}{4} =$

17. $4\frac{5}{10} - 3\frac{15}{10} =$

18. $5\frac{1}{2} + 5\frac{2}{4} =$

19. $\frac{8}{16} + \frac{7}{14} + \frac{9}{18} + \frac{11}{22} =$

20. $\frac{1}{3} + \frac{3}{9} + \frac{2}{6} + \frac{5}{15} =$

21. $\frac{10}{11} - \frac{20}{22} =$

22. $6\frac{29}{4} + \frac{3}{4} =$

23. $5\frac{18}{12} + \frac{1}{2} =$

24. $\frac{13}{16} - \frac{1}{2} =$

25. $5\frac{7}{8} - 1\frac{1}{4} =$

If you did these problems with less than 2 errors, you can do almost any problem involving fractions.

SUPPLEMENTARY ACTIVITIES WITH RATIONAL NUMBERS

1. In our ice cream store we use 4 ounces of vanilla flavoring in every can of 10 gallons of ice cream mix. One day we wanted to use only 5 gallons of mix. How much vanilla flavoring should we use?

2. In the stock market quotations you read in the paper, the values of the stocks are shown in dollars and a fractional part of a dollar. On the following page is a sample of the report from the financial page of the newspaper. The net change shows the difference between the closing quotation of the day before and the closing quotation on the day reported. From the information given, can you find the previous day's closing price or the net change?

3. Betty does baby-sitting for her neighbor. She stays with the children after school until their mother gets home from work. Her pay is $1.00 per hour. She keeps track of her time. Last week she worked Monday $\frac{3}{4}$ hour, Tuesday $\frac{1}{2}$ hour, Wednesday $\frac{3}{4}$ hour, Thursday $\frac{1}{4}$ hour, and Friday $\frac{3}{4}$ hour. How much did Betty earn last week?

4. John's teacher gave a test that had 25 questions, and John answered 20 of them correctly. The test Betty took had 50 questions. How many correct answers would she need so that she would have the same grade as John when the grades are based on 100%?

5. The weight in pounds of a crate of oranges is $41\frac{1}{2}$. The weight in pounds of the box alone is $2\frac{5}{6}$. What is the weight in pounds of the oranges, without the box?

* 6. Bill has a daily average of 78, which is to be $\frac{2}{3}$ of his final grade for the six weeks. His six weeks test grade will be the other $\frac{1}{3}$ of his score. Bill would like to have a grade of "C but to do that his average would have to be at least 80. What score would Bill have to make on his test to get the "C" he wants? Hint: Bill's scores must total 240 points in order to average 80.

* If you solved problem number 6 correctly, your understanding of word problems involving fractions is very good.

Corporation	Sales in 100's	High	Low	(Yesterday) Closing Quotes	(Today) Closing Quotes	Net Change
Am Motors	615	9 3/8	8 7/8	9	9	———
Am T T	3155	56 1/8	54 1/2	———	55 1/4	+ 1
Beth Stl	1167	28 3/4	27 1/2	———	27 7/8	- 1/8
Comsat	390	43 1/3	38 7/8	39 1/4	40	+ 3/4
East Air L	3913	68	58	———	59 1/4	- 7
Fla PL	185	77	71 1/2	———	74 5/8	+ 2 7/8
Ford Mot	1258	43 1/8	41 5/8	41 5/8	42	———
Gen Elec	1278	94 1/4	88	———	93	+ 6
Gen Mot	3928	74 1/4	71 1/8	———	72 1/4	+ 1 1/8
I B M	496	326	315 1/4	———	322	+ 6
Pan Am	1386	49 1/2	45	46 3/8	45 1/2	———
R C A	1871	44 1/4	41 1/2	———	43	+ 1 7/8
Repub Stl	648	36 1/8	32 1/4	———	34 3/8	+ 1 3/8
Unit Air C	562	68 1/2	66	66 5/8	66	———
U S Steel	1647	36 3/8	35 1/8	———	35 5/8	- 3/8
Xerox Corp	3429	171 1/2	154 3/4	158 1/2	160	———

7. Yelof works in the Liam office. Every day it is his job to sort the liams into bins. The liams are labeled with rational numbers, and Yelof's job is to place each liam into the bin that is the equivalent of the number on the liam. See if you can do Yelof's job.

Example: Put $\frac{2}{6}$ in the bin marked $\frac{1}{3}$. As you know, $\frac{2}{6} = \frac{2 \times 1}{2 \times 3} = \frac{1}{3}$

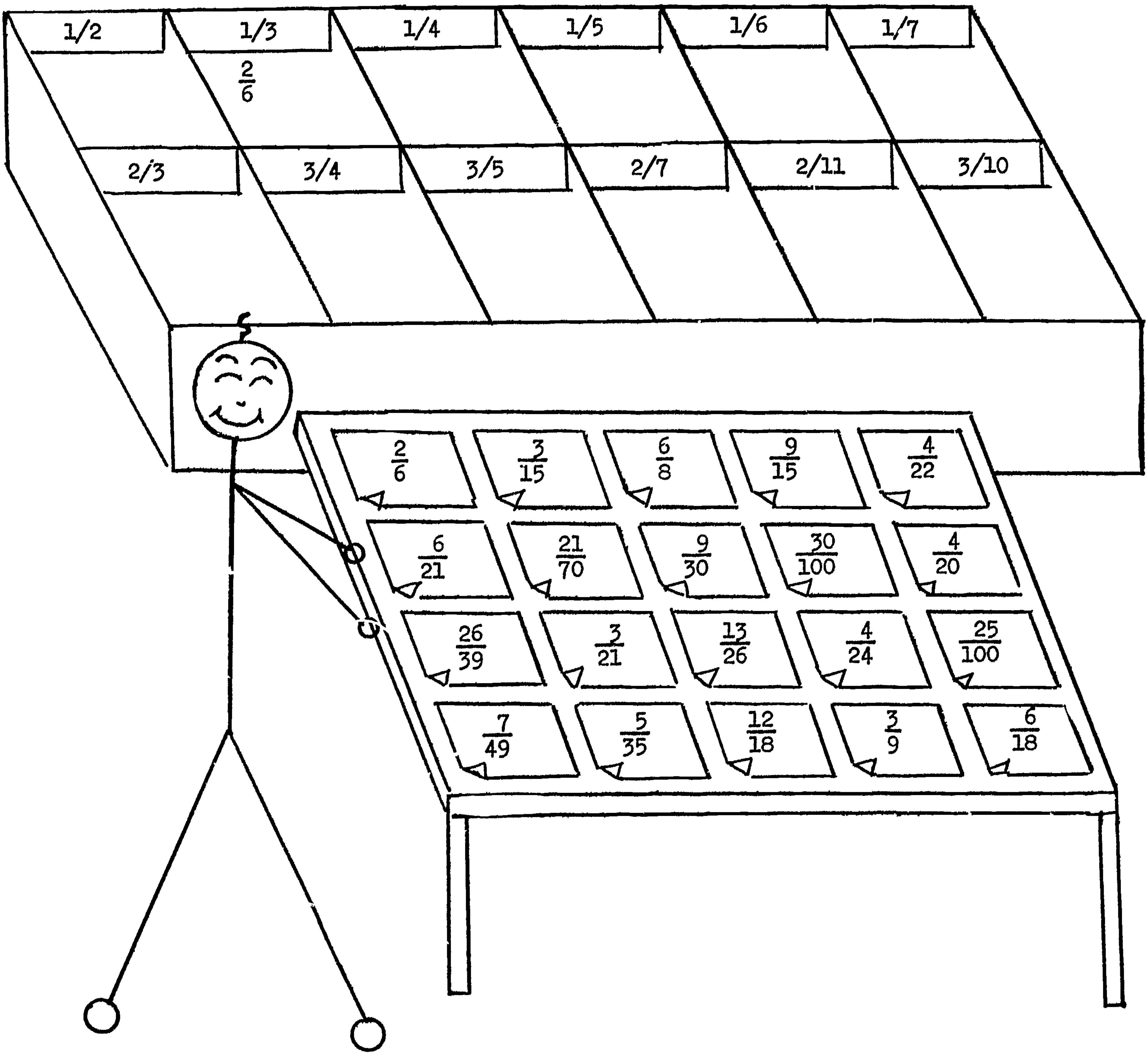


ILLUSTRATION OF TERMS

The examples given with some of the following definitions are meant to help you understand the meanings of words used in this booklet. They are merely samples, however, and many other examples could be used.

. . .--and so forth

Equivalent--any two or more symbols which express the same idea; equal in value. For instance, $\frac{6}{3}$ names the same number as 2, or $\frac{6}{3}$ is equivalent to 2.

Horizontal--parallel to, or in the direction of the line where the earth meets the sky; level, opposite of vertical (direction in which trees grow). When you are lying down or sleeping, you are in a horizontal position, and, when you are standing, you are in a vertical position.

Inadequate--not enough to meet the need. Less than that which is required.

Integers--a set which contains all the natural (counting) numbers, their opposites, and zero.

$\{ {}^{+}1, {}^{+}2, {}^{+}3, {}^{+}4, {}^{+}5 \ldots \}$	natural numbers combined with
$\{ \ldots {}^{-}5, {}^{-}4, {}^{-}3, {}^{-}2, {}^{-}1 \}$	opposites combined with
$\{ 0 \}$	zero

These three sets unite to form the set called integers.

$\{ \ldots {}^{-}5, {}^{-}4, {}^{-}3, {}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, {}^{+}3, {}^{+}4, {}^{+}5 \ldots \}$

Minuend--the number from which another number is to be subtracted. 10 - 5. 10 represents the minuend. It is the number from which 5 is to be subtracted.

Rational number--(general definition)--expresses the relation of one integer (number) with another. For instance, if John ate three of the five pieces of candy, he ate $\frac{3}{5}$ (rational number) of the candy.

Rational number--(mathematical definition):

1. A number which can be expressed as the quotient of integers. The divisor cannot be zero, but, rather, must be a natural (counting) number or its opposite number.
2. An ordered pair, (a, b) when a and b are elements of the set of integers, and b is $\neq$ (not equal) to zero.

Subtrahend--a number to be subtracted from another number. In the example 10 - 5, 5 is the subtrahend. It is to be subtracted from 10, the minuend.

Vulgar fraction--identical with the rational numbers we call common fractions.

In early American arithmetic the name "broken numbers" was used to name "fractions". "Vulgar" was applied to "common fractions" to distinguish them from "decimal fractions" or "decimals." At that time the words "vulgar" and "common" had almost the same meaning.

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